Higher twist effects in e^+e^- annihilation at high energies

Shu-yi Wei (魏树一)

Shandong University

WSY, Yu-kun Song, Zuo-tang Liang, **Phys.Rev.D89,(2014)014024**
WSY, Kai-bao Chen, Yu-kun Song, Zuo-tang Liang, **arXiv:1410.4314**

Content

Motivation

Process Involving Hadron States in Initial or Final State

$$\text{Cross Section} = \text{Hard Part} \otimes \text{PDFs/FFs} + \cdots$$

Motivation

Process Involving Hadron States in Initial or Final State

$$\text{Cross Section} = \text{Hard Part} \otimes \text{PDFs/FFs} + \cdots$$

PDFs and FFs

- Transverse Momentum Dependence
- Spin Dependence

$\Leftarrow$

Experiments

- Azimuthal Asymmetries
- Spin Asymmetries

Motivation

Process Involving Hadron States in Initial or Final State

$$\text{Cross Section} = \text{Hard Part} \otimes \text{PDFs/FFs} + \cdots$$

PDFs and FFs

- Transverse Momentum Dependence
- Spin Dependence

$\Leftarrow$

Experiments

- Azimuthal Asymmetries
- Spin Asymmetries

$\Uparrow$

Collinear Expansion $\Rightarrow$ Higher Twist Terms

Motivation

PDFs

R.K. Ellis, W. Furmanski and R. Petronzio

NPB 1982,1983

J.W. Qiu, G. Sterman NPB, 1991

TMD PDFs

Z.T. Liang and X.N. Wang

PRD, 2007

Collinear Expansion

$\swarrow$ $\downarrow$ $\searrow$

$e+p \rightarrow e+X$

$\Downarrow$

Gauge Invariant PDFs

Higher twist terms

Twist-4 in SIDIS

Y.K. Song,

J.H. Gao,

Z.T. Liang,

X.N. Wang

PRD, 2011

$e+p \rightarrow e+\mathrm{jet}+X$

$\Downarrow$

TMD PDFs

Azimuthal Asymmetries

Motivation

PDFs

R.K. Ellis, W. Furmanski and R. Petronzio

NPB 1982,1983

J.W. Qiu, G. Sterman NPB, 1991

TMD PDFs

Z.T. Liang and X.N. Wang

PRD, 2007

Collinear Expansion

$\swarrow$ $\downarrow$ $\searrow$

$e+p\to e+X$

$\Downarrow$

Gauge Invariant PDFs

Higher twist terms

Twist-4 in SIDIS

Y.K. Song,

J.H. Gao,

Z.T. Liang,

X.N. Wang

PRD, 2011

$e+p\to e+\mathrm{jet}+X$

$\Downarrow$

TMD PDFs

Azimuthal Asymmetries

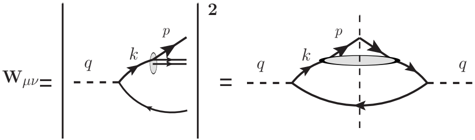
Fragmentation Functions (Leading Order Twist-3 Contributions)

$e^+e^-\to \mathrm{h}+X$	Tree Level	FFs	(PRD 89, (2014) 014024)
$e^+e^-\to \mathrm{h}+\bar{q}+X$	$\Rightarrow$	TMD FFs	(arXiv:1410.4314)

Collinear Expansion

$$d\sigma = \frac{g_Z^4}{32s} L_{\mu'\nu'}(l_1, l_2) D_F^{\mu'\mu}(q) D_F^{\nu'\nu *}(q) W_{\mu\nu}(q, p, S) \frac{d^3 p}{(2\pi)^2 2E_p}$$

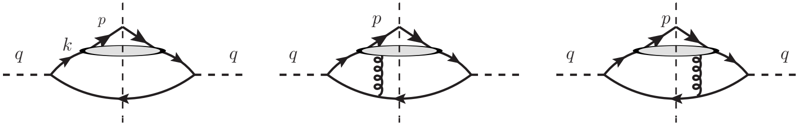
$$d\sigma = \frac{g_Z^4}{32s} L_{\mu'\nu'}(l_1, l_2) D_F^{\mu'\mu}(q) D_F^{\nu'\nu*}(q) W_{\mu\nu}(q,p,S) \frac{d^3p}{(2\pi)^2 2E_p}$$



$$W_{\mu\nu}^{(0)}(q,p,S) = \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\hat{H}_{\mu\nu}^{(0)}(k,q)\hat{\Pi}^{(0)}(k,p,S)\right]$$

$$\hat{H}_{\mu\nu}^{(0)}(k) = \Gamma_\mu^q (\not{q} - \not{k}) \Gamma_\nu^q (2\pi)\delta_+\left((q-k)^2\right)$$

$$\hat{\Pi}^{(0)}(k) = \frac{1}{2\pi} \sum_X \int d^4\xi\, e^{-ik\xi} \langle 0|\psi(0)|hX\rangle\langle hX|\bar{\psi}(\xi)|0\rangle$$



$$W_{\mu\nu} = W^{(0)}_{\mu\nu} + W^{(1)}_{\mu\nu} + W^{(2)}_{\mu\nu} + \cdots$$

Where,

$$W^{(1,L)}_{\mu\nu}(q,p,S) = \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4} \mathrm{Tr}[\hat{H}^{(1,L)\rho}_{\mu\nu}(k_1,k_2,q)\hat{\Pi}^{(1,L)}_{\rho}(k_1,k_2,p,S)]$$

$$\hat{H}^{(1,L)\rho}_{\mu\nu}(k_1,k_2) = \Gamma^q_\mu(\not{q} - \not{k}_1)\gamma^\rho \frac{\not{k}_2 - \not{q}}{(k_2-q)^2 - i\epsilon}\Gamma^q_\nu(2\pi)\delta_+\left((q-k_1)^2\right)$$

$$\hat{\Pi}^{(1,L)}_{\rho}(k_1,k_2) = \frac{1}{2\pi}\sum_X \int d^4\xi\, d^4\eta e^{-ik_1\xi} e^{-i(k_2-k_1)\eta} \langle 0|g{\color{red}A_\rho(\eta)\psi(0)}|hX\rangle\langle hX|{\color{red}\bar{\psi}(\xi)}|0\rangle$$

Collinear Expansion

Taylor Expansion

$$\hat{H}^{(0)}_{\mu\nu}(k) = \hat{H}^{(0)}_{\mu\nu}(z) + \frac{\partial \hat{H}^{(0)}_{\mu\nu}(k)}{\partial k_\rho}\bigg|_{k=p^+/z} \omega_\rho^{\ \rho'} k_{\rho'} + \cdots$$

$$= \hat{H}^{(0)}_{\mu\nu}(z) - (\hat{H}^{(1L)\rho}_{\mu\nu}(z,z) + \hat{H}^{(1R)\rho}_{\mu\nu}(z,z))\omega_\rho^{\ \rho'} k_{\rho'} + \cdots$$

$$\hat{H}^{(1L)\rho}_{\mu\nu}(k_1,k_2) = \hat{H}^{(1L)\rho}_{\mu\nu}(z_1,z_2) + \cdots$$

Gluon Fields

$$A_\rho = A^+ \bar{n}_\rho + \omega_\rho^{\ \rho'} A_{\rho'}$$

Ward Identities

$$p^+ \bar{n}_\rho \hat{H}^{(1L)\rho}_{\mu\nu}(z_1,z_2) = -\frac{z_1 z_2}{z_2 - z_1 - i\epsilon} \hat{H}^{(0)}_{\mu\nu}(z_1)$$

$$W^{(0)}_{\mu\nu} = H^{(0)}(k)\Phi_0 = H^{(0)}_{\mu\nu}(z) \times \phi_0 + H^{(1)}_{\mu\nu}(z) \times \phi_1 + \cdots$$

$$W^{(1)}_{\mu\nu} = H^{(1)}(k)\Phi_1 = H^{(0)}_{\mu\nu}(z) \times \phi_2 + H^{(1)}_{\mu\nu}(z) \times \phi_3 + \cdots$$

$$W_{\mu\nu} = W_{\mu\nu}^{(0)} + W_{\mu\nu}^{(1)} + W_{\mu\nu}^{(2)} + \cdots \quad \Rightarrow \quad W_{\mu\nu} = \tilde{W}_{\mu\nu}^{(0)} + \tilde{W}_{\mu\nu}^{(1)} + \tilde{W}_{\mu\nu}^{(2)} + \cdots$$

Inclusive Process

$$\tilde{W}_{\mu\nu}^{(0)} = \frac{1}{2}\mathrm{Tr}\left[\hat{\mathrm{h}}_{\mu\nu}^{(0)}\,\hat{\Xi}^{(0)}(z_B, p, S)\right] \quad \Rightarrow \quad \text{twist 2, twist 3, } \cdots$$

$$\tilde{W}_{\mu\nu}^{(1L)} = -\frac{1}{4p\cdot q}\mathrm{Tr}\left[\hat{\mathrm{h}}_{\mu\nu}^{(1)\rho}\,\omega_{\rho}^{\ \rho'}\hat{\Xi}_{\rho'}^{(1)}(z_B, p, S)\right] \quad \Rightarrow \quad \text{twist 3, } \cdots$$

Semi-inclusive Process

$$\tilde{W}_{\mu\nu}^{(0,\mathrm{si})} = \frac{1}{2}\mathrm{Tr}\left[\hat{\mathrm{h}}_{\mu\nu}^{(0)}\hat{\Xi}^{(0)}(z_B, k'_\perp, p, S)\right] \quad \Rightarrow \quad \text{twist 2, twist 3, } \cdots$$

$$\tilde{W}_{\mu\nu}^{(1,L,\mathrm{si})} = -\frac{1}{4p\cdot q}\mathrm{Tr}\left[\hat{\mathrm{h}}_{\mu\nu}^{(1)\rho}\omega_{\rho}^{\ \rho'}\hat{\Xi}_{\rho'}^{(1,L)}(z_B, k'_\perp, p, S)\right] \quad \Rightarrow \quad \text{twist 3, } \cdots$$

Inclusive Process

$$\hat{\Xi}^{(0)}(z,p,S) = \sum_X \int \frac{p^+ d\xi^-}{2\pi} e^{-ik^+\xi^-} \langle 0|\mathcal{L}^\dagger(0^-,\infty)\psi(0)|\mathrm{hX}\rangle\langle \mathrm{hX}|\bar{\psi}(\xi^-)\mathcal{L}(\xi^-,\infty)|0\rangle$$

$$\hat{\Xi}^{(1)}_\rho(z,p,S) = \sum_X \int \frac{p^+ d\xi^-}{2\pi} e^{-ik^+\xi^-} \langle 0|\mathcal{L}^\dagger(0,\infty)[D_\rho(0)\psi(0)]|\mathrm{hX}\rangle\langle \mathrm{hX}|\bar{\psi}(\xi^-)\mathcal{L}(\xi^-,\infty)|0\rangle$$

Semi-inclusive Process

$$\hat{\Xi}^{(0)}(z,k_\perp,p,S) = \sum_X \int \frac{p^+ d\xi^- d^2\xi_\perp}{2\pi} e^{-ik^+\xi^- + ik_\perp\cdot\xi_\perp} \langle 0|\mathcal{L}^\dagger(0,\infty)\psi(0)|\mathrm{hX}\rangle\langle \mathrm{hX}|\bar{\psi}(\xi)\mathcal{L}(\xi,\infty)|0\rangle$$

$$\hat{\Xi}^{(1)}_\rho(z,k_\perp,p,S) = \sum_X \int \frac{p^+ d\xi^- d^2\xi_\perp}{2\pi} e^{-ik^+\xi^- + ik_\perp\cdot\xi_\perp} \langle 0|\mathcal{L}^\dagger(0,\infty)\hat{D}_\rho(0)\psi(0)|\mathrm{hX}\rangle\langle \mathrm{hX}|\bar{\psi}(\xi)\mathcal{L}(\xi,\infty)|0\rangle$$

$$W_0^{\mu\nu} \propto \mathrm{Tr}[\gamma^\mu \not{n} \gamma^\nu \hat{\Xi}^{(0)}]$$

Gamma Matrices Expansion

$$\hat{\Xi}^{(0)}(z,p,S,k_\perp) = \Xi^{(0)}_\alpha(z,p,S,k_\perp)\gamma^\alpha + \tilde{\Xi}^{(0)}_\alpha(z,p,S,k_\perp)\gamma_5\gamma^\alpha + \ldots$$
$$\hat{\Xi}^{(1)}_\rho(z,p,S,k_\perp) = \Xi^{(1)}_{\rho\alpha}(z,p,S,k_\perp)\gamma^\alpha + \tilde{\Xi}^{(1)}_{\rho\alpha}(z,p,S,k_\perp)\gamma_5\gamma^\alpha + \ldots$$

Parity Conservation

$$\Xi^{(0)}_\alpha(V,A) = \Xi^{(0)\alpha}(\tilde{V},-\tilde{A})$$
$$\tilde{\Xi}^{(0)}_\alpha(V,A) = -\tilde{\Xi}^{(0)\alpha}(\tilde{V},-\tilde{A})$$

$\mathbf{V}$: Vector

$\mathbf{A}$: Axis Vector

$\tilde{V}^\mu = V_\mu$

Spin Density Matrix (Particle Rest Frame)

Spin $\frac{1}{2}$ $\quad \rho = \begin{pmatrix} \rho_{++} & \rho_{+-} \\ \rho_{-+} & \rho_{--} \end{pmatrix} = \frac{1}{2}(1 + S^i \sigma^i)$

Spin 1 $\quad \rho = \begin{pmatrix} \rho_{++} & \rho_{+0} & \rho_{+-} \\ \rho_{0+} & \rho_{00} & \rho_{0-} \\ \rho_{-+} & \rho_{-0} & \rho_{--} \end{pmatrix} = \frac{1}{3} + \frac{1}{2} S^i \sigma^i + T^{ij} \Sigma^{ij}$

Relativistic Case $p^\rho = (p^+, M^2/2p^+, \vec{0}_\perp)$

$$S^\rho = \lambda_h \left(\frac{p^+}{M} \bar{n}^\rho - \frac{M}{2p^+} n^\rho \right) + S_\perp^\rho$$

$$T^{\mu\nu} = \frac{1}{2} \Big\{ \frac{4}{3} S_{LL} \Big[(\frac{p^+}{M})^2 \bar{n}^\mu \bar{n}^\nu + (\frac{M}{2p^+})^2 n^\mu n^\nu - \frac{1}{2} (\bar{n}^{\{\mu} n^{\nu\}} - g_\perp^{\mu\nu}) \Big]$$
$$+ [\frac{p^+}{M} \bar{n}^{\{\mu} - \frac{M}{2p^+} n^{\{\mu}] S_{LT}^{\nu\}} + S_{TT}^{\mu\nu} \Big\}$$

Inclusive Process

$$\tilde{W}^{(0)}_{\mu\nu} = \frac{1}{2}\mathrm{Tr}\left[\hat{\mathrm{h}}^{(0)}_{\mu\nu}\gamma_\alpha\right]\Xi^{(0)\alpha}(z_B,p,S) + \frac{1}{2}\mathrm{Tr}\left[\hat{\mathrm{h}}^{(0)}_{\mu\nu}\gamma_5\gamma_\alpha\right]\tilde{\Xi}^{(0)\alpha}(z_B,p,S)$$

$$\tilde{W}^{(1L)}_{\mu\nu} = -\frac{1}{4p\cdot q}\mathrm{Tr}\left[\hat{\mathrm{h}}^{(1)\rho}_{\mu\nu}\gamma_\alpha\right]\omega_\rho^{\ \rho'}\Xi^{(1)\alpha}_{\rho'}(z_B,p,S) - \frac{1}{4p\cdot q}\mathrm{Tr}\left[\hat{\mathrm{h}}^{(1)\rho}_{\mu\nu}\gamma_5\gamma_\alpha\right]\omega_\rho^{\ \rho'}\tilde{\Xi}^{(1)}_{\rho'}(z_B,p,S)$$

$$\Xi^{(0)\alpha}(z,p,S) = \sum_X \int \frac{p^+ d\xi^-}{2\pi} e^{-ik^+\xi^-}\mathrm{Tr}[\gamma^\alpha\langle 0|\mathcal{L}^\dagger(0^-,\infty)\psi(0)|\mathrm{hX}\rangle\langle\mathrm{hX}|\bar{\psi}(\xi^-)\mathcal{L}(\xi^-,\infty)|0\rangle]$$

$$\Xi^{(1)\alpha}_\rho(z,p,S) = \sum_X \int \frac{p^+ d\xi^-}{2\pi} e^{-ik^+\xi^-}\mathrm{Tr}[\gamma^\alpha\langle 0|\mathcal{L}^\dagger(0,\infty)[D_\rho(0)\psi(0)]|\mathrm{hX}\rangle\langle\mathrm{hX}|\bar{\psi}(\xi^-)\mathcal{L}(\xi^-,\infty)|0\rangle]$$

$$\ldots$$

Lorentz Structures Up to Twist-3

Spin Independent	$z\Xi^{(0)\alpha} = p^\alpha D_1(z)$
Spin Vector	$z\Xi^{(0)\alpha} = M\epsilon_\perp^{\alpha\gamma} S_{\perp\gamma} D_T(z)$ $z\tilde{\Xi}^{(0)\alpha} = \lambda_\mathrm{h} p^\alpha \Delta D_{1L}(z) + M S_\perp^\alpha \Delta D_T(z)$ $z\Xi^{(1)\rho\alpha} = M\epsilon_\perp^{\rho\gamma} S_{\perp\gamma} p^\alpha \xi_{\perp S}^{(1)}(z)$ $z\tilde{\Xi}^{(1)\rho\alpha} = iM S_\perp^\rho p^\alpha \tilde{\xi}_{\perp S}^{(1)}(z)$
Spin Tensor	$z\Xi^{(0)\alpha} = S_{LL} p^\alpha D_{1LL}(z) + M S_{LT}^\alpha D_{LT}(z)$ $z\tilde{\Xi}^{(0)\alpha} = M\epsilon_\perp^{\alpha\gamma} S_{LT,\gamma} \Delta D_{LT}(z)$ $z\Xi^{(1)\rho\alpha} = M S_{LT}^\rho \xi_{LTS}^{(1)}(z) p^\alpha$ $z\tilde{\Xi}^{(1)\rho\alpha} = iM\epsilon_\perp^{\rho\gamma} S_{LT,\gamma} \tilde{\xi}_{LTS}^{(1)}(z) p^\alpha$

Leading twist fragmentation functions (possibility density)

$D_1(z)$ quark $\longrightarrow \sum_S$ hadron

$\Delta D_{1L}(z)$ longitudinally polarized quark $\longrightarrow$ longitudinally polarized hadron

$D_{1LL}(z)$ quark $\longrightarrow$ LL polarized hadron

Twist-3 fragmentation functions

Vector Polarized Hadron

$D_T(z)$, $\Delta D_T(z)$

Tensor Polarized Hadron

$D_{LT}(z)$, $\Delta D_{LT}(z)$

Gauge Invariant Hadronic Tensor

Spin-0

$$\tilde{W}_{\mu\nu} = -\frac{2}{z}\left(c_1^q d_{\mu\nu} + i c_3^q \epsilon_{\perp\mu\nu}\right) D_1(z)$$

Spin-$\frac{1}{2}$

$$\begin{aligned}\tilde{W}_{\mu\nu} = \frac{2}{z}\Big\{ & -\left(c_1^q d_{\mu\nu} + i c_3^q \epsilon_{\perp\mu\nu}\right) D_1(z) + \lambda_{\mathrm{h}} \left(c_3^q d_{\mu\nu} + i c_1^q \epsilon_{\perp\mu\nu}\right) \Delta D_{1L}(z) \\ & + \frac{M}{p\cdot q}\left[c_1^q (q - 2p/z)_{\{\mu} \epsilon_{\perp\nu\}\gamma} S_\perp^\gamma + i c_3^q (q - 2p/z)_{[\mu} S_{\perp\nu]}\right] D_T(z) \\ & - \frac{M}{p\cdot q}\left[c_3^q (q - 2p/z)_{\{\mu} S_{\perp\nu\}} - i c_1^q (q - 2p/z)_{[\mu} \epsilon_{\perp\nu]\gamma} S_\perp^\gamma\right] \Delta D_T(z)\Big\}\end{aligned}$$

Spin-1

$$\begin{aligned}\tilde{W}_{\mu\nu} = \frac{2}{z}\Big\{ & -\left(c_1^q d_{\mu\nu} + i c_3^q \epsilon_{\perp\mu\nu}\right) [D_1 + S_{LL} D_{1LL}] + \lambda_{\mathrm{h}} \left(c_3^q d_{\mu\nu} + i c_1^q \epsilon_{\perp\mu\nu}\right) \Delta D_{1L} \\ & + \frac{M}{p\cdot q}\left[c_1^q (q - 2p/z)_{\{\mu} \epsilon_{\perp\nu\}\gamma} S_\perp^\gamma + i c_3^q (q - 2p/z)_{[\mu} S_{\perp\nu]}\right] D_T(z) \\ & + \frac{M}{p\cdot q}\left[c_1^q (q - 2p/z)_{\{\mu} S_{LT,\nu\}} - i c_3^q (q - 2p/z)_{[\mu} \epsilon_{\perp\nu]\gamma} S_{LT}^\gamma\right] D_{LT}(z) \\ & - \frac{M}{p\cdot q}\left[c_3^q (q - 2p/z)_{\{\mu} S_{\perp\nu\}} - i c_1^q (q - 2p/z)_{[\mu} \epsilon_{\perp\nu]\gamma} S_\perp^\gamma\right] \Delta D_T(z) \\ & - \frac{M}{p\cdot q}\left[c_3^q (q - 2p/z)_{\{\mu} \epsilon_{\perp\nu\}\gamma} S_{LT}^\gamma + i c_1^q (q - 2p/z)_{[\mu} S_{LT,\nu]}\right] \Delta D_{LT}(z)\Big\}\end{aligned}$$

Spin-0 Hadrons - $e^+e^- \to \mathrm{Z}_0 \to \mathrm{hX}$

$$\frac{d^2\sigma}{dz\,dy} = \sum_q \frac{2\pi\alpha^2}{Q^2}\chi T_0^q(y) D_1^{q\to \mathrm{h}}(z).$$

$$D_1^{q\to \mathrm{h}}(z) = \frac{z}{4}\sum_X \int \frac{d\xi^-}{2\pi} e^{-ip^+\xi^-/z} \mathrm{Tr}\left[\gamma^+ \langle 0|\mathcal{L}^\dagger(0,\infty)\psi(0)|hX\rangle\langle hX|\bar{\psi}(\xi^-)\mathcal{L}(\xi^-,\infty)|0\rangle\right]$$

For $e^+e^- \to \gamma^* \to q\bar{q} \to \mathrm{hX}$,

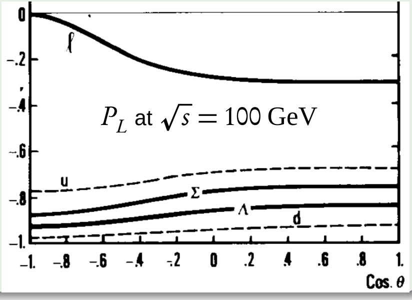
$$\frac{d\sigma^{\mathrm{em}}}{dz} = \sum_q \frac{4\pi\alpha^2}{3Q^2} e_q^2 D_1^{q\to \mathrm{h}}(z)$$

Inclusive Process $e^+ + e^- \to \mathrm{h} + \mathrm{X}$

Spin-1/2 Hadrons - $e^+e^- \to Z_0 \to \mathrm{hX}$

$$\frac{d^2\sigma}{dzdy} = \chi\frac{2\pi\alpha^2}{Q^2}\Big\{\big[T_0(y)D_1(z) + \lambda_h T_1(y)\Delta D_{1L}(z)\big] + \frac{4M}{zQ^2}\big[T_2(y)D_T(z)\epsilon_\perp^{l_\perp S_\perp} + T_3(y)\Delta D_T(z) l_\perp \cdot S_\perp\big]\Big\}$$

$$P_{L\mathrm{h}}(z,y) = \frac{\sum_q P_q(y) T_0^q(y)\Delta D_{1L}^{q\to \mathrm{h}}(z)}{\sum_q T_0^q(y) D_1^{q\to \mathrm{h}}(z)}$$



NPB, 1980, J.E. Augustin, F.M. Renard

P_q: Longitudinal polarization of quark

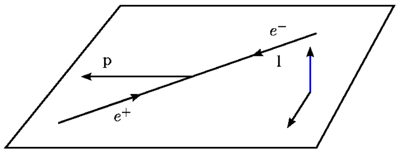
$\Delta D_{1L}^{q\to \mathrm{h}}(z)$: Longitudinally polarized quark $\longrightarrow$ Longitudinally polarized hadron

Spin-1/2 Hadrons - $e^+e^- \to Z_0 \to \mathrm{h}X$

$$\frac{d^2\sigma}{dz\,dy} = \chi \frac{2\pi\alpha^2}{Q^2}\Big\{\big[T_0(y)D_1(z) + \lambda_\mathrm{h} T_1(y)\Delta D_{1L}(z)\big] + \frac{4M}{zQ^2}\big[T_2(y)D_T(z)\epsilon_\perp^{l_\perp S_\perp} + T_3(y)\Delta D_T(z) l_\perp \cdot S_\perp\big]\Big\}$$

$$P_{\mathrm{hx}}(z,y) = -\frac{4M}{zQ}\frac{\sum_q \tilde{T}_3^q(y)\Delta D_T^{q\to\mathrm{h}}(z)}{\sum_q T_0^q(y) D_1^{q\to\mathrm{h}}(z)}$$

$$P_{\mathrm{hy}}(z,y) = \frac{4M}{zQ}\frac{\sum_q \tilde{T}_2^q(y) D_T^{q\to\mathrm{h}}(z)}{\sum_q T_0^q(y) D_1^{q\to\mathrm{h}}(z)}$$



P_q: Longitudinal polarization of quark

$\Delta D_{1L}^{q\to\mathrm{h}}(z)$: Longitudinally polarized quark $\longrightarrow$ Longitudinally polarized hadron

Spin-1/2 Hadrons - $e^+e^- \to \gamma^* \to \mathrm{hX}$

$$\frac{d\sigma^{\rm em}}{dzdy} = \frac{2\pi\alpha^2 e_q^2}{Q^2}\left\{A(y)D_1(z) + \frac{4M}{zQ^2}B(y)D_T(z)\epsilon_\perp^{l_\perp S_\perp}\right\}.$$

$$P_{L\mathrm{h}}(z,y) = 0 \qquad P_{\mathrm{hx}}^{\rm em}(z,y) = 0$$

$$P_{\mathrm{hy}}^{\rm em}(z,y) = \frac{4M}{zQ}\frac{\sqrt{y(1-y)}(1-2y)}{(1-y)^2+y^2}\frac{\sum_q e_q^2 D_T^{q\to\mathrm{h}}(z)}{\sum_q e_q^2 D_1^{q\to\mathrm{h}}(z)}$$

$$MS_\perp^2 D_T(z) = \frac{z}{4}\sum_X\int\frac{p^+d\xi^-}{2\pi}e^{-ip^+\xi^-/z}\epsilon_\perp^{\alpha\gamma}S_{\perp\gamma}\mathrm{Tr}[\gamma_\alpha\langle 0|\mathcal{L}^\dagger(0,\infty)\psi(0)|\mathrm{hX}\rangle\langle\mathrm{hX}|\bar{\psi}(\xi^-)\mathcal{L}(\xi^-,\infty)|0\rangle]$$

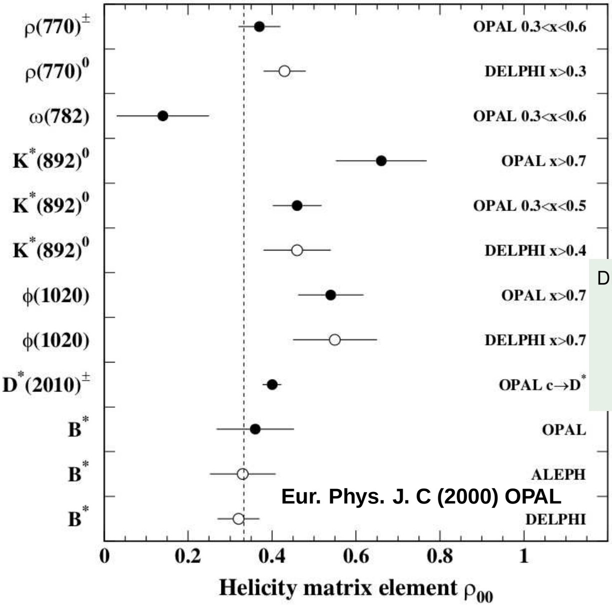
Inclusive Process $e^+ + e^- \to \mathrm{h} + \mathrm{X}$

Spin-1 Hadrons - $e^+e^- \to Z_0 \to \mathrm{hX}$

$$\frac{d\sigma}{dz\,dy} = \chi \frac{2\pi\alpha^2}{Q^2}\Big\{\big[T_0(y)D_1(z) + T_0(y)S_{LL}D_{1LL}(z) + \lambda_{\mathrm{h}} T_1(y)\Delta D_{1L}(z)\big]$$
$$+ \frac{4M}{zQ^2}\big[T_2(y)\epsilon_\perp^{l_\perp S_\perp} D_T(z) + T_3(y) l_\perp \cdot S_\perp \Delta D_T(z)\big]$$
$$+ \frac{4M}{zQ^2}\big[T_2(y) l_\perp \cdot S_{LT} D_{LT}(z) + T_3(y)\epsilon_\perp^{l_\perp S_{LT}} \Delta D_{LT}(z)\big]\Big\}$$

Spin alignment

$$\rho_{00} = \frac{1}{3} - \frac{1}{3}\frac{\sum_q t_0^q D_{1LL}^{q\to\mathrm{h}}(z)}{\sum_q t_0^q D_1^{q\to\mathrm{h}}(z)}$$



Eur. Phys. J. C (2000) OPAL

$)\Delta D_{1L}(z)]$

DELPHI Data for k^{*0}

0.1 $\sim$ 0.3	0.33 $\pm$ 0.05
z $\geq$ 0.3	0.41 $\pm$ 0.07
z $\geq$ 0.4	0.46 $\pm$ 0.08
z $\geq$ 0.5	0.47 $\pm$ 0.10

PLB (1997) DELPHI

Spin-1 Hadrons - $e^+e^- \to \gamma^* \to \mathrm{hX}$

$$\frac{d\sigma^{\mathrm{em}}}{dzdy} = \frac{2\pi\alpha^2 e_q^2}{Q^2}\Big\{A(y)\big[D_1(z) + S_{LL}D_{1LL}(z)\big] + \frac{M}{zQ}\sqrt{y(1-y)}B(y)\big[|\vec{S}_\perp|\sin\phi_s D_T(z) - |\vec{S}_{LT}|\cos\phi_{LT}D_{LT}(z)\big]\Big\}.$$

Spin alignment

$$\rho_{00}^{\mathrm{em}} = \frac{1}{3} - \frac{1}{3}\frac{\sum_q e_q^2 D_{1LL}^{q\to\mathrm{h}}(z)}{\sum_q e_q^2 D_1^{q\to\mathrm{h}}(z)}.$$

Semi-inclusive Process

$$\tilde{W}^{(0)}_{\mu\nu} = \frac{1}{2}\mathrm{Tr}\left[\hat{\mathrm{h}}^{(0)}_{\mu\nu}\gamma_\alpha\right]\Xi^{(0)\alpha}(z_B,p,S,k'_\perp) + \frac{1}{2}\mathrm{Tr}\left[\hat{\mathrm{h}}^{(0)}_{\mu\nu}\gamma_5\gamma_\alpha\right]\tilde{\Xi}^{(0)\alpha}(z_B,p,S,k'_\perp)$$

$$\tilde{W}^{(1L)}_{\mu\nu} = -\frac{1}{4p\cdot q}\mathrm{Tr}\left[\hat{\mathrm{h}}^{(1)\rho}_{\mu\nu}\gamma_\alpha\right]\omega_\rho^{\ \rho'}\Xi^{(1)\alpha}_{\rho'}(z_B,p,S,k'_\perp) - \frac{1}{4p\cdot q}\mathrm{Tr}\left[\hat{\mathrm{h}}^{(1)\rho}_{\mu\nu}\gamma_5\gamma_\alpha\right]\omega_\rho^{\ \rho'}\tilde{\Xi}^{(1)}_{\rho'}(z_B,p,S,k'_\perp)$$

$$\Xi^{(0)\alpha}(z,k_\perp,p,S) = \sum_X\int\frac{p^+d\xi^-d^2\xi_\perp}{2\pi}e^{-ik^+\xi^-+ik_\perp\cdot\xi_\perp}$$
$$\mathrm{Tr}[\gamma^\alpha\langle 0|\mathcal{L}^\dagger(0,\infty)\psi(0)|\mathrm{hX}\rangle\langle\mathrm{hX}|\bar{\psi}(\xi)\mathcal{L}(\xi,\infty)|0\rangle]$$

$$\Xi^{(1)\alpha}_\rho(z,k_\perp,p,S) = \sum_X\int\frac{p^+d\xi^-d^2\xi_\perp}{2\pi}e^{-ik^+\xi^-+ik_\perp\cdot\xi_\perp}$$
$$\mathrm{Tr}[\gamma^\alpha\langle 0|\mathcal{L}^\dagger(0,\infty)\hat{D}_\rho(0)\psi(0)|\mathrm{hX}\rangle\langle\mathrm{hX}|\bar{\psi}(\xi)\mathcal{L}(\xi,\infty)|0\rangle]$$

$$\cdots$$

New Lorentz Structures up to twist-3

Spin Independent

$$z\Xi^{(0)}_{\alpha}(z,k_\perp,p) = k_{\perp\alpha}\hat{D}^{\perp}(z,k_\perp)$$
$$z\tilde{\Xi}^{(0)}_{\alpha}(z,k_\perp,p) = \epsilon_{\perp\alpha k_\perp}\Delta\hat{D}^{\perp}(z,k_\perp)$$
$$z\Xi^{(1)}_{\rho\alpha}(z,k_\perp,p) = p_\alpha k_{\perp\rho}\,\xi^{(1)}_{\perp}(z,k_\perp)$$
$$z\tilde{\Xi}^{(1)}_{\rho\alpha}(z,k_\perp,p) = i\,p_\alpha\epsilon_{\perp\rho k_\perp}\,\tilde{\xi}^{(1)}_{\perp}(z,k_\perp)$$

Spin Vector

$$z\Xi^{(0)}_{\alpha} = p_\alpha\frac{\epsilon_\perp^{k_\perp S_\perp}}{M}\hat{D}^{\perp}_{1T} + k_{\perp\alpha}\frac{\epsilon_\perp^{k_\perp S_\perp}}{M}\hat{D}^{\perp}_{T} + \lambda_h\epsilon_{\perp\alpha k_\perp}\hat{D}^{\perp}_{L}$$
$$z\tilde{\Xi}^{(0)}_{\alpha} = p_\alpha\frac{k_\perp\cdot S_\perp}{M}\Delta\hat{D}^{\perp}_{1T} + \frac{\epsilon_\perp^{k_\perp S_\perp}}{M}\epsilon_{\perp\alpha k_\perp}\Delta\hat{D}^{\perp}_{T} + \lambda_h k_{\perp\alpha}\Delta\hat{D}^{\perp}_{L}$$
$$z\Xi^{(1)}_{\rho\alpha} = p_\alpha\left[k_{\perp\rho}\frac{\epsilon_\perp^{k_\perp S_\perp}}{M}\xi^{(1)\perp}_{T} + \lambda_h\epsilon_{\perp\rho k_\perp}\xi^{(1)\perp}_{L}\right]$$
$$z\tilde{\Xi}^{(1)}_{\rho\alpha} = i\,p_\alpha\left[\frac{\epsilon_\perp^{k_\perp S_\perp}}{M}\epsilon_{\perp\rho k_\perp}\tilde{\xi}^{(1)\perp}_{T} + \lambda_h k_{\perp\rho}\tilde{\xi}^{(1)\perp}_{L}\right]$$

New Lorentz Structures (Spin Tensor)

$$z\Xi^{(0)}_{\alpha} = p_\alpha \frac{S_{LT}\cdot k_\perp}{M}\hat{D}^{\perp}_{1LT} + p_\alpha \frac{k_{\perp\gamma}k_{\perp\delta}S^{\gamma\delta}_{TT}}{M^2}\hat{D}^{\perp}_{1TT}$$

$$+ k_{\perp\alpha}S_{LL}\hat{D}^{\perp}_{LL} + k_{\perp\alpha}\frac{k_\perp\cdot S_{LT}}{M}\hat{D}^{\perp}_{LT} + S_{TT\alpha\beta}k^{\beta}_{\perp}\hat{D}^{\perp A}_{TT} + k_{\perp\alpha}\frac{k_{\perp\gamma}k_{\perp\delta}S^{\gamma\delta}_{TT}}{M^2}\hat{D}^{\perp C}_{TT}$$

$$z\tilde{\Xi}^{(0)}_{\alpha} = p_\alpha \frac{\epsilon^{k_\perp S_{LT}}_{\perp}}{M}\Delta\hat{D}^{\perp}_{1LT} + p_\alpha \frac{\epsilon_{\perp k_\perp\gamma}k_{\perp\delta}S^{\gamma\delta}_{TT}}{M^2}\Delta\hat{D}^{\perp}_{1TT}$$

$$+\epsilon_{\perp\alpha k_\perp}\left[S_{LL}\Delta\hat{D}^{\perp}_{LL} + \frac{k_\perp\cdot S_{LT}}{M}\Delta\hat{D}^{\perp}_{LT} + \frac{k_{\perp\gamma}k_{\perp\delta}S^{\gamma\delta}_{TT}}{M^2}\Delta\hat{D}^{\perp C}_{TT}\right] + \epsilon_{\perp\alpha\beta}S^{\beta\gamma}_{TT}k_{\perp\gamma}\Delta\hat{D}^{\perp A}_{TT}$$

$$z\Xi^{(1)}_{\rho\alpha} = p_\alpha\left[k_{\perp\rho}S_{LL}\xi^{\perp}_{LL} + k_{\perp\rho}\frac{k_\perp\cdot S_{LT}}{M}\xi^{\perp}_{LT} + S_{TT\rho\beta}k^{\beta}_{\perp}\xi^{\perp A}_{TT} + k_{\perp\rho}\frac{k_{\perp\gamma}k_{\perp\delta}S^{\gamma\delta}_{TT}}{M^2}\xi^{\perp C}_{TT}\right]$$

$$z\tilde{\Xi}^{(1)}_{\rho\alpha} = i\,p^{\alpha}\left[\epsilon_{\perp\rho k_\perp}S_{LL}\tilde{\xi}^{\perp}_{LL} + \epsilon_{\perp\rho k_\perp}k_\perp\cdot S_{LT}\,\tilde{\xi}^{\perp}_{LT} + \epsilon_{\perp\rho\beta}S^{\beta\gamma}_{TT}k_{\perp\gamma}\tilde{\xi}^{\perp A}_{TT} + \epsilon_{\perp\rho k_\perp}\frac{k_{\perp\gamma}k_{\perp\delta}S^{\gamma\delta}_{TT}}{M^2}\tilde{\xi}^{\perp C}_{TT}\right]$$

New leading twist fragmentation functions (possibility density)

$\hat{D}_{1T}^{\perp}(z,k_\perp)$ quark $\longrightarrow$ transversely polarized hadron

$\Delta\hat{D}_{1T}^{\perp}(z,k_\perp)$ longitudinally polarized quark $\longrightarrow$ transversely polarized hadron

$\hat{D}_{1LT}^{\perp}(z,k_\perp)$ quark $\longrightarrow$ LT polarized hadron

$\Delta\hat{D}_{1LT}^{\perp}(z,k_\perp)$ longitudinally polarized quark $\longrightarrow$ LT polarized hadron

$\hat{D}_{1TT}^{\perp}(z,k_\perp)$ quark $\longrightarrow$ TT polarized hadron

$\Delta\hat{D}_{1TT}^{\perp}(z,k_\perp)$ longitudinally polarized quark $\longrightarrow$ TT polarized hadron

New Twist-3 fragmentation functions

unpolarized hadron: 2; polarization vector: 4; polarization tensor: 8;

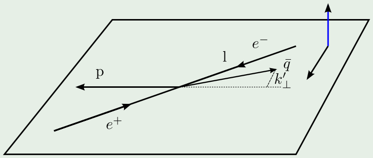
Spin-0 Hadrons - $e^+e^- \to Z_0 \to \mathrm{h} + \bar{\mathrm{q}} + \mathrm{X}$

$$\frac{d\sigma^{(\mathrm{si,unp})}}{dy\,dz\,d^2k'_\perp} = \frac{\alpha^2\chi}{2\pi Q^2}\Big\{T_0^q(y)\hat{D}_1(z,k'_\perp) + \frac{4}{zQ^2}\Big[T_2^q(y)l_\perp\cdot k'_\perp\hat{D}^\perp(z,k'_\perp) + T_3^q(y)\epsilon_\perp^{l_\perp k'_\perp}\Delta\hat{D}^\perp(z,k'_\perp)\Big]\Big\}$$

Azimuthal Asymmetries,

$$A_{\mathrm{unp}}^{\cos\varphi} = -\frac{2|\vec{k}'_\perp|}{zQ}\frac{\sum_q \tilde{T}_2^q(y)\hat{D}^{\perp q\to h}}{\sum_q T_0^q(y)\hat{D}_1^{q\to h}}$$

$$A_{\mathrm{unp}}^{\sin\varphi} = \frac{2|\vec{k}'_\perp|}{zQ}\frac{\sum_q \tilde{T}_3^q(y)\Delta\hat{D}^{\perp q\to h}}{\sum_q T_0^q(y)\hat{D}_1^{q\to h}}$$



Spin-0 Hadrons - $e^+e^- \to \gamma^* \to \mathrm{h} + \bar{\mathrm{q}} + \mathrm{X}$

$$\frac{d\sigma^{(\mathrm{si,unp,em})}}{dy\,dz\,d^2k'_\perp} = \frac{\alpha^2 e_q^2}{2\pi Q^2}\left\{A(y)\hat{D}_1(z,k'_\perp) + \frac{4l_\perp\cdot k'_\perp}{zQ^2}B(y)\hat{D}^\perp(z,k'_\perp)\right\}$$

Azimuthal Asymmetries,

$$A^{\cos\varphi}_{\mathrm{unp,em}}(z,y,k'_\perp) = -\frac{2|\vec{k}'_\perp|}{zQ}\frac{\tilde{B}(y)\sum_q e_q^2 D^{\perp q\to h}(z,k'_\perp)}{A(y)\sum_q e_q^2 D_1^{q\to h}(z,k'_\perp)}$$

$$A^{\sin\varphi}_{\mathrm{unp,em}}(z,y,k'_\perp) = 0$$

Spin-1/2 Hadrons - $e^+e^- \to Z_0 \to \mathrm{h} + \bar{\mathrm{q}} + \mathrm{X}$

$$\frac{d\sigma^{(\mathrm{si},1/2)}}{dydzd^2k'_\perp} = \frac{d\sigma^{(\mathrm{si},\mathrm{unp})}}{dydzd^2k'_\perp} + \frac{d\sigma^{(\mathrm{si},\mathrm{Vpol})}}{dydzd^2k'_\perp}$$

$$\begin{aligned}\frac{d\sigma^{(\mathrm{si},\mathrm{Vpol})}}{dydzd^2k'_\perp} = &\frac{\alpha^2\chi}{2\pi Q^2}\Big\{T_0^q(y)\frac{\epsilon_\perp^{k'_\perp S_\perp}}{M}\hat{D}_{1T}^\perp(z,k'_\perp) + T_1^q(y)\big[\lambda_h\Delta\hat{D}_{1L}(z,k'_\perp) + \frac{k'_\perp\cdot S_\perp}{M}\Delta\hat{D}_{1T}^\perp(z,k'_\perp)\big]\\
&+\frac{4\lambda_h}{zQ^2}\big[T_2^q(y)\epsilon_\perp^{l_\perp k'_\perp}\hat{D}_L^\perp(z,k'_\perp) + T_3^q(y)l_\perp\cdot k'_\perp\Delta\hat{D}_L^\perp(z,k'_\perp)\big]\\
&+\frac{4\epsilon_\perp^{k'_\perp S_\perp}}{zMQ^2}\big[T_2^q(y)l_\perp\cdot k'_\perp\hat{D}_T^\perp(z,k'_\perp) + T_3^q(y)\epsilon_\perp^{l_\perp k'_\perp}\Delta\hat{D}_T^\perp(z,k'_\perp)\big]\\
&+\frac{4M}{zQ^2}\big[T_2^q(y)\epsilon_\perp^{l_\perp S_\perp}\hat{D}_T(z,k'_\perp) + T_3^q(y)l_\perp\cdot S_\perp\Delta\hat{D}_T(z,k'_\perp)\big]\Big\}.\end{aligned}$$

Polarization

$$P_{\mathrm{Lh}}(y,z,k'_\perp) = \frac{\sum_q T_0^q(y) P_q(y) \Delta \hat{D}_{1L}(z,k'_\perp)}{\sum_q T_0^q(y) \hat{D}_1(z,k'_\perp)}$$

$$P_{\mathrm{hn}}(y,z,k'_\perp) = -\frac{|\vec{k}'_\perp|}{M} \frac{\sum_q T_0^q(y) \hat{D}_{1T}^\perp(z,k'_\perp)}{\sum_q T_0^q(y) \hat{D}_1(z,k'_\perp)}$$

$$P_{\mathrm{ht}}(y,z,k'_\perp) = -\frac{|\vec{k}'_\perp|}{M} \frac{\sum_q P_q(y) T_0^q(y) \Delta \hat{D}_{1T}^\perp(z,k'_\perp)}{\sum_q T_0^q(y) \hat{D}_1(z,k'_\perp)}$$

Polarization

$$P_{\mathrm{Lh}}(y,z,k'_\perp) = \frac{\sum_q T_0^q(y) P_q(y) \Delta\hat{D}_{1L}(z,k'_\perp)}{\sum_q T_0^q(y)\hat{D}_1(z,k'_\perp)} \left[1 + \frac{M}{Q}\hat{\Delta}(y,z,k'_\perp)\right] + \frac{4}{zQ}\frac{\sum_q \left[\tilde{T}_2^q(y) k'_y \hat{D}_L^\perp(z,k'_\perp) - \tilde{T}_3^q(y) k'_x \Delta\hat{D}_L^\perp(z,k'_\perp)\right]}{\sum_q T_0^q(y)\hat{D}_1(z,k'_\perp)}$$

$$P_{\mathrm{hn}}(y,z,k'_\perp) = -\frac{|\vec{k}'_\perp|}{M}\frac{\sum_q T_0^q(y)\hat{D}_{1T}^\perp(z,k'_\perp)}{\sum_q T_0^q(y)\hat{D}_1(z,k'_\perp)} + \cdots$$

$$P_{\mathrm{ht}}(y,z,k'_\perp) = -\frac{|\vec{k}'_\perp|}{M}\frac{\sum_q P_q(y) T_0^q(y)\Delta\hat{D}_{1T}^\perp(z,k'_\perp)}{\sum_q T_0^q(y)\hat{D}_1(z,k'_\perp)} + \cdots$$

Spin-1/2 Hadrons - $e^+e^- \to \gamma^* \to \mathrm{h} + \bar{\mathrm{q}} + \mathrm{X}$

$$\frac{d\sigma^{(\mathrm{si,Vpol,em})}}{dy\,dz\,d^2k'_\perp} = \frac{\alpha^2 e_q^2}{2\pi Q^2}\Big\{A(y)\frac{\epsilon_\perp^{k'_\perp S_\perp}}{M}\hat{D}_{1T}^\perp(z,k'_\perp)$$
$$+\frac{4B(y)}{zMQ^2}\big[\lambda_h M\epsilon_\perp^{l_\perp k'_\perp}\hat{D}_L^\perp(z,k'_\perp)+\epsilon_\perp^{k'_\perp S_\perp} l_\perp\cdot k'_\perp \hat{D}_T^\perp(z,k'_\perp)+M^2\epsilon_\perp^{l_\perp S_\perp}\hat{D}_T(z,k'_\perp)\big]\Big\}$$

Polarization (Leading Twist)

$$P_{\mathrm{Lh}}^{(0)(\mathrm{em})}(y,z,k'_\perp)=P_{\mathrm{ht}}^{(0)(\mathrm{em})}(y,z,k'_\perp)=0, \quad P_{\mathrm{hn}}^{(0)(\mathrm{em})}(y,z,k'_\perp)=-\frac{|\vec{k}'_\perp|}{M}\frac{\sum_q e_q^2\hat{D}_{1T}^\perp(z,k'_\perp)}{\sum_q e_q^2\hat{D}_1(z,k'_\perp)}$$

Twist-3

$$P_{\mathrm{Lh}}^{(\mathrm{em})}(y,z,k'_\perp)=\frac{4k'_y}{zQ}\frac{\tilde{B}(y)}{A(y)}\frac{\sum_q e_q^2\hat{D}_L^\perp(z,k'_\perp)}{\sum_q e_q^2\hat{D}_1(z,k'_\perp)}$$

Spin-1 Hadrons - $e^+e^- \to Z_0 \to \mathrm{h} + \bar{\mathrm{q}} + \mathrm{X}$

$$\frac{d\sigma^{(\mathrm{si},1)}}{dydzd^2k'_\perp} = \frac{d\sigma^{(\mathrm{si,unp})}}{dydzd^2k'_\perp} + \frac{d\sigma^{(\mathrm{si,Vpol})}}{dydzd^2k'_\perp} + \frac{d\sigma^{(\mathrm{si,Tpol})}}{dydzd^2k'_\perp}$$

$$\frac{d\sigma^{(\mathrm{si,Tpol})}}{dydzd^2k'_\perp} = \frac{d\sigma^{(\mathrm{si,LL})}}{dydzd^2k'_\perp} + \frac{d\sigma^{(\mathrm{si,LT})}}{dydzd^2k'_\perp} + \frac{d\sigma^{(\mathrm{si,TT})}}{dydzd^2k'_\perp}$$

$$[\mathbf{LL}] = \frac{\alpha^2\chi}{2\pi Q^2} S_{LL}\Big\{T_0^q(y)\hat{D}_{1LL}(z,k'_\perp) + \frac{4}{zQ^2}\big[T_2^q(y)(l_\perp\cdot k'_\perp)\hat{D}_{LL}^\perp(z,k'_\perp) + T_3^q(y)\epsilon_\perp^{l_\perp k'_\perp}\Delta\hat{D}_{LL}^\perp(z,k'_\perp)\big]\Big\}$$

Spin Alignment

$$\rho_{00} = \frac{1}{3} - \frac{1}{3}\frac{\sum_q T_0^q(y)\hat{D}_{1LL}(z,k'_\perp)}{\sum_q T_0^q(y)\hat{D}_1(z,k'_\perp)}\Big[1 + \frac{M}{Q}\hat{\Delta}(y,z,k'_\perp)\Big]$$
$$- \frac{4}{3}\frac{\sum_q \Big[\tilde{T}_2^q(y)k'_x\hat{D}_{LL}^\perp(z,k'_\perp) - \tilde{T}_3^q(y)k'_y\Delta\hat{D}_{LL}^\perp(z,k'_\perp)\Big]}{zQ\sum_q T_0^q(y)\hat{D}_1(z,k'_\perp)}.$$

$$[\textbf{LT Terms}] = \frac{\alpha^2\chi}{2\pi Q^2} S_{LT}^{\alpha}\Big\{ T_0^q(y)\frac{k'_{\perp\alpha}}{M}\hat{D}_{1LT}^{\perp}(z,k'_\perp) + T_1^q(y)\frac{\epsilon_{\perp k'\alpha}}{M}\Delta\hat{D}_{1LT}^{\perp}(z,k'_\perp)$$
$$+ \frac{4}{zQ^2}T_2^q(y)\big[(l_\perp\cdot k'_\perp)\frac{k'_{\perp\alpha}}{M}\hat{D}_{LT}^{\perp}(z,k'_\perp) + M l_{\perp\alpha}\hat{D}_{LT}(z,k'_\perp)\big]$$
$$+ \frac{4}{zQ^2}T_3^q(y)\big[\epsilon_\perp^{l_\perp k'_\perp}\frac{k'_{\perp\alpha}}{M}\Delta\hat{D}_{LT}^{\perp}(z,k'_\perp) + M\epsilon_{\perp l\alpha}\Delta\hat{D}_{LT}(z,k'_\perp)\big]\Big\}$$

$$[\textbf{TT Terms}] = \frac{\alpha^2\chi}{2\pi Q^2} S_{TT}^{\alpha\beta}\Big\{ T_0^q(y)\frac{k'_{\perp\alpha}k'_{\perp\beta}}{M^2}\hat{D}_{1TT}^{\perp}(z,k'_\perp) + T_1^q(y)\frac{\epsilon_{\perp k'\alpha}k'_{\perp\beta}}{M^2}\Delta\hat{D}_{1TT}^{\perp}(z,k'_\perp)$$
$$+ \frac{4}{zQ^2}T_2^q(y)\big[(l_\perp\cdot k'_\perp)\frac{k'_{\perp\alpha}k'_{\perp\beta}}{M^2}\hat{D}_{TT}^{\perp C}(z,k'_\perp) + l_{\perp\alpha}k'_{\perp\beta}\hat{D}_{TT}^{\perp A}(z,k'_\perp)\big]$$
$$+ \frac{4}{zQ^2}T_3^q(y)\big[\epsilon_\perp^{l_\perp k'_\perp}\frac{k'_{\perp\alpha}k'_{\perp\beta}}{M^2}\Delta\hat{D}_{TT}^{\perp C}(z,k'_\perp) + \epsilon_{\perp l\alpha}k'_{\perp\beta}\Delta\hat{D}_{TT}^{\perp A}(z,k'_\perp)\big]\Big\}$$

Summary

Inclusive $e^+e^- \to Z_0 \to \mathrm{h} + X$

- There is a leading twist longitudinal polarization for spin-1/2 hadrons and also spin alignment ($\rho_{00} \neq 1/3$) for vector mesons.
- On the twist-3 level, there are transverse polarizations for spin-1/2 hadrons that in and perpendicular to the leptonic plane.

Semi-inclusive $e^+e^- \to Z_0 \to \mathrm{h} + \bar{q}(jet) + X$

- For spin-0 hadrons, there are two azimuthal asymmetries on twist-3 level.
- For spin-1/2 hadrons, there is a longitudinal polarization and also transverse polarizations that in and transverse to the production plane on the leading twist.
- For vector mesons, all five tensor polarizations have leading twist contributions. S_{LL}, S_{LT}^n, S_{LT}^t, S_{TT}^{nn} and S_{TT}^{nt}

Summary

Inclusive $e^+e^- \to Z_0 \to \mathrm{h} + X$

- There is a leading twist longitudinal polarization for spin-1/2 hadrons and also spin alignment ($\rho_{00} \neq 1/3$) for vector mesons.
- On the twist-3 level, there are transverse polarizations for spin-1/2 hadrons that in and perpendicular to the leptonic plane.

Semi-inclusive $e^+e^- \to Z_0 \to \mathrm{h} + \bar{q}(jet) + X$

- For spin-0 hadrons, there are two azimuthal asymmetries on twist-3 level.
- For spin-1/2 hadrons, there is a longitudinal polarization and also transverse polarizations that in and transverse to the production plane on the leading twist.
- For vector mesons, all five tensor polarizations have leading twist contributions. S_{LL}, S_{LT}^n, S_{LT}^t, S_{TT}^{nn} and S_{TT}^{nt}

Thanks for your attention!