Proton Spin Content in Lattice QCD from Large Momentum Effective Field Theory

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Outline

- A physical sum rule for proton spin
- The LaMET approach to calculate parton observables in lattice QCD
- Matching conditions for the proton spin content

A Physical Sum Rule for Proton Spin

In high energy scattering, the proton is travelling at almost the speed of light.

The parton model: The proton can be regarded as a beam of almost interaction-free quark and gluon partons.

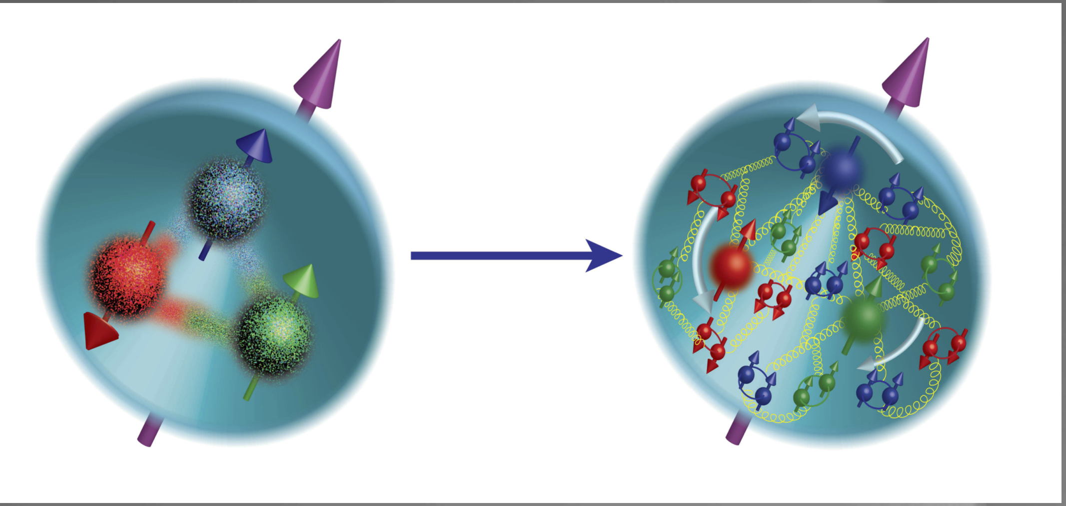


Image credit: A. Accardi et al., arXiv: 1212.1701 [nucl-ex]

With free quarks and gluons, one can talk about physically meaningful observables like the gluon spin and orbital angular momentum (OAM).

A naïve spin sum rule:

$$\frac{1}{2} = \Delta\Sigma(\mu) + \Delta L_q(\mu) + \Delta G(\mu) + \Delta L_g(\mu)$$

A Physical Sum Rule for Proton Spin

Jaffe-Manohar form of the QCD AM (1989):

$$\vec{J} = \int d^3x\, \psi^\dagger \frac{\vec{\Sigma}}{2}\psi + \int d^3x\, \psi^\dagger \vec{x} \times (-i\vec{\nabla})\psi$$
$$+ \int d^3x\, \vec{E}_a \times \vec{A}^a + \int d^3x\, E^i_a (\vec{x} \times \vec{\nabla}) A^{i,a}$$

- The parton spin and OAM are defined to be proton matrix elements of the free-field operators;
- Light-cone coordinates $x^{\pm}=(x^0 \pm x^3)/\sqrt{2}$ and gauge $A^+=0$ are used, and $E^i=F^{i+}$.

A Physical Sum Rule for Proton Spin

- Motivations:
  1. The free-field operators allow the parton observables to be expressed as simple addition of the physical properties of quarks and gluons;
  2. The gluon polarization ΔG in this expression can be probed in high energy scattering experiments (RHIC, JLab, EIC, …).

- Problems:
  1. Except for the quark spin, all the operators are gauge dependent;
  2. The Jaffe-Manohar expression must be fixed in the light-cone gauge.

A Physical Sum Rule for Proton Spin

How does the Jaffe-Manohar expression acquire physical meaning?

- In free electromagnetic theory, the fields are transverse. E×A ($E=F^{i0}$) is regarded as the photon spin (for more precise discussion, see van Enk and Nienhuis, 1994), and its projection along the propagation direction (z) can be measured in experiments (Beth, 1936);
- But for the fields generated by a static charge, E is longitudinal, and the meaning of E×A as photon spin is not clear.

A Physical Sum Rule for Proton Spin

Weiszacker-Williams (WW) approximation:

- If the charge moves at infinite momentum, the transverse EM fields dominate and can be regarded as free radiation (Jackson, CED). Again E×A can represent the photon spin.

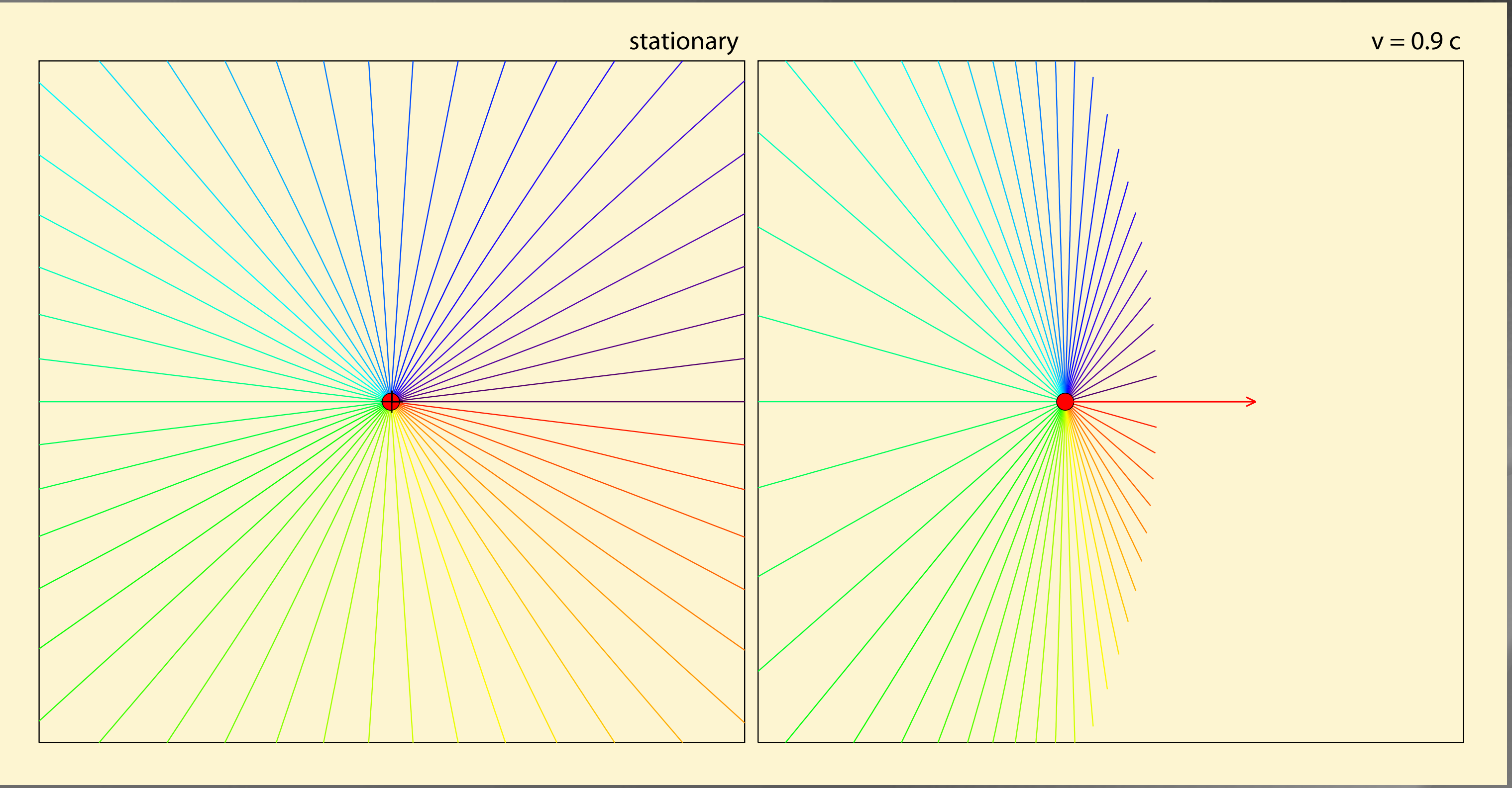


Image credit: Magnus Gålfalk

A Physical Sum Rule for Proton Spin

- Likewise, as the proton moves at extremely large momentum, the gluon fields can also be regarded as free radiation in the WW approximation, and thus E×A can represent the gluon spin with a physical gauge;
- A physical gauge is a gauge that does not affect the transverse component of the gluon field.

A Physical Sum Rule for Proton Spin

- In short, E×A can be regarded as the gluon spin if we work in

The infinite momentum frame (IMF) | Physical gauge

- This is equivalent to the definition in the light-cone gauge and coordinates because:

1. $F^{i0} \rightarrow F^{i+}$ in the IMF limit;
2. The physical gauge conditions, if not all, flow into the light-cone gauge $A^+=0$ in the IMF limit (Hatta, Ji, and Zhao, 2014);

- Similar arguments also apply to the OAM operators.

A Physical Sum Rule for Proton Spin

- Therefore, we establish the Jaffe-Manohar form of spin sum rule as physical, and will try to obtain it in lattice QCD.

But hold on…, it poses great difficulties for practical lattice calculations!

- Can't fix light-cone time and gauge on Euclidean lattice;
- Lattice can only calculate with finite momentum that is much smaller than the cut-off π/a.

The LaMET Approach

LaMET is an effective theory that allows direct calculation of parton properties in lattice QCD (Ji, 2014);

- Consider a quasi-observable ~O which depends on the hardon momentum P and can be directly calculated in lattice QCD;
- The WW approximation of ~O in the IMF limit is a parton observable O by construction;
- ~O is equivalent to O by taking the IMF limit before UV regularization, while for lattice calculation one needs to impose UV regularization first. Their difference is a matter of order of limits.

The LaMET Approach

In standard effective theory set-up,

$$\tilde{O}(P^z/\Lambda) = Z(P^z/\Lambda, \mu/\Lambda)O(\mu) + \frac{c_2}{(P^z)^2} + \frac{c_4}{(P^z)^4} + \ldots$$

- Taking the IMF limit does not change the IR and collinear divergence, and thus O fully captures the IR and collinear divergence of O~, so Z is completely perturbative;
- Higher order terms are suppressed by powers of $1/(P^z)^2$ (P^z must be large enough);

The LaMET Approach

- A possible choice of the quasi-observables (Chen et al., 2008):

$$\vec{J}\,' = \int d^3x\,\psi^\dagger \frac{\vec{\Sigma}}{2}\psi + \int d^3x\,\psi^\dagger \vec{x}\times(-i\vec{\nabla} - g\vec{A}_{//})\psi$$
$$+ \int d^3x\,\vec{E}_a \times \vec{A}_\perp^a + \int d^3x\,E_a^i(\vec{x}\times\vec{\nabla})A_\perp^{i,a}$$

where

$$\partial^i A_{//}^{j,a} - \partial^j A_{//}^{i,a} - g f^{abc} A_{//}^{i,b} A_{//}^{j,c} = 0$$
$$\partial^i A_\perp^i - ig[A^i, A_\perp^i] = 0$$

The LaMET Approach

- Under a gauge transformation,

$$\vec{A}_\perp \to U(x)\vec{A}_\perp U^\dagger(x), \quad \vec{A}_{//} \to U(x)\vec{A}_{//}U^\dagger(x) + \frac{i}{g}(\vec{\nabla}U(x))U^\dagger(x).$$

- So that the expression is gauge invariant.
- The physical meaning of the nonlocal expression is not clear, but $E\times A_{perp}$ has been proven to be equivalent to the gluon spin in the Jaffe-Manohar expression in the IMF limit (Ji, Zhang, Zhao, 2013).

Matching conditions

A "renovated sum rule":

$$\frac{1}{2} = \frac{1}{2}\Delta\tilde{\Sigma}(\mu, P^z) + \Delta\tilde{G}(\mu, P^z) + \Delta\tilde{L}_q(\mu, P^z) + \Delta\tilde{L}_g(\mu, P^z)$$

In the LaMET framework:

$$\Delta\tilde{\Sigma}(\mu, P^z) = \Delta\Sigma(\mu),$$

$$\Delta\tilde{G}(\mu, P^z) = z_{qg}\Delta\Sigma(\mu) + z_{gg}\Delta G(\mu) + O(\frac{M^2}{P_z^2}),$$

$$\Delta\tilde{L}_q(\mu, P^z) = P_{qq}\Delta L_q(\mu) + P_{gq}\Delta L_g(\mu) + p_{qq}\Delta\Sigma(\mu) + p_{gq}\Delta G(\mu) + O(\frac{M^2}{P_z^2}),$$

$$\Delta\tilde{L}_g(\mu, P^z) = P_{qg}\Delta L_q(\mu) + P_{gg}\Delta L_g(\mu) + p_{qg}\Delta\Sigma(\mu) + p_{gg}\Delta G(\mu) + O(\frac{M^2}{P_z^2}).$$

Matching conditions

Ji, Zhang, Zhao, arXiv: 1409.6329

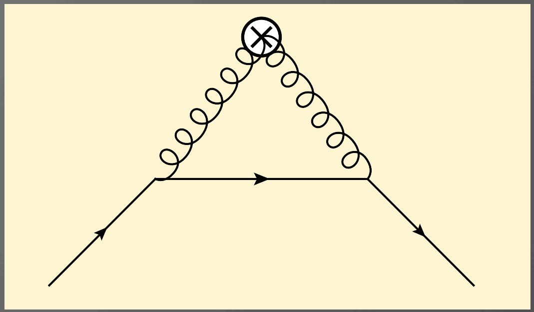
Calculation of the matching coefficients:

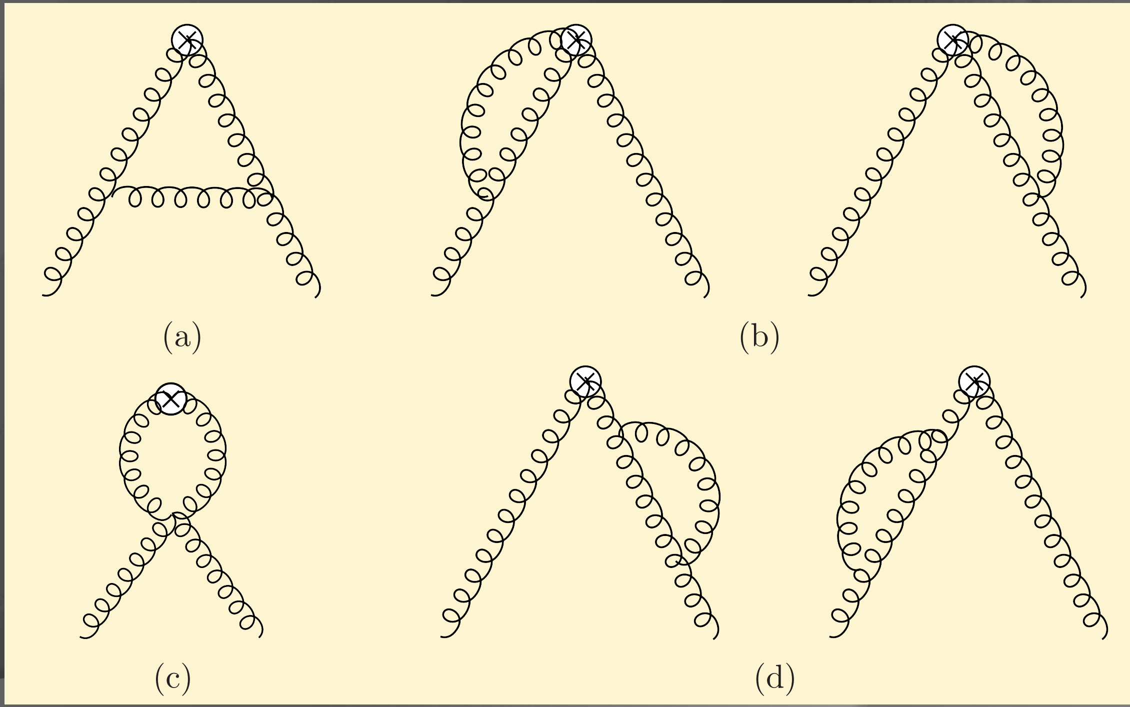
$$\Delta\tilde{G}(\mu, P^z) = z_{qg}\Delta\Sigma(\mu) + z_{gg}\Delta G(\mu) + O(\frac{M^2}{P_z^2})$$

- Compare the quark and gluon matrix elements of $E\times A_{perp}$ and E×A in the light-cone gauge.
- To guarantee the gauge invariance, we choose dimensional regularization with d=4-2ε and the MS-bar scheme, and the external states are onshell and massless.

Matching conditions

- At tree level, $$\Delta\tilde{G}^{\text{tree}} = \Delta G^{\text{tree}}.$$
- At one-loop level, consider the following Feynman diagrams:





Matching conditions

- The result

$$\Delta\tilde{G}^{\text{1-loop}} = \frac{\alpha_S C_F}{4\pi}\left[\frac{5}{3}\frac{1}{\varepsilon'_{UV}} + \frac{4}{3}\ln\frac{P_z^2}{\mu^2} - \frac{3}{\varepsilon'_{IR}} + R_1\right]\Delta\Sigma^{\text{tree}} + \frac{\alpha_S}{4\pi}\left[\frac{4C_A - 2n_f}{3}\frac{1}{\varepsilon'_{UV}} - \frac{11C_A - 2n_f}{3}\frac{1}{\varepsilon'_{IR}} + C_A\left(\frac{7}{3}\ln\frac{P_z^2}{\mu^2} + R_2\right)\right]\Delta G^{\text{tree}}$$

- The matrix element of E×A in the light-cone gauge is

$$\Delta G^{\text{1-loop}} = \frac{\alpha_S C_F}{4\pi}\left[\frac{3}{\varepsilon'_{UV}} - \frac{3}{\varepsilon'_{IR}}\right]\Delta\Sigma^{\text{tree}} + \frac{\alpha_S}{4\pi}\left[\frac{11C_A - 2n_f}{3}\frac{1}{\varepsilon'_{UV}} - \frac{11C_A - 2n_f}{3}\frac{1}{\varepsilon'_{IR}}\right]\Delta G^{\text{tree}}$$

Matching conditions

- Anomalous dimension different, but IR divergence the same!
- Subtract the UV poles, and replace the IR poles with ΔG

$$\Delta\tilde{G}^{\text{1-loop}} = \frac{\alpha_S C_F}{4\pi}\left(\frac{4}{3}\ln\frac{P_z^2}{\mu^2} + R_1\right)\Delta\Sigma^{\text{tree}} + \frac{\alpha_S C_A}{4\pi}\left(\frac{7}{3}\ln\frac{P_z^2}{\mu^2} + R_2\right)\Delta G^{\text{tree}} + \Delta G^{\text{1-loop}}$$

- Since

$$\Delta\tilde{G} \approx \Delta G^{\text{tree}} + \Delta\tilde{G}^{\text{1-loop}}, \quad \Delta G \approx \Delta G^{\text{tree}} + \Delta G^{\text{1-loop}}$$

finally

$$\Delta\tilde{G} = \frac{\alpha_S C_F}{4\pi}\left(\frac{4}{3}\ln\frac{P_z^2}{\mu^2} + R_1\right)\Delta\Sigma + \left[1 + \frac{\alpha_S C_A}{4\pi}\left(\frac{7}{3}\ln\frac{P_z^2}{\mu^2} + R_2\right)\right]\Delta G$$

Matching conditions

- The matching coefficients

$$z_{qg}(\mu/P^z)=\frac{\alpha_S C_F}{4\pi}(\frac{4}{3}\ln\frac{P_z^2}{\mu^2}+R_1),\quad z_{gg}(\mu/P^z)=1+\frac{\alpha_S C_A}{4\pi}(\frac{7}{3}\ln\frac{P_z^2}{\mu^2}+R_2).$$

- Similarly, we obtain

$$P_{qq}=1+\frac{\alpha_S C_F}{4\pi}(-2\ln\frac{P_z^2}{\mu^2}+R_3),\qquad P_{gq}=0,$$

$$P_{qg}=\frac{\alpha_S C_F}{4\pi}(2\ln\frac{P_z^2}{\mu^2}-R_3),\qquad P_{gg}=1,$$

$$p_{qq}=\frac{\alpha_S C_F}{4\pi}(-\frac{1}{3}\ln\frac{P_z^2}{\mu^2}+R_4),\qquad p_{gq}=0,$$

$$p_{qg}=\frac{\alpha_S C_F}{4\pi}(-\ln\frac{P_z^2}{\mu^2}-R_1-R_4),\qquad p_{qq}=\frac{\alpha_S C_F}{4\pi}(-\frac{7}{3}\ln\frac{P_z^2}{\mu^2}-R_2).$$

Matching conditions

- Finite constants:

$$R_1 = \frac{8}{3}\ln 2 - \frac{64}{9}, \quad R_2 = \frac{14}{3}\ln 2 - \frac{121}{9},$$

$$R_3 = -4\ln 2 + \frac{28}{3}, \quad R_4 = -\frac{2}{3}\ln 2 + \frac{13}{9}.$$

Next step:

We need to perform a similar matching procedure in lattice QCD (Capitani, 2003) to extract the physical result from simulations.

The finite constants are important because they are different in lattice renormalization and need to be precisely calculated.

Conclusion

- We justify the physical meaning of the Jaffe-Manohar sum rule for proton spin;
- The parton contributions in the Jaffe-Manohar sum rule can be related to certain quasi-observables through perturbative matching conditions in LaMET;
- We obtain the matching condition for proton spin content at one-loop order.

Back-up slides

Lattice QCD

- Euclidean Space

$$x_0^{\mathrm{E}} = \mathrm{i}x_0^{\mathrm{M}} \qquad \gamma_0^{\mathrm{E}} = \gamma_0^{\mathrm{M}}, \quad \gamma_i^{\mathrm{E}} = -\mathrm{i}\gamma_i^{\mathrm{M}} \quad \mathrm{e}^{\mathrm{i}S_{\mathrm{M}}} \longrightarrow \mathrm{e}^{-S_{\mathrm{E}}}$$

- Discretization

$$\int \mathrm{d}^4x \to a^4 \sum_x ,$$

$$\frac{1}{V}\sum_k \to \int_{-\pi/a}^{\pi/a} \frac{\mathrm{d}k_0}{2\pi} \int_{-\pi/a}^{\pi/a} \frac{\mathrm{d}k_1}{2\pi} \int_{-\pi/a}^{\pi/a} \frac{\mathrm{d}k_2}{2\pi} \int_{-\pi/a}^{\pi/a} \frac{\mathrm{d}k_3}{2\pi} .$$

- Derivatives

$$\nabla_\mu \psi(x) = \frac{\psi(x + a\hat{\mu}) - \psi(x)}{a} , \quad \nabla_\mu^\star \psi(x) = \frac{\psi(x) - \psi(x - a\hat{\mu})}{a}$$

Lattice QCD

- Gauge invariance?

$$\psi(x) \to \Omega(x)\psi(x) \ , \ \bar{\psi}(x) \to \bar{\psi}(x)\Omega^{-1}(x)$$

$$\nabla_\mu \psi(x) = \frac{\psi(x+a\hat{\mu}) - \psi(x)}{a} \ ,$$

Cannot form gauge-invariant bilinear operator

- Gauge link:

$$U_\mu(x) = \mathrm{e}^{\mathrm{i}g_0 a T^a A^a_\mu(x)} \quad (a = 1, \ldots, N_c^2 - 1) \ ,$$

$$U_\mu(x) \to \Omega(x) U_\mu(x) \Omega^{-1}(x+a\hat{\mu}) \ ,$$

so that

$$U_\mu(x)\psi(x+a\hat{\mu}) \to \Omega(x)U_\mu(x)\psi(x+a\hat{\mu})$$

Lattice QCD

- Wilson Action

$$\tilde{\nabla}_\mu \psi(x) = \frac{U_\mu(x)\psi(x+a\hat{\mu}) - \psi(x)}{a}$$

$$S_\mathrm{W} = S_\mathrm{W}^f + S_\mathrm{W}^g \ ,$$

$$S_\mathrm{W}^f = a^4 \sum_x \Big[-\frac{1}{2a} \sum_\mu [\bar{\psi}(x)(r-\gamma_\mu)U_\mu(x)\psi(x+a\hat{\mu})$$

$$+ \bar{\psi}(x+a\hat{\mu})(r+\gamma_\mu)U_\mu^\dagger(x)\psi(x)] + \bar{\psi}(x)\left(m_0 + \frac{4r}{a}\right)\psi(x)\Big]$$

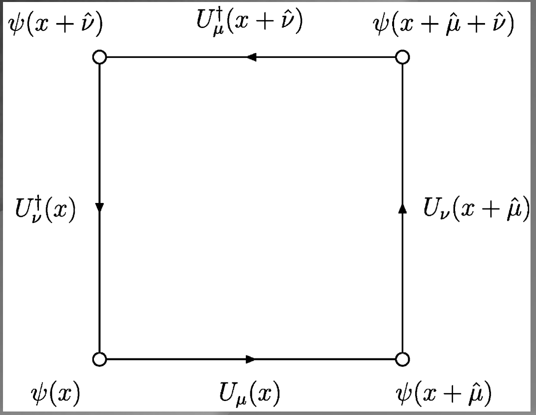
$$= a^4 \sum_x \bar{\psi}(x)\left[\frac{1}{2}(\gamma_\mu(\tilde{\nabla}_\mu^\star + \tilde{\nabla}_\mu) - ar\tilde{\nabla}_\mu^\star\tilde{\nabla}_\mu) + m_0\right]\psi(x) \ ,$$

Wilson term

$$S_\mathrm{W}^g = \frac{1}{g_0^2} a^4 \sum_{x,\mu\nu} [N_c - \mathrm{Re\,Tr}[U_\mu(x)U_\nu(x+a\hat{\mu})U_\mu^\dagger(x+a\hat{\nu})U_\nu^\dagger(x)]] \ ,$$

The plaquette P

$$\mathrm{Re\,Tr}\, P_{\mu\nu}(x) = N_c - \frac{1}{2} g_0^2 a^4 \,\mathrm{Tr}\, F_{\mu\nu}^2(x) + O(a^6)$$



Lattice QCD

- Fermion doubling problem

For r=0, $$S^{ab}(k, m_0) = \delta^{ab} \cdot a \frac{-\mathrm{i} \sum_\mu \gamma_\mu \sin ak_\mu + am_0}{\sum_\mu \sin^2 ak_\mu + (am_0)^2} .$$

If $m_0=0$, the propagator has 16 different poles including the origin.

divided into 8 particles with opposite chirality, which is more than that of the continuum theory, and can destroy its chiral property

The Wilson term $-\frac{1}{2} ar \tilde{\nabla}^{\star}_{\mu} \tilde{\nabla}_{\mu}$ contributes a momentum-dependent mass to the fermions, and thus removes the doublers.

Expense: adding new source of error to mass renormalization, destroying chiral symmetry.

Lattice QCD

- No-go theorem: Impossible to eliminate the doublers without at the same time breaking chiral symmetry or some other important properties of the field theory, i.e., locality or unitarity.

Lattice QCD

- Solution to the paradox: modified chiral symmetry:

$$S_F = a^4 \sum_{x,y} \bar{\psi}(x) D(x-y) \psi(y)$$

Exact chiral symmetry: $\gamma_5 D + D\gamma_5 = 0$

Ginsparg-Wilson relation (1982): $\gamma_5 D + D\gamma_5 = a \frac{1}{\rho} D\gamma_5 D$

$$\psi \to \psi + \epsilon \cdot \gamma_5 \left(1 - \frac{a}{\rho} D\right) \psi \ ,$$

$$\bar{\psi} \to \bar{\psi} + \epsilon \cdot \bar{\psi} \gamma_5 \ .$$

Overlap fermions

- The overlap Dirac operator (Neuberger, 1998):

$$D_{\text{ov}} = \frac{\rho}{a}\left(1 + \frac{X}{\sqrt{X^\dagger X}}\right), \qquad X = D_W - \frac{\rho}{a}$$

$$D_{\text{W}} = \frac{1}{2}(\gamma_\mu(\tilde{\nabla}^\star_\mu + \tilde{\nabla}_\mu) - ar\tilde{\nabla}^\star_\mu \tilde{\nabla}_\mu),$$

1. One solution to the Ginsparg-Wilson relation;
2. Apparently nonlocal, but the interaction decays exponentially over distance, and thus can be regarded as local;
3. Other solutions, like domain wall fermions, fixed-point fermions ("classically perfect" fermions).

Overlap fermions

- Why do we use overlap fermions for the perturbative matching?

A plain reason: our lattice collaborators use overlap fermions.

Deeper reason:

1. Success in dealing with certain problems in QCD (see Liu, Alexandru, Hovrath, 2007);
2. D_{ov} can be used to construct gauge field operators;
3. And perhaps more…

Overlap fermions

- In particular, it has been proven that

$$\mathrm{tr}_s \sigma_{\mu\nu} D_{\mathrm{ov}}(x,x) \; = \; a^2 C^T(\rho, r) F_{\mu\nu}(x) + \mathcal{O}(a^3)$$

Trace over spinor indices

A constant that can be calculated to arbitrary precisio

- Therefore, for perturbative calculations we use D_{ov} to obtain the Feynman rules for the field strength tensor with finite a.

Overlap fermions

- Expansion of D_{ov} in orders of g

$$X(q,p) \;=\; X_0(p)(2\pi)^4\delta_P(q-p) + X_1(q,p) + X_2(q,p) + \mathcal{O}(g^3),$$

$$\frac{1}{\sqrt{X^\dagger X}} = \int_{-\infty}^{\infty}\frac{dt}{\pi}\frac{1}{t^2+X^\dagger X},$$

$$\begin{aligned}\frac{1}{t^2+X^\dagger X} \;=\;& \frac{1}{t^2+X_0^\dagger X_0} - \frac{1}{t^2+X_0^\dagger X_0}\left(X_0^\dagger X_1 + X_1^\dagger X_0\right)\frac{1}{t^2+X_0^\dagger X_0} \\ &+ \left[\frac{1}{t^2+X_0^\dagger X_0}\left(X_0^\dagger X_1 + X_1^\dagger X_0\right)\frac{1}{t^2+X_0^\dagger X_0}\left(X_0^\dagger X_1 + X_1^\dagger X_0\right)\frac{1}{t^2+X_0^\dagger X_0}\right. \\ &\left. - \frac{1}{t^2+X_0^\dagger X_0}\left(X_0^\dagger X_2 + X_2^\dagger X_0 + X_1^\dagger X_1\right)\frac{1}{t^2+X_0^\dagger X_0}\right] + \mathcal{O}(g^3)\,.\end{aligned}$$

Overlap fermions

- Results

$$[\mathrm{tr}_s \sigma_{\mu\nu} D_{\mathrm{ov}}(x,x)]^0 = 0$$

$$[\mathrm{tr}_s \sigma_{\mu\nu} D_{\mathrm{ov}}(x,x)]^1 = -ig \int \frac{d^3p}{(2\pi)^3} \frac{d^3k}{(2\pi)^3} e^{ix\cdot p} \sum_\nu A_\nu(p) \int d\sigma \frac{1}{(t^2+\omega^2(k-p))} \frac{1}{(t^2+\omega^2(k))}$$
$$\times \mathrm{tr}_s \left\{ \left[\sum_\mu \gamma_\mu \left(-\frac{i}{a} \sin((k-p)_\mu a) \right) + \sum_\mu \frac{r}{a} (1-\cos((k-p)_\mu a)) \right] \right.$$
$$\times [\gamma_\nu i \sin((p_\mu/2 - k_\mu)a) - r\cos((p_\mu/2-k_\mu)a)]$$
$$\left. \times \left[-\sum_\rho \gamma_\rho \frac{i}{a} \sin(k_\rho a) + \sum_\rho \frac{r}{a}(1-\cos(k_\rho a)) \right] \right\}$$

In the continuum limit, $[\mathrm{tr}_s \sigma_{\mu\nu} D_{\mathrm{ov}}(x,x)]^1 \sim g\left(\partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)\right)$

- To obtain the 3-gluon part, we need to go to order g^2, which is much more complicated.