THE SPIN STRUCTURE OF THE NUCLEON

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Outline

- Introduction and theoretical issues
- Gluon helicity and frame-dependent spin sum rule
- How to probe orbital motion?
- How to calculate gluon helicty and orbital angular momentum on a lattice?
- Summary

The spin "crisis"

1. **An Investigation of the Spin Structure of the Proton in Deep Inelastic Scattering of Polarized Muons on Polarized Protons**
   European Muon Collaboration (J. Ashman *et al.*). Jun 1989. 46 pp.
   Published in **Nucl.Phys. B328 (1989) 1**
   CERN-EP-89-73
   DOI: 10.1016/0550-3213(89)90089-8
   References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
   CERN Document Server
   Data: INSPIRE | HepData
   Detailed record - Cited by 1493 records 1000+

2. **A Measurement of the Spin Asymmetry and Determination of the Structure Function g(1) in Deep Inelastic Muon-Proton Scattering**
   European Muon Collaboration (J. Ashman *et al.*). Dec 1987. 18 pp.
   Published in **Phys.Lett. B206 (1988) 364**
   CERN-EP-87-230
   DOI: 10.1016/0370-2693(88)91523-7
   References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
   CERN Document Server ; ADS Abstract Service
   Data: INSPIRE | HepData
   Detailed record - Cited by 1751 records 1000+

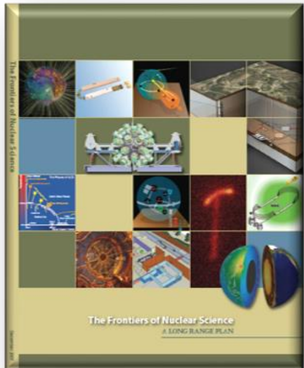
- Understanding the proton spin structure has been the driving force for hadronic spin physics for the last 25 years!

The spin structure of the proton is still an unsolved problem!

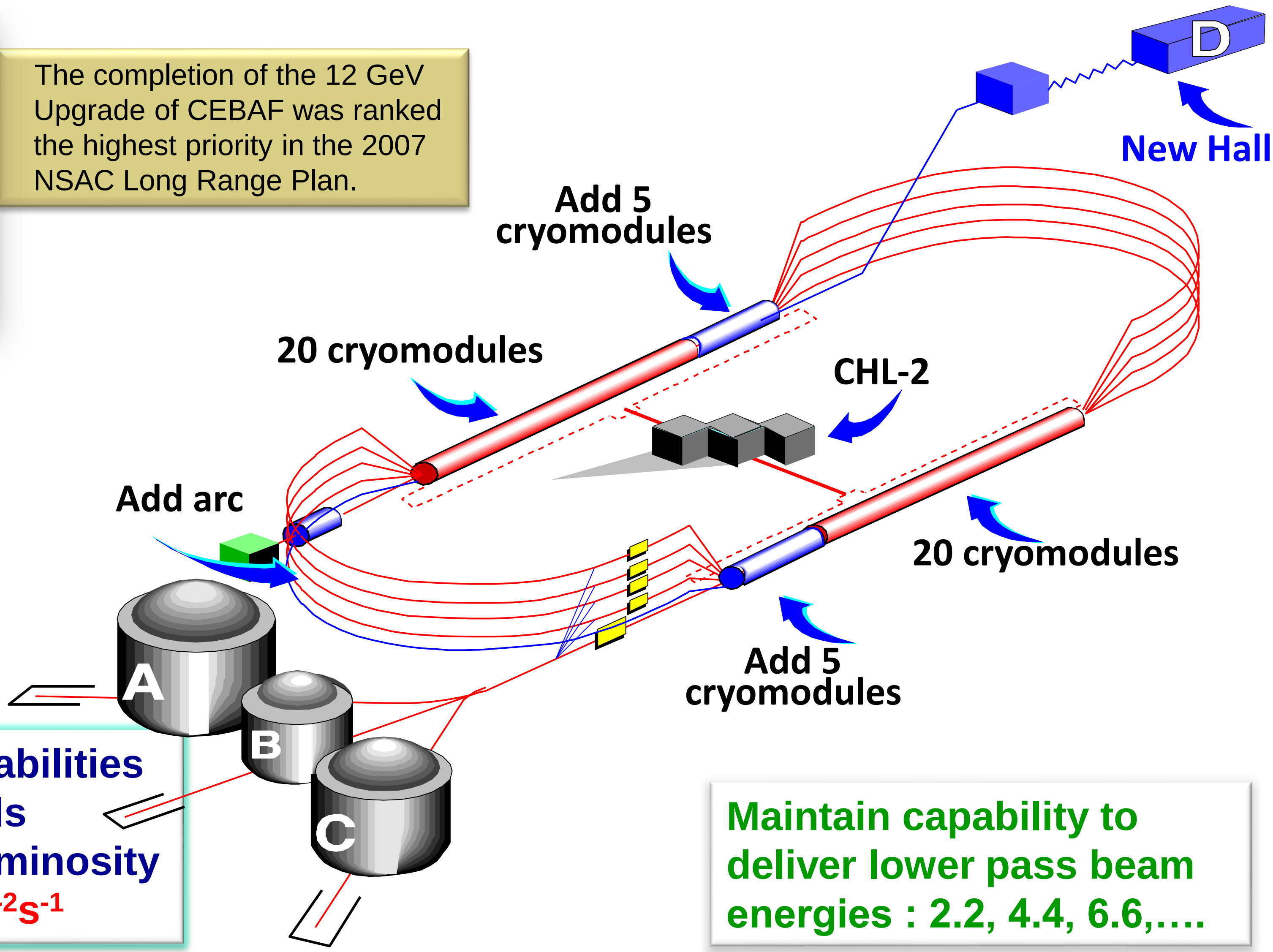
still the holy-grail in hadron spin physics!

Much experimental progress

12 GeV Upgrade



The completion of the 12 GeV Upgrade of CEBAF was ranked the highest priority in the 2007 NSAC Long Range Plan.

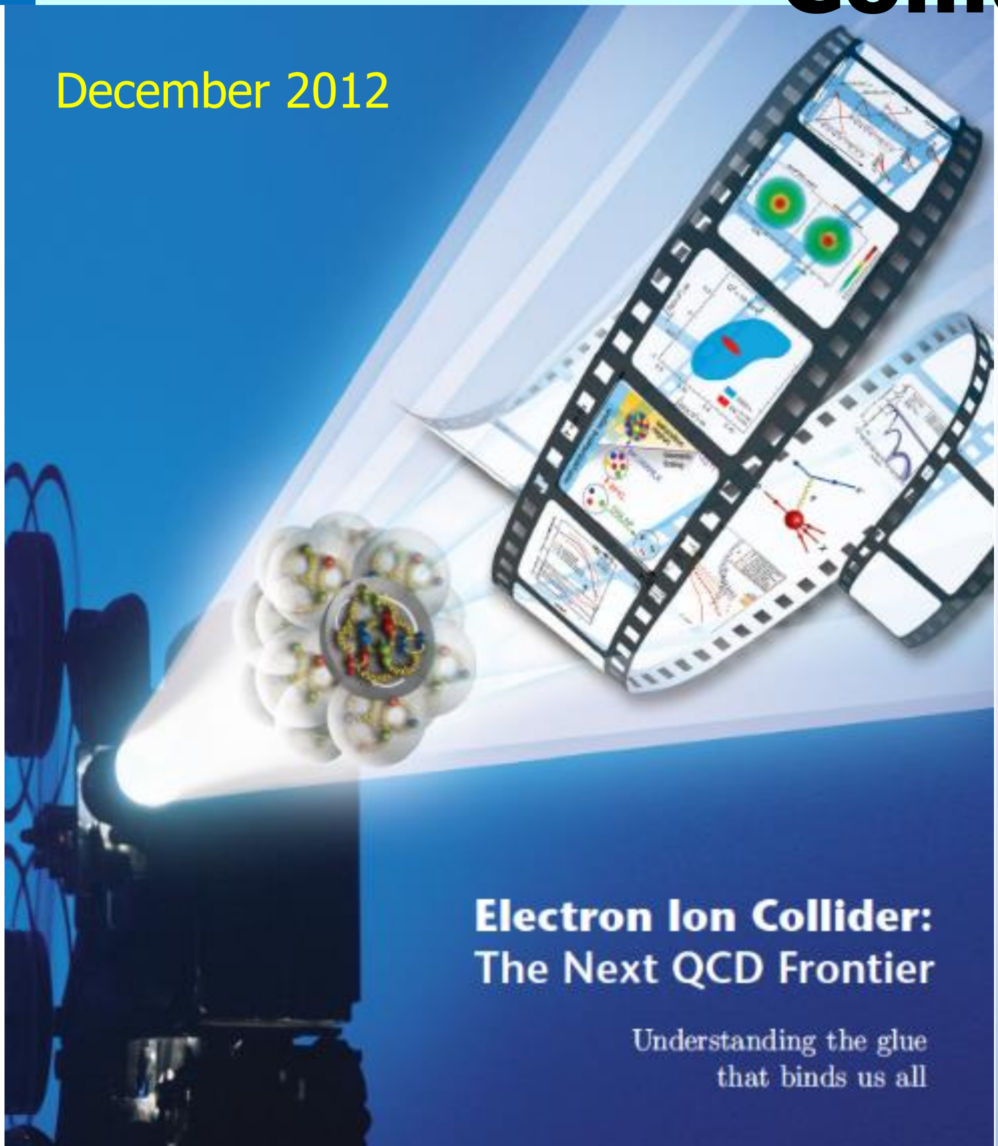


- **Enhanced capabilities in existing Halls**
- **Increase of Luminosity**
  10^{35} - ~10^{39} cm^{-2}s^{-1}

Maintain capability to deliver lower pass beam energies : 2.2, 4.4, 6.6,….

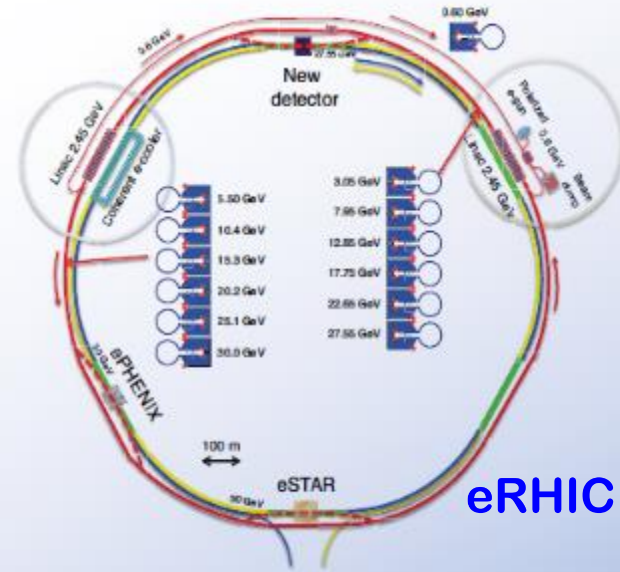
White Paper for the Electron-Ion Collider

December 2012



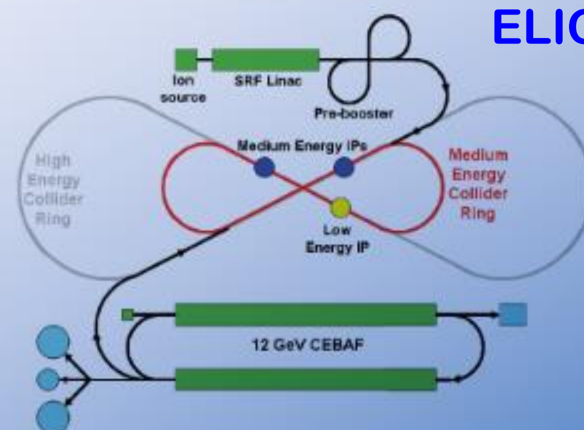
Ed. A. Deshpande, Z.-E. Meziani, J. Qiu

arXiv:1212.1701



eRHIC (BNL)

ELIC (JLab)



Theory: a tale of two sum rules

- Jaffe & Manohar, 1990

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + \ell_q^z + \ell_g^z$$

  - Simple parton picture
  - Valid in a particular frame (and gauge)

- X. Ji, 1996

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q^z + \mathrm{J_g}$$

  - Local and gauge symmetric
  - Works in general frame, including the rest and infinite momentum frame.

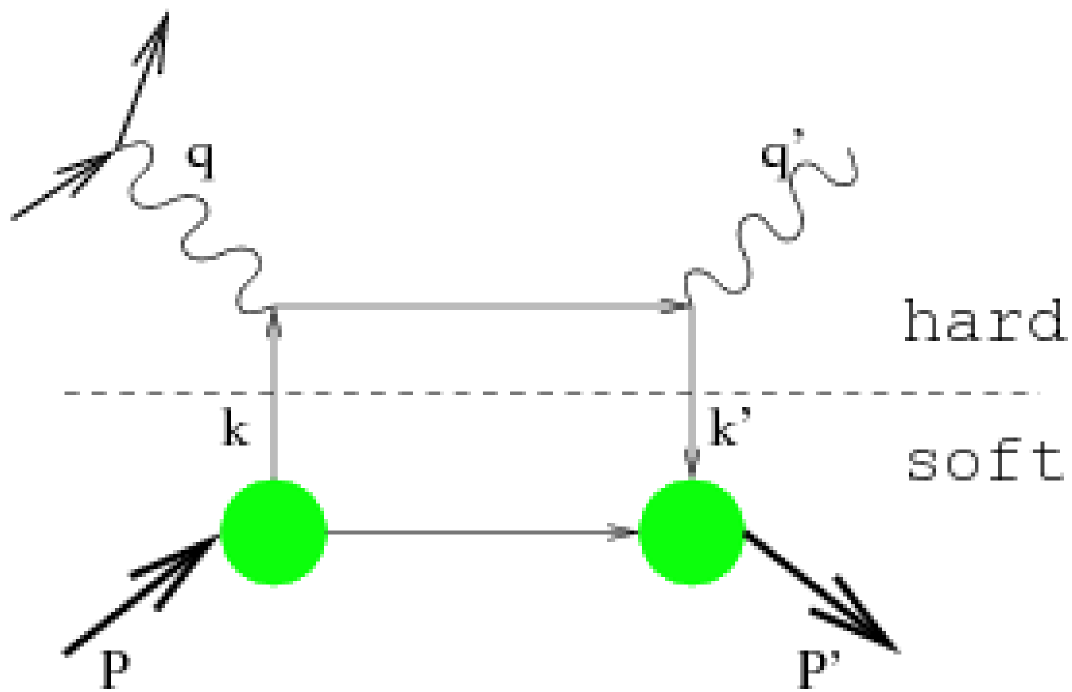
Progress in frame-indept. spin sum rule

- The sum rule contains three terms:
- The quark spin contribution (measurable in inclusive DIS)
- The quark orbital contribution: twist-2 GPDs
- The gluon contribution: twist-2 GPDs
- Measurable in deep exclusive processes:

  Jlab12 GeV upgrade

  EIC

Deeply virtual Compton scattering



Outstanding questions

- Is the Jaffe-Manohar sum rule physical?
- What is the relationship between the two sum rules?
- How to measure and calculate the orbital contributions
- More sum rules?
- …

Intense theoretical activities

- F. Wang, X. S. Chen, T. Goodman,…
- E. Leader, B. Bakker, T. Trueman,
- Wakamatsu, C. Lorce
- Y. Hatta
- X. Ji, P. Hoodbhoy, F. Yuan, ..

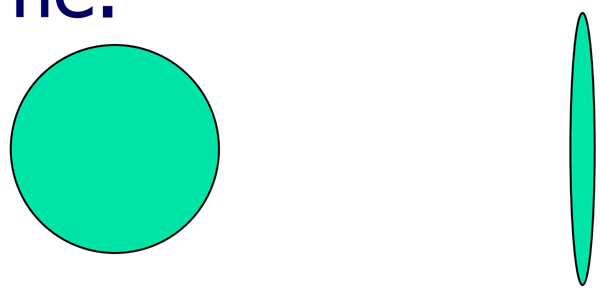
Much of the theoretical issues has been resolved!

Key Issue: FRAME-DEPENDENCE

- JM sum rule is frame dependent (Infinite Momentum Frame, or IMF), tailored for high-energy scattering, i.e. partons.
  - → IMF allows one to talk about the gluon helicity
- Ji's sum rule is frame-independent
  - → Does not allow gluon spin term
  - → Frame independence allows calculations done with the standard lattice method.
  - → GPD and spin calculations, K.F. Liu's talk

Frame Dependence of the proton WF

- The proton wave-function is a frame dependent concept, it is defined by observing different space points in a fixed time (simultaneously), i.e., a particular frame.



- Boost operators, K_i, are interaction-dependent

$$|P\rangle = \mathbf{U(\Lambda(p))}\, |p=0\rangle$$

U is not just kinematical, it is dynamical!

One frame is special!

- IMF, in which the proton is moving at the speed of light.
- This is the frame where
  - ➔ probes "see" the proton (expts).
  - ➔ probe and structure physics can be neatly separated (factorization).
  - ➔ partons become a useful language: constructing the bound state properties in terms of those of the individual particles, such as the momentum

Gluon spin ΔG

- The JM sum rule was motivated by the gluon spin contribution ΔG, which can be (has been) measured experimentally.
- However, in textbooks, it is usually remarked that the gauge field spin $\vec{E} \times \vec{A}$ is not gauge invariant, therefore not physical.

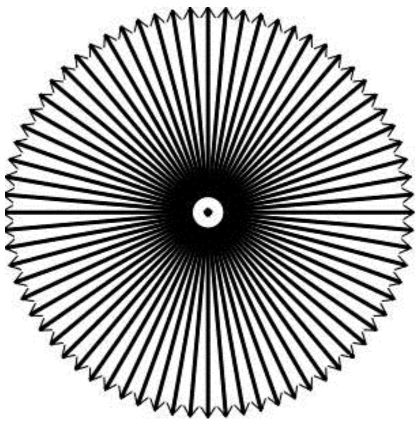
How does gluon spin becomes physical?

- There are two kinds of gluons inside the nucleon:
  - Coulomb (confining, binding) gluons
  - Radiation gluons

- Their roles can be separated in heavy-quark (non-relativistic) limit when we have a situation like H-atom.

Coulomb gluon

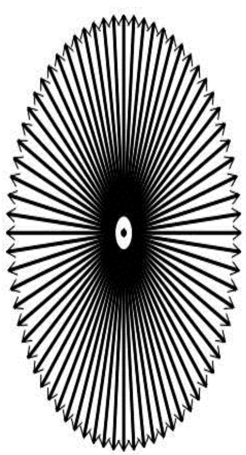
Consider a heavy quark at rest, it generates a gluonic Coulomb field, and the corresponding potential depends on gauge choices.



This contribution to the gluon spin is gauge dependent.

Radiation gluon

- When the heavy quark moves, it also generates a radiation gluon field, which has non-zero B compoent.



v = 0.5c

Radiation gluon's contribution to the gluon spin is gauge-invariant.

Frame dependence

- The relative sizes of the two types of gluons are frame dependent.
- In the IMF limit, the radiation gluon entirely dominates over the Coulomb gluon (Weizsacker-William's picture) The contribution from the latter becomes very small.
- Thus in the IMF, the gluon spin becomes a physical observable. This is exactly what high-energy scattering experiments measure!

Relationship of the two sum rules

- The gluon contribution J_g in the frame-indept sum rule can be further decomposed into three contributions in IMF,

$$J_g = \Delta G + \ell_g^z + J_{pot}$$

$$L_q^z = \ell_q^z - J_{pot} \qquad\qquad j_{pot} \sim \psi^+ \vec{r} \times \vec{A} \psi$$

Thus, knowing ΔG and frame-indept sum rule is not enough to determine ℓ_g^z, ℓ_q^z, one needs additional information to determine all the terms in the JM spin sum rule.

Measuring ℓ_q^z , ℓ_g^z and J_{pot}

- Parton orbital angular momentum is intrinsically a twist-three observable.
  - ➔ AM operator involves parton transverse momentum!
- In the case of L_q, it can be related to the twist-two process (DVCS) because of the combination of frame-independence and rotational symmetry.
- Following the development of GPDs, one needs to measure ℓ_q^z , ℓ_g^z and J_{pot} through twist-three GPD (not TMDs).

How to calculate the observables in IMF?

- Quantities like ΔG, ℓ_q^z , ℓ_g^z and J_{pot} are defined in the IMF and light-cone gauge. They are not accessible in the usual lattice QCD.
- However, recently we have proposed an approach how to do this (Yong Zhao's talk)

Lattice calculations of the spin content

- Starting from a finite mom. frame at a physical gauge.
- Calculate the angular momentum matrix elements in this specific gauge.
- Match the result to that in IMF.

e-Print: arXiv:1409.6329

Justifying the Naive Partonic Sum Rule for Proton Spin

X. Ji, J. H. Zhang, Y. Zhao

Matching formula to one-loop order

$$\Delta\widetilde{\Sigma}(\mu, P^z) = \Delta\Sigma(\mu) \ ,$$

$$\Delta\widetilde{G}(\mu, P^z) = z_{qg}\Delta\Sigma(\mu) + z_{gg}\Delta G(\mu) + O\left(\frac{M^2}{(P^z)^2}\right) \ ,$$

$$\Delta\widetilde{L}_q(\mu, P^z) = P_{qq}\Delta L_q(\mu) + P_{gq}\Delta L_g(\mu) + p_{qq}\Delta\Sigma(\mu) + p_{gq}\Delta G(\mu) + O\left(\frac{M^2}{(P^z)^2}\right) \ ,$$

$$\Delta\widetilde{L}_g(\mu, P^z) = P_{qg}\Delta L_q(\mu) + P_{gg}\Delta L_g(\mu) + p_{qg}\Delta\Sigma(\mu) + p_{gg}\Delta G(\mu) + O\left(\frac{M^2}{(P^z)^2}\right) \ ,$$

$$P_{qq} = 1 + \frac{\alpha_S C_F}{4\pi}\left(-2\ln\frac{(P^z)^2}{\mu^2} + R_3\right) \ , \quad P_{gq} = 0$$

$$P_{qg} = \frac{\alpha_S C_F}{4\pi}\left(2\ln\frac{(P^z)^2}{\mu^2} - R_3\right) \ , \quad P_{gg} = 1$$

$$p_{qq} = \frac{\alpha_S C_F}{4\pi}\left(-\frac{1}{3}\ln\frac{(P^z)^2}{\mu^2} + R_4\right) \ , \quad p_{gq} = 0$$

$$p_{qg} = \frac{\alpha_S C_F}{4\pi}\left(-\ln\frac{(P^z)^2}{\mu^2} - R_1 - R_4\right) \ ,$$

$$p_{gg} = \frac{\alpha_S C_A}{4\pi}\left(-\frac{7}{3}\ln\frac{(P^z)^2}{\mu^2} - R_2\right) \ , \qquad ($$

where

$$R_3 = -4\ln 2 + \frac{28}{3} \ , \qquad R_4 = -\frac{2}{3}\ln 2 + \frac{13}{9} \ . \qquad ($$

Summary

- To understand the spin structure of the nucleon in a frame-independent formalism, one needs to
  - ➔ Measure twist-2 GPDs in the hard exclusive processes
  - ➔ Making usual lattice QCD calculations.
- Further progress can be made in IMF by
  - ➔ Measuring ΔG accurately
  - ➔ Measuring twist-3 GPDs in the hard exclusive processes
  - ➔ Calculating the quasi-observables on lattice in the large momentum limit and matching.

Smog tip in Beijing

- 厚德载物，自强不息

Hold world with virtue, and strengthen self without stopping

- 厚德载雾，自强不吸

Hold smog with virtue, and strengthen self without breath in