Some new opportunities for spin physics at small x

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Based on: **Phys.Rev. D84 (2011) 051503. A. Metz and ZJ**
Phys.Rev. D85 (2012) 114004. A. Schäfer and ZJ
Phys.Rev. D87 (2013) 054010. E. Akcakaya, A. Schäfer and ZJ
arXiv:1308.4961. A. Schäfer and ZJ
Phys.Rev. D89 (2014) 074050. ZJ
Phys. Rev. D90(2014) A. Schäfer and ZJ

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Outline:

- **Gluon BM distribution inside a large nucleus**
- **SSAs at small x**
- **SSAs in polarized pA collisions**
- **Summary**

Gluon BM distribution inside a large nucleus

Gluon Boer-Mulders distribution: definition

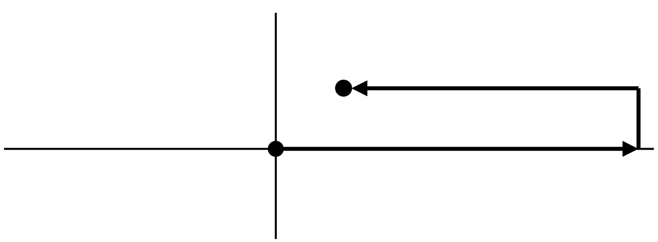
$$\int \frac{dr^- d^2 r_\perp}{(2\pi)^3 P^+} \, e^{-ix_1 P^+ r^- + i\vec{k}_{1\perp}\cdot\vec{r}_\perp} \langle A | F^{+i}(r^- + y^-, r_\perp + y_\perp) \, L^\dagger \, L \, F^{+j}(y^-, y_\perp) | A \rangle$$

$$= \frac{\delta_\perp^{ij}}{2} x_1 G(x_1, k_{1\perp}) + \left(\hat{k}^i_{1\perp} \hat{k}^j_{1\perp} - \frac{1}{2}\delta_\perp^{ij} \right) x_1 h_1^{\perp g}(x_1, k_{1\perp}) \, , \qquad \delta_\perp^{ij} = -g^{ij} + (p^i n^j + p^j n^i)/p \cdot n$$

Mulders, Rodrigues, 2001

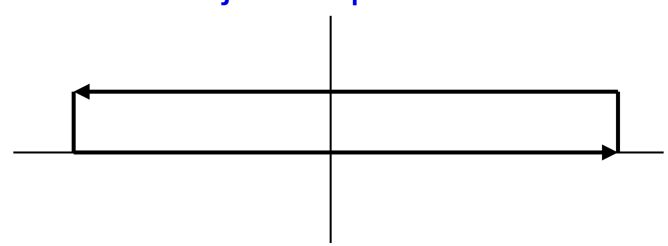
Gluon TMD distributions receive the contribution from ISI/FSI ⟶ make gauge link process dependent

in the adjoint representation



Weizsäcker-Williams distribution
(probability interpretation)

in the fundamental representation
or in the adjoint representation



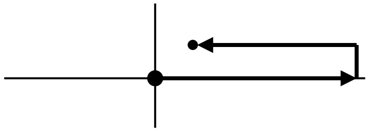
Dipole distribution

Dominguez, Marquet, Xiao & Yuan, 2011

Distribution of linearly polarized gluons: dynamics I

At small x, due to the presence of semi-hard scale(Q_S), usual unp. gluon distributions can be computed using MV model,

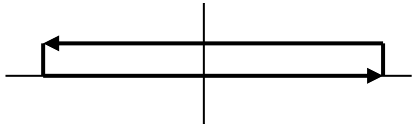
$$xG^g_{WW}(x,k_\perp) = \frac{N_c^2-1}{N_c}\frac{S_\perp}{4\pi^4\alpha_s}\int d^2\xi_\perp\, e^{-i\vec{k}_\perp\cdot\vec{\xi}_\perp}\,\frac{1}{\xi_\perp^2}\left(1-e^{-\frac{\xi_\perp^2 Q_s^2}{4}}\right)$$



Kovchegov, 96

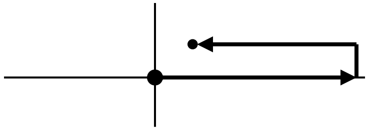
J. Marian, Kovner, Mclerran & Weigert, 97

$$xG^g_{DP}(x,k_\perp) = \frac{k_\perp^2 N_c}{2\pi^2\alpha_s}S_\perp\int\frac{d^2\xi_\perp}{(2\pi)^2}\,e^{-i\vec{k}_\perp\cdot\vec{\xi}_\perp}\,e^{-\frac{Q_{sq}^2\xi_\perp^2}{4}}$$



Following the similar procedure, linearly polarized gluon distribution also can be computed in the MV model.

Distribution of linearly polarized gluons: dynamics III

WW type: 

$$xh_{1,WW}^{\perp g}(x,k_\perp) = \frac{N_c^2-1}{8\pi^3} S_\perp \int d\xi_\perp \frac{K_2(k_\perp \xi_\perp)}{\frac{1}{4\mu_A}\xi_\perp Q_s^2}\left(1-e^{-\frac{\xi_\perp^2 Q_s^2}{4}}\right)$$

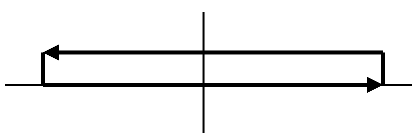
Metz, ZJ, 2011

Large k_t, dilute limit $(k_\perp \gg Q_s)$

$$xG_{WW}^{g}(x,k_\perp) = xh_{1,WW}^{\perp g}(x,k_\perp) \simeq S_\perp \frac{N_c^2-1}{4\pi^3}\frac{\mu_A}{k_\perp^2}$$ same perturbative tail

Small k_t, dense medium limit $(\Lambda_{QCD} \ll k_\perp \ll Q_s)$

$$xh_{1,WW}^{\perp g}(x,k_\perp) \simeq S_\perp \frac{N_c^2-1}{4\pi^3}\frac{\mu_A}{Q_s^2} \qquad xG_{WW}^{g}(x,k_\perp) \simeq S_\perp \frac{N_c^2-1}{4\pi^3}\frac{1}{\alpha_s N_c}\ln\frac{Q_s^2}{k_\perp^2}$$

Dipole type: 

$$xh_{1,DP}^{\perp g}(x,k_\perp) = xG_{DP}^{g}(x,k_\perp)$$ positivity bound saturated for any value of k_t

Remark: Small x evolution.

Dominguez, Qiu, Xiao & Yuan 2011

How to probe the gluon BM distribution ?

Many proposals:

(Boer, Mulders, Pisano, 2009 / Boer, Brodsky, Mulders, Pisano, 2010/
Boer, denDunnen, Pisano, Schlegel, Vogelsang, 2011, 2013 /
AM, Zhou, 2011 / Sun, Xiao, Yuan, 2011 / Dominguez, Qiu, Xiao, Yuan, 2011/
Schaefer, Zhou, 2012 / Akcakaya, Schaefer, Zhou, 2012 /
Pisano, Boer, Brodsky, Buffing, Mulders, 2013 / Lansberg, den Dunnen, Pisano, Schlegel, 2014 / ...)

We focus on:

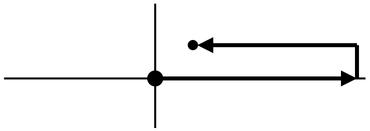
- Di-jet production in eA scatterings
- Photo-jet production in pA

Cos 2 Φ azimuthal asymmetries   **Φ: $k_t \wedge P_t$**

Cos2Φ asymmetries in eA and pA collisions

$$\text{Correlation limit}: \; k_\perp << l_\perp$$

Di-jet in eA

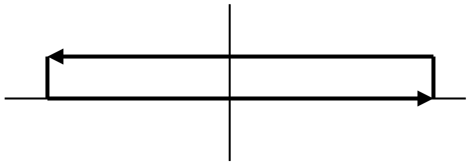


$$\frac{d\sigma^{\gamma_T^* A\to q\bar{q}+X}}{dP.S.} = \delta(x_{\gamma^*}-1)H_{\gamma_T^* g\to q\bar{q}}\Big\{xG_{WW}^g(x,k_\perp)$$
$$-2\frac{[z_q^2+(1-z_q)^2]\epsilon_f^2P_\perp^2-m_q^2P_\perp^2}{[z_q^2+(1-z_q)^2](\epsilon_f^4+P_\perp^4)+2m_q^2P_\perp^2}\cos(2\phi)xh_{1,WW}^{\perp g}(x,k_\perp)\Big\}$$
$$\frac{d\sigma^{\gamma_L^* A\to q\bar{q}+X}}{dP.S.} = \delta(x_{\gamma^*}-1)H_{\gamma_L^* g\to q\bar{q}}\Big\{xG_{WW}^g(x,k_\perp)+\cos(2\phi)xh_{1,WW}^{\perp g}(x,k_\perp)\Big\},$$

γ*- jet production in pA

Metz & ZJ 2011

$$\frac{d\sigma^{pA\to\gamma^* q+X}}{dP.S.} = \sum_q x_p f_1^q(x_p)\,xG_{DP}^g(x,k_\perp)H_{qg\to\gamma^* q}\Big\{1+\cos(2\phi)\frac{4Q^2\hat{t}}{\hat{s}^2+\hat{u}^2+2Q^2\hat{t}}\Big\}$$



parameter free!

also verified by the calculation performed in the position space

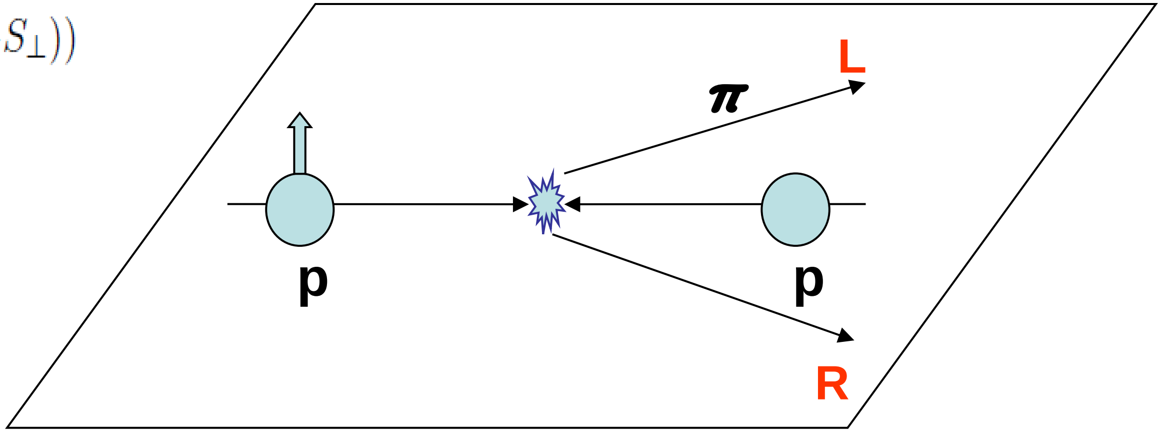
Dominguez, Qiu, Xiao & Yuan 2011

SSAs at small x

Transverse single spin asymmetris

$$p(\uparrow) + p \rightarrow \pi + X$$

$$A_N \equiv (\sigma(S_\perp) - \sigma(-S_\perp))/(\sigma(S_\perp) + \sigma(-S_\perp))$$



SSAs at small x in the CGC formalism

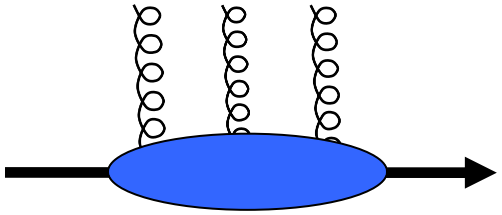
- The odderon origin of SSAs

Ahmedov, Akushevich, Kuraev & Ratcliffe 1999
Ahmedov, Antonov, Bartos, Kuraev & Zemlyanaya 2003

kovchegov & Sievert 2012

- **An odderon from unpolarized target nucleon/nucleus !**

In PQCD:



- BKP equation Bartels 1980, Kwiecinski & Praszalowicz 1980

BLV Solution Bartels, Lipatov & Vacca 2000

- Formulation in Mueller's dipole model

Kovchegov, Szymanowski & Wallon 2004

- Formulation in the CGC

Hatta, Iancu, Itakura & McLerran 2005

$$\hat{D}(R_\perp, r_\perp) = \frac{1}{N_c}\mathrm{Tr}\left[U(R_\perp + \frac{r_\perp}{2})U^\dagger(R_\perp - \frac{r_\perp}{2})\right]$$

$$= \hat{S}(R_\perp, r_\perp) + i\hat{O}(R_\perp, r_\perp)$$

Wilson line $U(x_\perp) = \mathrm{P}e^{ig\int_{-\infty}^{+\infty} dx^- A_+(x^-,\, x_\perp)}$

Antisymmetric part $\hat{O}(R_\perp, r_\perp) = \frac{1}{2i}\left[\hat{D}(R_\perp, r_\perp) - \hat{D}(R_\perp, -r_\perp)\right]$

Symmetric part $\hat{S}(R_\perp, r_\perp) = \frac{1}{2}\left[\hat{D}(R_\perp, r_\perp) + \hat{D}(R_\perp, -r_\perp)\right]$

Odderon in the MV model

The expectation value of the odderon operator

$$\int d^2R_\perp \theta(R_0 - |R_\perp|) < \hat{O}(R_\perp, r_\perp) >$$

$$= c_0 \alpha_s^3 \int d^2R_\perp \theta(R_0 - |R_\perp|) \int d^2 z_\perp \ln^3 \frac{|R_\perp + r_\perp/2 - z_\perp|}{|R_\perp - r_\perp/2 - z_\perp|} e^{-\frac{1}{4} r_\perp^2 Q_s^2} \frac{1}{3} \int dx_q f_q(x_q, z_\perp)$$

Valence quark distribution

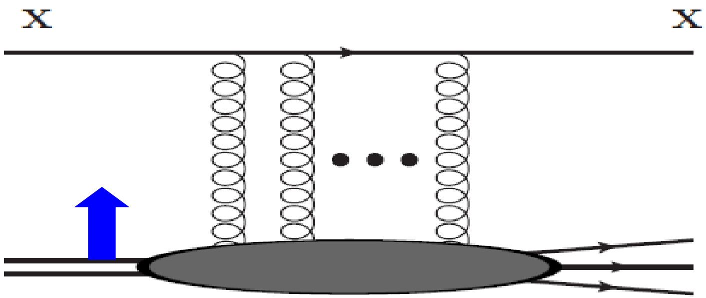
Jeon,Venugopalan 2005

$$\approx \frac{c_0 \alpha_s^3 \pi}{4R_0^2} r_\perp^2 e^{-\frac{1}{4} r_\perp^2 Q_s^2} \int dx_q d^2 z_\perp (r_\perp \cdot z_\perp) f_q(x_q, z_\perp)$$

- Impact parameter dependent valence quark distribution

$$f_q(x_q, z_\perp) = \sum_{u,d} \left\{ \mathcal{H}(x_q, z_\perp^2) - \frac{1}{2M} \epsilon_\perp^{ij} S_{\perp i} \frac{\partial \mathcal{E}(x_q, z_\perp^2)}{\partial z_\perp^j} \right\}$$

M. Burkardt 2000, 2003



Odderon in the MV model

$$\int d^2R_\perp \theta(R_0 - |R_\perp|) < \hat{O}(R_\perp, r_\perp) >$$

$$= \quad -\frac{c_0\alpha_s^3\pi}{8MR_0^2} e^{-\frac{1}{4}r_\perp^2 Q_s^2} r_\perp^2 \epsilon_\perp^{ij} S_{\perp i} r_{\perp j} \int dx_q d^2 z_\perp \sum_{u,d} \mathcal{E}(x_q, z_\perp^2)$$

$$= \quad -\frac{c_0\alpha_s^3\pi}{8MR_0^2} e^{-\frac{1}{4}r_\perp^2 Q_s^2} r_\perp^2 \epsilon_\perp^{ij} S_{\perp i} r_{\perp j} \left(\kappa_p^u + \kappa_p^d\right)$$

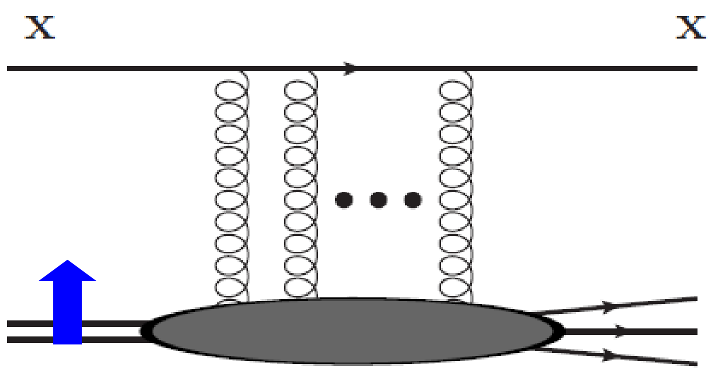
ZJ 2013

- An earlier attempt to connect SSA phenomena to GPD E has been made.

M. Burkardt 2003

✦ **Potential relation to the orbital angular momentum !**

SSA in jet production in the backward region of pp collisions



LO **Dumitru & J. Marian 2002**

NLO **Chirilli, Xiao & Yuan 2012**

$$\frac{d\sigma^{pA\to qX}}{d^2k_\perp dY} = \sum_f xq_f(x)\int\frac{d^2r_\perp}{(2\pi)^2}e^{-ik_\perp\cdot r_\perp}\int d^2R_\perp < \hat{D}(R_\perp, r_\perp) >_{x_g}$$
$$= \sum_f xq_f(x)\left\{F_{x_g}(k_\perp^2) + \frac{1}{M}\epsilon_\perp^{ij}S_{\perp i}k_{\perp j}O^\perp_{1T,x_g}(k_\perp^2)\right\}$$

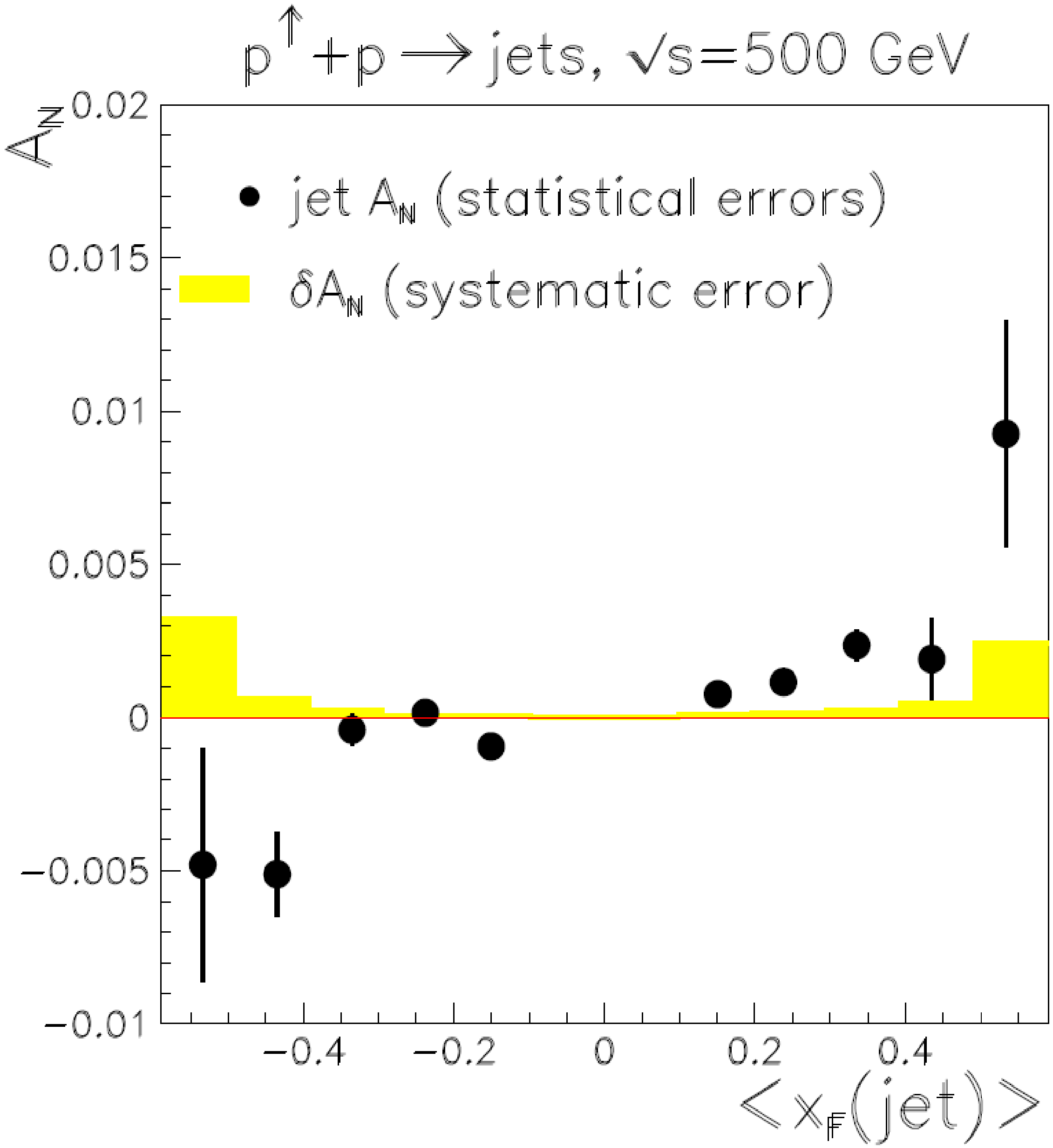
F_{xg} is just the Dipole type unpolarized gluon distribution.

Spin dependent odderon

$$O^\perp_{1T,x_g}(k_\perp^2) = \frac{-c_0\alpha_s^3\left(\kappa_p^u + \kappa_p^d\right)}{4R_0^4}\left[\frac{\partial}{\partial k_\perp^2}\frac{\partial}{\partial k_\perp^i}\frac{\partial}{\partial k_{\perp i}}F_{x_g}(k_\perp^2)\right]$$

- The asympotic behavior

$$A_{UT} \sim k_\perp O^\perp_{1T,x_g}/F_{x_g}$$ BLV solution → $$A_{UT} \sim (x_g)^{0.3}$$



AnDY at RHIC, 2012

K_T moment of the odderon & C-odd tri-gluon correlation

C-odd tri-gluon correlation:

$$O^{\alpha\beta\gamma}(x_1,x_2) = -gi^3 \int \frac{dy^- dz^-}{(2\pi)^2 P^+} e^{iy^- x_1 P^+} e^{iz^-(x_2-x_1)P^+} \langle pS| d^{bca} F_b^{\beta+}(0) F_c^{\gamma+}(z^-) F_a^{\alpha+}(y^-)|pS\rangle$$

$$\int d^2k_\perp k_\perp^\alpha k_\perp^\beta k_\perp^\gamma \frac{1}{M} \epsilon_\perp^{ij} S_{\perp i} k_{\perp j} O_{1T,x_g}^\perp(k_\perp^2) = \frac{-ig^2\pi^2}{2N_c} O^{\alpha\beta\gamma}(x_g)$$

The moment of the gluon Sivers function → **C-even tri-gluon**

The moment of the $O_{1T,x_g}^\perp(k_\perp^2)$ → **C-odd tri-gluon**

CGC v.s. Collinear twist-3

The soft parts related;

How about the hard coefficients? **Work in progress…**

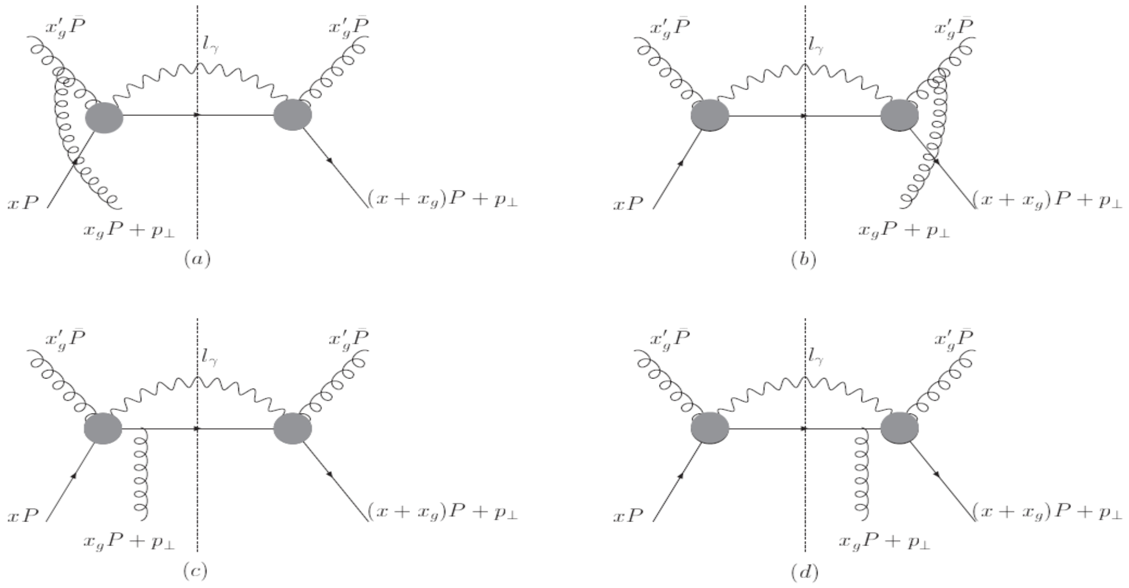
Small x

GPD

SSAs/Collinear twist-3

SSAs in polarized pA collisions

SSA for photon production in polarized pp collisions



Based on the collinear twist-3 approach

Qiu, Sterman 1992
Kouvaris, Qiu, Vogelsang, Yuan 2006

In the forward region or in pA collisions: gluon number density is large,
Single gluon exchange approximation is not sufficient.

Hybrid approach: Colliner twist-3 & CGC

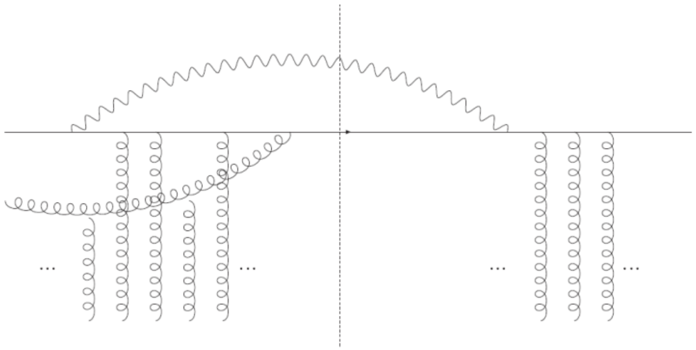
Formulated in a hybrid approach:

one extra gluon exchange with proton remnants;

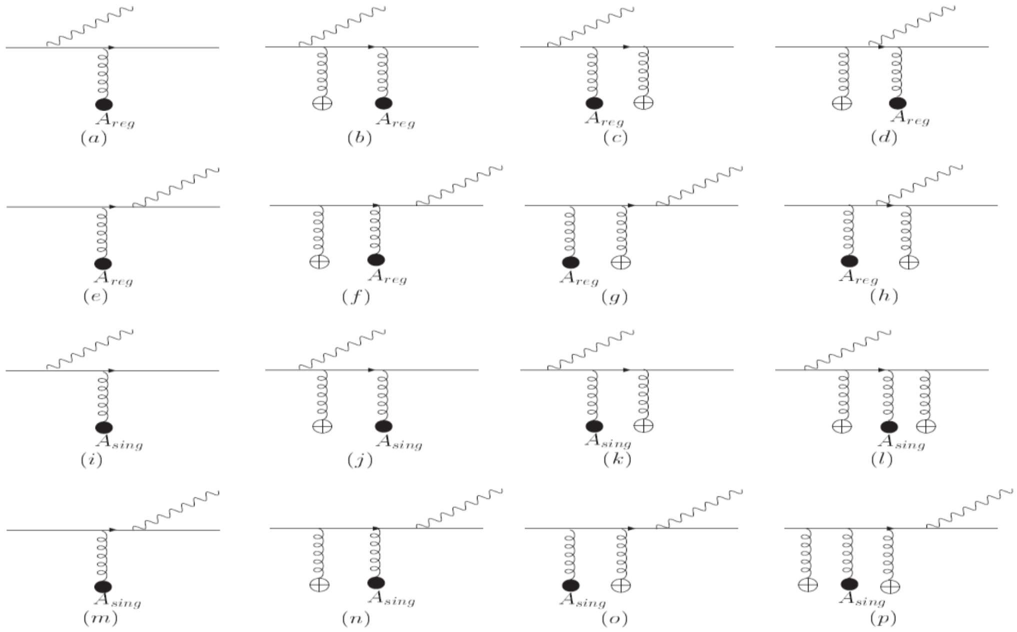
resuming multiple gluon scattering inside nucleus to all orders.

→ **Collinear twist-3 & CGC**

A typical diagram



In the hybrid approach:



Color entanglement effect for SSA

Derivative term contribution to the polarized cross section

$$\frac{d^3\Delta\sigma}{d^2l_{\gamma\perp}dz} = \frac{\alpha_s\alpha_{em}N_c}{N_c^2-1}\frac{1+(1-z)^2}{zl_{\gamma\perp}^2}(z-1)\int_{x_{min}}^1\frac{dx}{x}\int d^2k_\perp\frac{\left[\epsilon^{l_\gamma S_\perp np}-\epsilon^{k_\perp S_\perp np}\right]}{(k_\perp-l_{\gamma\perp}/z)^2\,(k_\perp-l_{\gamma\perp})^2}$$
$$\times\sum_q e_q^2\left[-x\frac{d}{dx}T_{F,q}(x,x)\right]\left[x'_gG_{DP}(x'_g,k_\perp)-x'_gG_4(x'_g,k_\perp)\right]$$

Schafer, ZJ 2014

G_4 is a new gluon distribution

$$x'_gG_4(x'_g,k_\perp) = \frac{k_\perp^2N_c}{2\pi^2\alpha_s}\int\frac{d^2x_\perp d^2y_\perp}{(2\pi)^2}e^{ik_\perp\cdot(x_\perp-y_\perp)}\frac{1}{N_c^2}\langle\mathrm{Tr_c}[U(x_\perp)]\mathrm{Tr_c}[U^\dagger(y_\perp)]\rangle_{x'_g}$$

In the MV model

$$x'_gG_4(x'_g,k_\perp) = \frac{1}{N_c^2}G_{DP}(x'_g,k_\perp)$$

Extrapolating to the kinematical limit $l_{\gamma\perp}\gg Q_{sq}\sim k_\perp$

$$\mathrm{Tr_C}\left[t^a\right]=0 \;\textbf{!}$$

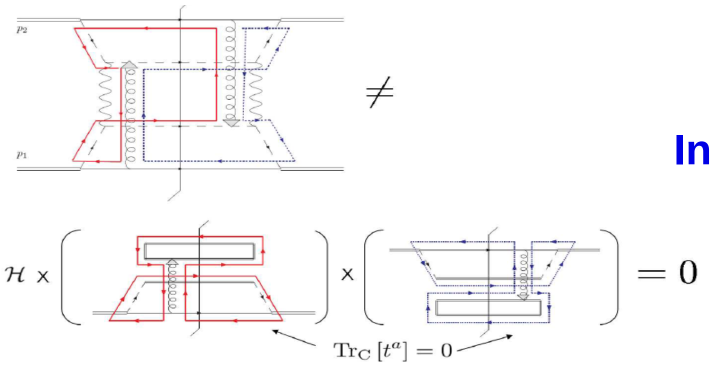
$$\frac{d^3\Delta\sigma}{d^2l_{\gamma\perp}dz} = \frac{\alpha_s\alpha_{em}N_c}{N_c^2-1}\epsilon^{l_\gamma S_\perp np}\frac{z[1+(1-z)^2]}{l_{\gamma\perp}^6}(z-1)$$
$$\times\sum_q e_q^2\int_{x_{min}}^1\frac{dx}{x}\left[-x\frac{d}{dx}T_{F,q}(x,x)\right]\left\{x'_gG(x'_g)-\frac{1}{N_c^2}x'_gG(x'_g)\right\}$$

Color entanglement effect

Collinear twist-3

Color entanglement effect

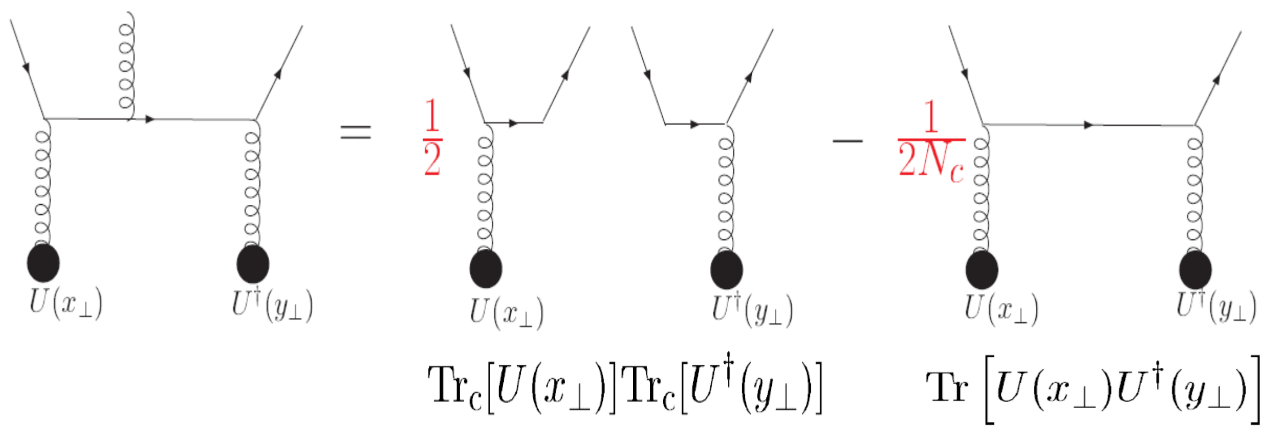
First discovered by **Rogers, Mulders, 2010**



In the double spin asymmtry case

How does G_4 emerge?

A simple illustration:



◆**Caution:** **the nontrivial color structure appears in the case for which the Ward identity argument does not apply.**

T-odd effect & coherent gluon rescattering & non abelian feature of QCD

Summary I

- gluon polarization at small x
- spin dependent odderon & the asymptotic behavior of SSAs at small x
- studying color entanglement effect in a hybrid approach

Summary II

Small x physics:

Powerfull theoretical tools

- Dipole approach & MV model
- BFKL and BK equations
- CGC and JIMWLK equation
- …

Spin physics:

Novel ideas/conceptions

- T-odd effect
- Universality v.s. process dependent TMDs
- Color entanglement effect
- …

The study of hardon/nucleus structure is benefiting from the joint force of small x physics and spin physics.