

# Limits for Spin-Dependent Short-Range Interaction of Axion-Like Particles



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# Outline

- Motivation
- Principle of measurements
- Experimental setup
- Results
- Summary

# Motivation

## The strong CP-Problem

The non-trivial vacuum structure of QCD predicts violation of CP-symmetry:

$$L_\theta = \bar{\theta} \frac{g^2}{32\pi^2} G_{\mu\nu}^b \tilde{G}_b^{\mu\nu}$$

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From neutron EDM we get:

$$|d_n| \approx \bar{\theta} \cdot 10^{-16} \text{ ecm} < 2.9 \cdot 10^{-26} \text{ ecm}$$

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**Original proposal for Axion** (*R. Peccei, H. Quinn PRL 38(1977),1440*)

as possible solution to the „strong CP-Problem“ that cancels the CP violating term in the QCD.

$$L_a = \zeta \frac{a}{f_a} \frac{g^2}{32\pi^2} G_{\mu\nu}^b \tilde{G}_b^{\mu\nu}$$

$$\langle a \rangle = -f_a \frac{\bar{\theta}}{\zeta}$$

$$\text{with: } m_a \propto \frac{1}{f_a}$$

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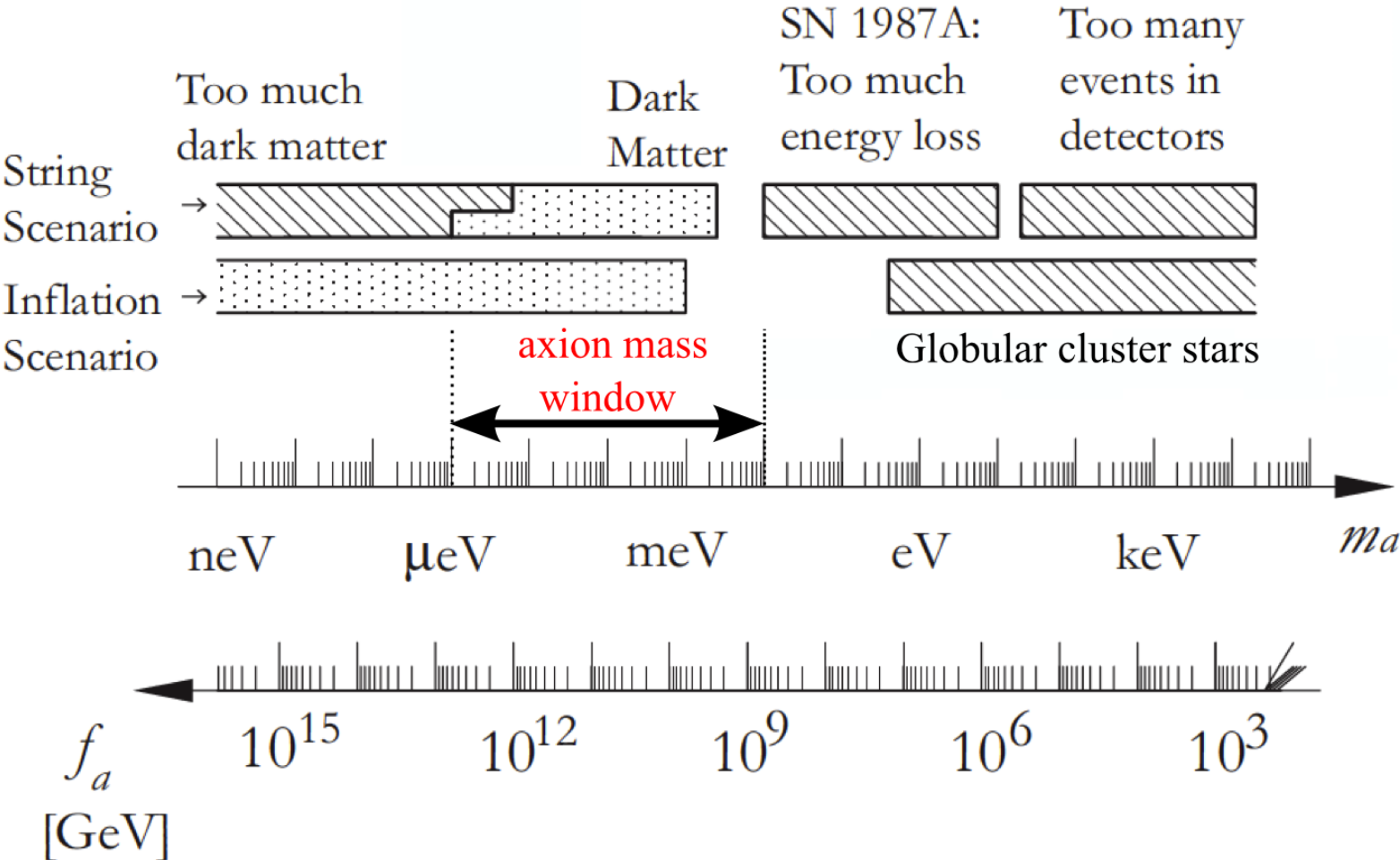
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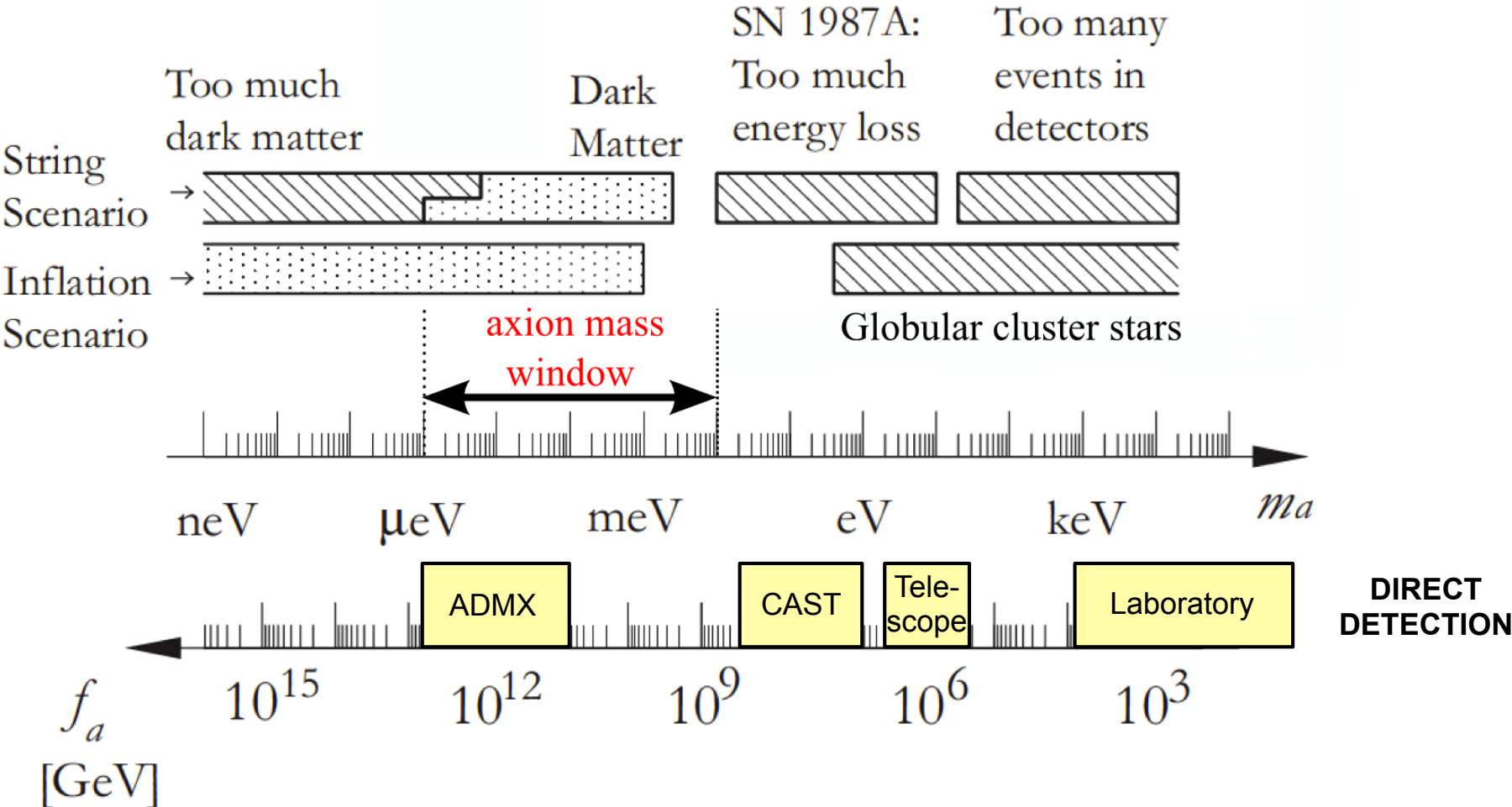
**AXIONS:** Light and weak interacting particles.

**Modern Interest:** Dark Matter candidate.

# Axion Mass



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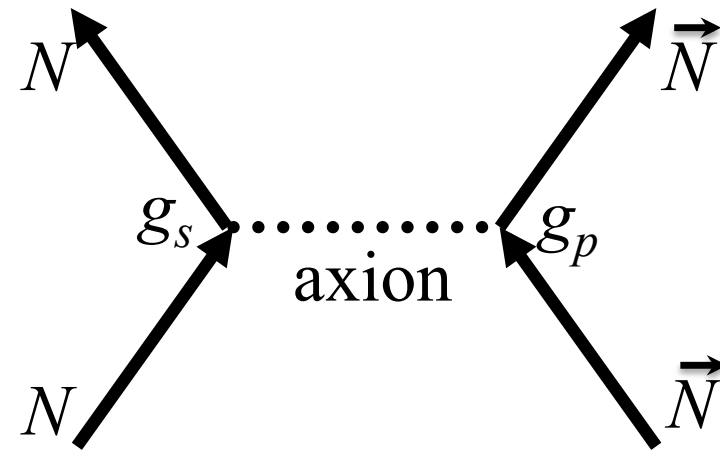


# Motivation

## Axion Potential:

Yukawa type potential with monopole-dipole coupling [1]

$$V(r) = \kappa \hat{n} \cdot \vec{\sigma} \left( \frac{1}{\lambda r} + \frac{1}{r^2} \right) e^{-r/\lambda}$$



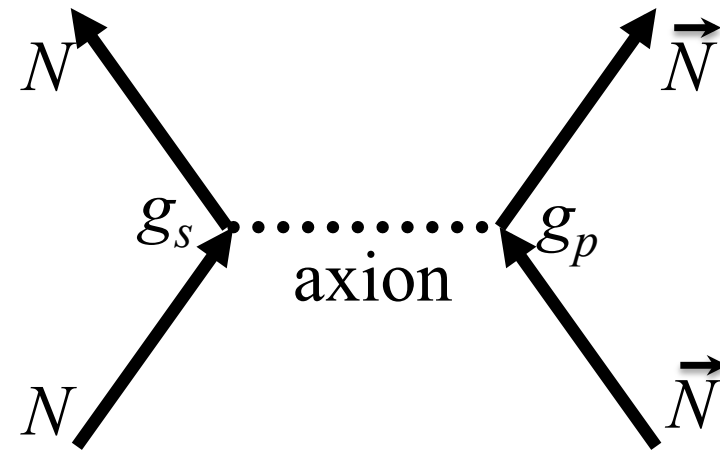
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$$\kappa = \frac{\hbar^2 g_s g_p}{8\pi m_n} \quad , \quad \lambda = \frac{\hbar}{m_a c}$$



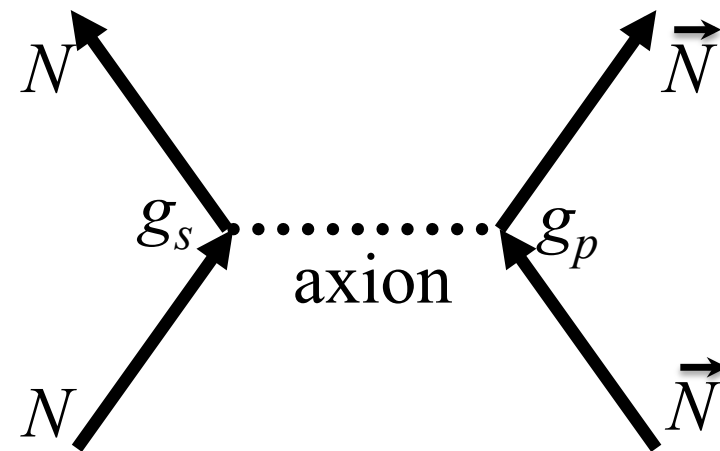
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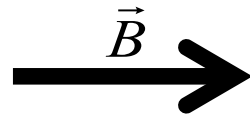
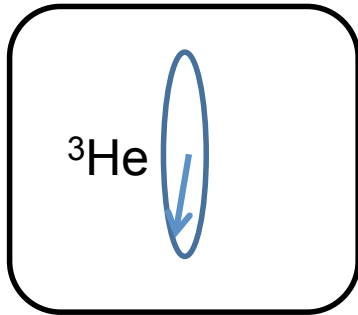


⇒ Indirect search of the axion via the axion potential in the range of the „axion mass window“:

$$10^{-6} \text{ eV} < m_a < 10^{-2} \text{ eV}$$
$$10^{-5} \text{ m} < \lambda < 10^{-1} \text{ m}$$

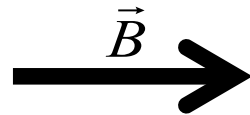
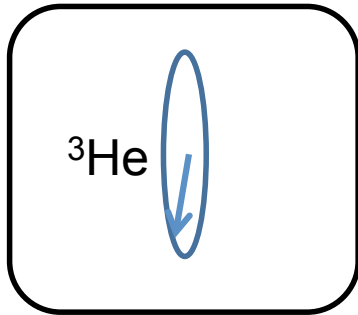
# Principle of measurements

How to measure?



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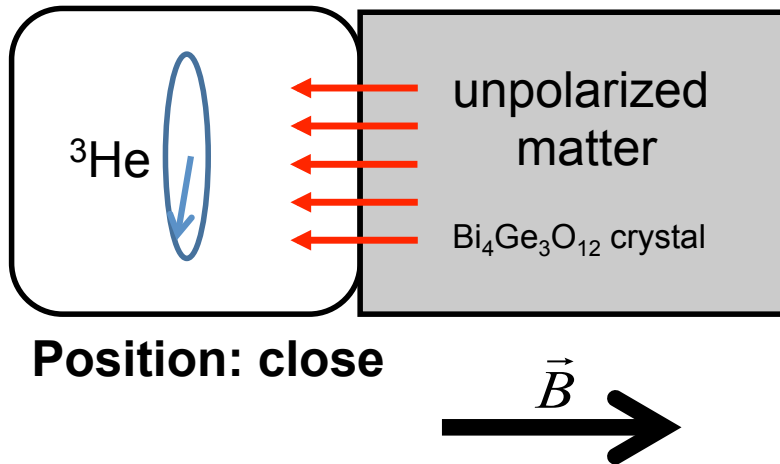
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$$\omega_{\text{L,He}}(t) = \gamma_{\text{He}} \cdot B(t)$$

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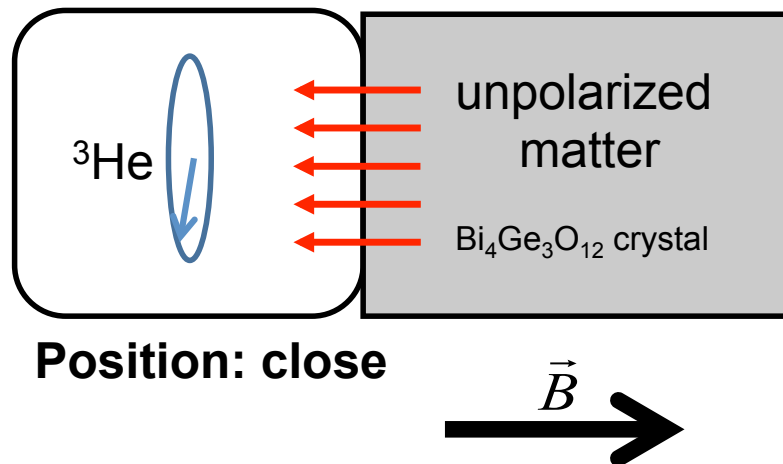
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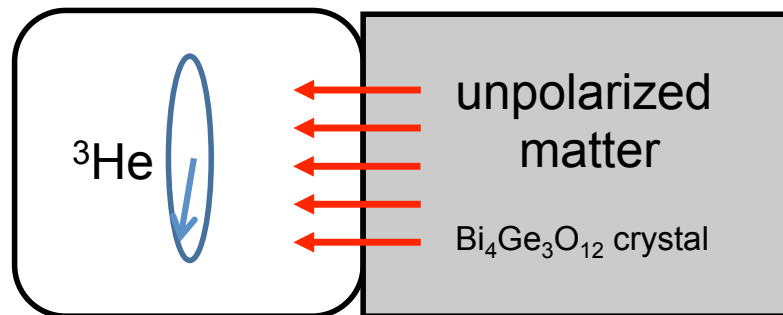
$$\omega_{\text{close}}(t) = \omega_{\text{L,He}}(t) + \omega_{\text{sp}}$$

with:  $\omega_{\text{L,He}}(t) = \gamma_{\text{He}} \cdot B(t)$

$$\omega_{\text{sp}} = 2\pi \cdot \nu_{\text{sp}} = 2 \cdot \bar{V} / \hbar$$

# Principle of measurements

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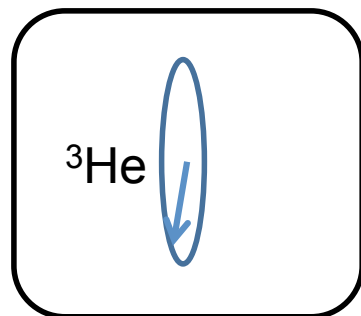
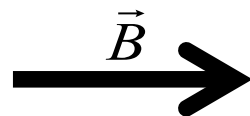


Position: close

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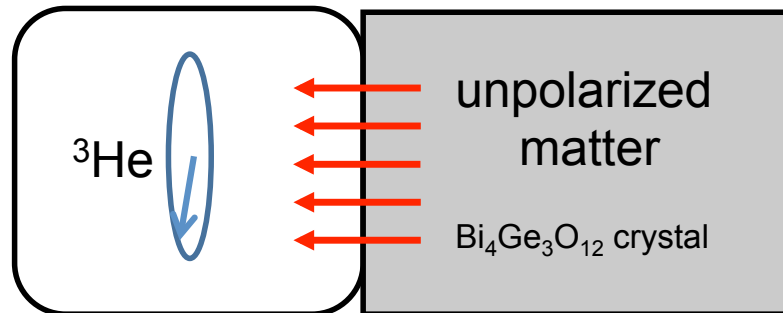
Position: distant

$$\omega_{\text{distant}}(t) = \omega_{\text{L,He}}(t)$$



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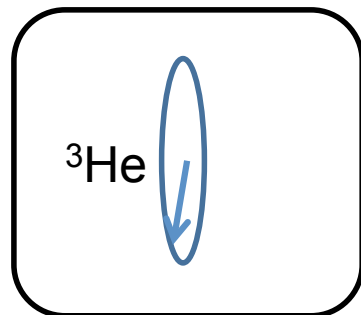
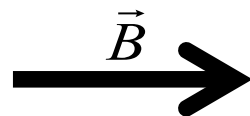


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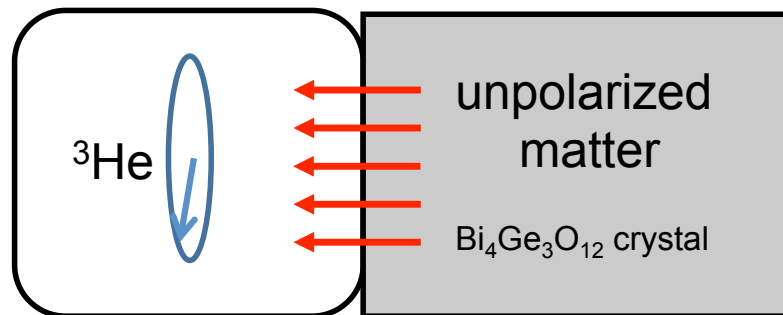
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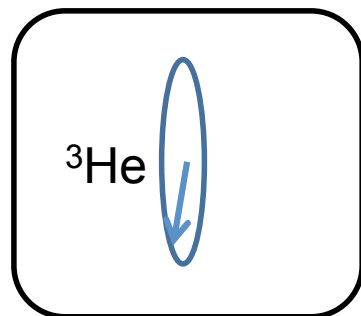
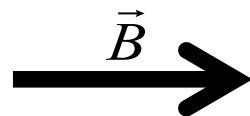


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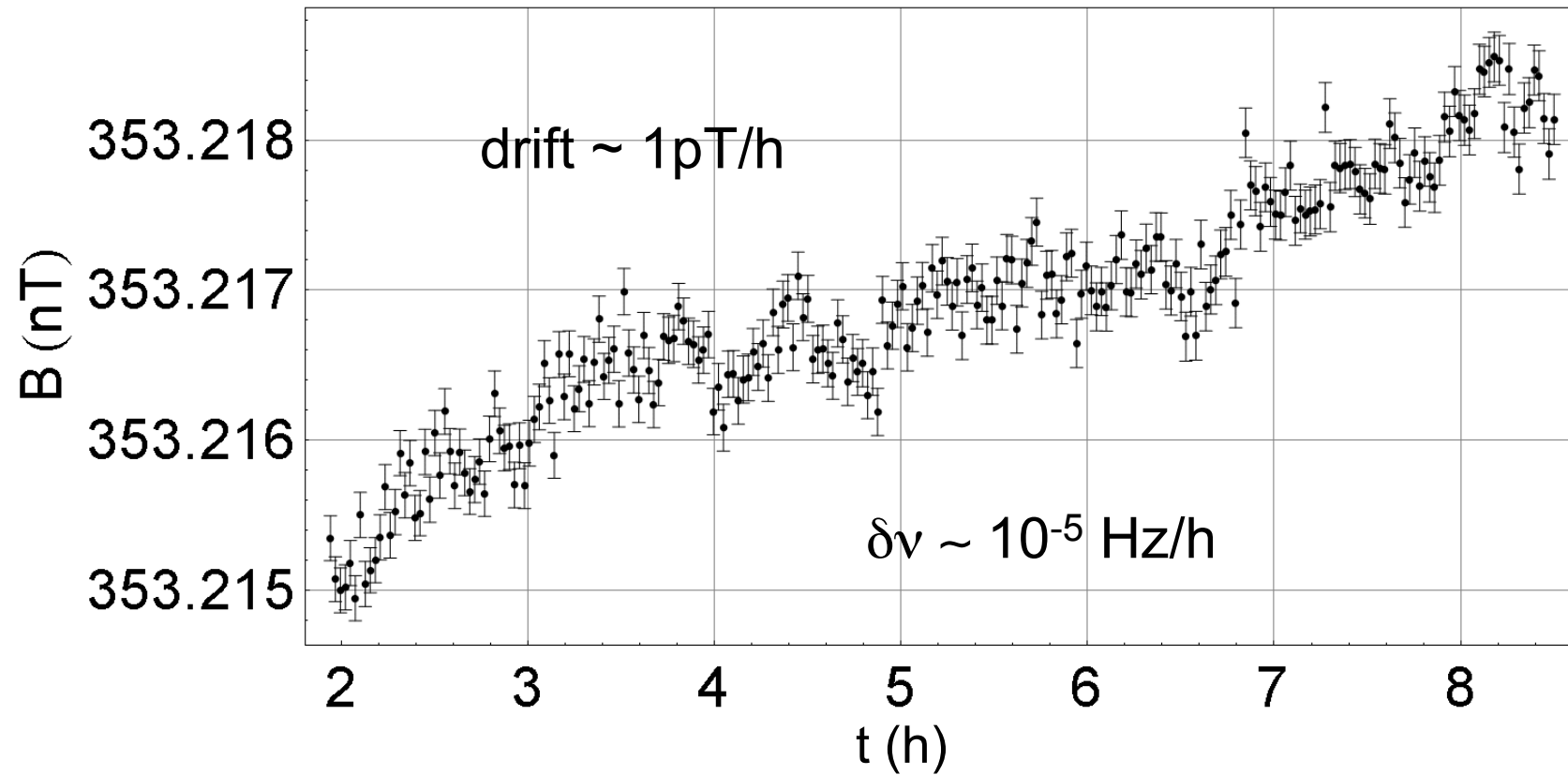
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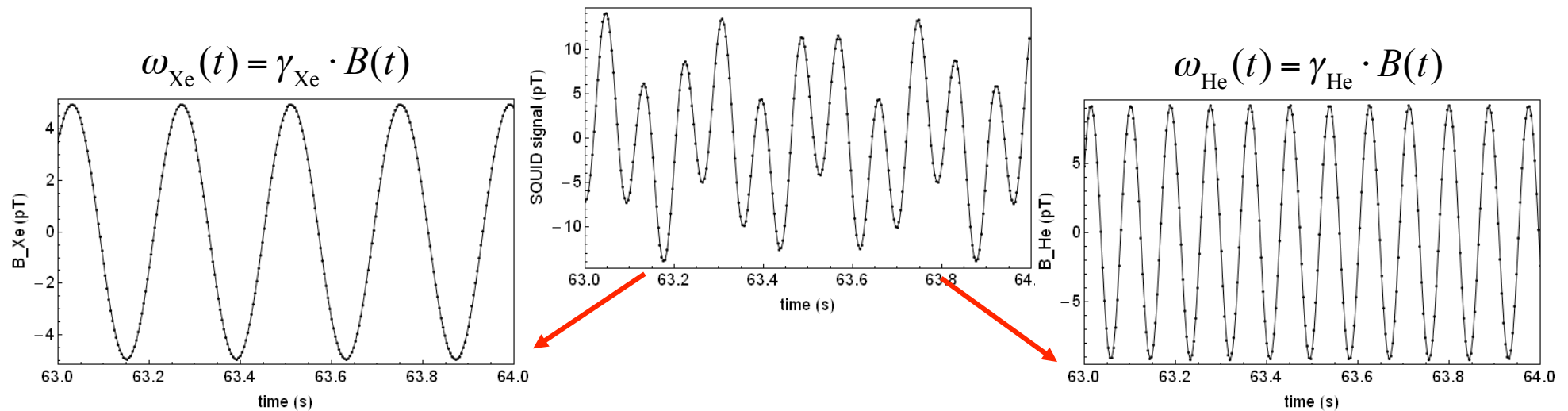
**Requirement:**  $\omega_{\text{L,He}}(t) = \text{const.}$

# Principle of measurements



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## $^3\text{He}/^{129}\text{Xe}$ Co-magnetometer

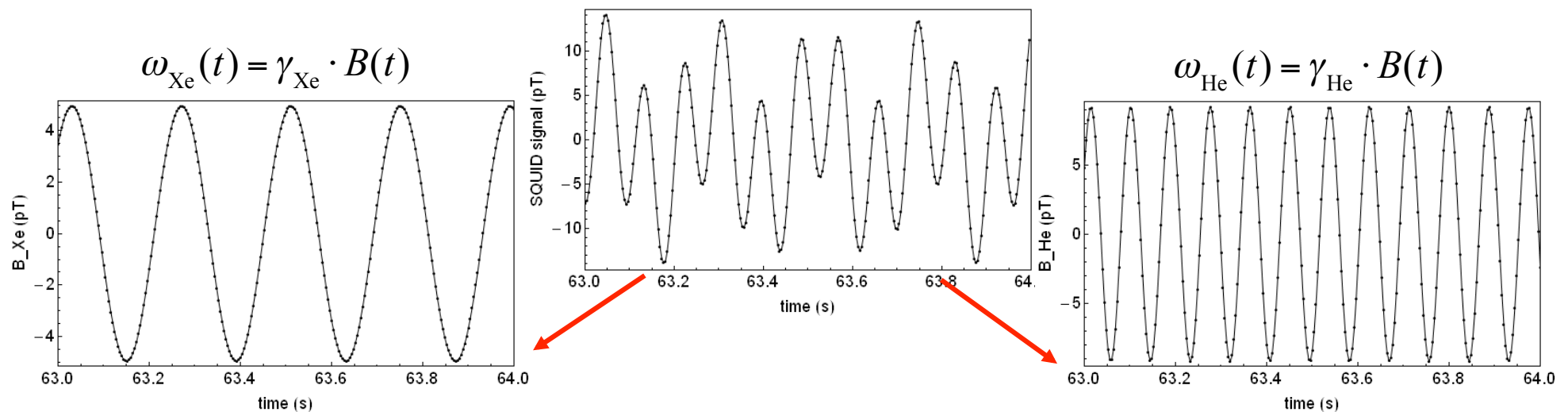


Elimination of magnetic field drifts (Zeeman term):

$$\text{Frequency difference: } \Delta\omega = \omega_{\text{He}} - \frac{\gamma_{\text{He}}}{\gamma_{\text{Xe}}} \omega_{\text{Xe}} = \left( \gamma_{\text{He}} - \frac{\gamma_{\text{He}}}{\gamma_{\text{Xe}}} \cdot \gamma_{\text{Xe}} \right) \cdot B(t) \stackrel{!}{=} 0$$

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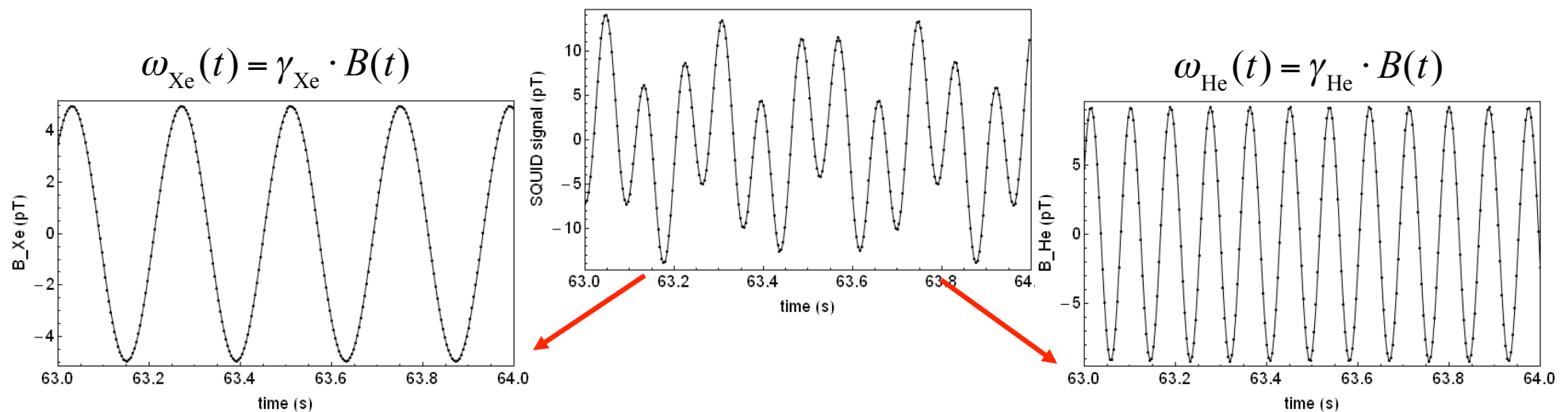
$$\Phi(t) = \int_0^t \omega(t') dt'$$

Phase difference:

$$\Delta\Phi(t) = \Phi_{\text{He}}(t) - \frac{\gamma_{\text{He}}}{\gamma_{\text{Xe}}} \cdot \Phi_{\text{Xe}}(t) = \Phi_0 \stackrel{!}{=} \text{const.}$$

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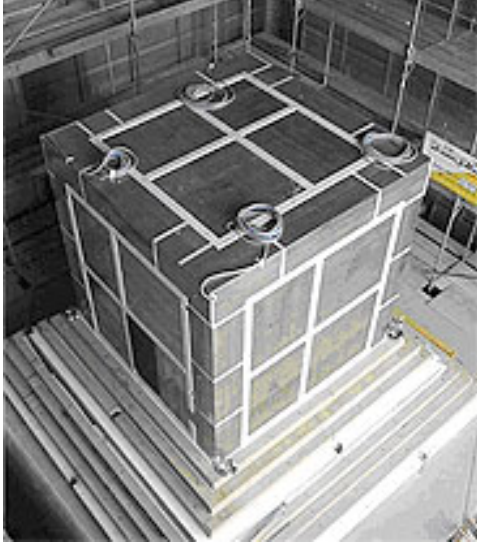
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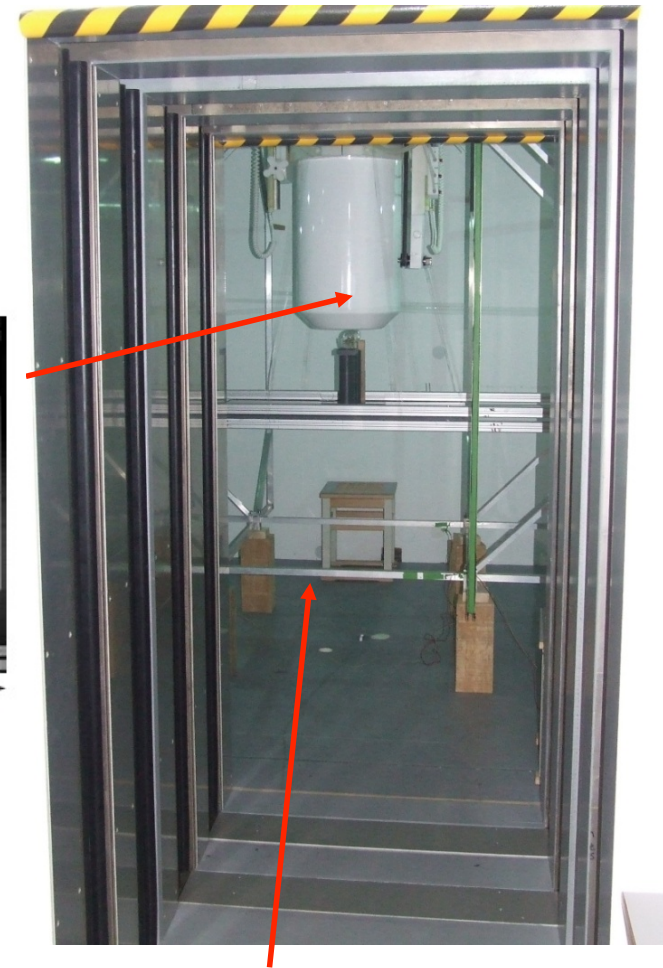
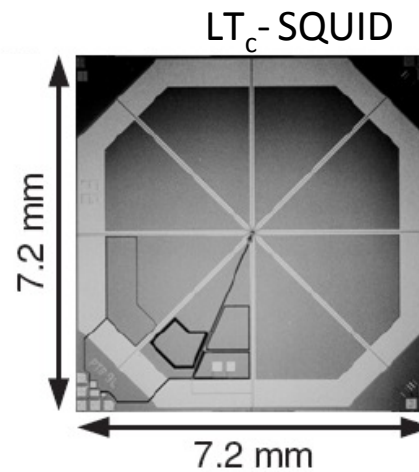
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# Experimental Setup



7 layered magnetically shielded room  
(residual field < 1 nT)

*J. Bork, et al., Proc. Biomag 2000, 970 (2000).*

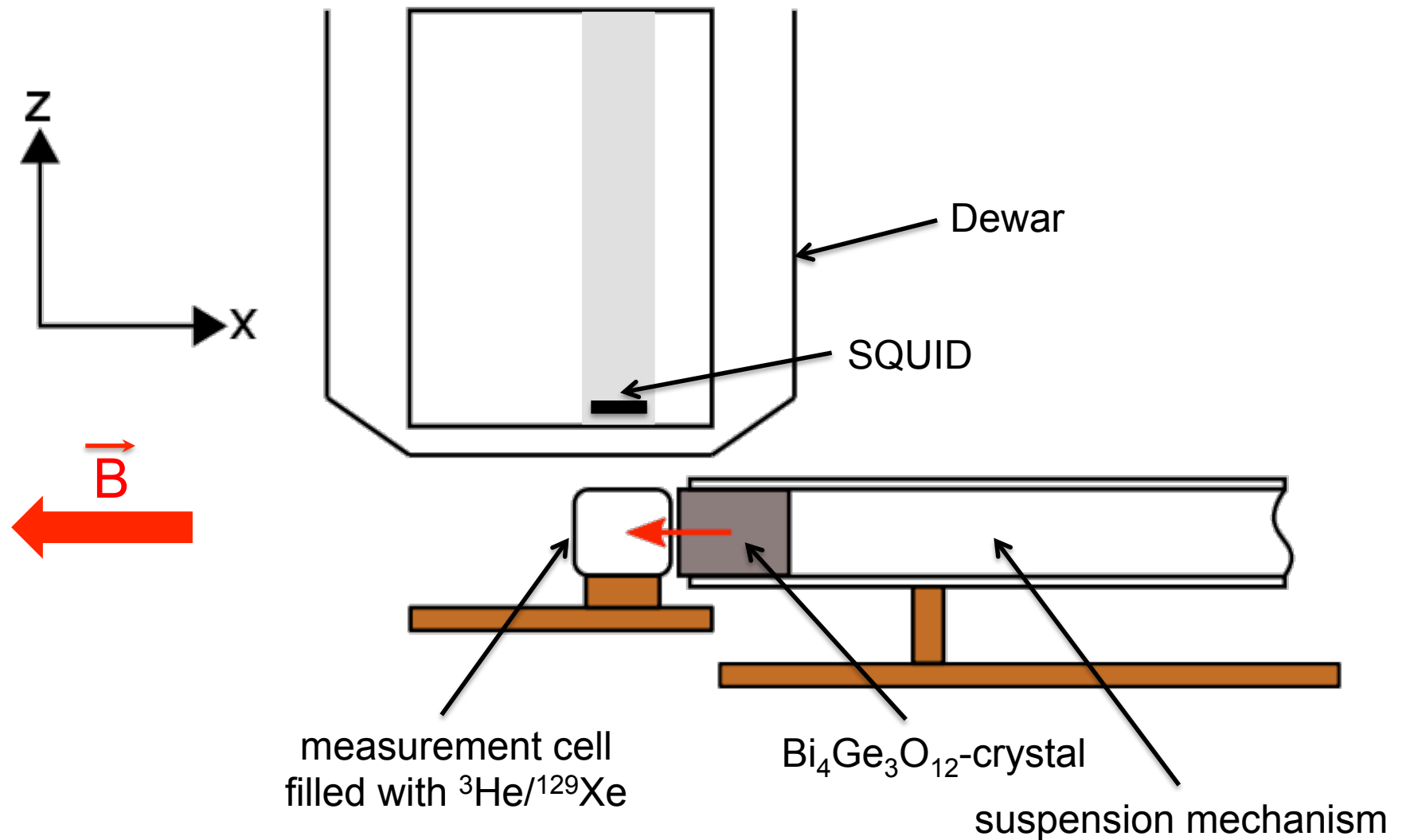


magn. guiding field  $\approx 350$  nT (Helmholtz-coils)

$$|\vec{\nabla} B_{x,y,z}| \approx 20 \text{ pT/cm}$$

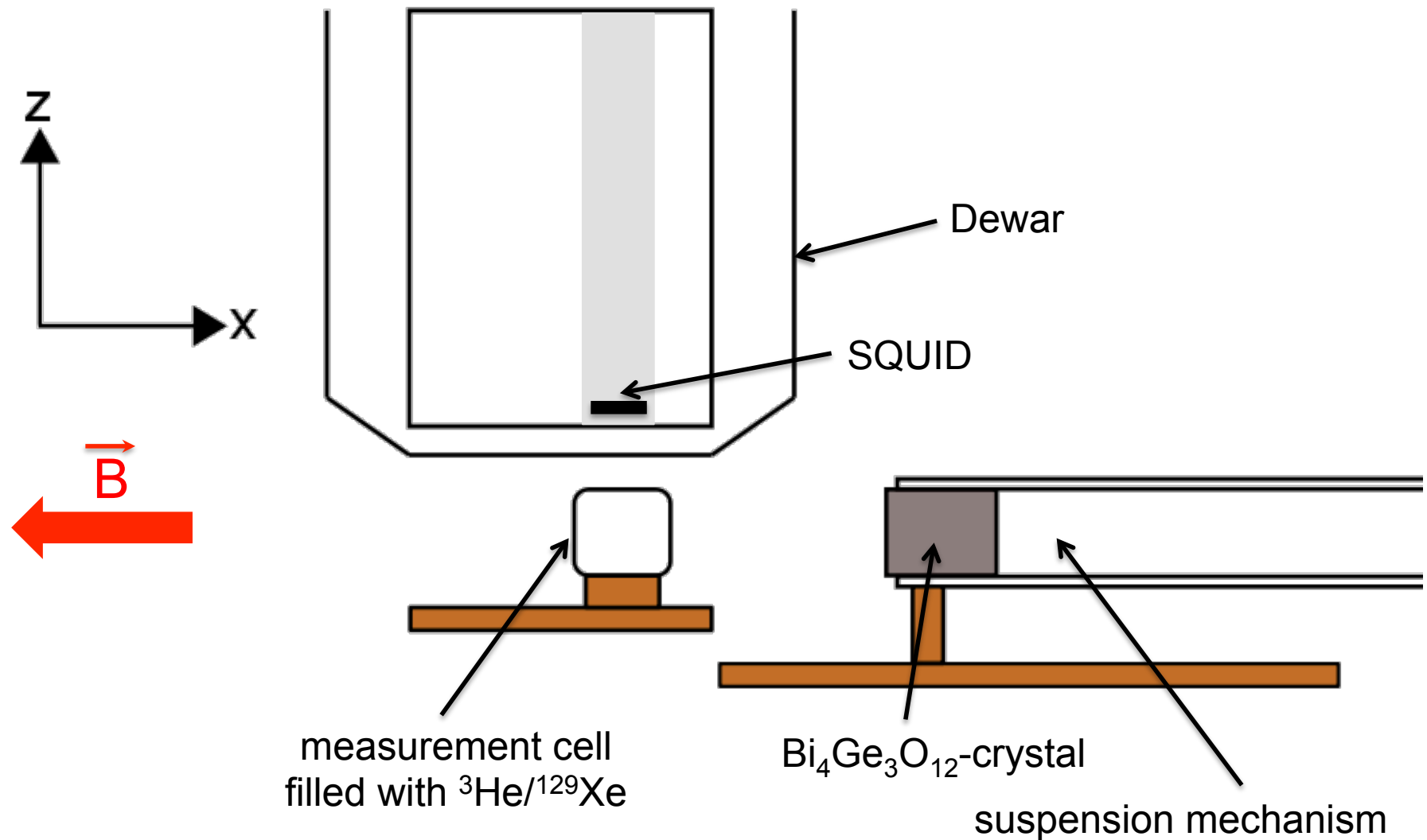


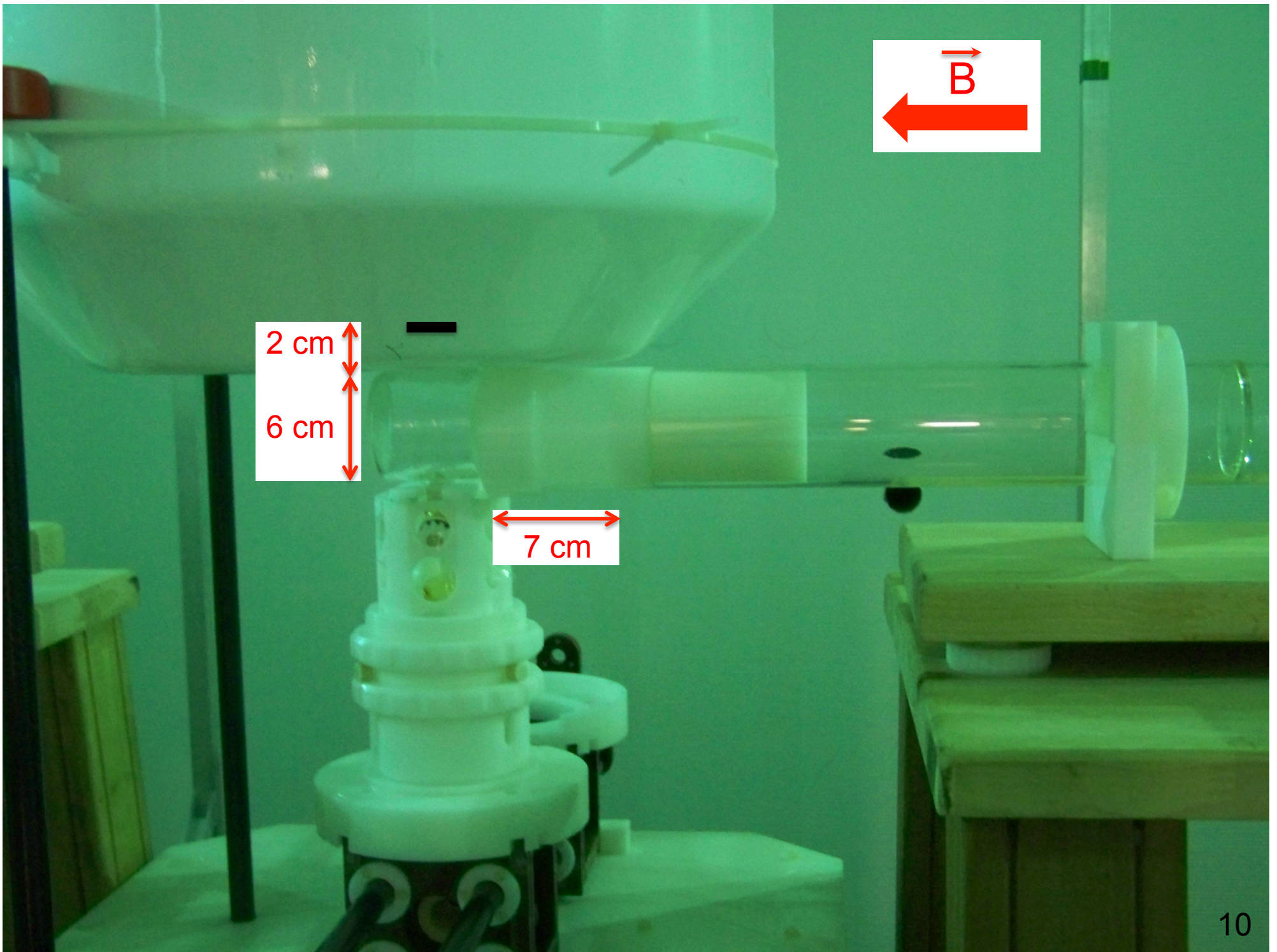
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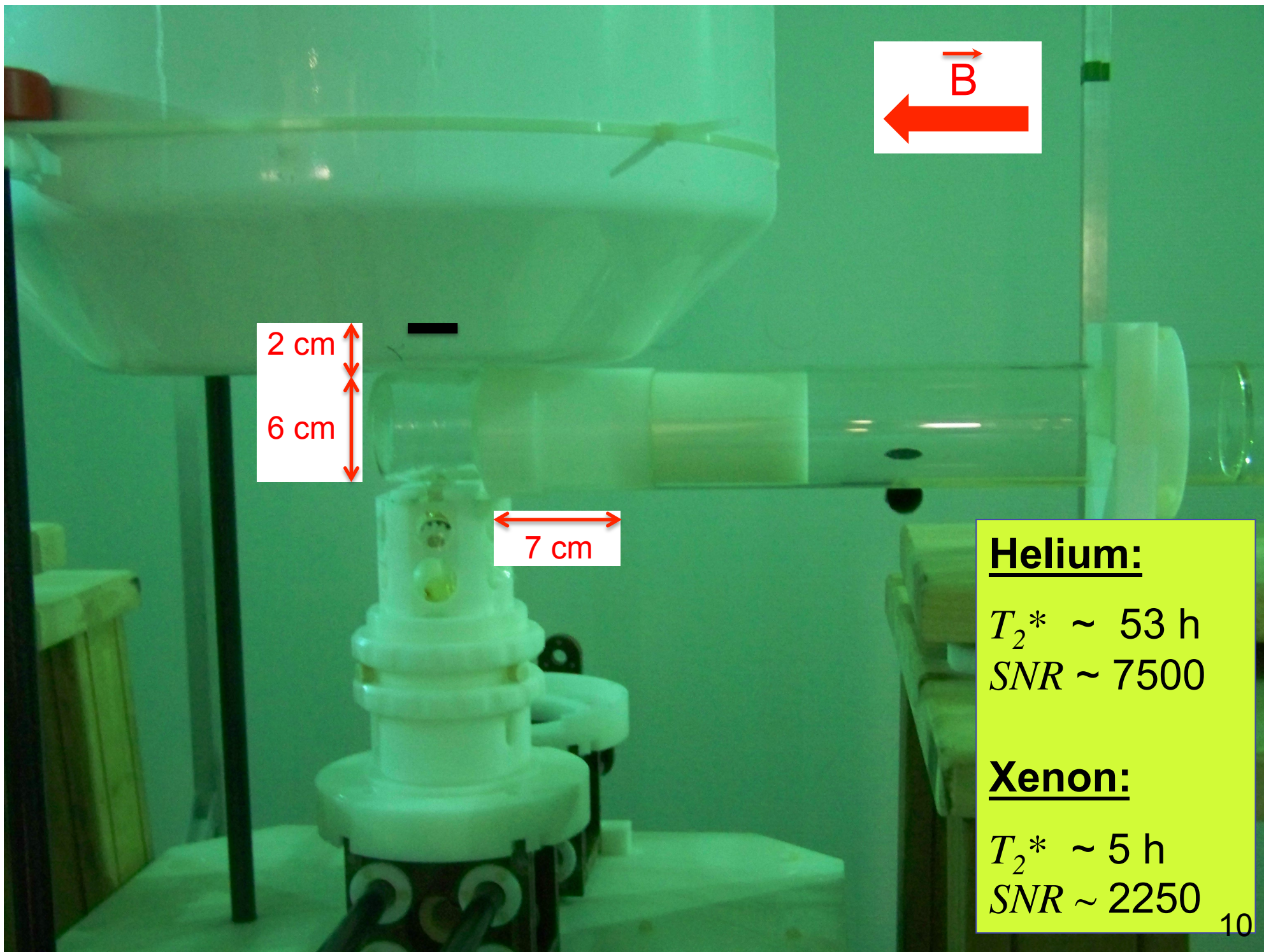




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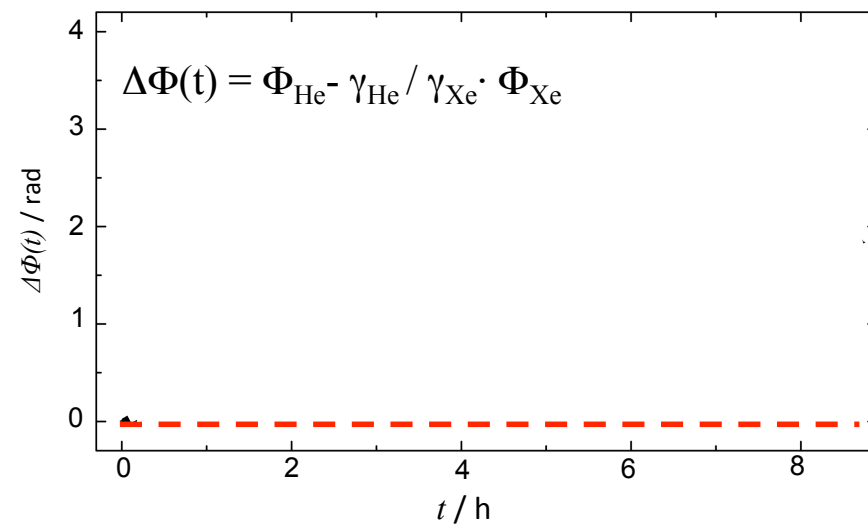




# Data Analysis

1. To cancel magnetic field influence we calculate the weighted phase difference:

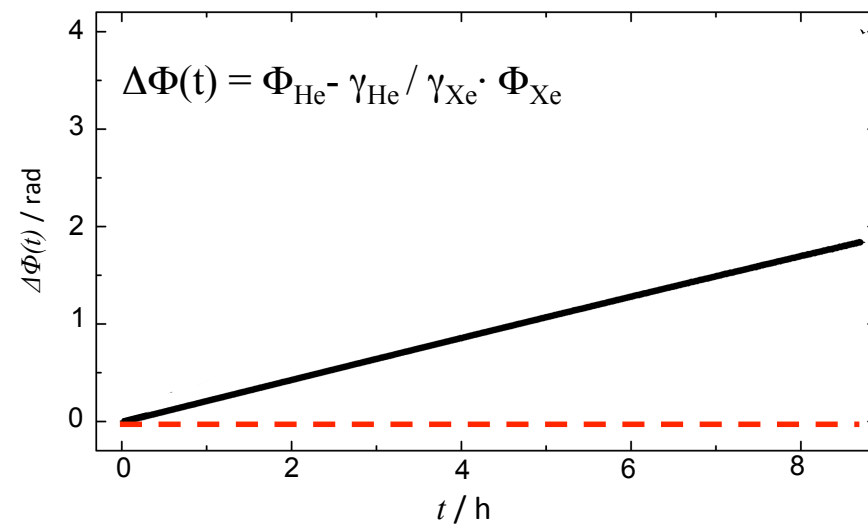
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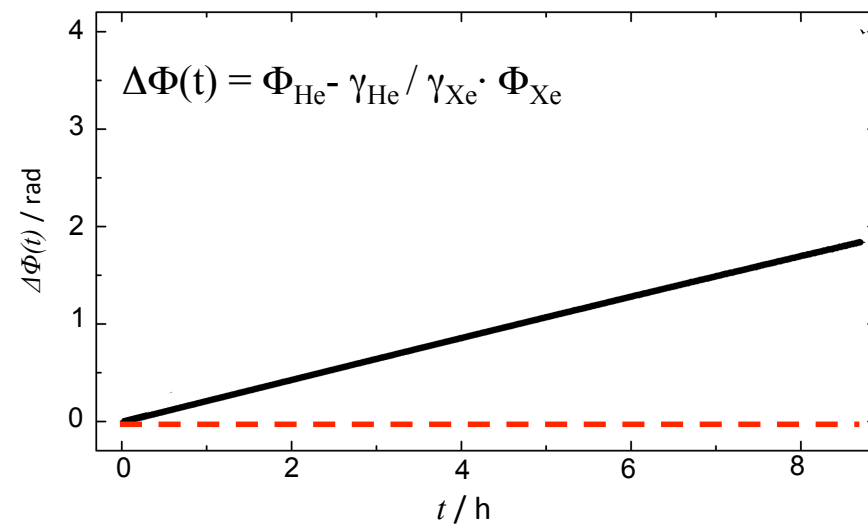
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2. Temporal dependence can be described by:

$$f(t) = c + a_{\text{lin}} \cdot t + a_{\text{He}} \cdot e^{-t/T_{2,\text{He}}^*} + a_{\text{Xe}} \cdot e^{-t/T_{2,\text{Xe}}^*}$$



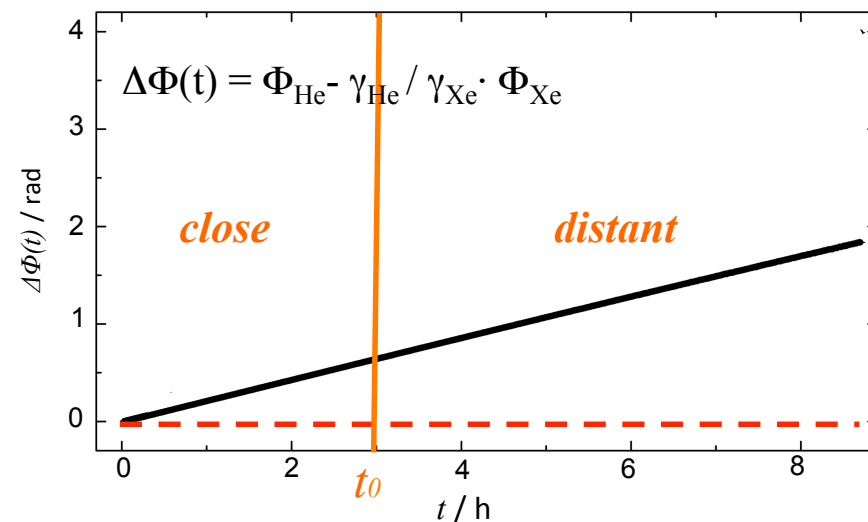
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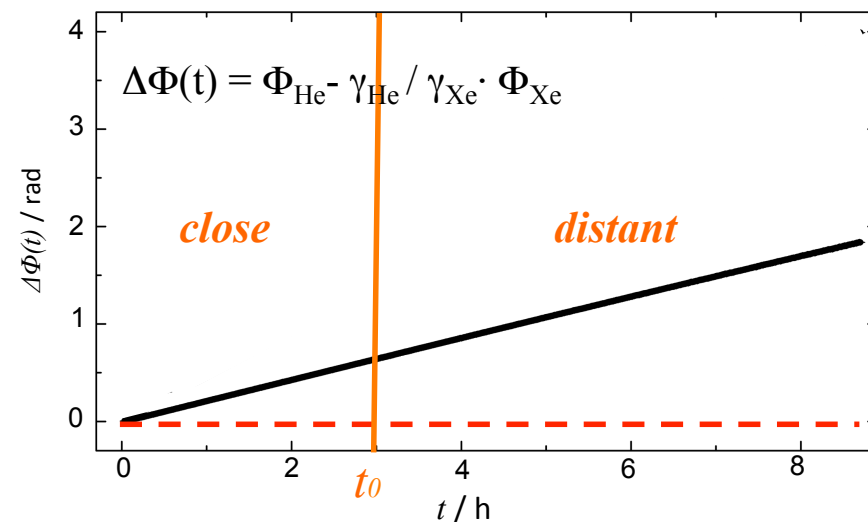
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$$\Rightarrow \Delta\nu_{\text{sp}} = \frac{\Delta\omega_{\text{sp}}}{2\pi(1 - \gamma_{\text{He}} / \gamma_{\text{Xe}})}$$





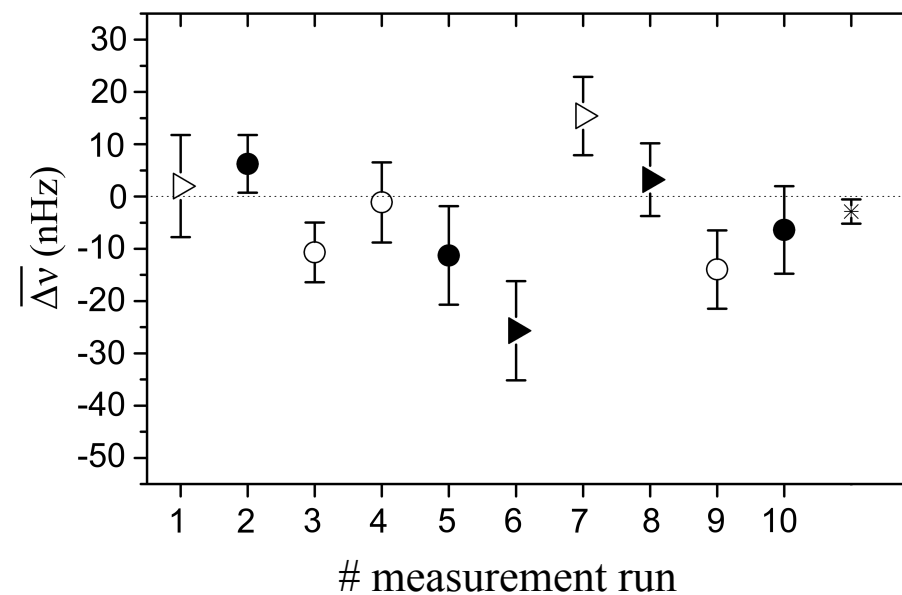
# Summary Results

## September 2010:

10 measurements (~9 hours)

gap = 2.2 mm

sample:  $\text{Bi}_4\text{Ge}_3\text{O}_{12}$  with  $\rho = 7.13 \text{ g/cm}^3$



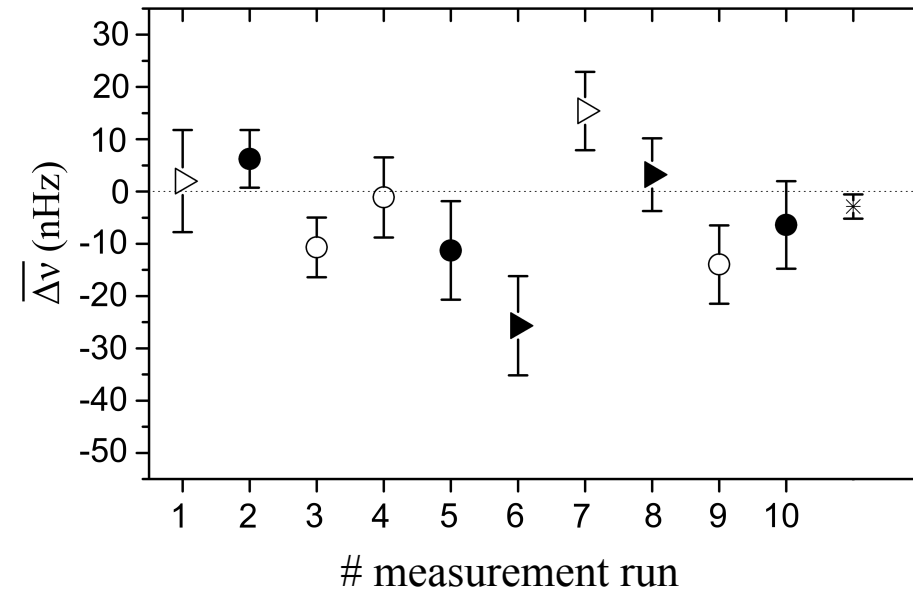
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$$\Rightarrow \Delta v_{\text{sp}} = (-2.9 \pm 3.5) \text{ nHz} \rightarrow \delta(\Delta v_{\text{sp}})_{\text{corr}} = 7.1 \text{ nHz (95\% CL)}$$

K. Tullney et al.  
PRL 111, 100801 (2013)

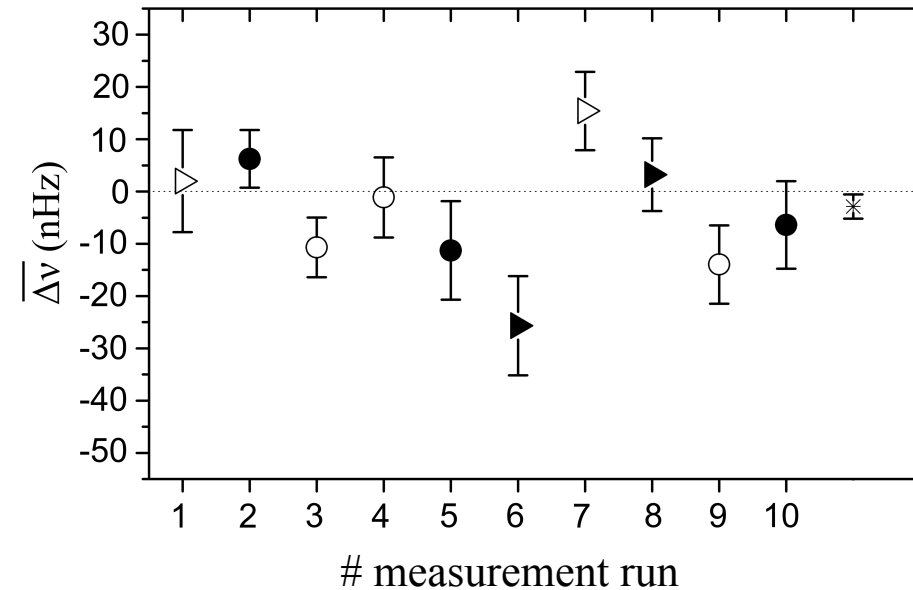
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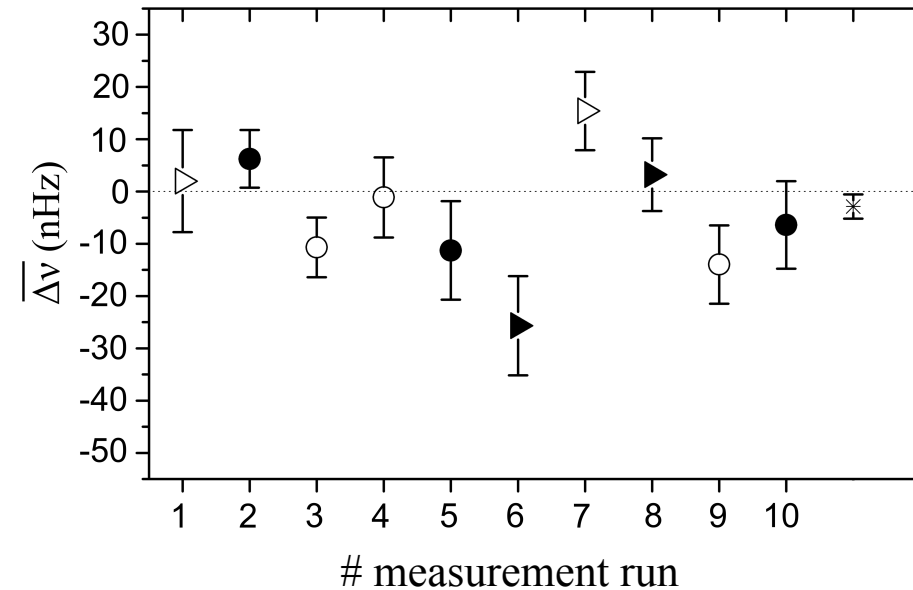
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**Potential:** 
$$V_{\text{sp}}(\vec{r}) = \frac{\hbar^2 g_s g_p}{8\pi m_n} \left( \frac{\vec{r}}{r} \cdot \vec{\sigma} \right) \left( \frac{1}{r\lambda} + \frac{1}{r^2} \right) \cdot e^{-r/\lambda}$$

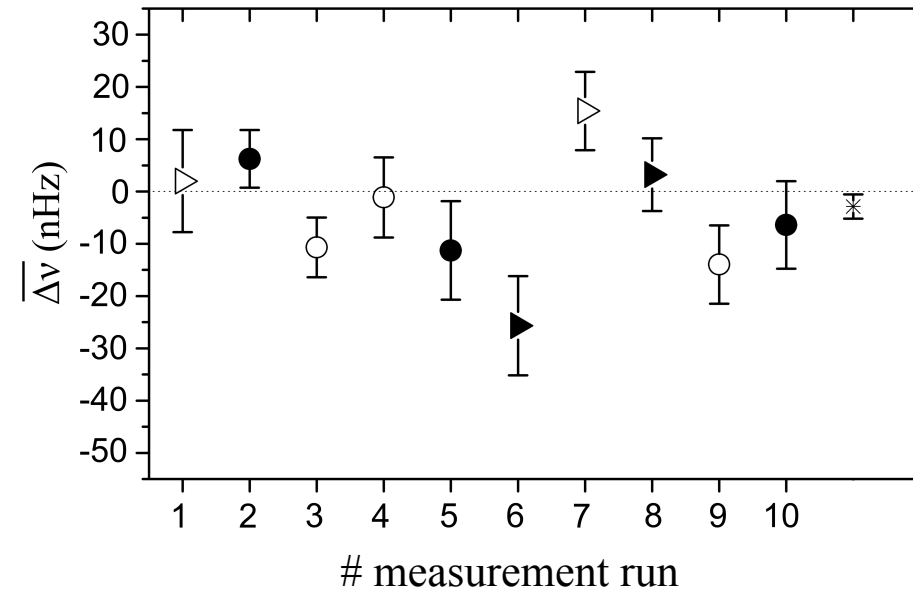
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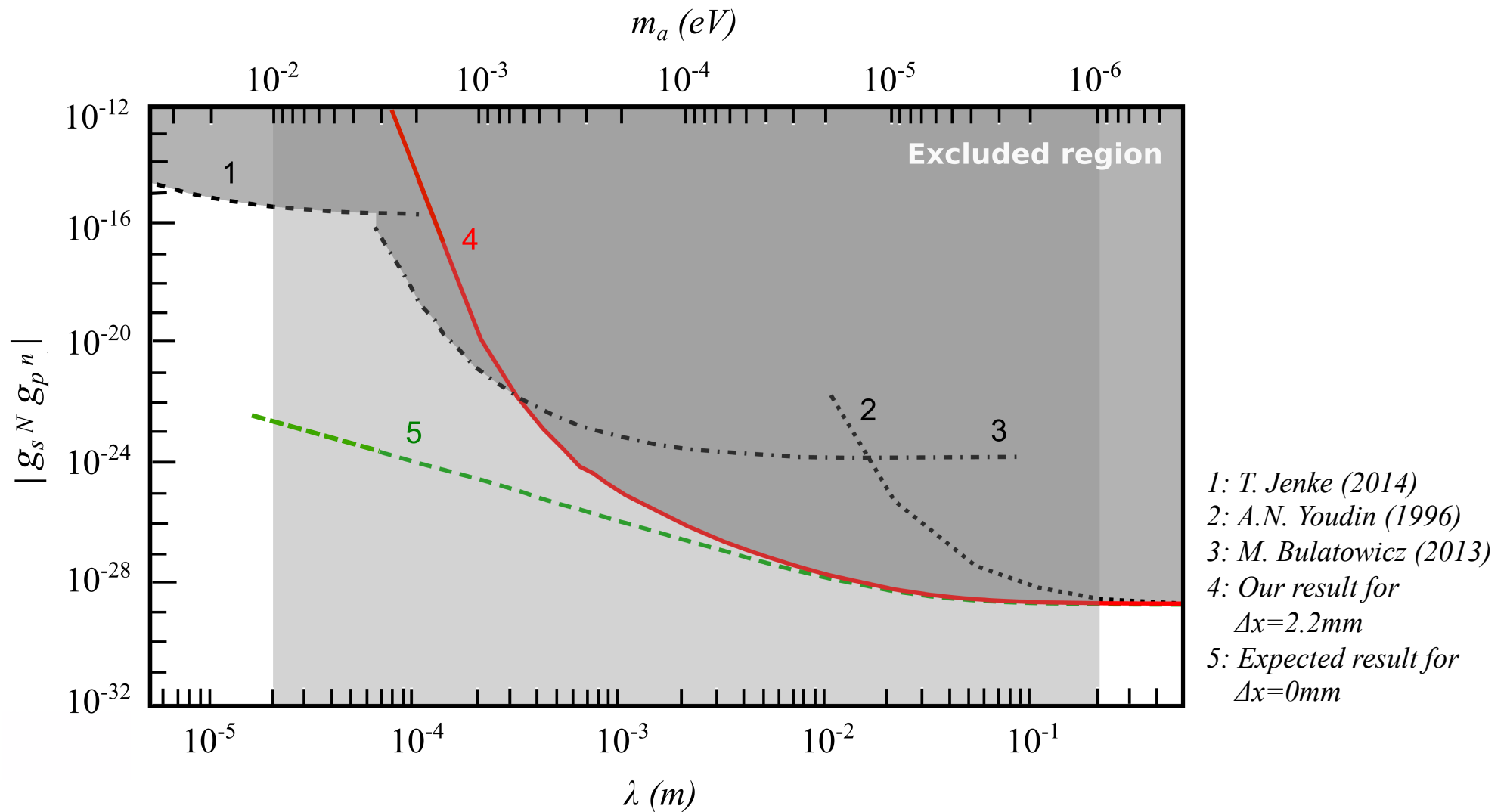
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$$\delta(\Delta v_{\text{sp}}) > 2 \langle V_{\text{sp}} \rangle / \hbar \Rightarrow g_s g_p(\lambda) < \frac{8\pi^2 \cdot m_n \cdot V_{\text{cell}} \cdot \delta(\Delta v_{\text{sp}})}{\hbar \cdot N \cdot \langle V^*(\lambda) \rangle}$$

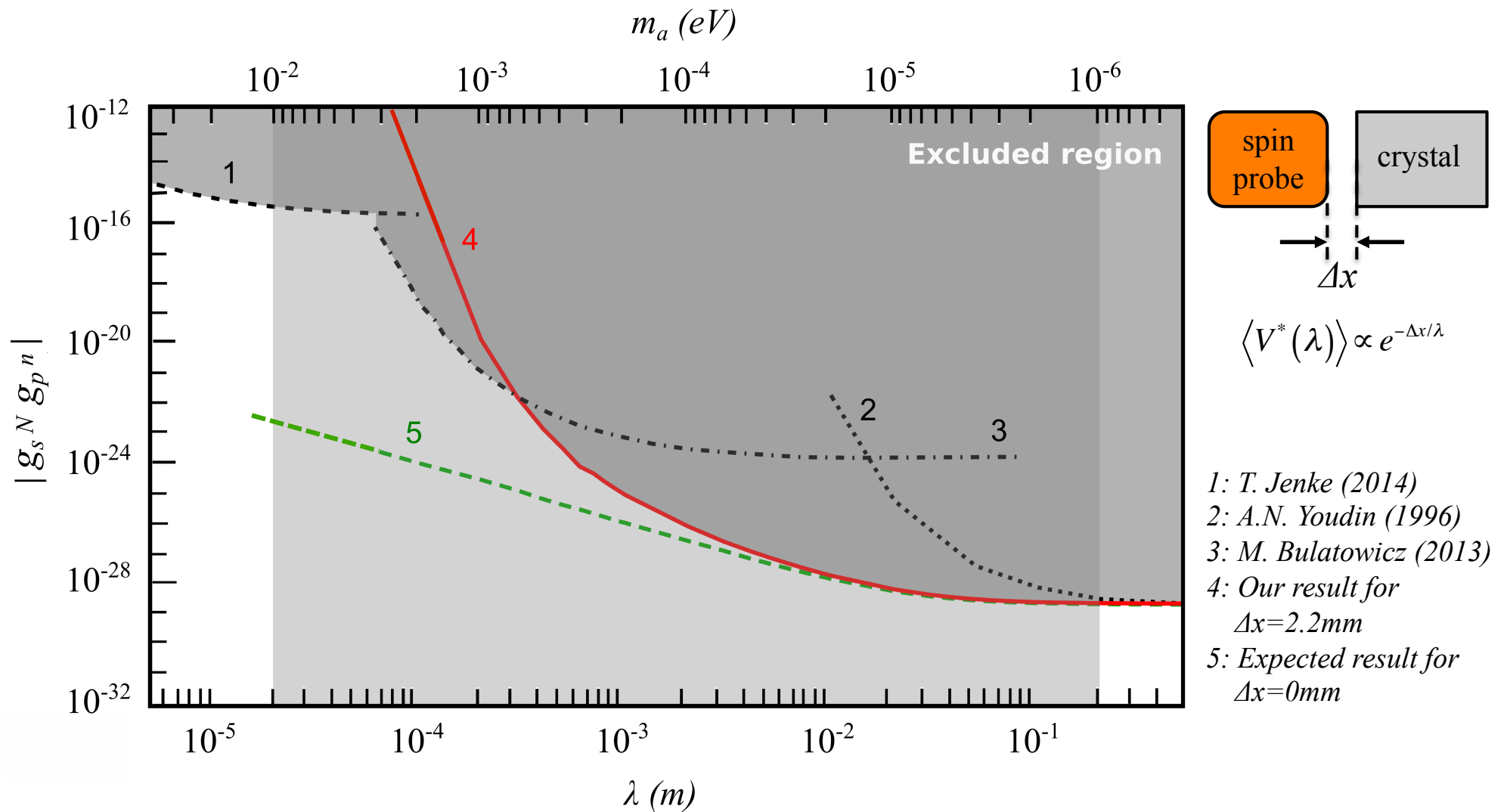
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PRL 111, 100801 (2013)

**Potential:** 
$$V_{\text{sp}}(\vec{r}) = \frac{\hbar^2 g_s g_p}{8\pi m_n} \left( \frac{\vec{r}}{r} \cdot \vec{\sigma} \right) \left( \frac{1}{r\lambda} + \frac{1}{r^2} \right) \cdot e^{-r/\lambda}$$

# Exclusion Plot



# Exclusion Plot



# Summary

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$$\Delta v_{\text{sp}} = (-2.9 \pm 3.6) \text{ nHz} \longrightarrow \delta(\Delta v_{\text{sp}})_{\text{corr}} = 7.1 \text{ nHz (95\% CL)}$$

Result: New upper limit for  $g_S g_P$  in the range  $3 \cdot 10^{-4} \text{ m} < \lambda < 10^{-1} \text{ m}$   
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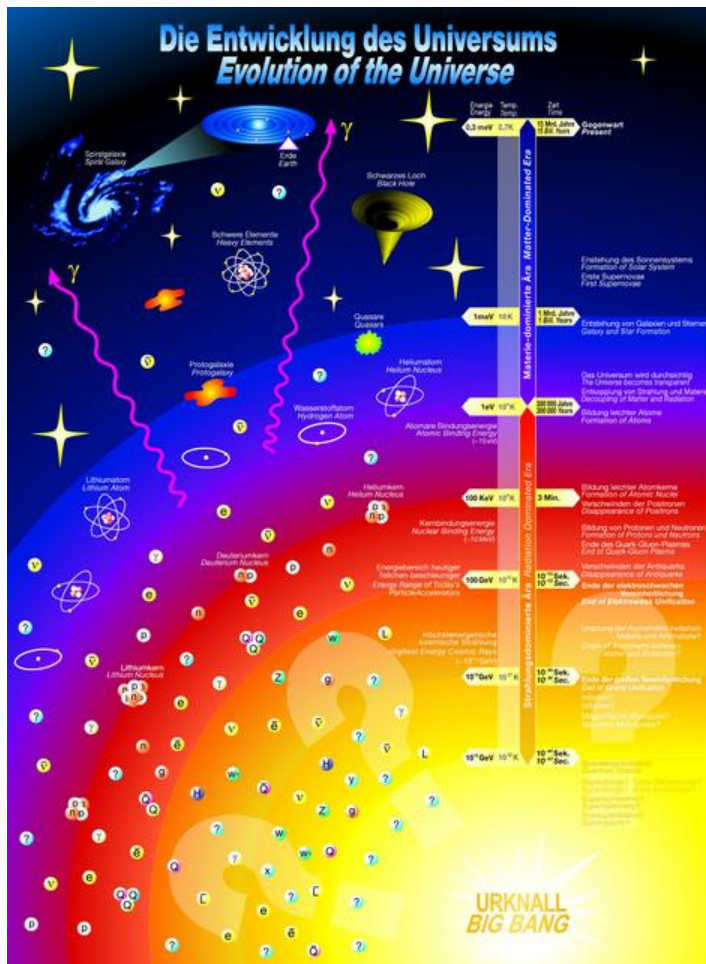
**THANK YOU!**





# Motivation

## Baryon/Anti-Baryon Puzzle

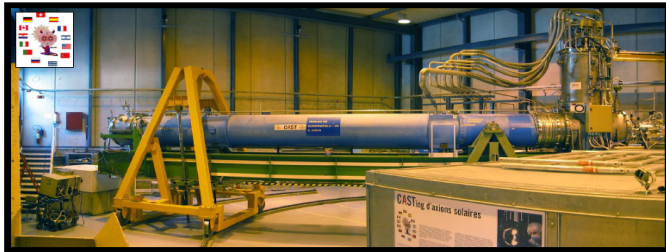
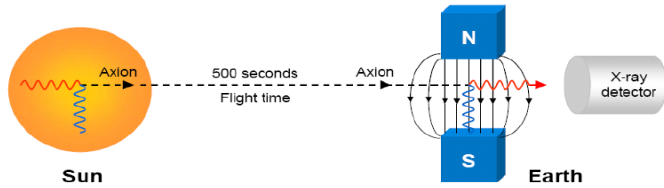


- Baryon/Anti-Baryon Asymmetry developed after  $10^{-10}$  s after the Big Bang
- Observed Asymmetry:  $n_B/n_\gamma=10^{-10}$
- Sacharow criteria:
  - ✓ Violation of baryon number
  - ✓ Thermal disequilibrium
  - ✓ CP-/C-Violation
- Calculated Asymmetry:  $n_B/n_\gamma=10^{-18}$

# Motivation

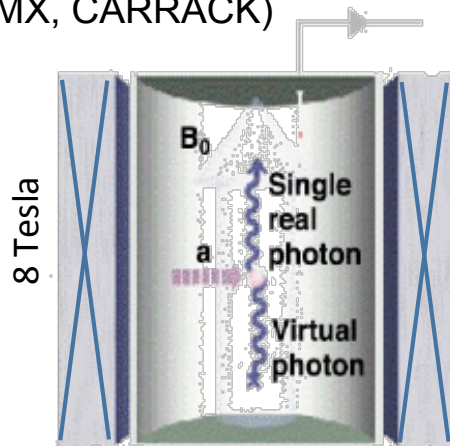
## Solar axions

Helioscope experiment (CAST)



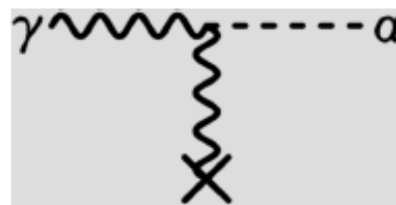
## Galactic axions

microwave cavity experiment  
(ADMX, CARRACK)



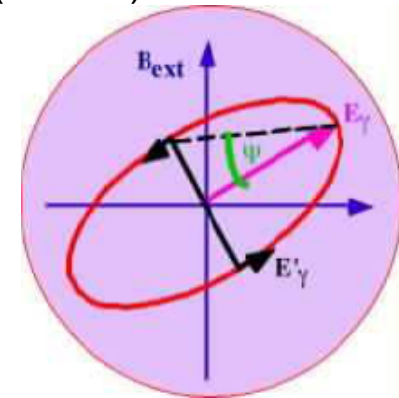
**DIRECT**  
search of the  
axion via the  
Primakoff effect

Primakoff effect  
Conversion of an  
axion into a photon

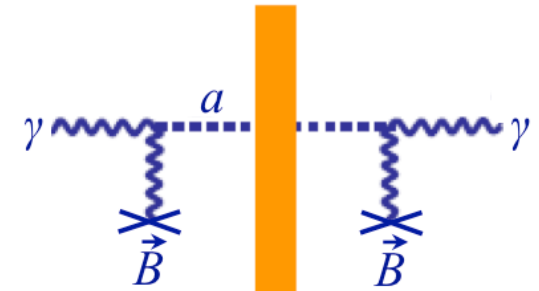


## Labor axions

- Polarization experiments (PVLAS)



- „Light shining through a wall“ experiment (BFRT, OSQAR, ALPS, LIPPS, GammaeV)





# Motivation

## Axion Potential:

Yukawa type potential with monopole-dipole coupling [1]

$$V(r) = \kappa \hat{n} \cdot \vec{\sigma} \left( \frac{1}{\lambda r} + \frac{1}{r^2} \right) e^{-r/\lambda}$$

with:  $\kappa = \frac{\hbar^2 g_s g_p}{8\pi m_n}$  ,  $\lambda = \frac{\hbar}{m_a c}$

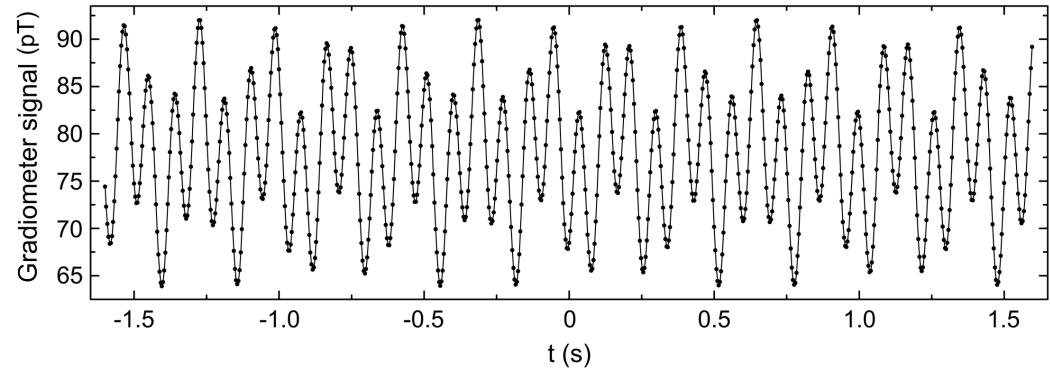
polar vector    axial vector

$$\hat{n} \cdot \vec{\sigma} \Rightarrow \begin{matrix} \uparrow & \nearrow \\ \hat{n} & \vec{\sigma} \end{matrix} \Rightarrow \begin{matrix} \text{P}(\hat{n} \cdot \vec{\sigma}) = -(\hat{n} \cdot \vec{\sigma}) \\ \text{T}(\hat{n} \cdot \vec{\sigma}) = -(\hat{n} \cdot \vec{\sigma}) \end{matrix} \Rightarrow \begin{matrix} \text{P-, T-violation} \\ \text{CP-violation} \end{matrix}$$

# Data Analysis

## Fit to raw data:

$$f^i(t) = a_{s,He}^i \sin(\omega_{He}^i \cdot t) + a_{c,He}^i \cos(\omega_{He}^i \cdot t) \\ + a_{s,Xe}^i \sin(\omega_{Xe}^i \cdot t) + a_{c,Xe}^i \cos(\omega_{Xe}^i \cdot t) \\ + (c_0^i + c_{lin}^i \cdot t)$$



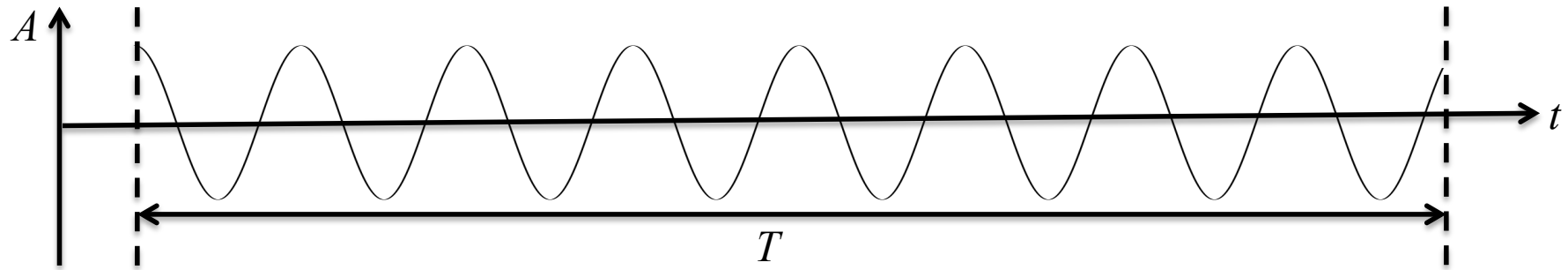
↓  $^3\text{He}$ ,  $^{129}\text{Xe}$  phase

$$\Phi_{He/Xe}^i = \arctan\left(\frac{a_{s,He/Xe}^i}{a_{c,He/Xe}^i}\right) + n_{He/Xe}^i \cdot 2\pi, \quad n_{He/Xe}^i \approx \bar{\omega}_{He/Xe} \cdot t \quad (\text{number of periods since begin of measurement})$$

↓ weighted phase difference

$$\Delta\Phi = \Phi_{He} - \frac{\gamma_{He}}{\gamma_{Xe}} \cdot \Phi_{Xe} = \Phi_0$$

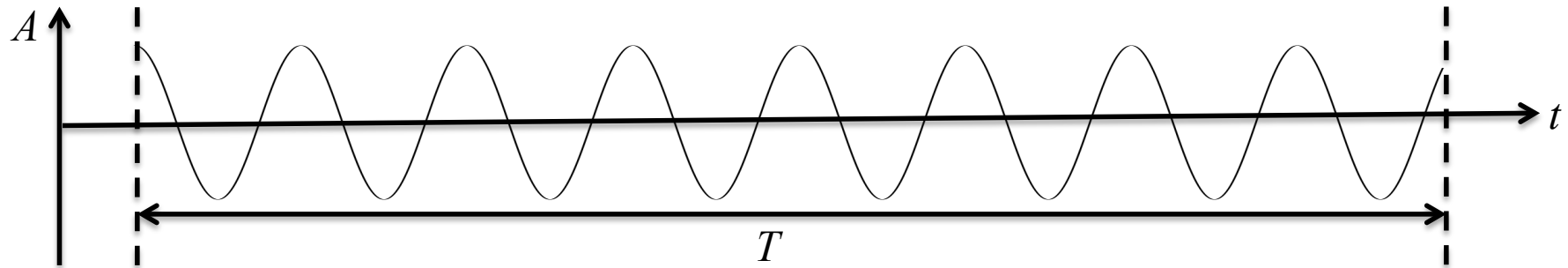
# Precision of measurement



If the noise  $w[n]$  is **Gaussian distributed**, the “Cramer-Rao-Lower Bound (CRLB)” sets the lower limit on the variance  $\sigma_v$ :

$$\sigma_v \geq \frac{\sqrt{12}}{2\pi \cdot \text{SNR} \cdot \sqrt{\nu_{\text{BW}}} \cdot T^{3/2}} \cdot \sqrt{C(T_2^*)}$$

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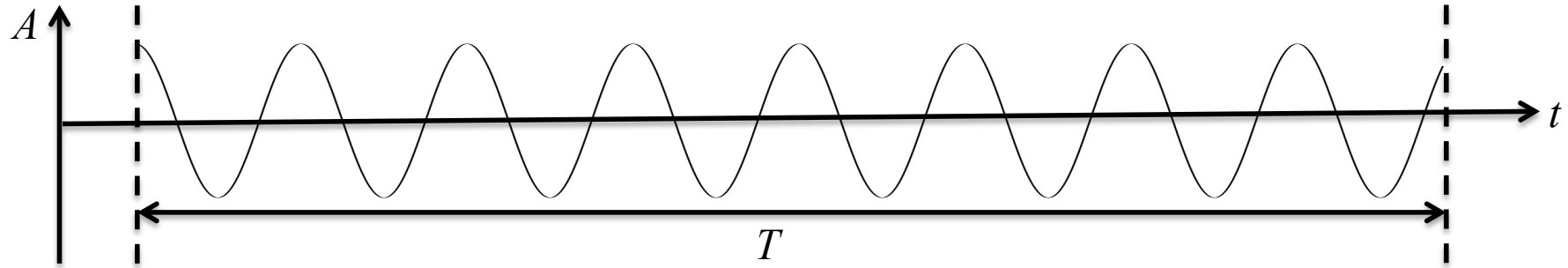
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**Example:**

SNR = 2300,  $\nu_{\text{BW}} = 1$  Hz,  $T = 9$  h

$$\Rightarrow \sigma_v \approx 10^{-10} \text{ Hz}$$

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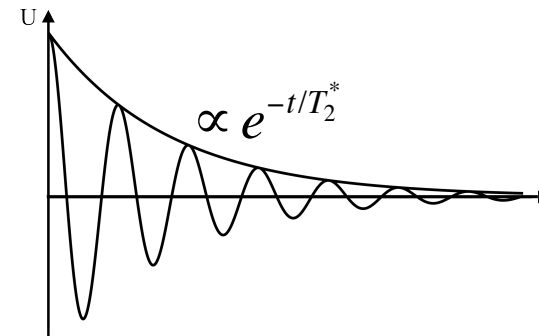
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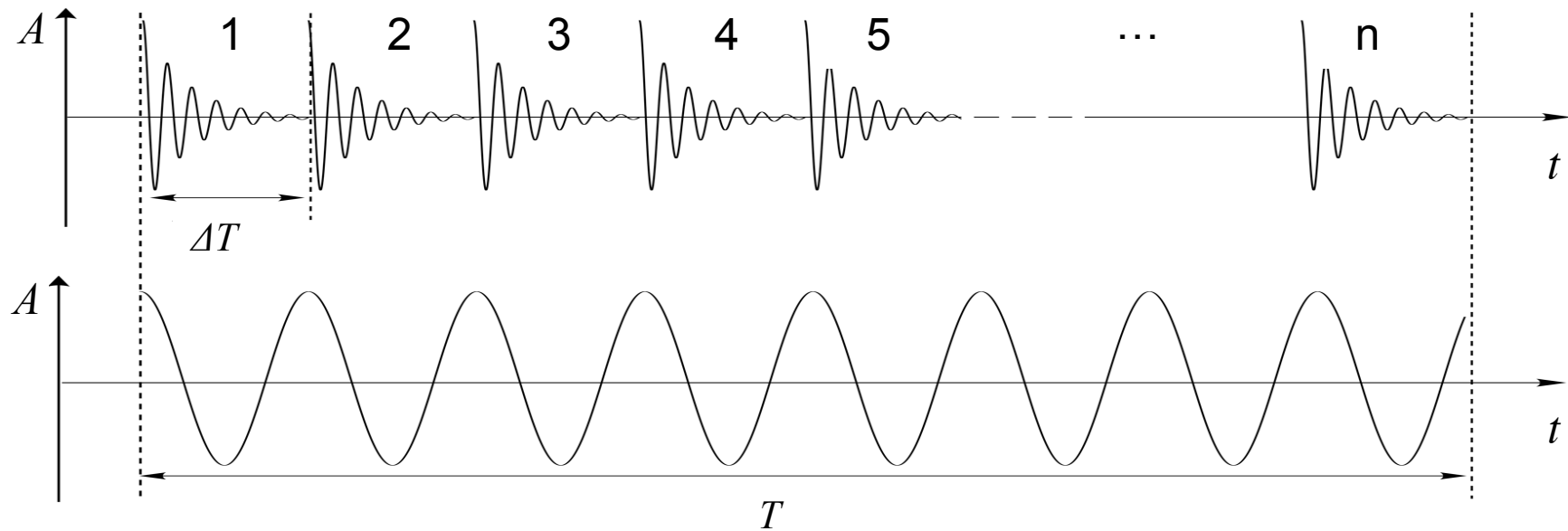


$$(T_2^*)^{-1} \propto V^{4/3} p |\vec{\nabla} B_1|^2 \quad \text{vs.} \quad \text{SNR} \propto pV$$

$$p \approx \text{mbar}, \quad V \approx 200 \text{ cm}^3, \quad B_1 \approx \mu\text{T}, \quad |\vec{\nabla} B_1|^2 \approx \frac{pT}{\text{cm}}$$

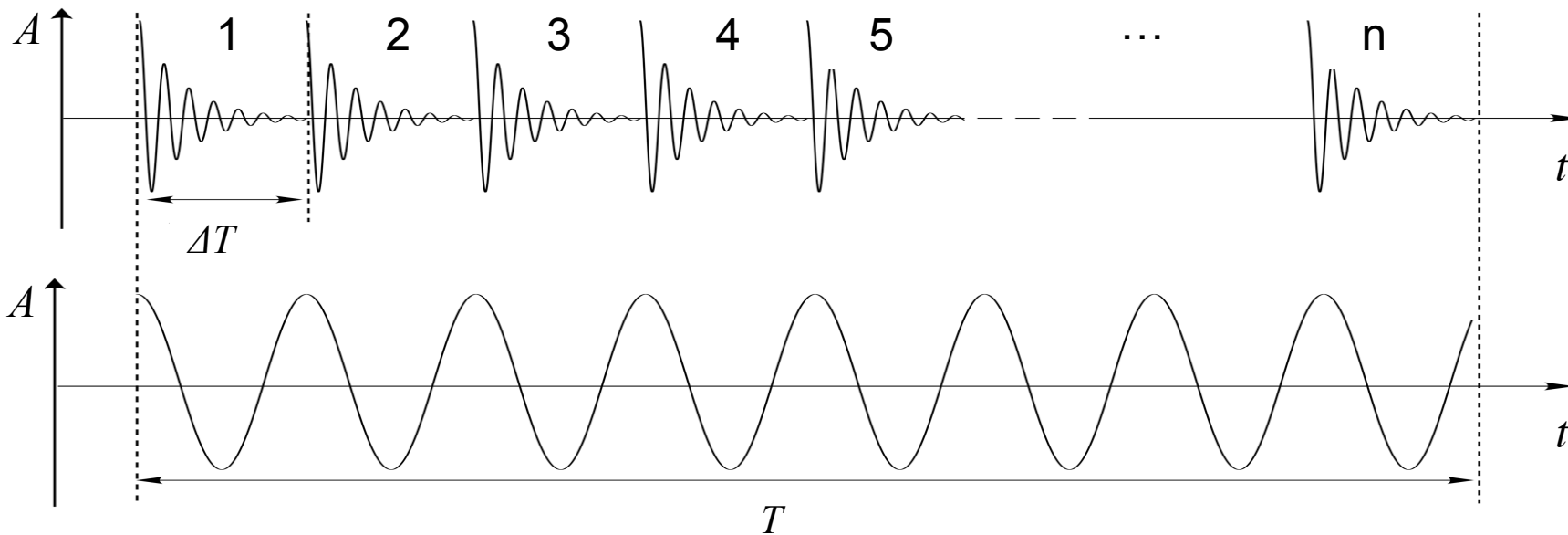
# Sensitivity

Spin coherence time: long ( $T$ ) vs. short ( $\Delta T$ )



# Sensitivity

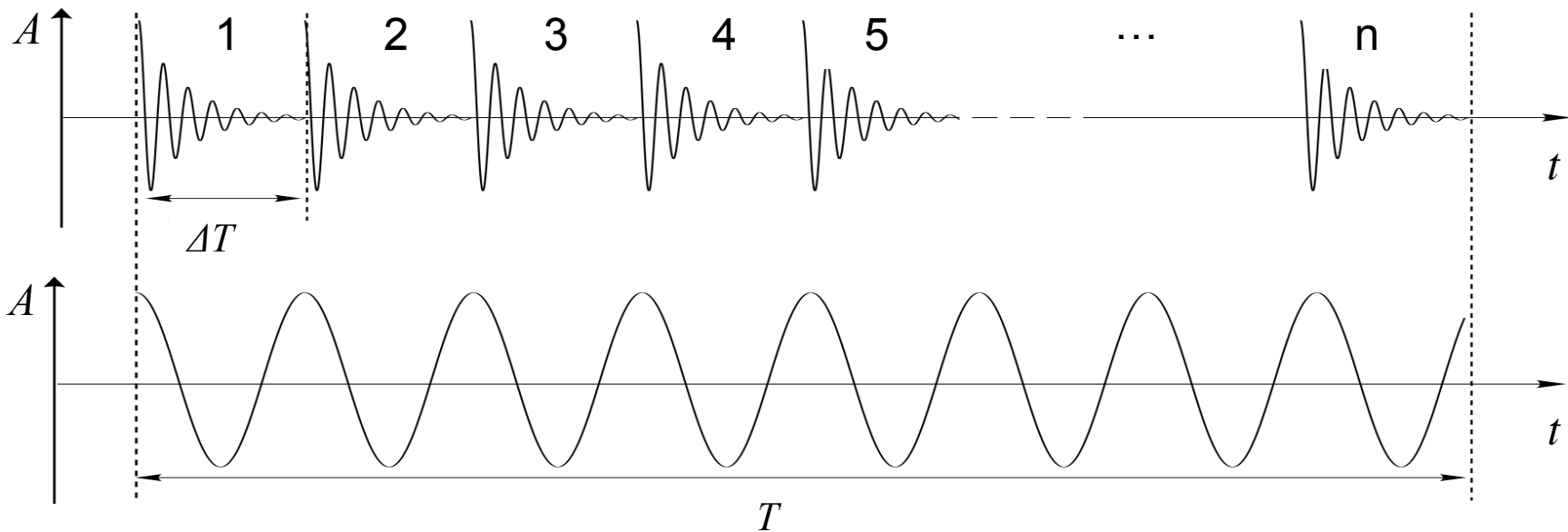
Spin coherence time: long ( $T$ ) vs. short ( $\Delta T$ )



$$\sigma_{v, \text{long}} \propto \frac{1}{T^{3/2}}$$

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Spin coherence time: long ( $T$ ) vs. short ( $\Delta T$ )



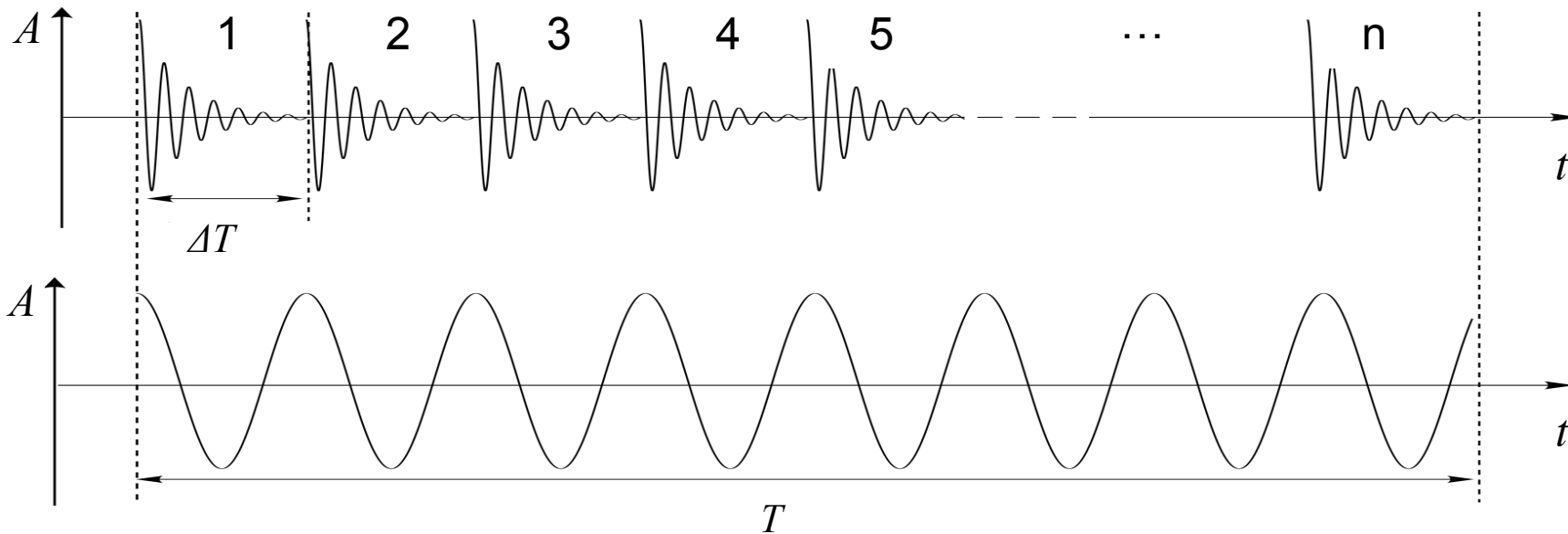
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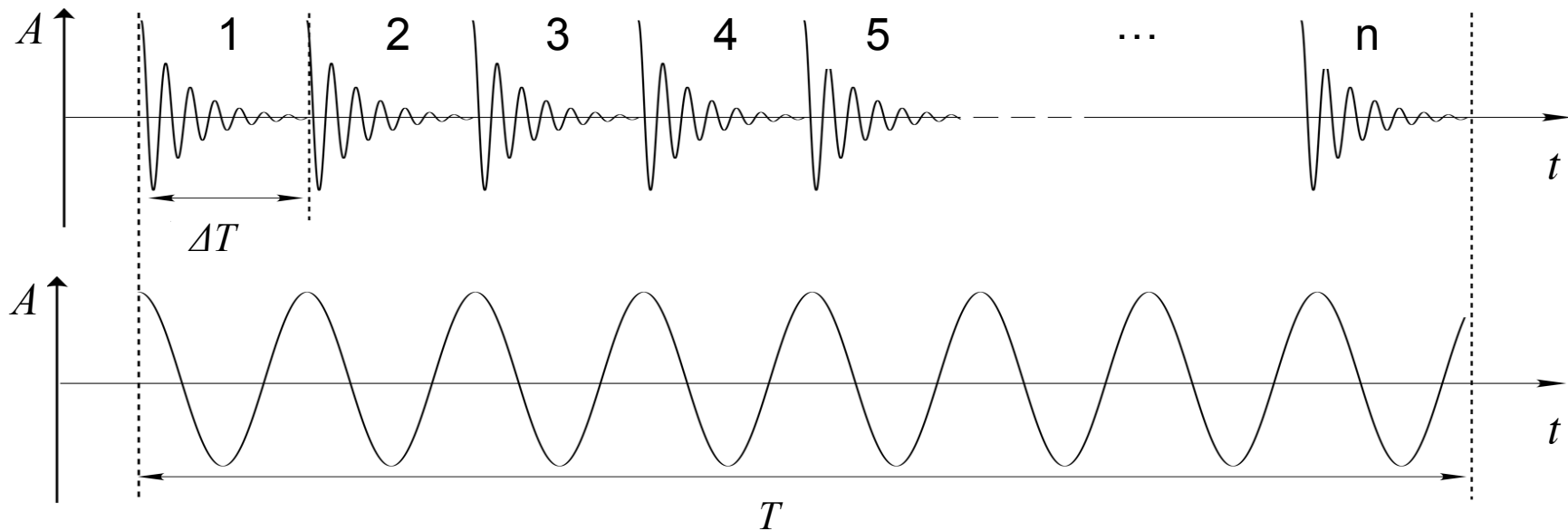
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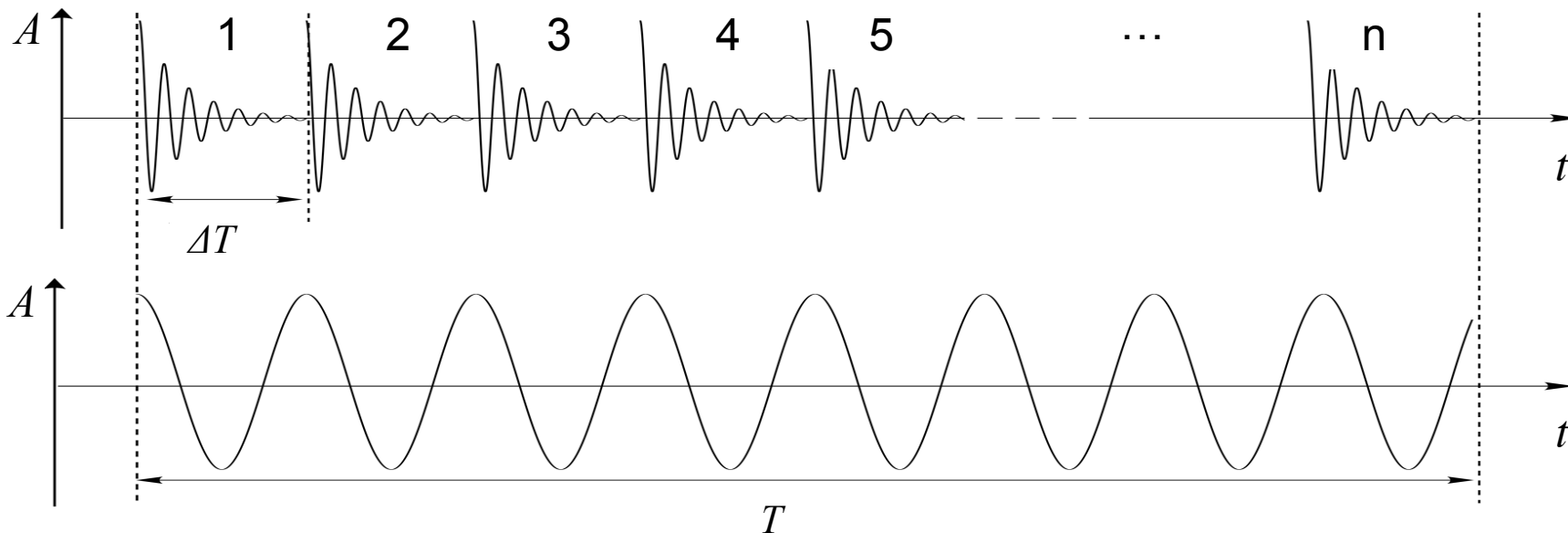
$$\sigma_{v, \text{long}} \propto \frac{1}{T^{3/2}}$$

$$\sigma_{v, \text{short}, n} \propto \frac{1}{\sqrt{n}} \frac{1}{\Delta T^{3/2}} = \left( \frac{1}{T^{3/2}} \right) \cdot \frac{T}{\Delta T}$$

$$T = n \cdot \Delta T$$

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Spin coherence time: long ( $T$ ) vs. short ( $\Delta T$ )



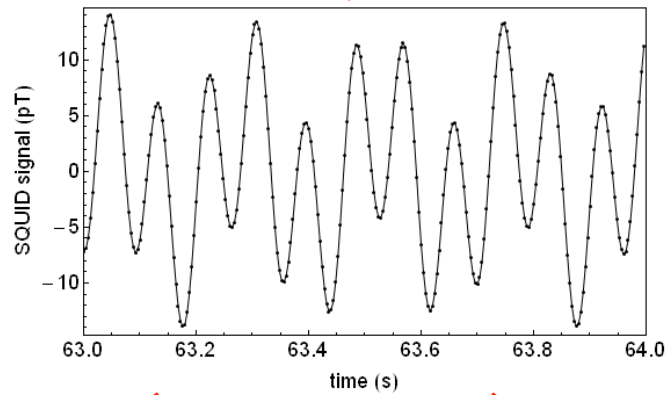
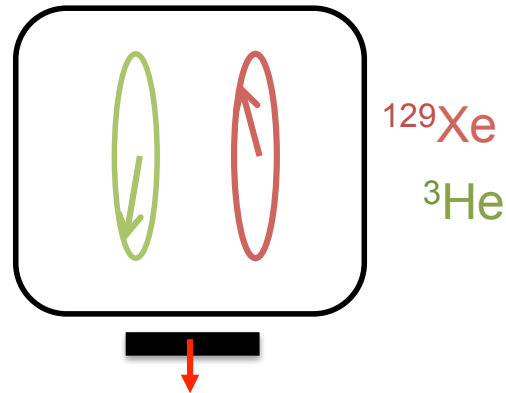
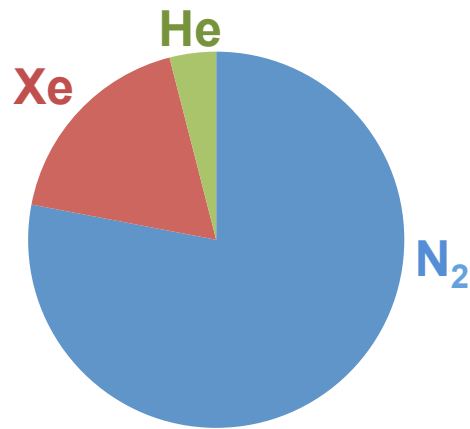
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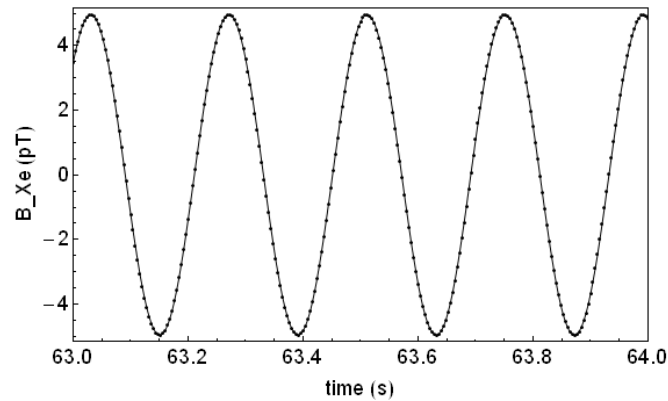
**Example:** SNR = 10000:1,  $f_{BW} = 1$  Hz,  $T = 24$  h,  $\Delta T = 5$  min  $\rightarrow \sigma_{v, \text{short}} = 300 \cdot \sigma_{v, \text{long}}$

# Data Acquisition

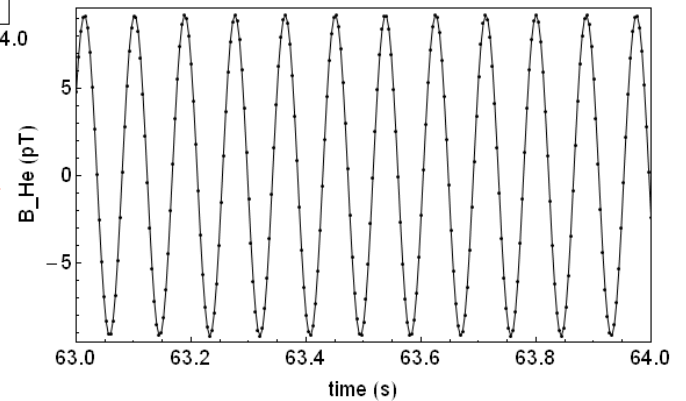
$^3\text{He}$ ,  $^{129}\text{Xe}$  +  $\text{N}_2$  (buffer gas)  
@  $p_{\text{tot}} = 31 \dots 37$  mbar



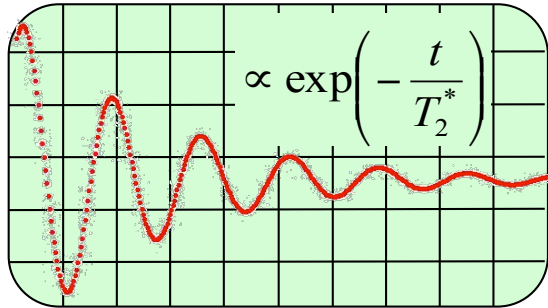
$$\omega_{\text{Xe}}(t) = \gamma_{\text{Xe}} \cdot B(t)$$



$$\omega_{\text{He}}(t) = \gamma_{\text{He}} \cdot B(t)$$



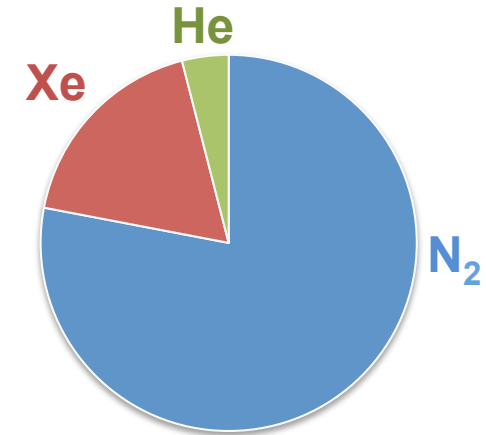
# Transversal Relaxationtime



**Free spin precession:**

$$\text{signal} \propto \exp\left(-t/T_2^*\right)$$

$T_2^*$ : ms  $\rightarrow$  h



wall collisions [1]

$$T_{1,\text{wand}} \approx 8 \text{ h} - 100 \text{ h}$$

$$\frac{1}{T_2^*} = \frac{1}{T_{1,\text{wand}}} + \frac{1}{T_{1,\text{vdW}}} + \frac{1}{T_{2,\text{grad}}}$$

van der Waals molecules [2]

$$T_{1,\text{vdW}}^{\text{Xe}} \approx 13 \text{ h}$$

field gradients [3]

$$T_{2,\text{grad}} \propto \frac{1}{V^{4/3} p |\vec{\nabla} B_1|^2}$$

**CRLB:**  $\sigma_v \propto \frac{1}{\text{SNR} \cdot T^{3/2}}$

$$\text{signal} \propto p \cdot V$$

$\Rightarrow$

$$p \sim \text{mbar}, V \sim 100 \text{ cm}^3, B_1 \sim \mu\text{T} \quad [4]$$

[1] J. Schmiedeskamp et al., Eur. Phys. J. D 38, 2006.

[2] B. Chann et al., PRL. 88(11), 2002.

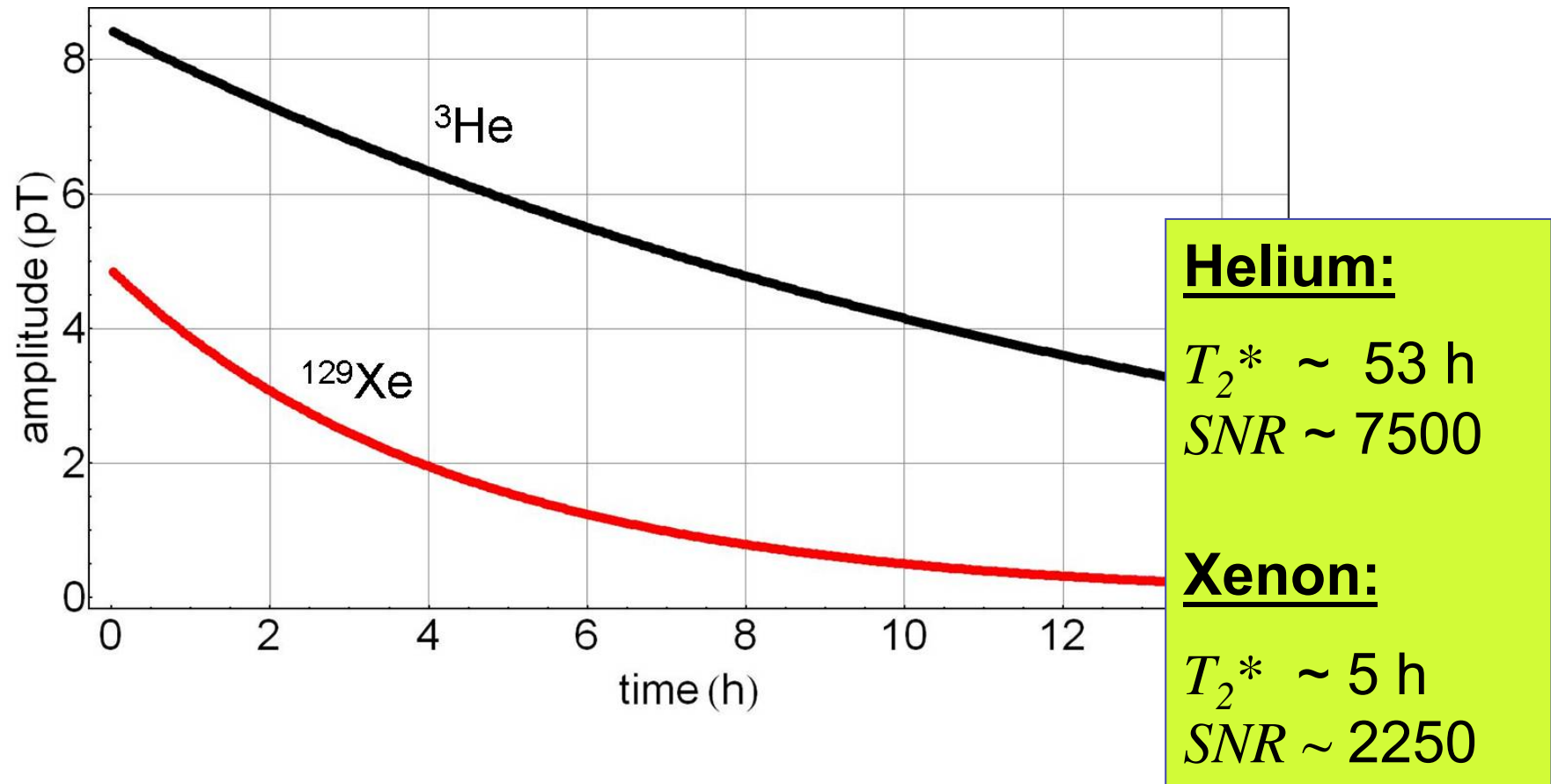
[3] G. D. Cates, S. R. Schaefer, W. Happer, Phys. Rev. A 37, 1988.

[4] C. Gemmel et al., Eur. Phys. J. D 57, 2010.

# Transversal Relaxationtime

$R \approx 3 \text{ cm}$

$p_{\text{He}} \approx 2 \text{ mbar}$ ,  $p_{\text{Xe}} \approx 8 \text{ mbar}$ ,  $p_{\text{N}_2} \approx 35 \text{ mbar}$



# Summary Results

## September 2010:

10 measurements (~9 hours)

gap = 2.2 mm

sample:  $\text{Bi}_4\text{Ge}_3\text{O}_{12}$  crystal with density  $\rho = 7.13 \text{ g/cm}^3$

*K. Tullney et al.  
PRL 111, 100801 (2013)*

$$\Rightarrow \Delta v_{\text{sp}} = (-2.9 \pm 3.5) \text{ nHz} \rightarrow \delta(\Delta v_{\text{sp}})_{\text{corr}} = 7.1 \text{ nHz (95\% CL)}$$

**Analysis:** 
$$V_{\text{sp}}(\vec{r}) = \frac{\hbar^2 g_s g_p}{8\pi m_n} \left( \frac{\vec{r}}{r} \cdot \vec{\sigma} \right) \left( \frac{1}{r\lambda} + \frac{1}{r^2} \right) \cdot e^{-r/\lambda}$$

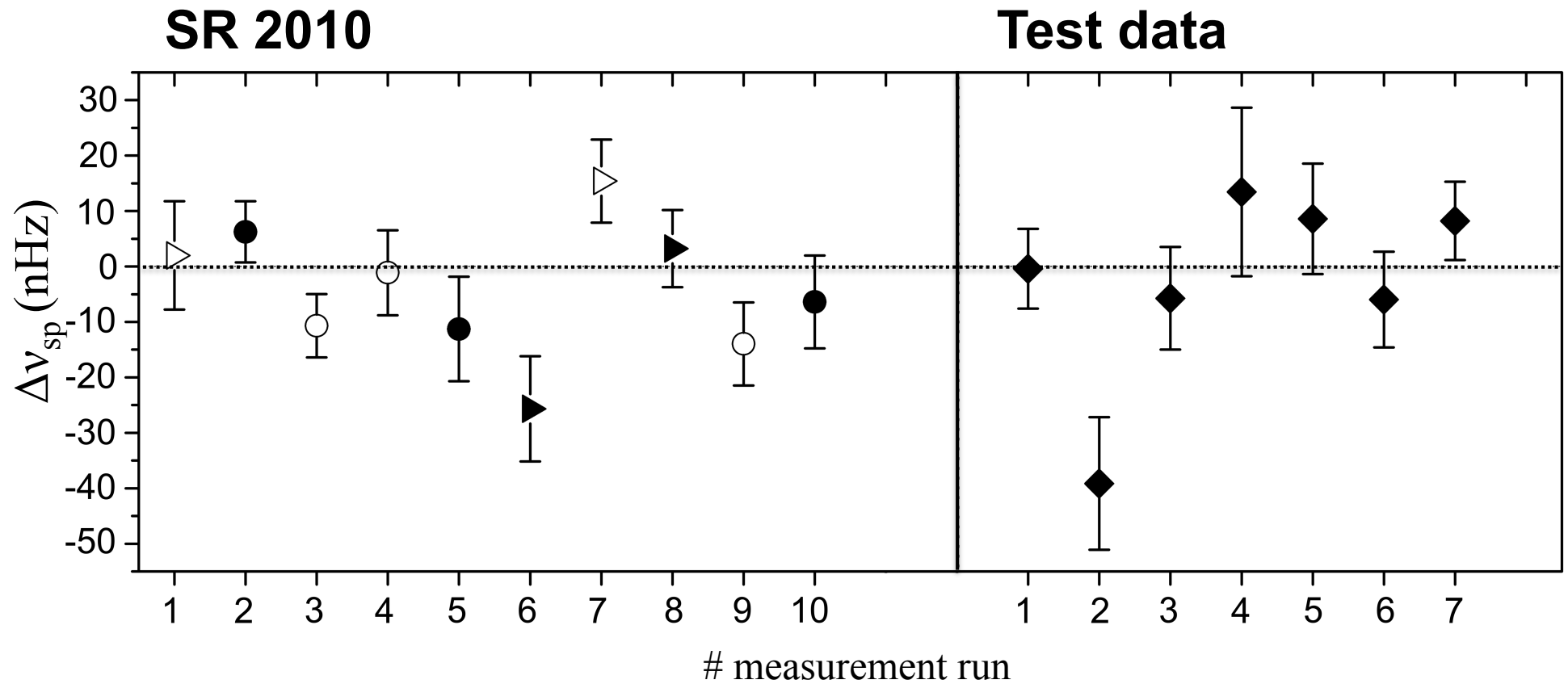
$$\langle V_{\text{sp}} \rangle = \frac{\hbar^2 g_s g_p}{8\pi m_n} \int_{\text{VBGO}} \int_{\text{Vcell}} \left( \frac{\vec{r}}{r} \cdot \vec{\sigma} \right) \left( \frac{1}{r\lambda} + \frac{1}{r^2} \right) \cdot e^{-r/\lambda} dV_{\text{cell}} dV_{\text{BGO}} = \frac{\hbar^2 g_s g_p}{8\pi m_n} \langle V^*(\lambda) \rangle$$

The mean potential  $\langle V_{\text{sp}} \rangle$  was calculated numerically for our measurement cells ( $\emptyset = 58$  mm,  $l = 58$  mm). The gap between the inner volume of the measurement cell and the  $\text{Bi}_4\text{Ge}_3\text{O}_{12}$  crystal ( $\emptyset = 60$  mm,  $l = 70$  mm) was 2.2 mm.

$$\Delta(\Delta v_{\text{sp}}) > 2 \langle V_{\text{sp}} \rangle / \hbar \Rightarrow g_s g_p < \frac{8\pi^2 m_n V_{\text{cell}} \Delta(\Delta v_{\text{sp}})}{\hbar N \langle V^*(\lambda) \rangle}$$

# Results

$$\Delta\nu_{sp} = \frac{\Delta\omega_{sp}^w}{2\pi \cdot (1 - \gamma_{\text{He}} / \gamma_{\text{Xe}})}$$

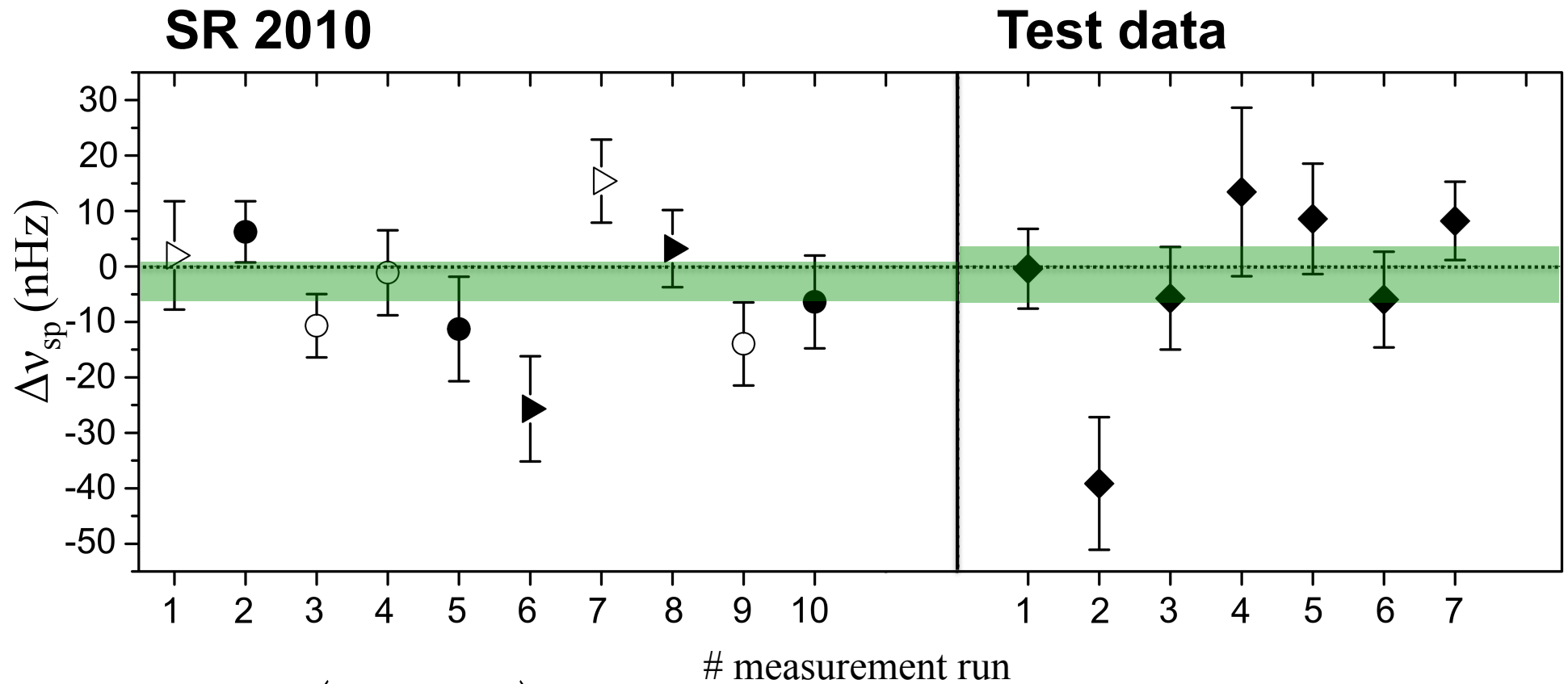


- c → d Left
- d → c Left
- ▶ c → d Right
- ▷ d → c Right



# Results

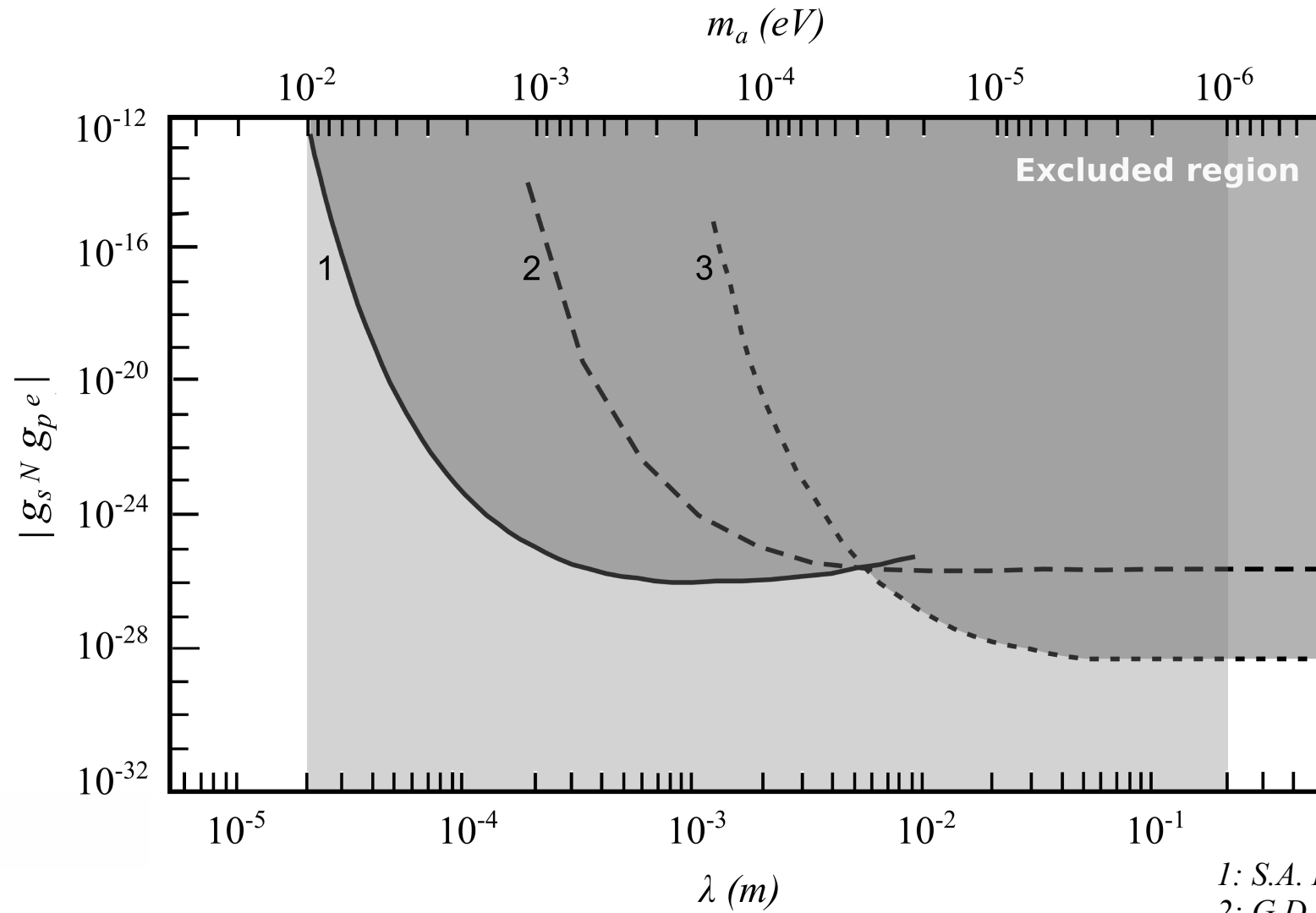
$$\Delta\nu_{sp} = \frac{\Delta\omega_{sp}^w}{2\pi \cdot (1 - \gamma_{\text{He}} / \gamma_{\text{Xe}})}$$



$$\Delta\nu_{\text{Test}} = (-1.4 \pm 5.2) \text{ nHz}$$

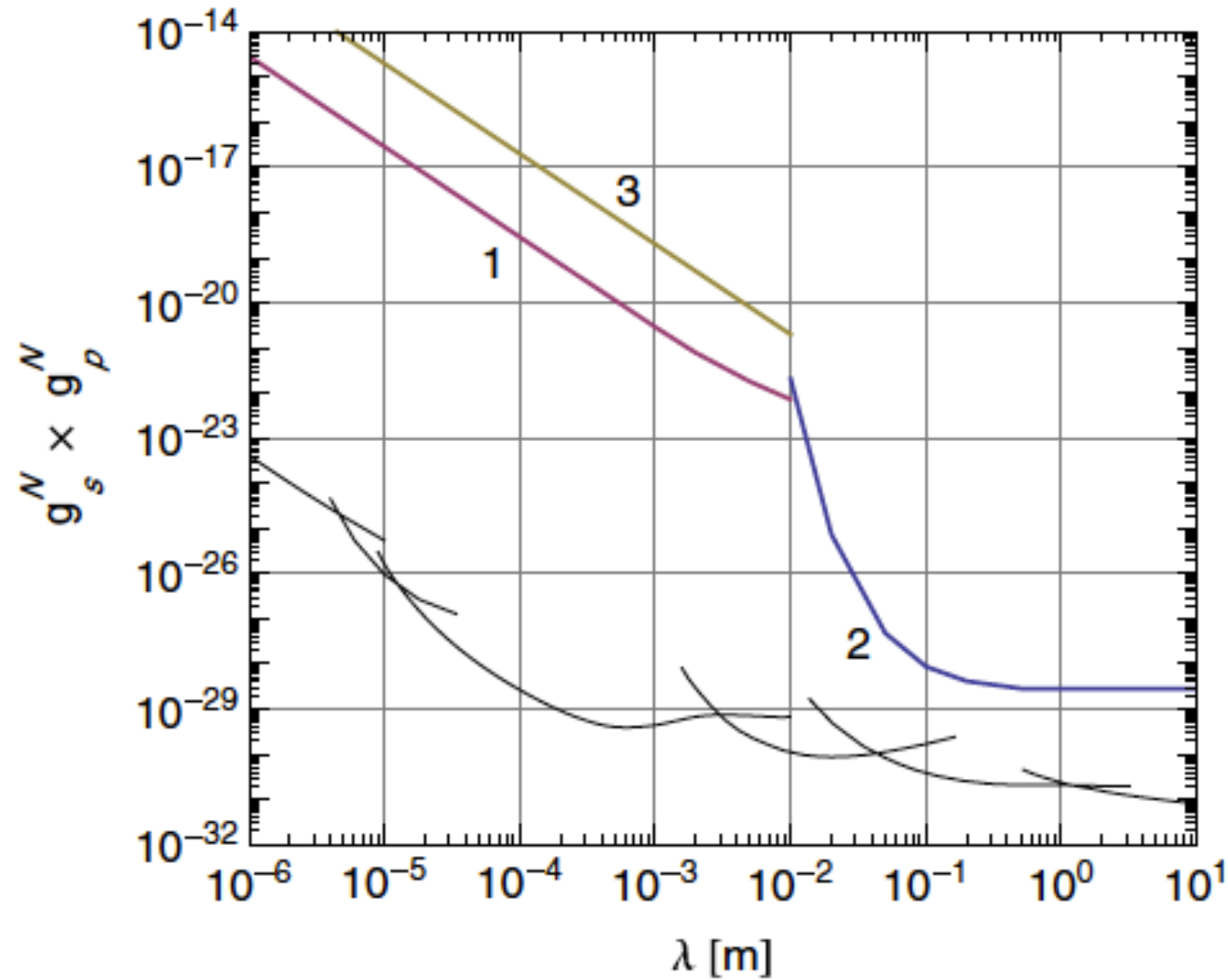
$$\Delta\nu_{sp} = (-2.9 \pm 3.5) \text{ nHz}$$

# Exclusion Plot



- 1: S.A. Hoedel (2011)
- 2: G.D. Hammond (2007)
- 3: W.-T. Ni (1998)

# Exclusion Plot



- 1: A.K. Petukhov (2010)
- 2: A.N. Youdin (1996)
- 3: A.P. Serebrov (2010)

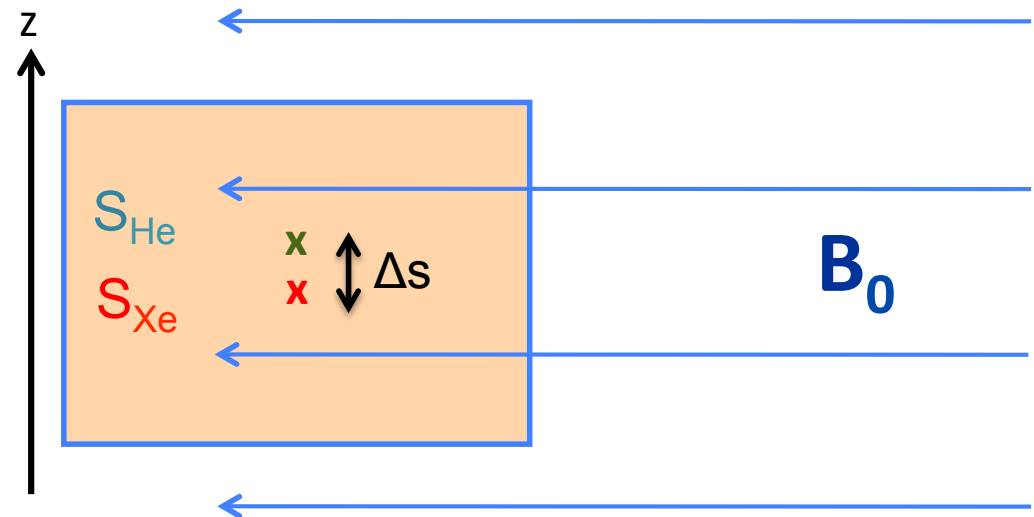
Plot taken from: G. Raffelt, *Phys. Rev. D* 86, 015001 (2012)

Influence of the  
 $\text{Bi}_4\text{Ge}_3\text{O}_{12}$  crystal on  $T_2^*$

# Systematic errors

## Transversal field gradients

$$\Delta\omega = \omega_{\text{He}} - \frac{\gamma_{\text{He}}}{\gamma_{\text{Xe}}} \omega_{\text{Xe}}$$



# Systematic errors

## Transversal field gradients

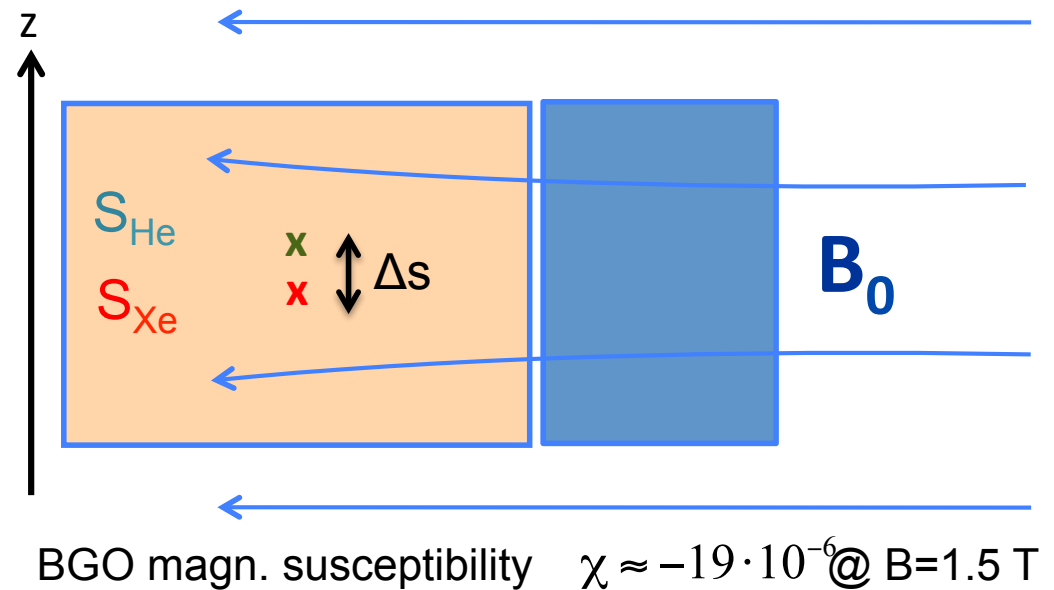
$$\Delta\omega = \omega_{\text{He}} - \frac{\gamma_{\text{He}}}{\gamma_{\text{Xe}}} \omega_{\text{Xe}} \Rightarrow \Delta\omega^{\text{CM}} = \gamma_{\text{He}} \cdot (B_{\text{Xe}} + \Delta B) - \gamma_{\text{He}} \cdot B_{\text{Xe}} = \gamma_{\text{He}} \cdot \Delta B \cdot \mathbf{H}(t-t_0)$$

$$\text{simulated: } \frac{\Delta B}{\Delta s} = \overline{\frac{\partial}{\partial z} B_{\text{BGO}}(z)} \leq 0.08 \frac{\text{pT}}{\text{cm}}$$

Barometric formula:

$$p(z) = p_0 \cdot \exp\left(-\frac{Mg_0}{R_G T} z\right)$$

$$\Rightarrow \Delta s = S_{\text{He}} - S_{\text{Xe}} \approx 1.2 \cdot 10^{-7} \text{ m}$$



$$\Rightarrow \Delta\nu_{\text{sys}}^{\text{CM}} = \Delta s \cdot \overline{\frac{\partial}{\partial z} B_{\text{BGO}}(z)} \cdot \frac{\gamma_{\text{He}}}{2\pi(1 - \gamma_{\text{He}} / \gamma_{\text{Xe}})} \leq 0.03 \text{ nHz}$$

*S. Yamamoto, IEEE,  
Transactions  
on Nuclear Science,  
Vol. 50, No. 5, 2003*

# Systematic errors

## Field gradients (general)

$T_{2,\text{field}}$  in low magnetic fields (G. D. Cates, et al., Phys. Rev. A 37, 2877)

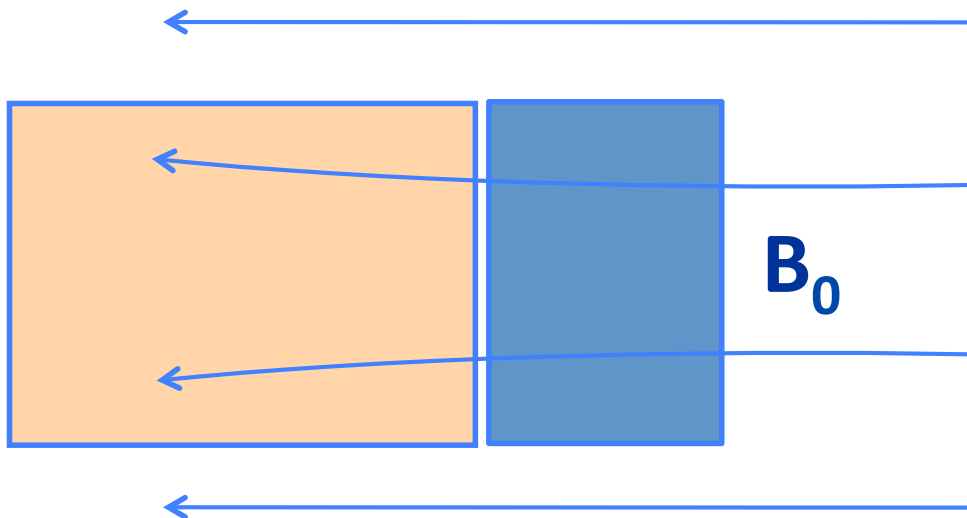
$$\frac{1}{T_2^*} = \frac{1}{T_1} + \frac{1}{T_{2,\text{field}}}$$

$$\frac{1}{T_{2,\text{field}}} \approx \frac{4R^4\gamma^2}{175D} \left( |\vec{\nabla}B_{1,z}|^2 + |\vec{\nabla}B_{1,y}|^2 + 2|\vec{\nabla}B_{1,x}|^2 \right) \propto R^4 \cdot \rho \cdot |\vec{\nabla}B|^2$$

BGO magn. susceptibility  $\chi \approx -19 \cdot 10^{-6}$  @ B=1.5 T

*S. Yamamoto, IEEE, Transactions on Nuclear Science, Vol. 50, No. 5, 2003*

⇒ additional field gradients



# Systematic errors

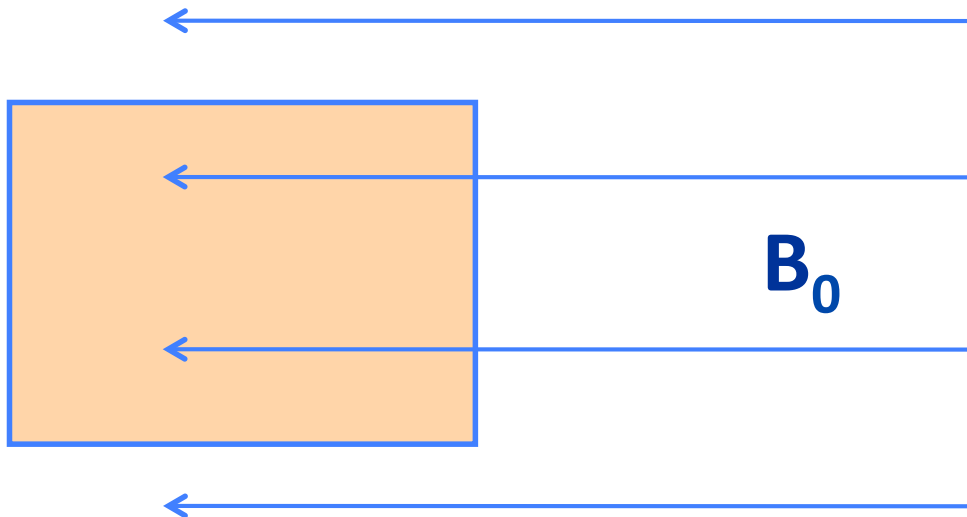
## Field gradients (general)

$$\frac{1}{T_2^*} = \frac{1}{T_1} + \frac{1}{T_{2, \text{field}}}$$

$$\frac{1}{T_{2, \text{field}}} \approx \frac{4R^4\gamma^2}{175D} \left( \left| \vec{\nabla} B_{1,z} \right|^2 + \left| \vec{\nabla} B_{1,y} \right|^2 + 2 \left| \vec{\nabla} B_{1,x} \right|^2 \right) \propto R^4 \cdot p \cdot \left| \vec{\nabla} B \right|^2$$

BGO magn. susceptibility  $\chi \approx -19 \cdot 10^{-6}$  @ B=1.5 T

⇒ additional field gradients



Error due to changes of  $T_2^*$ :

$$\left| \Delta \nu_{\text{sys}}^{T_2} \right| \leq \frac{\frac{\Delta T_{2,\text{He}}^*}{(T_{2,\text{He}}^*)^2} \cdot \left( \frac{a_{\text{He}}}{T_{2,\text{He}}^*} - \frac{1}{2} \frac{a_{\text{Xe}}}{T_{2,\text{Xe}}^*} \right) \cdot \frac{t_0}{2}}{2\pi(1 - \gamma_{\text{He}} / \gamma_{\text{Xe}})}$$

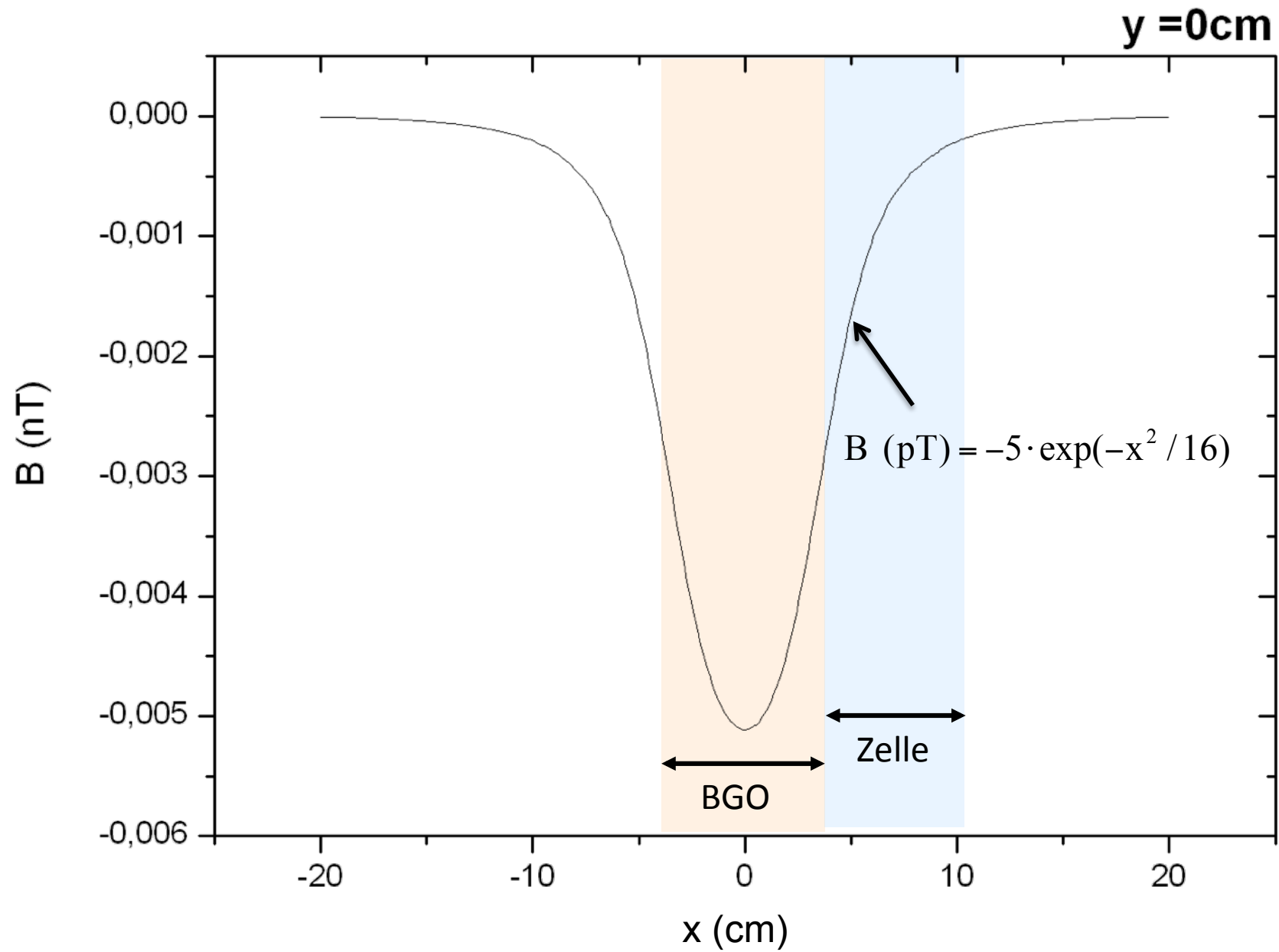
with:  $\langle a_{\text{He}} \rangle \approx 11.5$  rad,  $\langle a_{\text{Xe}} \rangle \approx 0.1$  rad,  $t_0 = 3$  h,

$$T_{2,\text{He}}^* = 53 \text{ h}, T_{2,\text{Xe}}^* = 5 \text{ h}, \left| \Delta T_{2,\text{He}}^* \right| \leq 160 \text{ s}$$

$$\Rightarrow \left| \Delta \nu_{\text{sys}}^{T_2} \right| \leq 0.1 \text{ nHz}$$



# Comsol Simulation



# Comsol Simulation

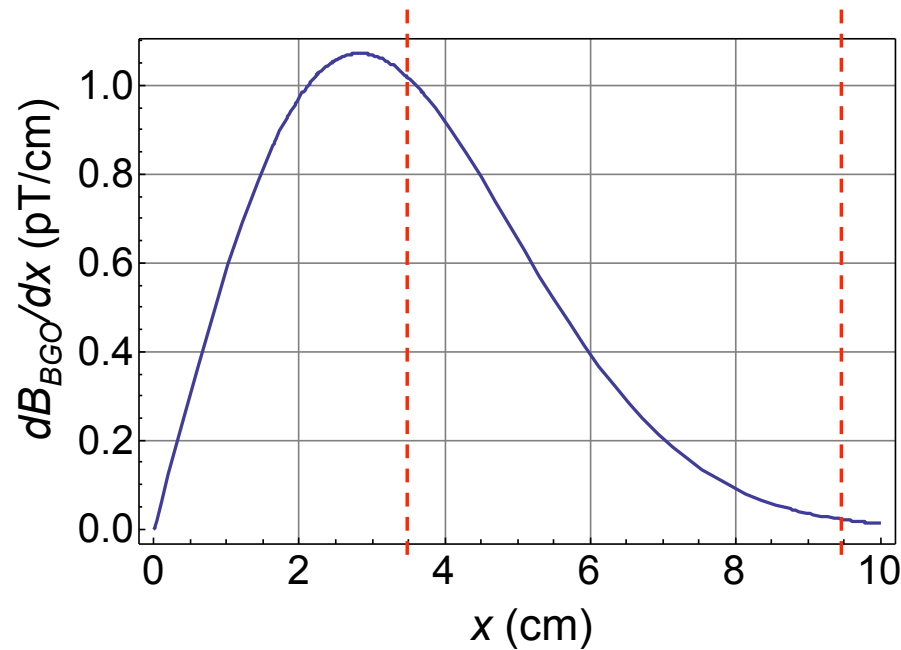
Assumption:

$$B_{\text{BGO}} (\text{pT}) = -5 \cdot \exp(-x^2 / 16) \Rightarrow \partial B_{\text{BGO}} / \partial x (\text{pT} / \text{cm}) = +10 / 16 \cdot x \cdot \exp(-x^2 / 16)$$

# Comsol Simulation

Assumption:

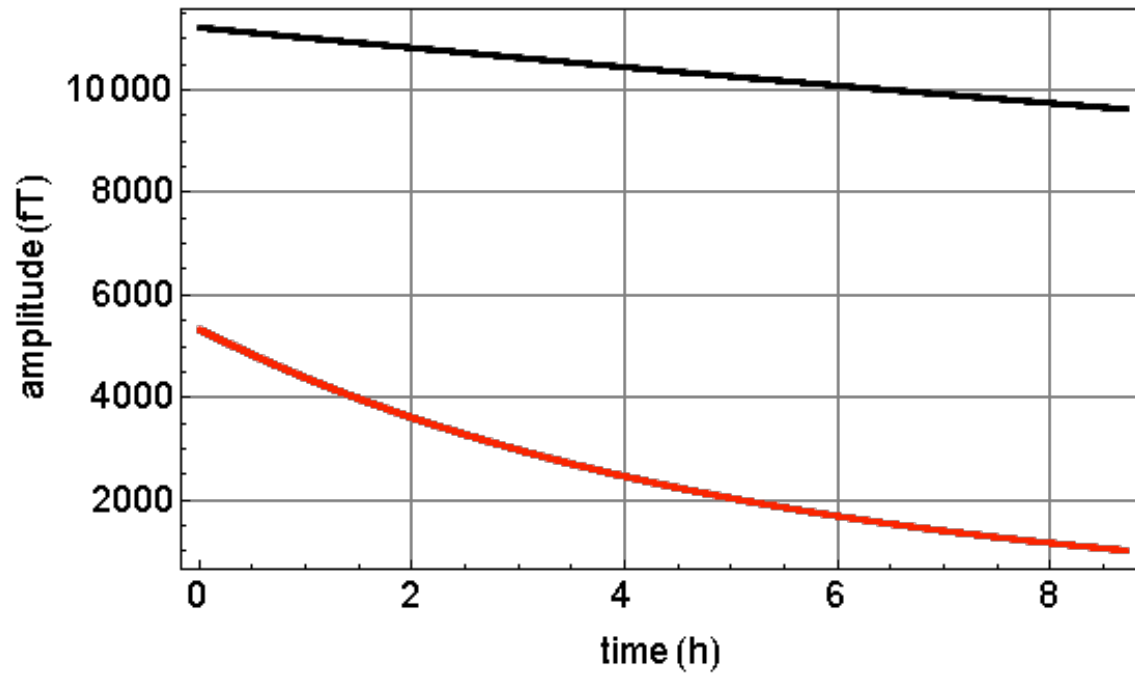
$$B_{BGO} \text{ (pT)} = -5 \cdot \exp(-x^2 / 16) \Rightarrow \partial B_{BGO} / \partial x \text{ (pT/cm)} = +10 / 16 \cdot x \cdot \exp(-x^2 / 16)$$



$$\overline{\partial B_{BGO} / \partial x} \approx 0.47 \text{ pT/cm}$$

$$\partial B_{BGO} / \partial y, \partial B_{BGO} / \partial z \ll \partial B_{BGO} / \partial x$$

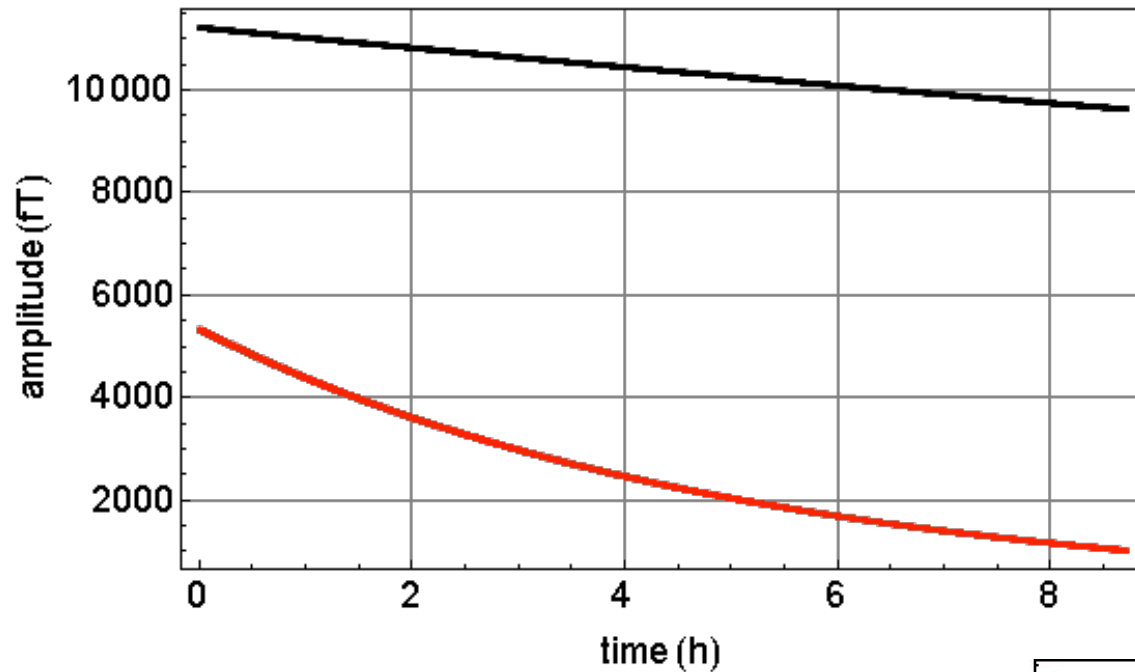
# Determination of $T_2^*$



**Exponential fit of amplitudes:**

$$fit(t) = a_0 \cdot e^{-t/T_2}$$

# Determination of $T_2^*$

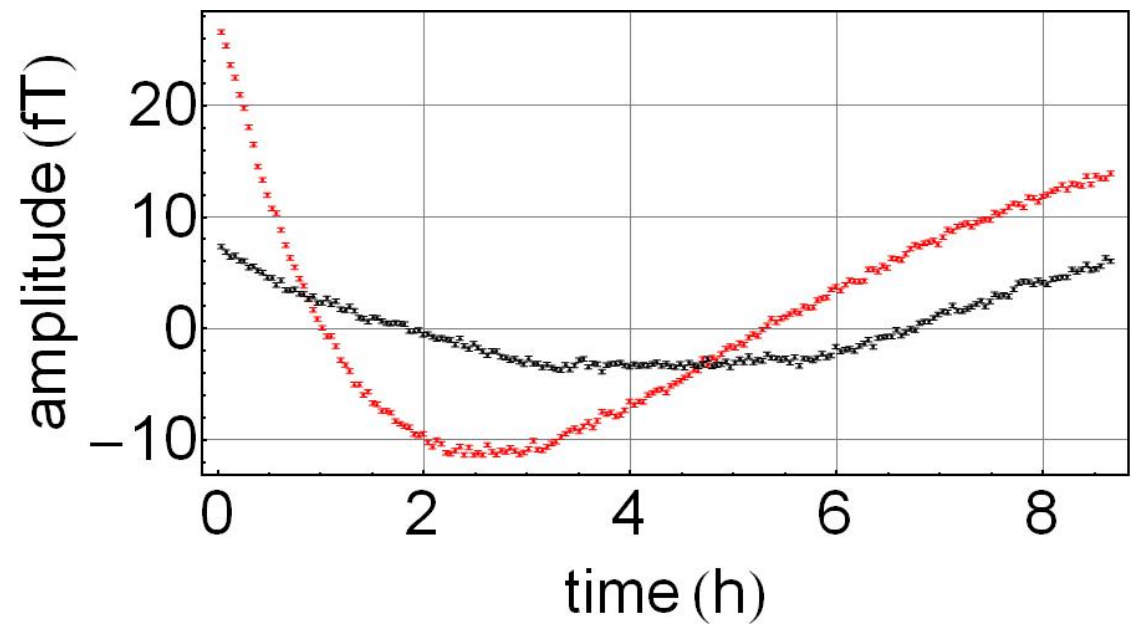


**Exponential fit of amplitudes:**

$$fit(t) = a_0 \cdot e^{-t/T_2}$$

**Residuals:**

$$residuals(t) = a(t) - fit(t)$$



# Decay of Amplitudes

$$A(t) = f_a(t) \cdot f_b(t) \cdot f_c(t) \cdot \dots \cdot A_0 \cdot e^{-t/T_2}$$

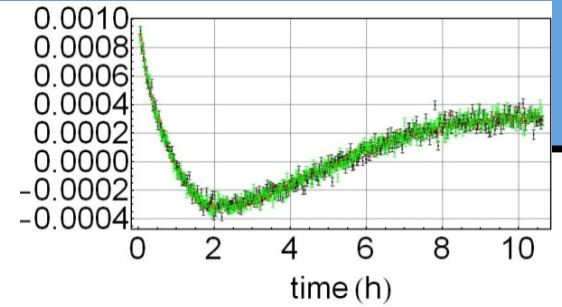
a.) Drift of SQUIDs and motion of SQUIDs in the magnetic field  $f_a(t)$

b.) Changes of the field gradients

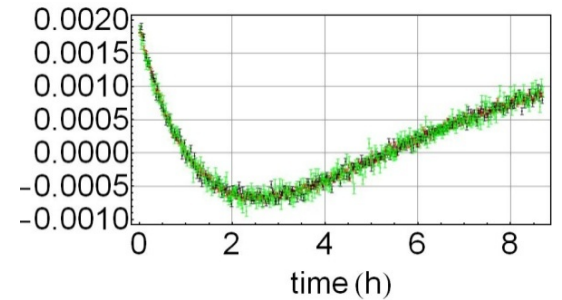
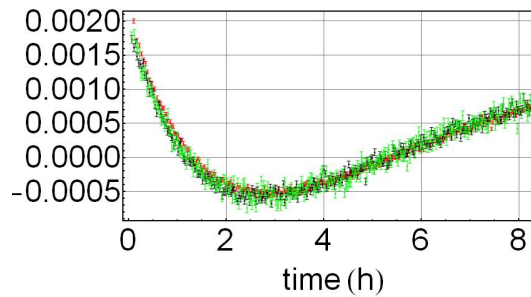
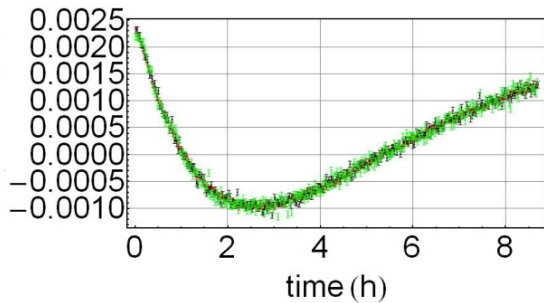
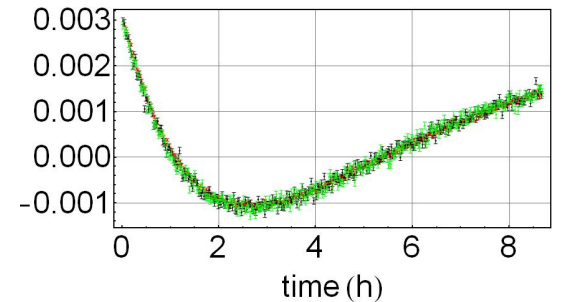
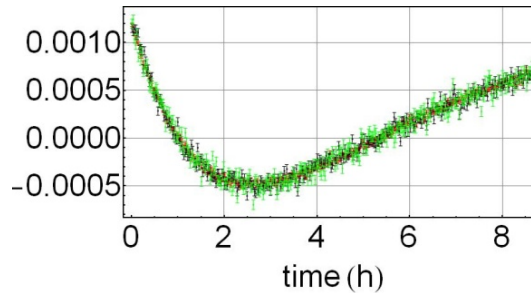
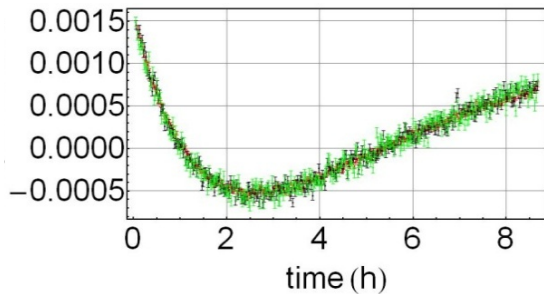
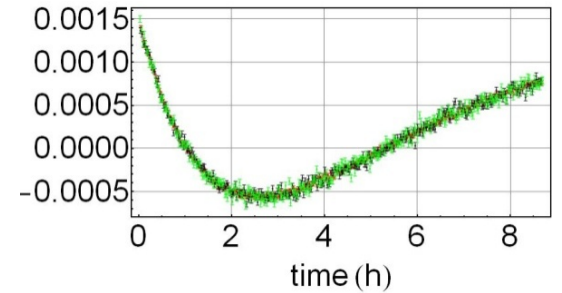
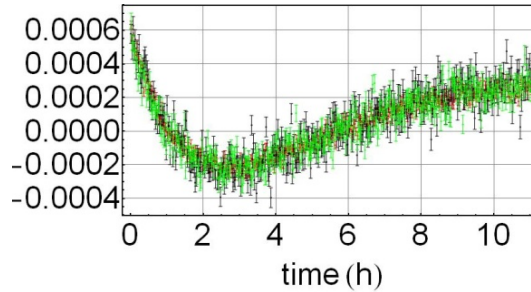
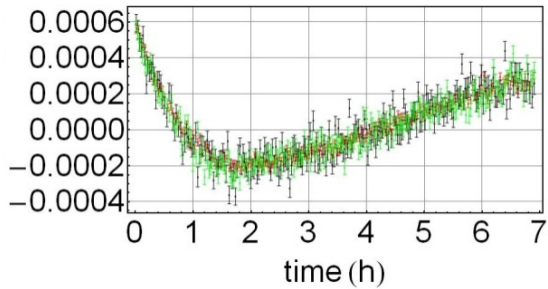
$$\text{Ratio}(t) = \frac{f_{b,Xe}(t)}{f_{b,He}(t)} \cdot \frac{f_{c,Xe}(t)}{f_{c,He}(t)} \cdot \frac{A_{0,Xe}}{A_{0,He}} \cdot e^{-t/T} \quad \text{mit: } \frac{1}{T} = \frac{1}{T_{2,Xe}^*} - \frac{1}{T_{2,He}^2}$$

⇒ Drift  $f_a(t)$  due to SQUIDs and motion of SQUIDs in the magnetic drops out.

# SR Measurements september 2010



green: Z2E+Z5S, red: Z3D-Z2S, black: Z3I-Z1S



# LV Measurements march 2009

