Modeling the pion Generalized Parton Distribution

C. Mezrag

CEA Saclay - IRFU/SPhN

October 21st, 2014

In collaboration with:
L. Chang, H. Moutarde, C. Roberts,
J. Rodriguez-Quintero, F. Sabatié, P. Tandy

Formal Definition

- Proton case:

$$
\begin{aligned}
& \frac{1}{2}\int \frac{e^{ixP^+z^-}}{2\pi}\langle P+\frac{\Delta}{2}|\bar{q}(-\frac{z}{2})\gamma^+ q(\frac{z}{2})|P-\frac{\Delta}{2}\rangle \mathrm{d}z^-|_{z^+=0,\mathbf{z}=0} \\
= \quad & \frac{1}{2P^+}[H^q(x,\xi,t)\bar{u}(P+\frac{\Delta}{2})\gamma^+ u(P-\frac{\Delta}{2}) \\
& +E^q(x,\xi,t)\bar{u}(P+\frac{\Delta}{2})\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u(P-\frac{\Delta}{2})].
\end{aligned}
$$

X. Ji,1997
D. Müller et al.,1994
A. Radyushkin,1997

Formal Definition

- Proton case:

$$\frac{1}{2}\int \frac{e^{ixP^+z^-}}{2\pi}\langle P+\frac{\Delta}{2}|\bar{q}(-\frac{z}{2})\gamma^+q(\frac{z}{2})|P-\frac{\Delta}{2}\rangle \mathrm{d}z^-|_{z^+=0,\mathbf{z}=0}$$

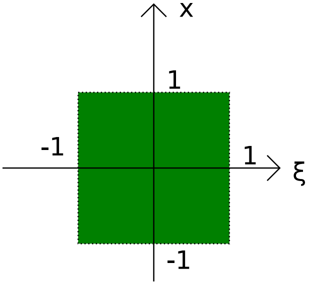
$$= \frac{1}{2P^+}[H^q(x,\xi,t)\bar{u}(P+\frac{\Delta}{2})\gamma^+u(P-\frac{\Delta}{2})$$

$$+E^q(x,\xi,t)\bar{u}(P+\frac{\Delta}{2})\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u(P-\frac{\Delta}{2})].$$

- Pion case:

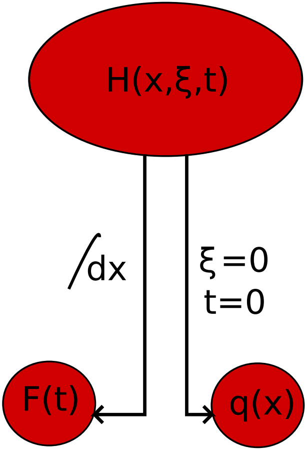
$$H(x,\xi,t)=\frac{1}{2}\int\frac{\mathrm{d}z^-}{2\pi}\,e^{ixP^+z^-}\langle P+\frac{\Delta}{2}|\bar{q}\left(-\frac{z}{2}\right)\gamma^+q\left(\frac{z}{2}\right)|P-\frac{\Delta}{2}\rangle_{z^+=0,z_\perp=0}$$

GPD properties



- **Support properties**:
  $|x| \leq 1$ and $|\xi| \leq 1$
  Valence case: $-\xi \leq x \leq 1$

GPD properties



- **Support properties**:
  $|x| \leq 1$ and $|\xi| \leq 1$
  Valence case: $-\xi \leq x \leq 1$
- **Special limits**:
  - if $t = 0$ and $\xi = 0 \rightarrow$ PDF,
  - if $\int \mathrm{d}x \rightarrow$ form factor.

GPD properties

$$\mathcal{M}_m(\xi, t) = \int \mathrm{d}x \; x^m H(x, \xi, t) = \sum_{i=0}^{m+1} c_i(t)\xi^i$$

- **Support properties**:
  $|x| \leq 1$ and $|\xi| \leq 1$
  Valence case: $-\xi \leq x \leq 1$
- **Special limits**:
  - if $t = 0$ and $\xi = 0 \rightarrow$ PDF,
  - if $\int \mathrm{d}x \rightarrow$ form factor.
- **Polynomiality**:
  - The Mellin Moments are polynomials in ξ
  - This comes from Lorentz invariance.

GPD properties

$$\mathcal{M}_m(\xi, t) = \sum_{i=0}^{\frac{m}{2}} c_{2i}(t)\xi^{2i} + mod(m,2)c_{m+1}\xi^{m+1}$$

- **Support properties**:
  $|x| \leq 1$ and $|\xi| \leq 1$
  Valence case: $-\xi \leq x \leq 1$
- **Special limits**:
  - if $t = 0$ and $\xi = 0 \rightarrow$ PDF,
  - if $\int \mathrm{d}x \rightarrow$ form factor.
- **Polynomiality**:
  - The Mellin Moments are polynomials in ξ
  - This comes from Lorentz invariance.
- **Time reversal invariance**:
  - $H(x, \xi, t) = H(x, -\xi, t)$

GPD properties

- **Support properties**:
  $|x| \leq 1$ and $|\xi| \leq 1$
  Valence case: $-\xi \leq x \leq 1$
- **Special limits**:
  - if $t = 0$ and $\xi = 0 \to$ PDF,
  - if $\int \mathrm{d}x \to$ form factor.
- **Polynomiality**:
  - The Mellin Moments are polynomials in ξ
  - This comes from Lorentz invariance.
- **Time reversal invariance**:
  - $H(x, \xi, t) = H(x, -\xi, t)$

Those properties make GPD modeling a challenge.

Models of GPDs

- Double Distribution models:

Models of GPDs

- Double Distribution models:
  - Vanderhaeghen-Guichon-Guidal model (VGG) (1999),
  - Goloskokov-Kroll model (GK) (2005).

Models of GPDs

- Double Distribution models:
  - Vanderhaeghen-Guichon-Guidal model (VGG) (1999),
  - Goloskokov-Kroll model (GK) (2005).
- Dual model:
  - M.V. Polyakov (1999), M.V. Polyakov and A.G. Shuvaev (2002), M.V. Polyakov and K.M. Semenov-Tian-Shansky (2009).

Models of GPDs

- Double Distribution models:
  - Vanderhaeghen-Guichon-Guidal model (VGG) (1999),
  - Goloskokov-Kroll model (GK) (2005).
- Dual model:
  - M.V. Polyakov (1999), M.V. Polyakov and A.G. Shuvaev (2002), M.V. Polyakov and K.M. Semenov-Tian-Shansky (2009).
- Mellin-Barnes approach:
  - D. Müller and A. Schäfer (2006), K. Kumericki and D. Müller (2010).

Alternative ideas

- Lattice QCD:
  - Computations of Mellin Moments.
  - Until now, only the very first Mellin moments have been computed.
  - Still, new proposals done by X. Ji (*X. Ji*, 2013).

Alternative ideas

- Lattice QCD:
  - Computations of Mellin Moments.
  - Until now, only the very first Mellin moments have been computed.
  - Still, new proposals done by X. Ji (*X. Ji*, 2013).
- Dyson-Schwinger Equations:
  - Very powerful non pertubative method,
  - Approximations scheme have been developed for QCD.

Alternative ideas

- Lattice QCD:
  - Computations of Mellin Moments.
  - Until now, only the very first Mellin moments have been computed.
  - Still, new proposals done by X. Ji (*X. Ji*, 2013).
- Dyson-Schwinger Equations:
  - Very powerful non pertubative method,
  - Approximations scheme have been developed for QCD.

Dyson-Schwinger equations seems to be a very promising approach to model GPDs!

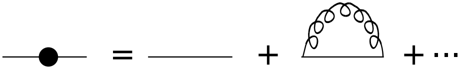
Dyson-Schwinger Equations

- Equations between non pertubative Green functions.
- Infinite number of coupled equations $\rightarrow$ no one has solved it until now!
- This requires approximations. In QCD, there are mainly two:
  - Rainbow Ladder (RL), resumming over a certain class of diagrams,
  - Dynamical Chiral Symmetry Breaking (DCSB).

  See for instance *L.Chang et al.*,PRC87,2013 for details about truncation schemes.

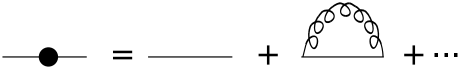
Example: the quark propagator

Pertubative case:



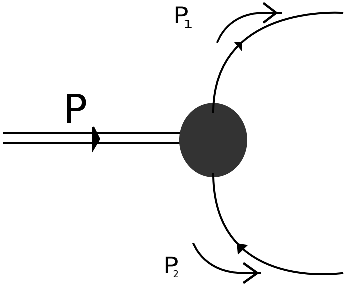
Example: the quark propagator

Pertubative case:



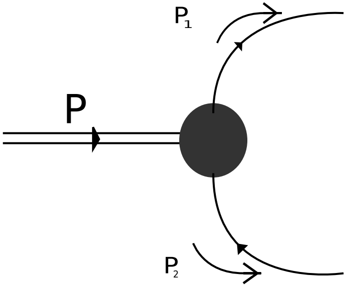
Dyson-Schwinger case:

The pion? But why?



- Advantages :
  - Two-body system.
  - Pseudo-scalar meson.
  - Valence quarks u and d.
  - Isospin symmetry.
- Drawbacks
  - Very few experimental data available.
  - No data at $\xi \neq 0$
    $\rightarrow$ The model can be compared only at $\xi = 0$, *i.e.* to the Parton Distribution Function (PDF) and to the form factor.
  - *Amrath et al., Eur. Phys. J. C58 179*

The pion? But why?



- Advantages :
  - Two-body system.
  - Pseudo-scalar meson.
  - Valence quarks u and d.
  - Isospin symmetry.
- Drawbacks
  - Very few experimental data available.
  - No data at $\xi \neq 0$
    $\rightarrow$ The model can be compared only at $\xi = 0$, *i.e.* to the Parton Distribution Function (PDF) and to the form factor.
  - *Amrath et al., Eur. Phys. J. C58 179*

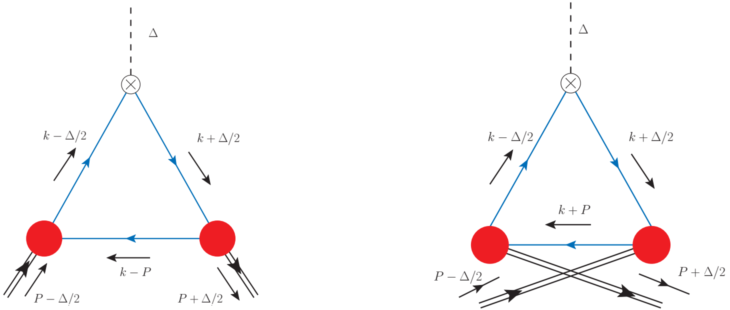
Good starting point before dealing with more complicated objects.

Pion GPD model

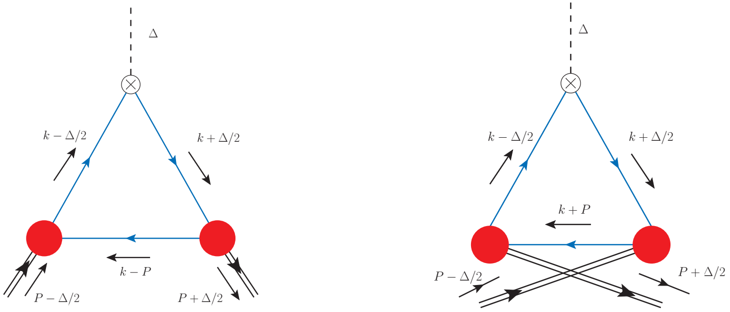
$$\begin{aligned}\mathcal{M}_m(\xi,t) &= \int_{-1}^{1} \mathrm{d}x\ x^m\ H(x,\xi,t) \\ &= \frac{1}{2(P\cdot n)^{m+1}}\left\langle \pi, P+\frac{\Delta}{2}\left|\bar{\psi}(0)\gamma\cdot n(i\overleftrightarrow{D}\cdot n)^m\psi(0)\right|\pi, P-\frac{\Delta}{2}\right\rangle .\end{aligned}$$

Pion GPD model

$$\mathcal{M}_m(\xi, t) = \int_{-1}^{1} \mathrm{d}x \; x^m \; H(x, \xi, t)$$
$$= \frac{1}{2(P\cdot n)^{m+1}} \left\langle \pi, P + \frac{\Delta}{2} \left| \bar{\psi}(0)\gamma\cdot n(i\overleftrightarrow{D}\cdot n)^m \psi(0) \right| \pi, P - \frac{\Delta}{2} \right\rangle .$$



Pion GPD model



$$2(P\cdot n)^{m+1}\mathcal{M}_m(\xi,t) = \mathrm{tr}_{CFD}\int \frac{\mathrm{d}^4k}{(2\pi)^4}(k\cdot n)^m\ i\Gamma_\pi\left(k-\frac{\Delta}{2},P-\frac{\Delta}{2}\right)\ S\left(k-\frac{\Delta}{2}\right)$$
$$i\gamma\cdot n\ S\left(k+\frac{\Delta}{2}\right)\ i\bar{\Gamma}_\pi\left(k+\frac{\Delta}{2},P+\frac{\Delta}{2}\right)\ S(k-P)$$

Analytic Model

Propagator :

$$S(p^2) = \frac{-ip\cdot\gamma + M}{p^2 + M^2}$$

- p is the quark momentum,
- M is the effective mass of the constituent quark.

Vertex :

$$\Gamma_\pi \propto i\gamma_5 \int \frac{\mathrm{d}z \; M^2 \rho_\nu(z)}{(q(k,\Delta,P)^2 + M^2)^\nu}$$

- $\rho_\nu(z) \propto (1-z^2)^\nu$ is the z distribution.
- $q(k,\Delta,P) = k - \frac{1-z}{2}\left(P \pm \frac{\Delta}{2}\right)$ deals with the momentum fraction carried by the quark.

Analytic Model

Propagator :

$$S(p^2) = \frac{-ip \cdot \gamma + M}{p^2 + M^2}$$

- p is the quark momentum,
- M is the effective mass of the constituent quark.

Vertex :

$$\Gamma_\pi \propto i\gamma_5 \int \frac{\mathrm{d}z \ M^2 \rho_\nu(z)}{(q(k,\Delta,P)^2 + M^2)^\nu}$$

- $\rho_\nu(z) \propto (1-z^2)^\nu$ is the z distribution.
- $q(k,\Delta,P) = k - \frac{1-z}{2}\left(P \pm \frac{\Delta}{2}\right)$ deals with the momentum fraction carried by the quark.

Those functions are inspired by realistic Bethe-Salpeter computations.

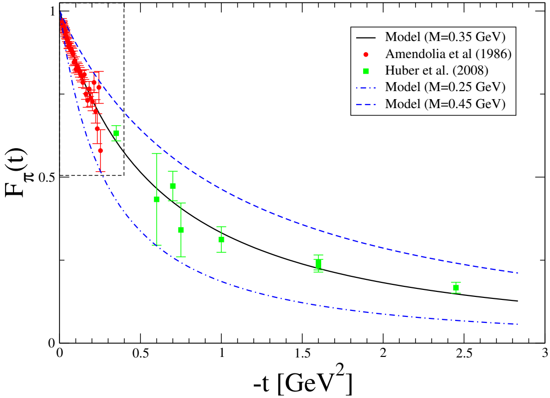
Chang et al.,2013

Form factor

$$\mathcal{F}_\pi^q(t) = \mathcal{M}_0(t) = \int_{-1}^{1} \mathrm{d}x \; H^q(x, \xi, t)$$

Form factor

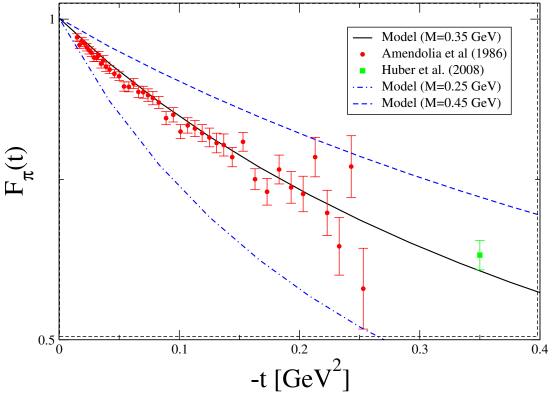
$$\mathcal{F}_\pi^q(t) = \mathcal{M}_0(t) = \int_{-1}^{1} \mathrm{d}x \; H^q(x, \xi, t)$$



C. Mezrag et al., arXiv 1406.7425

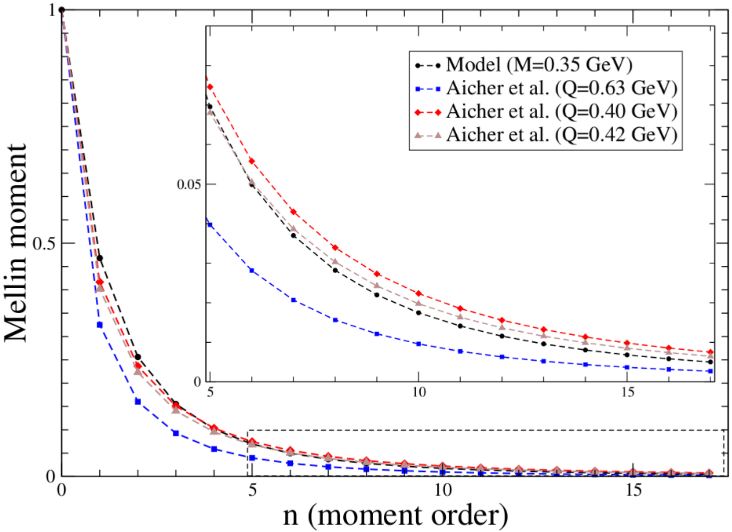
Form factor

$$\mathcal{F}_\pi^q(t) = \mathcal{M}_0(t) = \int_{-1}^{1} \mathrm{d}x \; H^q(x,\xi,t)$$



C. Mezrag et al., arXiv 1406.7425

PDF's Mellin moments

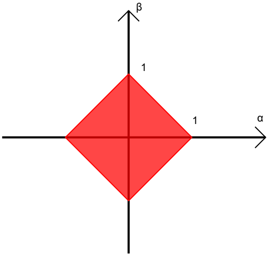


C.Mezrag et al, arXiv 1406.7425

Double Distributions

Double Distributions are formally the Radon transform of the GPDs.

$$H(x,\xi) = \int_\Omega \mathrm{d}\alpha \mathrm{d}\beta (F(\beta,\alpha) + \xi G(\beta,\alpha))\delta(x - \beta - \xi\alpha)$$



$$\Omega = \{(\alpha,\beta) | |\alpha| + |\beta| \leq 1\}$$

Advantage:
Easy way to respect the polynomiality in ξ

$$\int_{-1}^{1} x^n H(x,\xi)\mathrm{d}x$$
$$= \int_\Omega (\beta + \xi\alpha)^n (F(\beta,\alpha) + \xi G(\beta,\alpha))\mathrm{d}\Omega$$

Properties of Mellin moments

Polynomiality:

$$
\begin{aligned}
&\mathcal{M}_m(\xi, t) \\
= \quad & \frac{1}{2(P\cdot n)^{m+1}} \left\langle \pi, P + \frac{\Delta}{2} \middle| \bar{\psi}(0)\gamma\cdot n(i\overleftrightarrow{D}\cdot n)^m\psi(0) \middle| \pi, P - \frac{\Delta}{2} \right\rangle \\
= \quad & \frac{n_\mu n_{\mu_1}\ldots n_{\mu_m}}{(P\cdot n)^{m+1}} P^{\{\mu} \sum_{j=0}^{m} \binom{m}{j} F_{m,j}(t) P^{\mu_1}\ldots P^{\mu_j} \left(-\frac{\Delta}{2}\right)^{\mu_{j+1}} \ldots \left(-\frac{\Delta}{2}\right)^{\mu_m\}} \\
& - n_\mu n_{\mu_1}\ldots n_{\mu_m} \frac{\Delta}{2}^{\{\mu} \sum_{j=0}^{m} \binom{m}{j} G_{m,j}(t) P^{\mu_1}\ldots P^{\mu_j} \left(-\frac{\Delta}{2}\right)^{\mu_{j+1}} \ldots \left(-\frac{\Delta}{2}\right)^{\mu_m\}}
\end{aligned}
$$

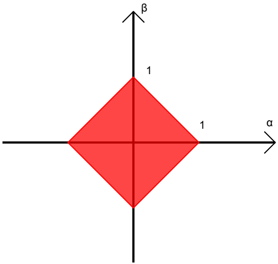
$\xi = -\frac{\Delta\cdot n}{2P\cdot n} \Rightarrow \mathcal{M}_m(\xi, t)$ is a polynomial in ξ of order $m+1$.

Properties of Mellin moments

Double distributions:

$$F_{m,j}(t) = \int_{-1}^{1} \mathrm{d}\beta \int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \; \beta^{m-j}\alpha^{j} \; F(\beta,\alpha,t)$$

$$G_{m,j}(t) = \int_{-1}^{1} \mathrm{d}\beta \int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \; \beta^{m-j}\alpha^{j} \; G(\beta,\alpha,t)$$



We want to rewrite our results for the Mellin moment as:

$$\mathcal{M}_m(\xi,t) = n_\mu n_{\mu_1}...n_{\mu_m} \sum_{j=0}^{m} \binom{m}{j} \int_{-1}^{1} \mathrm{d}\beta \int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \; \beta^{m-j}\alpha^j$$
$$F(\beta,\alpha,t) P^{\{\mu} P^{\mu_1}...P^{\mu_j} \left(-\frac{\Delta}{2}\right)^{\mu_{j+1}} ... \left(-\frac{\Delta}{2}\right)^{\mu_m\}}$$
$$-G(\beta,\alpha,t) \frac{\Delta}{2}^{\{\mu} P^{\mu_1}...P^{\mu_j} \left(-\frac{\Delta}{2}\right)^{\mu_{j+1}} ... \left(-\frac{\Delta}{2}\right)^{\mu_m\}}$$

We want to rewrite our results for the Mellin moment as:

$$
\begin{aligned}
\mathcal{M}_m(\xi,t) = \; & n_\mu n_{\mu_1} \dots n_{\mu_m} \sum_{j=0}^{m} \binom{m}{j} \int_{-1}^{1} \mathrm{d}\beta \int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \; \beta^{m-j} \alpha^j \\
& F(\beta,\alpha,t) P^{\{\mu} P^{\mu_1} \dots P^{\mu_j} \left(-\frac{\Delta}{2}\right)^{\mu_{j+1}} \dots \left(-\frac{\Delta}{2}\right)^{\mu_m\}} \\
& - G(\beta,\alpha,t) \frac{\Delta}{2}^{\{\mu} P^{\mu_1} \dots P^{\mu_j} \left(-\frac{\Delta}{2}\right)^{\mu_{j+1}} \dots \left(-\frac{\Delta}{2}\right)^{\mu_m\}}
\end{aligned}
$$

We can identify F and G in our computations!

Analytic Results

$$
\begin{aligned}
F^u(\beta,\alpha,t) = \frac{48}{5}\Bigg\{ & -\frac{18M^4 t(\beta-1)(\alpha-\beta+1)(\alpha+\beta-1)\left(\left(\alpha^2-(\beta-1)^2\right)\tanh^{-1}\left(\frac{2\beta}{-\alpha^2+\beta^2+1}\right)+2\beta\right)}{\left(4M^2+t\left((\beta-1)^2-\alpha^2\right)\right)^3} \\
& + \frac{9M^4(\alpha-\beta+1)\left(-4\beta\left(-\alpha^2+\beta^2+1\right)+2\tanh^{-1}\left(\frac{2\beta}{-\alpha^2+\beta^2+1}\right)\right)}{4(\alpha-\beta-1)\left(4M^2+t\left((\beta-1)^2-\alpha^2\right)\right)^2} \\
& + \frac{9M^4(\alpha-\beta+1)\left(\left(\alpha^4-2\alpha^2\left(\beta^2+1\right)+\beta^2\left(\beta^2-2\right)\right)\log\left(\frac{(\alpha-\beta-1)(\alpha+\beta+1)}{\alpha^2-(\beta-1)^2}\right)\right)}{4(\alpha-\beta-1)\left(4M^2+t\left((\beta-1)^2-\alpha^2\right)\right)^2} \\
& + \frac{9M^4(\alpha+\beta-1)\left(-4\beta\left(-\alpha^2+\beta^2+1\right)+2\tanh^{-1}\left(\frac{2\beta}{-\alpha^2+\beta^2+1}\right)\right)}{4(\alpha+\beta+1)\left(4M^2+t\left((\beta-1)^2-\alpha^2\right)\right)^2} \\
& + \frac{9M^4(\alpha+\beta-1)\left(\left(\alpha^4-2\alpha^2\left(\beta^2+1\right)+\beta^4-2\beta^2\right)\log\left(\frac{(\alpha-\beta-1)(\alpha+\beta+1)}{\alpha^2-(\beta-1)^2}\right)\right)}{4(\alpha+\beta+1)\left(4M^2+t\left((\beta-1)^2-\alpha^2\right)\right)^2} \\
& + \frac{9M^4\beta(\alpha-\beta+1)^2(\alpha+\beta-1)^2\left(\frac{2\left(\alpha^2\beta-\beta^3+\beta\right)}{\alpha^4-2\alpha^2(\beta^2+1)+(\beta^2-1)^2}\right)}{\left(4M^2+t\left((\beta-1)^2-\alpha^2\right)\right)^2} \\
& + \frac{9M^4\beta(\alpha-\beta+1)^2(\alpha+\beta-1)^2\left(-\tanh^{-1}(\alpha-\beta)+\tanh^{-1}(\alpha+\beta)\right)}{\left(4M^2+t\left((\beta-1)^2-\alpha^2\right)\right)^2}\Bigg\},
\end{aligned}
$$

Analytic Results

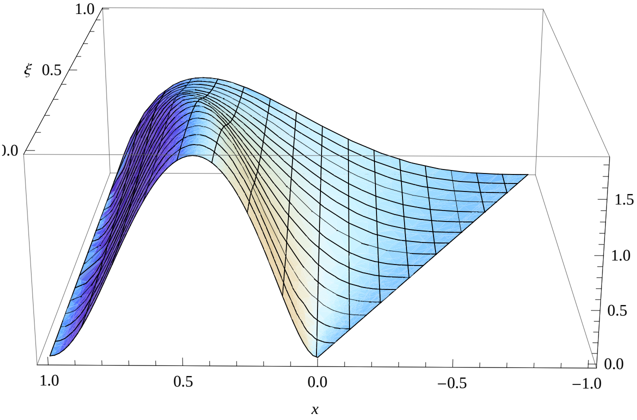
$$
\begin{aligned}
H^{u}_{x \geq \xi}(x,\xi,0) \quad = \quad & \frac{48}{5} \Bigg\{ \frac{3\left(-2(x-1)^4\left(2x^2-5\xi^2+3\right)\log(1-x)\right)}{20\left(\xi^2-1\right)^3} \\
& \frac{3\left(+4\xi\left(15x^2(x+3)+(19x+29)\xi^4+5(x(x(x+11)+21)+3)\xi^2\right)\tanh^{-1}\left(\frac{(x-1)\xi}{x-\xi^2}\right)\right)}{20\left(\xi^2-1\right)^3} \\
& + \frac{3\left(x^3(x(2(x-4)x+15)-30)-15(2x(x+5)+5)\xi^4\right)\log\left(x^2-\xi^2\right)}{20\left(\xi^2-1\right)^3} \\
& + \frac{3\left(-5x(x(x(x+2)+36)+18)\xi^2-15\xi^6\right)\log\left(x^2-\xi^2\right)}{20\left(\xi^2-1\right)^3} \\
& + \frac{3\left(2(x-1)\left((23x+58)\xi^4+(x(x(x+67)+112)+6)\xi^2+x(x((5-2x)x+15)+3)\right)\right)}{20\left(\xi^2-1\right)^3} \\
& + \frac{3\left(\left(15(2x(x+5)+5)\xi^4+10x(3x(x+5)+11)\xi^2\right)\log\left(1-\xi^2\right)\right)}{20\left(\xi^2-1\right)^3} \\
& + \frac{3\left(2x(5x(x+2)-6)+15\xi^6-5\xi^2+3\right)\log\left(1-\xi^2\right)}{20\left(\xi^2-1\right)^3} \Bigg\},
\end{aligned}
$$

Analytic Results

$$
\begin{aligned}
H^{u}_{|x|\leq\xi}(x,\xi,0) = \frac{48}{5}\Bigg\{ & \frac{6\xi(x-1)^4\left(-\left(2x^2-5\xi^2+3\right)\right)\log(1-x)}{40\xi\left(\xi^2-1\right)^3} \\
&+\frac{6\xi\left(-4\xi\left(15x^2(x+3)+(19x+29)\xi^4+5(x(x(x+11)+21)+3)\xi^2\right)\log(2\xi)\right)}{40\xi\left(\xi^2-1\right)^3} \\
&+\frac{6\xi(\xi+1)^3\left((38x+13)\xi^2+6x(5x+6)\xi+2x(5x(x+2)-6)+15\xi^3-9\xi+3\right)\log(\xi+1)}{40\xi\left(\xi^2-1\right)^3} \\
&+\frac{6\xi(x-\xi)^3\left((7x-58)\xi^2+6(x-4)x\xi+x(2(x-4)x+15)+15\xi^3+75\xi-30\right)\log(\xi-x)}{40\xi\left(\xi^2-1\right)^3} \\
&+\frac{3(\xi-1)(x+\xi)\left(4x^4\xi-2x^3\xi(\xi+7)+x^2(\xi((119-25\xi)\xi-5)+15)\right)}{40\xi\left(\xi^2-1\right)^3} \\
&+\frac{3(\xi-1)(x+\xi)\left(x\xi(\xi(\xi(71\xi+5)+219)+9)+2\xi(\xi(2\xi(34\xi+5)+9)+3)\right)}{40\xi\left(\xi^2-1\right)^3}\Bigg\}.
\end{aligned}
$$

Let's make beautiful pictures from horrible formulas!

Reconstruction ($t = 0$)



We get back the support properties!

Advantages of DDs

- Support is stricly respected.

Advantages of DDs

- Support is stricly respected.
- Reconstruction is exact (no numerical noise).

Advantages of DDs

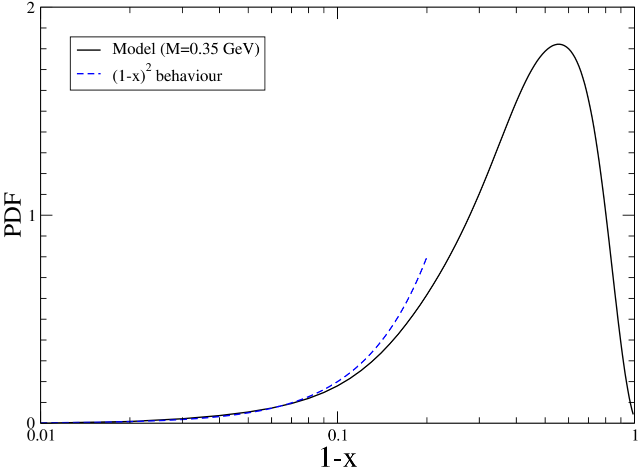
- Support is stricly respected.
- Reconstruction is exact (no numerical noise).
- Prove the continuity at $x = \xi$.

Advantages of DDs

- Support is stricly respected.
- Reconstruction is exact (no numerical noise).
- Prove the continuity at $x = \xi$.
- We can get analytic expressions. For the PDF($\nu = 1$):

$$
\begin{aligned}
q(x) &= \frac{72}{25}\left(x^3(x(-2(x-4)x-15)+30)\log(x)\right. \\
&\quad + \left(2x^2+3\right)(x-1)^4\log(1-x) \\
&\quad \left. + x(x(x(2x-5)-15)-3)(x-1)\right)
\end{aligned}
$$

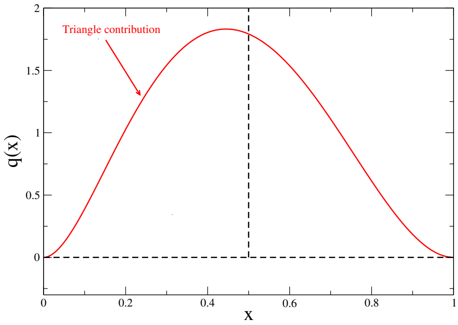
Large x behavior



At large x:

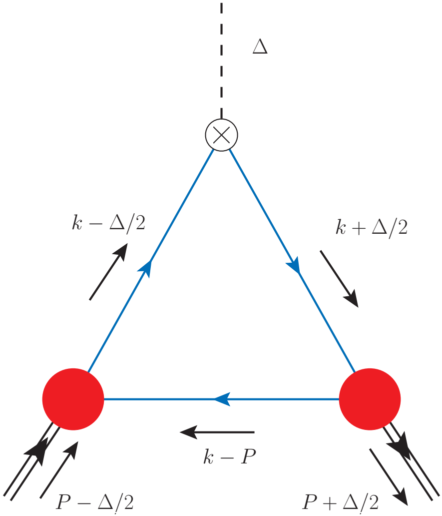
$$q(x) \approx (1-x)^2$$

Limits of the triangle diagrams



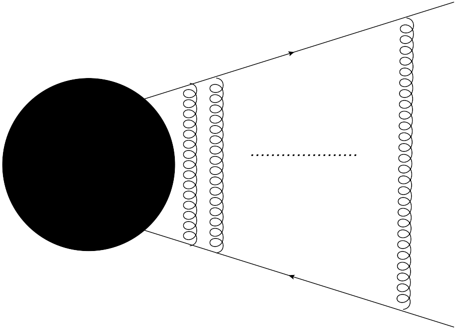
- The PDF appears not to be symmetric around $x = \frac{1}{2}$.

Limits of the triangle diagrams



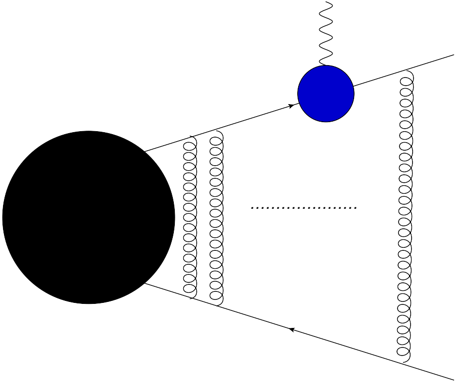
- The PDF appears not to be symmetric around $x = \frac{1}{2}$.
- Part of the gluons contribution is neglected in the triangle diagram approach.

Limits of the triangle diagrams



- The PDF appears not to be symmetric around $x = \frac{1}{2}$.
- Part of the gluons contribution is neglected in the triangle diagram approach.

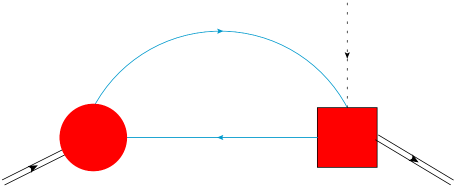
Limits of the triangle diagrams



$$\Gamma_\pi(k,P)\; S(k)\; \gamma^\mu \rightarrow \frac{\partial \Gamma_\pi}{\partial k_\mu}(k,P)$$

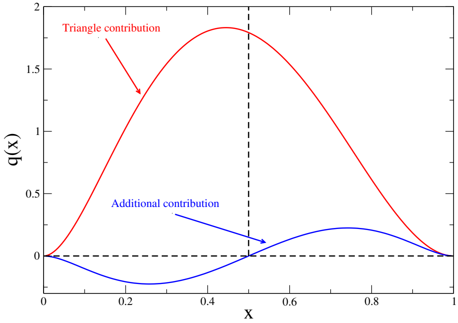
- The PDF appears not to be symmetric around $x = \frac{1}{2}$.
- Part of the gluons contribution is neglected in the triangle diagram approach.

Limits of the triangle diagrams



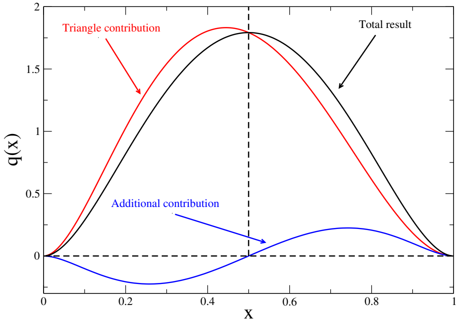
- The PDF appears not to be symmetric around $x = \frac{1}{2}$.
- Part of the gluons contribution is neglected in the triangle diagram approach.

Limits of the triangle diagrams



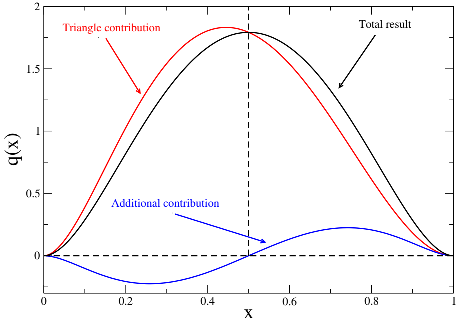
- The PDF appears not to be symmetric around $x = \frac{1}{2}$.
- Part of the gluons contribution is neglected in the triangle diagram approach.
- Adding this contribution allows us to recover a symmetric PDF (*L. Chang et al.*,2014).

Limits of the triangle diagrams



- The PDF appears not to be symmetric around $x = \frac{1}{2}$.
- Part of the gluons contribution is neglected in the triangle diagram approach.
- Adding this contribution allows us to recover a symmetric PDF (*L. Chang et al.*,2014).

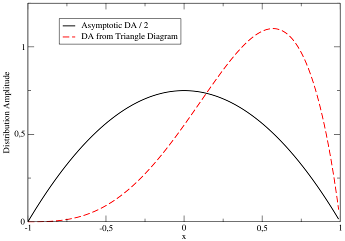
Limits of the triangle diagrams



- The PDF appears not to be symmetric around $x = \frac{1}{2}$.
- Part of the gluons contribution is neglected in the triangle diagram approach.
- Adding this contribution allows us to recover a symmetric PDF (*L. Chang et al.*,2014).

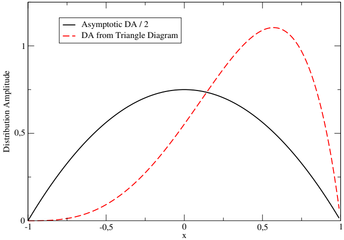
No obvious way to generalize this to the non forward case.

Soft Pion Theorem



- Polyakov soft pion theorem: if $\xi = 1$ and $t = 0$ then $H \propto$ Pion DA.

Soft Pion Theorem

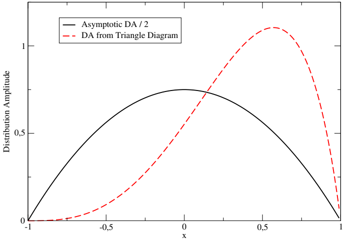


$$S^{-1}(k) = i\gamma \cdot kA(k^2) + B(k^2)$$

$$f_\pi \Gamma(k,0) = B(k^2)$$

- Polyakov soft pion theorem: if $\xi = 1$ and $t = 0$ then $H \propto$ Pion DA.
- Our algebraic model does not respect the Axial-Vector Ward-Takahashi Identities (AVWTI).

Soft Pion Theorem



$$S^{-1}(k) = i\gamma \cdot kA(k^2) + B(k^2)$$

$$f_\pi \Gamma(k,0) = B(k^2)$$

- Polyakov soft pion theorem: if $\xi = 1$ and $t = 0$ then $H \propto$ Pion DA.
- Our algebraic model does not respect the Axial-Vector Ward-Takahashi Identities (AVWTI).

The triangle diagram approximation is sufficient to get back the soft pion theorem, providing a good implementation of AVWTI.

It should be cured using the numerical solutions of BSE and DSE.

Summary and conclusions

- We presented a new model for pion GPD which fulfills most of the required symmetry properties.
- Using the DD method, we managed to fully reconstruct the GPD, avoiding the problems of polynomial reconstruction.
- DDs make the full problem analytic.
- Our comparisons with available experimental data are very encouraging.
- Limitations highlights physics key points (gluons, AVWTI...).

Summary and conclusions

- We presented a new model for pion GPD which fulfills most of the required symmetry properties.
- Using the DD method, we managed to fully reconstruct the GPD, avoiding the problems of polynomial reconstruction.
- DDs make the full problem analytic.
- Our comparisons with available experimental data are very encouraging.
- Limitations highlights physics key points (gluons, AVWTI...).

If the GPDs remain the good objects to understand the physics, DDs are the good objects to deal with support properties and full reconstruction.

Outlooks

- Going beyond the triangle diagram approximation in the non-forward case.
- We want to reconstruct the GPD thanks to DD in the realistic case, *i.e.* with vertices and propagators coming from numerical solutions of the Dyson-Schwinger equations.
- Compare our model with the existing phenomenological DD models, *i.e.* Radyushkin Ansatz.
- The proton case remains the Holy Grail...

Outlooks

- Going beyond the triangle diagram approximation in the non-forward case.
- We want to reconstruct the GPD thanks to DD in the realistic case, *i.e.* with vertices and propagators coming from numerical solutions of the Dyson-Schwinger equations.
- Compare our model with the existing phenomenological DD models, *i.e.* Radyushkin Ansatz.
- The proton case remains the Holy Grail... which may be reached in the valence region using a quark-diquark model.

Thank You!

Back up

More realistic model

- Change the propagator (fitted on DSE numerical solutions in *L. Chang et al.*, 2013):

$$S(k) = \sum_{j=1}^{m} \left(\frac{z_j}{i\not{k} + m_j} + \frac{z_j^*}{i\not{k} + m_j^*} \right)$$

More realistic model

- Change the propagator (fitted on DSE numerical solutions in *L. Chang et al.*, 2013):

$$S(k) = \sum_{j=1}^{m} \left(\frac{z_j}{i\not{k} + m_j} + \frac{z_j^*}{i\not{k} + m_j^*} \right)$$

- Change the Pion amplitude (also fitted on DSE numerical solutions):

$$\Gamma_\pi(k, P) = c_j \int_{-1}^{1} \mathrm{d}z \rho_\nu(z) \Lambda^2 \Delta^2(k, P, \Lambda) + \dots$$

More realistic model

- Change the propagator (fitted on DSE numerical solutions in *L. Chang et al.*, 2013):

$$S(k) = \sum_{j=1}^{m} \left(\frac{z_j}{i\not{k} + m_j} + \frac{z_j^*}{i\not{k} + m_j^*} \right)$$

- Change the Pion amplitude (also fitted on DSE numerical solutions):

$$\Gamma_\pi(k,P) = c_j \int_{-1}^{1} \mathrm{d}z \rho_\nu(z) \Lambda^2 \Delta^2(k,P,\Lambda) + \ldots$$

- New Ansatz for the inserted operator.

Kroll - Goloskokov model.

- **Factorized Ansatz**. For $i = g$, sea or val :

$$H_i(x,\xi,t) = \int_{|\alpha|+|\beta|\leq 1} d\beta d\alpha\, \delta(\beta+\xi\alpha-x) f_i(\beta,\alpha,t)$$

$$f_i(\beta,\alpha,t) = e^{b_i t}\frac{1}{|\beta|^{\alpha' t}} h_i(\beta)\pi_{n_i}(\beta,\alpha)$$

$$\pi_{n_i}(\beta,\alpha) = \frac{\Gamma(2n_i+2)}{2^{2n_i+1}\Gamma^2(n_i+1)}\frac{(1-|\beta|)^2-\alpha^2]^{n_i}}{(1-|\beta|)^{2n_i+1}}$$

- Expressions for h_i and n_i :

$$\begin{aligned} h_g(\beta) &= |\beta| g(|\beta|) & n_g &= 2 \\ h^q_{\mathrm{sea}}(\beta) &= q_{\mathrm{sea}}(|\beta|)\mathrm{sign}(\beta) & n_{\mathrm{sea}} &= 2 \\ h^q_{\mathrm{val}}(\beta) &= q_{\mathrm{val}}(\beta)\Theta(\beta) & n_{\mathrm{val}} &= 1 \end{aligned}$$

Goloskokov and Kroll, Eur. Phys. J. **C42**, 281 (2005)

- Comparison to **existing DVCS measurements** at LO.

Kroll *et al.*, Eur. Phys. J. **C73**, 2278 (2013)

Double Distribution Ambiguity

Teryaev Phys. Lett. B **510** (2001) 125
Tiburzi Phys. Rev. D **70** (2004) 057504

Rewrite the non forward matrix element in terms of DD :

$$\begin{aligned} &\langle P - \frac{r}{2} | \bar{\psi}(-\frac{z}{2}) \not{z} \psi(\frac{z}{2}) | P + \frac{r}{2} \rangle \\ = \quad & \int_{\Omega} e^{-i\beta(Pz) - i\alpha\frac{(rz)}{2}} \left(2(Pz)F(\beta,\alpha) + (rz)G(\beta,\alpha)\right) \mathrm{d}\alpha \mathrm{d}\beta \end{aligned}$$

Matrix element **invariant** under the following transformation :

$$\begin{aligned} F(\beta,\alpha) &\rightarrow F(\beta,\alpha) + \frac{\partial \sigma}{\partial \alpha} \\ G(\beta,\alpha) &\rightarrow G(\beta,\alpha) - \frac{\partial \sigma}{\partial \beta} \\ \sigma(\beta,\alpha) &= -\sigma(\beta,-\alpha) \end{aligned}$$

This invariance allows for **different** methods to parametrise GPDs.

Positivity

- Positivity condition in the DGLAP region:

$$|H(x,\xi,t)| \leq \sqrt{q(\frac{x-\xi}{1-\xi})q(\frac{x+\xi}{1+\xi})}$$

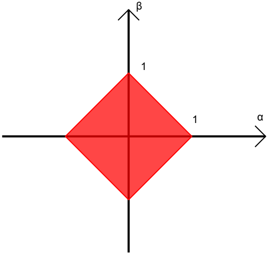
Pire, Soffer, Teryaev, 1999

- In our two-body problem, $q(x) \propto x^2$ at small x.
- Consequently $H(x,\xi,t)$ should vanish on the line $x=\xi$.
- We'll see how the more realistic model behaves.

Double Distributions

Double Distributions are formally the Radon transform of the GPDs.

$$H(x,\xi) = \int_\Omega \mathrm{d}\alpha \mathrm{d}\beta (F(\beta,\alpha) + \xi G(\beta,\alpha))\delta(x - \beta - \xi\alpha)$$



$$\Omega = \{(\alpha,\beta) | |\alpha| + |\beta| \leq 1\}$$

Advantage:
Easy way to respect the polynomiality in ξ

$$\int_{-1}^{1} x^n H(x,\xi)\mathrm{d}x$$
$$= \int_\Omega (\beta + \xi\alpha)^n (F(\beta,\alpha) + \xi G(\beta,\alpha))\mathrm{d}\Omega$$

From Mellin moments to Double Distributions (DD)

$$H(x,\xi,t) = \int_{-1}^{1} \mathrm{d}\beta \int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \; \left(F(\beta,\alpha,t) + \xi G(\beta,\alpha,t)\right) \delta(x-\beta-\alpha\xi)$$

- Time reversal invariance is encoded in the parity in α:
  - $F(\beta,\alpha)$ must be even in α
  - $G(\beta,\alpha)$ must be odd in α

From Mellin moments to Double Distributions (DD)

$$H(x,\xi,t) = \int_{-1}^{1} \mathrm{d}\beta \int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \, \left(F(\beta,\alpha,t) + \xi G(\beta,\alpha,t)\right) \delta(x-\beta-\alpha\xi)$$

- Time reversal invariance is encoded in the parity in α:
  - $F(\beta,\alpha)$ must be even in α
  - $G(\beta,\alpha)$ must be odd in α
- PDF case:

$$q(x) = H(x,0,0) = \int_{-1}^{1} \mathrm{d}\beta \int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \, F(\beta,\alpha,t)\delta(x-\beta)$$

From Mellin moments to Double Distributions (DD)

$$H(x,\xi,t) = \int_{-1}^{1} \mathrm{d}\beta \int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \, \left(F(\beta,\alpha,t) + \xi G(\beta,\alpha,t)\right) \delta(x-\beta-\alpha\xi)$$

- Time reversal invariance is encoded in the parity in α:
  - $F(\beta,\alpha)$ must be even in α
  - $G(\beta,\alpha)$ must be odd in α
- PDF case:

$$q(x) = H(x,0,0) = \int_{-1}^{1} \mathrm{d}\beta \int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \, F(\beta,\alpha,t)\delta(x-\beta)$$

- Form Factor case:

$$\mathcal{F}(t) = \int_{-1}^{1} \mathrm{d}x \, H(x,\xi,t) = \int_{-1}^{1} \mathrm{d}\beta \int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \, F(\beta,\alpha,t)$$

From Mellin moments to Double Distributions (DD)

$$H(x,\xi,t) = \int_{-1}^{1} \mathrm{d}\beta \int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \, \left(F(\beta,\alpha,t) + \xi G(\beta,\alpha,t)\right) \delta(x-\beta-\alpha\xi)$$

- Time reversal invariance is encoded in the parity in α:
  - $F(\beta,\alpha)$ must be even in α
  - $G(\beta,\alpha)$ must be odd in α
- PDF case:

$$q(x) = H(x,0,0) = \int_{-1}^{1} \mathrm{d}\beta \int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \, F(\beta,\alpha,t)\delta(x-\beta)$$

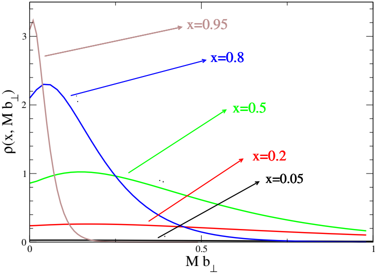
- Form Factor case:

$$\mathcal{F}(t) = \int_{-1}^{1} \mathrm{d}x \, H(x,\xi,t) = \int_{-1}^{1} \mathrm{d}\beta \int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \, F(\beta,\alpha,t)$$

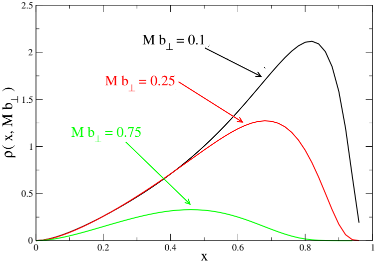
$G(\beta,\alpha)$ **does not** play any role in those cases.

2D Projection

Comparison at fixed x.

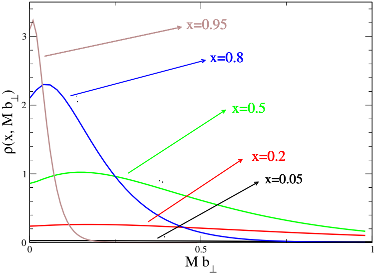


Comparison at fixed $b_\perp$.

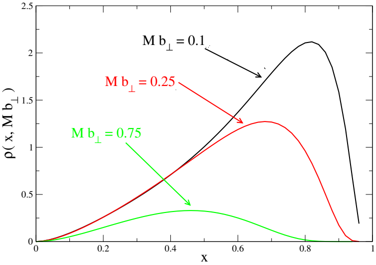


2D Projection

Comparison at fixed x.



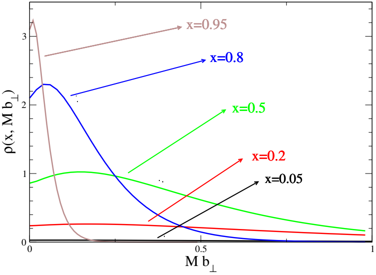
Comparison at fixed $b_\perp$.



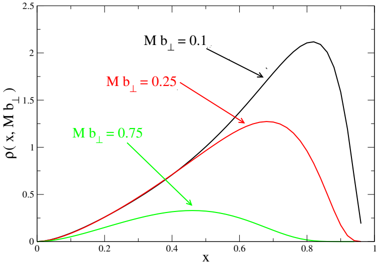
Fail to reproduce the t-x correlations at large t.

2D Projection

Comparison at fixed x.



Comparison at fixed $b_\perp$.



Fail to reproduce the t-x correlations at large t.

But the effect happens at t presumably too large to be noticed on the Form Factor.