Generalized parton distributions for nucleon in the soft-wall model of AdS/QCD

Neetika Sharma

Indian Institute of Science Education and Research Mohali, India

SPIN2014, Peking University
October 20-24, 2014

Quantum Chromodynamics (QCD)

- One of the challenging problem in high energy physics is to understand the hadron structure in terms of confined quark and gluon quanta of QCD.
- It is essential to compute the detailed hadronic properties, such as, electromagnetic form factors (EFFs), structure functions, distribution amplitudes, excitation dynamics of hadron resonances, etc., starting from the QCD Lagrangian.
- The strong coupling in the infrared region makes it has been difficult to find analytic solutions.
- This gives impetus on the formulation of alternative approaches to calculate the hadronic properties and to make precise predictions in the nonperturbative regime.

Light-front holography (LFH)

- LFH is an important approach based on the AdS/CFT correspondence between the string theory on a higher-dimensional anti-de Sitter (AdS) space and conformal field theory (CFT) in physical space-time.
- It is based on a mapping of string modes in the AdS fifth dimension to hadron light-front wave functions (LFWFs) in physical space-time. The AdS/CFT correspondence has led to a semiclassical approximation for strongly-coupled QFTs which provides physical insights into its nonperturbative dynamics.
- The models based upon AdS/QCD holography incorporate the essential features of QCD, such as, confinement and chiral symmetry breaking, and successfully explain the general properties of mesons, such as, mass spectra (Regge trajectories), EFFs, decay constants, decay widths, and other physical quantities.

Generalized parton distributions (GPDs)

- The idea of matching the matrix elements of AdS modes to the light-front QCD has been successfully applied to the baryonic sector.
- One can constrain the information on GPDs for valence quarks indirectly via the sum rules that connect them with form factors.
- The first moments of GPDs are related to the electromagnetic form factors.

  $$F_1^q(q^2) = \int dx\ H^q(x, q^2)\,, \qquad F_2^q(q^2) = \int dx\ E^q(x, q^2)\,.$$

  The Dirac and Pauli form factors for the nucleon are given by charge weighted sum

  $$F_i^N(Q^2) = \sum_q e_q^N\ F_i^q(Q^2)\,, \tag{1}$$

  with appropriate coefficients $e_u^p = e_d^n = \frac{2}{3}$, and $e_d^p = e_u^n = -\frac{1}{3}$.
- The GPDs contain much more information about the nucleon structure and spin compared to the ordinary parton distribution functions (PDFs) which are function of x only. The GPDs reduce to the ordinary PDFs in the forward limit.

- At zero skewness, the Fourier transform (FT) of the GPDs with respect to the momentum transfer in the transverse direction $(q_\perp)$ gives the impact parameter dependent GPDs.
- Impact parameter GPDs provide us information about partonic distributions in the impact parameter or the transverse position space as the impact parameter $(b_\perp)$ is a measure of the transverse distance between the struck parton and the center of momentum of the hadron.
- They obey certain positivity constraints and have probabilistic interpretation in terms of distribution functions.
- Experimental information on GPDs can be extracted from the exclusive processes like deeply virtual compton scattering and vector meson production.
- Recent experiments at DESY and JLab are planning to determine of GPDs in the valence quarks region, whereas measurements at COMPASS will explore the region of sea quarks, gluons, and transverse single spin asymmetries.

GPDs in the AdS/QCD holography

- The AdS/QCD action which generates the nucleon form factors

$$S^V_{\rm int} = \int d^4x\, dz\, \sqrt{g}\, e^{-\varphi(z)}\, \mathcal{L}^V_{\rm int}(x,z)\,,$$

  where $g = |g_{MN}|$ and $\varphi(z) = \kappa^2 z^2$ is the quadratic dilaton field with κ as the free scale parameter taking care of the soft-wall breaking of conformal symmetry.
- The interaction Lagrangian is given as

$$\mathcal{L}_{\rm int}(x,z) = \sum_{i=+,-} \sum_{\tau} c_\tau\, \bar{\Psi}_{i,\tau}(x,z)\, \hat{\mathcal{V}}_i(x,z)\, \Psi_{i,\tau}(x,z)\,,$$

$$\hat{\mathcal{V}}_\pm(x,z) = Q\,\Gamma^M V_M(x,z) \pm \frac{i}{4}\,\eta_V\,[\Gamma^M,\Gamma^N]\,V_{MN}(x,z) \pm g_V\,\tau_3\,\Gamma^M\, i\Gamma^z\, V$$

Here $\Psi_{\pm,\tau}(x,z)$ is five-Dimensional fermion fields with spin $J = 1/2$ and scaling dimension τ,

$V_M(x,z)$ is the vector fields which is holographic dual of the electromagnetic field,

$V_{MN} = \partial_M V_N - \partial_N V_M$ is the stress tensor of the vector field, $Q = \mathrm{diag}(1,0)$ is the nucleon charge matrix, $\tau_3 = \mathrm{diag}(1,-1)$ is the Pauli isospin matrix, $\Gamma^M = \epsilon^M_a \Gamma^a$, and $\Gamma^a = (\gamma^\mu, -i\gamma^5)$.

- The Dirac and Pauli form factors for nucleon in soft-wall model of AdS/QCD

$$\begin{aligned}
F_1^p(Q^2) &= C_1(Q^2) + g_V C_2(Q^2) + \eta_V^p C_3(Q^2)\,, \\
F_2^p(Q^2) &= \eta_V^p C_4(Q^2)\,, \\
F_1^n(Q^2) &= -g_V C_2(Q^2) + \eta_V^n C_3(Q^2)\,, \\
F_2^n(Q^2) &= \eta_V^n C_4(Q^2)\,.
\end{aligned}$$

- The structure integrals $C_i(Q^2)$

$$\begin{aligned}
C_1(Q^2) &= \frac{1}{2}\int dz\, V(a,z)\, \sum_\tau c_\tau \left([f_\tau^L(z)]^2 + [f_\tau^R(z)]^2\right), \\
C_2(Q^2) &= \frac{1}{2}\int dz\, V(a,z)\, \sum_\tau c_\tau \left([f_\tau^R(z)]^2 - [f_\tau^L(z)]^2\right), \\
C_3(Q^2) &= \frac{1}{2}\int dz\, z\, \partial_z V(a,z) \sum_\tau c_\tau \left([f_\tau^L(z)]^2 - [f_\tau^R(z)]^2\right), \\
C_4(Q^2) &= 2m_N \int dz\, z\, V(a,z) \sum_\tau c_\tau\, f_\tau^L(z) f_\tau^R(z)\,.
\end{aligned}$$

- The functions $f_\tau^L(z)$ and $f_\tau^R(z)$ are the bulk profiles of fermions corresponding to the left and right handed ground-state nucleon with radial quantum number $n = 0$. The nucleon wavefunctions are expressed as

$$\begin{aligned} f_\tau^L(z) &= \sqrt{\frac{2}{\Gamma(\tau)}}\,\kappa^\tau\, z^{\tau-1/2}\, e^{-\kappa^2 z^2/2}, \\ f_\tau^R(z) &= \sqrt{\frac{2}{\Gamma(\tau-1)}}\,\kappa^{\tau-1}\, z^{\tau-3/2}\, e^{-\kappa^2 z^2/2}. \end{aligned}$$

- The $V(Q,z)$ is the bulk-to-boundary propagator of the transverse massless vector bulk field $V(Q,z) = \Gamma\left(1+\frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2},\, 0,\, \kappa^2 z^2\right)$, can be written in a integral representation

$$V(Q,z) = \kappa^2\, z^2 \int \frac{dx}{(1-x)^2}\, x^{\frac{Q^2}{4\kappa^2}} \exp\left[\frac{-\kappa^2 z^2 x}{(1-x)}\right].$$

- One can calculate the analytical expressions for nucleon form factors by substituting the structure integrals, the hadronic states with twist τ, and integral representation of the bulk-to-boundary propagator[1].

[1] T. Gutsche, V.E. Lyubovitskij, I. Schmidt, and A. Vega, Phys. Rev. D **86**, 036007 (2012)

- We perform a matching of expressions for the nucleon form factors to calculate the GPDs using the integral representation of bulk-to-boundary propagator $V(Q,z)$. The GPDs for up and down quarks is given as

$$H^q(x,Q^2) = q(x)\, x^{Q^2/4\kappa^2}\,, \qquad E^q(x,Q^2) = e^q(x)\, x^{Q^2/4\kappa^2}\,.$$

- The quark distribution functions $q(x)$ and $e^q(x)$:

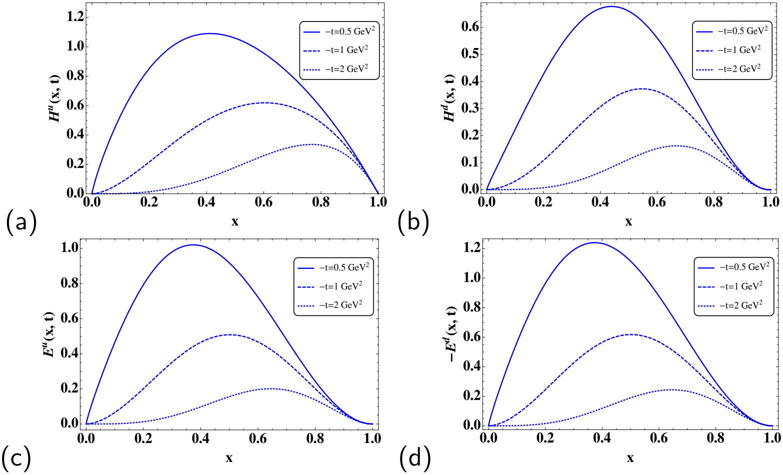
$$q(x) = \alpha_1^q \gamma_1(x) + \alpha_2^q \gamma_2(x) + \alpha_3^q \gamma_3(x)\,, \qquad e^q(x) = \alpha_3^q \gamma_4(x)\,.$$

- The flavor coupling parameters α_i^q and $\gamma_i(x)$:

$$\alpha_1^u = 2,\ \alpha_2^u = g_V,\ \alpha_3^u = 2\eta_V^p + \eta_V^n\,, \alpha_1^d = 1,\ \alpha_2^d = -g_V,\ \alpha_3^d = \eta_V^p + 2\eta_V^n\,,$$

$$\begin{aligned}
\gamma_1(x) &= -\frac{1}{2}\sum_\tau c_\tau\,(1-2\tau+x\tau)(1-x)^{\tau-2}\,,\\
\gamma_2(x) &= \frac{1}{2}\sum_\tau c_\tau\,(1-x\tau)(1-x)^{\tau-2}\,,\\
\gamma_3(x) &= \sum_\tau c_\tau\,(1+x\tau(-3+x+x\tau))(1-x)^{\tau-2}\,,\\
\gamma_4(x) &= \frac{2M_n}{\kappa}\sum_\tau c_\tau\,\tau\sqrt{\tau-1}(1-x)^{\tau-1}\,.
\end{aligned}$$

Plots of GPDs $H^{u,d}(x,t)$ and $E^{u,d}(x,t)$ vs x for fixed values of $(t=-Q^2)$



- We have presented the GPD $H^{u/d}(x,t)$ and $E^{u/d}(x,t)$ as a function of x for different values of t for the up and down quark.
- The GPDs $H(x,t)$ increases with x till a maxima is obtained and then falls to zero at $x \to 1$. One can see that the qualitative behavior of the GPDs is same for up and down quarks, however the fall off behavior is faster with the increasing values of x for the down quark.
- The GPDs $E(x,t)$ increases to a maximum value and then decreases, however, the fall off behavior with x is same for both up and down quark.
- For all cases the peak of GPDs shifted towards a higher value of x as the value of t increases.

Mellin Moments of GPDs:

- We now use these GPDs to compute higher moments in x defined as $H_n^q(Q^2)$ and $E_n^q(Q^2)$

$$H_n^q(Q^2) = \int \mathrm{d}x\, x^{n-1} H^q(x, Q^2)\,, \;\; E_n^q(Q^2) = \int \mathrm{d}x\, x^{n-1} E^q(x, Q^2)\,.$$

- Integrating over the parameter x gives the moment of GPDs

$$H_n^q(Q^2) = \alpha_1^q \beta_1(Q^2) + \alpha_2^q \beta_2(Q^2) + \alpha_3^q \beta_3(Q^2)\,,$$

$$E_n^q(Q^2) = \alpha_3^q \beta_4(Q^2)\,.$$

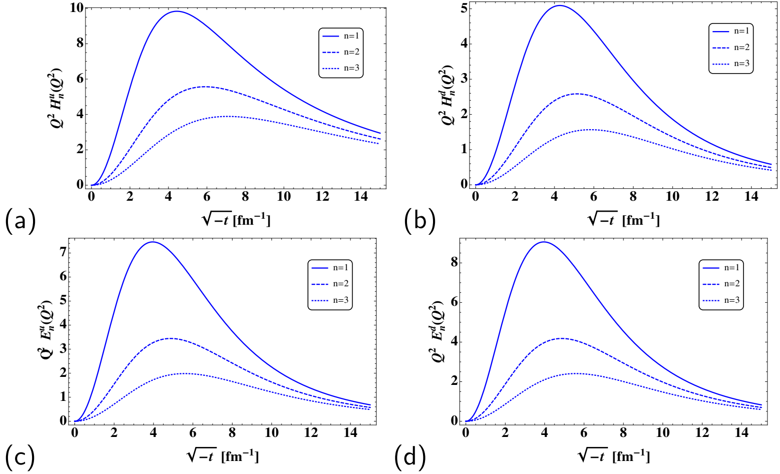
Here we have defined the parameters $\beta_i(Q^2)$

$$\beta_1(Q^2) = \sum_\tau c_\tau \left(\tau + \frac{a+n-1}{2}\right) B(a+n, \tau)\,, \;\; \beta_2(Q^2) = -\left(\frac{a+n-1}{2}\right) \sum_\tau c_\tau\, B(a+n, \tau)\,,$$

$$\beta_3(Q^2) = (a+n-1) \sum_\tau c_\tau \left(a+n-1-\frac{a+n}{\tau}\right) B(a+n, \tau+1)\,,$$

$$\beta_4(Q^2) = \frac{2M_n}{\kappa} \sum_\tau c_\tau\, \tau\sqrt{\tau-1}\, B(a+n, \tau)\,.$$

Plots of three moments of generalized parton distributions $Q^2 H_n^{u/d}(Q^2)$ and $Q^2 E_n^{u/d}(Q^2)$ vs $\sqrt{-t}$ for up and down quark



- The first moment of GPDs give the electromagnetic form factors, the second moment H_2^q and E_2^q correspond to gravitational form factors, and the third moments H_3^q and E_3^q are form factors of a twist-two operator.
- We have plotted the behavior of first three moments of GPDs $Q^2 H_n^{u/d}(Q^2)$ and $Q^2 E_n^{u/d}(Q^2)$ with momentum $\sqrt{-t}$ for up and down quarks.
- The qualitative behavior of moments of GPDs is same for the up and down quarks. We observe a decrease in the behavior of moments with the index n, which can be understood from the decrease of the profile functions with the momentum fraction x. The same trend has been observed in lattice calculations of moments.

GPDs in impact parameter space

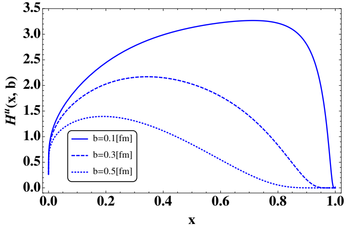
- GPDs in the momentum space are related to their impact parameter dependent parton distribution by the Fourier transform.

$$
\begin{aligned}
q(x,b) &= \frac{1}{(2\pi)^2}\int \mathrm{d}^2 q_\perp e^{-b_\perp\cdot q_\perp} H^q(x,t)\,,\\
e^q(x,b) &= \frac{1}{(2\pi)^2}\int \mathrm{d}^2 q_\perp e^{-b_\perp\cdot q_\perp} E^q(x,t)\,.
\end{aligned}
$$

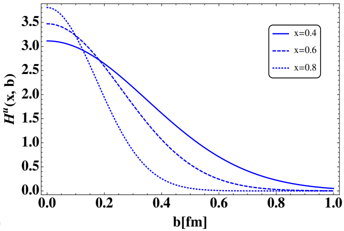
- The transverse impact parameter $b = |b_\perp|$ is a measure of the transverse distance between the struck parton and the center of momentum of the hadron.
- In a holographic soft-wall model the expressions for GPDs in transverse impact parameter space are given as

$$
\begin{aligned}
q(x,b_\perp) &= q(x)\frac{\kappa^2}{\pi\log(1/x)} e^{\frac{b^2\kappa^2}{\log(x)}}\,,\\
e^q(x,b_\perp) &= e^q(x)\frac{\kappa^2}{\pi\log(1/x)} e^{\frac{b^2\kappa^2}{\log(x)}}\,.
\end{aligned}
$$

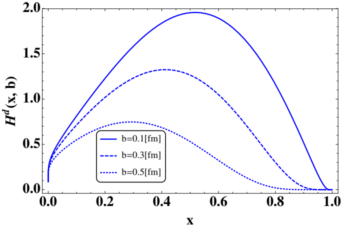
Plots of $H^{u/d}(x,b)$ vs x and plots of $H^{u/d}(x,b)$ vs b



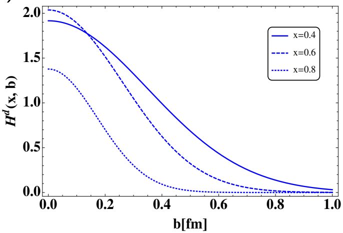
(a)



(b)

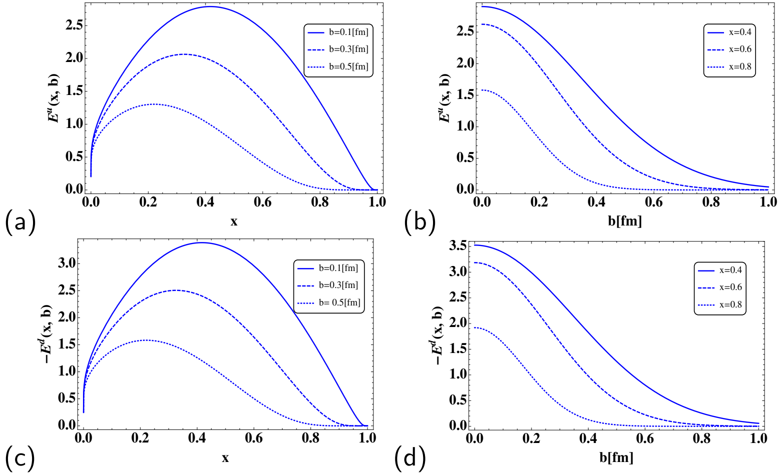


(c)



(d)

Plots of $E^{u/d}(x,b)$ vs x and plots of $E^{u/d}(x,b)$ vs b



- We have plotted the behavior of $H(x, b)$ with x for fixed values of $b = 0.1, 0.3, 0.5$ fm, and we have plotted the behavior of same GPD with the impact parameter b for the fixed values of $x = 0.4, 0.6, 0.8$. We plot the same GPDs for the down quarks for the same set of parameters. Similar plots showing the behavior of GPDs $E^{u/d}(x, b)$ are shown.
- In both cases, the maxima of GPDs shifted towards a lower value of x as b increases, therefore the transverse profile is peaked at $b = 0$ and falls off further.
- Also, for the small values of impact parameter b, the magnitude of GPD $H(x, b)$ is larger for up quark than down quark, whereas the magnitude of the GPD $E(x, b)$ is larger for down quark than up quark.

Summary and Conclusions

- We presented a numerical analysis of helicity independent GPDs for up and down quarks in the soft-wall model of AdS/QCD holography.
- This approach is based upon the light-front holography principle to match the matrix element of form factors in AdS modes and Light-front QCD.
- A detailed comparison of behavior has been made in the momentum space and transverse impact parameter space.
- The magnitude of GPD $H(x, b)$ is larger for up quarks than down quarks, whereas magnitude of GPD $E(x, b)$ is larger for down quarks than up quarks. It seems to a model independent result.
- Though we have considered only valence quarks contribution in GPDs, it is interesting to observe that the qualitative behavior of GPDs is same as the other phenomenological models in both momentum and impact parameter space.

Thank you very much for your kind attention!