Extracting PDFs by Global Fit of Lattice QCD Calculations

Yan-Qing Ma

University of Maryland

Based on works done with: Jian-Wei Qiu

YQM, Qiu, 1404.6860

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PDFs in QCD

- **Confinement: how to study hadrons?**
  - Only colorless hadrons can be found experimentally
  - But QCD is a quantum field theory of quarks and gluons

- **Parton distribution functions: relate hadrons to partons**

$$f_{q/p}(x,\mu^2) = \int \frac{d\xi_-}{4\pi} e^{-ix\xi_- P_+} \langle P|\overline{\psi}(\xi_-)\,\gamma_+ \exp\left\{-ig\int_0^{\xi_-} d\eta_- A_+(\eta_-)\right\}\psi(0)|P\rangle$$

- **Boost invariant along "+" direction**

- **Parton interpretation emerges in $A_+ \equiv A\cdot n = 0$ gauge.**

PDFs by fitting data

- **QCD factorization**

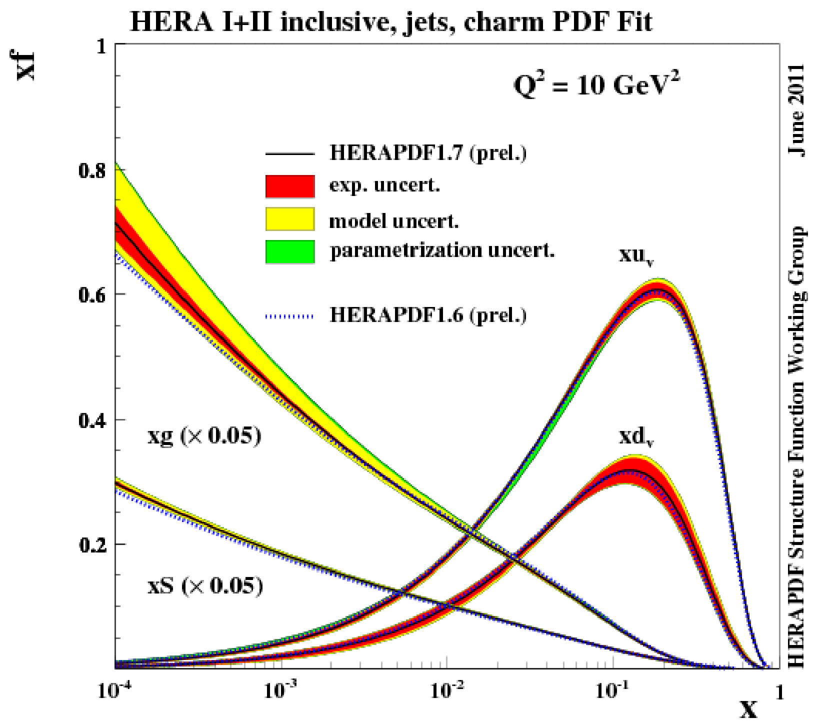
DIS: $F_2(x_B, Q^2) = \sum_i C_i(x_B/x, \mu^2/Q^2) \otimes f_{i/p}(x, \mu^2)$

HH: $\frac{d\sigma}{dy dp_T^2} = \sum_{i,j} \frac{d\hat{\sigma}_{ij}}{dy dp_T^2} \otimes f_{i/p}(x, \mu^2) \otimes f_{j/p}(x', \mu^2)$

- **DGLAP evolution:**

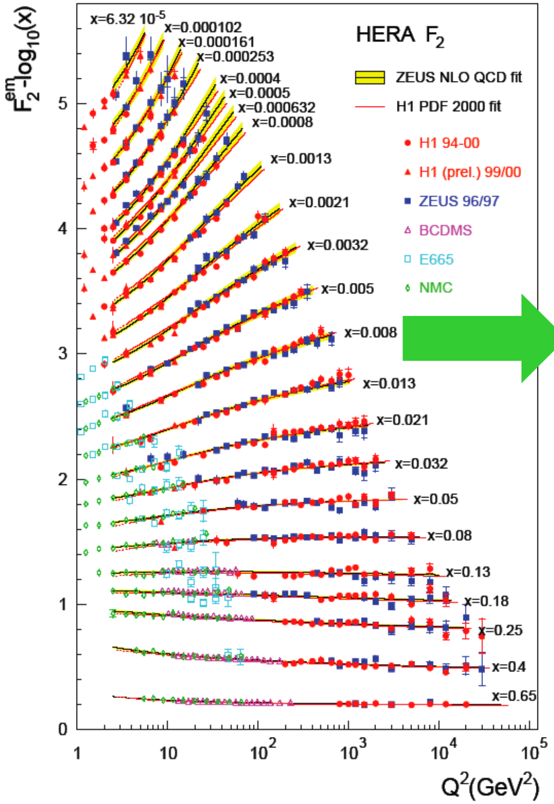
$$\frac{\partial f_{i/p}(x,\mu^2)}{\partial \ln\mu^2} = \sum_j P_{ij}(x/x') \otimes f_{j/p}(x', \mu^2)$$

- **Extract PDFs by fitting data**

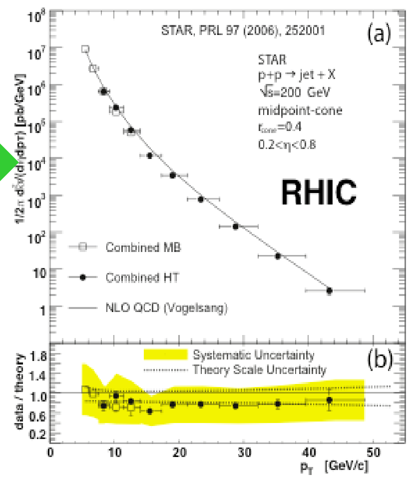


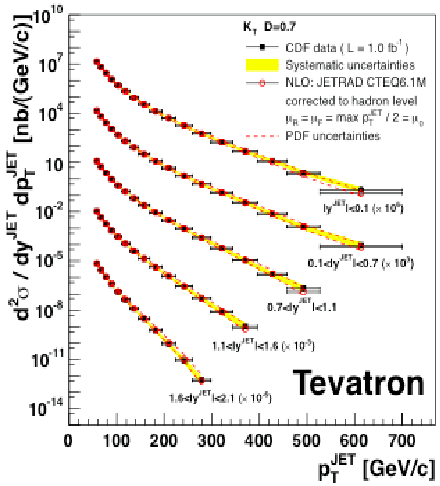
Successful of QCD factorization

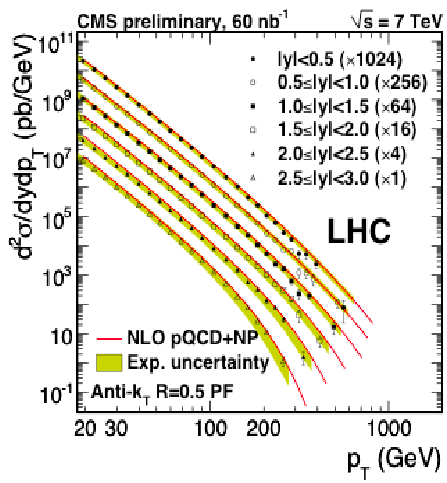
- **Successful**



Measure e-p at 0.3 TeV (HERA)
Predict p-p at 0.2, 1.96, and 7 TeV







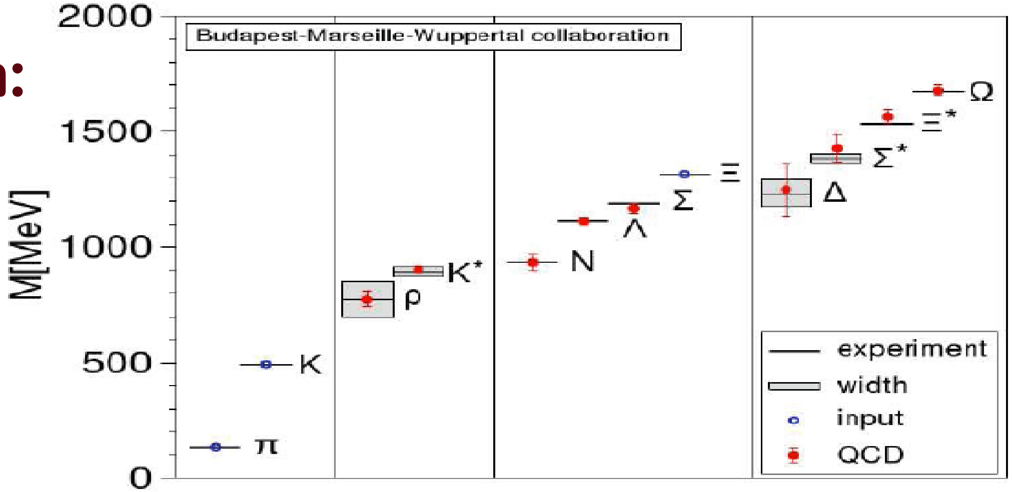
- **But, how to determine PDFs nonperturbatively from first principle?**

Lattice QCD

➢ **The main non-perturbative approach to solve QCD**

➢ **Hadron mass spectrum:**

Predict the spectrum
with limited inputs



➢ **An intrinsically Euclidean approach**

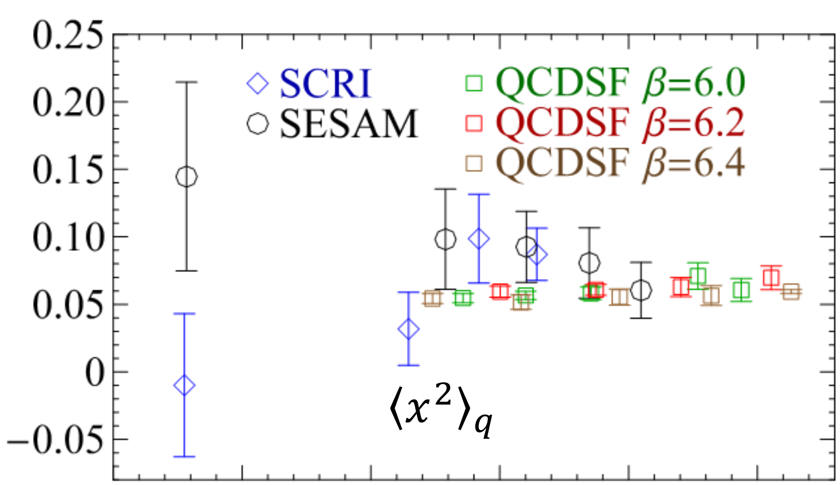
- "time" is Euclidean $\tau = i\, t$
- No direct implementation of physical time

➢ **Cannot calculate PDFs directly, whose operators are time dependent**

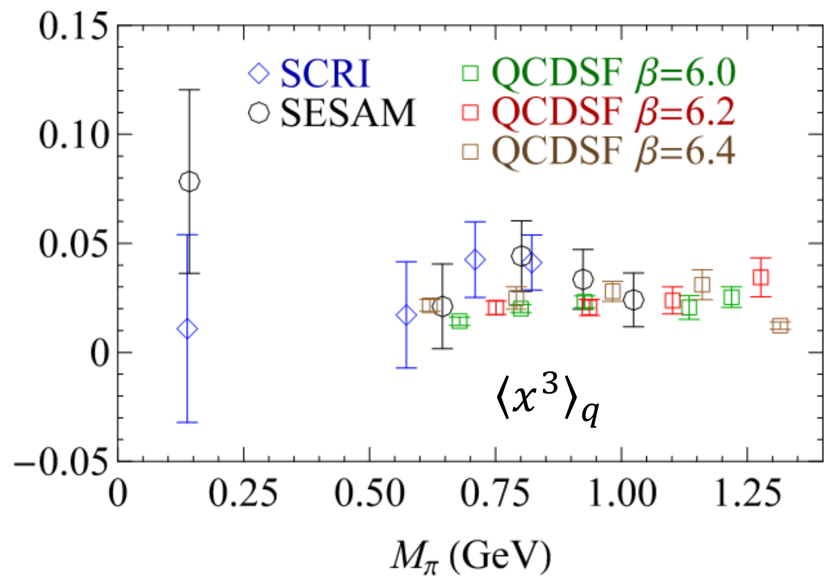
PDF from lattice

➢ **Only moments of PDFs can be calculated at present: local operator**

$$\langle x^n(\mu^2)\rangle_q \equiv \int_0^1 dx\, x^n f_{q/p}(x,\mu^2)$$



Dolgov et al., hep-lat/0201021



Gockeler et al., hep-ph/0410187

Limitation of lattice calculation

- **Only limited moments can be calculated on lattice**
  - Cannot give good predictions for large x behavior of PDFs, which are sensitive to higher moments
- **Leading power QCD factorization is not good for small x region**
  - Lattice calculated lower moments, which are sensitive to small x behavior of PDFs, can not compare with experimental data precisely

Ji's idea

Ji, 1305. 1539

- **"Quasi" quark distribution:**

$$\tilde{f}_{q/p}(x, \mu^2, P_z) = \int \frac{d\xi_z}{4\pi} e^{-ix\xi_z P_z} \langle P|\overline{\psi}(\xi_z)\,\gamma_z \exp\left\{-ig\int_0^{\xi_z} d\eta_z A_z(\eta_z)\right\}\psi(0)|P\rangle$$

- **Features of "Quasi" PDF**

- Quark fields separated along the z-direction
- No time dependence: can be calculated using standard lattice method
- "quasi" PDFs → normal PDFs, as $P_z \to \infty$.

Large Momentum Effective Theory

Ji, 1404. 6680

- **Relation between "Quasi" PDFs and normal PDFs at finite P_z :**

$$F(P^z/\Lambda) = Z(P^z/\Lambda, \Lambda/\mu) f(\mu) + \mathcal{O}(1/(P^z)^2) + \ldots$$

Quasi PDFs

Normal PDFs

Higher dimensional operators in LaMET

Normal PDFs: an effective field theory of "quasi" PDFs

- **Finite P_z effects can be improved by solving RG equation :**

$$\gamma(\alpha_s) = \frac{1}{Z}\frac{\partial Z}{\partial \ln P^z} \qquad \frac{\partial F(P^z)}{\partial \ln P^z} = \gamma(\alpha_s) F(P^z) + \mathcal{O}(1/(P^z)^2)$$

Exploratory study

Lin et al. 1402.1462

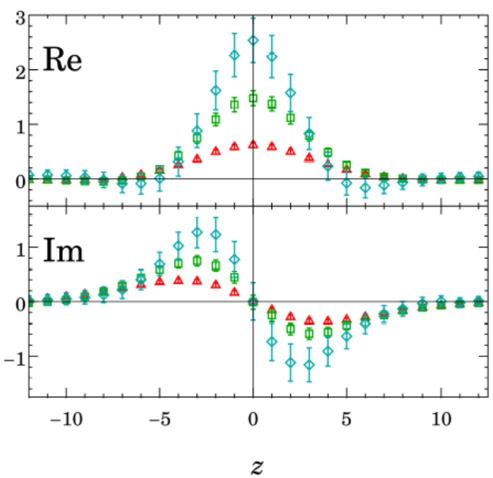


FIG. 1. The real (top) and imaginary (bottom) parts of the nonlocal isovector matrix element h of Eq. 3 computed on a lattice with the nucleon momentum P_z (in units of $2\pi/L$) = 1 (red triangles), 2 (green squares), 3 (cyan diamonds).

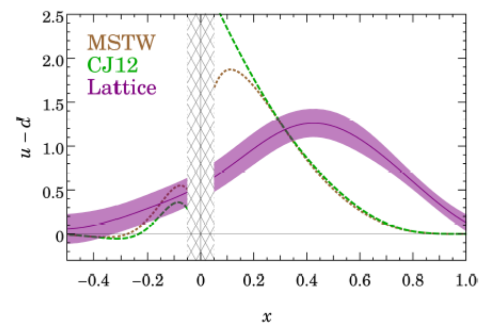


FIG. 2. The unpolarized isovector quark distribution $u(x) - d(x)$ computed on the lattice (purple band), compared with the global analyses by MSTW [13] (brown dotted line), and CTEQ-JLab (CJ12, green dashed line) [14] with medium nuclear correction near $(1.3\text{GeV})^2$. The negative x region is the sea quark distribution with $\bar{q}(x) = -q(-x)$.

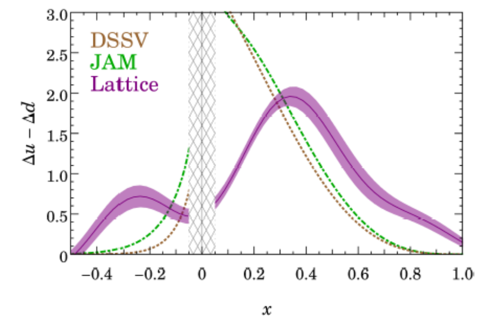


FIG. 3. (top) The isovector helicity distribution $\Delta u(x) - \Delta d(x)$ (purple band) computed on the lattice, along with selected global polarized analyses by JAM [19] (green dot-dashed) and DSSV09 [3] (brown dotted line). The corresponding sea-quark distributions are $\Delta\bar{q}(x) = \Delta q(-x)$.

- Good convergence, but results are not consistent with experimental data
- Improvement? $O(1/P_z)$ correction?

Our proposal: a generalized idea

YQM, Qiu, 1404.6860

What we need from lattice calculation?

- Minimum information: nonperturbative information of PDFs
- Calculate the exact PDFs on lattice QCD is OK, but not necessary

A relaxed condition:

- Any quantity calculated on lattice can be used to determine PDFs, as far as it has the same nonperturbative structure as PDFs
- Unlike "quasi" PDFs, the quantity is not demanded to go to PDF in any limit

Factorization: relate different quantities which have the same nonperturbative structure

- Coefficients are IR safe and perturbatively calculable

Lattice "cross section"

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- **Definition:** $\tilde{\sigma}_{\mathrm{E}}^{\mathrm{Lat}}(\tilde{x}, 1/a, P_z)$

  Fourier transform of a hadronic matrix element, $\langle h(P)|\mathcal{O}(\psi, A)|h(P)\rangle$

  With: $P^\mu = (P^0, 0_\perp, P_z)$

  - P_z mimics the "collision energy"
  - Hard scale to enable the factorization: $1/a \sim \mu \sim Q$

- **Condition for a good lattice "cross section"**

  ① Calculable in lattice QCD with an Euclidean time

  ② UV and IR safe perturbatively (renormalizable)

  ③ All CO divergences of its continuum limit can be factorized into normal PDFs with perturbatively calculable hard coefficient functions

$$\tilde{\sigma}_{\mathrm{M}}(\tilde{x}, \tilde{\mu}^2, P_z) \approx \sum_i \int_0^1 \frac{dx}{x} f_{i/h}(x, \mu^2)\, \mathcal{C}_i(\frac{\tilde{x}}{x}, \tilde{\mu}^2, \mu^2, P_z) + O(1/\mu^2)$$

- **Factorization is the essential question in our method!**

Case study—factorization of quasi PDFs

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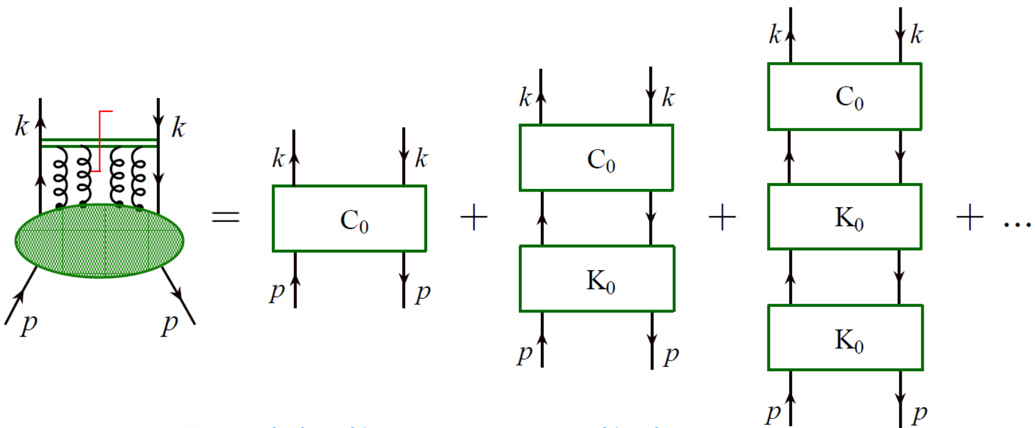
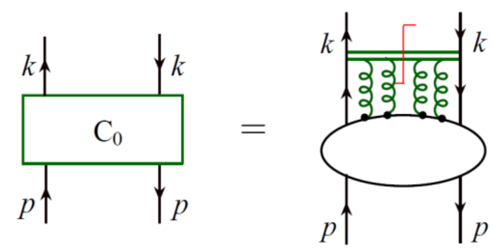
- **"Quasi quark" PDF as an example**

$$\tilde{f}_{q/p}(x,\mu^2,P_z) = \int \frac{d\xi_z}{4\pi} e^{-ix\xi_z P_z} \langle P|\overline{\psi}(\xi_z)\,\gamma_z \exp\left\{-ig\int_0^{\xi_z} d\eta_z A_z(\eta_z)\right\}\psi(0)|P\rangle$$

- **Soft pole cancellation**
  - Summing over all intermediate states, soft pole cancelled by unitarity
- **Generalized ladder diagrams decomposition: physical gauge**



- Double line: gauge link
- Blob: matrix element
- C_0, K_0: 2PI kernels
- Ordering in virtuality $p^2 \ll k^2 \sim \mu^2$



Factorization

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➢ Using physical gauge, 2PI diagrams are finite

Ellis, Georgi, Machacek, Politzer, Ross, 1978, 1979

- Landau rule: infrared divergence are associated with physical, kinematically allowed subprocesses
- 2PI diagrams: from interference terms (other than the lowest order one)

➢ Factorize the last kernel, and then recursively:

$\widehat{\mathcal{P}}$: pick up the singular part of integration

$$\tilde{f}_{q/p} = \lim_{m\to\infty} C_0 \sum_{i=0}^{m} K^i + \mathrm{UVCT}$$

$$= \lim_{m\to\infty} C_0 \left[1 + \sum_{i=0}^{m-1} K^i (1-\widehat{\mathcal{P}})K\right]_{\mathrm{ren}} + \tilde{f}_{q/p}\,\widehat{\mathcal{P}}\,K$$

$$= \lim_{m\to\infty} C_0 \left[1 + \sum_{i=1}^{m} \left[(1-\widehat{\mathcal{P}})K\right]^i\right]_{\mathrm{ren}} + \tilde{f}_{q/p}\,\widehat{\mathcal{P}}\,K\,,$$

$$\tilde{f}_{q/p} = \left[C_0 \frac{1}{1-(1-\widehat{\mathcal{P}})K}\right]_{\mathrm{ren}} \left[\frac{1}{1-\widehat{\mathcal{P}}K}\right]$$

Normal PDFs

All CO divergence of quasi PDF

Finite

$$\tilde{\sigma}_{\mathrm{M}}(\tilde{x},\tilde{\mu}^2,P_z) \approx \sum_i \int_0^1 \frac{dx}{x}\, f_{i/h}(x,\mu^2)\, \mathcal{C}_i\!\left(\frac{\tilde{x}}{x},\tilde{\mu}^2,\mu^2,P_z\right) + O(1/\mu^2)$$

One loop example: quark→quark

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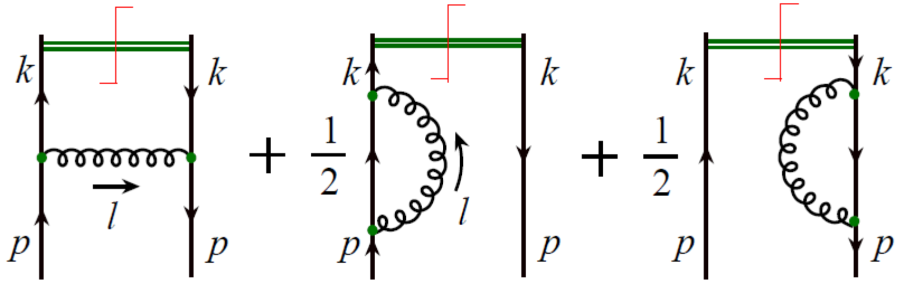
- **Expand the factorization formula**

$$\tilde{f}^{(1)}_{q/q}(\tilde{x}) = f^{(0)}_{q/q}(x) \otimes \mathcal{C}^{(1)}_{q/q}(\tilde{x}/x) + f^{(1)}_{q/q}(x) \otimes \mathcal{C}^{(0)}_{q/q}(\tilde{x}/x)$$

$$\Rightarrow \quad \mathcal{C}^{(1)}_{q/q}(t, \tilde{\mu}^2, \mu^2, P_z) = \tilde{f}^{(1)}_{q/q}(t, \tilde{\mu}^2, P_z) - f^{(1)}_{q/q}(t, \mu^2)$$

- **Feynman diagrams**

Same diagrams for both, but with different gauge



- **Gauge choice**

$A_z = 0$ **for** $\tilde{f}_{q/q}$

Gluon propagator:

$$\tilde{d}^{\alpha\beta}(l) = -g^{\alpha\beta} + \frac{l^\alpha n_z^\beta + n_z^\alpha l^\beta}{l_z} - \frac{n_z^2 l^\alpha l^\beta}{l_z^2}$$

$A_+ = 0$ **for** $f_{q/q}$

$$d^{\alpha\beta}(l) = -g^{\alpha\beta} + \frac{l^\alpha n^\beta + n^\alpha l^\beta}{l_+}$$

One-loop expression

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- **After the integration of energy component by using residue theory**

$$\tilde{f}^{(1)}_{q/q}(\tilde{x},\tilde{\mu}^2,P_z) = C_F\frac{\alpha_s}{2\pi}\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\int_0^{\tilde{\mu}^2}\frac{dl_\perp^2}{l_\perp^{2+2\epsilon}}\int_{-\infty}^{+\infty}\frac{dl_z}{P_z}\left[\delta\left(1-\tilde{x}-y\right)-\delta\left(1-\tilde{x}\right)\right]\left\{\frac{1}{y}\left(1-y+\frac{1-\epsilon}{2}y^2\right)\right.$$

$$\left.\times\left[\frac{y}{\sqrt{\lambda^2+y^2}}+\frac{1-y}{\sqrt{\lambda^2+(1-y)^2}}\right]+\frac{(1-y)\lambda^2}{2y^2\sqrt{\lambda^2+y^2}}+\frac{\lambda^2}{2y\sqrt{\lambda^2+(1-y)^2}}+\frac{1-\epsilon}{2}\frac{(1-y)\lambda^2}{[\lambda^2+(1-y)^2]^{3/2}}\right\}$$

where $y = l_z/P_z,\ \lambda^2 = l_\perp^2/P_z^2,\ C_F = (N_c^2-1)/(2N_c)$

- **Cancellation of CO divergence**

$$\frac{y}{\sqrt{\lambda^2+y^2}}+\frac{1-y}{\sqrt{\lambda^2+(1-y)^2}} = 2\theta(0<y<1)-\left[\mathrm{Sgn}(y)\frac{\sqrt{\lambda^2+y^2}-|y|}{\sqrt{\lambda^2+y^2}}+\mathrm{Sgn}(1-y)\frac{\sqrt{\lambda^2+(1-y)^2}-|1-y|}{\sqrt{\lambda^2+(1-y)^2}}\right]$$

Only the first term is CO divergent for $0<y<1$, which is the same as normal PDF - necessary!

One-loop coefficient functions

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- $\overline{MS}$ **scheme for normal PDF**

$$\mathcal{C}^{(1)}_{q/q}(t,\tilde{\mu}^2,\mu^2,P_z)=\tilde{f}^{(1)}_{q/q}(t,\tilde{\mu}^2,P_z)-f^{(1)}_{q/q}(t,\mu^2)$$

$$t=\tilde{x}/x$$

$$\frac{\mathcal{C}^{(1)}_{q/q}(t)}{C_F\frac{\alpha_s}{2\pi}}=\left[\frac{1+t^2}{1-t}\ln\frac{\tilde{\mu}^2}{\mu^2}+1-t\right]_+ +\left[\frac{t\Lambda_{1-t}}{(1-t)^2}+\frac{\Lambda_t}{1-t}\right.$$
$$\left.+\frac{\mathrm{Sgn}(t)\Lambda_t}{\Lambda_t+|t|}-\frac{1+t^2}{1-t}\left[\mathrm{Sgn}(t)\ln\left(1+\frac{\Lambda_t}{2|t|}\right)+\mathrm{Sgn}(1-t)\ln\left(1+\frac{\Lambda_{1-t}}{2|1-t|}\right)\right]\right]_N$$

where $\Lambda_t=\sqrt{\tilde{\mu}^2/P_z^2+t^2}-|t|$, $\mathrm{Sgn}(t)=1$ if $t\geq 0$, and -1 otherwise

Generalized "+" description: Integration in all region

- $P_z\to\infty$, **Ji's result** $\quad\Lambda_t\to 0$

- **Explicit verification of the factorization at one-loop**

Coefficient functions for all partonic channels are IR safe and finite

To do list

- **Additional matching:**

$$\tilde{\sigma}_{\rm E}^{\rm Lat}(\tilde{x}, 1/a, P_z) \overset{\mathcal{Z}}{\longleftrightarrow} \tilde{\sigma}_{\rm E}(\tilde{x}, \tilde{\mu}^2, P_z)$$
$$\Updownarrow$$
$$\tilde{\sigma}_{\rm M}(\tilde{x}, \tilde{\mu}^2, P_z) \overset{\mathcal{C}}{\longleftrightarrow} f_{i/h}(x, \mu^2)$$

  - Lattice perturbation theory
  - Nonperturbative matching

S. Yoshida's talk

- **Finding more good lattice "cross section" to calculate PDFs**

- **Finding good lattice "cross section" for other nonperturbative quantities: GPDs, TMDs, …**

Summary

- "Lattice cross sections" = hadronic matrix elements that are calculabe in Lattice QCD and factorizable to PDFs

  E.g. quasi-PDFs proposed by Ji

- PDFs can be extracted by global analysis of data on "Lattice cross sections". Same for other distributions.

- "Lattice cross sections"- complementary to colliders
  - High energy scattering experiments: sensitive to small x physics
  - "Lattice cross sections": sensitive to large x physics

- Lattice QCD can calculate PDFs now, but, more works are needed!

Thank you!