Overview of TMD Evolution

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Spin2014, Beijing, October 20, 2014

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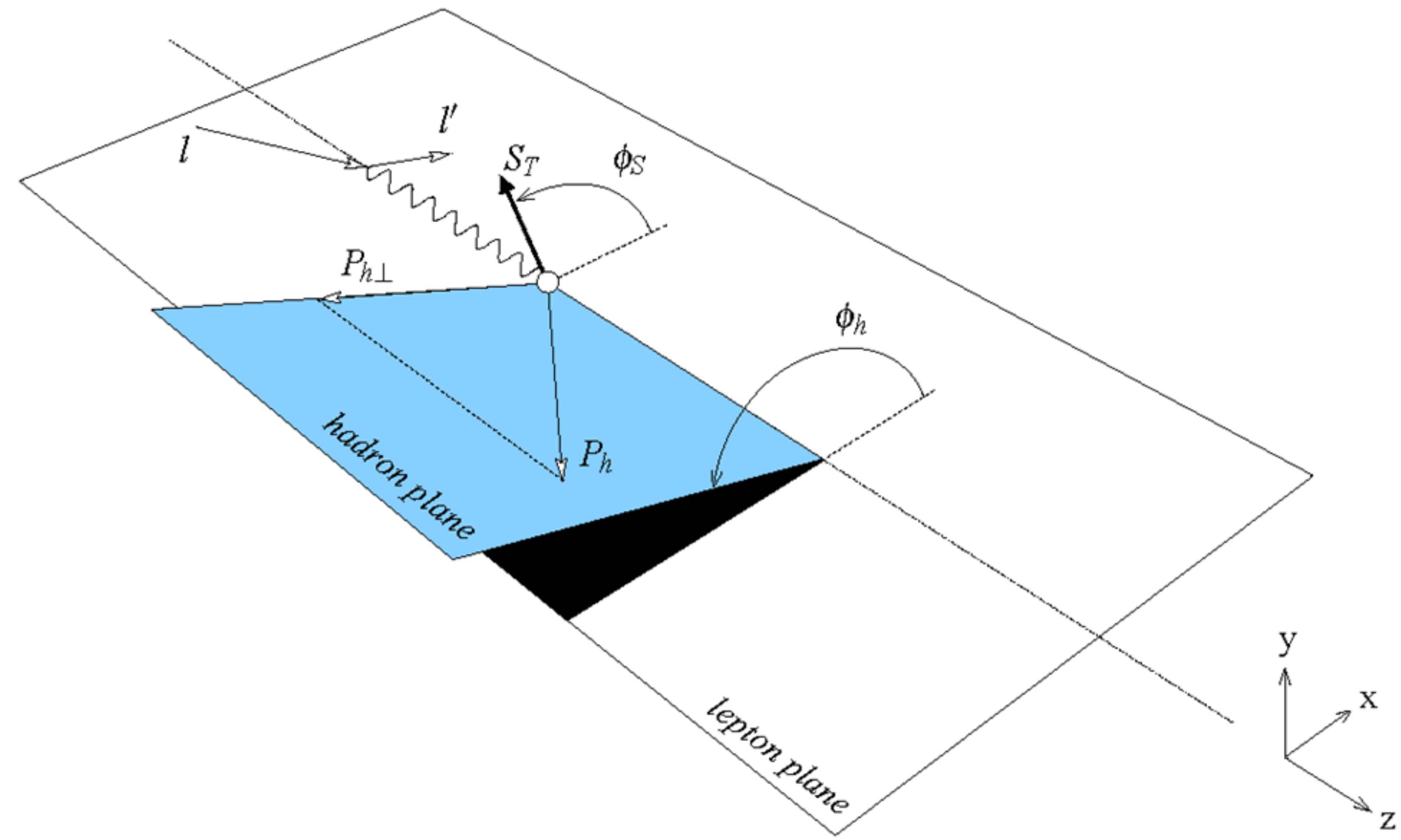
Introduction

SIDIS: a typical TMD process



$$e\,p \to e'\,h\,X$$

Semi-inclusive DIS is a process sensitive to transverse momentum of quarks

$P_{h\perp}$ = the observed transverse momentum of the produced hadron = $z_h\,Q_T$

Q_T = the transverse momentum of the virtual photon w.r.t. p and h

Many transverse momentum dependent *angular distributions* have been measured in SIDIS by HERMES, COMPASS, and JLab experiments

Evolution is needed to compare these results, factorization dictates the evolution

Transverse Momentum of Quarks

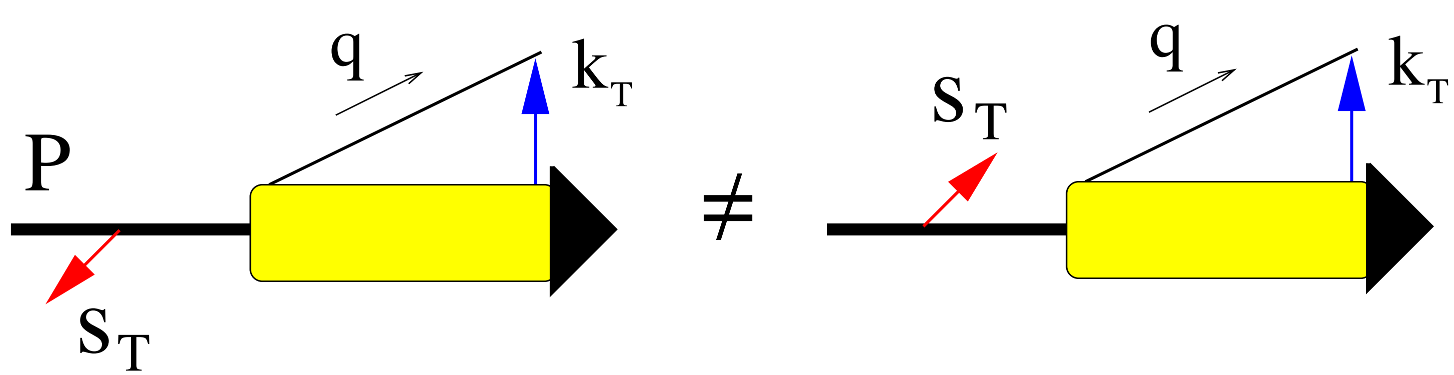
Including transverse momentum of quarks involves much more than replacing $f_1(x) \rightarrow f_1(x,k_T^2)$ in collinear factorization expressions

One deals with less inclusive processes and with TMD factorization

TMD = *transverse momentum dependent* parton distribution

Here the transverse momentum dependence can be correlated with the spin, e.g.

D. Sivers ('90):



$k_T \times S_T$

Sivers function

$$\Phi(x, \boldsymbol{k}_T) = \frac{M}{2} \left\{ f_1(x, \boldsymbol{k}_T^2) \frac{\not{P}}{M} + f_{1T}^{\perp}(x, \boldsymbol{k}_T^2) \frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu P^\nu k_T^\rho S_T^\sigma}{M^2} + g_{1s}(x, \boldsymbol{k}_T^2) \frac{\gamma_5 \not{P}}{M} \right.$$

$$\left. + h_{1T}(x, \boldsymbol{k}_T^2) \frac{\gamma_5 \not{S}_T \not{P}}{M} + h_{1s}^{\perp}(x, \boldsymbol{k}_T^2) \frac{\gamma_5 \not{k}_T \not{P}}{M^2} + h_1^{\perp}(x, \boldsymbol{k}_T^2) \frac{i \not{k}_T \not{P}}{M^2} \right\}$$

[Ralston, Soper '79; Sivers '90; Collins '93; Kotzinian '95; Mulders, Tangerman '95; D.B., Mulders '98]

TMD factorization

"Evolution" of TMD Factorization

- Collins & Soper, 1981: $e^+e^- \to h_1\, h_2\, X$ [NPB 193 (1981) 381]
- X. Ji, J.-P. Ma & F. Yuan, 2004/5: SIDIS & Drell-Yan (DY) [PRD 71 (2005) 034005 & PLB 597 (2004) 299]
- Collins (JCC), 2011: "Foundations of perturbative QCD" [Cambridge Univ. Press]
- P. Sun, B.-W. Xiao & F. Yuan, 2011: Higgs prod. (gluon TMDs)[PRD 84 (2011) 094005]
- Echevarria, Idilbi & Scimemi (EIS), 2012/4: DY & SIDIS (SCET)[JHEP 1207 (2012) 002 & PRD 90 (2014) 014003]
- J.P. Ma, J.X. Wang & S. Zhao, 2012: quarkonium prod. 1-loop [PRD 88 (2013) 014027]
- J.P. Ma, J.X. Wang & S. Zhao, 2014: breakdown of factorization in P-wave quarkonium production beyond 1-loop [PLB 737 (2014) 103]

Main differences among the various approaches:
- treatment of rapidity/LC divergences, in order to make each factor well-defined
- redistribution of terms to avoid large logarithms

TMD factorization

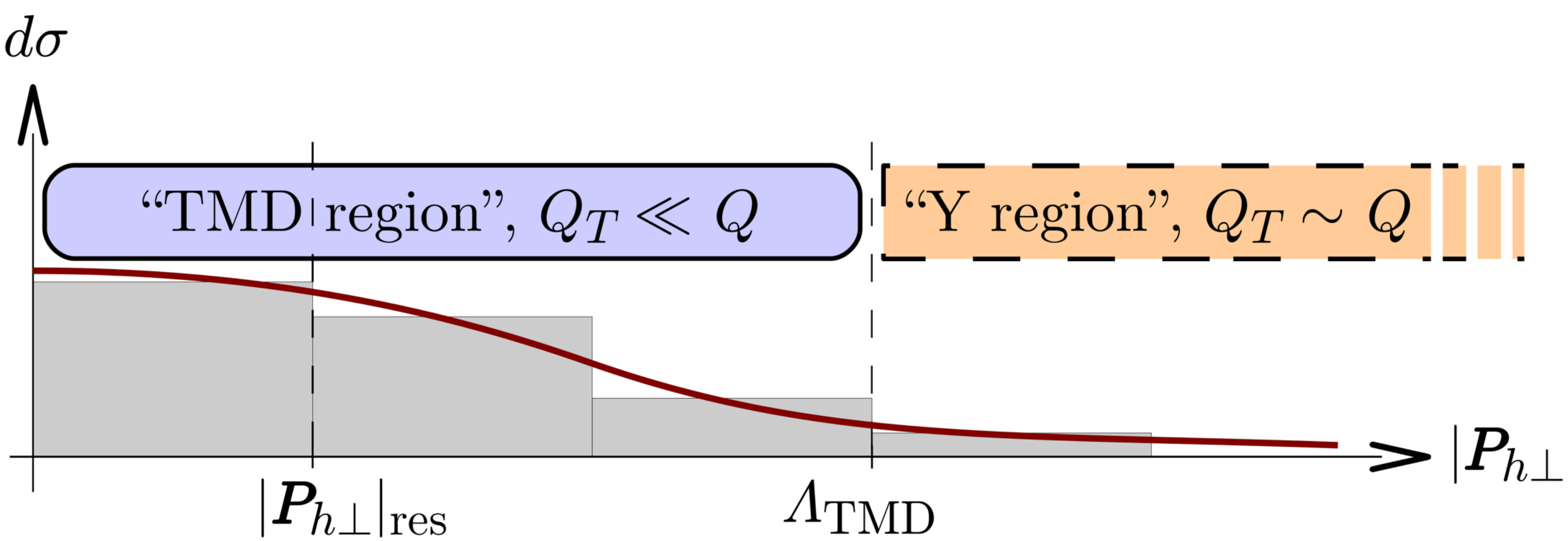
TMD factorization proven for SIDIS, $e^+e^- \to h_1\, h_2\, X$ and Drell-Yan (DY)

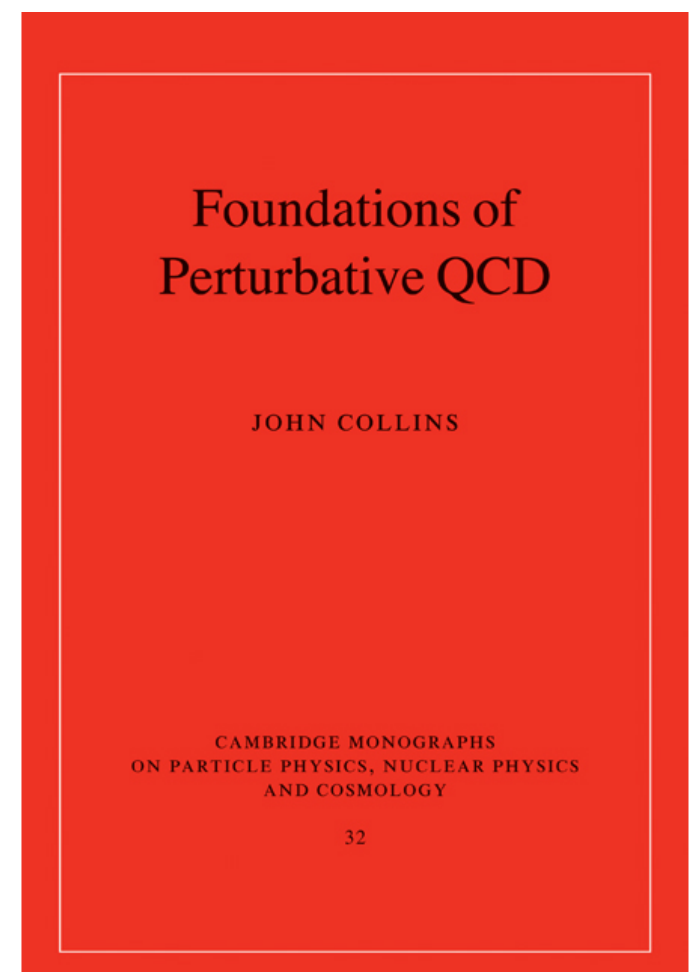
Schematic form of (new) TMD factorization "JCC" [Collins 2011]:

$$d\sigma = H \times \text{convolution of } A\,B + \text{high-}q_T \text{ correction } (Y) + \text{power-suppressed}$$

A & B are TMD pdfs or FFs
(a soft factor has been absorbed in them)

Details in book by J.C. Collins
Summarized in arXiv:1107.4123





Convolution in terms of A and B best
deconvoluted by Fourier transform

New TMD factorization expressions

$$\frac{d\sigma}{d\Omega d^4q} = \int d^2b\, e^{-i\boldsymbol{b}\cdot\boldsymbol{q}_T}\tilde{W}(\boldsymbol{b}, Q; x, y, z) + \mathcal{O}\left(Q_T^2/Q^2\right)$$

$$\tilde{W}(\boldsymbol{b}, Q; x, y, z) = \sum_a \tilde{f}_1^a(x, \boldsymbol{b}^2; \zeta_F, \mu)\tilde{D}_1^a(z, \boldsymbol{b}^2; \zeta_D, \mu)H\left(y, Q; \mu\right)$$

Fourier transforms of the TMDs are functions of the momentum fraction *x* (or *z*), the transverse coordinate b, a rapidity variable ζ , and the renormalization scale μ

$$\zeta_F = M^2x^2e^{2(y_P - y_s)} \qquad \zeta_D = M_h^2e^{2(y_s - y_h)}/z^2$$

y_s is an arbitrary rapidity that drops out of the final answer

$$\zeta_F\zeta_D \approx Q^4 \qquad\qquad \zeta_F \approx \zeta_D \approx Q^2$$

The TMDs in principle also depend on the Wilson line U

$$\tilde{f}^{[\mathcal{U}]}(x, b_T^2; \zeta, \mu)$$

New TMD factorization expressions

Y term

$$\frac{d\sigma}{d\Omega d^4q} = \int d^2b\, e^{-i\boldsymbol{b}\cdot\boldsymbol{q}_T}\tilde{W}(\boldsymbol{b},Q;x,y,z) + \mathcal{O}\left(Q_T^2/Q^2\right)$$

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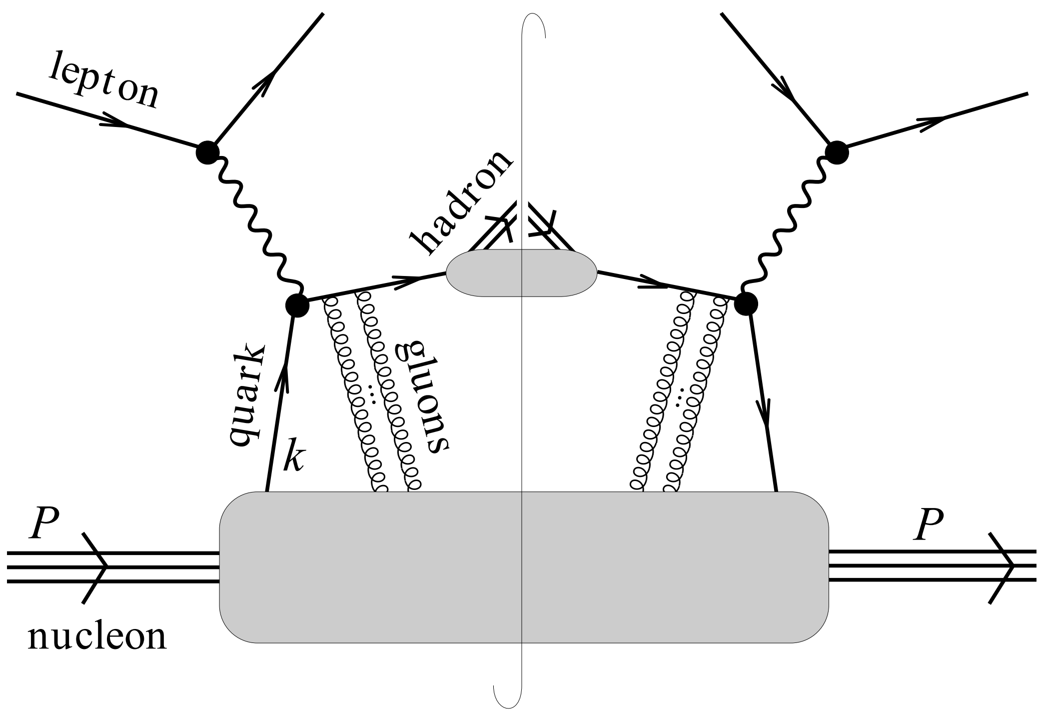
Gauge invariance of TMD correlators



summation of all gluon insertions leads to path-ordered exponentials in the correlators

$$\mathcal{L}_{\mathcal{C}}[0,\xi] = \mathcal{P}\exp\left(-ig\int_{\mathcal{C}[0,\xi]} ds_\mu\, A^\mu(s)\right)$$

$$\Phi \propto \langle P|\overline{\psi}(0)\mathcal{L}_{\mathcal{C}}[0,\xi]\psi(\xi)|P\rangle$$

Efremov & Radyushkin, Theor. Math. Phys. 44 ('81) 774

Resulting Wilson lines depend on whether the color is incoming or outgoing

[Collins & Soper, 1983; DB & Mulders, 2000; Brodsky, Hwang & Schmidt, 2002; Collins, 2002; Belitsky, X. Ji & F. Yuan, 2003; DB, Mulders & Pijlman, 2003]

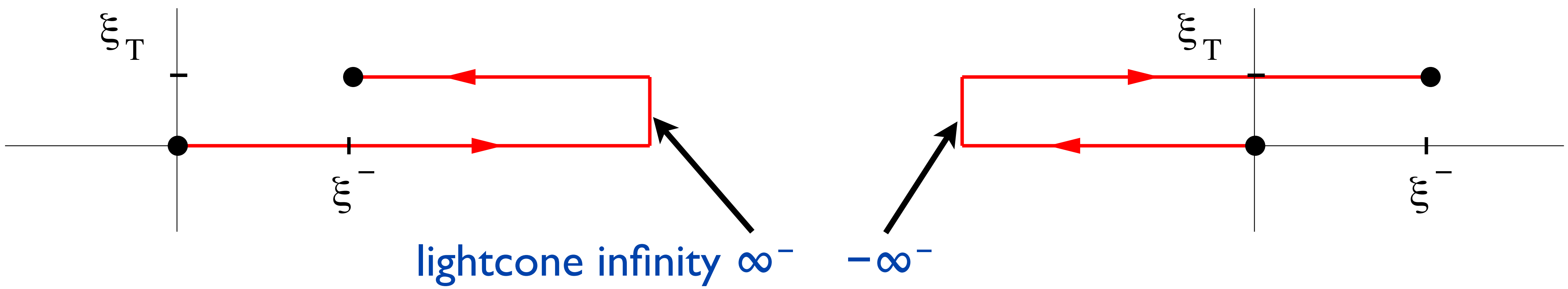
This does not automatically imply that this affects observables, but it turns out that it does in certain cases, for example, Sivers asymmetries

[Brodsky, Hwang & Schmidt, 2002; Collins, 2002; Belitsky, Ji & Yuan, 2003]

Process dependence of Sivers TMD

Gauge invariant definition of TMDs in semi-inclusive DIS contains a future pointing Wilson line, whereas in Drell-Yan (DY) it is past pointing
[Belitsky, X. Ji & F. Yuan '03]

$\gamma^* p \to h\, X$ (SIDIS) $\qquad$ $pp \to \gamma^* X$ (Drell-Yan)



One can use parity and time reversal invariance to relate the Sivers functions:

$$f_{1T}^{\perp [\mathrm{SIDIS}]} = - f_{1T}^{\perp [\mathrm{DY}]}$$

[Collins '02]

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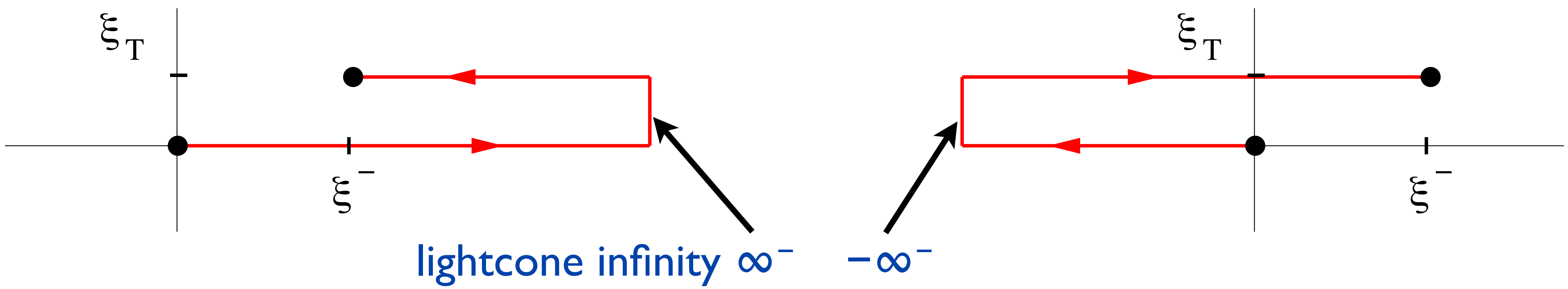

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The more hadrons are observed in a process, the more complicated the end result: more complicated N_c-dependent prefactors
[Bomhof, Mulders & Pijlman '04; Buffing, Mulders '14]

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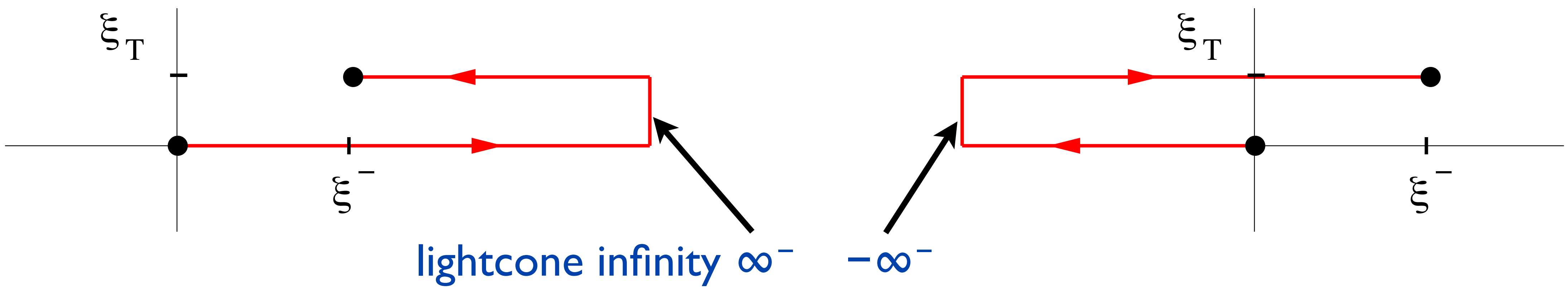

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When color flow is in too many directions: *factorization breaking*

[Collins & J. Qiu '07; Collins '07; Rogers & Mulders '10]

Scale dependence of TMDs

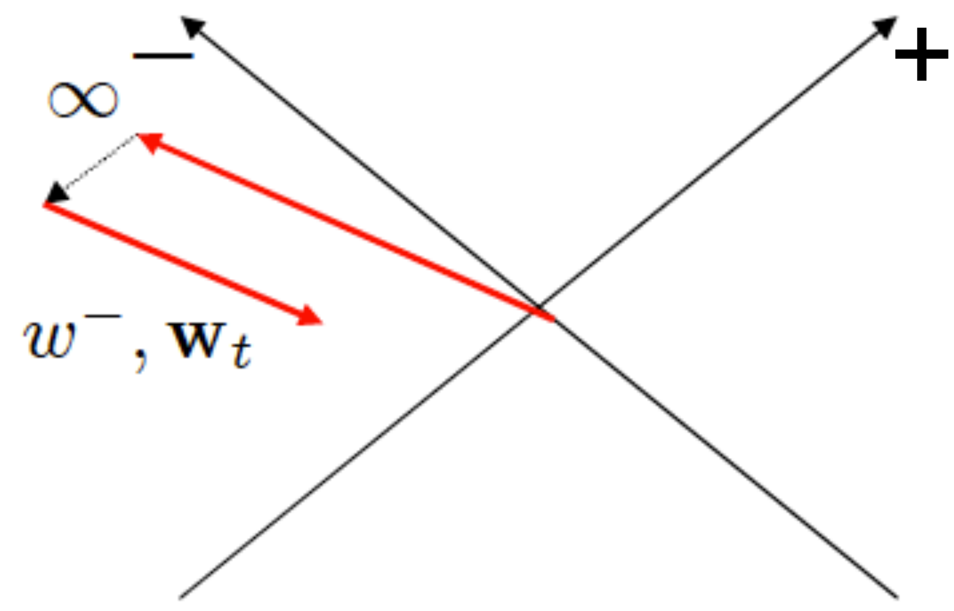
QCD corrections will also attach to the Wilson line, which needs renormalization

Wilson lines not smooth: cusp anomalous dimension
[Polyakov '80; Dotsenko & Vergeles ’80; Brandt, Neri, Sato '81; Korchemsky, Radyushkin '87]

This determines the change with μ

As a regularization of LC divergences, in JCC’s TMD factorization the path is taken off the lightfront, the variation in rapidity determines the change with ζ

$$\tilde{f}^{[\mathcal{U}]}(x, b_T^2; \zeta, \mu)$$



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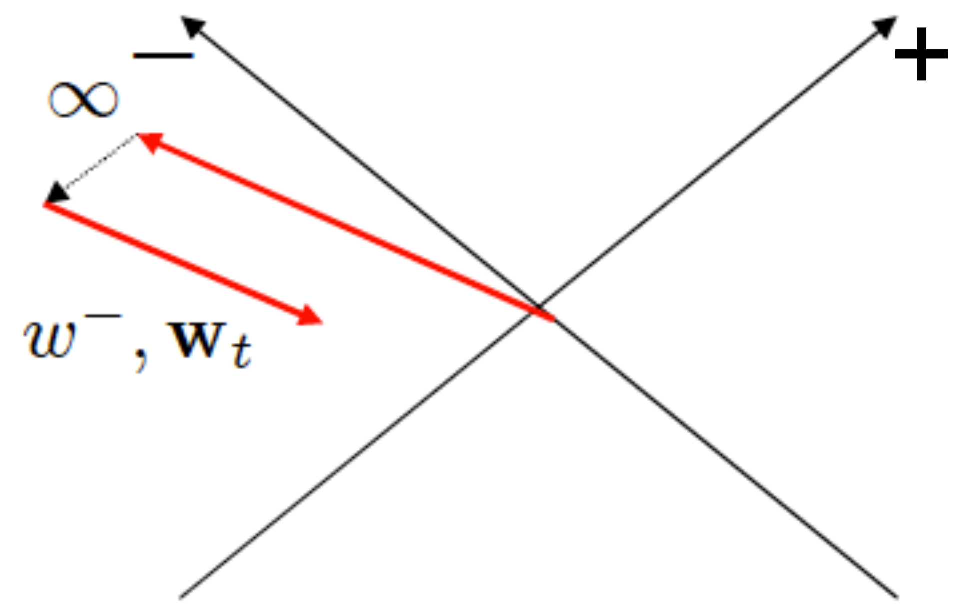

$$\tilde{f}^{[\mathcal{U}]}(x, b_T^2; \zeta, \mu)$$

Two important consequences:

- yields energy evolution of TMD observables
- allows for calculation of the Sivers and Boer-Mulders effect on the lattice
  Musch, Hägler, Engelhardt, Negele & Schäfer, 2012

New TMD factorization expressions

$$\frac{d\sigma}{d\Omega d^4q} = \int d^2b\, e^{-i\boldsymbol{b}\cdot\boldsymbol{q}_T}\tilde{W}(\boldsymbol{b}, Q; x, y, z) + \mathcal{O}\left(Q_T^2/Q^2\right)$$

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Take $\mu = Q$

$$H\left(Q; \alpha_s(Q)\right) \propto e_a^2\left(1 + \alpha_s(Q^2)F_1 + \mathcal{O}(\alpha_s^2)\right)$$

Avoids large logarithms in H, but now they do appear in the TMDs

Use renormalization group equations to evolve the TMDs to the scale:

$$\mu_b = C_1/b = 2e^{-\gamma_E}/b \quad (C_1 \approx 1.123)$$

Or to a fixed low (but still perturbative) scale Q_0, although that only works for not too large Q

RG and CS equations

$$\frac{d\ln \tilde{f}(x,b;\zeta,\mu)}{d\ln\sqrt{\zeta}} = \tilde{K}(b;\mu)$$ Collins-Soper equation

$$\frac{d\ln \tilde{f}(x,b;\zeta,\mu)}{d\ln\mu} = \gamma_F(g(\mu);\zeta/\mu^2)$$ RG equation

$$d\tilde{K}/d\ln\mu = -\gamma_K(g(\mu))$$

$$\gamma_F(g(\mu);\zeta/\mu^2) = \gamma_F(g(\mu);1) - \frac{1}{2}\gamma_K(g(\mu))\ln(\zeta/\mu^2)$$

Using these equations one can evolve the TMDs to the scale μ_b

$$\tilde{f}_1^a(x,b^2;\zeta_F,\mu)\,\tilde{D}_1^b(z,b^2;\zeta_D,\mu) = e^{-S(b,Q)}\,\tilde{f}_1^a(x,b^2;\mu_b^2,\mu_b)\,\tilde{D}_1^b(z,b^2;\mu_b^2,\mu_b)$$

with Sudakov factor

$$S(b,Q) = -\ln\left(\frac{Q^2}{\mu_b^2}\right)\tilde{K}(b,\mu_b) - \int_{\mu_b^2}^{Q^2}\frac{d\mu^2}{\mu^2}\left[\gamma_F(g(\mu);1) - \frac{1}{2}\ln\left(\frac{Q^2}{\mu^2}\right)\gamma_K(g(\mu))\right]$$

Perturbative expressions

At leading order in α_s

$$\tilde{K}(b,\mu) = -\alpha_s(\mu)\frac{C_F}{\pi}\ln(\mu^2 b^2/C_1^2) + \mathcal{O}(\alpha_s^2)$$

$$\gamma_K(g(\mu)) = 2\alpha_s(\mu)\frac{C_F}{\pi} + \mathcal{O}(\alpha_s^2)$$

$$\gamma_F(g(\mu),\zeta/\mu^2) = \alpha_s(\mu)\frac{C_F}{\pi}\left(\frac{3}{2} - \ln\left(\zeta/\mu^2\right)\right) + \mathcal{O}(\alpha_s^2)$$

Such that the perturbative expression for the Sudakov factor becomes:

$$S_p(b,Q) = \frac{C_F}{\pi}\int_{\mu_b^2}^{Q^2}\frac{d\mu^2}{\mu^2}\alpha_s(\mu)\left(\ln\frac{Q^2}{\mu^2} - \frac{3}{2}\right) + \mathcal{O}(\alpha_s^2)$$

It can be used whenever the restriction $b^2 << 1/\Lambda^2$ is justified (e.g. at very large Q^2)

If also larger b contributions are important, at moderate Q and small Q_T for instance, then one needs to include a *nonperturbative* Sudakov factor

Nonperturbative Sudakov factor

$$\tilde{W}(b) \equiv \tilde{W}(b_*)\, e^{-S_{NP}(b)} \qquad b_* = b/\sqrt{1 + b^2/b_{\rm max}^2} \leq b_{\rm max}$$

$$b_{\rm max} = 1.5\ {\rm GeV}^{-1} \Rightarrow \alpha_s(b_0/b_{\rm max}) = 0.62$$

such that W(b_*) can be calculated within perturbation theory
In general the nonperturbative Sudakov factor is Q dependent and of the form:

$$S_{NP}(b,Q) = \ln(Q^2/Q_0^2) g_1(b) + g_A(x_A,b) + g_B(x_B,b) \qquad Q_0 = \frac{1}{b_{\rm max}}$$

Collins, Soper & Sterman, NPB 250 (1985) 199

The $g_{..}$ functions need to be fitted to data

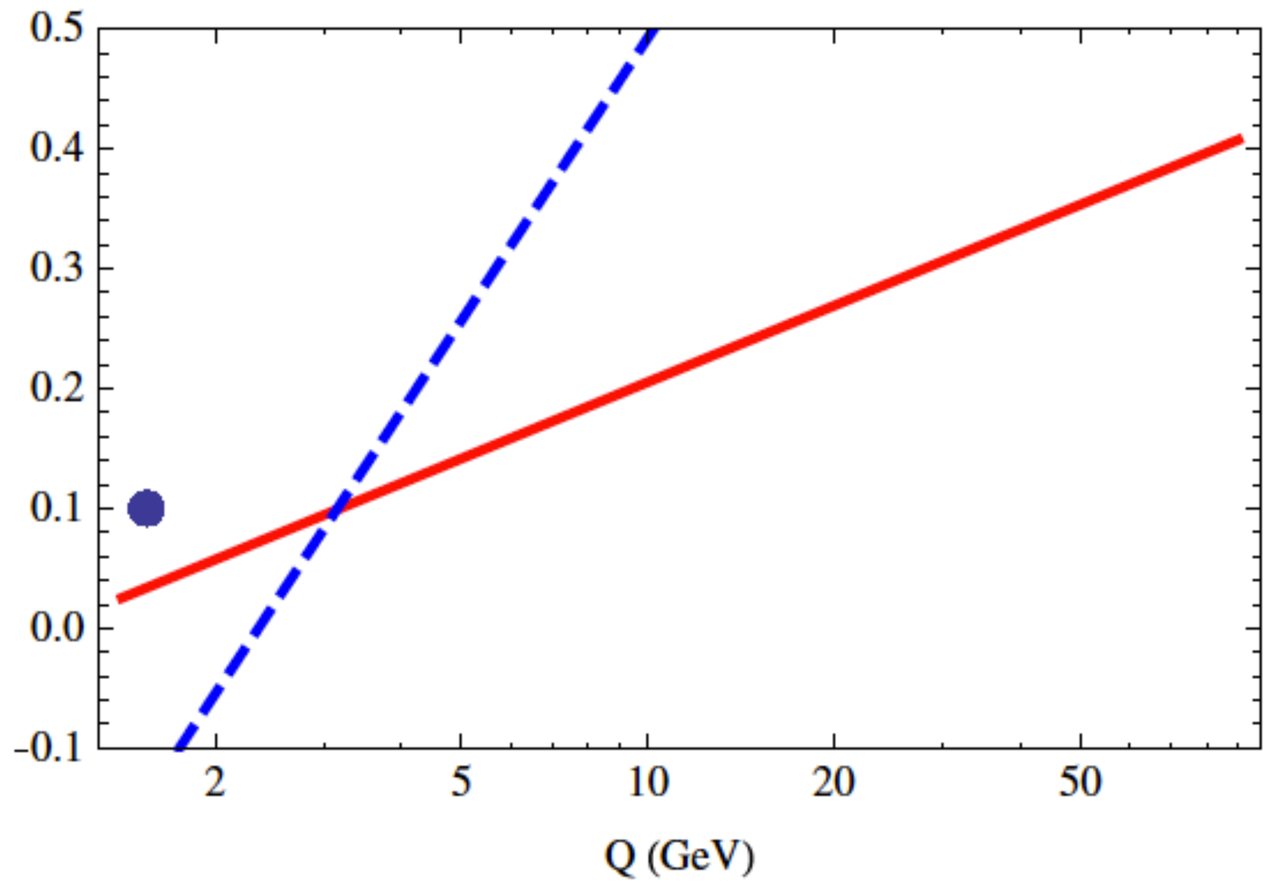
Until recently S_{NP} typically chosen as a Gaussian, e.g. Aybat & Rogers (x=0.1):

$$S_{NP}(b,Q,Q_0) = \left[0.184 \ln \frac{Q}{2Q_0} + 0.332\right] b^2$$

Recently alternatives considered in: P. Sun & F. Yuan, PRD 88 (2013) 034016
P. Sun, Isaacson, C.-P. Yuan & F. Yuan, arXiv:1406.3073

New form suggested by Collins (QCD evolution workshop 2013): $e^{-m\left(\sqrt{b^2+b_0^2}-b_0\right)}$

S_{NP}



Problem is to find one single universal S_{NP} that describes both SIDIS and DY/Z data

Figure 6. Coefficient of $-b_T^2$ in the exponent in Eq. (6), from Sun and Yuan [13], as a function of Q at $x = 0.1$. The blue dashed line is for the BLNY fit, and the red solid line for a KN fit with $b_{max} = 1.5\,\text{GeV}^{-1}$. The dot represents the value needed for SIDIS at HERMES.

From Collins, 1409.5408 based on P. Sun & F. Yuan, PRD 88 (2013) 034016

BLNY = Brock, Landry, Nadolsky, C.-P. Yuan, PRD67 (2003) 073016
KN = Konychev & Nadolsky, PLB 633 (2006) 710

Further resummations

$$\tilde{F}(x, b_T; \zeta_f, \mu_f) = \tilde{R}(b_T; \zeta_i, \mu_i, \zeta_f, \mu_f)\tilde{F}(x, b_T; \zeta_i, \mu_i)$$

$$\tilde{R}(b_T; \zeta_i, \mu_i, \zeta_f, \mu_f) = \exp\left\{\int_{\mu_i}^{\mu_f} \frac{d\bar{\mu}}{\bar{\mu}}\gamma_F\left(\alpha_s(\bar{\mu}), \ln\frac{\zeta_f}{\bar{\mu}^2}\right)\right\}\left(\frac{\zeta_f}{\zeta_i}\right)^{-D(b_T;\mu_i)}$$

$$\boxed{D(b_T, \mu) = -\frac{1}{2}\tilde{K}(b_T, \mu)} \qquad \frac{dD(b_T, \mu)}{d\ln\mu} = \Gamma_{\rm cusp} = \frac{1}{2}\gamma_K$$

$$\begin{aligned}D^R(b_T;\mu) = &-\frac{\Gamma_0}{2\beta_0}\ln(1-X) + \frac{1}{2}\left(\frac{a_s}{1-X}\right)\left[-\frac{\beta_1\Gamma_0}{\beta_0^2}(X+\ln(1-X)) + \frac{\Gamma_1}{\beta_0}X\right] \\ &+ \frac{1}{2}\left(\frac{a_s}{1-X}\right)^2\left[2d_2(0) + \frac{\Gamma_2}{2\beta_0}(X(2-X)) + \frac{\beta_1\Gamma_1}{2\beta_0^2}\left(X(X-2) - 2\ln(1-X)\right) + \frac{\beta_2\Gamma_0}{2\beta_0^2}X^2\right. \\ &\left.+ \frac{\beta_1^2\Gamma_0}{2\beta_0^3}(\ln^2(1-X) - X^2)\right],\end{aligned}$$

where we have used the notation

$$a_s = \frac{\alpha_s(\mu)}{4\pi}, \qquad X = a_s\beta_0 L_T, \qquad L_T = \ln\frac{\mu^2 b_T^2}{4e^{-2\gamma_E}} = \ln\frac{\mu^2}{\mu_b^2}.$$

Echevarria, Idilbi, Schäfer, Scimemi, EPJC 73 (2013) 2636

Convergence fails as b approaches b_X which to leading order is $b_X = \frac{C_1}{\mu_i} \exp\left(\frac{2\pi}{\beta_0 \alpha_s(\mu_i)}\right)$

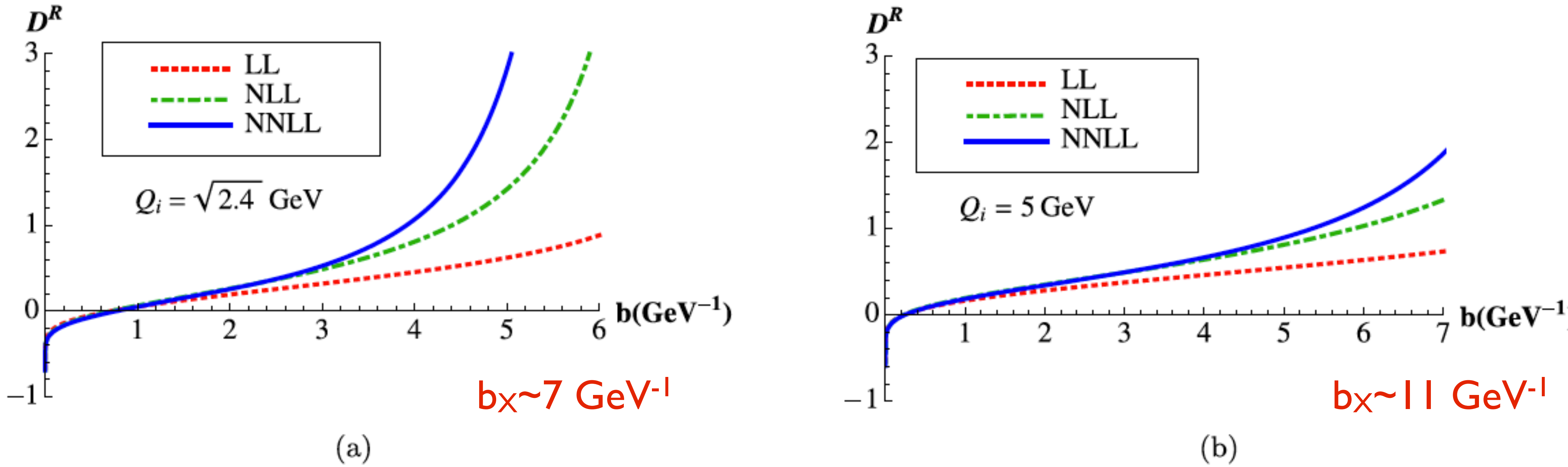


Fig. 1 Resummed D at $Q_i = \sqrt{2.4}$ GeV with $n_f = 4$ (**a**) and $Q_i = 5$ GeV with $n_f = 5$ (**b**)

Echevarria, Idilbi, Schäfer, Scimemi, EPJC 73 (2013) 2636

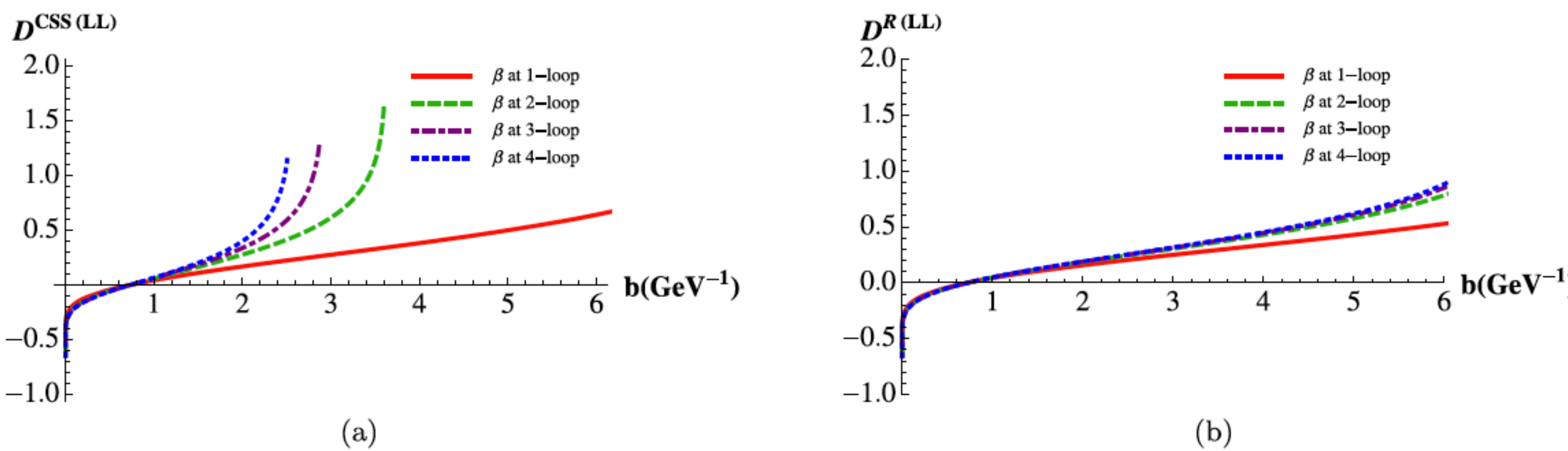


Fig. 3 Resummed $D(b; Q_i = \sqrt{2.4})$ at LL of Eqs. (25), (**a**), and (26), (**b**), with the running of the strong coupling at various orders and decoupling coefficients included

Evolutor R vanishes well before b ~ b_X if Q_f >>Q_i, reduces need for large b regularization

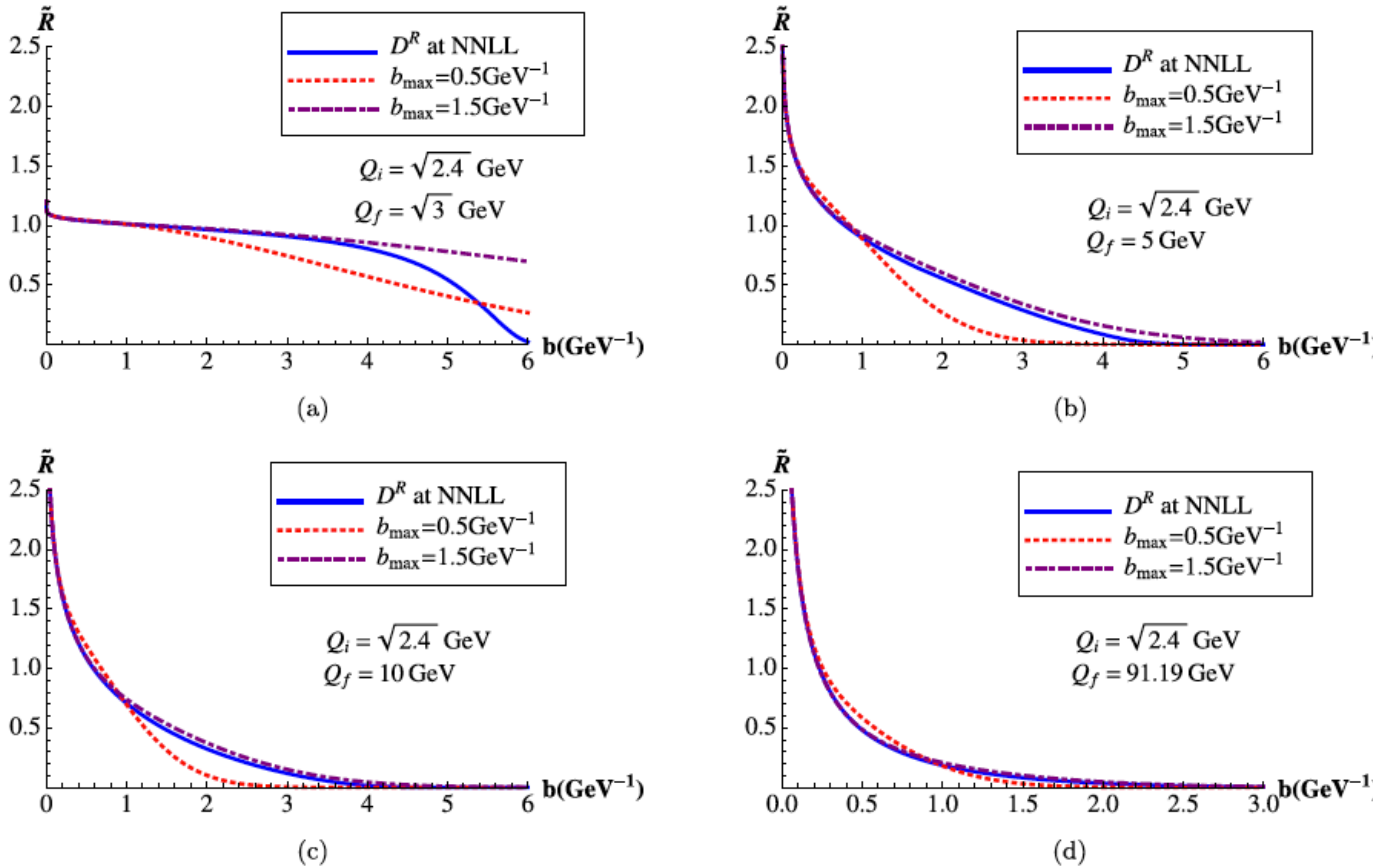
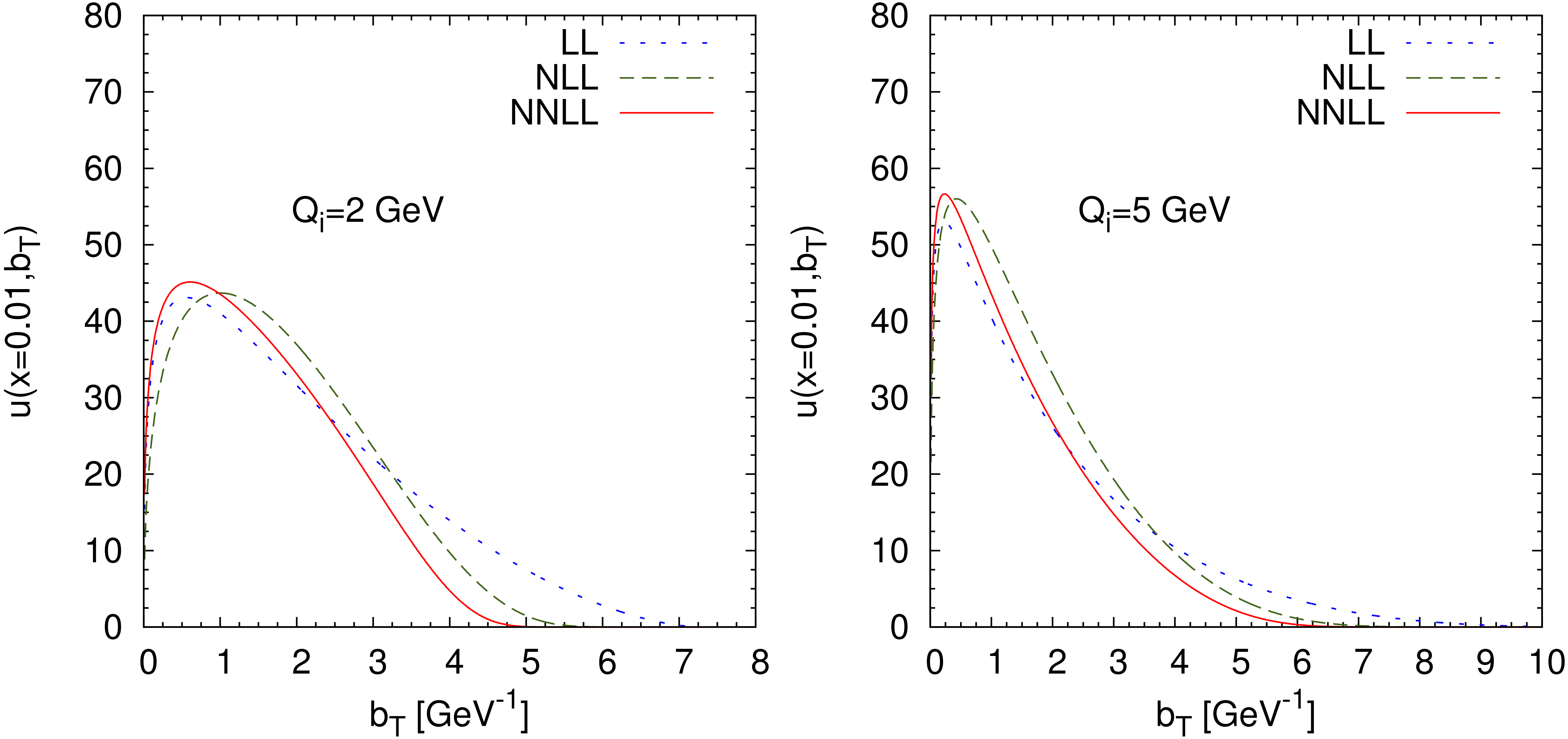


Fig. 4 Evolution kernel from $Q_i = \sqrt{2.4}$ GeV up to $Q_f = \{\sqrt{3}, 5, 10, 91.19\}$ GeV using ours and CSS approaches, both at NNLL

Echevarria, Idilbi, Schäfer, Scimemi, EPJC 73 (2013) 2636

This approach favors b_{max} = 1.5 GeV^{-1}

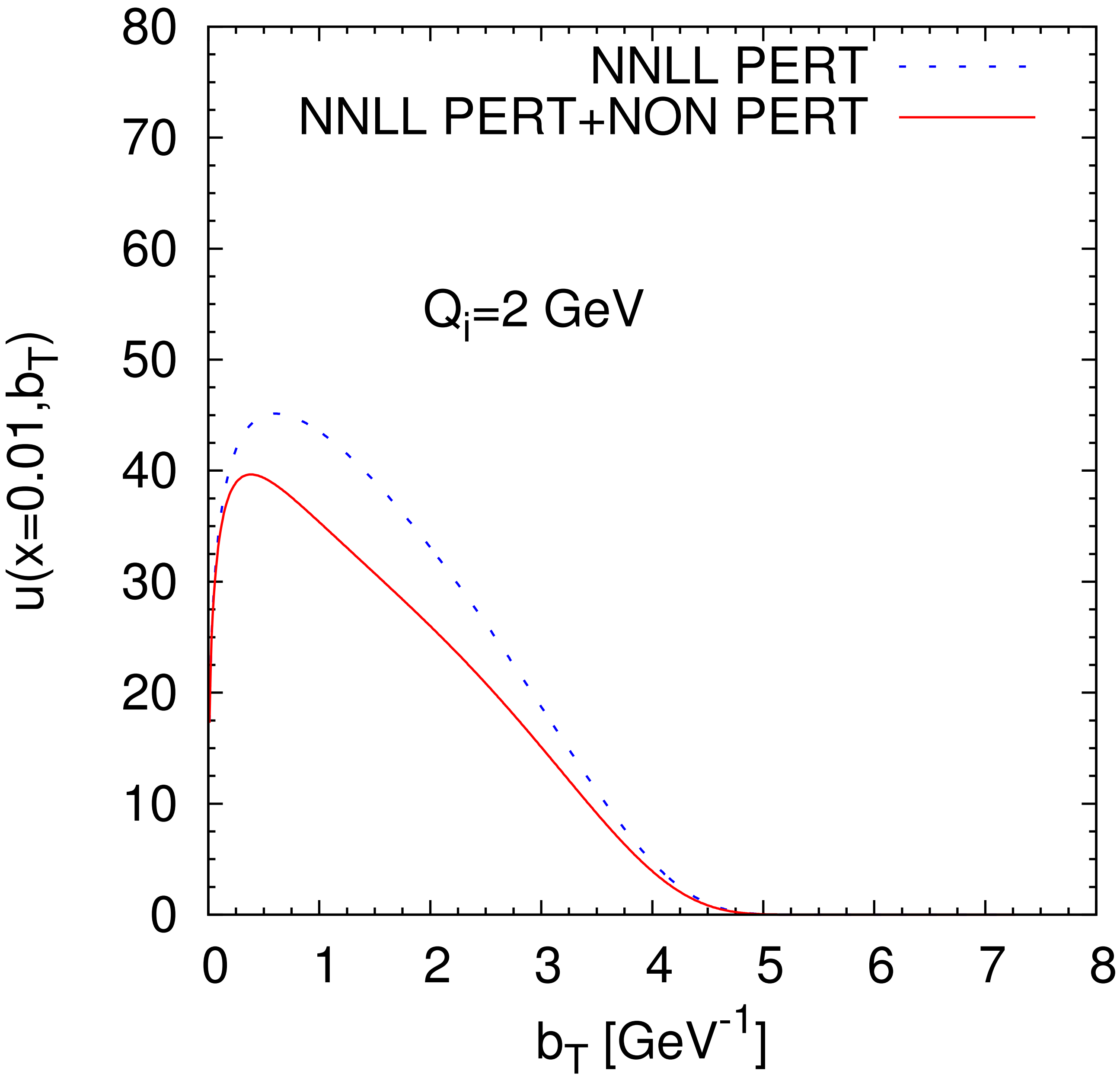
Further resummations



D'Alesio, Echevarria, Melis, Scimemi, arXiv:1407.3311

Resummed TMD at low scales very small at large b_T where $\alpha_s(\mu_b)$ is very large

New approach to Landau pole problem



Sensitivity to Landau pole minimized by using $Q_i = Q_0 + q_T$ rather than μ_b

Correspondingly a new F^{NP} form is considered

High Q data (DY/Z) need only λ_1 & λ_2
Low Q (SIDIS) needs modification (λ_3)

$$\tilde{F}^{\mathrm{NP}}_{q/N}(x, b_T; Q) = e^{-\lambda_1 b_T} \left(1 + \lambda_2 b_T^2\right) \left(\frac{Q^2}{Q_0^2}\right)^{-\frac{\lambda_3}{2} b_T^2}$$

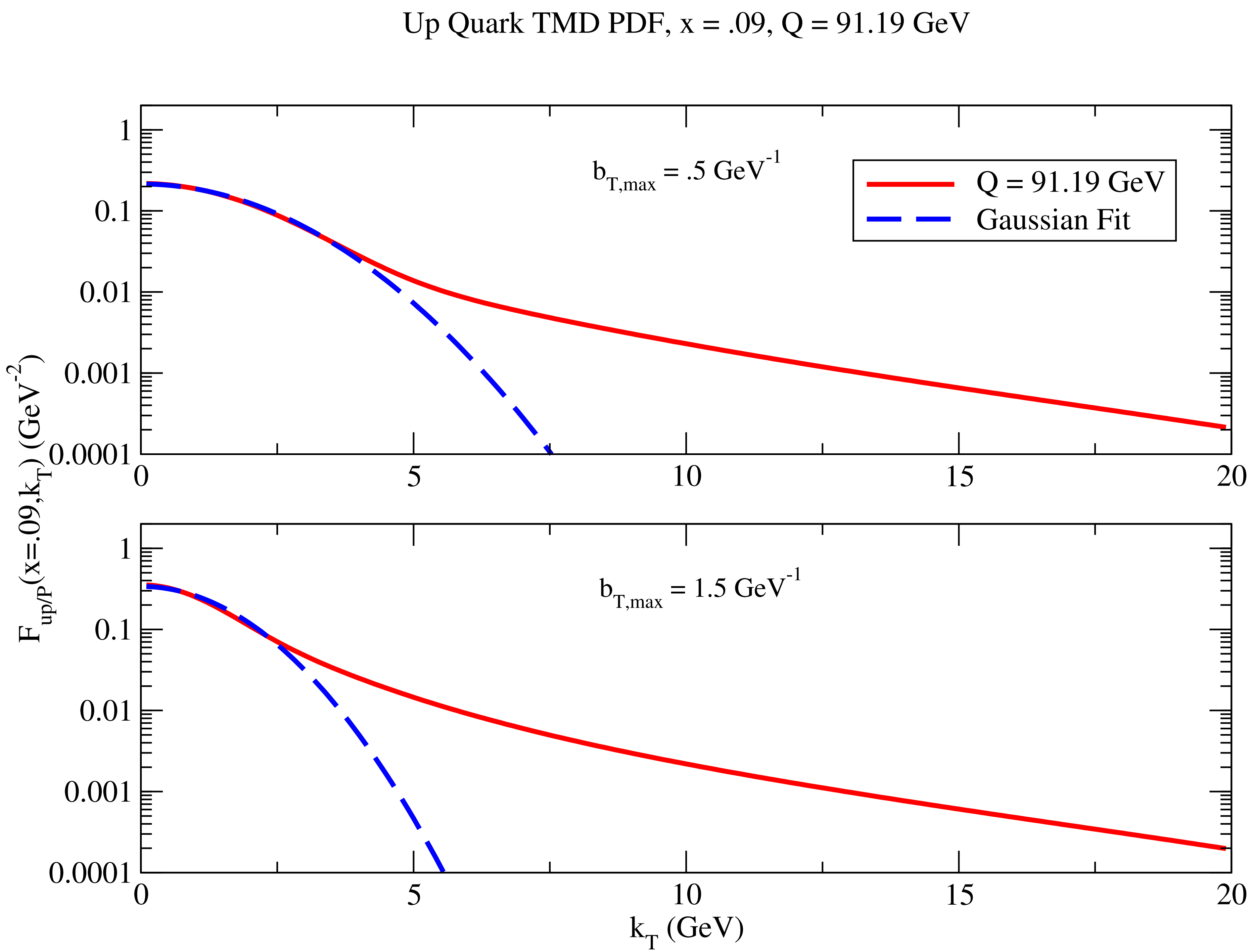
D'Alesio, Echevarria, Melis, Scimemi, arXiv:1407.3311

TMD evolution

Large p_T tail

Factorization dictates the evolution:

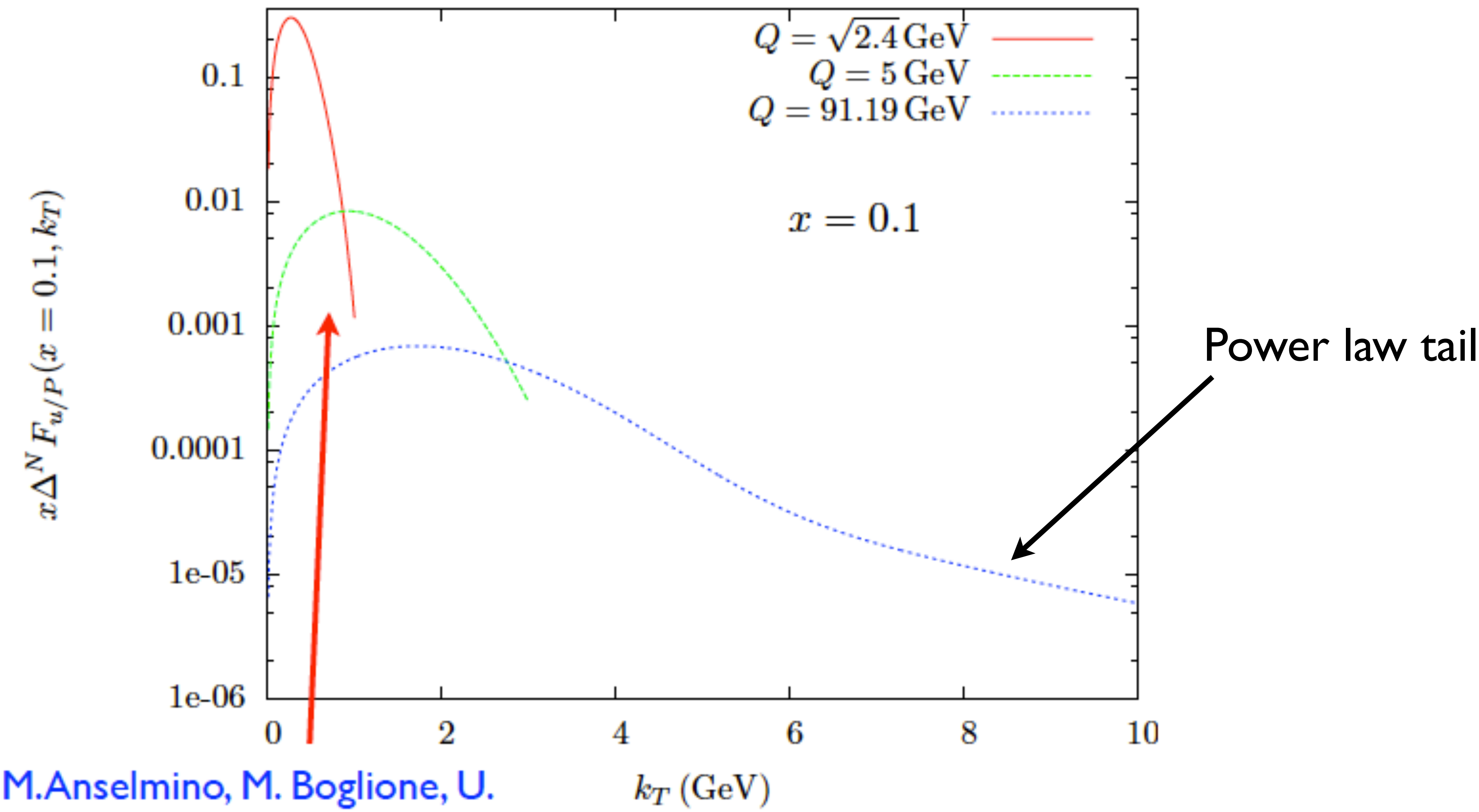
Under evolution TMDs develop a power law tail



Aybat & Rogers, PRD 83 (2011) 114042

Evolution of Sivers function

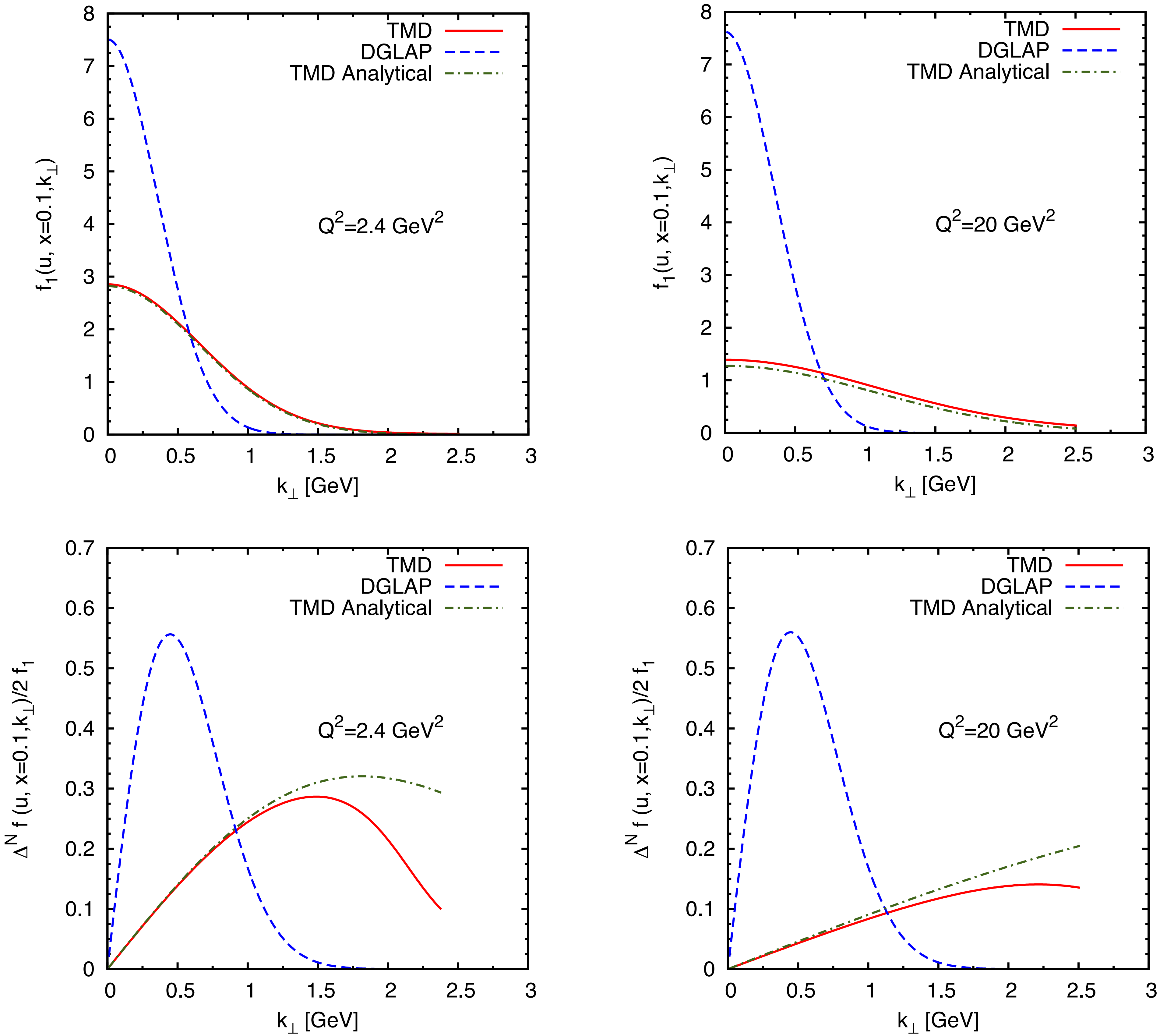
TMDs and their asymmetries become broader and smaller with increasing energy



M.Anselmino, M. Boglione, U. D'Alesio, A.Kotzinian, S.Melis, F. Murgia, A. Prokudin, C.Turk; 2009

Aybat & Rogers, PRD 83 (2011) 114042
Aybat, Collins, Qiu, Rogers, PRD 85 (2012) 034043

Comparing TMD and DGLAP evolution



Anselmino, Boglione, Melis
PRD 86 (2012) 014028

All curves evolved from
$Q^2 = 1\ GeV^2$

Makes quite a difference
in this limited range of Q:
from 1.5 to 4.5 GeV

S_{NP} dominates evolution

TMD evolution of azimuthal asymmetries

- Sivers effect in SIDIS and DY
  [Idilbi, Ji, Ma & Yuan, 2004; Aybat, Prokudin & Rogers, 2012; Anselmino, Boglione, Melis, 2012; Sun & Yuan, 2013; D.B., 2013; Echevarria, Idilbi, Kang & Vitev, 2014]

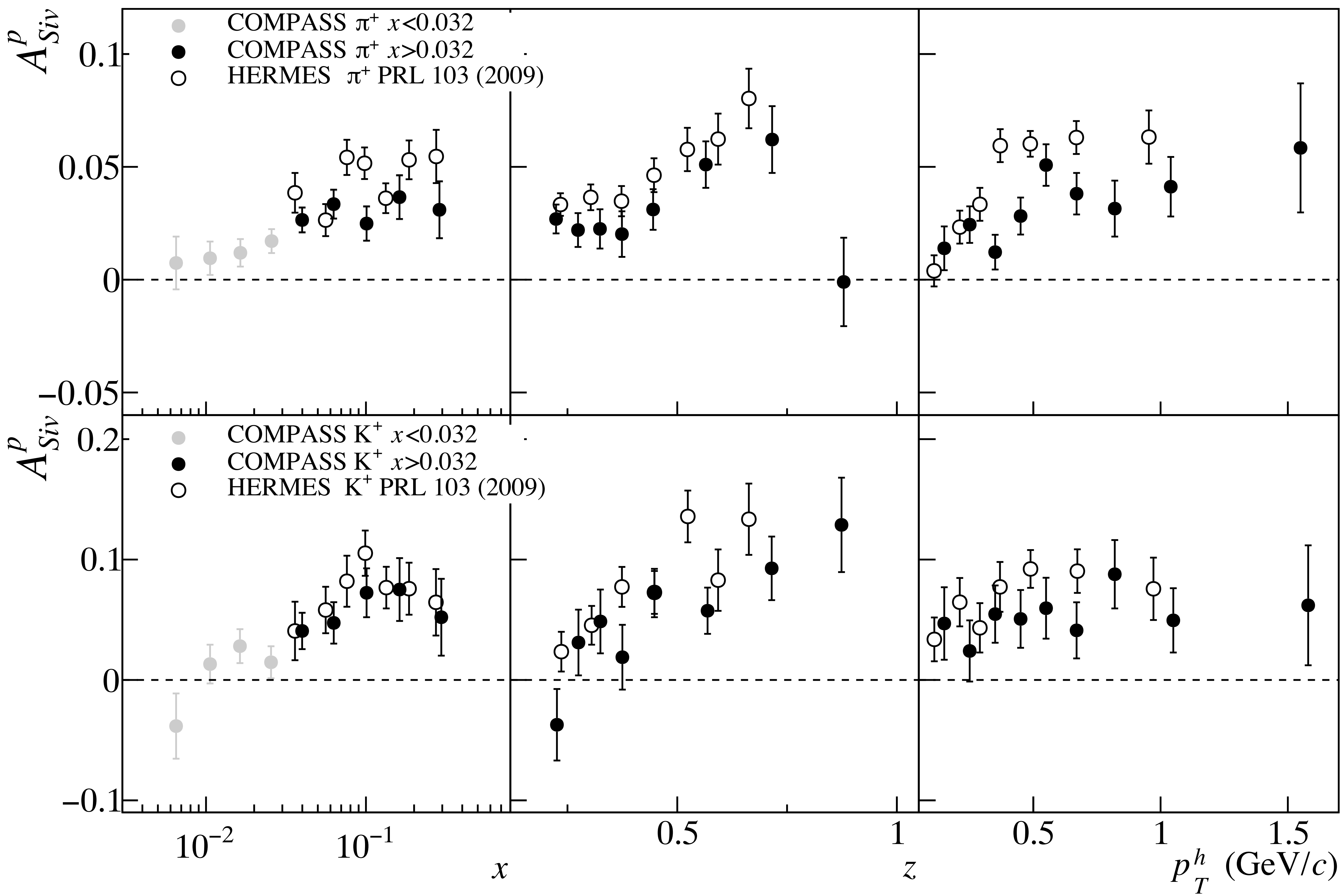
- Collins effect in e^+e^- and SIDIS
  [D.B., 2001 & 2009; Echevarria, Idilbi, Scimemi, 2014]

- Sivers effect in J/ψ production
  [Godbole, Misra, Mukherjee, Rawoot, 2013; Godbole, Kaushik, Misra, Rawoot, 2014]

Main differences among the various approaches:
- treatment of nonperturbative Sudakov factor
- treatment of leading logarithms, i.e. the level of perturbative accuracy

TMD evolution
of the Sivers asymmetry

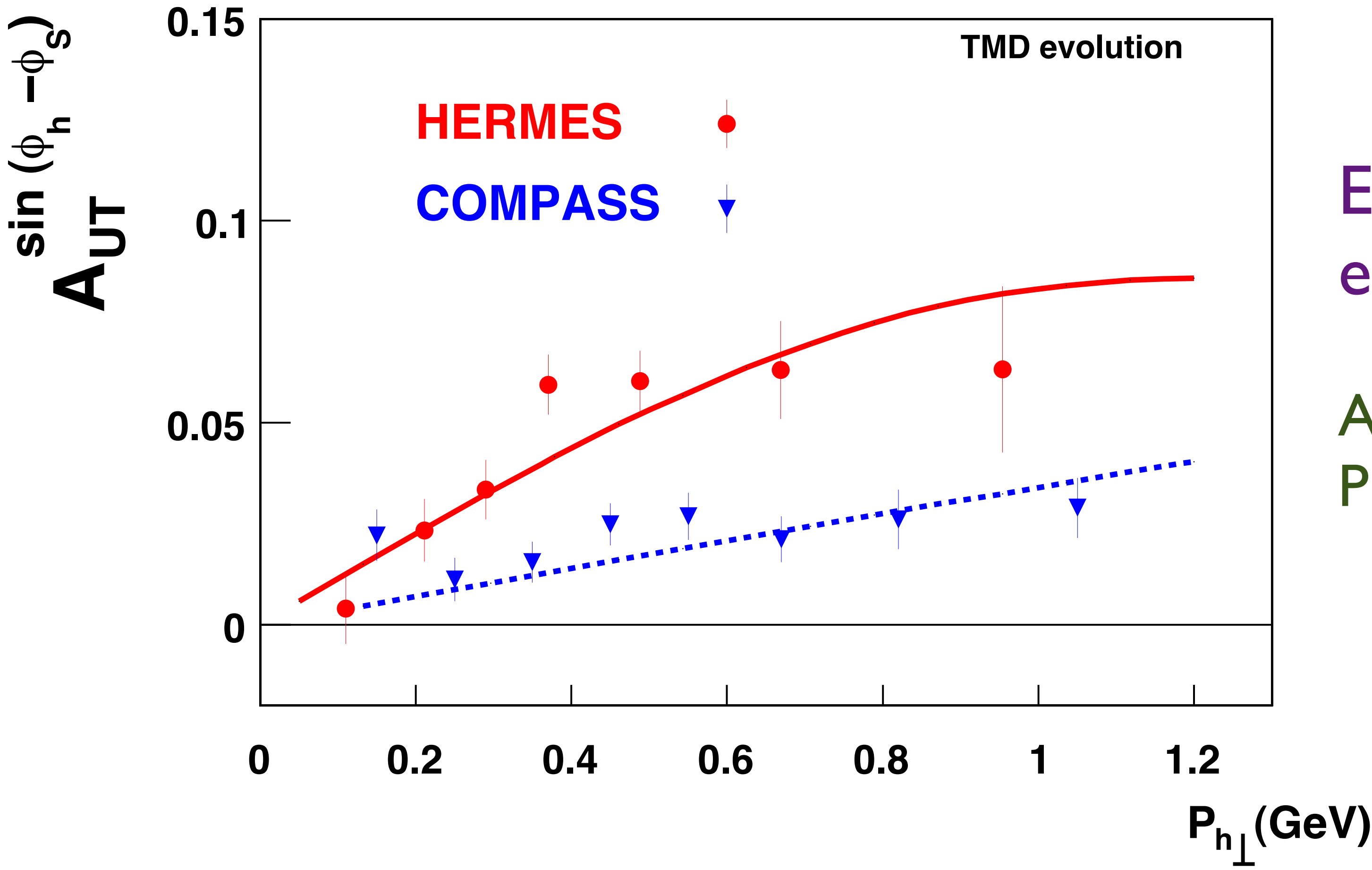
Sivers Asymmetry



[COMPASS, arXiv:1408.4405]

HERMES data (<Q^2> ~ 2.4 GeV^2) mostly above COMPASS data (<Q^2> ~ 3.8 GeV^2)

Evolution of the Sivers Asymmetry



Evolution from HERMES to COMPASS energy scale seems to work well

Aybat, Prokudin & Rogers, PRL 108 (2012) 242003

This is obtained using the 2011 TMD factorization, including some approximations that should be applicable at small Q:

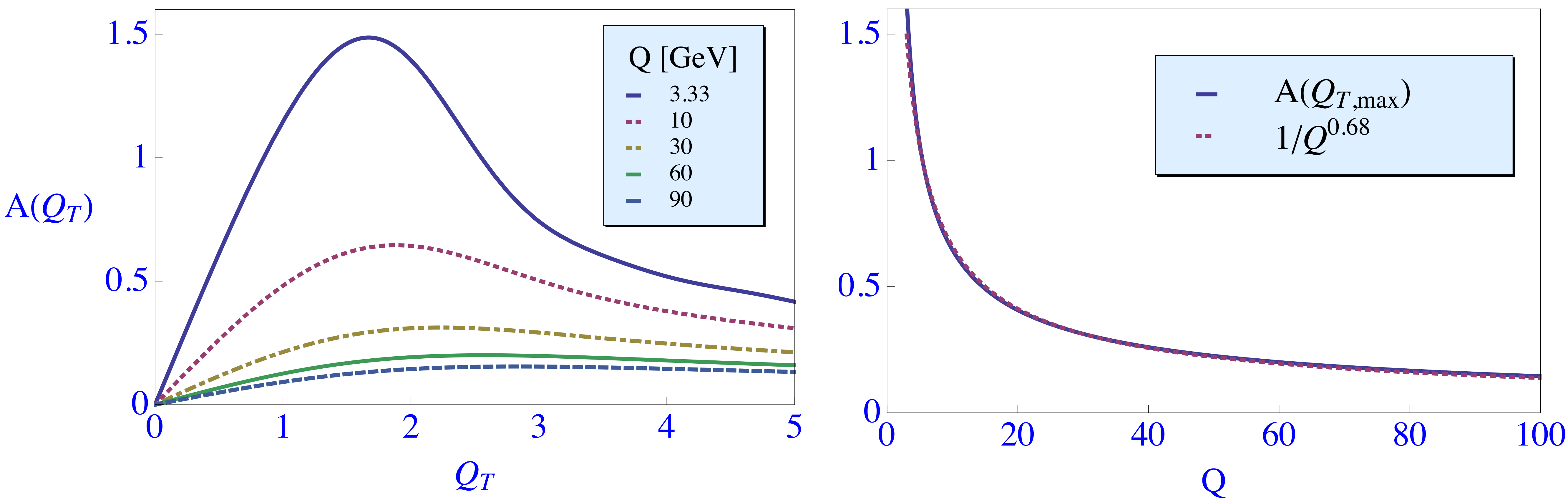
- Y term is dropped (or equivalently the perturbative tail)
- evolve from a fixed starting Q_0 rather than μ_b
- Gaussian TMDs at starting scale Q_0

TMD evolution of the Sivers asymmetry

Under very similar assumptions, the Q dependence of the Sivers asymmetry resides in an overall factor:

$$A_{UT}^{\sin(\phi_h - \phi_S)} \propto \mathcal{A}(Q_T, Q)$$

[D.B., NPB 874 (2013) 217]

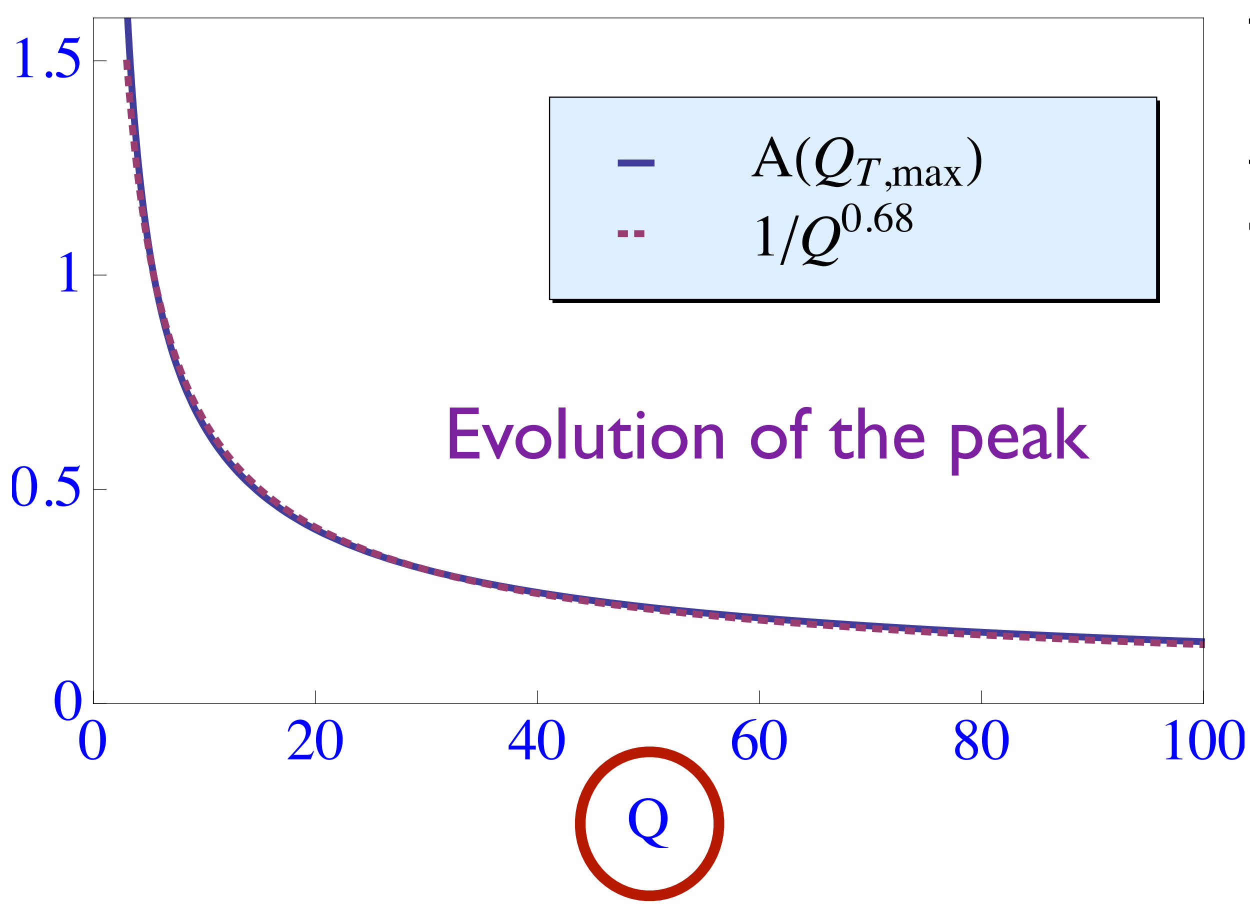


Observations:

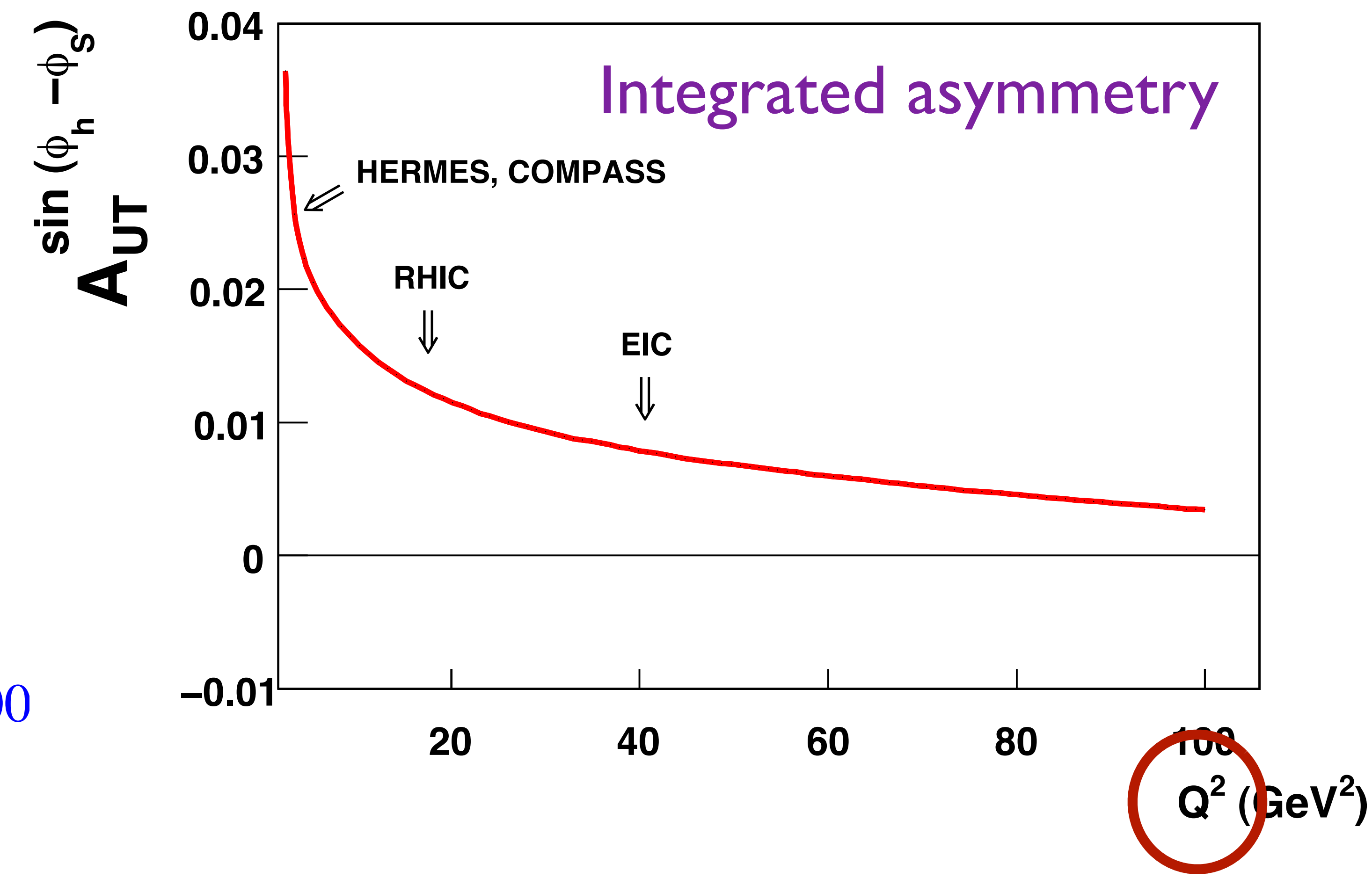
- the peak of the Sivers asymmetry decreases as $1/Q^{0.7\pm0.1}$ ("Sudakov suppression")
- the peak of the asymmetry shifts slowly towards higher Q_T, also offers a test

Testing these features needs a larger Q range, requiring a high-energy EIC

TMD evolution of the Sivers asymmetry



D.B., NPB 874 (2013) 217



Aybat, Prokudin & Rogers,
PRL 108 (2012) 242003

Both approaches use the same formalism (2011 TMD factorization), very similar approximations and ingredients, the key difference is in the integration over x, z, $P_{h\perp}$

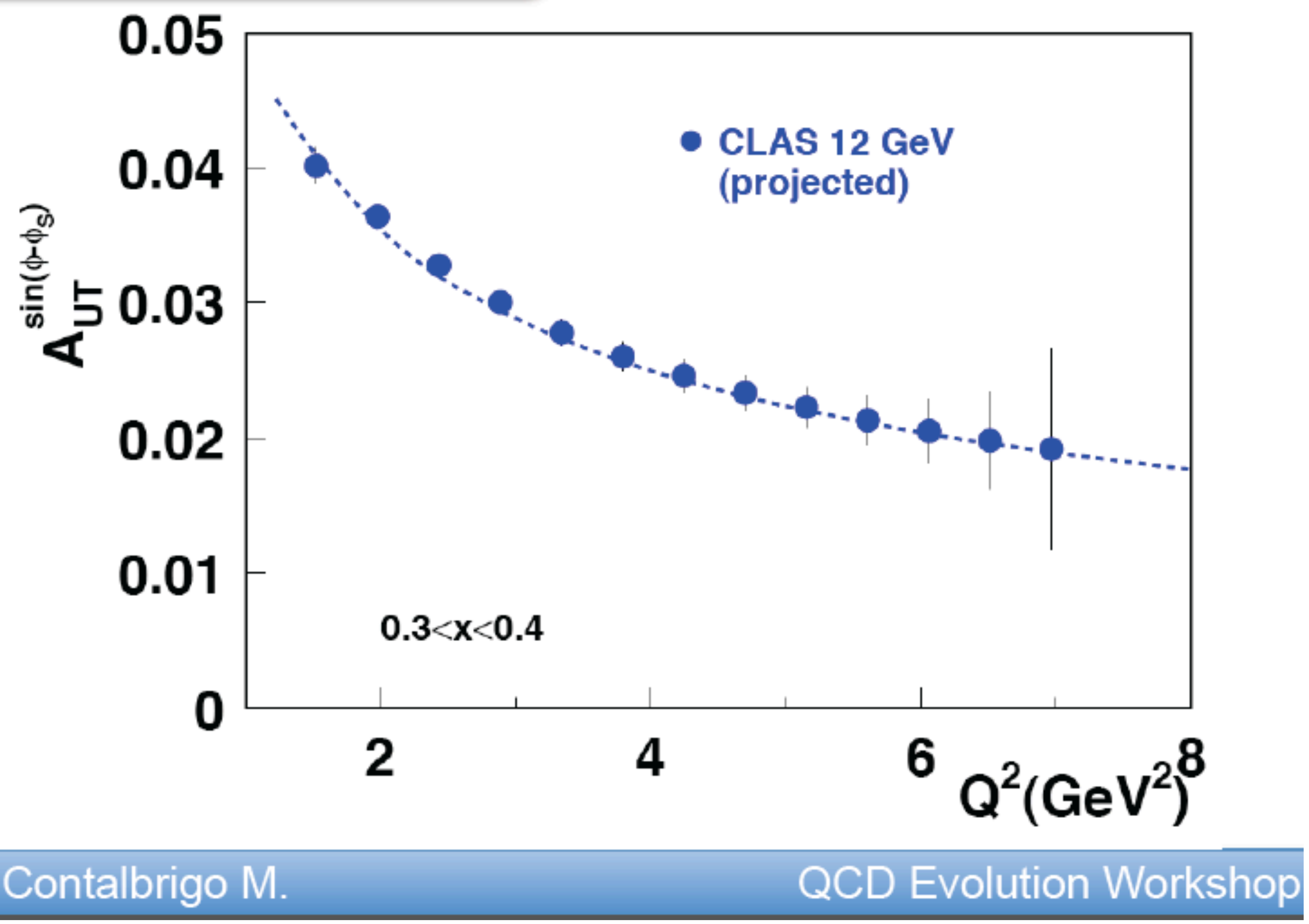
The *integrated* asymmetry falls off fast, not of form $1/Q^{\alpha}$, but in the considered range it falls off faster than 1/Q but slower than $1/Q^2$

TMD evolution of the Sivers asymmetry

At low Q^2 (up to ~20 GeV^2), the Q^2 evolution is dominated by S_{NP}

[Anselmino, Boglione, Melis,PRD 86 (2012) 014028]

Q^2 dependence of
Sivers asymmetry
Test of TMDs evolution



Precise low Q^2 data can help to determine the form and size of S_{NP}

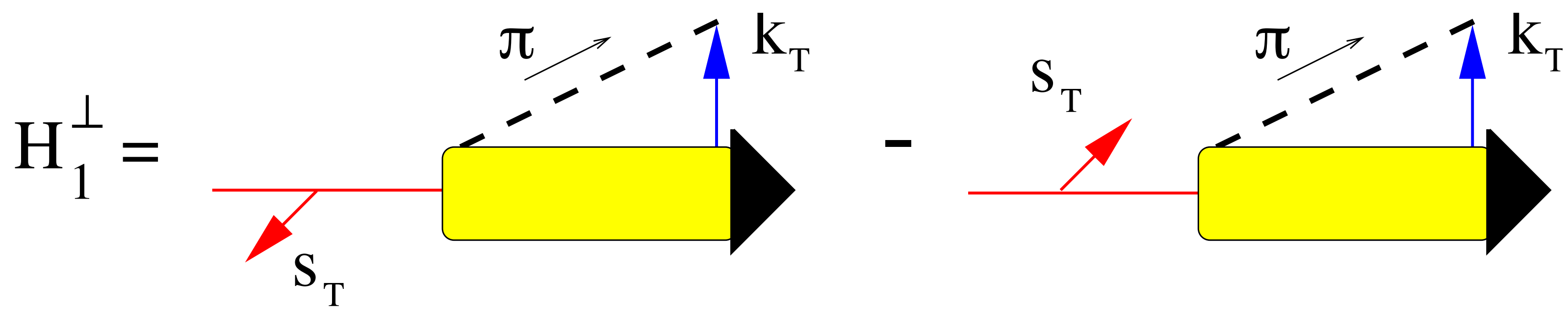
Uncertainty in S_{NP} determines the ±0.1 in $1/Q^{0.7\pm0.1}$

TMD evolution of Collins asymmetries

Collins Effect

Collins effect is described by a TMD fragmentation function:

[NPB 396 (1993) 161]



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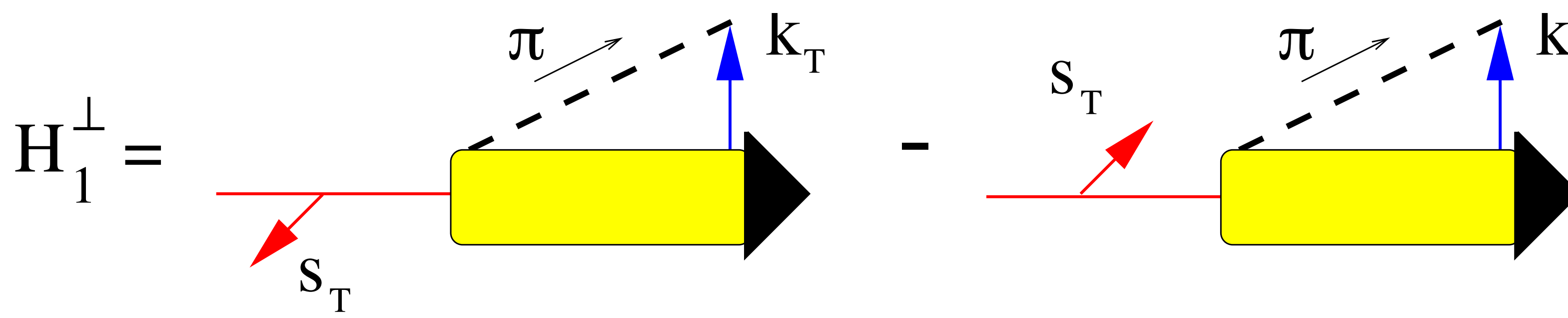

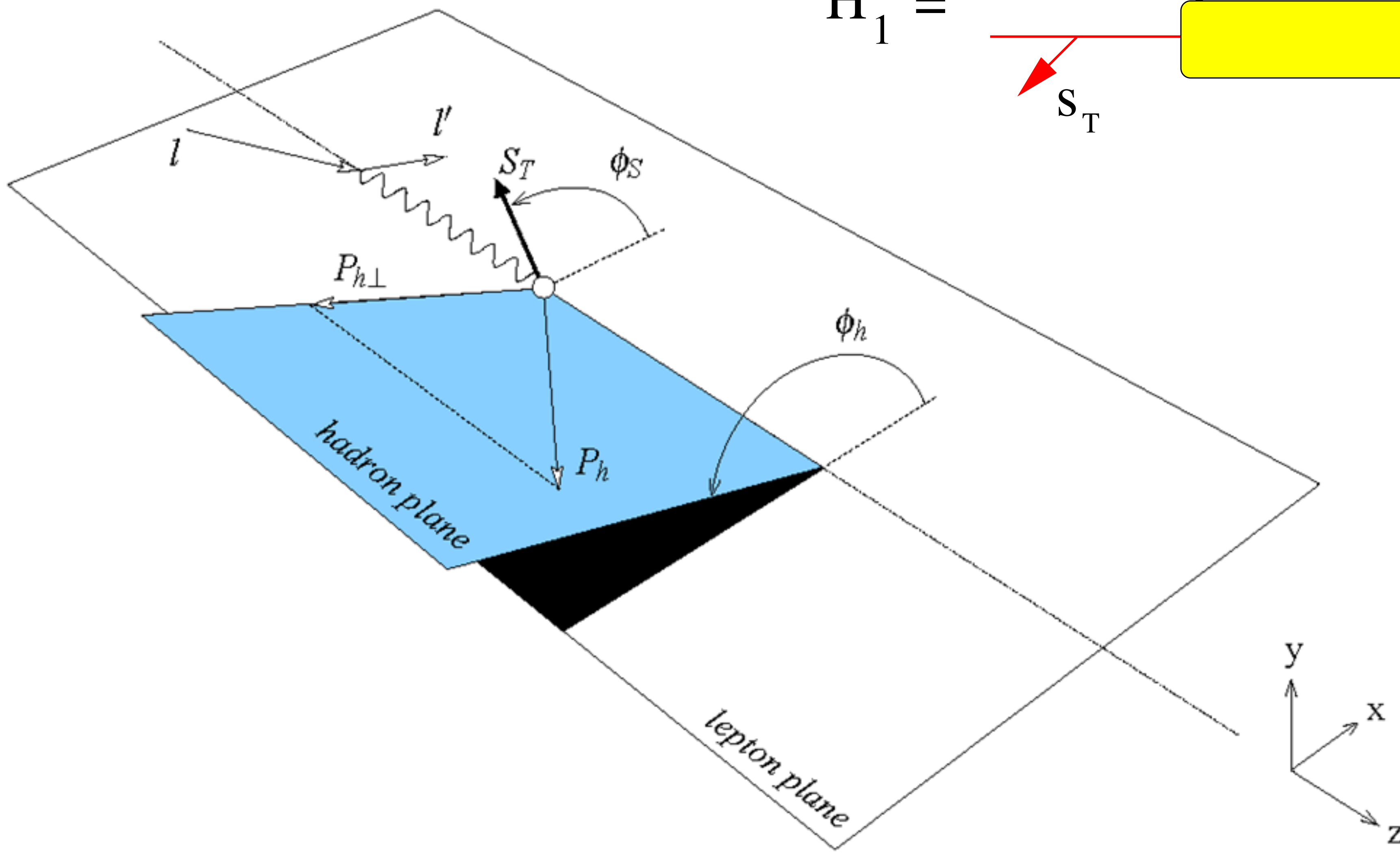

It gives rise to a $\sin(\varphi_h+\varphi_S)$ asymmetry in SIDIS:

$$\frac{d\sigma(e\,p^{\uparrow} \to e'\pi X)}{d\phi_\pi^e d|\boldsymbol{P}_\perp^\pi|^2} \propto \{1 + |\boldsymbol{S}_T|\ \sin(\phi_\pi^e - \phi_S^e)\ f_{1T}^\perp D_1 + |\boldsymbol{S}_T|\ \sin(\phi_\pi^e + \phi_S^e)\ h_1 H_1^\perp\}$$

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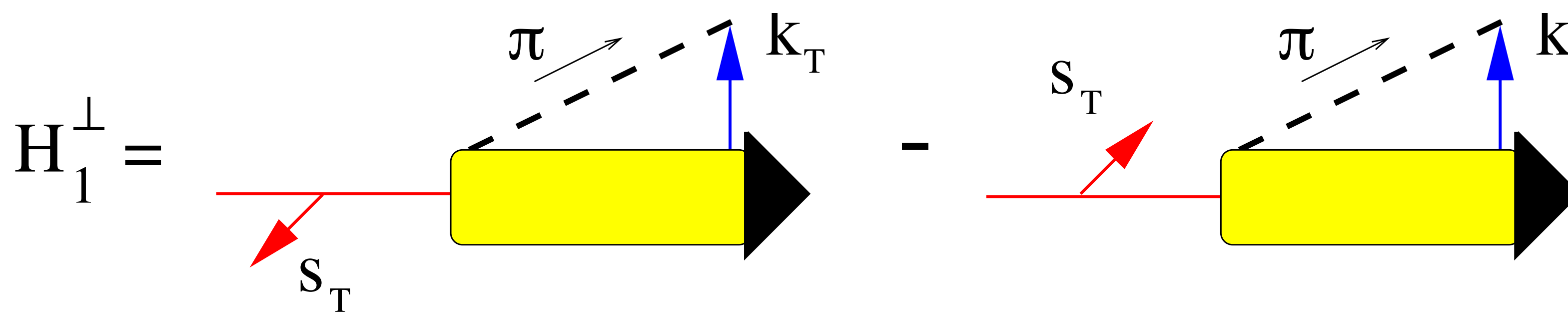
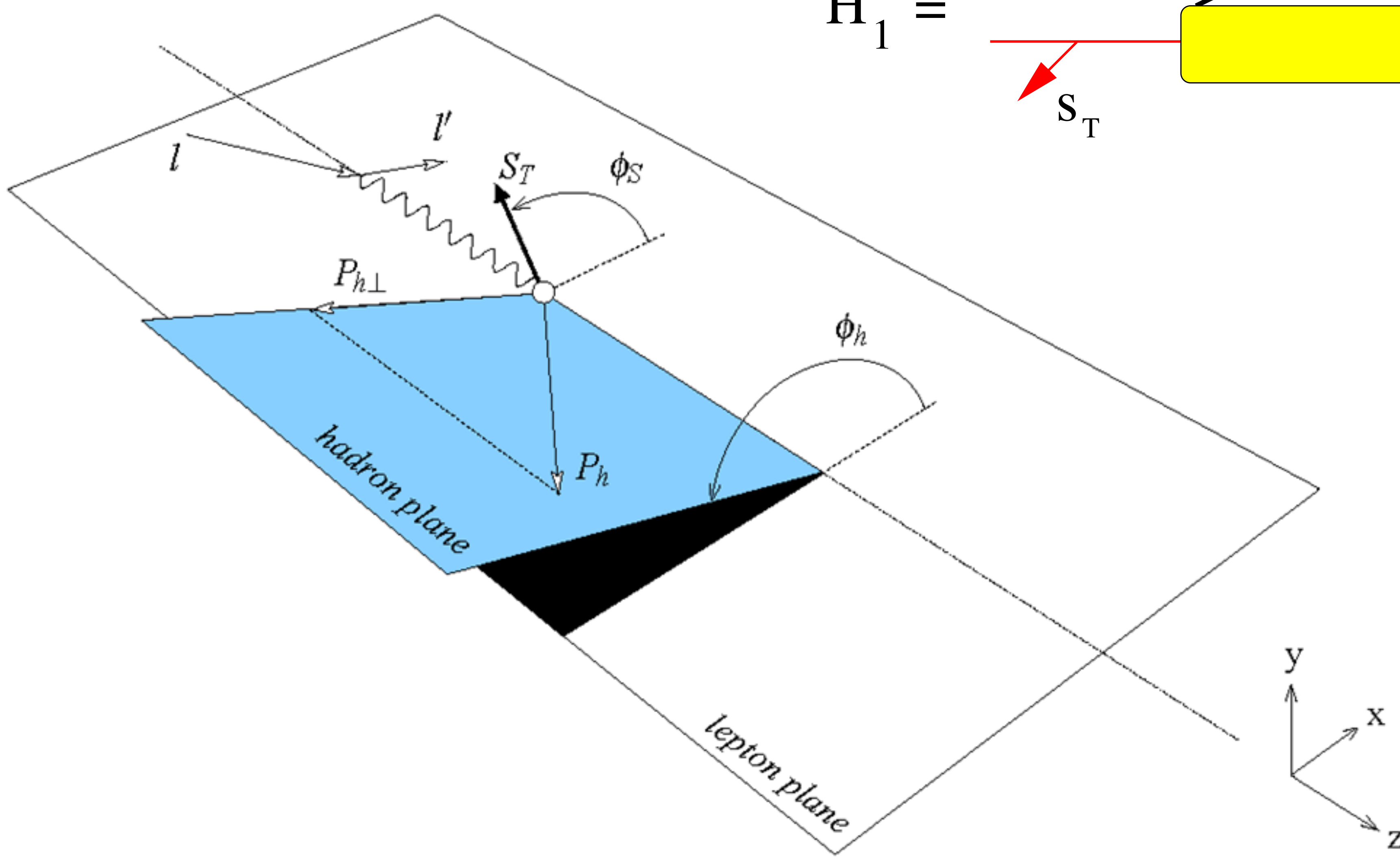


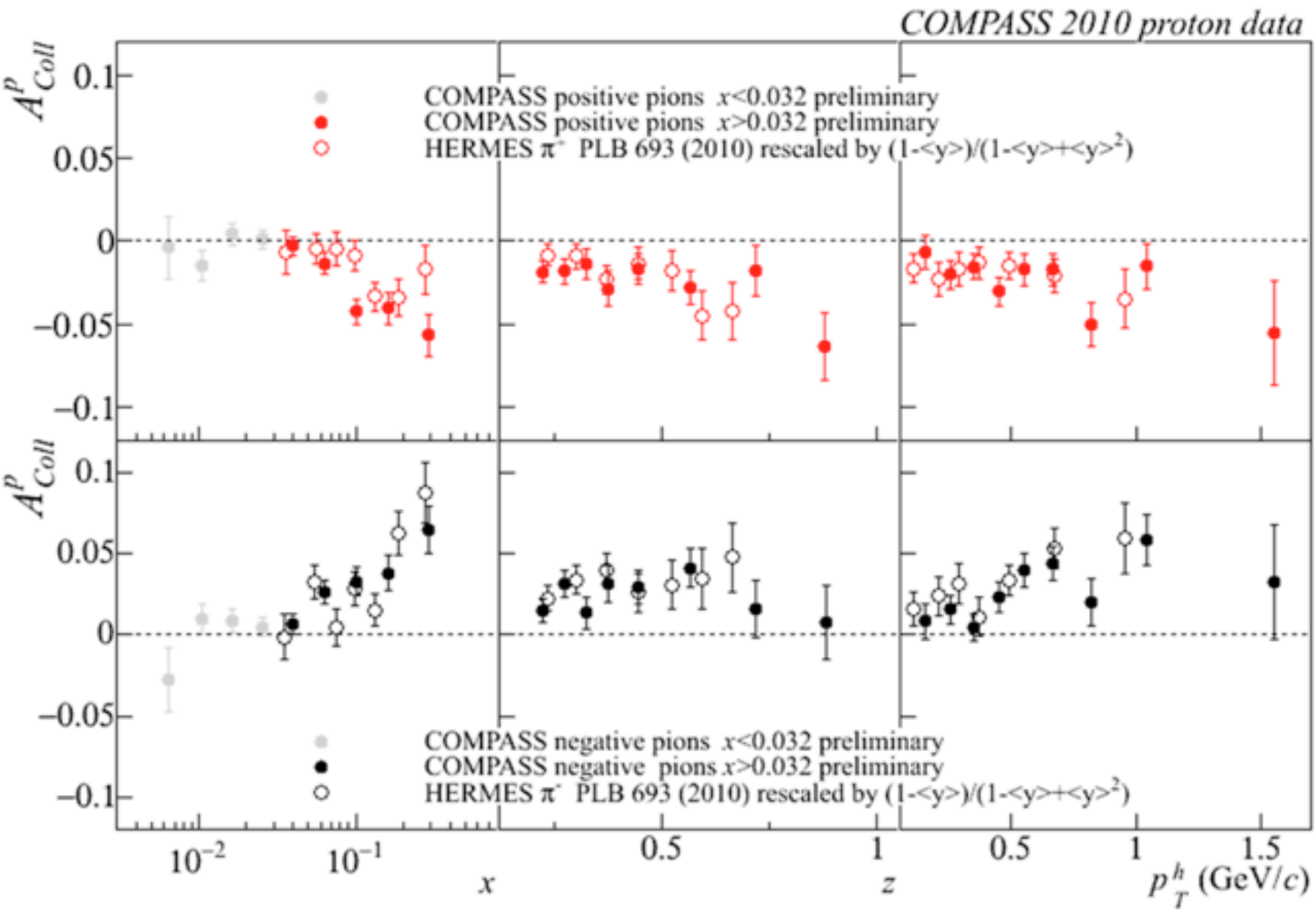


It gives rise to a sin(φh+φS) asymmetry in SIDIS:

transversity ⊗
Collins function

$$\frac{d\sigma(e\,p^{\uparrow} \to e'\pi X)}{d\phi^e_\pi d|\boldsymbol{P}^\pi_\perp|^2} \propto \{1 + |\boldsymbol{S}_T|\ \sin(\phi^e_\pi - \phi^e_S)\ f^\perp_{1T} D_1 + |\boldsymbol{S}_T|\ \sin(\phi^e_\pi + \phi^e_S)\ h_1 H^\perp_1\}$$

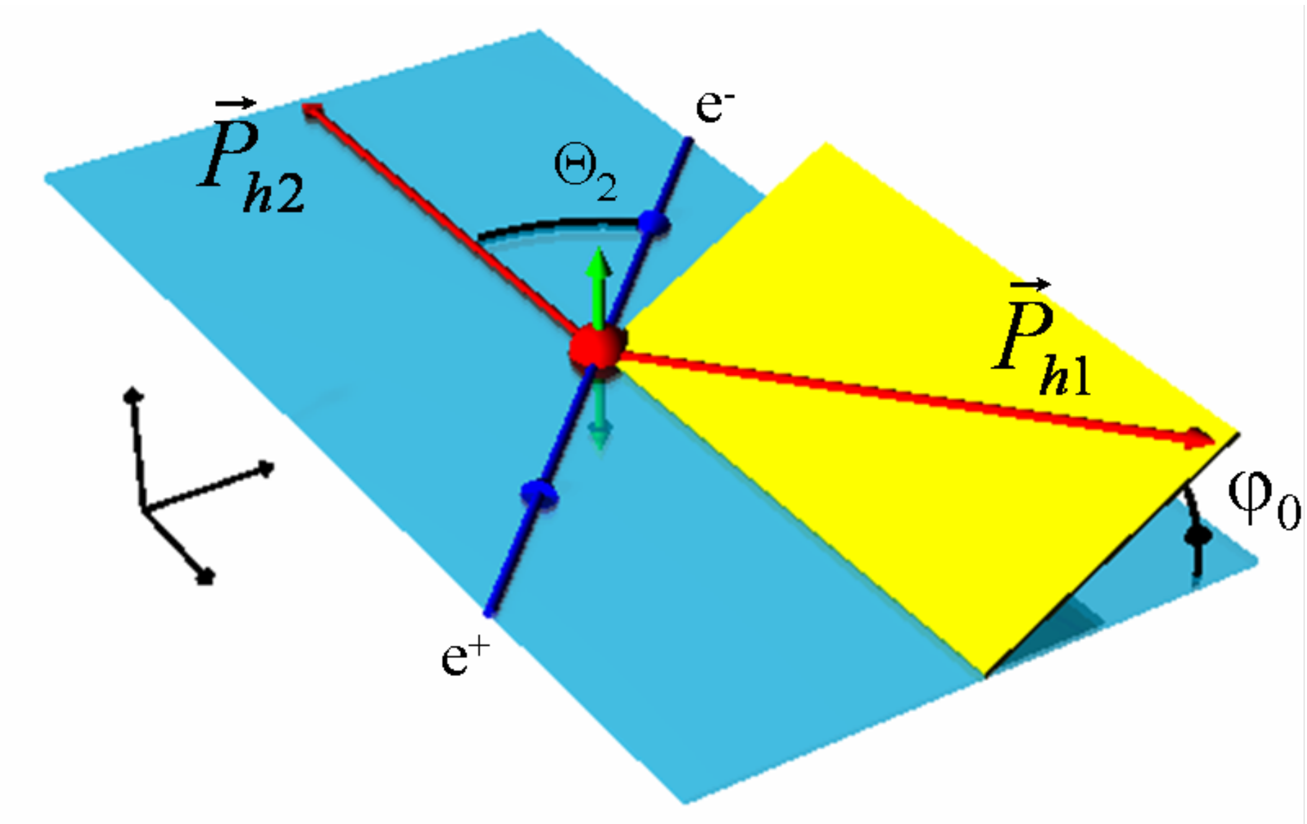
Collins Asymmetry in SIDIS



No clear need for TMD evolution from HERMES to COMPASS

Double Collins Effect

The Collins fragmentation function provides a way to probe transversity (h_1), if measured independently in another process



Double Collins effect gives rise to a cos 2φ asymmetry in $e^+e^- \rightarrow h_1\ h_2\ X$
[D.B., Jakob, Mulders, NPB 504 (1997) 345]

Clearly observed in experiment by BELLE (R. Seidl *et al.*, PRL '06; PRD '08) and BaBar (I. Garzia at Transversity 2011 & J.P. Lees et al., arXiv:1309.527)

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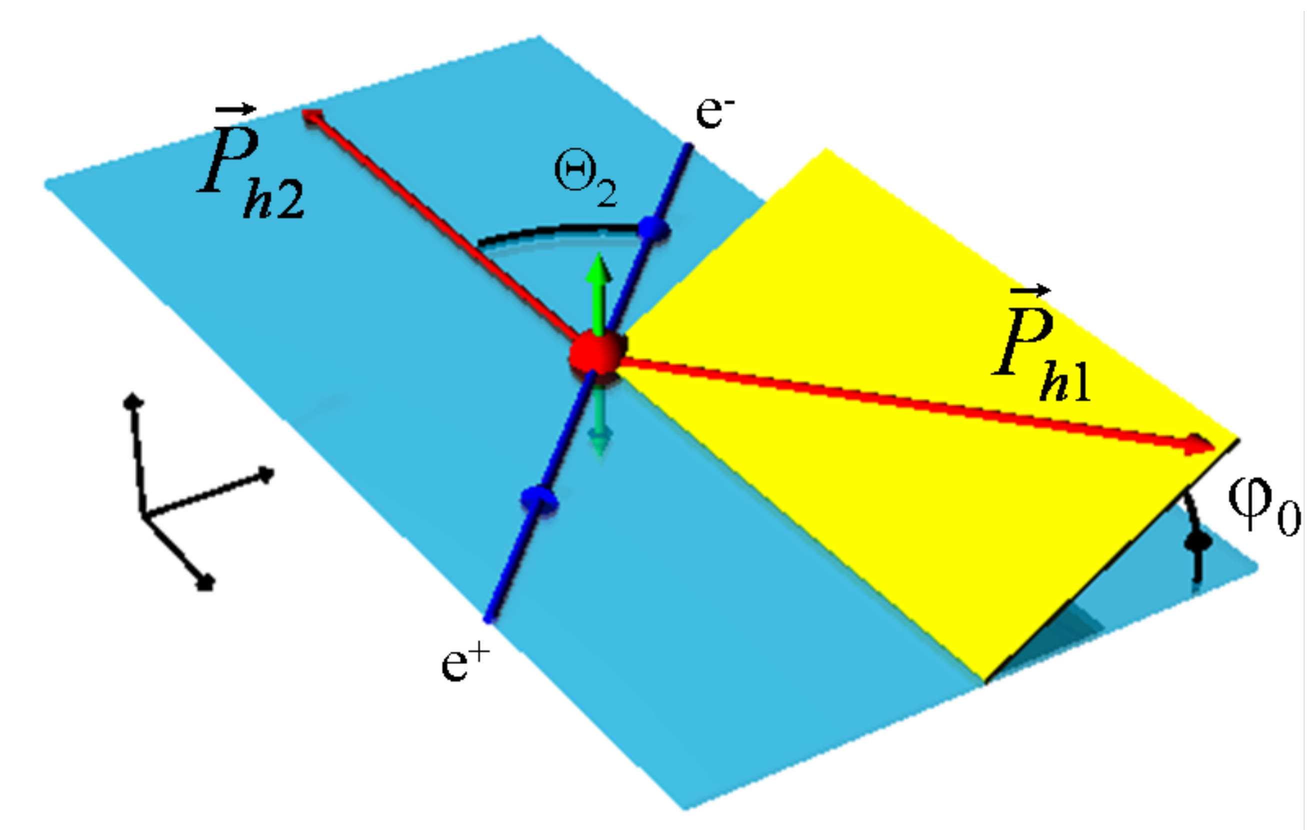
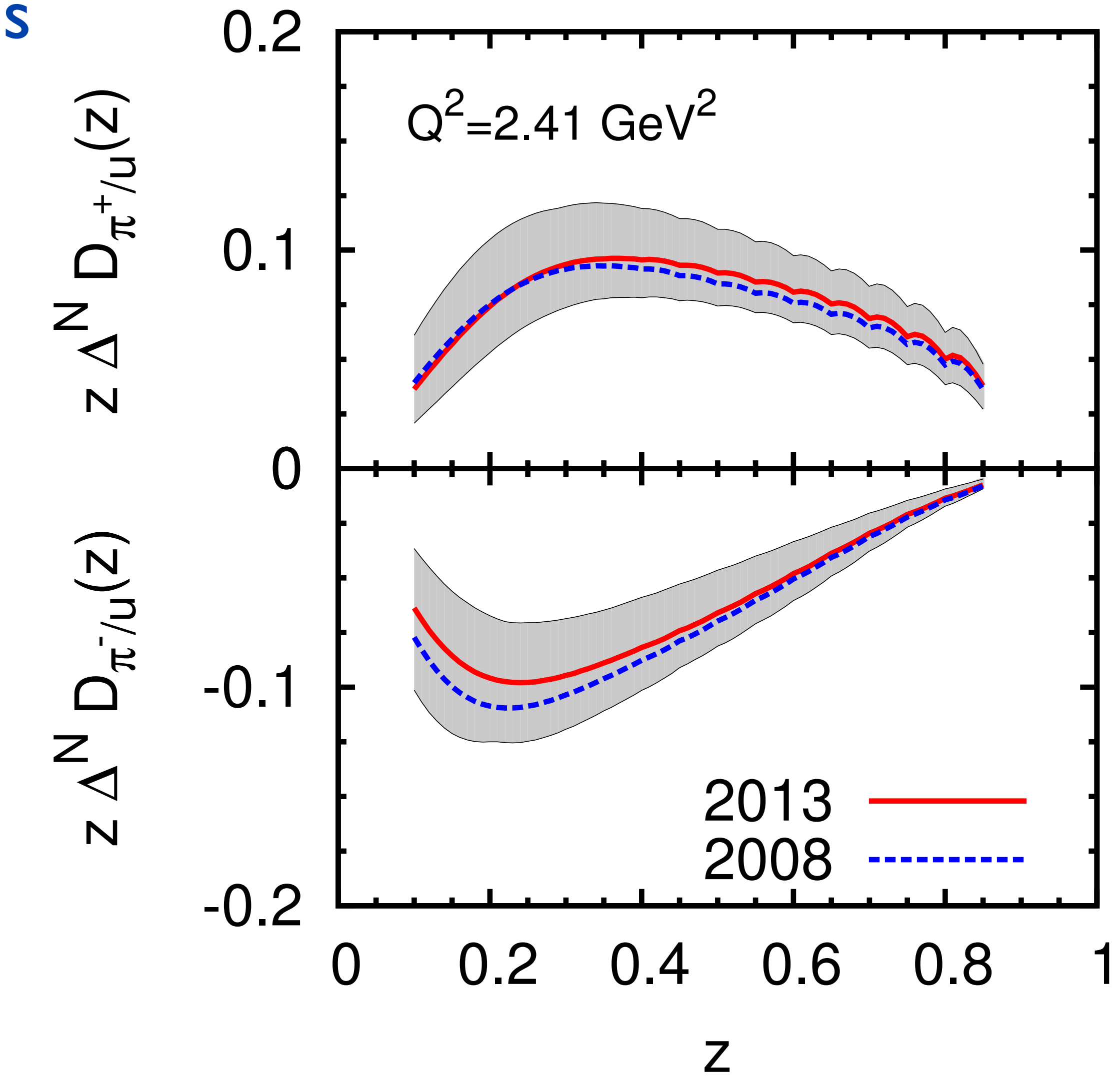




Anselmino *et al.*, PRD 87 (2013) 094019

Double Collins effect gives rise to a cos 2φ asymmetry in $e^+e^- \rightarrow h_1\ h_2\ X$
[D.B., Jakob, Mulders, NPB 504 (1997) 345]

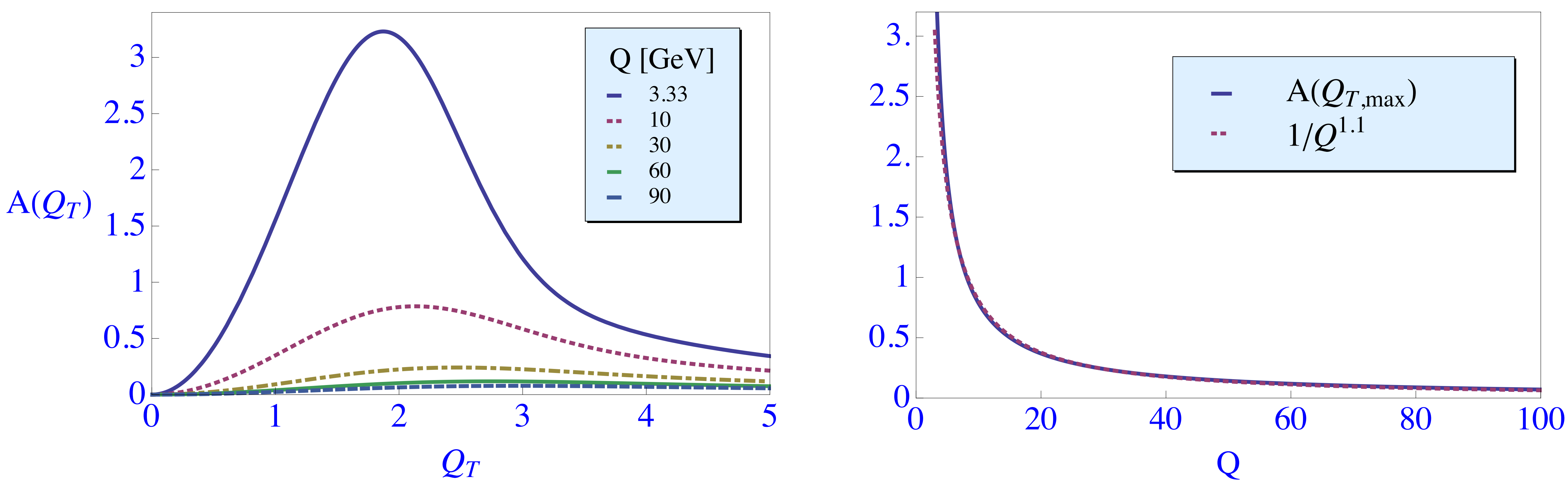
Clearly observed in experiment by BELLE (R. Seidl *et al.*, PRL '06; PRD '08) and BaBar (I. Garzia at Transversity 2011 & J.P. Lees et al., arXiv:1309.527)

Double Collins Asymmetry

$$\frac{d\sigma(e^+e^- \to h_1 h_2 X)}{dz_1 dz_2 d\Omega d^2\boldsymbol{q}_T} \propto \{1 + \cos 2\phi_1 A(\boldsymbol{q}_T)\}$$

Under similar assumptions as for the Sivers asymmetry:

$$A(Q_T) = \frac{\sum_a e_a^2 \sin^2\theta \; H_1^{\perp(1)a}(z_1;Q_0)\; \overline{H}_1^{\perp(1)a}(z_2;Q_0)}{\sum_b e_b^2 (1+\cos^2\theta)\; D_1^b(z_1;Q_0)\; \overline{D}_1^b(z_2;Q_0)}\; \mathcal{A}(Q_T)$$

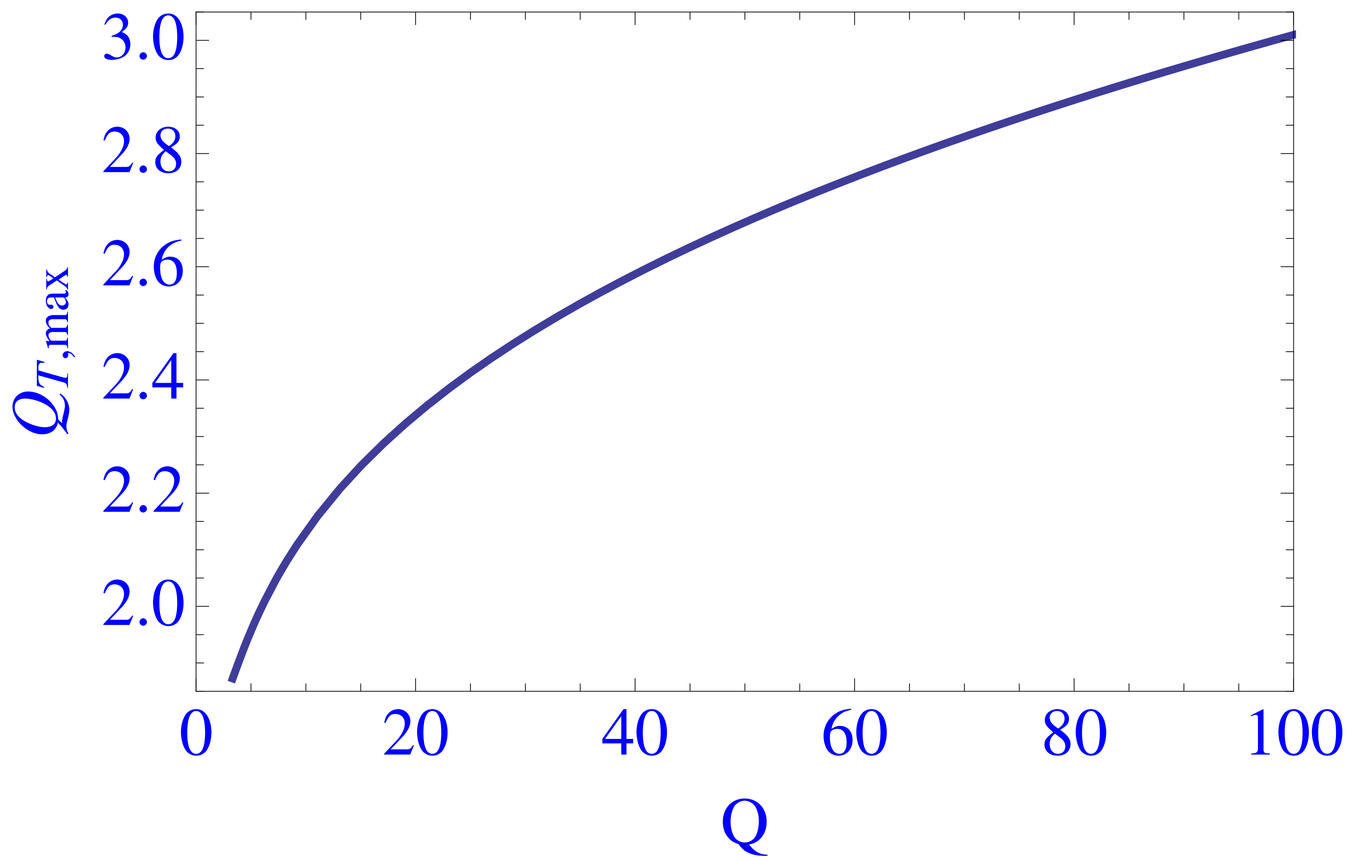


Considerable Sudakov suppression ~1/Q (effectively twist-3)

D.B., NPB 603 (2001) 195 & NPB 806 (2009) 23 & NPB 874 (2013) 217 & arXiv:1308.4262

Next steps

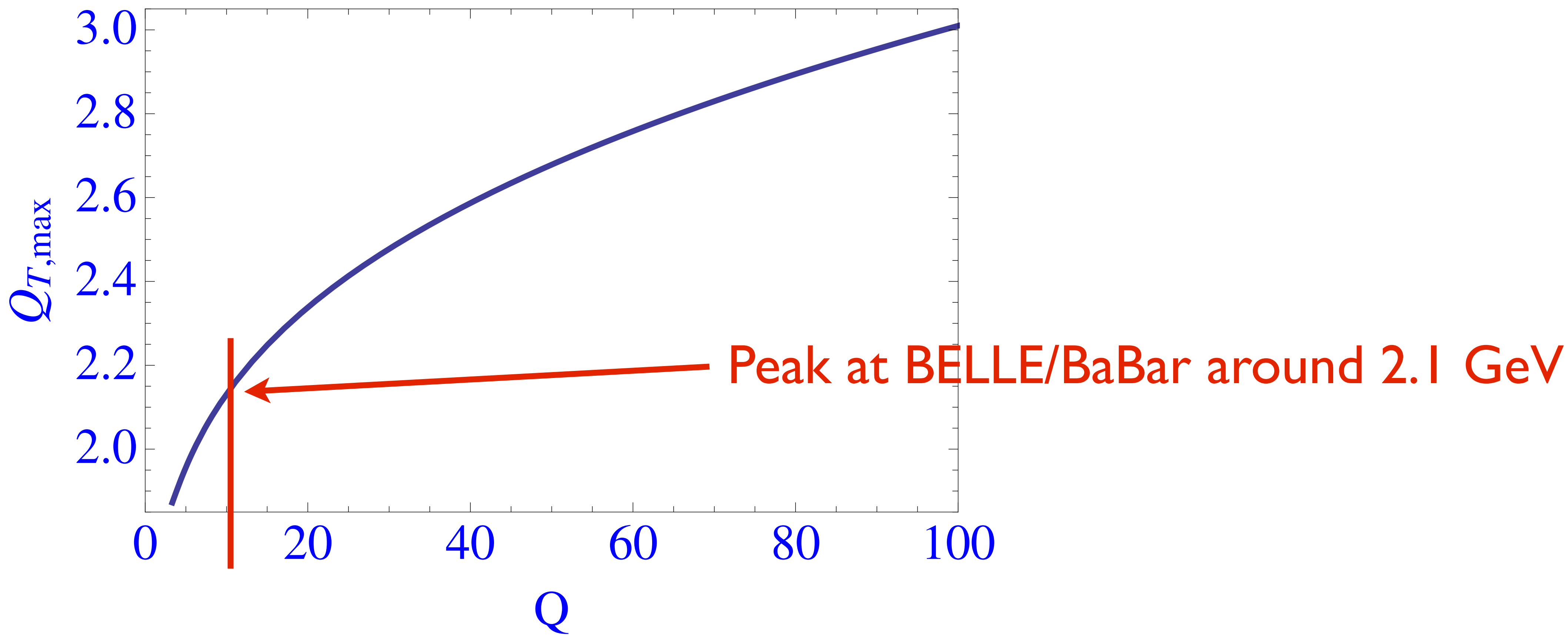
Peak of the asymmetry shifts slowly towards higher Q_T, offers a test



Data from charm factory (BEPC) important by providing data around Q≈4 GeV

Next steps

Peak of the asymmetry shifts slowly towards higher Q_T, offers a test



Data from charm factory (BEPC) important by providing data around Q≈4 GeV

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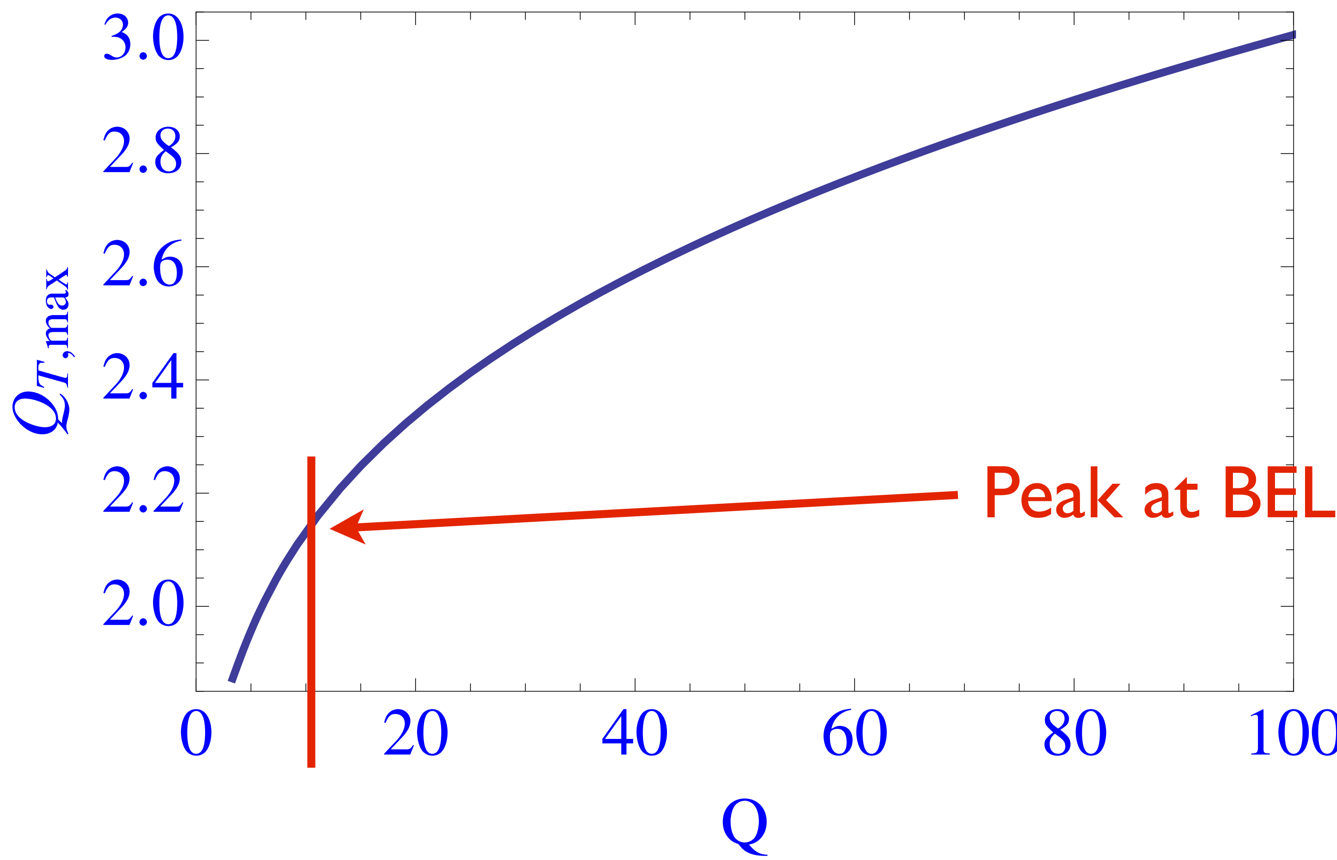


Data from charm factory (BEPC) important by providing data around Q≈4 GeV

The 1/Q behavior should modify the transversity extraction using Collins effect, full TMD evolution still to be implemented (for Q ~ 10 GeV S_{pert} is important)

Need to check the TMD evolution of the Collins asymmetry in SIDIS, which is slower than that of the double Collins asymmetry (Jefferson Lab & possibly EIC)

Double Collins Asymmetry

Data from BES important by providing data at lower Q

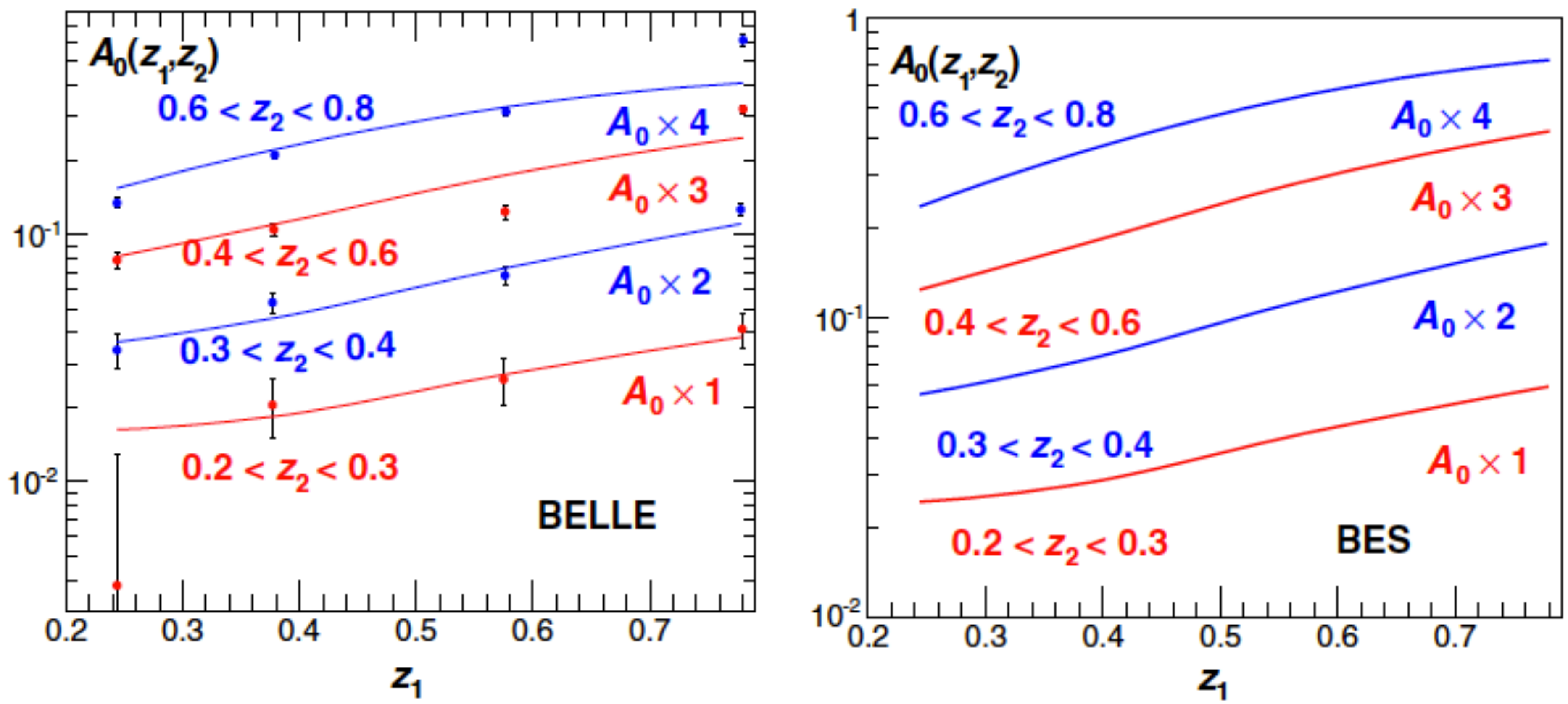


FIG. 4 (color online). The Collins asymmetries in di-hadron azimuthal angular distributions in e^+e^- annihilation processes: fit to the BELLE experiment at $\sqrt{S} = 10.6$ GeV Ref. [8], and predictions for the experiment at BEPC at $\sqrt{S} = 4.6$ GeV.

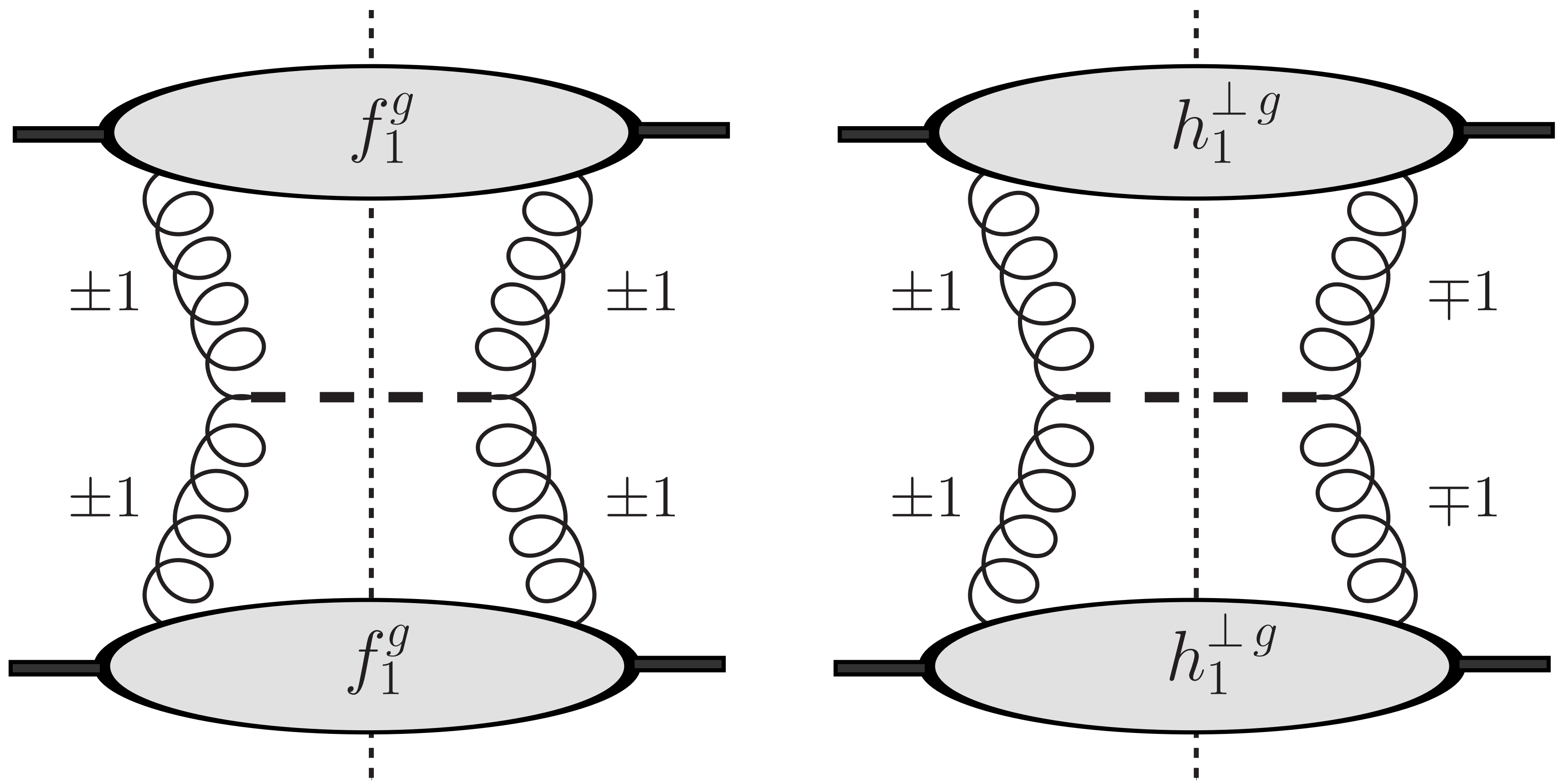
P. Sun & F. Yuan, PRD 88 (2013) 034016

One does have to worry about $1/Q^2$ corrections (analogue of the Cahn effect), which can be bounded by study simultaneously the $1/Q$ $\cos\varphi$ asymmetry

E.L. Berger, ZPC 4 (1980) 289; Brandenburg, Brodsky, Khoze & D. Mueller, PRL 73 (1994) 939

Higgs transverse momentum distribution

Higgs transverse momentum



The transverse momentum distribution in Higgs production at LHC is also a TMD factorizing process

P. Sun, B.-W. Xiao & F. Yuan, PRD 84 (2011) 094005

In this case starting the evolution from a fixed scale Q_0 is not appropriate due to the large Q/Q_0 ratio

The linear polarization of gluons inside the unpolarized protons plays a role

[Catani & Grazzini, '10; D.B., Den Dunnen, Pisano, Schlegel, Vogelsang, '12]

TMD factorization expressions

$$\frac{d\sigma}{dx_A dx_B d\Omega d^2\boldsymbol{q}_T} = \int d^2b\, e^{-i\boldsymbol{b}\cdot\boldsymbol{q}_T}\tilde{W}(\boldsymbol{b}, Q; x_A, x_B) + \mathcal{O}\left(\frac{Q_T^2}{Q^2}\right)$$

$$\tilde{W}(\boldsymbol{b}, Q; x_A, x_B) = \tilde{f}_1^g(x_A, \boldsymbol{b}^2; \zeta_A, \mu)\, \tilde{f}_1^g(x_B, \boldsymbol{b}^2; \zeta_B, \mu) H\,(Q; \mu)$$

TMD factorization expressions

$$\frac{d\sigma}{dx_A\, dx_B\, d\Omega\, d^2\boldsymbol{q}_T} = \int d^2b\, e^{-i\boldsymbol{b}\cdot\boldsymbol{q}_T}\, \tilde{W}(\boldsymbol{b}, Q; x_A, x_B) + \mathcal{O}\left(\frac{Q_T^2}{Q^2}\right)$$

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This is a naive expression, since gluons can be polarized inside unpolarized protons

[Mulders, Rodrigues '01]

$$\begin{aligned} \Phi_g^{\mu\nu}(x, \boldsymbol{p}_T) &= \frac{n_\rho\, n_\sigma}{(p\cdot n)^2} \int \frac{d(\xi\cdot P)\, d^2\xi_T}{(2\pi)^3}\, e^{ip\cdot\xi}\, \langle P|\, \mathrm{Tr}\big[\, F^{\mu\rho}(0)\, F^{\nu\sigma}(\xi)\, \big]\, |P\rangle\, \big|_{\mathrm{LF}} \\ &= -\frac{1}{2x}\left\{ g_T^{\mu\nu}\, f_1^g - \left(\frac{p_T^\mu p_T^\nu}{M^2} + g_T^{\mu\nu}\, \frac{\boldsymbol{p}_T^2}{2M^2} \right) h_1^{\perp\, g} \right\} \end{aligned}$$

Second term requires nonzero k_T, but is k_T even, chiral even and T even

$$\tilde{\Phi}_g^{ij}(x, \boldsymbol{b}) = \frac{1}{2x}\left\{ \delta^{ij}\, \tilde{f}_1^g(x, b^2) - \left(\frac{2b^i b^j}{b^2} - \delta^{ij} \right) \tilde{h}_1^{\perp\, g}(x, b^2) \right\}$$

Cross section

$$\left.\frac{E\, d\sigma^{pp\to HX}}{d^3\vec{q}}\right|_{q_T\ll m_H} = \frac{\pi\sqrt{2}G_F}{128 m_H^2 s}\left(\frac{\alpha_s}{4\pi}\right)^2 |\mathcal{A}_H(\tau)|^2$$
$$\times\left(\mathcal{C}\left[f_1^g\, f_1^g\right] + \mathcal{C}\left[w_H\, h_1^{\perp g}\, h_1^{\perp g}\right]\right) + \mathcal{O}\left(\frac{q_T}{m_H}\right)$$

$$w_H = \frac{(\boldsymbol{p}_T\cdot\boldsymbol{k}_T)^2 - \frac{1}{2}\boldsymbol{p}_T^2\boldsymbol{k}_T^2}{2M^4} \qquad\qquad \tau = m_H^2/(4m_t^2)$$

The relative effect of linearly polarized gluons:

$$\mathcal{R}(Q_T) \equiv \frac{\mathcal{C}[w_H\, h_1^{\perp g}\, h_1^{\perp g}]}{\mathcal{C}[f_1^g\, f_1^g]}$$

$$\mathcal{R}(Q_T) = \frac{\int d^2\boldsymbol{b}\, e^{i\boldsymbol{b}\cdot\boldsymbol{q}_T} e^{-S_A(b_*,Q)-S_{NP}(b,Q)}\ \tilde{h}_1^{\perp g}(x_A, b_*^2; \mu_{b_*}^2, \mu_{b_*})\ \tilde{h}_1^{\perp g}(x_B, b_*^2; \mu_{b_*}^2, \mu_{b_*})}{\int d^2\boldsymbol{b}\, e^{i\boldsymbol{b}\cdot\boldsymbol{q}_T}\ e^{-S_A(b_*,Q)-S_{NP}(b,Q)}\,\tilde{f}_1^g(x_A, b_*^2; \mu_{b_*}^2, \mu_{b_*})\,\tilde{f}_1^g(x_B, b_*^2; \mu_{b_*}^2, \mu_{b_*})}$$

CSS approach

Consider now only the perturbative tails:

$$\tilde{f}_1^g(x, b^2; \mu_b^2, \mu_b) = f_{g/P}(x; \mu_b) + \mathcal{O}(\alpha_s)$$

$$\tilde{h}_1^{\perp g}(x, b^2; \mu_b^2, \mu_b) = \frac{\alpha_s(\mu_b) C_A}{2\pi} \int_x^1 \frac{d\hat{x}}{\hat{x}} \left(\frac{\hat{x}}{x} - 1 \right) f_{g/P}(\hat{x}; \mu_b) + \mathcal{O}(\alpha_s^2)$$

This coincides with the CSS approach

[Nadolsky, Balazs, Berger, C.-P. Yuan, ’07; Catani, Grazzini, ’10; P. Sun, B.-W. Xiao, F. Yuan, ’11]

PHYSICAL REVIEW D **86,** 094026 (2012)

Improved resummation prediction on Higgs boson production at hadron colliders

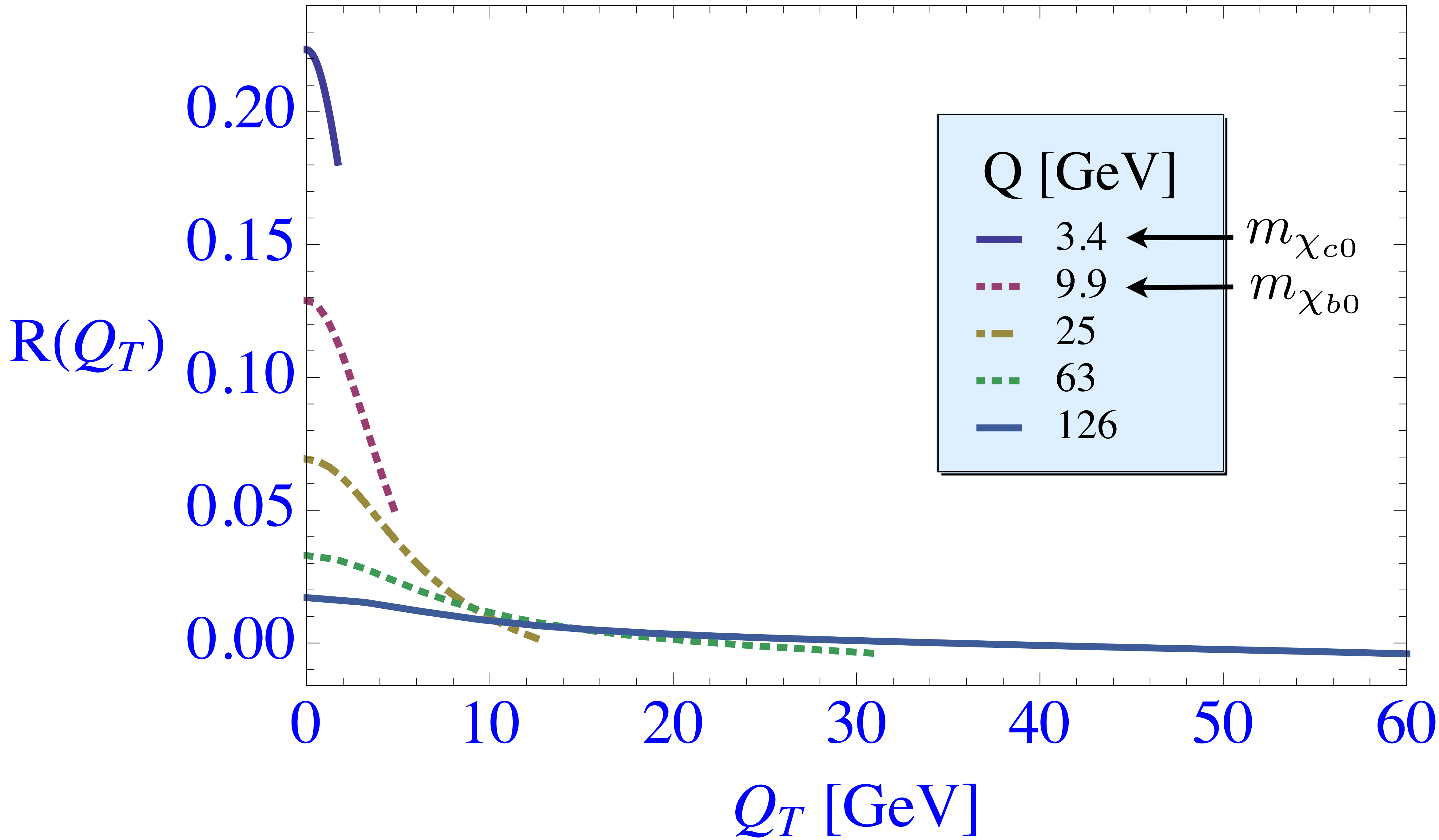
Jian Wang,[1] Chong Sheng Li,[1,2,*] Hai Tao Li,[1] Zhao Li,[3,†] and C.-P. Yuan[2,3,‡]

They find permille level effects at the Higgs scale, but using the TMD approach at the LL level yields percent level effects

D.B. & den Dunnen, NPB 886 (2014) 421

Wang et al. also use a different S_{NP}

TMD / CSS evolution effects



$x_A = x_B = Q/(8\text{TeV})$

MSTW08 LO gluon distribution

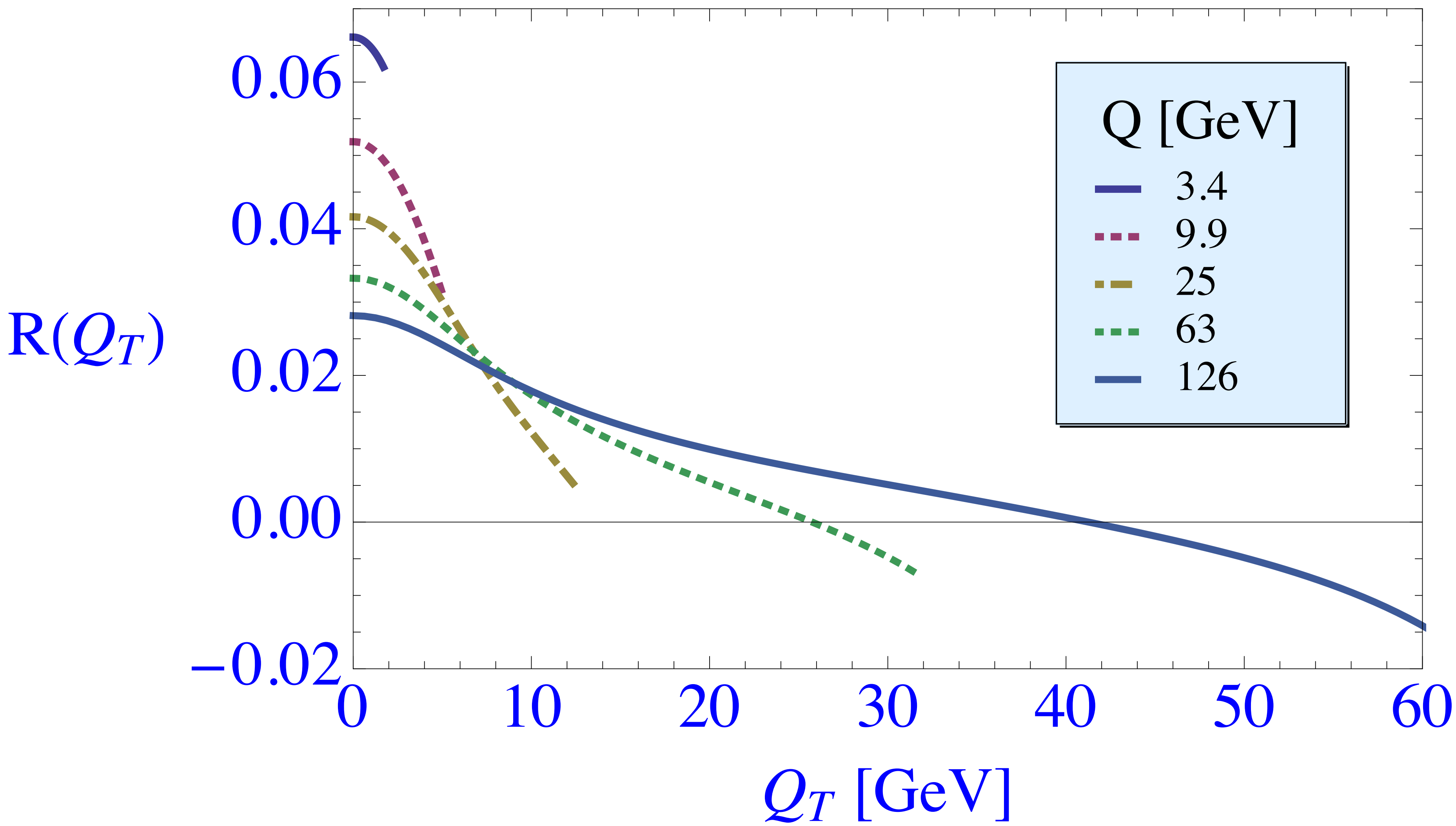
D.B. & den Dunnen, NPB 886 (2014) 421

Beyond CSS

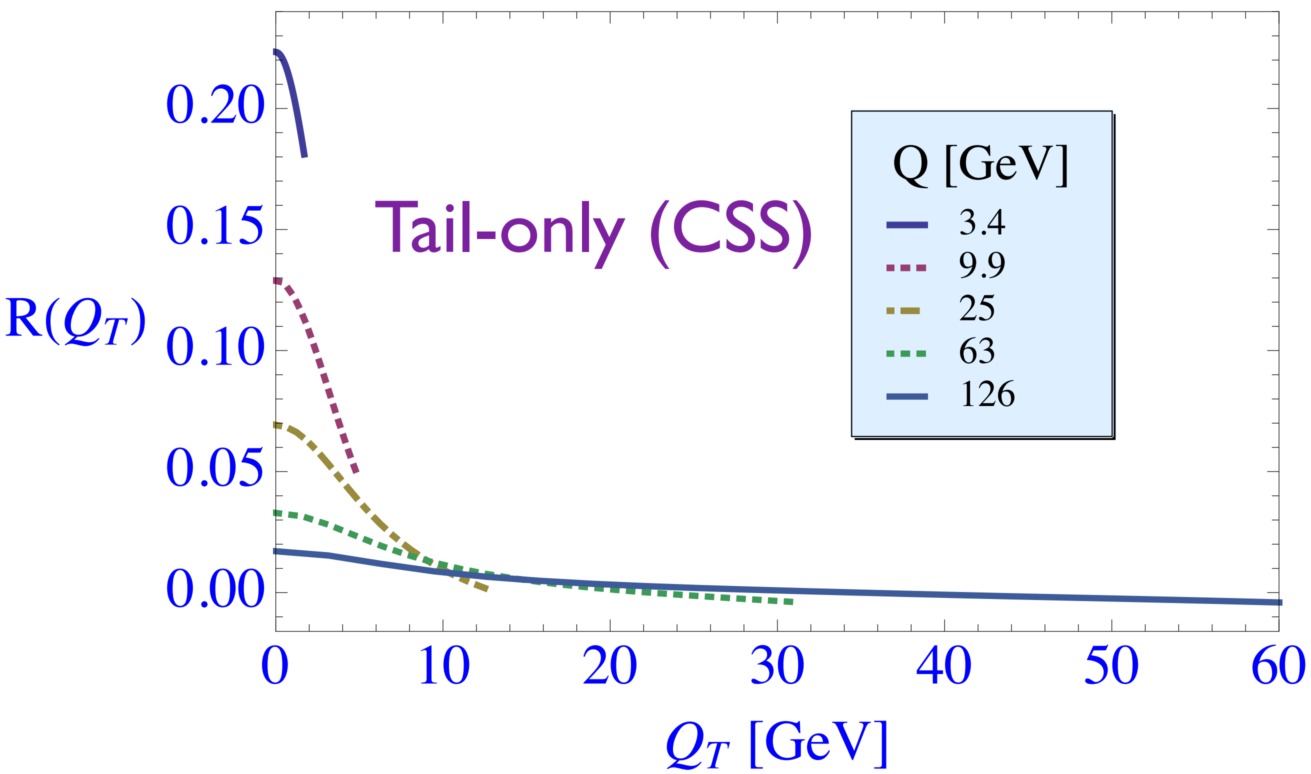
In the TMD factorized expression there may be nonperturbative contributions from small p_T which mainly affect large b

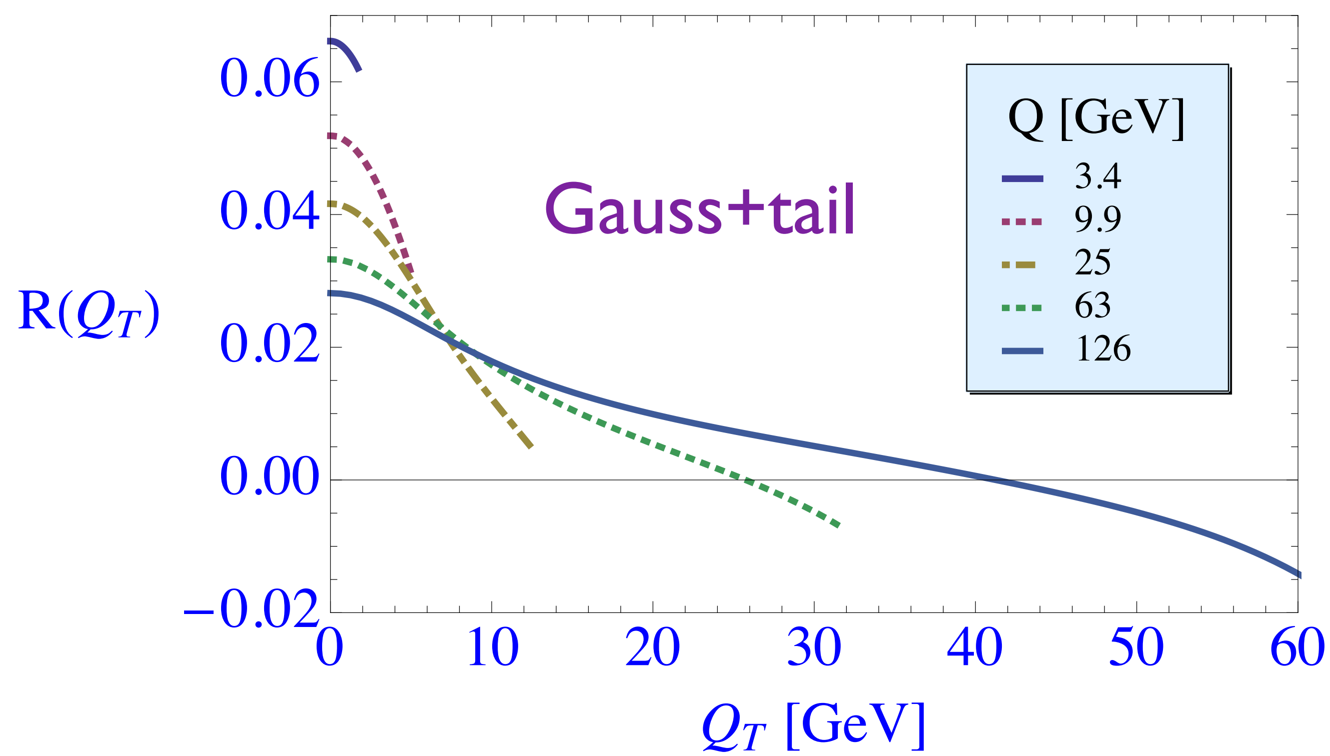
CSS only allows NP contribution via S_{NP} and does not allow all possibilities of the TMD approach

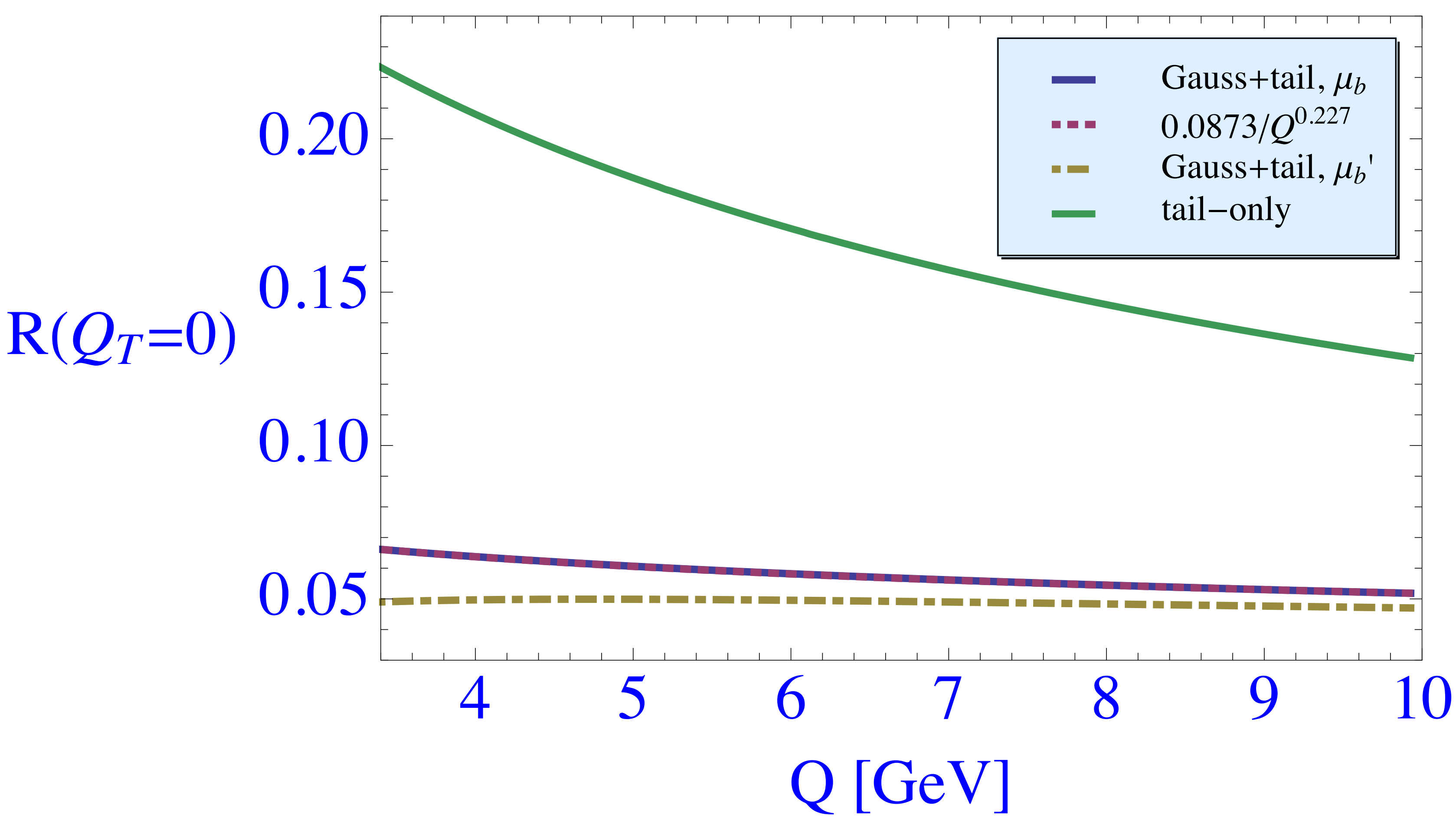
To illustrate this we consider a model which is approximately Gaussian at low p_T and has the correct tail at high p_T or small b



Comparison



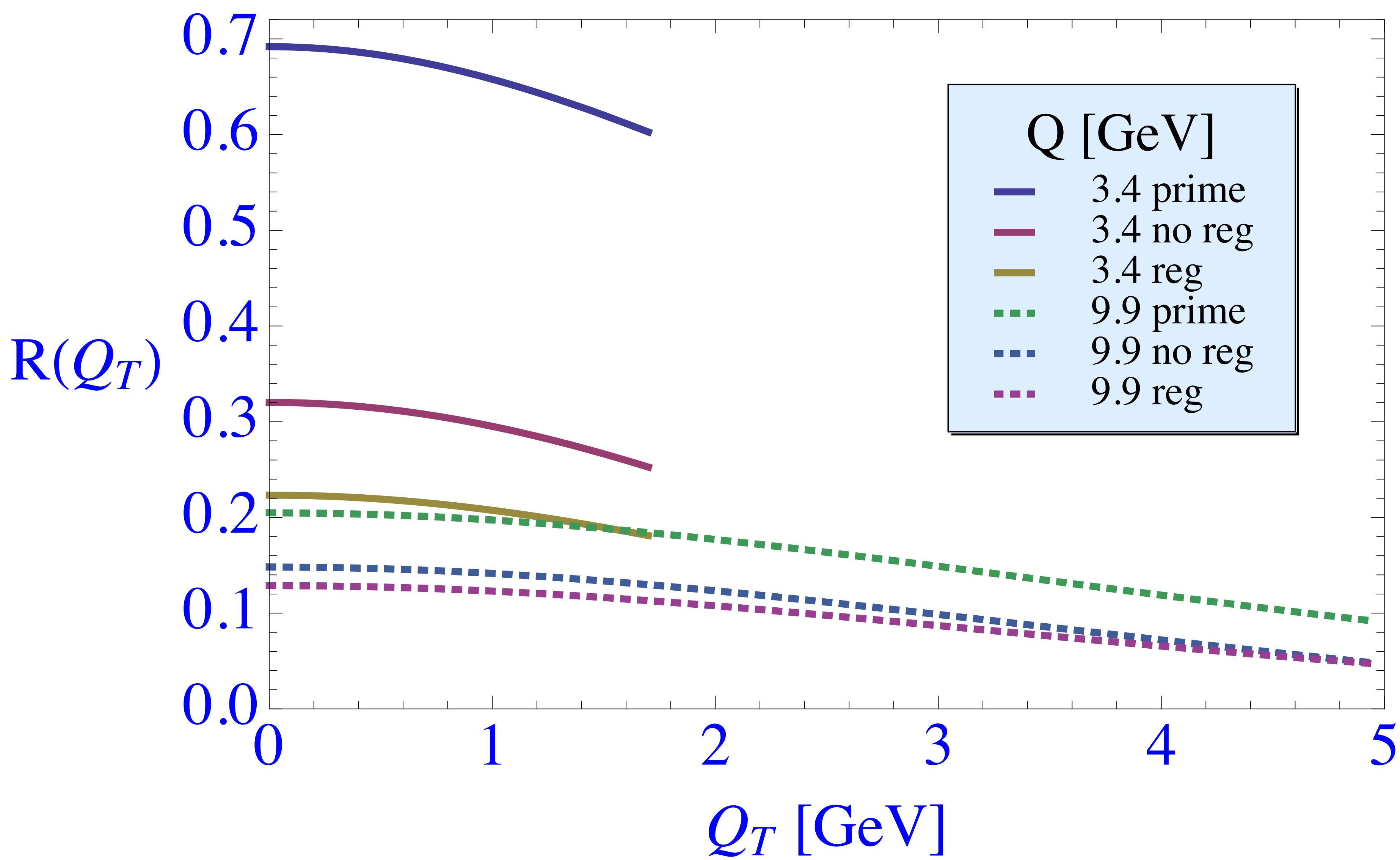




Gaussian+tail evolves much more slowly than tail-only (CSS) expression

Very small b region

At low Q there is quite some uncertainty from the very small b region (b << 1/Q) where the perturbative expressions for S_A are all incorrect (don't satisfy S(0)=0)



Standard regularization:

$$Q^2/\mu_b^2 = b^2 Q^2/b_0^2 \to Q^2/\mu_b'^2 \equiv (bQ/b_0 + 1)^2$$

Parisi, Petronzio, 1985

Very small b region

For very small b region (b << 1/Q) the perturbative expressions for S_A are all incorrect

$$S_A(b,Q) = \frac{C_A}{\pi}\int_{\mu_b^2}^{Q^2}\frac{d\mu^2}{\mu^2}\alpha_s(\mu)\,[\ldots] \stackrel{b\ll 1/Q}{\longrightarrow} -\frac{C_A}{\pi}\int_{Q^2}^{\mu_b^2}\frac{d\mu^2}{\mu^2}\alpha_s(\mu)\,[\ldots]$$

Sudakov suppression ($e^{-\#}$) becomes an *unphysical* Sudakov enhancement ($e^{+\#}$)

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Sudakov suppression ($e^{-\#}$) becomes an *unphysical* Sudakov enhancement ($e^{+\#}$)

$$\frac{d\sigma}{dq_T^2} = Y(q_T^2) + \int \frac{d^2\mathbf{b}}{4\pi} e^{-i\mathbf{q}_T\cdot\mathbf{b}}\sigma_0(1+A)\exp S(b)$$

where

$$S(b) = \int_0^{A_T^2}\frac{dk^2}{k^2}(J_0(bk)-1)\left(B\ln\frac{Q^2}{k^2}+C\right).$$

$$A_T^2 = A_T^2(y) = \frac{(S+Q^2)^2}{4S\cosh^2 y} - Q^2.$$

$$\exp S = \exp\int_0^{A_T^2} \approx \left(1+\int_{Q^2}^{A_T^2}\right)\exp\int_0^{Q^2}$$

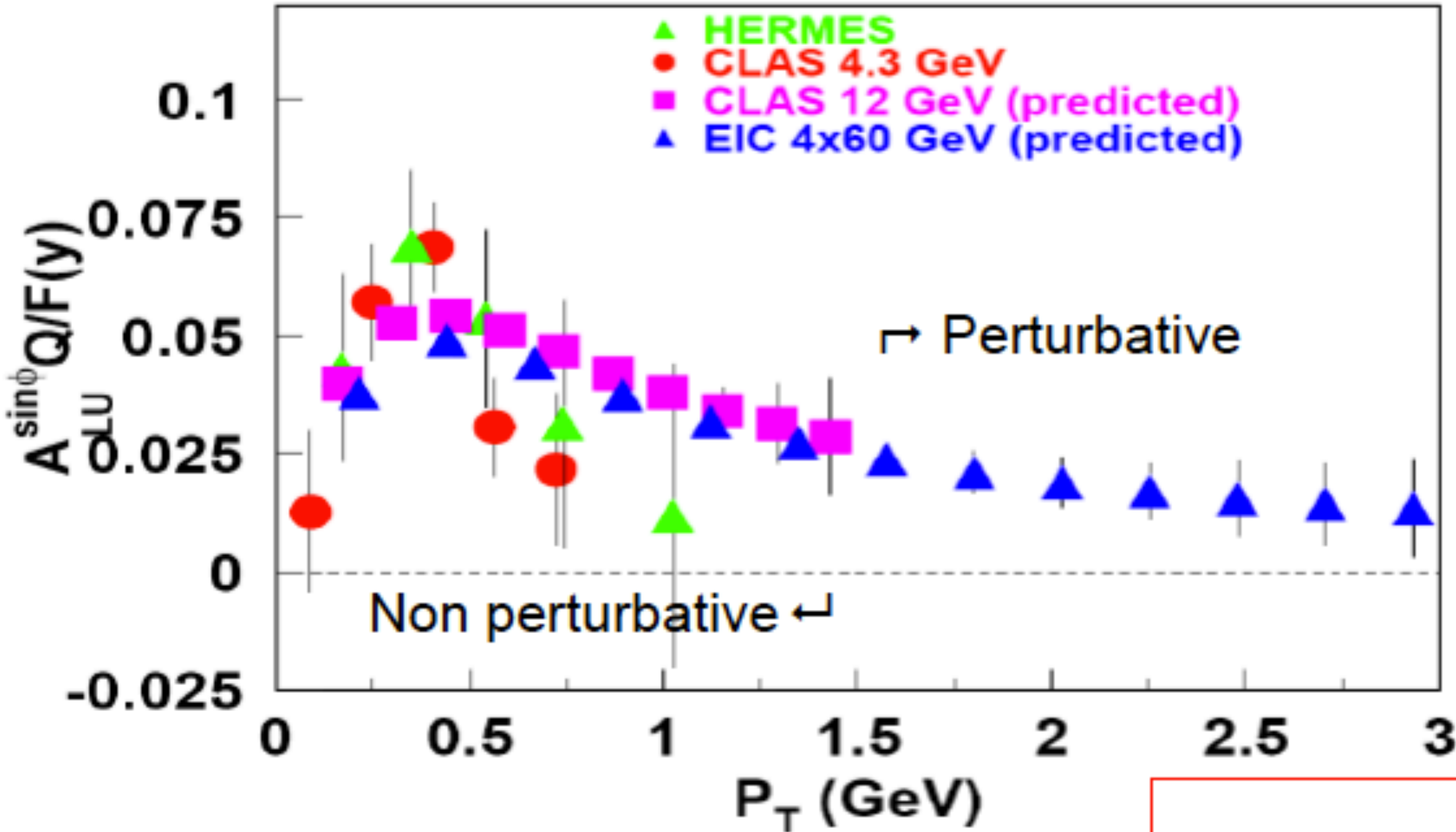
Altarelli, Ellis, Martinelli, 1985

Does satisfy S(0)=0

Not yet clear what is the exact expression to take in TMD factorization

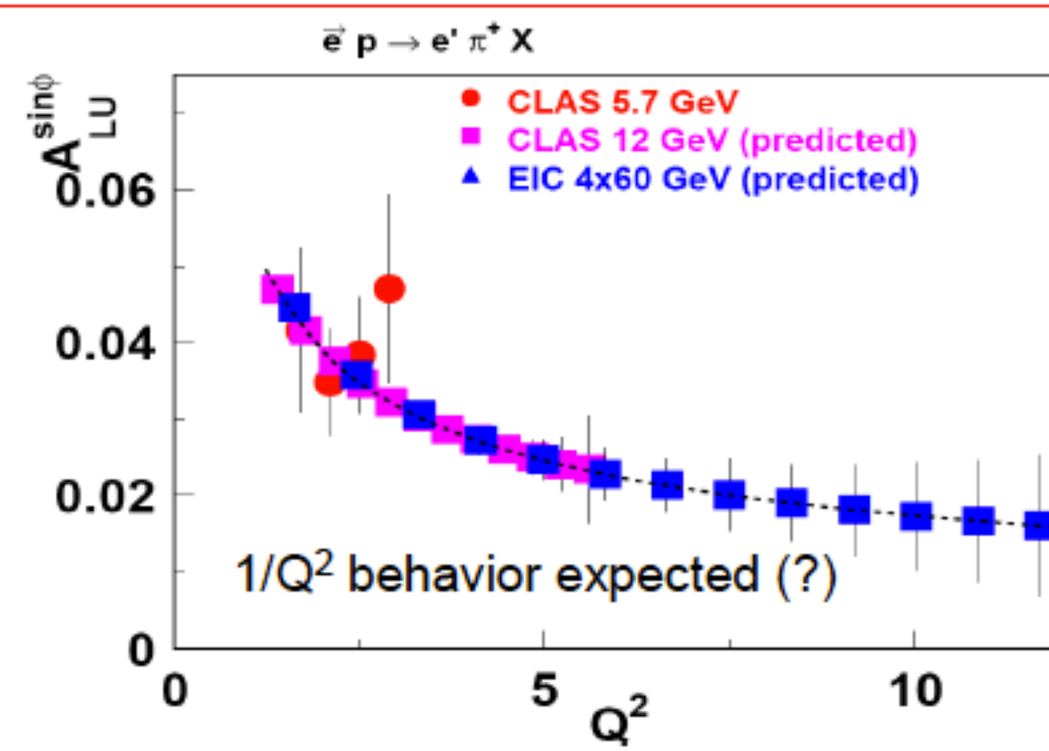
Higher twist

P_T and Q^2-dep Higher Twist $A_{LU}^{\sin\phi}$



Study for SSA transition from non-perturbative to perturbative regime.

E12-06-112



Study for Q^2 dependence of beam SSA allows to check the higher twist nature and access quark-gluon correlations

Conclusions

Conclusions

- Significant recent developments on TMD factorization and evolution:
  - New TMD factorization expressions by JCC (2011) & EIS (2012)
  - Improvements through additional resummations (Echevarria *et al.*) lifts analyses to the NNLL level (2013/4)
  - Progress towards describing SIDIS, DY & Z production data by a universal non-perturbative function (2013/4)

- Consequences of TMD evolution studied (in varying levels of accuracy) for:
  - Sivers & (single and double) Collins effect asymmetries
  - Higgs production including the effect of linear gluon polarization

- Future data from JLab12 and BES and perhaps a high-energy EIC can help to map out the Q dependence of Sivers and Collins asymmetries in greater detail

- Future data from LHC on Higgs and $\chi_{c/b0}$ production could do the same for gluon dominated TMD processes

- TMD (non-)factorization at next-to-leading twist remains entirely unexplored

Back-up slides

Further resummations

For the TMD at small b one often considers the perturbative tail, which is calculable

$$\tilde{f}_{g/P}(x, b^2; \mu, \zeta) = \sum_{i=g,q} \int_x^1 \frac{d\hat{x}}{\hat{x}} C_{i/g}(x/\hat{x}, b^2; g(\mu), \mu, \zeta) f_{i/P}(\hat{x}; \mu) + \mathcal{O}((\Lambda_{\mathrm{QCD}} b)^a)$$

To extend it to be valid at larger b values one can perform further resummation:

$$\tilde{F}^{\mathrm{pert}}_{q/N}(x, b_T; \zeta, \mu) = \left(\frac{\zeta b_T^2}{4e^{-2\gamma_E}}\right)^{-D^R(b_T;\mu)} e^{h^R_\Gamma(b_T;\mu) - h^R_\gamma(b_T;\mu)} \sum_j \int_x^1 \frac{dz}{z} \hat{C}_{q\leftarrow j}(x/z, b_T; \mu)\, f_{j/N}(z; \mu)$$

$$\tilde{F}_{q/N}(x, b_T; Q_i^2, \mu_i) = \tilde{F}^{\mathrm{pert}}_{q/N}(x, b_T; Q_i^2, \mu_i)\, \tilde{F}^{\mathrm{NP}}_{q/N}(x, b_T; Q_i)$$

$$\tilde{F}^{\mathrm{NP}}_{q/N}(x, b_T; Q_i) \equiv \tilde{F}^{\mathrm{NP}}_{q/N}(x, b_T) \left(\frac{Q_i^2}{Q_0^2}\right)^{-D^{\mathrm{NP}}(b_T)}$$

D'Alesio, Echevarria, Melis, Scimemi, arXiv:1407.3311

Tool to compare different methods: The L function

(JCC & Rogers, in preparation)

- Shape change of transverse momentum distribution comes only from b_{T}-dependence of $\tilde{K}$
- So define scheme independent

$$L(b_{\mathrm{T}}) = -\frac{\partial}{\partial \ln b_{\mathrm{T}}^2} \frac{\partial}{\partial \ln Q^2} \ln \tilde{W}(b_{\mathrm{T}}, Q, x_A, x_B) \overset{\text{CSS}}{=} -\frac{\partial}{\partial \ln b_{\mathrm{T}}^2} \tilde{K}(b_{\mathrm{T}}, \mu)$$

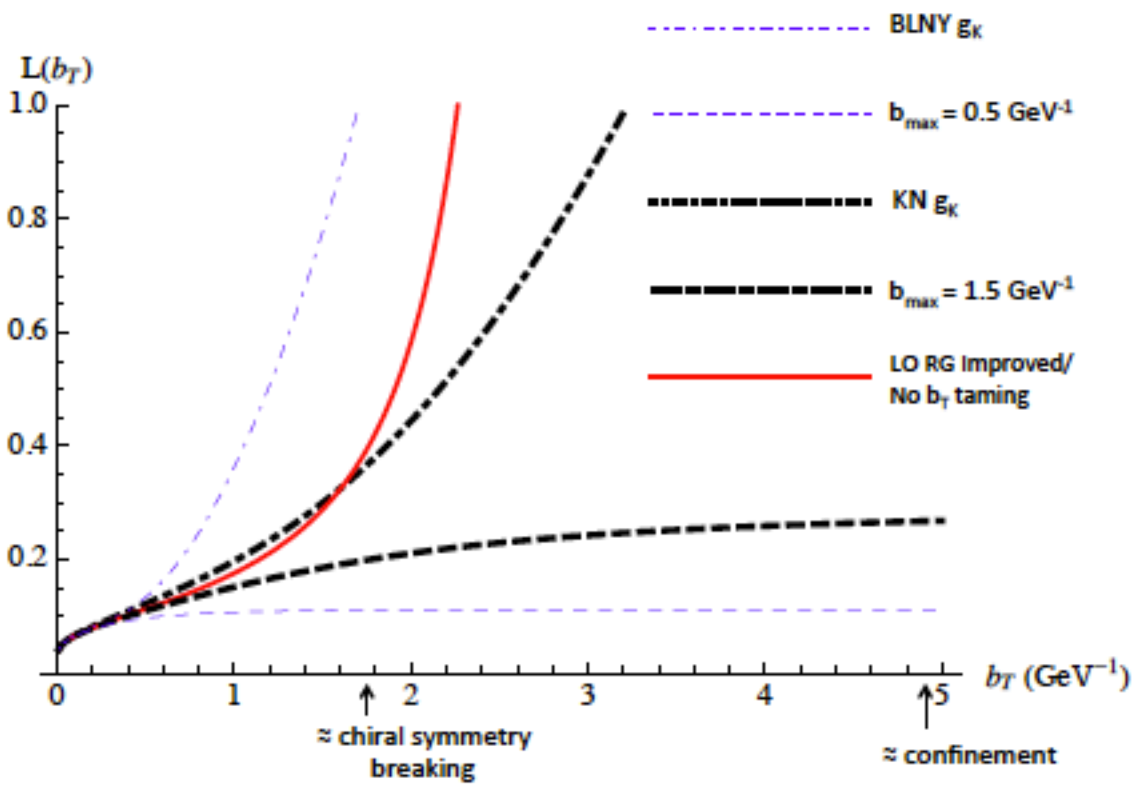
- QCD predicts it is
  - independent of Q, x_A, x_B
  - independent of light-quark flavor
  - RG invariant
  - perturbatively calculable at small b_{T}
  - non-perturbative at large b_{T}

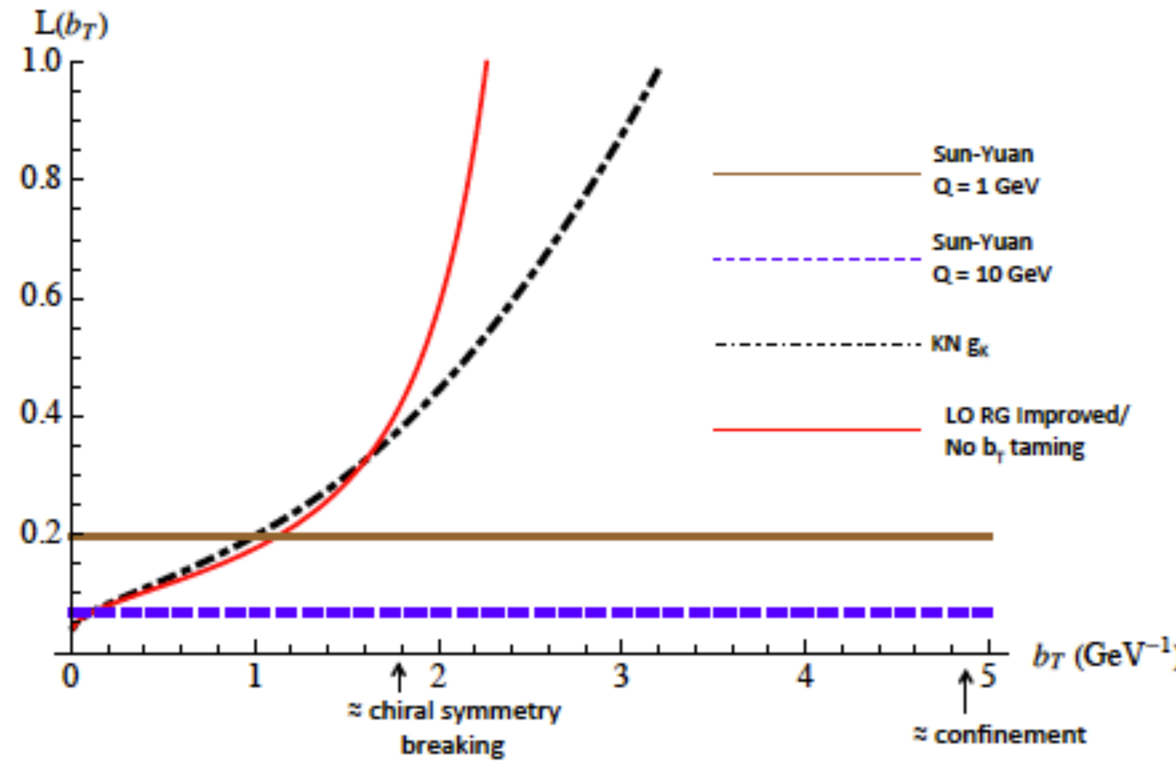
Collins, QCD Evolution workshop, May 12, 2014

L is called A in Collins, 1409.5408

Comparing different results using the L function

(Preliminary)





Q	Typical b_T
$2\,\mathrm{GeV}$	$3\,\mathrm{GeV}^{-1}$
$10\,\mathrm{GeV}$	$1.2\,\mathrm{GeV}^{-1}$
m_Z	$0.5\,\mathrm{GeV}^{-1}$

SY = Sun & Yuan (PRD 88, 114012 (2013)):

$$L_{\mathrm{SY}} = C_F \frac{\alpha_s(Q)}{\pi}$$

Depends on Q: contrary to QCD

Collins, QCD Evolution workshop, May 12, 2014

Sivers asymmetry expression

$$\mathcal{A}_{ab}(x,z,Q_T) \equiv \frac{\int db\, b^2\, J_1(bQ_T)\, \tilde{f}_{1T}^{\perp\prime\, a}(x,b_*^2;Q_0^2,Q_0)\, \tilde{D}_1^a(z,b_*^2;Q_0^2,Q_0)\, \exp\left(-S_p(b_*,Q,Q_0) - S_{NP}(b,Q/Q_0)\right)}{MQ_T \int db\, b\, J_0(bQ_T)\tilde{f}_1^b(x,b_*^2;Q_0^2,Q_0)\tilde{D}_1^b(z,b_*^2;Q_0^2,Q_0)\, \exp\left(-S_p(b_*,Q,Q_0) - S_{NP}(b,Q/Q_0)\right)}$$

Assume that the TMDs of b_* are slowly varying functions of b in the dominant b region ($b \sim 1/Q_T >> 1/Q$, hence $b_* \approx b_{max} = 1/Q_0$): $\Phi(x,b_*) \approx \Phi(z,1/Q_0)$

This approximation means dropping the perturbative tail of TMDs and leads to a decoupling of x and b dependence

$$\mathcal{A}_{ab}(x,z,Q_T) = \frac{f_{1T}^{\perp\prime\, a}(x;Q_0) D_1^a(z;Q_0)}{M^2 f_1^b(x;Q_0) D_1^b(z;Q_0)} \mathcal{A}(Q_T)$$

$$\mathcal{A}(Q_T) \equiv M \frac{\int db\, b^2\, J_1(bQ_T)\, \exp\left(-S_p(b_*,Q,Q_0) - S_{NP}(b,Q/Q_0)\right)}{\int db\, b\, J_0(bQ_T)\, \exp\left(-S_p(b_*,Q,Q_0) - S_{NP}(b,Q/Q_0)\right)}$$

DB, NPB 874 (2013) 217

Under this assumption, the same factor appears in $e^+e^- \to h_1\, h_2\, X$, SIDIS and DY and in all asymmetries involving one b-odd TMD, such as the Collins asymmetry

Claim: this captures the dominant Q dependence for Q_T and Q not too large

TMD factorization expressions

Differential cross section of $e^+e^- \to h_1\, h_2\, X$ process at small Q_T:

$$\frac{d\sigma}{dz_1 dz_2 d\Omega d^2 q_T} = \int d^2b\, e^{-ib\cdot q_T}\, \tilde{W}(b,Q;z_1,z_2) + \mathcal{O}\left(Q_T^2/Q^2\right)$$

$$\tilde{W}(b,Q;z_1,z_2) \quad = \quad \sum_{a,b} \tilde{D}_1^a(z_1,b;Q_0,\alpha_s(Q_0))\tilde{D}_1^b(z_2,b;Q_0,\alpha_s(Q_0))$$
$$\times e^{-S(b,Q,Q_0)}\, H_{ab}\left(Q;\alpha_s(Q)\right)$$

$b = |\boldsymbol{b}|$

TMDs are taken at a fixed scale Q_0, the smallest perturbative scale

$$\tilde{\Delta}(z,\boldsymbol{b}) = \frac{M}{4}\left\{\tilde{D}_1(z,b^2)\,\frac{\not{P}}{M} + \left(\frac{\partial}{\partial b^2}\tilde{H}_1^{\perp}(z,b^2)\right)\frac{2\,\not{b}\,\not{P}}{M^2}\right\}$$

$b = |\boldsymbol{b}|$

Assume that the TMDs of b_* are slowly varying functions of b in the dominant b region ($b \sim 1/Q_T \gg 1/Q$, hence $b_* \approx b_{max} = 1/Q_0$): $\Delta(z,b_*) \approx \Delta(z,1/Q_0)$

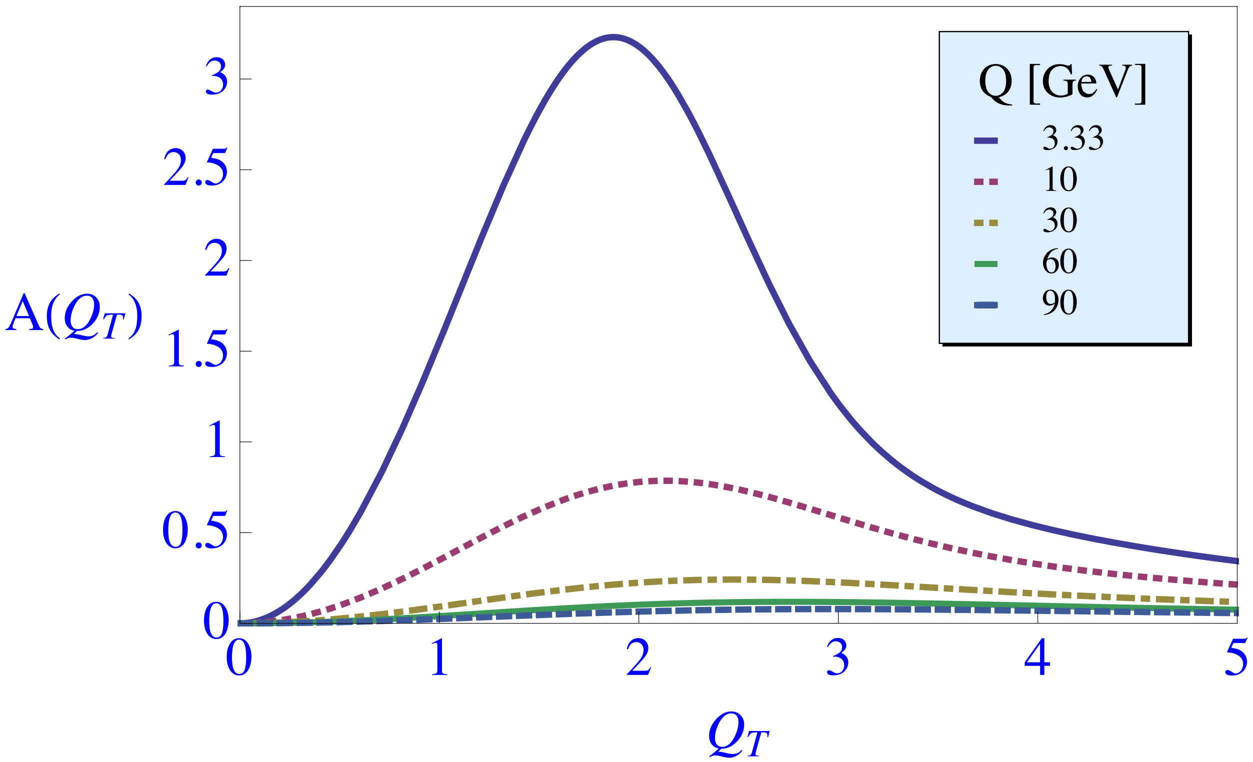
This approximation means dropping the perturbative tails and leads to a decoupling of z and b dependence (gives same result for SIDIS & DY)

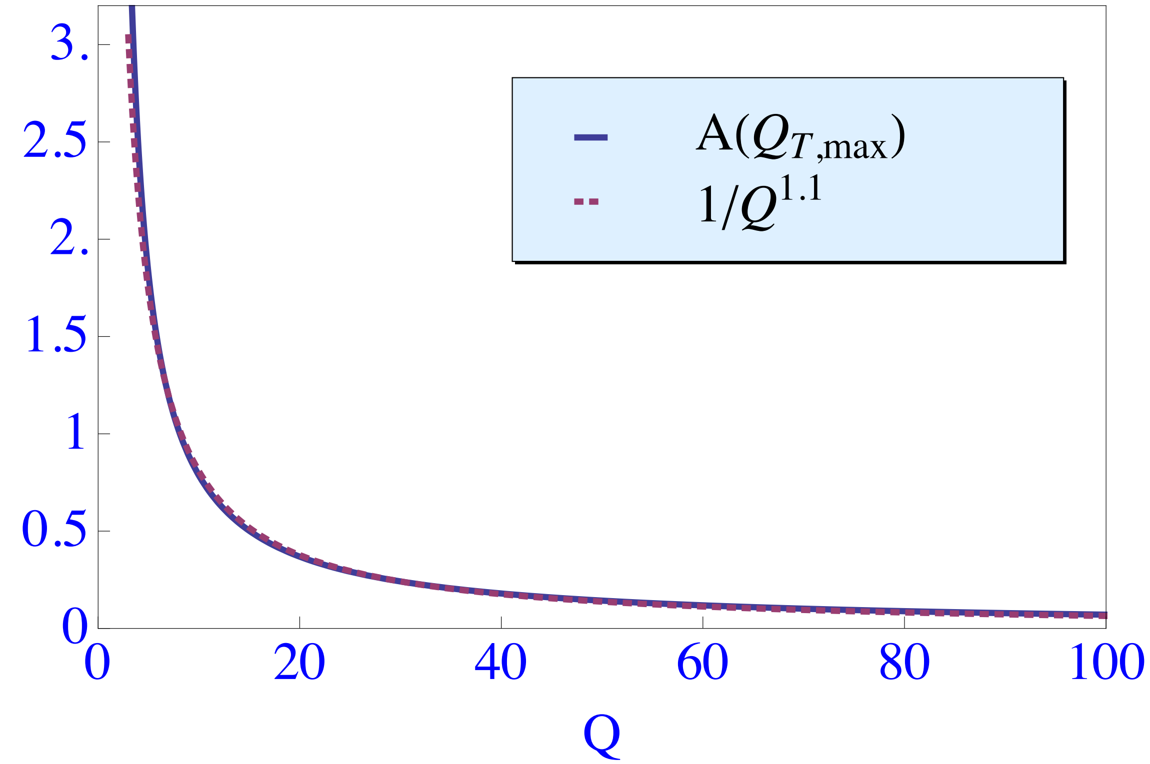
Double Collins Asymmetry

$$\frac{d\sigma(e^+e^- \to h_1 h_2 X)}{dz_1 dz_2 d\Omega d^2\boldsymbol{q}_T} \propto \{1 + \cos 2\phi_1 A(\boldsymbol{q}_T)\}$$

$$A(Q_T) = \frac{\sum_a e_a^2 \sin^2\theta\ H_1^{\perp(1)a}(z_1;Q_0)\ \overline{H}_1^{\perp(1)a}(z_2;Q_0)}{\sum_b e_b^2 (1+\cos^2\theta)\ D_1^b(z_1;Q_0)\ \overline{D}_1^b(z_2;Q_0)}\ \mathcal{A}(Q_T)$$

$$\mathcal{A}(Q_T) = M^2 \frac{\int db\, b^3\, J_2(bQ_T)\ \exp\left(-S_p(b_*,Q,Q_0) - S_{NP}(b,Q/Q_0)\right)}{\int db\, b\, J_0(bQ_T)\ \exp\left(-S_p(b_*,Q,Q_0) - S_{NP}(b,Q/Q_0)\right)}$$





Considerable Sudakov suppression ~1/Q (effectively twist-3)

D.B., NPB 603 (2001) 195 & NPB 806 (2009) 23 & NPB 874 (2013) 217 & arXiv:1308.4262

Tree level expression

$$\frac{E\, d\sigma^{pp\to HX}}{d^3\vec{q}}\bigg|_{q_T\ll m_H} = \frac{\pi\sqrt{2}G_F}{128 m_H^2 s}\left(\frac{\alpha_s}{4\pi}\right)^2 |\mathcal{A}_H(\tau)|^2 \times \left(\mathcal{C}\left[f_1^g\, f_1^g\right] + \mathcal{C}\left[w_H\, h_1^{\perp g}\, h_1^{\perp g}\right]\right) + \mathcal{O}\left(\frac{q_T}{m_H}\right)$$

The gluon TMDs enter in convolutions:

$$\mathcal{C}[w\, f\, f] \equiv \int d^2\boldsymbol{p}_T \int d^2\boldsymbol{k}_T\, \delta^2(\boldsymbol{p}_T + \boldsymbol{k}_T - \boldsymbol{q}_T)\, w(\boldsymbol{p}_T, \boldsymbol{k}_T)\, f(x_A, \boldsymbol{p}_T^2)\, f(x_B, \boldsymbol{k}_T^2)$$

$$w_H = \frac{(\boldsymbol{p}_T\cdot\boldsymbol{k}_T)^2 - \frac{1}{2}\boldsymbol{p}_T^2\boldsymbol{k}_T^2}{2M^4} \qquad\qquad \tau = m_H^2/(4m_t^2)$$

The relative effect of linearly polarized gluons:

$$\mathcal{R}(Q_T) \equiv \frac{\mathcal{C}[w_H\, h_1^{\perp g}\, h_1^{\perp g}]}{\mathcal{C}[f_1^g\, f_1^g]}$$

Beyond tree level

$$\mathcal{C}\left[f_1^g\, f_1^g\right] = \int \frac{d^2\boldsymbol{b}}{(2\pi)^2}\, e^{i\boldsymbol{b}\cdot\boldsymbol{q}_T}\, \tilde{f}_1^g(x_A, b^2; \zeta_A, \mu)\, \tilde{f}_1^g(x_B, b^2; \zeta_B, \mu)$$

$$= \int \frac{d^2\boldsymbol{b}}{(2\pi)^2}\, e^{i\boldsymbol{b}\cdot\boldsymbol{q}_T}\, e^{-S_A(b,Q)}\, \tilde{f}_1^g(x_A, b^2; \mu_b^2, \mu_b)\, \tilde{f}_1^g(x_B, b^2; \mu_b^2, \mu_b)$$

Perturbative Sudakov factor:

$$S_A(b,Q) = \frac{C_A}{\pi}\int_{\mu_b^2}^{Q^2} \frac{d\mu^2}{\mu^2}\alpha_s(\mu)\left[\ln\left(\frac{Q^2}{\mu^2}\right) - \frac{11 - 2n_f/C_A}{6}\right] + \mathcal{O}(\alpha_s^2)$$

$$= -\frac{36}{33-2n_f}\left[\ln\left(\frac{Q^2}{\mu_b^2}\right) + \ln\left(\frac{Q^2}{\Lambda^2}\right)\,\ln\left(1 - \frac{\ln\left(Q^2/\mu_b^2\right)}{\ln\left(Q^2/\Lambda^2\right)}\right)\right.$$

$$\left. + \frac{11 - 2n_f/C_A}{6}\ln\left(\frac{\ln\left(Q^2/\Lambda^2\right)}{\ln\left(\mu_b^2/\Lambda^2\right)}\right)\right]$$

$$\frac{d\sigma}{dx_A dx_B d\Omega d^2\boldsymbol{q}_T} = \int d^2b\, e^{-i\boldsymbol{b}\cdot\boldsymbol{q}_T} \tilde{W}(\boldsymbol{b}, Q; x_A, x_B) + \mathcal{O}\left(\frac{Q_T^2}{Q^2}\right)$$

The integral is over all b, including nonperturbatively large b

$$\tilde{W}(b) \equiv \tilde{W}(b_*)\, e^{-S_{NP}(b)}$$

$$b_* = b/\sqrt{1 + b^2/b_{\mathrm{max}}^2} \leq b_{\mathrm{max}}$$

$$b_{\mathrm{max}} = 1.5\ \mathrm{GeV}^{-1} \Rightarrow \alpha_s(b_0/b_{\mathrm{max}}) = 0.62$$

No extraction of S_{NP} exists, e.g. use a modified Aybat-Rogers S_{NP}

$$S_{NP}(b, Q, Q_0) = \frac{C_A}{C_F}\left[0.184 \ln\frac{Q}{2Q_0} + 0.332\right] b^2$$

Beyond tree level

$$\mathcal{R}(Q_T) = \frac{\int d^2\boldsymbol{b}\, e^{i\boldsymbol{b}\cdot\boldsymbol{q}_T} e^{-S_A(b_*,Q)-S_{NP}(b,Q)}\ \tilde{h}_1^{\perp g}(x_A, b_*^2; \mu_{b_*}^2, \mu_{b_*})\ \tilde{h}_1^{\perp g}(x_B, b_*^2; \mu_{b_*}^2, \mu_{b_*})}{\int d^2\boldsymbol{b}\, e^{i\boldsymbol{b}\cdot\boldsymbol{q}_T}\ e^{-S_A(b_*,Q)-S_{NP}(b,Q)}\, \tilde{f}_1^g(x_A, b_*^2; \mu_{b_*}^2, \mu_{b_*})\ \tilde{f}_1^g(x_B, b_*^2; \mu_{b_*}^2, \mu_{b_*})}$$

$$\tilde{h}_1^{\perp\, g}(x, b^2) = \int d^2\boldsymbol{p}_T\ \frac{(\boldsymbol{b}\cdot\boldsymbol{p}_T)^2 - \frac{1}{2}\boldsymbol{b}^2\boldsymbol{p}_T^2}{b^2 M^2}\ e^{-i\boldsymbol{b}\cdot\boldsymbol{p}_T}\ h_1^{\perp g}(x, p_T^2)$$
$$= -\pi \int dp_T^2 \frac{p_T^2}{2M^2} J_2(bp_T) h_1^{\perp g}(x, p_T^2)$$

Consider now only the perturbative tails:

$$\tilde{f}_1^g(x, b^2; \mu_b^2, \mu_b) = f_{g/P}(x; \mu_b) + \mathcal{O}(\alpha_s)$$

$$\tilde{h}_1^{\perp g}(x, b^2; \mu_b^2, \mu_b) = \frac{\alpha_s(\mu_b) C_A}{2\pi} \int_x^1 \frac{d\hat{x}}{\hat{x}} \left(\frac{\hat{x}}{x} - 1\right) f_{g/P}(\hat{x}; \mu_b) + \mathcal{O}(\alpha_s^2)$$

This coincides with the CSS approach

[Nadolsky, Balazs, Berger, C.-P. Yuan, '07; Catani, Grazzini, '10; P. Sun, B.-W. Xiao, F. Yuan, '11]

PHYSICAL REVIEW D **86,** 094026 (2012)

Improved resummation prediction on Higgs boson production at hadron colliders

Jian Wang,[1] Chong Sheng Li,[1,2,*] Hai Tao Li,[1] Zhao Li,[3,†] and C.-P. Yuan[2,3,‡]

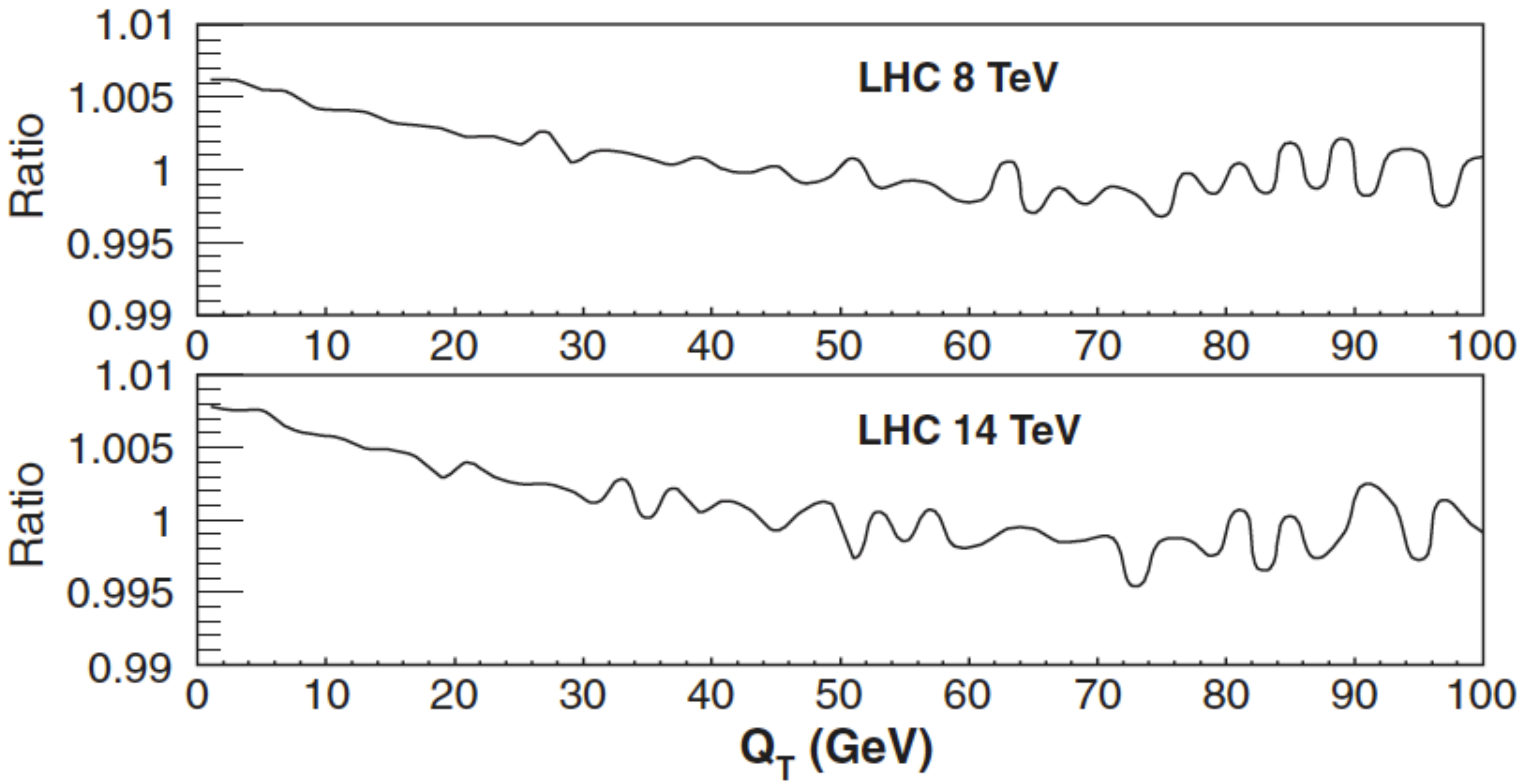


FIG. 3. The ratios between the transverse momentum distributions with and without G functions at the Tevatron (1.96 TeV) and the LHC (7, 8, and 14 TeV). The oscillations of the ratio curves in the figure are due to numerical uncertainties.

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$$\tilde{f}_{g/P}(x, b^2; \mu, \zeta) = \sum_{i=g,q} \int_x^1 \frac{d\hat{x}}{\hat{x}} C_{i/g}(x/\hat{x}, b^2; g(\mu), \mu, \zeta) f_{i/P}(\hat{x}; \mu) + \mathcal{O}((\Lambda_{\rm QCD} b)^a)$$

$$\frac{G^{(1)}G^{(1)}\alpha_s^2 + 2G^{(1)}G^{(2)}\alpha_s^3}{C^{(0)}C^{(0)} + 2C^{(0)}C^{(1)}\alpha_s + (C^{(1)}C^{(1)} + 2C^{(0)}C^{(2)})\alpha_s^2} \approx$$

$$\frac{G^{(1)}G^{(1)}\alpha_s^2}{C^{(0)}C^{(0)}} \left(1 + \frac{2G^{(1)}G^{(2)}}{G^{(1)}G^{(1)}}\alpha_s + \mathcal{O}(\alpha_s^2)\right) \left(1 - \frac{2C^{(0)}C^{(1)}}{C^{(0)}C^{(0)}}\alpha_s + \mathcal{O}(\alpha_s^2)\right)$$

They include third factor, but not second
May explain suppression partly

Wang et al. use also different S_{NP}

Beyond CSS

In the TMD factorized expression there may be nonperturbative contributions from small p_T which mainly affect large b

The perturbative tail holds for small b which is dominated by large p_T, but there is an intermediate region

CSS only allows NP contribution via S_{NP} and does not allow all possibilities of the TMD approach

To illustrate this we consider a model which is approximately Gaussian at low p_T and has the correct tail at high p_T or small b:

$$f_1^g(x, p_T^2) = f_1^g(x) \frac{R^2}{2\pi} \frac{1}{1 + p_T^2 R^2} \qquad R = 2\,\mathrm{GeV}^{-1}$$

$$h_1^{\perp g}(x, p_T^2) = c f_1^g(x) \frac{M^2 R_h^4}{2\pi} \frac{1}{(1 + p_T^2 R_h^2)^2}$$

To satisfy Soffer-like bound: $R_h^2 = 3R^2/2 \qquad c = 2$

Gaussian+tail model

$$\tilde{f}_1^g(x, b^2; \mu_b^2, \mu_b) = f_1^g(x; \mu_b) K_0(b/R) / \ln(R b_0/b + 1)$$

$$\tilde{h}_1^{\perp g}(x, b^2; \mu_b^2, \mu_b) = \frac{c}{4} f_1^g(x; \mu_b) \frac{b}{R_h} K_1(b/R_h) / \ln(R_h b_0/b + 1)$$