Perturbative Matching of the Quasi-PDFs in Continuum Space and Lattice Space

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Quasi-PDF

X. Ji, PRL110 (2013) 262002

- definition

$$\tilde{f}(\tilde{x}, \mu^2, P_z) = \int \frac{dz}{4\pi} e^{i\tilde{x}P_z z} \langle P|\bar{\psi}(z) \exp\left(-ig\int_0^z dz' A^z(z')\right)\psi(0)|P\rangle$$

  - Quasi-PDF does not depend on real-time in Minkowski space
  - Measurable on the Euclidean lattice

- Matching with light-cone PDF

$$\tilde{f}(\tilde{x}, \mu^2, P_z) = \int_{\tilde{x}}^1 \frac{dx}{x} Z\left(\frac{\tilde{x}}{x}, \frac{\mu}{P_z}\right) q(x, \mu^2) + \cdots$$

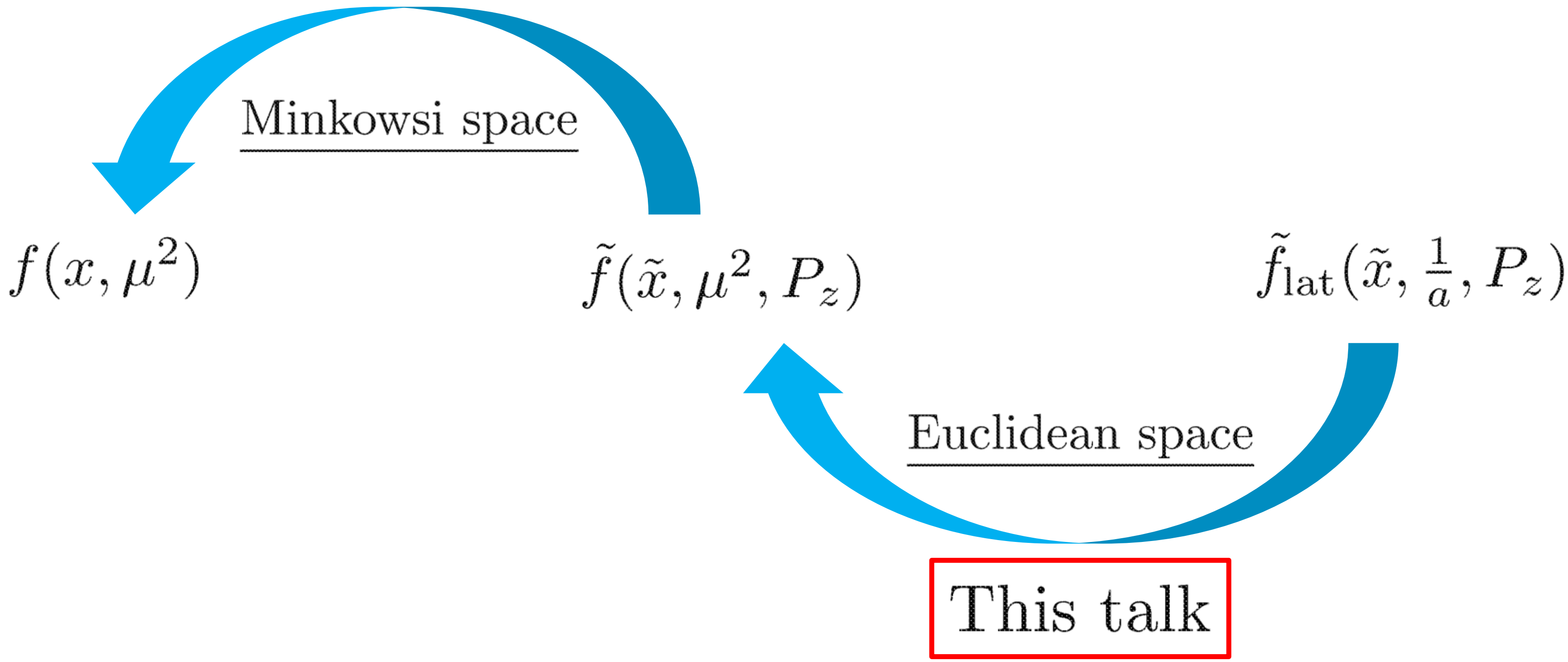
Lattice QCD simulation gives independent data points for the PDF fitting

Two matching procedures

- PDF has the renormalization scheme dependence
- Precise determination of quasi-PDF requires the matching between different schemes

X. Xiong et al., PRD90 (2014) 014051

Y. -Q. Ma and J. Qiu, arXiv:1404.6860



Lattice QCD

Lattice QCD is formulated in the descretized Euclidean space

$$S^f = a^4 \sum_x \Big[\frac{1}{2a} \sum_\mu [\bar{\psi}(x)\gamma_\mu U_\mu(x)\psi(x+a\hat{\mu}) - \bar{\psi}(x+a\hat{\mu})\gamma_\mu U_\mu^\dagger(x)\psi(x)] + m\bar{\psi}(x)\psi(x) \Big]$$

$$S^g = \frac{1}{g_0^2} a^4 \sum_{x,\mu\nu} \Big[N_c - \mathrm{ReTr}[U_\mu(x)U_\nu(x+a\hat{\mu})U_\mu^\dagger(x+a\hat{\nu})U_\nu^\dagger(x)] \Big]$$

$$U_\mu(x) = e^{-igaT^a A_\mu^a(x+\frac{1}{2})}$$

Boundary condition is imposed on each field in finite volume system

Momemtum space is restricted in finite Brillouin zone $\{-\frac{\pi}{a}, \frac{\pi}{a}\}$

UV finite theory

Lattice action is not unique, above action is the simplest one

Many improvements have been proposed to reduce discretization error

Doubling problem

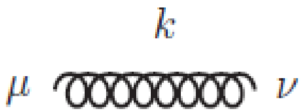
Naive discretized fermion shown in previous slide is useless for numerical simulation because of the doubling problem.

Wilson fermion was proposed to avoid the problem.

But we first use naive discretized fermion to demonstrate the cancelation of the infrared divergence in the simplest case.

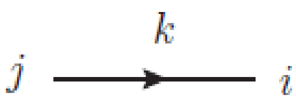
Feynman rules of lattice PT (Feynman gauge)

○gluon propagator



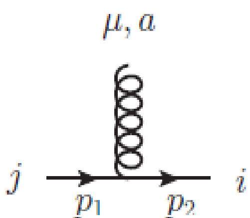
$$\frac{1}{k^2}\delta_{\mu\nu} \longrightarrow \frac{1}{4\sum_\mu \sin^2(\frac{ak_\mu}{2})}\delta_{\mu\nu}$$

○quark propagator



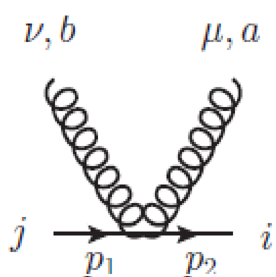
$$-i\frac{\not{k}}{k^2}\delta_{ij} \longrightarrow -ia\frac{\sum_\mu \gamma_\mu \sin(ak_\mu)}{\sum_\mu \sin^2(ak_\mu)}\delta_{ij}$$

○quark-gluon vertex



$$-ig(T^a)_{ij}\gamma_\mu \longrightarrow -ig(T^a)_{ij}\gamma_\mu \cos(\frac{a(p_1+p_2)_\mu}{2})$$

○quark-gluon vertex 2



$$\frac{1}{2}g(T^aT^b)_{ij}\gamma_\mu \sin(\frac{a(p_1+p_2)_\mu}{2})\delta_{\mu\nu}$$

Discretized quasi-PDF

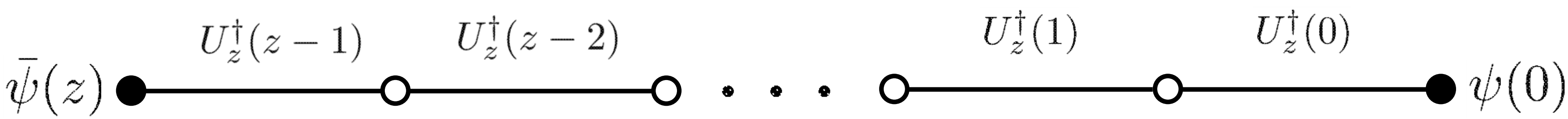
$$\tilde{f}(\tilde{x}, \mu^2, P_z) = \int \frac{dz}{4\pi} e^{i\tilde{x}P_z z} \langle P|\bar{\psi}(z) \exp\left(-ig \int_0^z dz' A^z(z')\right)\psi(0)|P\rangle$$

- Discretized operator is not unique
- We use the simplest discretized operator

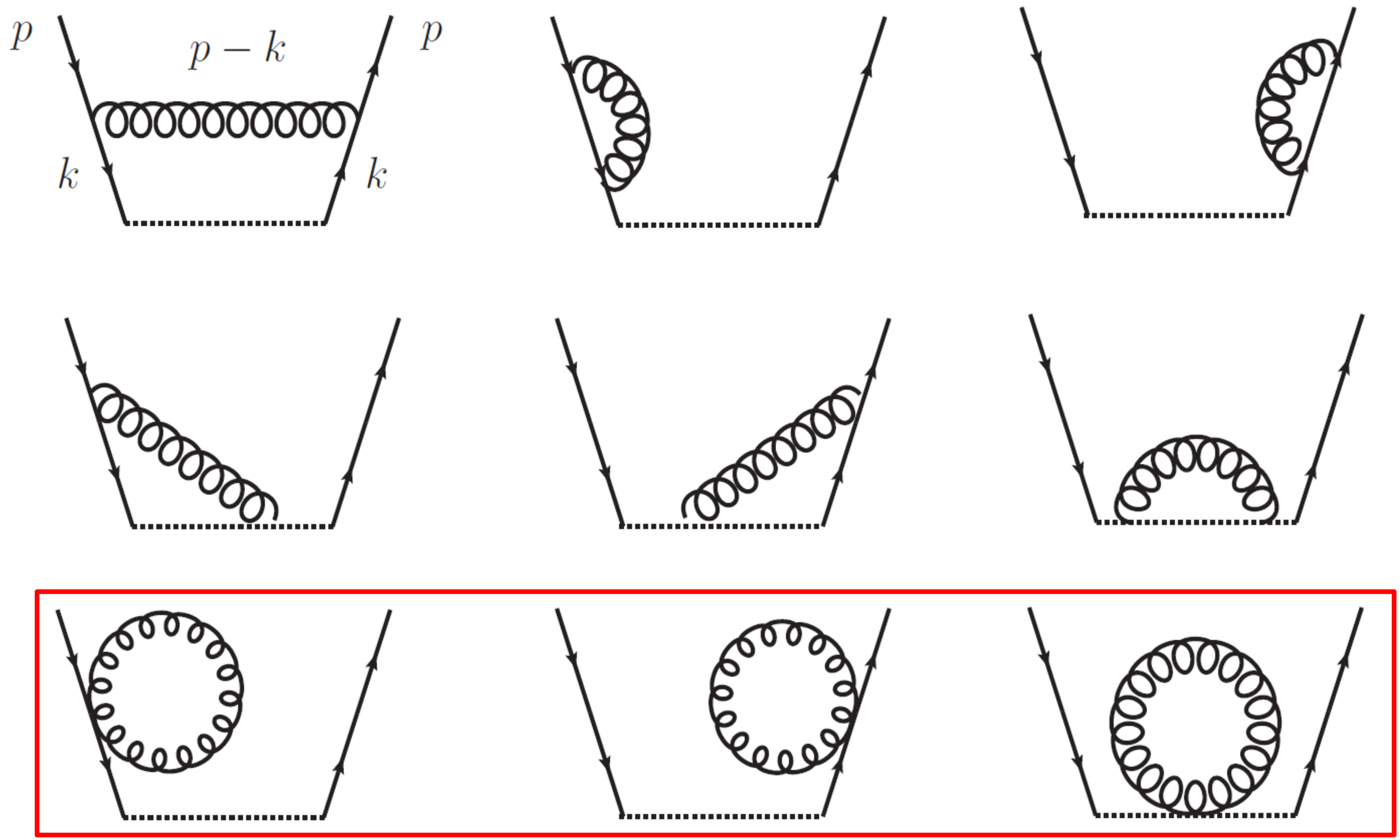
$$\bar{\psi}(z) \exp\left(-ig \int_0^z dz' A^z(z')\right)\psi(0)$$

$$\Downarrow$$

$$\bar{\psi}(z)U_z^\dagger(z-1)U_z^\dagger(z-2)\cdots U_z^\dagger(1)U_z^\dagger(0)\psi(0)$$

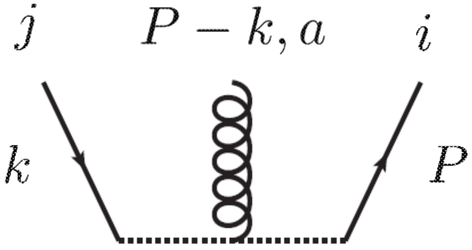


Diagrams in Feynman gauge

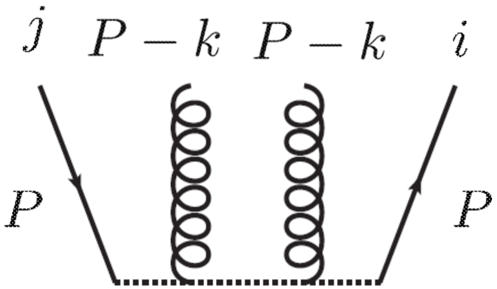


Only appear in LPT(Tadpole diagrams)

Soft divergence



$$ga(T^a)_{ij}(\gamma^z)_{ij}\frac{e^{-iP_z z}-e^{-ik_z z}}{2\sin(\frac{a(P_z-k_z)}{2})}$$



$$-g^2C_F\delta_{ij}(\gamma^z)_{ij}\left(a^2\frac{e^{-iP_z z}-e^{-ik_z z}}{4\sin^2(\frac{a(P_z-k_z)}{2})}-a\frac{ze^{-iP_z z}}{1-e^{-i(P_z-k_z)a}}\right)$$

No singularity in $k_z \to P_z$!

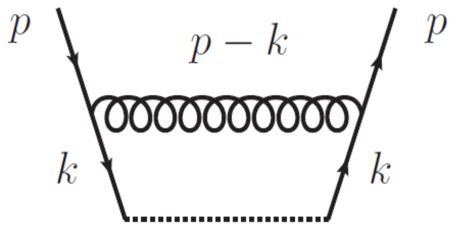
$$-\frac{g^2}{2}C_F\delta_{ij}(\gamma^z)_{ij}ze^{-iP_z z}$$

Cancelation of the soft divergence is manifested in Feynman gauge

Calculation technique

H. Kawai, NPB189 (1981) 40

S. Capitani, Phys. Rept. 382 (2003) 113



$$\Gamma_\mu = \sin(k_\mu) \quad C_\mu = \cos\left(\frac{a(P+k)_\mu}{2}\right)$$

$$2W = 4\sum_\mu \sin^2\left(\frac{a(P-k)_\mu}{2}\right)$$

$$a^4 \int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} \frac{d^4k}{(2\pi)^4} \delta\left(\tilde{x} - \frac{k_z}{P_z}\right) \frac{1}{4P_z} \mathrm{Tr}[\gamma^z \not{\Gamma} \gamma_\mu \not{P} \gamma_\mu \not{\Gamma}] \frac{C_\mu^2}{2W(\Gamma)^2}$$

$$= a^4 \int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} \frac{d^4k}{(2\pi)^4} \delta\left(\tilde{x} - \frac{k_z}{P_z}\right) \frac{1}{4P_z} \mathrm{Tr}[\gamma^z \not{\Gamma} \gamma_\mu \not{P} \gamma_\mu \not{\Gamma}] \left(\frac{C_\mu^2}{2W(\Gamma)^2} - \frac{C_\mu^2}{2W(\Gamma)^2}\bigg|_{P=0} + \frac{C_\mu^2}{2W(\Gamma)^2}\bigg|_{P=0} \right)$$

UV finite

$$= a^4 \int_{-\infty}^{\infty} \frac{d^4k}{(2\pi)^4} \delta\left(\tilde{x} - \frac{k_z}{P_z}\right) \frac{1}{4P_z} \mathrm{Tr}[\gamma^z \not{k} \gamma_\mu \not{P} \gamma_\mu \not{k}] \left(\frac{1}{(P-k)^2 (k^2)^2} - \frac{1}{(k^2)^3} \right) + O(a)$$

- analytically calculable
- easy to identify collinear divergence

$$+a^4 \int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} \frac{d^4k}{(2\pi)^4} \delta\left(\tilde{x} - \frac{k_z}{P_z}\right) \frac{1}{4P_z} \mathrm{Tr}[\gamma^z \not{\Gamma} \gamma_\mu \not{P} \gamma_\mu \not{\Gamma}] \frac{C_\mu^2}{2W(\Gamma)^2}\bigg|_{P=0}$$

- no P-dependence

Preliminary result

same collinear divergence with X. Xiong et al., PRD90 (2014) 014051

Y. -Q. Ma and J. Qiu, arXiv:1404.6860

$$\tilde{f}^{(1)}_{\rm lat}(\tilde{x},\mu^2,P_z) = \int dy\Big\{\delta(\tilde{x}-y)-\delta(\tilde{x}-1)\Big\}\Big\{\boxed{\frac{1+y^2}{1-y}\log(\frac{P_z^2}{m^2})} + 2\frac{1+y^2}{1-y}\log\Big(4(1-y)\Big)$$

$$-\frac{1}{16\pi^2 y}(1-2y^2)+\frac{1}{8\pi^2}\frac{1-3y}{1-y}+a^4\int_{-\frac{\pi}{a}}^{\frac{\pi}{a}}\frac{d^3k}{(2\pi)^4}\frac{4C_3^2\Gamma_3^2}{2W(\Gamma^2)^2}\Big|_{P=0,k_z=yP_z}$$

$$-\frac{a^4}{P_z}\frac{\cos(\frac{a(1+y)P_z}{2})}{\sin(\frac{a(1-y)P_z}{2})}\int_{-\frac{\pi}{a}}^{\frac{\pi}{a}}\frac{\Gamma_3}{2W\Gamma^2}\Big|_{P=0,k_z=yP_z}\Big\}$$

$$-\int dy\Big\{\delta(\tilde{x}-y)-\delta(\tilde{x}-1)-(1-y)\frac{\partial}{\partial\tilde{x}}\delta(1-\tilde{x})\Big\}\Big\{\frac{1}{4\pi^2(1-y)}\Big\}$$

$$+\frac{a^4}{2}\int_{-\frac{\pi}{aP_z}}^{\frac{\pi}{aP_z}}dy\int_{-\frac{\pi}{a}}^{\frac{\pi}{a}}\frac{d^3k}{(2\pi)^4}\Big\{\frac{1}{4\sin^2(\frac{a(1-y)p_z}{2})}\Big(\delta(\tilde{x}-y)-\delta(\tilde{x}-1)\Big)$$

$$-\frac{1}{iaP_z}\frac{1}{e^{-i(1-y)P_z}-1}\frac{\partial}{\partial\tilde{x}}\delta(1-\tilde{x})\Big\}\Big\{\frac{1}{2W}\Big\}$$

$$+\frac{Z_0}{2}\Big\{1+\frac{i}{aP_z}\frac{\partial}{\partial\tilde{x}}\Big\}\delta(\tilde{x}-1)$$

Summary

- We discussed the calculation of discretized quasi-PDF in momemtum space based on the lattice perturbation theory and demonstrated how the infrared divergence is canceled through the matching.

- Matching factor is conventionally calculated in the coordinate space because direct observable on the lattice is matrix element itself.
  We consider the matching also in the coordinate space.

- We proceed with the calculation using lattice fermions to avoid the doubling problem.

  ◦ Wilson ◦ domain wall ◦ overlap

- Our framework can be applied to spin dependent parton distribution functions.

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