Recent Progress of Spin-Isospin Excitations in Nuclei

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Two topics
- Gamow-Teller Collective states in $^8\text{He}(p,n)^8\text{Li}$ and $^{12}\text{Be}(p,n)^{12}\text{B}$
- spin-isospin SU(4) symmetry in N=Z+2 nuclei in pf shell nuclei
Spin-isospin physics: Gamow-Teller responses

Progress in Last century

- 1963 GT giant resonance predicted, GT(Ikeda) sum rule $3(N-Z)$
  collectivity?
- ~1980 GT giant resonances established
- Strength quenched/missing: 50-60% of $3(N-Z)$ due to $\Delta$-h or 2p-2h?
- 1997 ~90% of $3(N-Z)$ found (2p-2h dominance)

- Charge-exchange reactions on stable target nuclei
- CHEX reactions: $(p,n)/(n,p)$ and $(^3\text{He},t)/(t,^3\text{He})$ reactions at intermediate energy


Wakasa et al., PR C55, 2909 (1997)

GT strength quenching problem

- Wakasa et al., PR C 55, 2909 (1997)
This century

- **Unstable** beams → extend the horizon of spin-isospin responses
- Charge-exchange reactions in inverse kinematics

Gamow-Teller giant resonance under extreme condition

1. Spin-isospin correlations in $N \gg Z$ nuclei
2. Quenching of Spin-orbit interaction (tensor correlations)
3. Coupling to the continuum
4. Isoscalar spin-triplet pairing in $N \sim Z$ nuclei

Recent observations at $(N-Z)/A$ extreme at RIKEN/CNS

**Gamow-Teller Giant Resonances in very light neutron rich nuclei, $^8$He & $^{12}$Be**
Spin-isospin correlations in schematic model

- GTGR (IAS) induced by $ph$ residual interaction:
  \[ V_{12} = \kappa_{\sigma\tau} \bar{\sigma}_1 \bar{\sigma}_2 \tau_1 \tau_2 \quad (\kappa_{\tau} \tau_1 \tau_2) \]

- Dispersion relation for the collective state (GTGR)
  \[ \frac{\sqrt{2i(N_j|Z|\epsilon_f^r/|N_j| - Z|\epsilon_f^i/|N_j|})^2}{\epsilon_i - \epsilon + \epsilon_f - \Delta_{\ell s} - \epsilon} = \frac{1}{\kappa_{\sigma\tau}} \]

- Nakayama et al., PLB114 (1982) 217

- C. Garrde, NPA396 (1982) 127c.

\( (N-Z)/A < 0.21 \) was missing in 20th century

\( E_{GT} - E_{IAS} > 0 \)

\( E_{GT} - E_{IAS} = \Delta_{ls} + 2 \left( \frac{\kappa_{GT} - \kappa_F}{A} \right) (N - Z) \)

\( (\kappa_F - \kappa_{GT})A = 9.25 \text{ MeV} \)

Data:

one-p-\( h \) configuration for GT
Predicted in 1993 by Sagawa-Hamamoto-Ishihara, PLB303,215

Hartree-Fock + RPA (TDA) calculation with (BKN+spin-orbit \((^{16}\text{O})\))

- Relatively large \(E_{GT} - E_{IAS} < 0\)
- \(8\text{He} : E_{GT} - E_{IAS} = -4.5\text{ MeV} (f=0.44)\)

\[
f = \frac{B(GT j_\uparrow \to j_\downarrow)}{B(GT j_\uparrow \to j_\downarrow) + B(GT j_\downarrow \to j_\uparrow)} = \frac{4}{9} = 0.44 \quad l = 1 \\
= \frac{8}{15} = 0.53 \quad l = 2 \\
= \frac{12}{21} = 0.57 \quad l = 3 \\
= \frac{16}{27} = 0.59 \quad l = 4
\]
GT responses in very neutron rich light nuclei

- Target nuclei: $^8\text{He}$ and $^{12}\text{Be}$
  \((N-Z)=4\)
- Large neutron to proton ratio
  \(-\frac{(N-Z)}{A} = 0.33^{(12}\text{Be}), 0.5^{(8}\text{He})\)
- (p,n) reaction in inverse kinematics
- $^8\text{He}(p,n)$ by Kobayashi et al.,
- $^{12}\text{Be}(p,n)$ by Yako et al.,
- $^8\text{He}(p,n)$ at 200 MeV/u (Kobayashi)

\[ E_{\text{GT}} - E_{\text{IAS}} = -2.5 \pm 0.5 \text{ MeV} \]

- $^{12}\text{Be}(p,n)$ at 200 MeV/u (Yako)

\[ E_{\text{GT}} - E_{\text{IAS}} = -1.2 \pm 0.4 \text{ MeV} \]
Shell structure characteristics

- p-shell dominance for $^8\text{He}$
- Deformation or 2p-2h state mixing in the ground state in $^{12}\text{Be}$

Mixing of 2s1d configurations in $^{12}\text{Be}$

SFO' interaction (2s1/2 s.p.e. is lowered to obtain $B(\text{GT:}^{12}\text{Be} \rightarrow ^{12}\text{B}(\text{g.s.}))$:

- Large 2hw excitation components in the ground state

$|^{12}\text{Be}> = 0.55|p^8> + 0.82|p^6(sd)^2>$

Occupation probabilities of neutrons (# of particles)
- p-orbits: 4.61
- s-orbit: 0.68
- d-orbit: 0.71
TABLE I: Calculated $E_{GT} - E_{IAS}$ values for $\kappa_{\sigma\tau} = \frac{21}{A}$, $\frac{22}{A}$ and $\frac{23}{A}$ MeV with several assumed neutron-orbit configurations for $^8$He and $^{12}$Be together with experimental values. For comparison purpose, the results for $^{208}$Pb is also given. In all calculations, $\kappa_{\tau} = \frac{28}{A}$ MeV is assumed.

<table>
<thead>
<tr>
<th>$\kappa_{\sigma\tau}$ (MeV)</th>
<th>$\frac{21}{A}$</th>
<th>$\frac{22}{A}$</th>
<th>$\frac{23}{A}$</th>
<th>adopted $\nu$ configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^8$He</td>
<td>-3.01</td>
<td>-2.03</td>
<td>-1.16</td>
<td>$(1p_{3/2})^4$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Exp. $-2.5 \pm 0.5$ [10]</td>
</tr>
<tr>
<td>$^{12}$Be</td>
<td>-2.20</td>
<td>-1.58</td>
<td>-0.95</td>
<td>$(1p_{3/2})^4(1p_{1/2})^2$</td>
</tr>
<tr>
<td></td>
<td>+0.96</td>
<td>+1.75</td>
<td>+2.55</td>
<td>$(1p_{3/2})^4(2s_{1/2})^2$</td>
</tr>
<tr>
<td></td>
<td>+0.09</td>
<td>+0.73</td>
<td>+1.37</td>
<td>$(1p_{3/2})^4(2d_{5/2})^2$</td>
</tr>
<tr>
<td></td>
<td>-1.55</td>
<td>-0.91</td>
<td>-0.26</td>
<td>SFO configuration [20]</td>
</tr>
<tr>
<td></td>
<td>-1.73</td>
<td>-1.10</td>
<td>-0.46</td>
<td>WBP’ configuration [22]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Exp. $-1.2 \pm 0.4$ [11]</td>
</tr>
<tr>
<td>$^{208}$Pb</td>
<td>-0.29</td>
<td>+0.10</td>
<td>+0.50</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Exp. $+0.4 \pm 0.2$ [7]</td>
</tr>
</tbody>
</table>
TABLE II: Calculated results of excitation energies of GT and IAS and B(GT) values in $^8\text{Li}$ and $^{12}\text{B}$. The $E_{\text{IAS}}$ values for $^8\text{Li}$ and $^{12}\text{B}$ are taken from [15] and [16], respectively.

<table>
<thead>
<tr>
<th></th>
<th>$^8\text{Li}$</th>
<th>$^{12}\text{B}$</th>
<th>$E_{\text{GT}}$ (MeV)</th>
<th>$E_{\text{IAS}}$ (MeV)</th>
<th>$\Delta E$ (MeV)</th>
<th>$B(\text{GT})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0+2)$\hbar\omega$</td>
<td>(8-16)POT</td>
<td>7.5</td>
<td>11.7</td>
<td>-4.2</td>
<td>10.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(6-16)2BME</td>
<td>8.3</td>
<td>11.1</td>
<td>-2.8</td>
<td>9.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>SFO</td>
<td>7.8</td>
<td>12.1</td>
<td>-4.3</td>
<td>8.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>WBT'</td>
<td>5.9</td>
<td>10.8</td>
<td>-4.9</td>
<td>5.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>SFO(6-16)</td>
<td>8.2</td>
<td>11.1</td>
<td>-2.9</td>
<td>8.3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Eq.(5)[9]</td>
<td>-</td>
<td>-</td>
<td>-7.5</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td></td>
<td>SHI[2]</td>
<td>9.0</td>
<td>13.7</td>
<td>-4.7</td>
<td>9.4</td>
<td></td>
</tr>
<tr>
<td>(0+2)$\hbar\omega$</td>
<td>$^8\text{He}(\beta^-)[23]$</td>
<td>~9</td>
<td>10.8</td>
<td>-1.8</td>
<td>~3.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(p,n) exp.[10, 12]</td>
<td>$8.3\pm0.5$</td>
<td>10.8</td>
<td>$-2.5\pm0.5$</td>
<td>$(8 \pm 4)$</td>
<td></td>
</tr>
</tbody>
</table>

\[ S_{\beta^-} - S_{\beta^+} = \langle i | 2T_2 \cdot 3 | i \rangle = 3(N - Z) \]

=12 for $^8\text{He}$
<table>
<thead>
<tr>
<th>$^{12}\text{B}$</th>
<th>$E_{GT}$ (MeV)</th>
<th>$E_{IAS}$ (MeV)</th>
<th>$\Delta E$ (MeV)</th>
<th>$B$(GT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(8-16)POT</td>
<td>11.0</td>
<td>13.8</td>
<td>$-2.8$</td>
<td>9.3</td>
</tr>
<tr>
<td>(6-16)2BME</td>
<td>12.3</td>
<td>14.4</td>
<td>$-2.1$</td>
<td>7.4</td>
</tr>
<tr>
<td>SFO</td>
<td>11.6</td>
<td>13.8</td>
<td>$-2.2$</td>
<td>8.9</td>
</tr>
<tr>
<td>WBT’</td>
<td>9.5</td>
<td>13.2</td>
<td>$-3.7$</td>
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<tr>
<td>Eq.(5)[9]</td>
<td>—</td>
<td>—</td>
<td>$-2.5$</td>
<td>—</td>
</tr>
<tr>
<td>$(p,n)$ exp.[11]</td>
<td>$11.5\pm0.4$</td>
<td>$12.7$</td>
<td>$-1.2\pm0.4$</td>
<td>$(10\pm2)$</td>
</tr>
</tbody>
</table>

$$S_{\beta^-} - S_{\beta^+} = \langle i | 2T_z \cdot 3 | i \rangle = 3(N - Z)$$

$=12$ for $^{12}\text{Be}$
Nakayama fitting eq.

$$GT_{IAS} = 7.0 - 57.8 T_0 / A$$
Pairing interactions and Spin-Isospin response

T=1 pairing (n-n, p-p pairing correlations) $\Rightarrow$ T=1 superfluidity

- mass (odd-even staggering)
- energy spectra (gap between the first excited state and the ground state in even-even nuclei)
- moment of inertia
- n-n or p-p Pair transfer reactions
- fission barrier (large amplitude collective motion)
T=1 S=0 pairing and T=0 S=1 pairing interactions

T=1 pairing (n-n, p-p pairing correlations)  ➔ T=1 superfluidity
- mass (odd-even staggering)
- energy spectra (gap between the first excited state and the ground state in even-even nuclei)
- moment of inertia
- n-n or p-p Pair transfer reactions
- fission barrier (large amplitude collective motion)

T=0 pairing (p-n pairing with S=1)  ➔ S=1 superfluidity
- N=Z Wigner energy (still controversial)
- Energy spectra in nuclei with N=Z (T=0 and J=J_{\text{max}})
- n-p pair transfer reaction
- Super-allowed Gamow-Teller transition between SU(4) supermultiples (C.L. Bai et al.)
Two particle systems

T=1, S=0 pair

\[ |(L = S = 0)J = 0, T = 1\rangle \quad |(j = j')J = 0, T = 1\rangle \]

T=0, S=1 pair

\[ |(L = 0, S = 1)J = 1, T = 0\rangle \]

\[ a |(l = l', j = j')J = 1, T = 0\rangle + b |(l = l', j, j' = j ± 1)J = 1, T = 0 \rangle \]

If there is strong spin orbit splitting, it is difficult to make (T=0, S=1) pair.

\[ \Rightarrow \text{two kinds of superfluidity?} \]

But, T=0 J= 1\(^+\) state could be Gamow-Teller states in nuclei with N~Z

\[ \Rightarrow \text{strong GT states in N=Z+2 nuclei} \]

SU(4) supermultiplet in spin isospin space

Well-known in light p-shell nuclei (LS coupling dominance)
Low-Energy Collective Gamow-Teller States and Isoscalar Pairing Interaction

C.L. Bai\textsuperscript{1)}, H. Sagawa\textsuperscript{2,3)}, G. Colo\textsuperscript{4)}, Y. Fujita\textsuperscript{5,6)}, H.Q. Zhang\textsuperscript{7,8)}, X.Z. Zhang\textsuperscript{7),} and F.R. Xu\textsuperscript{8)}

HFB+QRPA with $T=1$ and $T=0$ pairing
$T=1$ pairing in HFB
$T=0$ pairing in QRPA

$$\hat{O}(GT) = \pm$$

, and are generators of SU(4)

Does Supermultiplet Wigner SU(4) symmetry revive in pf shell?
(E. Wigner 1937, F. Hund 1937)
$(T=1, S=0) \rightarrow (T=0, S=1)$ GT transition is allowed and enhanced.

$$V_{T=1}(r_1, r_2) = V_0 \frac{1 - P_\sigma}{2} \left(1 - \frac{\rho(r)}{\rho_\sigma}\right) \delta(r_1 - r_2), \quad (1)$$

$$V_{T=0}(r_1, r_2) = fV_0 \frac{1 + P_\sigma}{2} \left(1 - \frac{\rho(r)}{\rho_\sigma}\right) \delta(r_1 - r_2), \quad (2)$$
Gamow-Teller transitions in $N=Z+2$ $pf$ nuclei

\[ \hat{O}(GT) = \pm \]

**Supermultiplet**: Wigner SU(4) symmetry

$\langle T=1, S=0 \rangle \rightarrow \langle T=0, S=1 \rangle$ GT transition is allowed and enhanced.
$^{42}\text{Ca}(^{3}\text{He},t)^{42}\text{Sc}$

$E=140$ MeV/nucleon

$\Theta=0^\circ$

$N=Z+2$

Y. Fujita et al., PRL112, 112502 (2014)
Spin-spin interaction is strongly repulsive and induces higher energy IAS collective Gamow-Teller states. Spin-triplet T=0 pairing is attractive and shifts down GT strength near IAS.


HFB+QRPA with T=1 and T=0 pairing
  T=1 pairing in HFB
  T=0 pairing in QRPA
$^{46}\text{Ti}(^3\text{He},t)^{46}\text{V}$

$^{50}\text{Cr}(^3\text{He},t)^{50}\text{Mn}$

$^{54}\text{Fe}(^3\text{He},t)^{54}\text{Co}$
Summary

1. Collectivity enhancement in N>>Z nuclei.
2. Role of T=0 pairing is studied in Gamow-Teller transitions of N=Z+2 nuclei.
3. It is pointed out the enhancement of GT strength in the low energy peak just above IAS state is due to the strong T=0 pairing correlations in the final states.

→ Revive of Supermultiplets of (T=1,S=0) and (T=0 ,S=1) pairs in pf shell nuclei

4. A cooperative effect of T=0 pairing and tensor interactions are found in nuclei at the middle of pf shell.
5. The T=0 pairing strength is determined to be almost the same strength as the T=1 pairing from the observed relative energies of IAS and the low-energy GT states.
6. Energy spectra and M1 transitions in odd-odd N=Z nuclei show a manifestation of strong T=0 pairing correlations.
To establish firmly the collectivity in very neutron rich light nuclei, the measurements of GTGR as well as IAS in the neutron drip line nuclei such as $^{14}$Be ($(N - Z)/A=0.429$), $^{20,22}$C ($(N - Z)/A=0.400, 0.455$) and $^{24}$O ($(N - Z)/A=0.333$) are of highly desired.

1. What happens on spin-orbit splitting due to the existence of more neutrons?
2. Coupling to the continuum?
3. Deformation vs. 2hw and 4hw mixings?
4. RPA or shell model (IAS: self consistency is important)
Existence of Gamow-Teller giant resonance


IAS is predicted higher than GTGR in energy! Isospin consideration.

Anderson and Wong, 1961

\[ [H, \sigma_T] \neq 0 \]

\[ [H, T_z] \approx 0 \]
Super-allowed transitions

Transitions between states which are well approximated by Wigner super-multiplet scheme, and the spatial symmetry is conserved. Both Fermi and Gamow-Teller transitions

Example: $^6\text{He} \rightarrow ^6\text{Li}$

$B(\text{GT})_{\text{exp}} = 4.75$

$B(\text{GT})_{\text{cal}} = 5.59$

**Sum-rule value** = 6.00

$\beta^-$

Deuteron on $^4\text{He}$
Other candidates

Target: two neutrons on an $LS$-closed core, sum-rule = 6.00

From $^{18}\text{O} (p,n) ^{18}\text{F}$

$B(\text{GT}) = 3.23$

From $^{42}\text{Ca} (p,n) ^{40}\text{Ca}$

$\theta = 0^\circ$

$E_p = 160$ MeV

$B(\text{GT}) = 2.67$

The lowest $1^+$ state exhausts about 90% of the observed GT strength.
Particle-particle vs particle-hole

Particle-particle (hole-hole) is attractive. Particle-hole is repulsive.

\[ ^{42}\text{Ca} \rightarrow ^{42}\text{Sc} \]

\[ ^{48}\text{Ca} \rightarrow ^{48}\text{Sc} \]

\[ ^{54}\text{Fe} \rightarrow ^{54}\text{Co} \]

Particle-particle

\[ f_{5/2} \]

\[ f_{7/2} \]

\[ ^{42}\text{Sc} \ 1^+ \]

\[ ^{48}\text{Sc} \ 1^+ \]

\[ ^{54}\text{Co} \ 1^+ \]
A super-allowed GT transition to a high-lying state?

M.J.G. Borge et al. (ISOLDE Collaboration)

The Wigner super-multiplet scheme could be good in p-shell nuclei.

Table 2. Strong Gamow-Teller beta-decay branches of drip-line nuclei

<table>
<thead>
<tr>
<th>Decay</th>
<th>Level energy (MeV)</th>
<th>Level width (MeV)</th>
<th>Emitted particle</th>
<th>$B_{GT}$</th>
<th>log $ft$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^6\text{He} \rightarrow ^6\text{Li}$</td>
<td>0.0</td>
<td>Stable</td>
<td>$-$</td>
<td>4.75</td>
<td>2.91</td>
</tr>
<tr>
<td>$^8\text{He} \rightarrow ^8\text{Li}$</td>
<td>$\sim$ 9</td>
<td>$\sim$ 1</td>
<td>$n, t$</td>
<td>3.14</td>
<td>3.09</td>
</tr>
<tr>
<td>$^9\text{Li} \rightarrow ^9\text{Be}$</td>
<td>11.81</td>
<td>0.40</td>
<td>$n, \alpha$</td>
<td>5.6</td>
<td>2.84</td>
</tr>
<tr>
<td>$^{11}\text{Li} \rightarrow ^{11}\text{Be}^a$</td>
<td>$\sim$ 18.5</td>
<td>$\sim$ 0.5</td>
<td>$xn, t$</td>
<td>$&gt; 0.5$</td>
<td>$&lt; 4$</td>
</tr>
</tbody>
</table>

$^a$ Only the triton branch is included in the calculation of $B_{GT}$
**SUMMARY**

The SU(4) super-multiplet scheme is best realized in \( p \)-shell nuclei. Super-allowed GT transitions are expected not only to the lowest state in \( N \sim Z \) nuclei but also to high-lying states in neutron-rich nuclei.

<table>
<thead>
<tr>
<th>Decay</th>
<th>Level energy (MeV)</th>
<th>Level width (MeV)</th>
<th>Emitted particle</th>
<th>( B_{GT} )</th>
<th>( \log f t )</th>
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Isoscalar Spin-Triplet Pairing correlations and Spin-Isospin Response

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Low-Energy Collective Gamow-Teller States and Isoscalar Pairing Interaction

C.L. Bai¹, H. Sagawa²,³, G. Colè⁴, Y. Fujita⁵,⁶, H.Q. Zhang⁷,⁸, X.Z. Zhang⁷, and F.R. Xu⁸

PRC(2014) in press.
T=1, S=0 pair

\[ |(L = S = 0)J = 0, T = 1\rangle \]

T=0, S=1 pair

\[ |(L = 0, S = 1)J = 1, T = 0\rangle \]

How we can disentangle in quantum many-body systems.  
\[ \Rightarrow \text{two kinds of superfluidity?} \]
Gamow-Teller transitions in $N=Z+2$ nuclei

$p$-$h$ type excitation

Fermi energy

$\hat{O}(GT) = \pm$

Supermultiplet: Wigner SU(4) symmetry

$(T=1, S=0) \rightarrow (T=0, S=1)$ GT transition is allowed and enhanced.
Gamow-Teller transitions in N=Z+2 nuclei

Fermi energy

T=0 S=1

p-p type excitation

BCS vacuum

\[ v^2 \]

protons

neutrons

\[ ^{42}\text{Ca} \]
Results of MDA for $^{90}\text{Zr}(p,n)$ & $(n,p)$ at 300 MeV

T. Wakasa et al., PRC 55, 2909 (1997) \quad K. Yako et al., PLB 615, 193 (2005)

- Multipole Decomposition (MD) Analyses
  - $(p,n)/(n,p)$ data have been analyzed with the same MD technique
  - $(p,n)$ data have been re-analyzed up to 70 MeV

- Results
  - $(p,n)$
    - Almost L=0 for GTGR region (No Background)
    - Fairly large L=0 (GT) strength up to 50 MeV excitation
  - $(n,p)$
    - L=0 strength up to 30 MeV
Model-independent sum rule: GT(Ikeda) sum rule

\[ S_{\beta^-} - S_{\beta^+} = \frac{1}{2J_i + 1} \sum_f | \langle f || \sum_{i=1}^{A} t_-(i) \sigma_i || i \rangle |^2 \]

\[ - \frac{1}{2J_i + 1} \sum_f | \langle f || \sum_{i=1}^{A} t_+(i) \sigma_i || i \rangle |^2 \]

\[ = \langle i | \sum_{i,j=1}^{A} (t_+(j)t_-(i) - t_-(i)t_+(j)) \sigma_i \cdot \sigma_j | i \rangle \]

\[ [t_+(j), t_-(i)] = \delta_{ij} 2t_z(i), \quad \sum_{i=1}^{A} 2t_z(i) = 2T_z \quad \sigma_i \cdot \sigma_i = 3 \]

\[ S'_{\beta^-} - S'_{\beta^+} = \langle i | 2T_z \cdot 3 | i \rangle = 3(N - Z) \]

\[ = 12 \text{ for } ^8\text{He and } ^{12}\text{Be} \]

cf: Fermi transition

\[ S_{F^-} - S_{F^+} = \langle i | 2T_z | i \rangle = N - Z \]