

# Relativistic Studies of Spin-Isospin Resonances --- Towards exotic deformed nuclei

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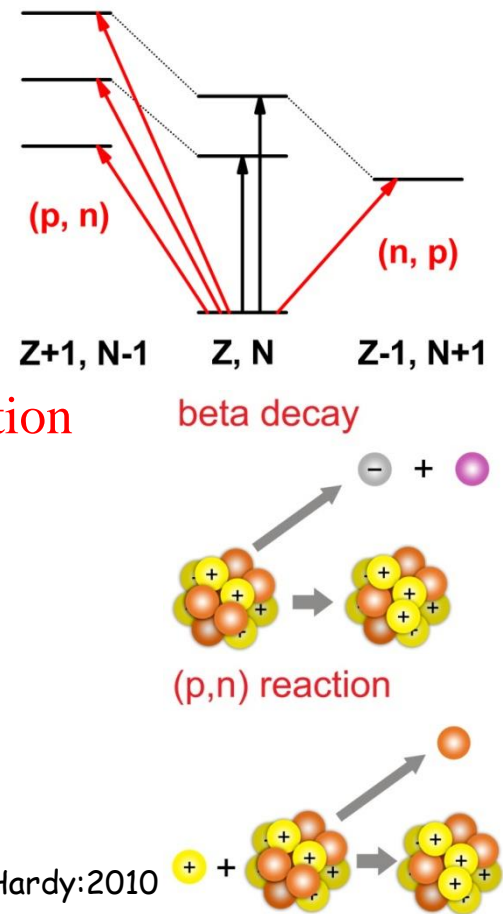
# Nuclear spin-isospin excitations

## Nuclear spin-isospin excitations

- $\beta$ -decays in nature
- charge-exchange reactions in lab

## These excitations are important to understand

- spin and isospin properties of in-medium nuclear interaction
- effective nucleon-nucleon tensor forces Bai:2010
- neutron skin thickness Krasznahorkay:1999, Vretenar:2003, Yako:2006
- $\beta$ -decay rates of  $r$ -process nuclei Engel:1999, Borzov:2006
- neutrino-nucleus cross sections Kolbe:2003, Vogel:2006, Paar:2008
- $\beta\beta$ -decay rates Ejiri:2000, Avignone:2008
- unitarity of Cabibbo-Kobayashi-Maskawa matrix Towner&Hardy:2010

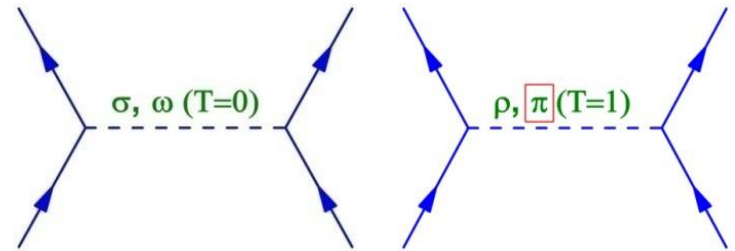


The theories achieving consistent treatment of the spin and isospin degrees of freedom are highly desired.

# Covariant density functional theory

- Fundamental: Kohn-Sham Density Functional Theory
- **CDFT** starting point: effective Lagrangian density [Walecka & Serot:1986](#), [Long:2006](#)

$$\begin{aligned} \mathcal{L} = & \bar{\psi} \left[ i\gamma^\mu \partial_\mu - M - g_\sigma \sigma - \gamma^\mu \left( g_\omega \omega_\mu + g_\rho \vec{\tau} \cdot \vec{\rho}_\mu + e \frac{1 - \tau_3}{2} A_\mu \right) - \frac{f_\pi}{m_\pi} \gamma_5 \gamma^\mu \partial_\mu \vec{\pi} \cdot \vec{\tau} \right] \psi \\ & + \frac{1}{2} \partial^\mu \sigma \partial_\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{4} \Omega^{\mu\nu} \Omega_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4} \vec{R}_{\mu\nu} \cdot \vec{R}^{\mu\nu} + \frac{1}{2} m_\rho^2 \vec{\rho}^\mu \cdot \vec{\rho}_\mu \\ & + \frac{1}{2} \partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi} - \frac{1}{2} m_\pi^2 \vec{\pi} \cdot \vec{\pi} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \end{aligned}$$



## Comparing to traditional non-relativistic DFT

- **Effective Lagrangian**  
connections to underlying theories, QCD at low energy
- **Dirac equation**  
consistent treatment of **spin** d.o.f. & nuclear saturation properties
- **Lorentz covariant symmetry**  
consistent treatment of **isospin** d.o.f. & unification of time-even and time-odd components

# Dirac equations and RPA

- Energy functional of the system

$$E = \langle \Phi_0 | H | \Phi_0 \rangle = E_k + E_\sigma^D + E_\omega^D + E_\rho^D + E_A^D + E_\sigma^E + E_\omega^E + E_\rho^E + E_\pi^E + E_A^E + E_{\text{pair}}$$

- Dirac equations for the ground-state properties

$$\int d\mathbf{r}' h(\mathbf{r}, \mathbf{r}') \psi(\mathbf{r}') = \varepsilon \psi(\mathbf{r}), \quad \text{with} \quad h^{\text{kin}}(\mathbf{r}, \mathbf{r}') = [\boldsymbol{\alpha} \cdot \mathbf{p} + \beta M] \delta(\mathbf{r} - \mathbf{r}'),$$
$$h^D(\mathbf{r}, \mathbf{r}') = [\Sigma_T(\mathbf{r}) \gamma_5 + \Sigma_0(\mathbf{r}) + \beta \Sigma_S(\mathbf{r})] \delta(\mathbf{r} - \mathbf{r}'),$$
$$h^E(\mathbf{r}, \mathbf{r}') = \begin{pmatrix} Y_G(\mathbf{r}, \mathbf{r}') & Y_F(\mathbf{r}, \mathbf{r}') \\ X_G(\mathbf{r}, \mathbf{r}') & X_F(\mathbf{r}, \mathbf{r}') \end{pmatrix}.$$

- RPA equations for the vibrational excitation properties [Ring & Schuck:1980](#)

$$\begin{pmatrix} A & B \\ -B & -A \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \omega_\nu \begin{pmatrix} X \\ Y \end{pmatrix}$$

- ◆  $\delta E / \delta \rho \rightarrow$  equation of motion for nucleons: **Dirac (-Bogoliubov) equations**
- ◆  $\delta^2 E / \delta \rho^2 \rightarrow$  linear response equation: **(Q)RPA equations**

# CDFT+RPA in charge-exchange channel

**Particle-hole residual interactions** HZL, Giai, Meng, *Phys. Rev. Lett.* **101**, 122502 (2008)

➤  $\sigma$ -meson 
$$V_\sigma(1, 2) = -[g_\sigma \gamma_0]_1 [g_\sigma \gamma_0]_2 D_\sigma(1, 2)$$

➤  $\omega$ -meson 
$$V_\omega(1, 2) = [g_\omega \gamma_0 \gamma^\mu]_1 [g_\omega \gamma_0 \gamma_\mu]_2 D_\omega(1, 2)$$

➤  $\rho$ -meson 
$$V_\rho(1, 2) = [g_\rho \gamma_0 \gamma^\mu \vec{T}]_1 \cdot [g_\rho \gamma_0 \gamma_\mu \vec{T}]_2 D_\rho(1, 2)$$

➤ pseudovector  $\pi$ - $N$  coupling

$$V_\pi(1, 2) = -\left[\frac{f_\pi}{m_\pi} \vec{T} \gamma_0 \gamma_5 \gamma^k \partial_k\right]_1 \cdot \left[\frac{f_\pi}{m_\pi} \vec{T} \gamma_0 \gamma_5 \gamma^l \partial_l\right]_2 D_\pi(1, 2)$$

➤ zero-range counter-term of  $\pi$ -meson

$$V_{\pi\delta}(1, 2) = g' \left[\frac{f_\pi}{m_\pi} \vec{T} \gamma_0 \gamma_5 \gamma\right]_1 \cdot \left[\frac{f_\pi}{m_\pi} \vec{T} \gamma_0 \gamma_5 \gamma\right]_2 \delta(\mathbf{r}_1 - \mathbf{r}_2), \quad g' = 1/3$$

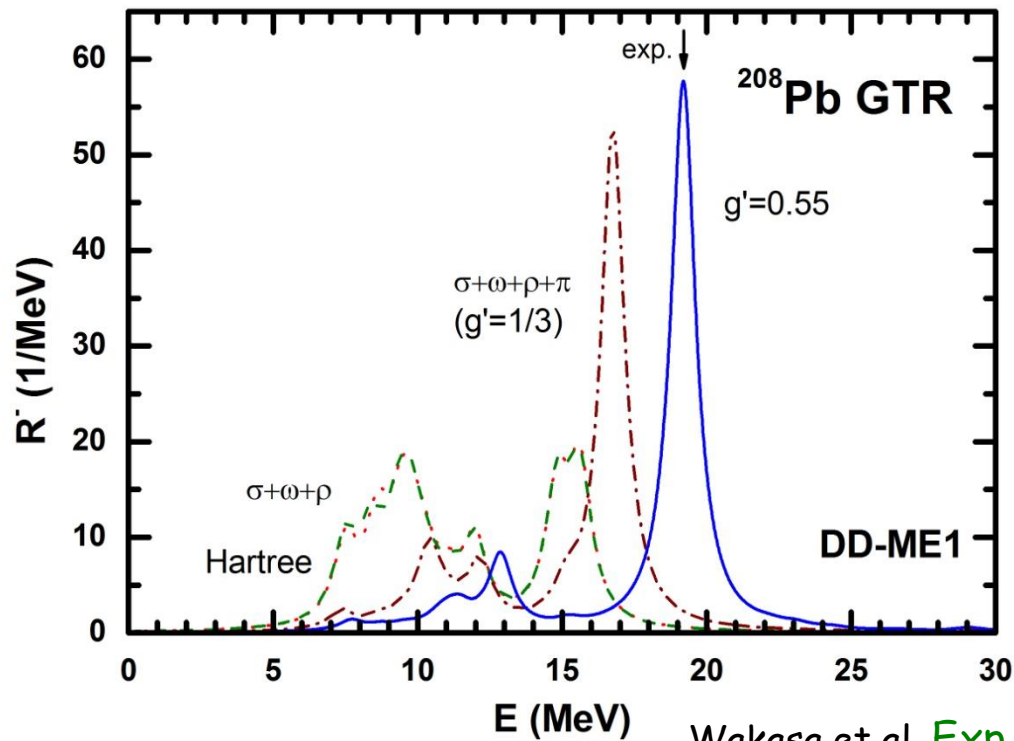
- ◆ For the correct asymptotic behavior at high  $q$ ,  $g'$  is not a parameter, but must be **1/3**.

# Previous results in Hartree level

**CDFT+RPA** for spin-isospin resonances (with only Hartree terms)

De Conti:1998, 2000, Vretenar: 2003, Ma:2004, Paar:2004, Niksic:2005

**example:** Gamow-Teller resonance (GTR) in  $^{208}\text{Pb}$  ( $\Delta S = 1, \Delta L = 0, J^\pi = 1^+$ )

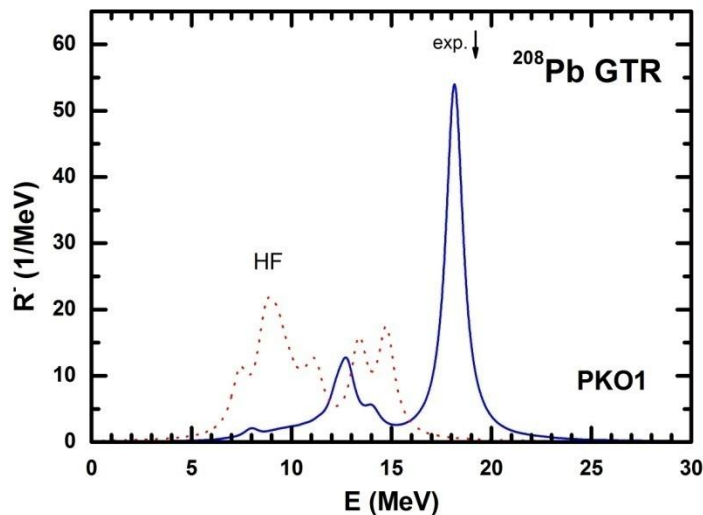
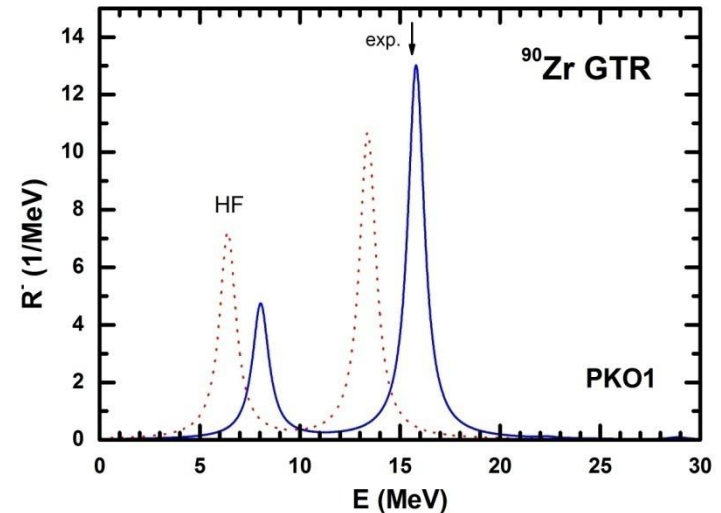
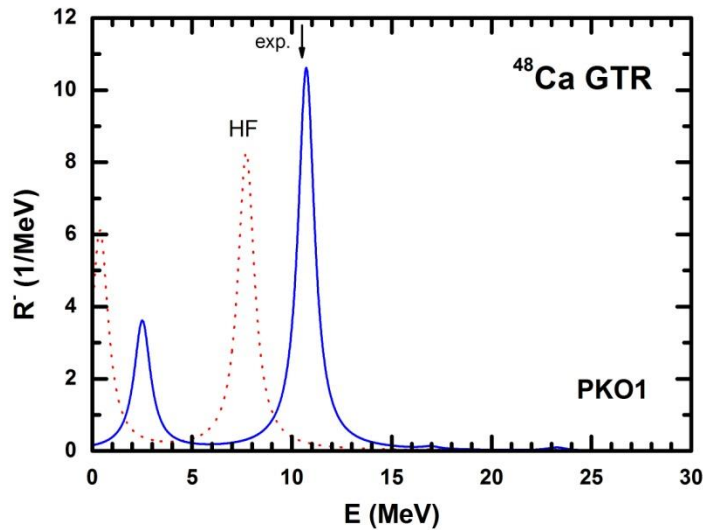


Wakasa et al. Exp. @ RCNP, Osaka U.

- ◆ One has to add  $\pi$  and fit  $g' \rightarrow$  **Self-consistency is lost**

# Gamow-Teller resonances

**CDFT+RPA** for Gamow-Teller resonances (with both Hartree & Fock terms)

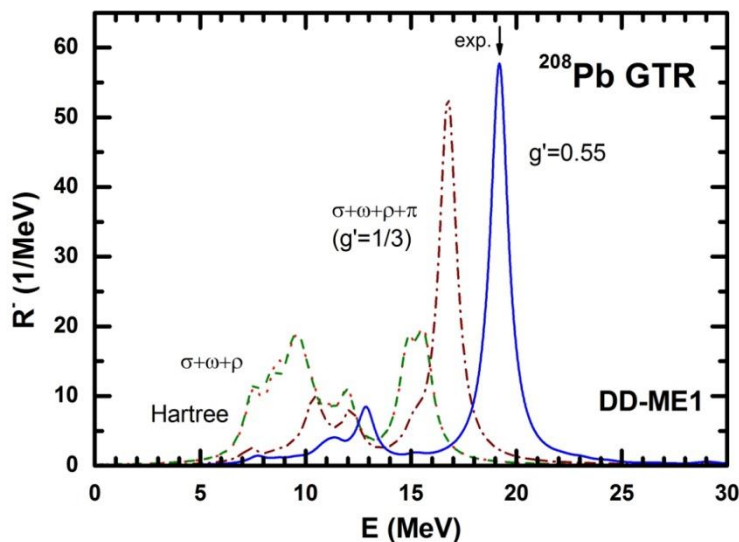


- ◆ GTR excitation energies can be reproduced in a fully self-consistent way.

HZL, Giai, Meng, *Phys. Rev. Lett.* **101**, 122502 (2008)

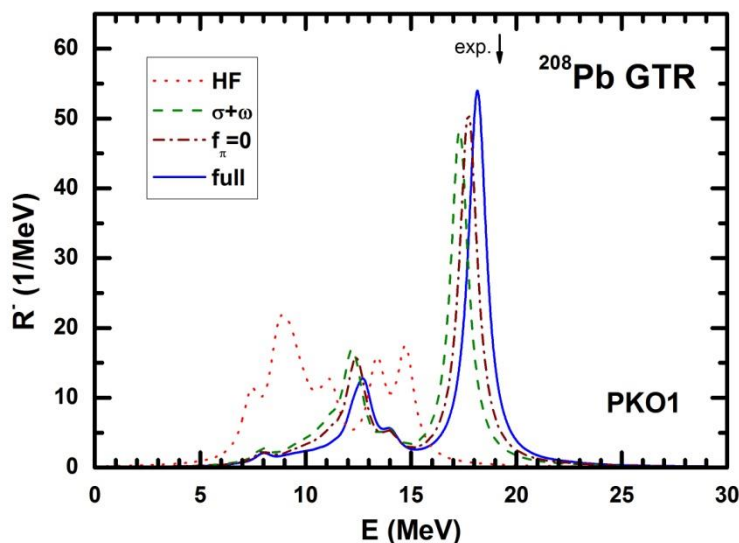


# Physical mechanisms of GTR



## With only Hartree terms

- ◆ No contribution from isoscalar  $\sigma$  and  $\omega$  mesons, because exchange terms are missing.
- ◆  $\pi$ -meson is dominant in this resonance.
- ◆  $g'$  has to be retted to reproduce the experimental data.



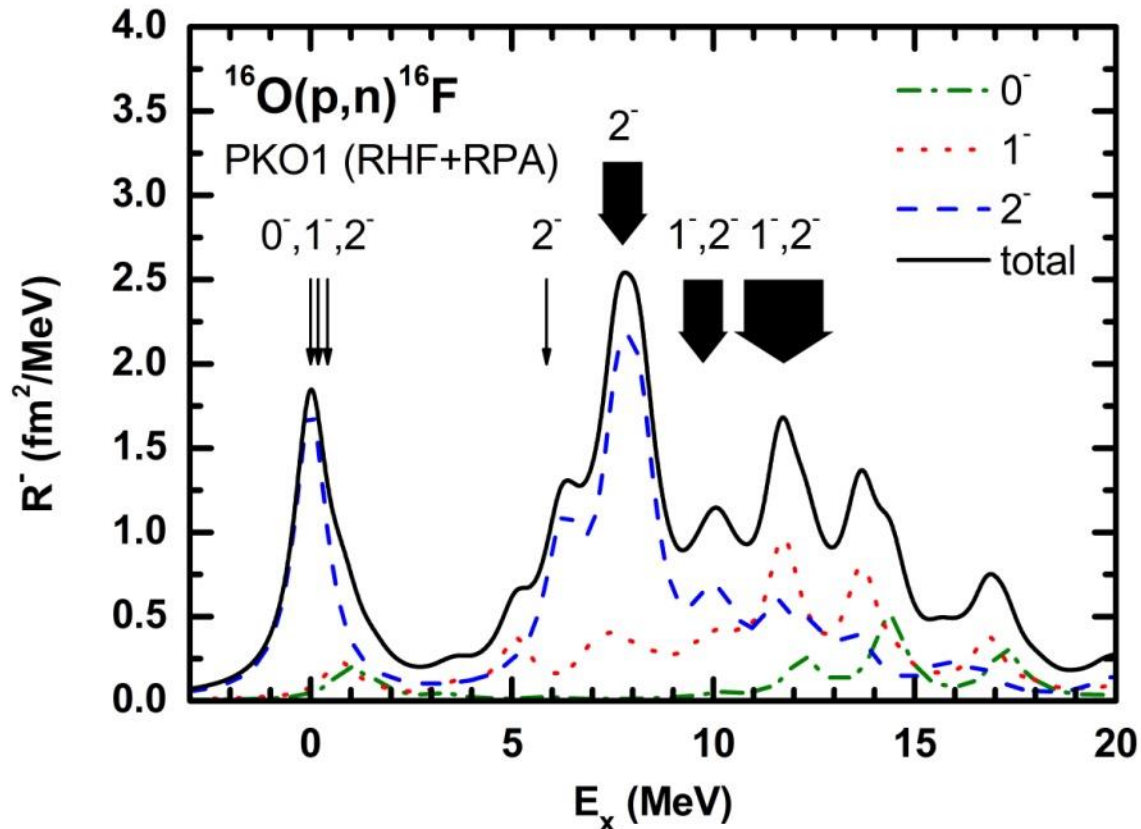
## With both Hartree & Fock terms

- ◆ Isoscalar  $\sigma$  and  $\omega$  mesons play an essential role via the exchange terms.
- ◆  $\pi$ -meson plays a minor role.
- ◆  $g' = 1/3$  is kept for self-consistency.

HZL, Giai, Meng, *PRL* **101**, 122502 (2008)  
 HZL, Zhao, Ring, Roca-Maza, Meng,  
*PRC* **86**, 021302(R) (2012)

# Spin-dipole resonances

CDFT+RPA for spin-dipole resonances ( $\Delta S = 1, \Delta L = 1, J^\pi = 0^-, 1^-, 2^-$ )



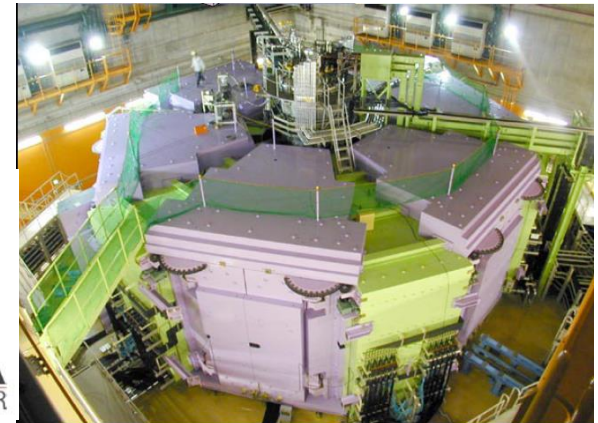
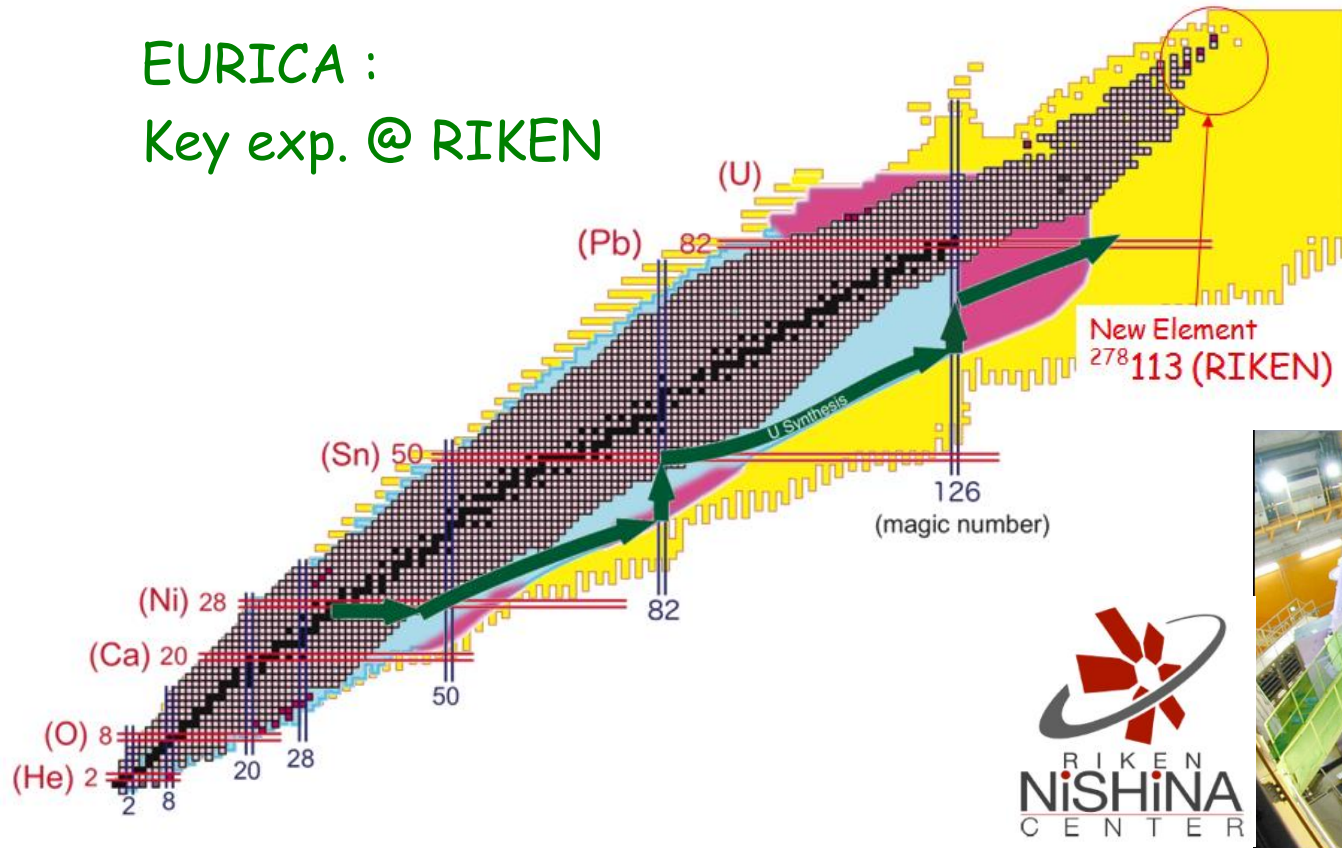
(Exp.) Wakasa et al., *PRC* 84, 014614 (2011); (Theory) HZL, Zhao, Meng, *PRC* 85, 064302 (2012)

- ◆ A crucial test for the theoretical predictive power.

# $\beta$ decays and $r$ -process

## Nuclear $\beta$ decays and $r$ -process nucleosynthesis

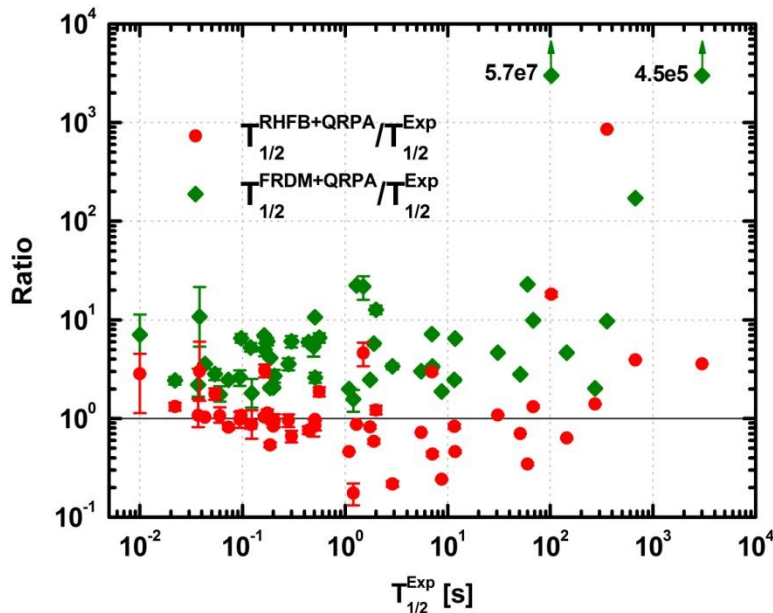
EURICA :  
Key exp. @ RIKEN



- ◆ EURICA project is providing lots of new  $\beta$ -decay data towards  $r$ -process path.

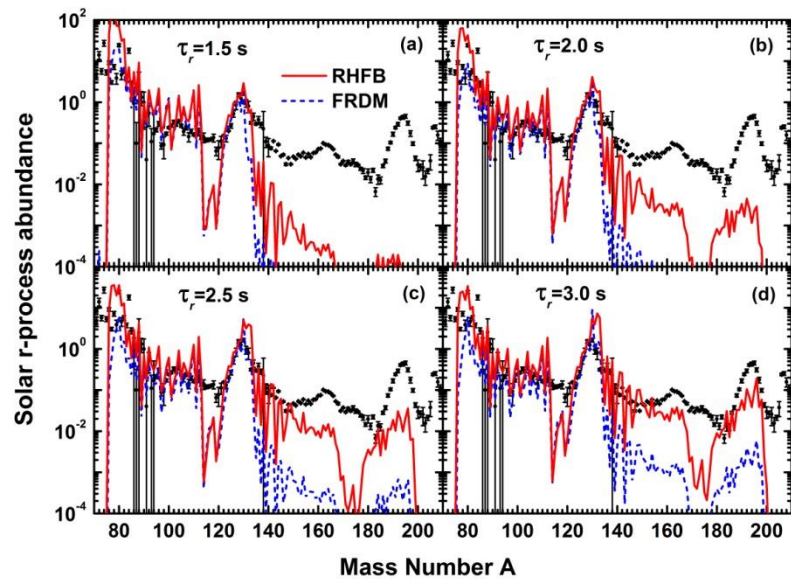
# $\beta$ decays and $r$ -process

## Nuclear $\beta$ -decay rates and $r$ -process flow ( $Z = 20 \sim 50$ region)



FRDM: widely used nuclear input

RHF: CDFT results with  $T=0$  pairing



Niu, Niu, HZL, Long, Niksic, Vretenar, Meng,  
*Phys. Lett. B* 723, 172 (2013)


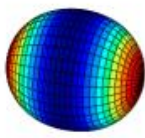
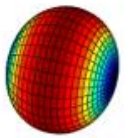
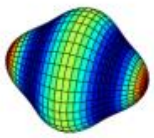
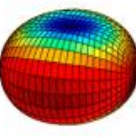
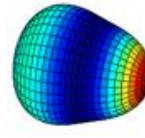
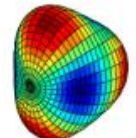
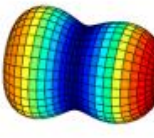
- ◆ Classical  $r$ -process calculation shows a faster  $r$ -matter flow at the  $N = 82$  region and higher  $r$ -process abundances of elements with  $A \sim 140$ .

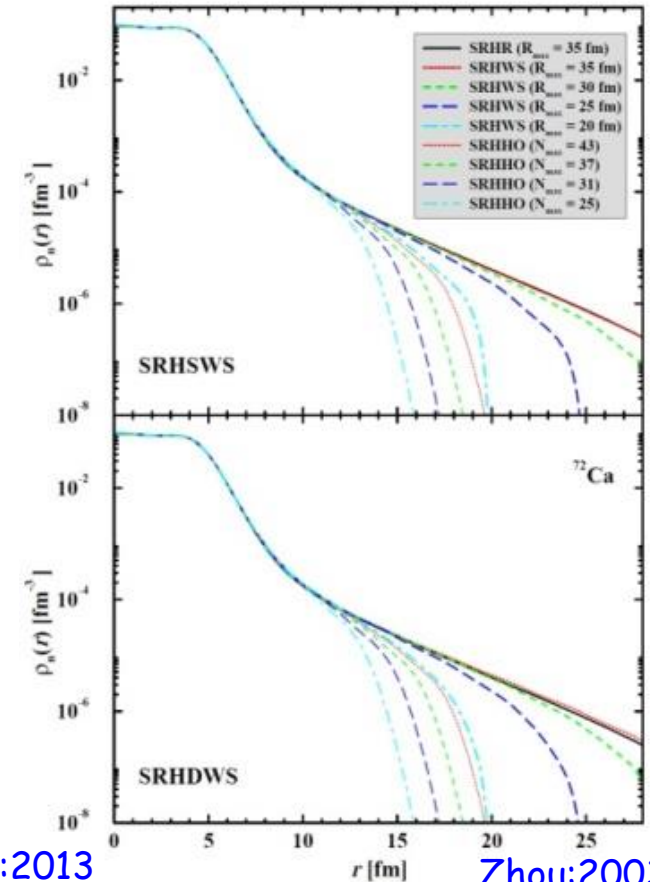
# Towards deformed nuclei

Goal: To study spin-isospin excitations in exotic deformed nuclei

- correct asymptotic behavior
- to break all geometric symmetries
- exotic shapes / exotic excitation modes
- reasonable computational time

CDFT on 3D mesh:  
ground states & excitations

$\beta_{\lambda\mu} = 0$	$\beta_{20} > 0$	$\beta_{20} < 0$	$\beta_{40} > 0$
			
$\beta_{22} \neq 0$	$\beta_{30} \neq 0$	$\beta_{32} \neq 0$	$\beta_{20} \gg 0$
			

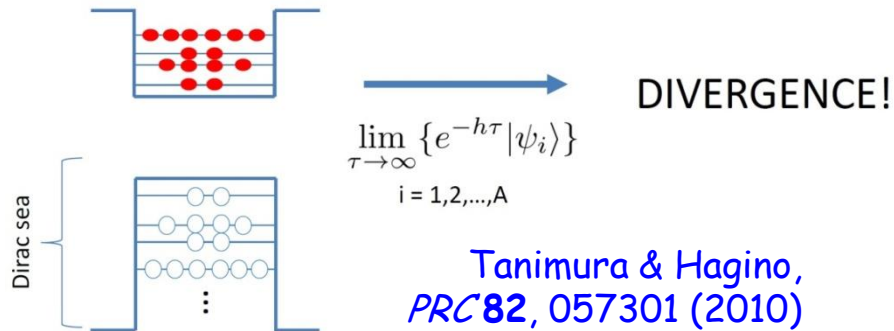


Lu:2013

Zhou:2003

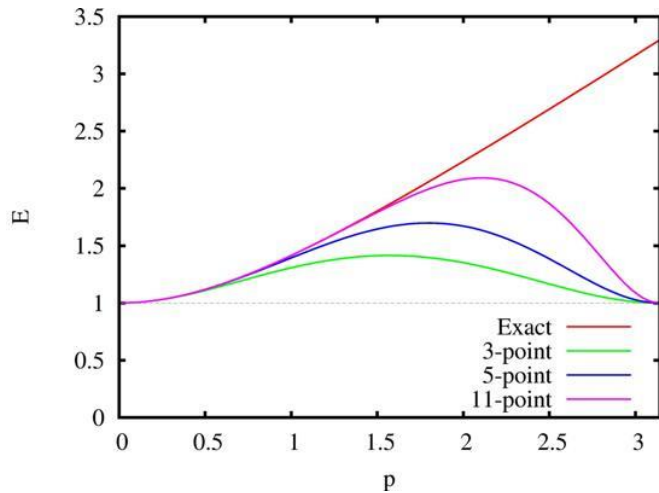
# CDFT on 3D mesh: ground states

## Challenge 1: Variational collapse



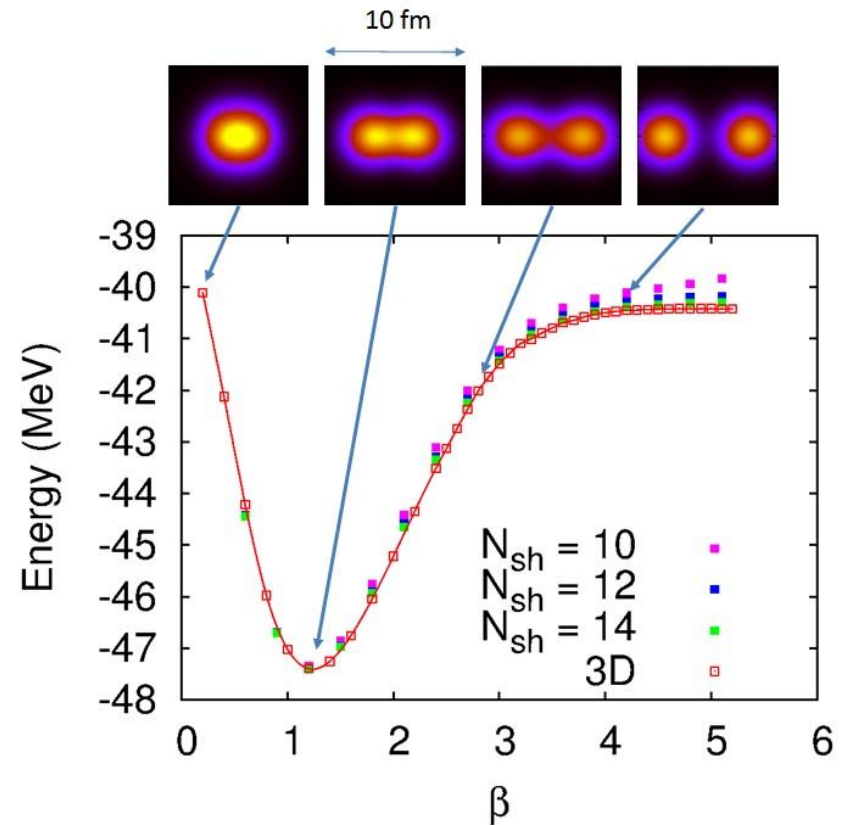
Solution: Inverse Hamiltonian method

## Challenge 2: Fermion doubling



Solution: High-order Wilson terms

## Preliminary results: ${}^8\text{Be}$



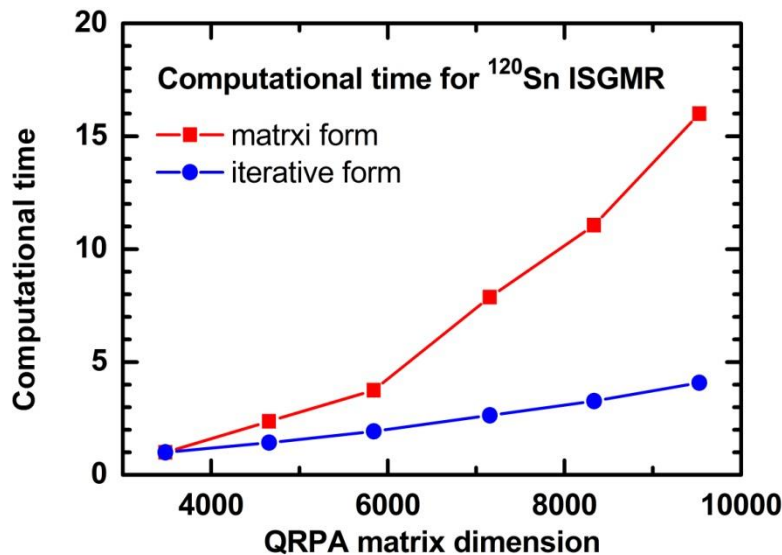
Tanimura, Hagino, HZL, in preparation

# CDFT on 3D mesh: excitations

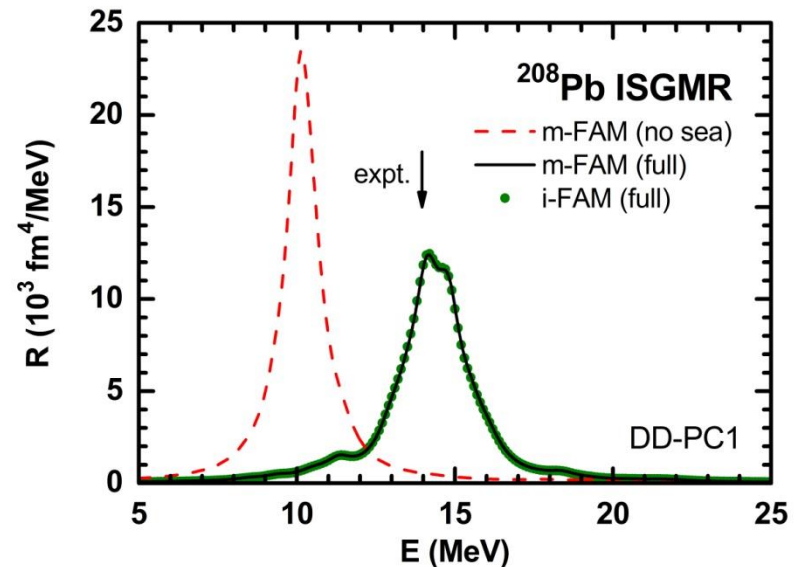
## Computational challenge for the excitations in deformed systems

A promising solution:

Finite amplitude method Nakatsukasa, Inakura, Yabana, *PRC*76, 024318 (2007)



(Left: data from Avogadro:2013)



HZL, Nakatsukasa, Niu, Meng, *PRC*87, 054310 (2013)

- ◆ The combination CDFT + FAM works well for the spherical and also the axially deformed systems. Niksic *et al.*, *PRC*88, 044327 (2013)
- ◆ To develop CDFT + FAM for the 3D deformed systems.

# Summary and Perspectives

- ✓ **Self-consistent and covariant descriptions of nuclear spin-isospin excitations for spherical cases.**

spin-isospin resonances,  $r$ -process, CKM unitarity, ....

Towards deformed cases: CDFT on 3D mesh

- **Ground states: Solving Dirac equations on 3D mesh**

inverse Hamiltonian method, high-order Wilson terms

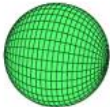
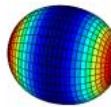
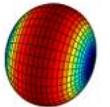
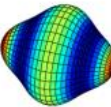
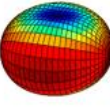
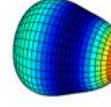
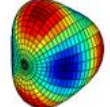
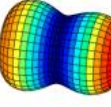
- **Excitations: Finite amplitude method for relativistic RPA**

In mesh representation, the effects of Dirac sea can be included implicitly and automatically.

Exotic shapes

Exotic excitation modes

.....

$\beta_{\lambda\mu} = 0$	$\beta_{20} > 0$	$\beta_{20} < 0$	$\beta_{40} > 0$
			
$\beta_{22} \neq 0$	$\beta_{30} \neq 0$	$\beta_{32} \neq 0$	$\beta_{20} \gg 0$
			



# Acknowledgments

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<b>RISP</b>	Youngman Kim, Yeunhwan Lim
<b>Peking U.</b>	Jie Meng, Shihang Shen
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<b>Zagreb U.</b>	Tamara Nikšić, Dario Vretenar
<b>Anhui U.</b>	Zhongming Niu
<b>TU München</b>	Peter Ring
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