Quark and Glue Spins of the Nucleon

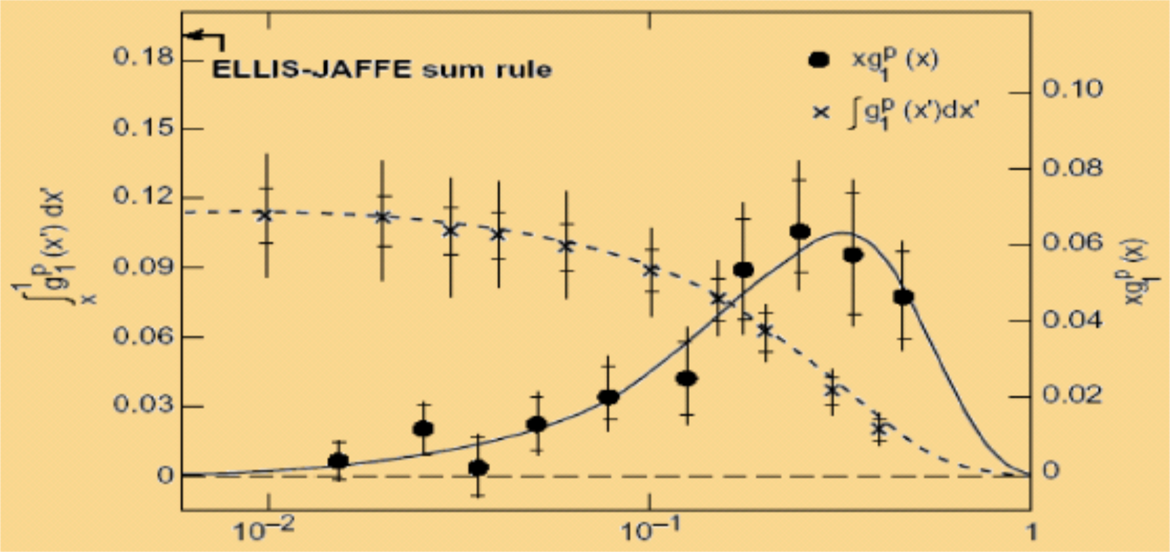
- Status of nucleon spin components
- Gauge field tensor operator
- Momentum and angular momentum sum rules
- Lattice results
- Quark spin from anomalous Ward identity
- Glue spin

SPIN2014, PKU,Oct. 21, 2014

Where does the spin of the proton come from?

Twenty^5 years since the "spin crisis"

❑ **EMC experiment in 1988/1989 – "the plot":**

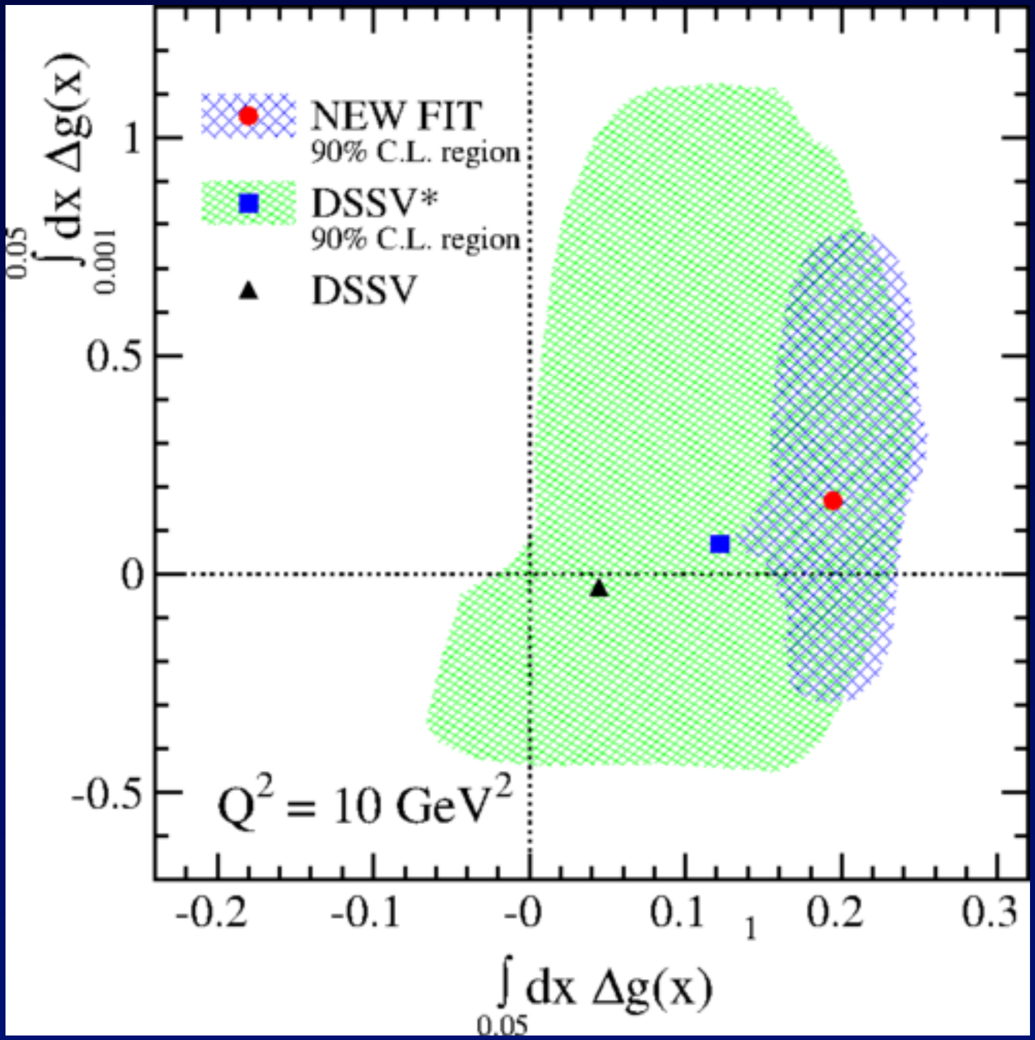


$$g_1(x) = \frac{1}{2}\sum_q e_q^2\left[\Delta q(x) + \Delta\bar{q}(x)\right] + \mathcal{O}(\alpha_s) + \mathcal{O}(1/Q)$$

$$\Delta q = \int_0^1 dx \Delta q(x) = \langle P, s_{||}|\overline{\psi}_q(0)\gamma^+\gamma_5\psi_q(0)|P, s_{||}\rangle$$

❑ **"Spin crisis" or puzzle:** $\Delta\Sigma = \sum_q \Delta q + \Delta\bar{q} = 0.2 - 0.3$

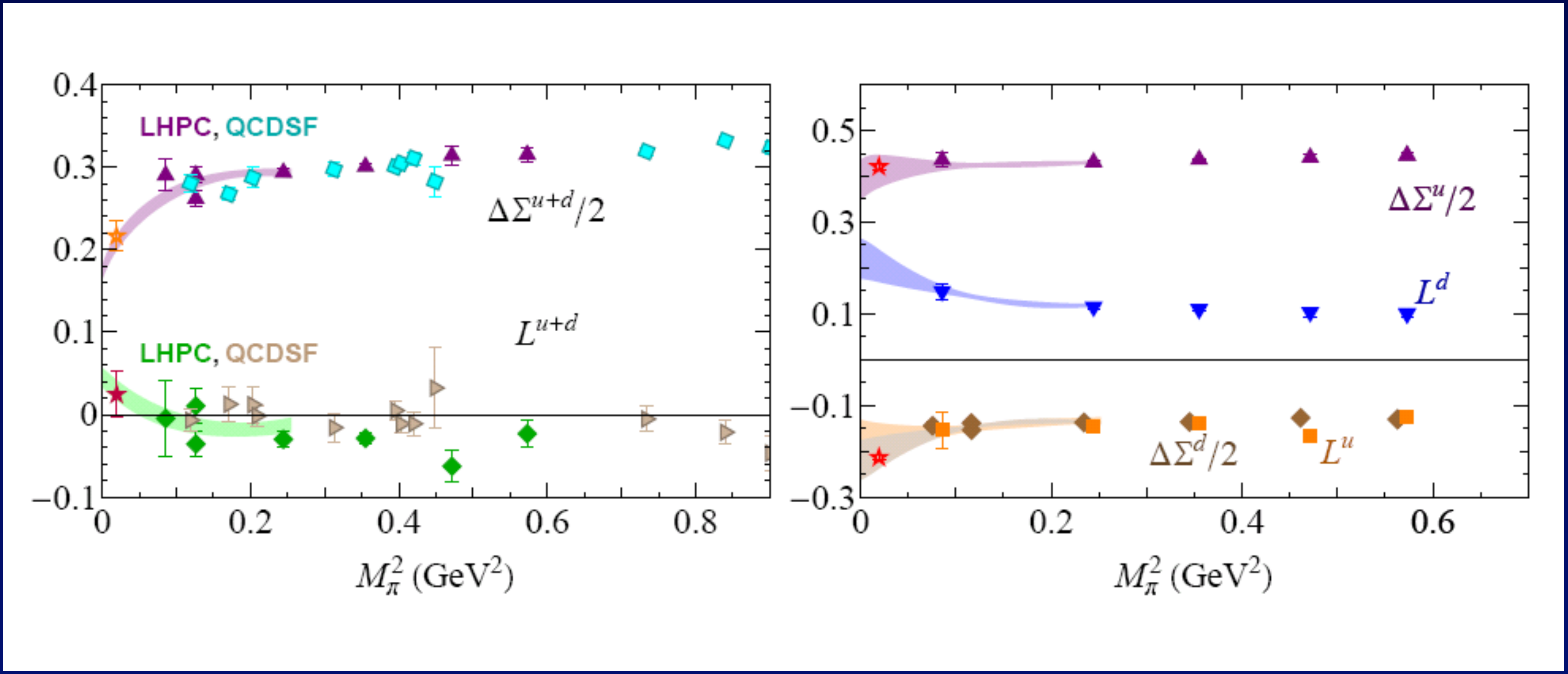
Glue Polarization ΔG



Experimental results from
STAR [1404.5134]
PHENIX [1402.6296]
COMPASS [1001.4654]

D. de Florian, R. Sassot, M. Stratmann, W. Vogelsang, PRL 113, 012001 (2014)

Quark Orbital Angular Momentum (connected insertion)



LHPC, S. Syritsyn et al., [111.0718]
QCDSF, A. Sternbeck et al, [1203.6579]

Status of Proton Spin

- Quark spin $\Delta\Sigma \sim 20 - 30\%$ of proton spin (DIS, Lattice)

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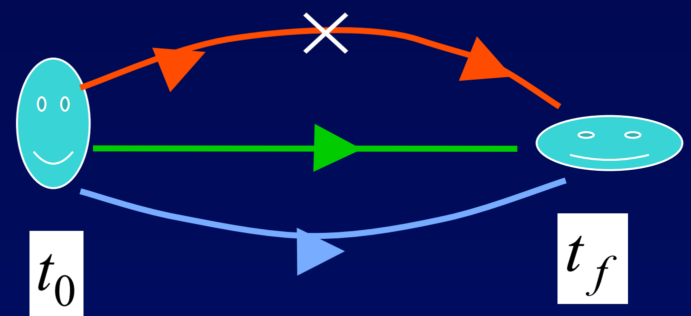
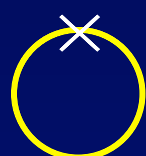
→ Dark Spin ?

Hadron Structure with Quarks and Glue

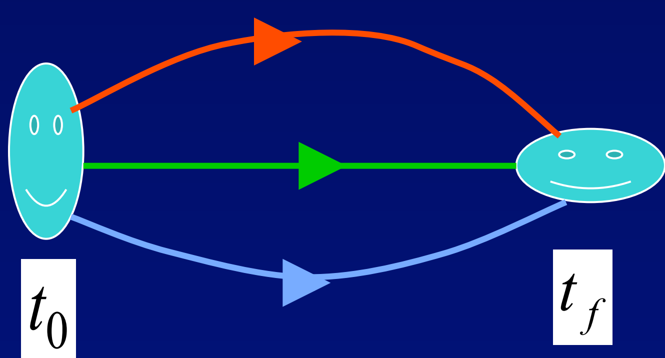
- Quark and Glue Momentum and Angular Momentum in the Nucleon

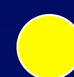
$$(\bar{u}\gamma_\mu D_\nu u + \bar{d}\gamma_\mu D_\nu d)(t)$$



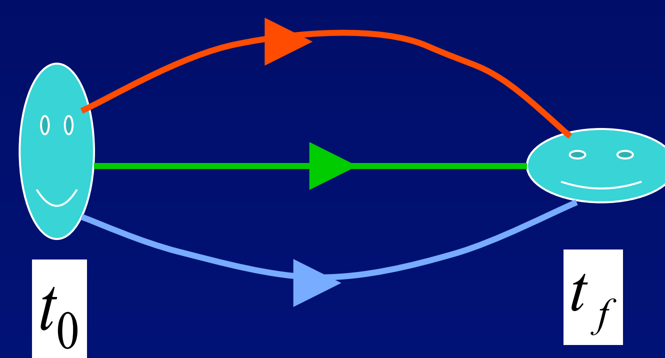


$$\bar{\Psi}\gamma_\mu D_\nu \Psi(t)(u,d,s)$$





$$F_{\mu\alpha}F_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}F^2$$



Momenta and Angular Momenta of Quarks and Glue

- Energy momentum tensor operators decomposed in quark and glue parts gauge invariantly --- Xiangdong Ji (1997)

$$T^q_{\mu\nu} = \frac{i}{4}\left[\bar{\psi}\gamma_\mu \overleftrightarrow{D}_\nu \psi + (\mu \leftrightarrow \nu)\right] \rightarrow \vec{\mathrm{J}}_q = \int d^3x \left[\frac{1}{2}\bar{\psi}\vec{\gamma}\gamma_5\psi + \vec{x}\times\bar{\psi}\gamma_4(-i\vec{D})\psi\right]$$

$$T^g_{\mu\nu} = F_{\mu\lambda}F_{\lambda\nu} - \frac{1}{4}\delta_{\mu\nu}F^2 \rightarrow \vec{\mathrm{J}}_g = \int d^3x\left[\vec{x}\times(\vec{E}\times\vec{B})\right]$$

- Nucleon form factors

$$\left\langle p,s \,|\, T_{\mu\nu} \,|\, p's'\right\rangle = \bar{u}(p,s)[T_1(q^2)\gamma_\mu \bar{p}_\nu - T_2(q^2)\bar{p}_\mu \sigma_{\nu\alpha} q_\alpha / 2m$$

$$-\mathrm{i}T_3(q^2)(q_\mu q_\nu - \delta_{\mu\nu}q^2)/m + T_4(q^2)\delta_{\mu\nu} m/2]u(p's')$$

- Momentum and Angular Momentum

$$Z_{q,g} T_1(0)_{q,g} \;\; \left[\mathrm{OPE}\right] \rightarrow \left\langle x \right\rangle_{q/g} (\mu, \overline{\mathrm{MS}}) \,, \quad Z_{q,g}\left[\frac{T_1(0)+T_2(0)}{2}\right]_{q,g} \rightarrow J_{q/g}(\mu, \overline{\mathrm{MS}})$$

Renormalization and Quark-Glue Mixing

Momentum and Angular Momentum Sum Rules

$$\langle x\rangle_q^R = Z_q \langle x\rangle_q^L, \quad \langle x\rangle_g^R = Z_g \langle x\rangle_g^L,$$

$$J_q^R = Z_q J_q^L, \quad J_g^R = Z_g J_g^L,$$

$$Z_q \langle x\rangle_q^L + Z_g \langle x\rangle_g^L = 1,$$

$$Z_q J_q^L + Z_g J_g^L = \frac{1}{2} \quad \Rightarrow \begin{cases} Z_q T_1^q(0) + Z_g T_1^g(0) = 1, \\ Z_q (T_1^q + T_2^q)(0) + Z_g (T_1^g + T_2^g)(0) = 1, \\ Z_q T_2^q(0) + Z_g T_2^g(0) = 0 \end{cases}$$

Mixing

$$\begin{bmatrix} \langle x\rangle_q^{\overline{MS}}(\mu) \\ \langle x\rangle_g^{\overline{MS}}(\mu) \end{bmatrix} = \begin{bmatrix} C_{qq}(\mu) & C_{qg}(\mu) \\ C_{gq}(\mu) & C_{gg}(\mu) \end{bmatrix} \begin{bmatrix} \langle x\rangle_q^R \\ \langle x\rangle_g^R \end{bmatrix}$$

M. Glatzmaier, KFL
PRD arXiv:1403.7211

Gauge Operators from the Overlap Dirac Operator

- Overlap operator

$$D_{ov} = 1 + \gamma_5 \varepsilon(H); \quad H = \gamma_5 D_W(m_0)$$

- Index theorem on the lattice (Hasenfratz, Laliena, Niedermayer, Lüscher)

$$\text{index } D_{ov} = -Tr\gamma_5 (1 - \frac{a}{2} D_{ov})$$

- Local version (Kikukawa & Yamada, Adams, Fujikawa, Suzuki)

$$q_L(x) = -tr\gamma_5 (1 - \frac{a}{2} D_{ov}(x,x)) \xrightarrow[a \to 0]{} a^4 q(x) + O(a^6)$$

- Study of topological structure of the vacuum
  - Sub-dimensional long range order of coherent charges (Horvàth et al; Thacker talk in Lattice 2006)
  - Negativity of the local topological charge correlator (Horvàth et al)

- We obtain the following result

$$\mathbf{tr}_s \sigma_{\mu\nu} a D_{ov}(x,x) = c^T a^2 F_{\mu\nu}(x) + O(a^3),$$

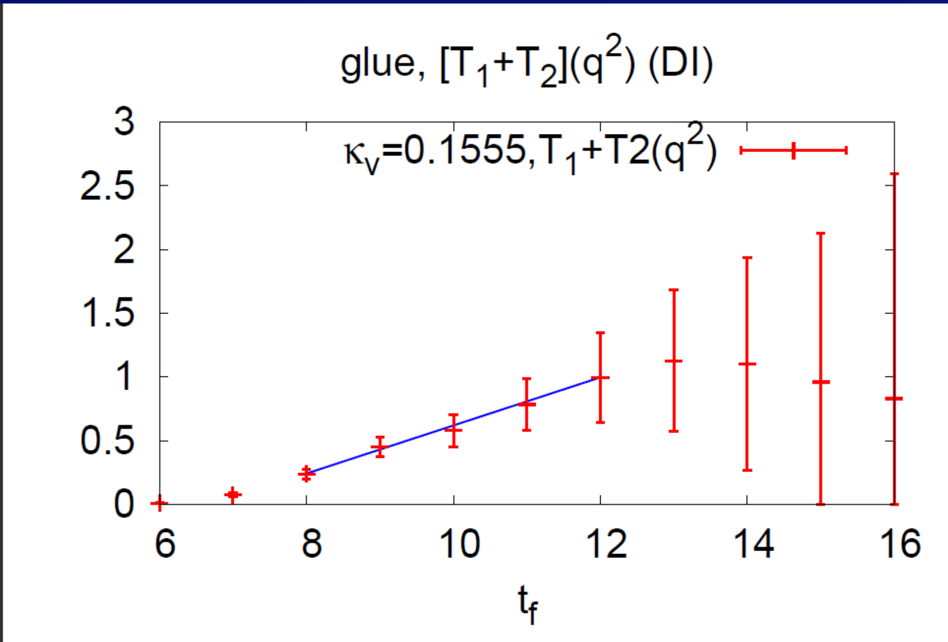
$$c^T = \rho \int_{-\pi}^{\pi} \frac{d^4k}{(2\pi)^4} \frac{2\left[(\rho + r\sum_{\lambda}(c_\lambda - 1))c_\mu c_\nu + 2rc_\mu s_\nu^2\right]}{(\sum_\mu s_\mu^2 + [\rho + \sum_\nu (c_\nu - 1)]^2)^{3/2}}$$

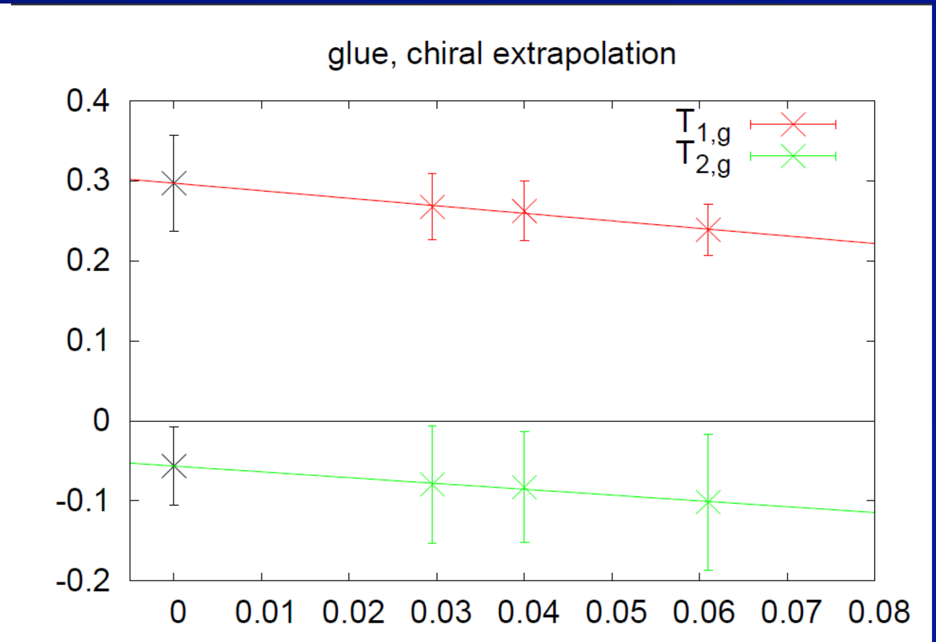
where, $r = 1, \quad \rho = 1.368, \quad c^T = 0.11157$

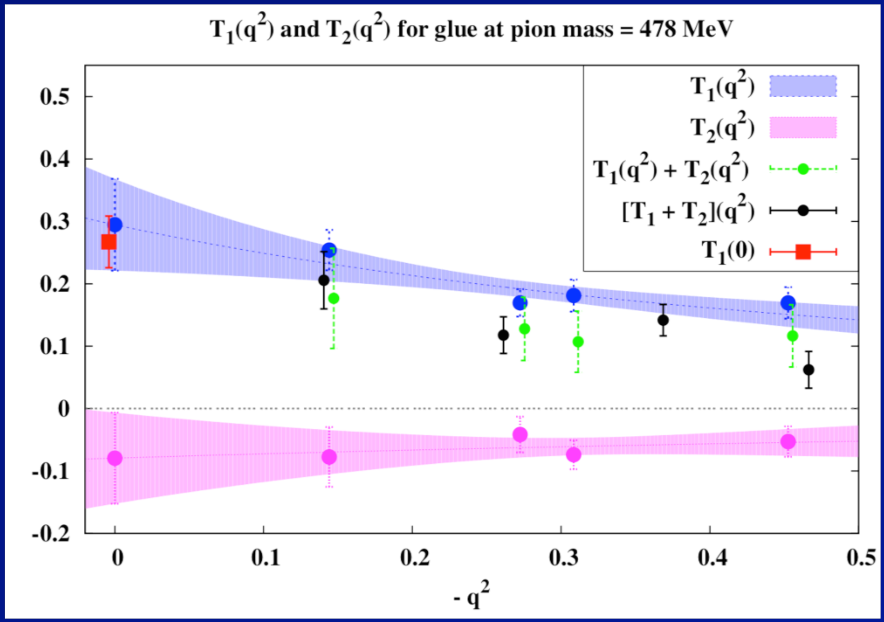
Liu, Alexandru, Horvath – PLB 659, 773 (2007)

- Noise estimation $D_{ov}(x,x) \to \left\langle \eta_x^\dagger (D_{ov}\eta)_x \right\rangle$
  with Z_4 noise with color-spin dilution and some dilution in
  space-time as well.

Glue $T_1(q^2)$ and $T_2(q^2)$







M. Deka et al., 1312.4816

(χ QCD Collaboration)

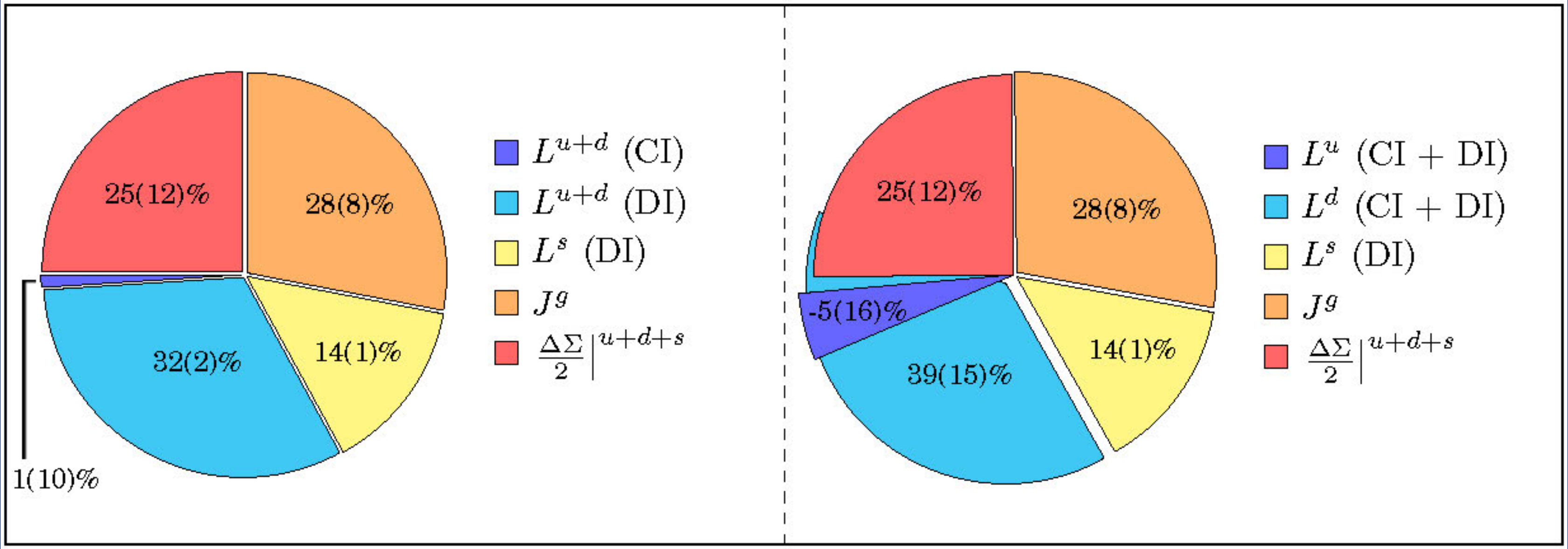
Quenched $16^3 \times 24$ lattice, β=6.0, $m_\pi \geq 478$ MeV, 500 configurations

Renormalized results: $Z_q = 1.05$, $Z_g = 1.05$

	CI(u)	CI(d)	CI(u+d)	DI(u/d)	DI(s)	Glue
<x>	0.416 (40)	0.151 (20)	0.567 (45)	0.037 (7)	0.023 (6)	0.334 (56)
$T_2(0)$	0.283 (112)	-.217 (80)	0.061 (22)	-0.002 (2)	-.001 (3)	-0.056 (52)
2J	0.704 (118)	-.070 (82)	0.629 (51)	0.035 (7)	0.022 (7)	0.278 (76)
g_A	0.91 (11)	-0.30 (12)	0.62 (9)	-0.12 (1)	-0.12 (1)	
2 L	-0.21 (16)	0.23 (15)	0.01 (10)	0.16 (1)	0.14 (1)	

Quark Spin, Orbital Angular Momentum, and Gule Angular Momentum (M. Deka *et al*, 1312.4816)

pizza cinque stagioni



$$\Delta q \approx 0.25;$$

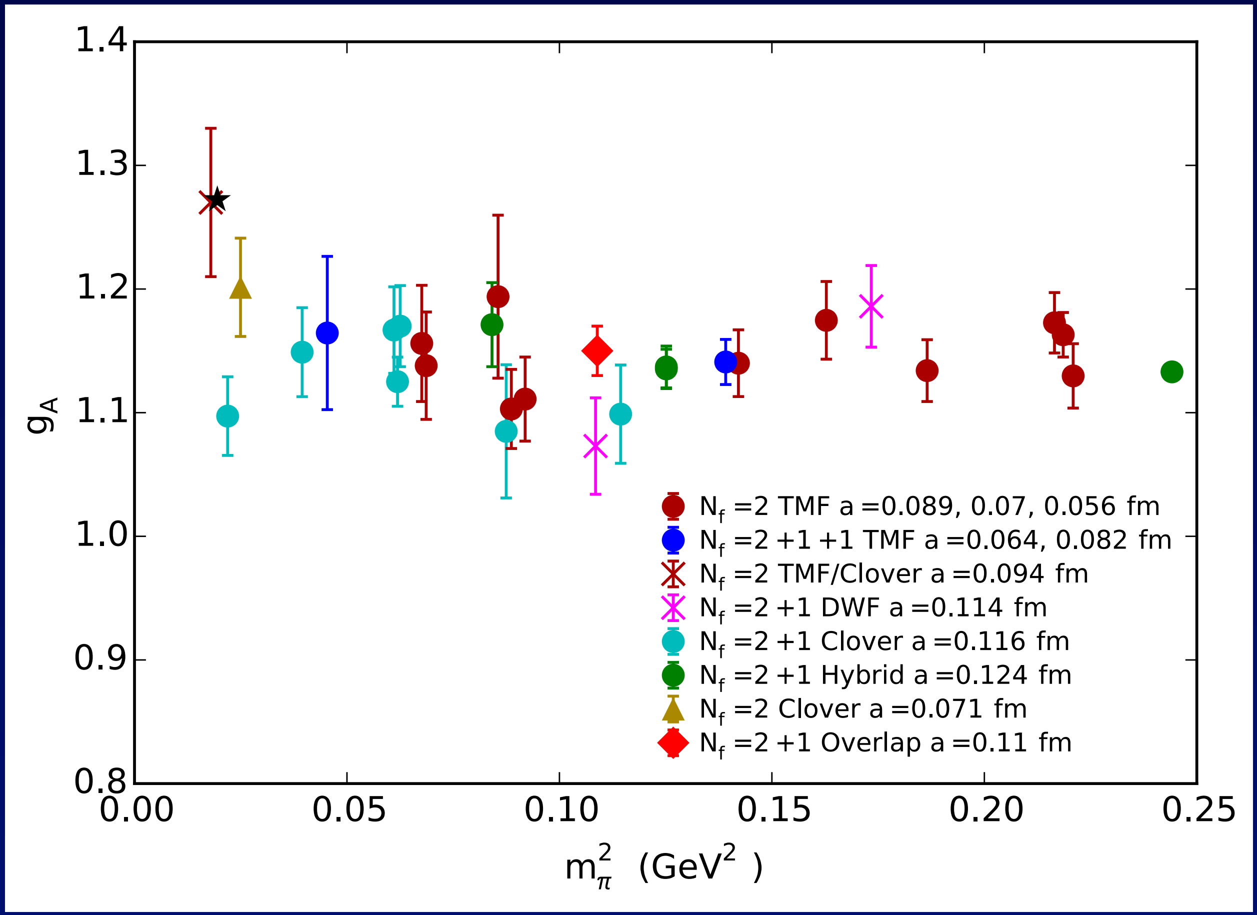
$$2\,L_q \approx 0.47\ (0.01(\mathrm{CI})+0.46(\mathrm{DI}));$$

$$2\,\mathrm{J}_g \approx 0.28$$

These are quenched results so far.

Dynamical Fermions

Isovector g_A^3 from recent lattice calculations with dynamical fermions



Concerns about excited state contamination and volume dependence

James Zanotti, Parallel-VIII:S4

Quark Spin Calculation with Axial-vector current

- Recent calculation of strange quark spin with dynamical fermions
  - R. Babich et al. (1012.0562)

    $$\Delta s = -0.019(11)$$

  - QCDSF (G. Bali et al. 1206.4205) gives

    $$\Delta s = -0.020(10)(4)$$

  - M. Engelhardt (1210.0025)

    $$\Delta s = -0.031(17)$$

  - C. Alexandrou et al. (arXiv:1310.6339)

    $$\Delta s \sim -0.0227(34)$$

Quark Spin from Anomalous Ward Identify

- Calculation of the axial-vector in the DI is very noisy
- Instead, try AWI $\partial_\mu A_\mu^0 = i2mP + \frac{iN_f}{8\pi^2} G_{\mu\nu}\tilde{G}_{\mu\nu}$

$$Z_A \langle p',s | A_\mu | p,s \rangle = \lim_{q\to 0} \frac{i|s|}{\vec{q}\cdot\vec{s}} \langle p',s | 2\sum_{f=1}^{N_f} m_f \vec{q}_f i\gamma_5 q_f + 2iN_f q | p,s \rangle$$

  - Overlap fermion --> mP is RGI ($Z_m Z_P = 1$)
  - Overlap operator for $q(x) = -1/2\ \mathrm{Tr}\gamma_5 D_{ov}(x,x)$ is RGI.
  - P is totally dominated by small eigenmodes.
  - q(x) from overlap is exponentially local and captures the high modes from A_μ^0.
  - Direct check the origin of `proton spin crisis'.

2+1 flavor DWF configurations (RBC-UKQCD)

La ~ 4.5 fm
m_π ~ 170 MeV

32^3 x 64, a =0.137 fm

($O(a^2)$ extrapolation)

La ~ 2.8 fm
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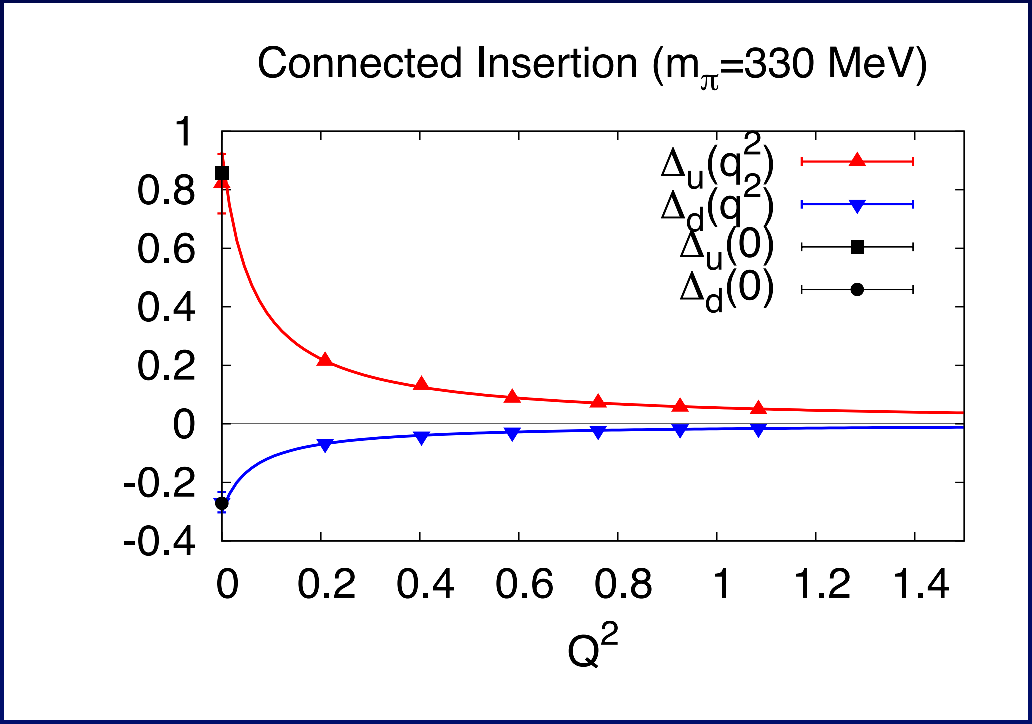
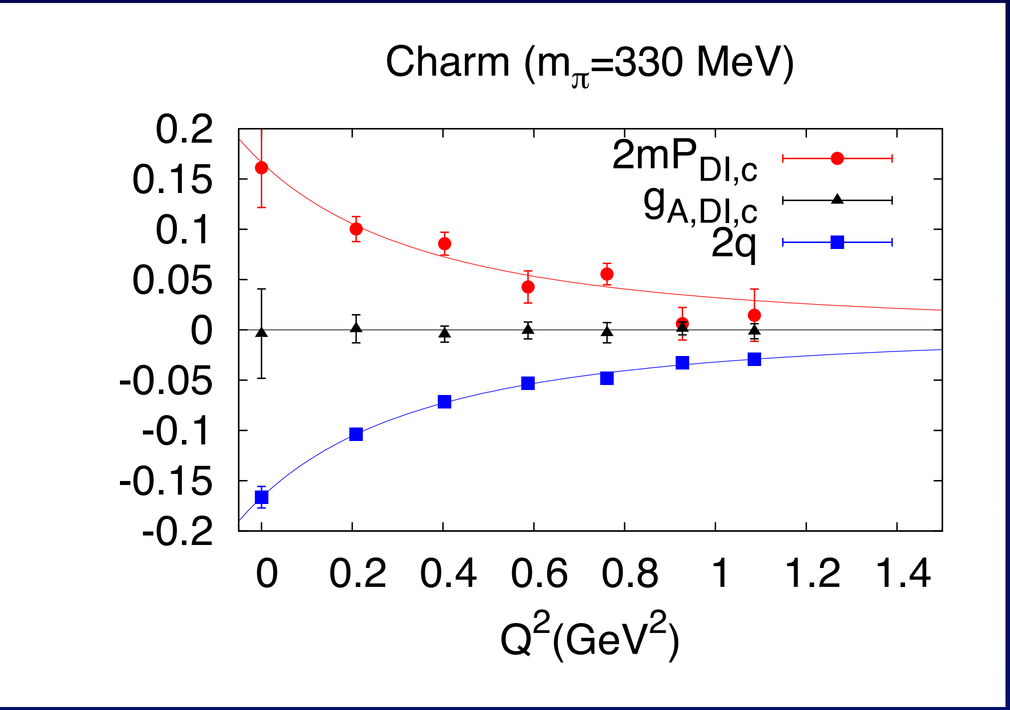
Connected Insertion from Ward Identity



χ QCD Collaboration

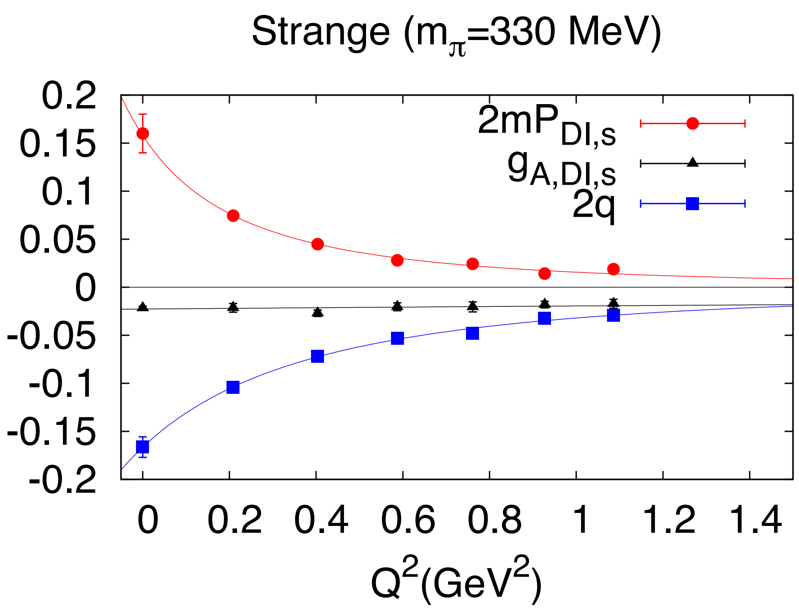
Y. Yang, M. Gong *et al*

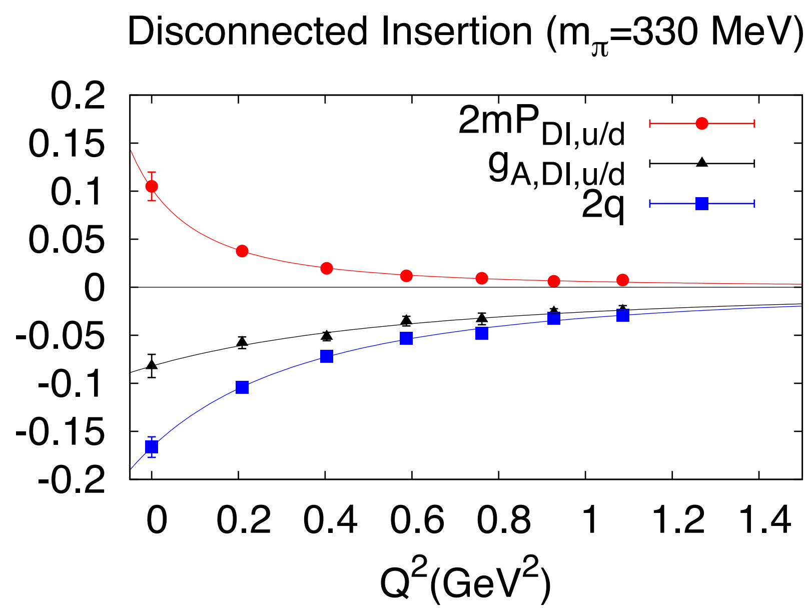
Disconnected Insertion for the Charm Quark



- Topological term is large and negative
- Pseudoscalar term and the topological term cancel

Disconnected Insertion for the Strange and u/d Quarks





Strange

u/d (DI)

Quark Spin from AWI

Overlap fermion on 2+1 flavor $24^3 \times 64$ DWF lattice (L=2.8 fm)

g_A^0 compt	**m_π=330 MeV (m_V=m_{sea})**	**m_π=140 MeV (Valence only)**
$\Delta u+\Delta d$ (CI)	0.57(2)	0.54(3)
Δc	~0	~0
Δs	-0.026(5)	-0.06(3)
Δu(DI)=Δd(DI)	-0.09(2)	-0.17(3)
g_A^0	0.36(5)	0.14(7)

The triangle anomaly (topological charge) is responsible for the smallness of quark spin in the proton (`proton spin crisis).

Glue Spin ΔG

- Jaffe and Manohar

$$S_g = \int d^3x \, \vec{E} \times \vec{A} \text{ in light-cone gauge } (A^+ = 0) \text{ and IMF frame.}$$

- Collins, Soper; Manohar

$$\Delta G \, S^+ = \int dx \, \frac{i}{2xP^+} \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle PS | F_a^{+\alpha}(\xi^-) L^{ab}(\xi^-, 0) \tilde{F}^+_{\alpha,b}(0) | PS \rangle$$

- X.S. Chen, T. Goldman, F. Wang; Wakamatsu; Hatta, etc.

$$S_g = \int d^3x \, \vec{E} \times \vec{A}_{nC}, \quad A^\mu = A^\mu_{nC} + A^\mu_{pure}, \quad F^{\mu\nu}_{pure} = 0;$$

$$A^\mu_{nC} \to U^\dagger A^\mu_{nC} U, \quad A^\mu_{pure} \to U^\dagger A^\mu_{pure} U - \frac{i}{g} U^\dagger \partial^\mu U$$

$$D^i A^i_{nC} = \partial^i A^i_{nC} - ig[A^i, A^i_{nC}] = 0; \quad A_{nC} = A_{phys} = A_\perp, \quad A_{pure} = A_{||}$$

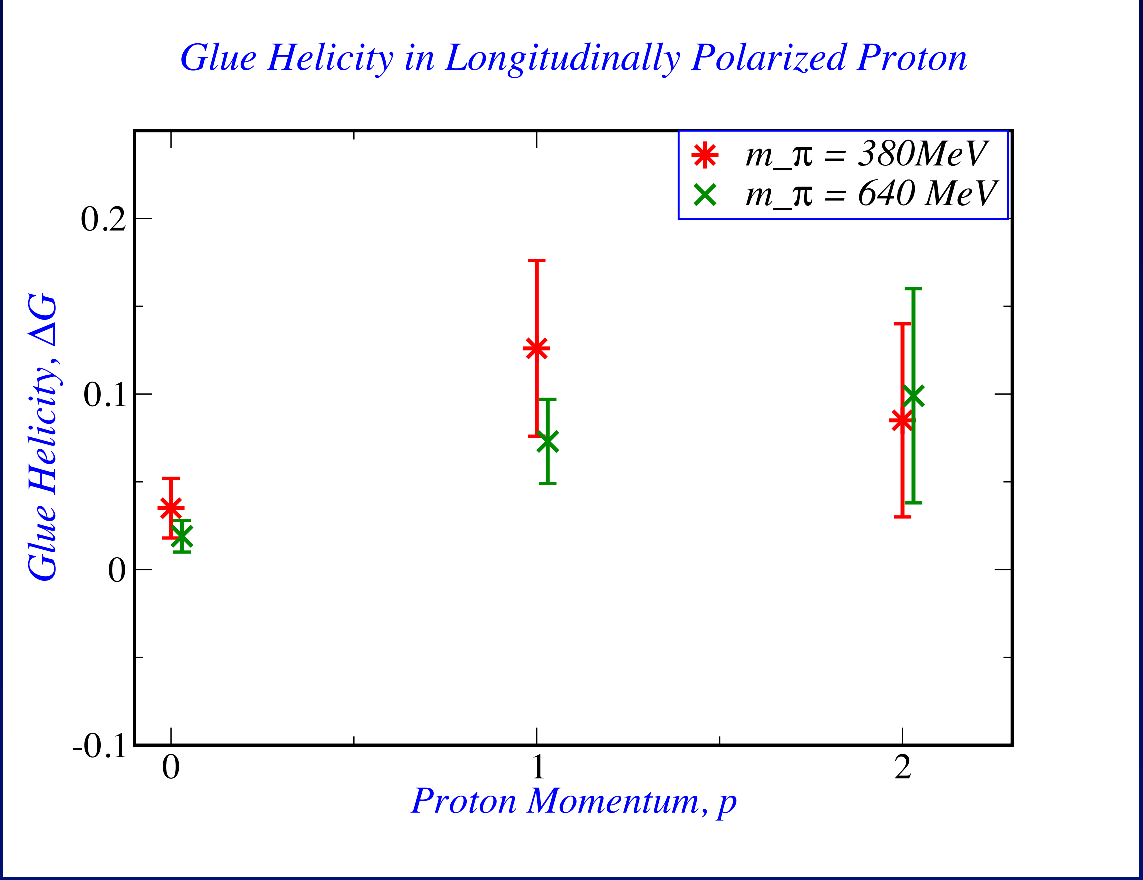
Gauge invariant decompostion

- X. Ji, J.H. Zhang, Y. Zhao; Y. Hatta, X. Ji, Y. Zhao

$$\Delta G \, S_z = \frac{\langle PS | \int d^3x \, (\vec{E} \times \vec{A}_{nC})_z | PS \rangle}{2E_P}$$

Large momentum limit

ΔG in Coulomb gauge at p =0, 400 MeV, 800 MeV on the 24^3 x 64 lattice



Check to see how noisy it is

Summary and Challenges

- Decomposition of proton spin into quark spin, quark orbital angular momentum, glue spin, and glue orbital angular momentum on the lattice is becoming feasible, pending on better understanding of the local glue spin operator.

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Summary and Challenges

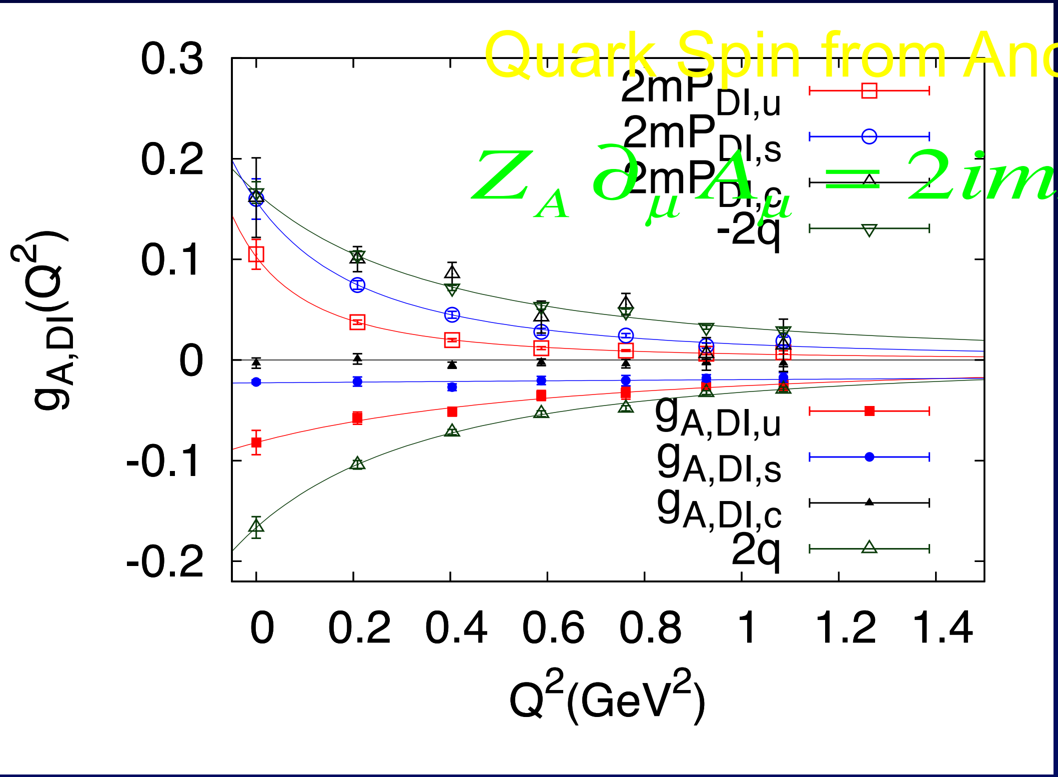
- Decomposition of proton spin into quark spin, quark orbital angular momentum, glue spin, and glue orbital angular momentum on the lattice is becoming feasible, pending on better understanding of the local glue spin operator.

- `Proton Spin Crisis' is likely to be the second example of observable U(1) anomaly.

- Continuum limit at physical pion mass and with large lattice volume (5.5 fm) with chiral fermions are being carried out.

Quark Spin from Anomalous Ward Identity

$$Z_A\, \partial_\mu A_\mu = 2imP + 2iN_f q$$



$24^3 \times 64$ 2+1-flavor DWF Conf.

$m_\pi \sim 330$ MeV (L = 2.8 fm)

Overlap valence fermion

Δu (CI) $+\Delta d$ (CI) = 0.54(3)

$\Delta c \sim 0$

Δs = - 0.06(3)

$\Delta u(\mathrm{DI}) = \Delta d(DI)$ = - 0.17(3)

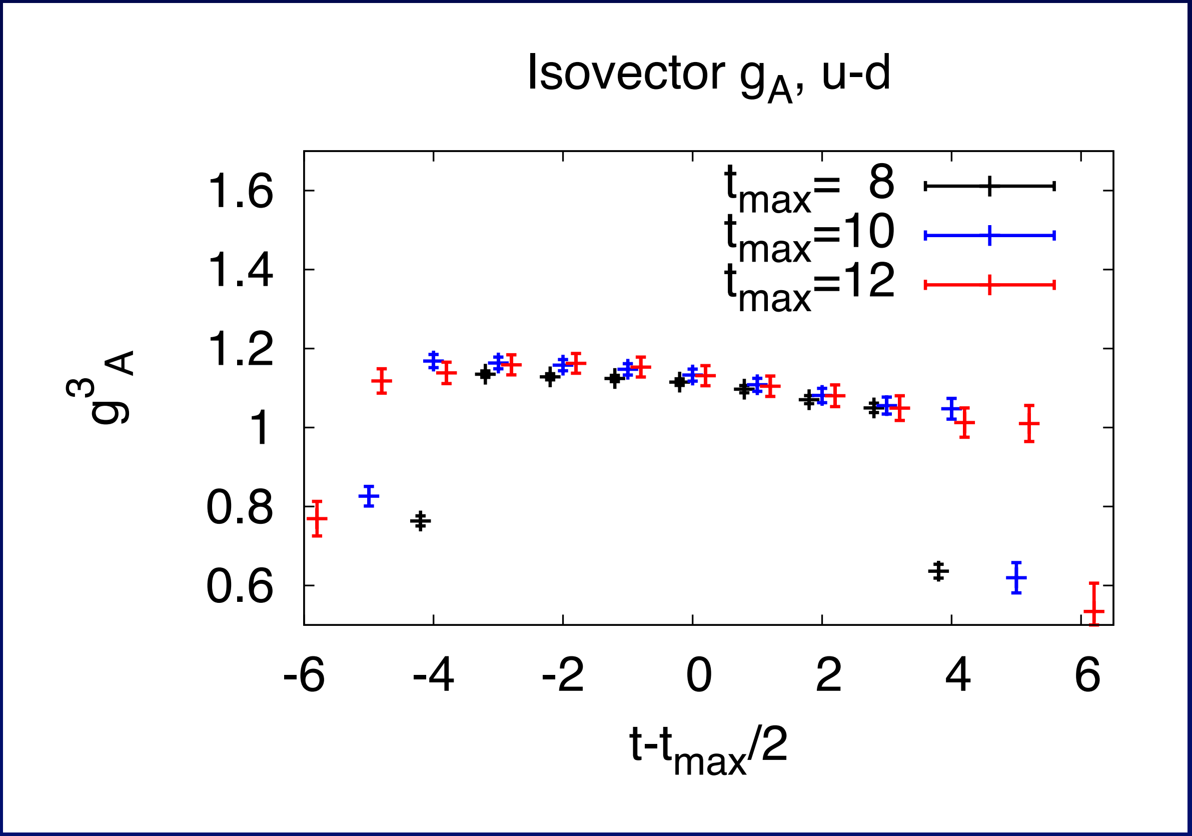
$$g_A^0 = \Delta u + \Delta d + \Delta s = 0.14(7)$$

$48^3 \times 96$, $m_\pi = 140$ MeV, L = 5.5 fm $\Rightarrow$ 100 Mhrs

$64^3 \times 128$, $m_\pi = 140$ MeV, L = 5.5 fm $\Rightarrow$ 300 Mhrs

$80^2 \times 96 \times 192$, $m_\pi = 140$ MeV, $L_x = 5.4$ fm $\Rightarrow$ 1100 Mhrs (?)

Excited state contamination and large volume dependence



Lattice Parameters

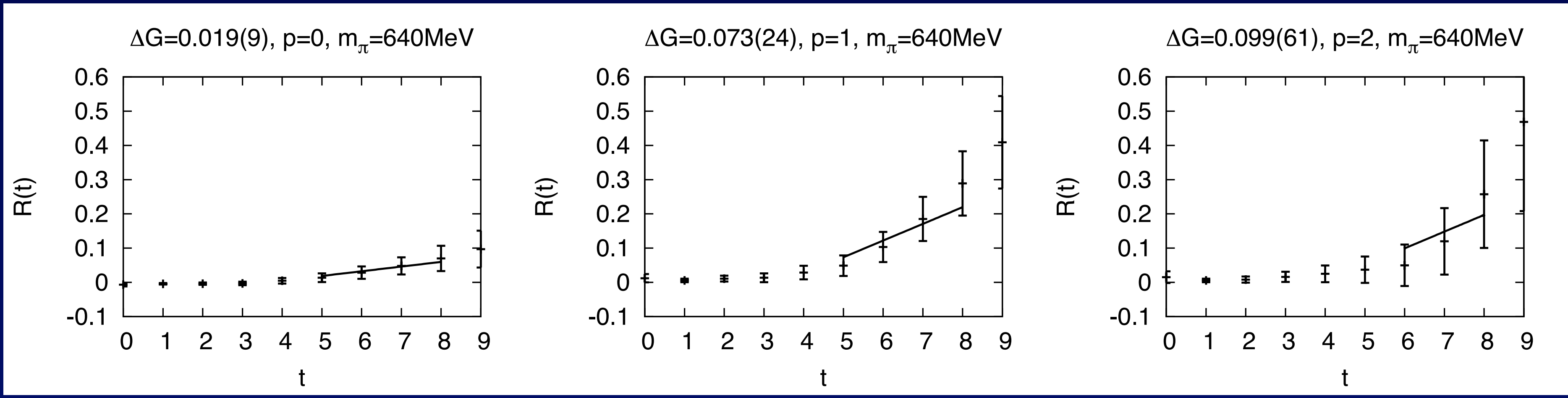
- Quenched 16^3 x 24 lattice with Wilson fermion
- Quark spin and <x> were calculated before for both the C.I. and D.I.
- κ = 0.154, 0.155, 0.1555 (m_π = 650, 538, 478 MeV)
- 500 gauge configurations
- 400 noises (Optimal Z_4 noise with unbiased subtraction) for DI
- 16 nucleon sources

Summary of Quenched Lattice Calculations

- Complete calculation of momentum fractions of quarks (both valence and sea) and glue have been carried out for a quenched lattice:
  - Glue momentum fraction is ~ 33% (CTEQ~40%)
  - $g_A^0 \sim 0.25$ in agreement with expt.
  - Glue angular momentum is ~ 28%.
  - Quark orbital angular momentum is large for the sea quarks (~ 47%).

- These are quenched results so far.

Calculation of ΔG in Coulomb gauge at p =0, 400 MeV, 800 MeV on the 24^3 x 64 lattice



$$\Delta G\ S_z = \frac{\langle PS \,|\, \int d^3x\ (\vec{E} \times \vec{A}_C)_z \,|\, PS \rangle}{2E_P}$$

Check to see how noisy it is

2+1 flavor DWF configurations (RBC-UKQCD)

La ~ 4.5 fm
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$$= \frac{1}{2P^+}\langle PS|\varepsilon^{ij}F^{i+}A^j_\perp|PS\rangle;$$

where $A^\mu_\perp \equiv \frac{1}{D^+}F^{+\mu}$

However, at IMF

$$A^\mu_\perp \equiv \frac{1}{D^+}F^{+\mu} = A^\mu - \frac{1}{D^+}\partial^\mu A^+ \to A^\mu - A^\mu_{||} = A^\mu_{nC}$$

- Therefore,

$$\Delta G\ S_z = \frac{\langle PS|\int d^3x\ (\vec{E}\times\vec{A}_{nC})_z\,|PS\rangle}{2E_P}$$

At large P

- Fist, try with A_C

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