Parton distribution functions and matching

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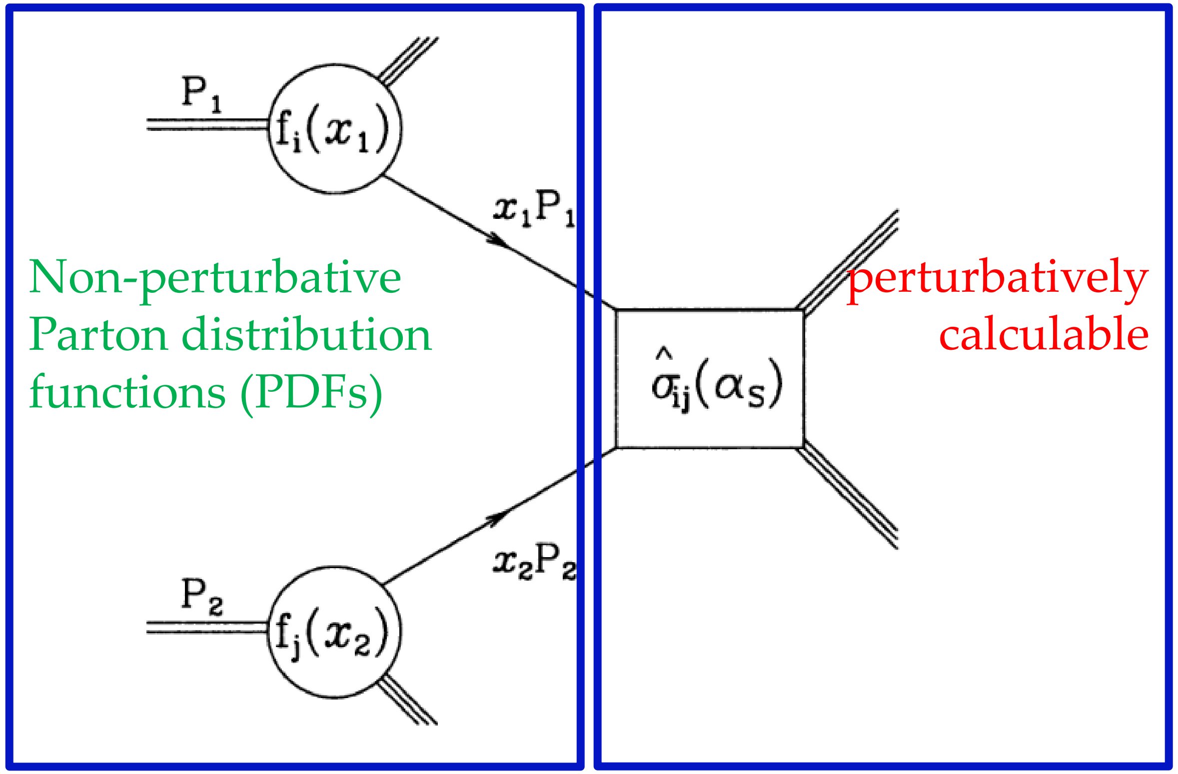
Contents

Hadronic collision

$$\sigma(P_1, P_2) = \sum_{i,j} \int dx_1 dx_2 f_i(x_1) f_j(x_2) \hat{\sigma}_{ij}(p_1, p_2)$$

- PDFs
  - Hadrons viewed as constituted by free point-like partons in high energy collision
  - Characterize the probability of a parton having a given fraction x of the longitudinal momentum of parent hadron
  - Defined on the light-cone



Hadronic collision

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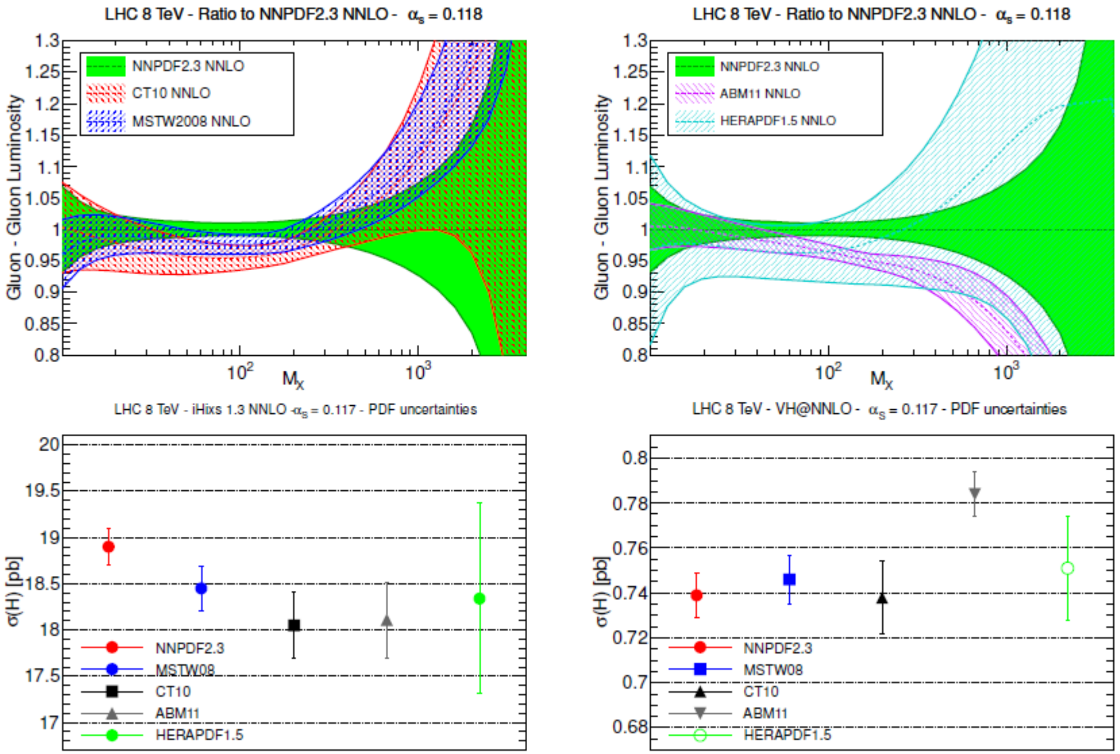
  - Extracted from experimental data
  - DGLAP evolution
  - Different PDF sets
  - Uncertainty in theoretical predictions



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- Parameterization and parameters determined from lattice computed moments
- Number of calculable moments limited

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Other possibility to directly access PDFs?

Lesson from proton spin

- Proton spin arises from quark and gluon angular momentum
- Spin sum rule

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

  - There have been a lot of discussions on the decomposition of proton spin in the past few decades [Jaffe and Manohar, Ji, Chen et. al., Wakamatsu, Hatta, Lorce, Leader……]

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  - Jaffe-Manohar decomposition

$$\vec{J} = \int d^3\xi\ \psi^\dagger \frac{\vec{\Sigma}}{2}\psi + \int d^3\xi\ \psi^\dagger \vec{\xi} \times (-i\vec{\nabla})\psi + \int d^3\xi\ \vec{E}_a \times \vec{A}^a + \int d^3\xi\ E_a^i\ \vec{\xi} \times \vec{\nabla} A^{i,a}$$

    - Complete decomposition into quark/gluon spin & orbital
    - Gauge-dependent, but with a clear partonic picture

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Gluon helicity

- Light-cone correlation [Manohar, 90']

$$\Delta G = \int dx \frac{i}{2xP^+} \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle PS | F_a^{+\alpha}(\xi^-) \mathcal{L}^{ab}(\xi^-, 0) \tilde{F}_{\alpha,b}^{\;+}(0) | PS \rangle$$

  - Difficult to access on the lattice
  - However [Ji, Zhang and Zhao, 13' and 14']
    - It is proven to be equivalent to the infinite momentum limit of a static gauge invariant gluon spin $\vec{E} \times \vec{A}_\perp$ (see Y. Zhao's and also Y. Hatta's talk), which is accessible on the lattice (see K. F. Liu's talk)
    - Actually not only gluon helicity, but all terms in Jaffe-Manohar sum rule can be related to the infinite momentum limit of gauge invariant operators (see Y. Zhao's talk)

Gluon helicity

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    - This provides a practical way of obtaining the light-cone matrix element from the matrix element of gauge invariant operator computed at finite momentum and then boosted to infinite momentum
    - Caution: the boosted result does not directly yield the desired one, but it can be converted to the latter through a perturbative matching factor

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    - Gluon helicity is the integral of gluon helicity distribution, is the same strategy also applicable at the level of parton distribution?

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YES!

(Quasi) parton distribution

- Operator definition of parton distribution

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P|\overline{\psi}(\xi^-)\gamma^+ \exp\left(-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)\right)\psi(0)|P\rangle$$

  - $P^\mu = (P^0, 0,0, P^z), \xi^\pm = (t \pm z)/\sqrt{2}$
  - Light-cone correlation, expectation of light-front number operator
- Look instead at an off-light-cone quasi parton distribution [Ji, 13′]

$$\tilde{q}(x,\Lambda,P^z) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{izk^z} \langle P|\overline{\psi}(0,0_\perp,z)\gamma^z \exp\left(-ig\int_0^z dz' A^z(0,0_\perp,z')\right)\psi(0)|P\rangle$$

  - Quark fields separated along z-direction, no time dependence, $x = k^z/P^z$
  - Light-cone distribution can be approached by this up to power suppressed corrections in large momentum limit

(Quasi) parton distribution

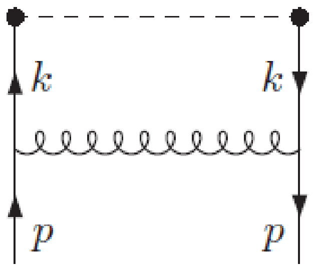
- As in gluon helicity, this is plagued by existence of divergences
- How to recover parton distribution from the quasi one? [Ji, 13', Ji, Xiong, Zhang and Zhao, 13']

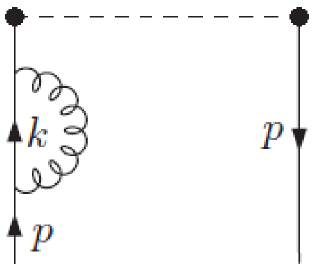
$$\tilde{q}(x, \Lambda, P^z) = \int_{-1}^{1} \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\Lambda}{P^z}, \frac{\mu}{P^z}\right) q(y, \mu)$$

  - Can be viewed as a factorization formula
    - The factorization can be shown to hold to all orders in perturbation theory (see Y. Q. Ma's talk)
  - Z factor sensitive to UV physics only, and thus perturbatively calculable

One-loop example

- LO simply a delta function
- NLO (in axial gauge $A^z = 0$)





- Remark:
  - In infinite momentum frame, support of quark density comes from requirement of positivity of cut external legs, or from integration over k^-
    - On-shell partons cannot have negative plus-momentum fraction, 0<x<1
  - In finite momentum frame, support of quasi quark density comes from integration over k^0, and the momentum fraction $-\infty < x < \infty$

One-loop example

- One-loop quasi distribution

$$\tilde{q}(x,\Lambda,P^z) = (1+\tilde{Z}_F^{(1)}(\Lambda,P^z))\delta(x-1) + \tilde{q}^{(1)}(x,\Lambda,P^z) + \dots$$

$$\tilde{q}^{(1)}(x,\Lambda,P^z) = \frac{\alpha_S C_F}{2\pi}\begin{cases} \frac{1+x^2}{1-x}\ln\frac{x}{x-1} + 1 + \frac{\Lambda}{(1-x)^2 P^z}\,, & x>1\,,\\ \frac{1+x^2}{1-x}\ln\frac{(P^z)^2}{m^2} + \frac{1+x^2}{1-x}\ln\frac{4x}{1-x} - \frac{4x}{1-x} + 1 + \frac{\Lambda}{(1-x)^2 P^z}\,, & 0<x<1\,,\\ \frac{1+x^2}{1-x}\ln\frac{x-1}{x} - 1 + \frac{\Lambda}{(1-x)^2 P^z}\,, & x<0\,,\end{cases}$$

$$\tilde{Z}_F^{(1)}(\Lambda,P^z) = \frac{\alpha_S C_F}{2\pi}\int dy \begin{cases} -\frac{1+y^2}{1-y}\ln\frac{y}{y-1} - 1 - \frac{\Lambda}{(1-y)^2 P^z}\,, & y>1\,,\\ -\frac{1+y^2}{1-y}\ln\frac{(P^z)^2}{m^2} - \frac{1+y^2}{1-y}\ln\frac{4y}{1-y} + \frac{4y^2}{1-y} + 1 - \frac{\Lambda}{(1-y)^2 P^z}\,, & 0<y<1\,,\\ -\frac{1+y^2}{1-y}\ln\frac{y-1}{y} + 1 - \frac{\Lambda}{(1-y)^2 P^z}\,, & y<0\,.\end{cases}$$

No logarithmic UV divergence, but ln P^z instead in 0<x<1 region
Momentum fraction not restricted to [0,1]

One-loop example

- One-loop light-cone distribution

$$q(x,\Lambda) = (1 + Z_F^{(1)}(\Lambda) + \dots)\delta(x-1) + q^{(1)}(x,\Lambda) + \dots$$

$$q^{(1)}(x,\Lambda) = \frac{\alpha_S C_F}{2\pi}\begin{cases} 0\,, & x>1 \text{ or } x<0\,, \\ \frac{1+x^2}{1-x}\ln\frac{\Lambda^2}{m^2} - \frac{1+x^2}{1-x}\ln(1-x)^2 - \frac{2x}{1-x}\,, & 0<x<1\,, \end{cases}$$

$$Z_F^{(1)}(\Lambda) = \frac{\alpha_S C_F}{2\pi}\int dy \begin{cases} 0\,, & y>1 \text{ or } y<0\,, \\ -\frac{1+y^2}{1-y}\ln\frac{\Lambda^2}{m^2} + \frac{1+y^2}{1-y}\ln(1-y)^2 + \frac{2y}{1-y}\,, & 0<y<1\,, \end{cases}$$

Logarithmic UV divergence in 0<x<1 region
Momentum fraction restricted to [0,1]
Same mass singularity as the quasi distribution

One-loop example

- Matching between quasi and light-cone quark distribution

$$\tilde{q}(x,\Lambda,P^z) = \int_{-1}^{1} \frac{dy}{|y|} Z\left(\frac{x}{y},\frac{\Lambda}{P^z},\frac{\mu}{P^z}\right) q(y,\mu)$$

- Z factor $Z\left(\xi,\frac{\Lambda}{P^z},\frac{\mu}{P^z}\right) = \delta(\xi-1) + \frac{\alpha_s}{2\pi} Z^{(1)}\left(\xi,\frac{\Lambda}{P^z},\frac{\mu}{P^z}\right) + \ldots$

$$Z^{(1)}(\xi)/C_F = \left(\frac{1+\xi^2}{1-\xi}\right)\ln\frac{\xi}{\xi-1} + 1 + \frac{1}{(1-\xi)^2}\frac{\Lambda}{P^z}, \qquad \xi > 1$$

$$Z^{(1)}(\xi)/C_F = \left(\frac{1+\xi^2}{1-\xi}\right)\ln\frac{(P^z)^2}{\mu^2} + \left(\frac{1+\xi^2}{1-\xi}\right)\ln\left[4\xi(1-\xi)\right] - \frac{2\xi}{1-\xi} + 1 + \frac{\Lambda}{(1-\xi)^2 P^z}, \qquad 0 < \xi < 1$$

$$Z^{(1)}(\xi)/C_F = \left(\frac{1+\xi^2}{1-\xi}\right)\ln\frac{\xi-1}{\xi} - 1 + \frac{\Lambda}{(1-\xi)^2 P^z}. \qquad \xi < 0$$

One-loop example

- Matching between quasi and light-cone quark distribution

- Eventually one needs the matching between quasi distribution computed on the lattice and light-cone distribution
  - This can be divided into two steps:
    - Matching between finite and infinite momentum in the continuum
    - Matching at finite momentum between lattice and continuum (see S. Yoshida's talk)

Summary

- Light-cone parton distribution or other quantities can be studied by investigating a related time-independent quantity at large momentum
  - Static operator for gluon helicity
  - Space-like correlation for parton distribution
- Allows calculation of parton distribution and related quantities on the lattice
- Matching required to connect quasi and light-cone distributions, but it depends on UV physics only, and therefore is perturbatively calculable

BACKUP SLIDES