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Azimuthal Asymmetries of the Drell-Yan Process in pA Collisions

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1. L. Chen, JHG, and Z. T. Liang   Phys.Rev. C89:035204,2014

2. JHG, Z. T. Liang and X. N. Wang   Phys.Rev. C81:065211,2010

Outline

- Introduction
- Gauge link in TMD distributions & nuclear effects
- Azimuthal asymmetry of DY in pA process
- Summary

QCD and Factorization Theorem

Factorization theorem in QCD：

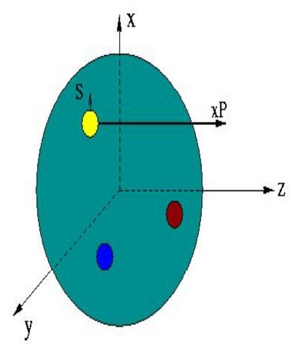
$$\sigma(Q,m) = \frac{1}{Q^2}\left[H(Q/\mu, \alpha_s(\mu)) \otimes S(\mu, m) + O(1/Q^n) \right]$$

Hard part: $H(Q/\mu, \alpha_s(\mu))$ Perturbative

Soft part: $S(\mu, m)$ Non-perturbative but universal

Collinear factorization：

	Unpolarized	Polarized
Parton distribution functions：	$f_q(x)$	$\Delta f_q(x)$
Parton fragmentation functions：	$D_h(z)$	$\Delta D_h(z)$

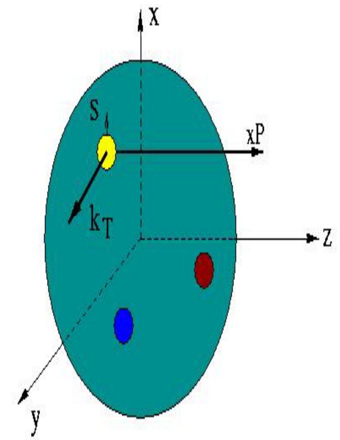


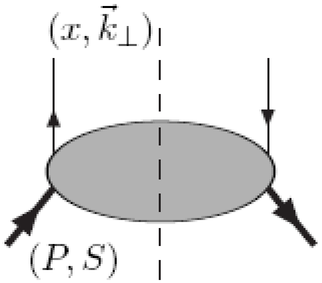
QCD and Factorization Theorem

Transverse Momentum Dependent (TMD) Factorization：

$$f_q(x, k_\perp) \quad \Delta f_q(x, k_\perp) \qquad D_h(z, k_\perp) \quad \Delta D_h(z, \perp)$$

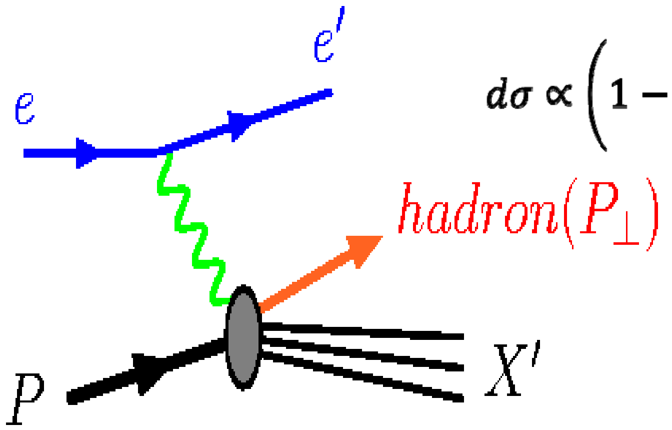
→ Nucleon Spin → Quark Spin





Nucleon Polarization	Quark polarization: Un-Polarized	Quark polarization: Longitudinally Polarized	Quark polarization: Transversely Polarized
U	$f_1 =$		$h_1^{\perp} =$ − Boer-Mulder
L		$g_1 =$ − Helicity	$h_{1L}^{\perp} =$ − Worm Gear
T	$f_{1T}^{\perp} =$ − Sivers	$g_{1T} =$ − Worm Gear	$h_{1T} =$ − Transversity $h_{1T}^{\perp} =$ − Pretzelosity

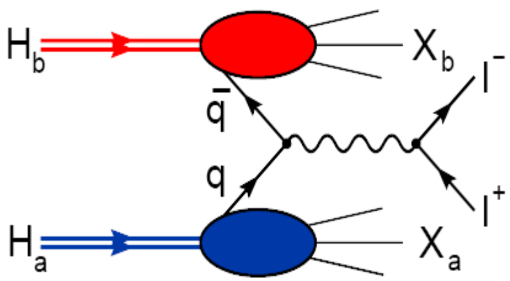
TMD factorization in SIDIS and DY



$$d\sigma \propto \left(1-y+\frac{y^2}{2}\right)x_B\left(F_{UU}+\sin(\phi_h-\phi_S)\left|\vec{S}_T\right|F_{UT}\right)+\lambda\lambda'\left(1-\frac{y}{2}\right)x_B F_{LL}\cdots$$

$$F_{UU}\propto f_q(x_B,k_\perp)\otimes D_h(z_h,p_\perp)\otimes S\,(l_\perp)\otimes H$$

$$\Lambda_{QCD}\ll Q \qquad P_\perp \ll Q$$



$$d\sigma \propto 2F_{UU}+\cos 2\phi\,\tilde{F}_{UU}+\lambda\sin 2\phi\,F_{LU}-2|\vec{S}_T|\sin\phi_S\,F_{TU}\cdots$$

$$F_{UU}\propto f_q(x_1,k_{1\perp})\otimes f_{\bar{q}}(x_2,k_{2\perp})\otimes S\,(l_\perp)\otimes H$$

X.D. Ji,J.P. Ma and F. Yuan, Phys.Rev.D71,034005(2005); Phys.Lett.B597,299-308(2004)

Operator Definition

Quark Correlation Matrix:

$$\Phi_{\alpha\beta}(x,\vec{k}_T,s) \equiv \int \frac{p^+ dy^-}{2\pi}\frac{d^2 y_\perp}{(2\pi)^2} e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp}\langle N,s\mid\bar{\psi}_\beta(0)\mathcal{L}(0,y)\psi_\alpha(y)\mid N,s\rangle$$

Gauge Link:

$$\mathcal{L}(0,y) \equiv P\exp\left(-ig\int_0^{y^-} d\xi^- A^+(\xi^-,\vec{y}_\perp)\right)$$

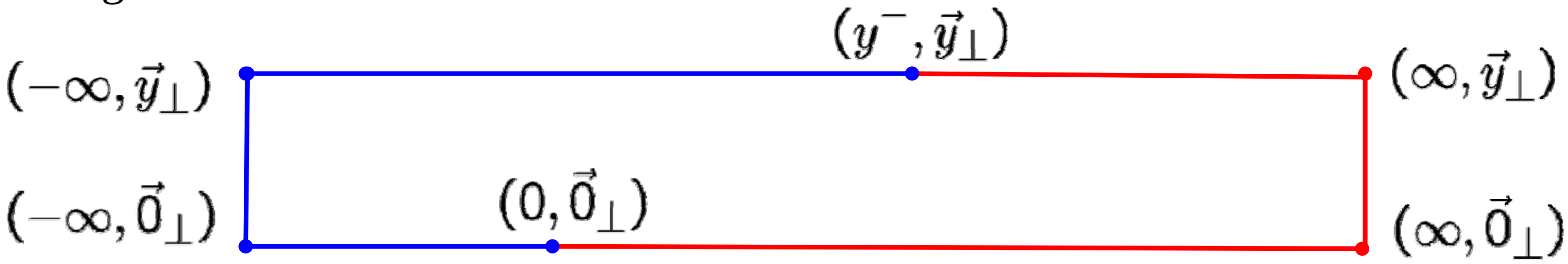
Twist-2 Decomposition in terms of Dirac matrices:

$$\begin{aligned}\Phi(x,\vec{k}_T,s) = \; & f_1\frac{\not{n}_+}{2} - f_{1T}^\perp \frac{\epsilon_T^{\rho\sigma}k_{T\rho}s_{T\sigma}}{M}\frac{\not{n}_+}{2} + g_{1L}\lambda_s\frac{\gamma_5\not{n}_+}{2} \\ & - g_{1T}\frac{k_T\cdot s_T}{M}\frac{\gamma_5\not{n}_+}{2} + h_{1T}\frac{\gamma_5\not{s}_T\not{n}_+}{2} + h_{1L}^\perp\lambda_s\frac{1}{M}\frac{\gamma_5\not{k}_T\not{n}_+}{2} \\ & - h_{1T}^\perp\frac{k_T\cdot s_T}{M}\frac{1}{M}\frac{\gamma_5\not{k}_T\not{n}_+}{2} + h_1^\perp\frac{1}{M}\frac{i\not{k}_T\not{n}_+}{2}\end{aligned}$$

$n_+ = [1,0,\vec{0}_\perp], n_- = [0,1,\vec{0}_\perp]$

Gauge Link in SIDIS and DY

Gauge link in SIDIS and DY:



$$q(x, \vec{k}_\perp) = \int \frac{dy^-}{4\pi} \frac{d^2\vec{y}_\perp}{(2\pi)^2} e^{-ixp^+y^- + i\vec{k}_\perp \cdot \vec{y}_\perp} \times \langle P|\bar{\psi}(y^-, \vec{y}_\perp)\gamma^+ \mathcal{L}(y^-, \vec{y}_\perp; 0, \vec{0}_\perp)\psi(0, \vec{0}_\perp)|P\rangle$$

$$q(x, \vec{k}_\perp) = f(x, k_\perp) + \vec{S} \cdot (\hat{\vec{p}} \times \vec{k}_\perp) f_{1T}^\perp(x, k_\perp)/M$$

$$f(x, k_\perp)\Big|_{DIS} = f(x, k_\perp)\Big|_{DY} \qquad f_{1T}^\perp(x, k_\perp)\Big|_{DIS} = -\, f_{1T}^\perp(x, k_\perp)\Big|_{DY}$$

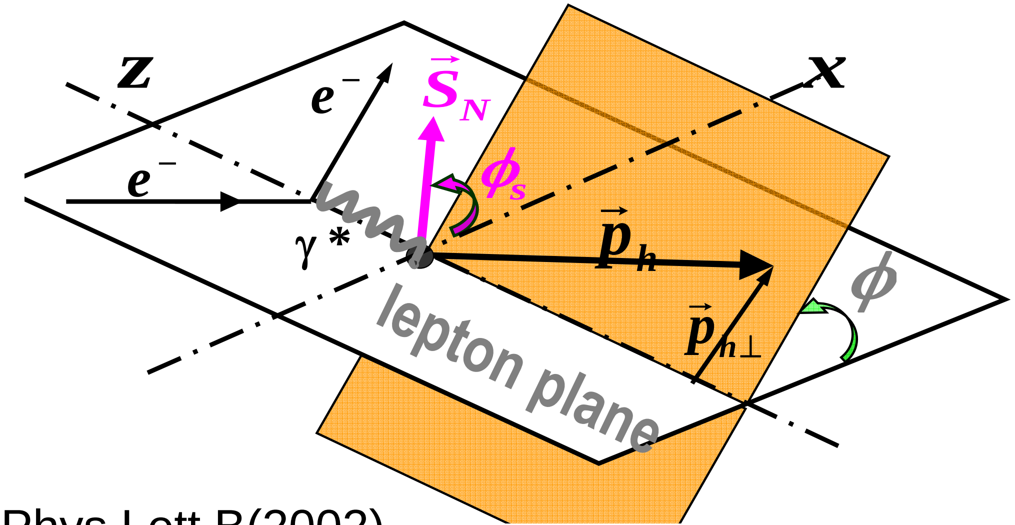
X. Ji and F. Yuan Phys.Lett.B543,66 (2002); A. Belitsky, X. Ji and F. Yuan Nucl. Phys. B 656,165 (2003)

Gauge link and SSA

$$q(x,\vec{k}_\perp) = \int \frac{dy^-}{4\pi}\frac{d^2\vec{y}_\perp}{(2\pi)^2} e^{-ixp^+y^- + i\vec{k}_\perp\cdot\vec{y}_\perp} \times \langle P|\bar{\psi}(y^-,\vec{y}_\perp)\gamma^+ \mathcal{L}(y^-,\vec{y}_\perp;0,\vec{0}_\perp)\psi(0,\vec{0}_\perp)|P\rangle$$

Sivers function

$$f_{1T}^{\perp}(x,k_\perp) \neq 0$$

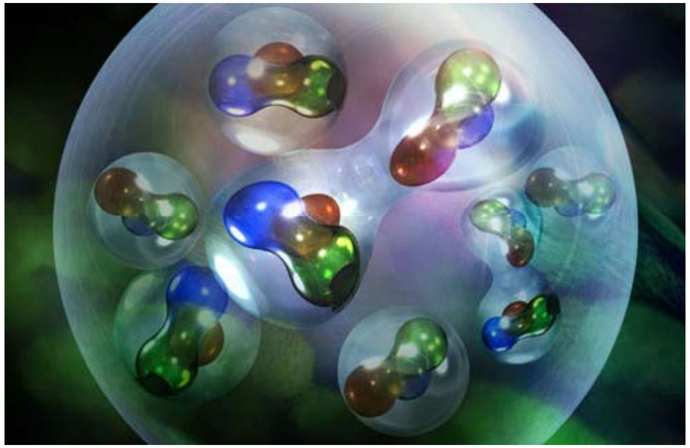


J.C. Collins, Nucl.Phys.B(1993);Phys.Lett.B(2002)

Nuclear effects from gauge link?

Nuclear effects from Gauge Link

- Assume the nucleus:



Very large,

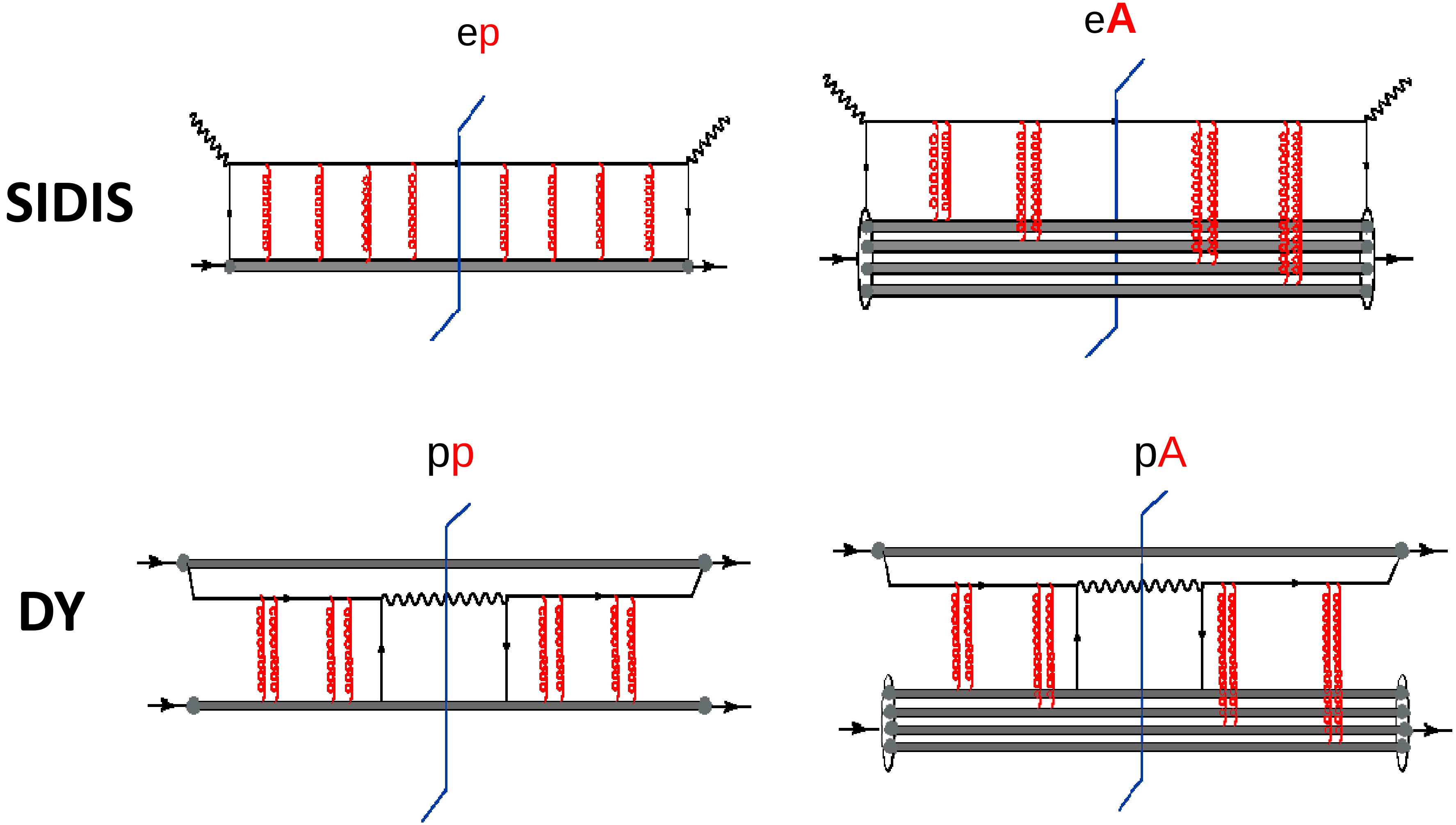
Uniform,

Weakly bound,

No Multiple-nucleon correlation

Nuclear effects will be mainly from gauge link.

Nuclear effects in SIDIS and DY



Maximal two-gluon correlation approximation

Nuclear dependence of TMD PDF

Integral form:

$$\Phi_\alpha^A(x,k_\perp) = A\exp\left[\int d\xi_N^- \hat{q}_F(\xi_N^-)\frac{\nabla_{k_\perp}^2}{4}\right]\Phi_\alpha^N(x,k_\perp)$$

Differential form:

$$\frac{\partial\Phi_\alpha(\xi_N^-,x,k_\perp)}{\partial\xi_N^-} = \frac{1}{4}\hat{q}_F(\xi_N^-)\nabla_{k_\perp}^2\Phi_\alpha(\xi_N^-;x,k_\perp)$$

$$\Phi_\alpha(0;x,k_\perp) = \Phi_\alpha^N(x,k_\perp),\quad \Phi_\alpha(\infty;x,k_\perp) = \Phi_\alpha^A(x,k_\perp)$$

Gluon distribution function in a nucleon

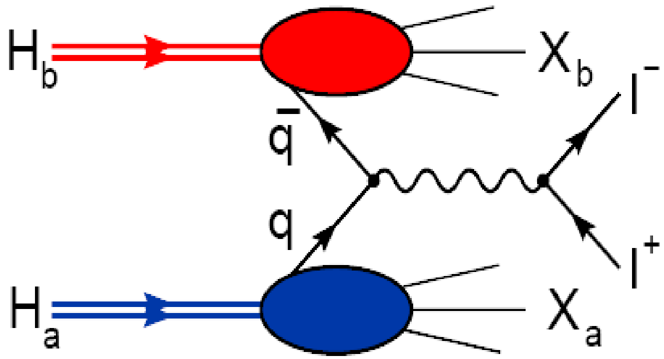
Quark transport coefficient

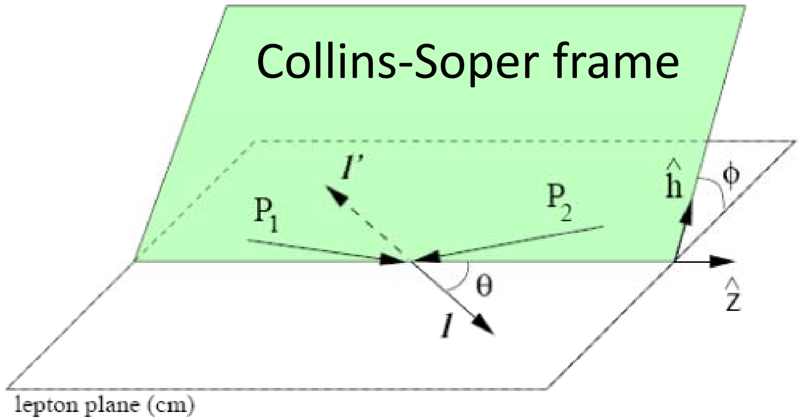
$$\hat{q}_F(\xi_N^-) = \frac{2\pi^2\alpha_s}{N_c}\rho_N^A(\xi_N^-)[xf_g^N(x)]_{x=0}$$

Spatial nucleon number density in the nucleus

Z.T. Liang, X.N. Wang and J. Zhou, PRD(2008); JHG, Z. T. Liang and X. N. Wang PRC(2010)

Azimuthal Asymmetry of DY in pp or pA





$$\frac{d\sigma}{d\phi d^2 q_T dx_1 dx_2} = \frac{\alpha_{em}^2}{3Q^2}\{2\mathcal{F}_0[f_1, f_1] + \cos 2\phi \mathcal{F}_1[h_1^\perp, h_1^\perp]$$

$$+ \lambda_s \sin 2\phi \mathcal{F}_1[h_{1L}^\perp, h_1^\perp] - 2|\vec{s}_T| \sin\phi_s \mathcal{F}_2[f_{1T}^\perp, f_1]$$

$$+|\vec{s}_T|\sin(2\phi - \phi_s)\mathcal{F}_3[h_1, h_1^\perp] + |\vec{s}_T|\sin(2\phi+\phi_s)\mathcal{F}_4[h_{1T}^\perp, h_1^\perp]\}$$

$$h_1(x, \vec{k}_T) \equiv h_{1T}(x, \vec{k}_T) + \frac{\vec{k}_T^2}{2M^2} h_{1T}^\perp(x, \vec{k}_T).$$

$$\mathcal{F}_j[f, h] \equiv \frac{1}{3}\sum_a e_a^2 \int \frac{d^2k_{1T}}{(2\pi)^2}\frac{d^2k_{2T}}{(2\pi)^2}\delta^2(\vec{k}_{1T} + \vec{k}_{2T} - \vec{q}_T)\chi_j(\vec{q}_T, \vec{k}_{1T}, \vec{k}_{2T})$$

$$\times \left[f^N(x_1, \vec{k}_{1T}; a) h^A(x_2, \vec{k}_{2T}; \bar{a}) + f^N(x_1, \vec{k}_{1T}; \bar{a}) h^A(x_2, \vec{k}_{2T}; a)\right]$$

Definitions of Azimuthal asymmetry

$$A_{NA}^{\cos 2\phi} = \frac{\mathcal{F}_1\left[h_1^\perp, h_1^\perp\right]}{4\mathcal{F}_0\left[f_1, f_1\right]},$$ Boer-Mulders

$$A_{NA}^{\sin 2\phi} = \lambda_s \frac{\mathcal{F}_1\left[h_{1L}^\perp, h_1^\perp\right]}{4\mathcal{F}_0\left[f_1, f_1\right]},$$ Long-Transversity

$$A_{NA}^{\sin \phi_s} = -|\vec{s}_T| \frac{\mathcal{F}_2\left[f_{1T}^\perp, f_1\right]}{2\mathcal{F}_0\left[f_1, f_1\right]},$$ Sivers

$$A_{NA}^{\sin(2\phi - \phi_s)} = |\vec{s}_T| \frac{\mathcal{F}_3\left[h_1, h_1^\perp\right]}{4\mathcal{F}_0\left[f_1, f_1\right]},$$ Transversity

$$A_{NA}^{\sin(2\phi + \phi_s)} = |\vec{s}_T| \frac{\mathcal{F}_4\left[h_{1T}^\perp, h_1^\perp\right]}{4\mathcal{F}_0\left[f_1, f_1\right]}.$$ Pretzelosity

$$R^{\cos 2\phi} \equiv \frac{A_{NA}^{\cos 2\phi}}{A_{NN}^{\cos 2\phi}} \qquad R^{\sin 2\phi} \equiv \frac{A_{NA}^{\sin 2\phi}}{A_{NN}^{\sin 2\phi}} \qquad R^{\sin \phi_s} \equiv \frac{A_{NA}^{\sin \phi_s}}{A_{NN}^{\sin \phi_s}} \qquad \ldots$$

Azimuthal Asymmetry in unpolarized DY

$$\frac{d\sigma}{d\phi d^2q_T dx_1 dx_2} = \frac{\alpha_{em}^2}{3Q^2}\left\{2\mathcal{F}_0\left[f_1, f_1\right] + \cos 2\phi \mathcal{F}_1\left[h_1^\perp, h_1^\perp\right]\right\}$$

Gaussian asumption

$$f_1^N(x, \vec{k}_T) = \frac{1}{\pi\alpha} f_1^N(x) e^{-\frac{\vec{k}_T^2}{\alpha}},$$

$$h_1^{N\perp}(x, \vec{k}_T) = \frac{1}{\pi\beta} h_1^{N\perp}(x) e^{-\frac{\vec{k}_T^2}{\beta}}$$

Different flavors have the same Gaussian widths for the same TMD distributions

$$\Phi_\alpha^A(x, k_\perp) = A \exp\left[\int d\xi_N^- \hat{q}_F(\xi_N^-) \frac{\nabla_{k_\perp}^2}{4}\right] \Phi_\alpha^N(x, k_\perp)$$

$$f_1^A(x, \vec{k}_T) = \frac{A}{\pi(\alpha + \Delta_{2F})} f_1^N(x) e^{-\frac{\vec{k}_T^2}{\alpha+\Delta_{2F}}},$$

$$h_1^{A\perp}(x, \vec{k}_T) = \frac{A\beta}{\pi(\beta + \Delta_{2F})^2} h_1^{N\perp}(x) e^{-\frac{\vec{k}_T^2}{\beta+\Delta_{2F}}}$$

$\Delta_{2F} = \int d\xi_N^- \hat{q}_F\left(\xi_N^-\right)$: transverse momentum broadening squared

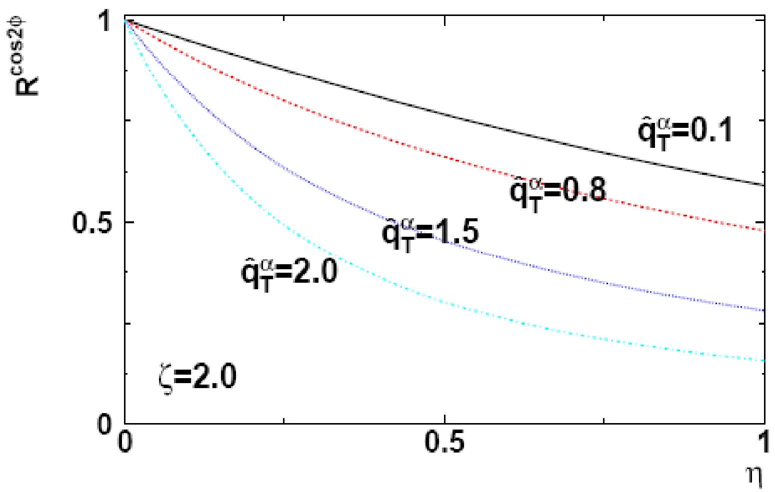
Nuclear effect in unpolarized DY

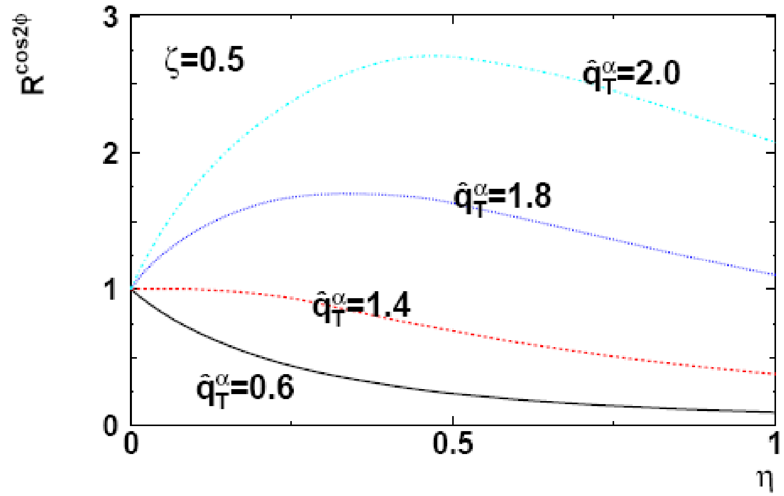
$$R^{\cos 2\phi} \equiv \frac{A_{NA}^{\cos 2\phi}}{A_{NN}^{\cos 2\phi}} = \frac{2\alpha + \Delta_{2F}}{2\alpha} \left(\frac{2\beta}{2\beta + \Delta_{2F}} \right)^3 e^{\frac{(\alpha-\beta)(2\alpha+2\beta+\Delta_{2F})\Delta_{2F}}{2\alpha\beta(2\alpha+\Delta_{2F})(2\beta+\Delta_{2F})} \vec{q}_T^{\,2}}$$

Special case: $\alpha = \beta$

$$R^{\cos 2\phi} = \left(\frac{2\alpha}{2\alpha + \Delta_{2F}} \right)^2$$

$$\eta \equiv \Delta_{2F}/2\alpha, \quad \hat{q}_T^\alpha \equiv |\vec{q}_T|/\sqrt{2\alpha}, \quad \zeta \equiv \beta/\alpha,$$





Au: $\Delta_{2F} \sim 0.12\text{GeV}^2, \eta \sim 0.24$ if: $\alpha = 0.25\text{GeV}^2, \hat{q} \approx 0.0025\text{GeV}^2/\text{fm}$

Nuclear effect in Single Polarized DY

$$h_{1L}^{N\perp}(x,\vec{k}_T) = \frac{1}{\pi\sigma_1} h_{1L}^{N\perp}(x) e^{\frac{-\vec{k}_T^2}{\sigma_1}}, \quad f_{1T}^{N\perp}(x,\vec{k}_T) = \frac{1}{\pi\sigma_2} f_{1T}^{N\perp}(x) e^{\frac{-\vec{k}_T^2}{\sigma_2}},$$

$$h_1^N(x,\vec{k}_T) = \frac{1}{\pi\sigma_3} h_1^N(x) e^{\frac{-\vec{k}_T^2}{\sigma_3}}, \quad h_{1T}^{N\perp}(x,\vec{k}_T) = \frac{1}{\pi\sigma_4} h_{1T}^{N\perp}(x) e^{\frac{-\vec{k}_T^2}{\sigma_4}}.$$

Special case: $\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4 = \alpha = \beta$

$$R^{\cos 2\phi} = \left(\frac{2\alpha}{2\alpha + \Delta_{2F}}\right)^2$$

$$R^{\sin 2\phi} = \left(\frac{2\alpha}{2\alpha + \Delta_{2F}}\right)^2, \quad R^{\sin \phi_s} = \frac{2\alpha}{2\alpha + \Delta_{2F}},$$

$$R^{\sin(2\phi - \phi_s)} = \frac{2\alpha}{2\alpha + \Delta_{2F}}, \quad R^{\sin(2\phi + \phi_s)} = \left(\frac{2\alpha}{2\alpha + \Delta_{2F}}\right)^3.$$

Summary

- The relation between the TMD distribution functions of the nucleon and nucleus can be obtained under some simple assumption.
- The azimuthal asymmetry is suppressed by multiple parton scattering for most cases of TMD distribution while in some kinematic regions the azimuthal asymmetry can be even enhanced.
- Nuclear effect of azimuthal asymmetry in DY is a very sensitive probe of twist-2 TMD parton distributions.

Thanks for your attention！