Transverse single-spin asymmetries of pion production in semi-inclusive DIS at subleading twist

Wenjuan Mao

E-Mail: wjmao@seu.edu.cn

Southeast University, Nanjing, China

2014.10

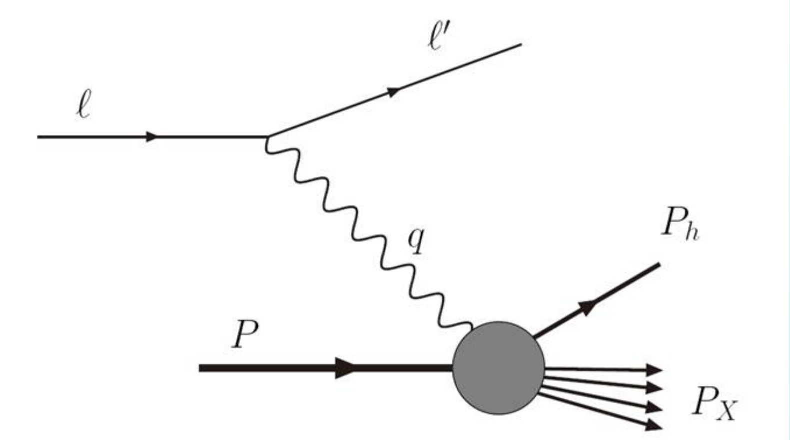
Outline

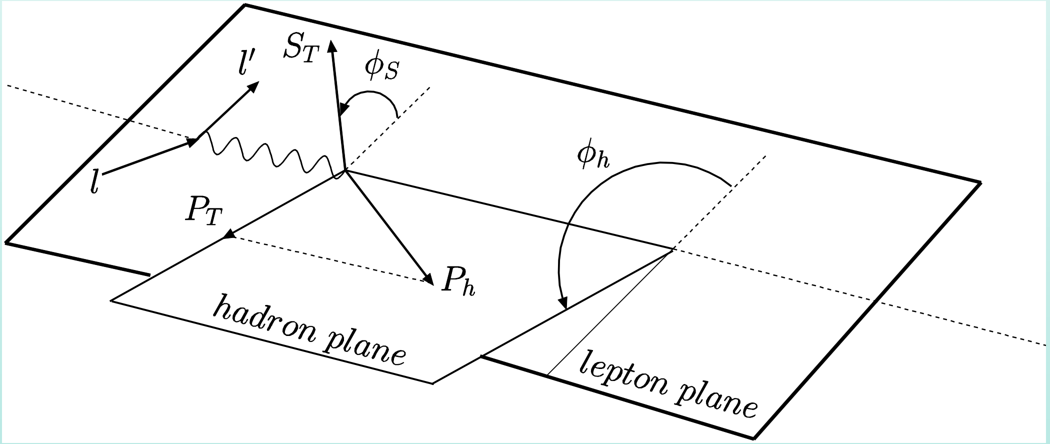
- Formalism
- The calculation of the four involved twist-3 PDFs $f_T(x, \boldsymbol{k_T^2})$, $f_T^\perp(x, \boldsymbol{k_T^2})$, $h_T(x, \boldsymbol{k_T^2})$, and $h_T^\perp(x, \boldsymbol{k_T^2})$ in the spectator model
- Prediction on the transverse SSAs for pion production in semi-inclusive DIS
- Conclusion

Formalism

- Semi-inclusive DIS by unpolarized lepton beam off the transversely polarized nucleon target:

$$e(l) + p^{\uparrow}(P) \rightarrow e(l') + h(P_h) + X$$





- The invariant variables used to express the differential cross section of SIDIS are defined as

$$x = \frac{Q^2}{2\,P \cdot q}, \quad y = \frac{P \cdot q}{P \cdot l}, \quad z = \frac{P \cdot P_h}{P \cdot q}, \quad \gamma = \frac{2Mx}{Q}$$
$$Q^2 = -q^2, \quad s = (P + \ell)^2, \quad W^2 = (P + q)^2,$$

Formalism

- The general form for the differential cross section with a transversely polarized nucleon target (Bacchetta *et al.*, JHEP0702, 093 (2007))

$$\frac{d\sigma}{dx\,dy\,dz\,d\phi_S\,d\phi_h\,dP_T^2} = \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\{F_{UU} + |S_T|[\sqrt{2\varepsilon(1+\varepsilon)}(\sin\phi_S F_{UT}^{\sin\phi_S} + \sin(2\phi_h-\phi_S)F_{UT}^{\sin(2\phi_h-\phi_S)})] + \cdots\}.$$

- The P_T dependent transverse SSAs can be defined as

$$A_{UT}^{\sin\phi_S}(P_T) = \frac{\int dx \int dy \int dz \, \frac{1}{xyQ^2}\frac{y^2}{2(1-\varepsilon)} \times \left(1+\frac{\gamma^2}{2x}\right)\sqrt{2\varepsilon(1+\varepsilon)}\; F_{UT}^{\sin\phi_S}}{\int dx \int dy \int dz \, \frac{1}{xyQ^2}\frac{y^2}{2(1-\varepsilon)} \times \left(1+\frac{\gamma^2}{2x}\right) F_{UU}}$$

$$A_{UT}^{\sin(2\phi_h-\phi_S)}(P_T) = \frac{\int dx \int dy \int dz \, \frac{1}{xyQ^2}\frac{y^2}{2(1-\varepsilon)} \times \left(1+\frac{\gamma^2}{2x}\right)\sqrt{2\varepsilon(1+\varepsilon)}\; F_{UT}^{\sin(2\phi_h-\phi_S)}}{\int dx \int dy \int dz \, \frac{1}{xyQ^2}\frac{y^2}{2(1-\varepsilon)} \times \left(1+\frac{\gamma^2}{2x}\right) F_{UU}}$$

- Sizable SSAs cannot be explained by pQCD (Ahmed & Gehrmann, PLB465, 297 (1999)).
- In the (assumed) TMD factorization (Bacchetta, Mulders, Pijlman PLB595, 309 (2004); Bacchetta *et al.*, JHEP0702, 093 (2007))

Formalism

- The structure functions in the numerator are given as

$$F_{UT}^{\sin\phi_S} = \frac{2M}{Q}\mathcal{C}\left\{\left(xf_T D_1 - \frac{M_h}{M}h_1\frac{\tilde{H}}{z}\right) + \frac{\boldsymbol{p_T}\cdot\boldsymbol{k_T}}{2zMM_h}\left[xh_T H_1^\perp + \frac{M_h}{M}g_{1T}\frac{\tilde{G}^\perp}{z}\right) - \left(xh_T^\perp H_1^\perp - \frac{M_h}{M}f_{1T}^\perp\frac{\tilde{D}^\perp}{z}\right)\right]\right\}$$

$$F_{\rm UT}^{\sin(2\phi_h-\phi_S)} \approx \frac{2M}{Q}\mathcal{C}\left\{\frac{2(\hat{\boldsymbol{P}}_T\cdot\boldsymbol{k}_T)^2-\boldsymbol{k}_T{}^2}{2M^2}\left(xf_T^\perp D_1 - \frac{M_h}{M}h_{1T}^\perp\frac{\tilde{H}}{z}\right)\right.$$

$$\left.+\frac{2(\hat{\boldsymbol{P}}_T\cdot\boldsymbol{p}_T)(\hat{\boldsymbol{P}}_T\cdot\boldsymbol{k}_T)-\boldsymbol{p}_T\cdot\boldsymbol{k}_T}{2zMM_h}\times\left[\left(xh_T H_1^\perp + \frac{M_h}{M}g_{1T}\frac{\tilde{G}^\perp}{z}\right) + \left(xh_T^\perp H_1^\perp - \frac{M_h}{M}f_{1T}^\perp\frac{\tilde{D}^\perp}{z}\right)\right]\right\}$$

- Using the WW (Wandzura-Wilczek) approximation to ignore the contributions from the twist-3 TMD fragmentation functions $\tilde{H},\ \tilde{G}^\perp,\ \tilde{D}^\perp$

$$F_{\rm UT}^{\sin\phi_S} \approx \frac{2M}{Q}\mathcal{C}\left\{xf_T D_1 + \frac{\boldsymbol{p}_T\cdot\boldsymbol{k}_T}{2zMM_h}\left(xh_T H_1^\perp - xh_T^\perp H_1^\perp\right)\right\}$$

$$F_{\rm UT}^{\sin(2\phi_h-\phi_S)} \approx \frac{2M}{Q}\mathcal{C}\left\{\frac{2(\hat{\boldsymbol{P}}_T\cdot\boldsymbol{k}_T)^2-\boldsymbol{k}_T{}^2}{2M^2}\left(xf_T^\perp D_1\right) + \frac{2(\hat{\boldsymbol{P}}_T\cdot\boldsymbol{p}_T)(\hat{\boldsymbol{P}}_T\cdot\boldsymbol{k}_T)-\boldsymbol{p}_T\cdot\boldsymbol{k}_T}{2zMM_h}\times\left[xh_T H_1^\perp + xh_T^\perp H_1^\perp\right]\right\}$$

- Nonzero contributions with the collinear twist-3 factorization (Z.-B. Kang, F. Yuan and J. Zhou, Phys. Lett. B 691, 243(2010); A. Metz and D. Pitonyak, Phys. Lett. B 723, 365 (2013); K. Kanazawa, Y. Koike, A. Metz, and D. Pitonyak, PRD 89, 111501(R) (2014)).

- Formalism
- The calculation of the four involved twist-3 PDFs $f_T(x, \boldsymbol{k_T^2})$, $f_T^\perp(x, \boldsymbol{k_T^2})$, $h_T(x, \boldsymbol{k_T^2})$, and $h_T^\perp(x, \boldsymbol{k_T^2})$ in the spectator model
- Prediction on the transverse SSAs for pion production in semi-inclusive DIS
- Conclusion

Calculation of the TMD twist-3 distribution functions

$$F_{\rm UT}^{\sin\phi_S} \approx \frac{2M}{Q}\mathcal{C}\left\{xf_T D_1 + \frac{\boldsymbol{p}_T\cdot\boldsymbol{k}_T}{2zMM_h}\left(xh_T H_1^\perp - xh_T^\perp H_1^\perp\right)\right\}$$

$$F_{\rm UT}^{\sin(2\phi_h-\phi_S)} \approx \frac{2M}{Q}\mathcal{C}\left\{\frac{2(\hat{\boldsymbol{P}}_T\cdot\boldsymbol{k}_T)^2-\boldsymbol{k}_T{}^2}{2M^2}\left(xf_T^\perp D_1\right) + \frac{2(\hat{\boldsymbol{P}}_T\cdot\boldsymbol{p}_T)(\hat{\boldsymbol{P}}_T\cdot\boldsymbol{k}_T)-\boldsymbol{p}_T\cdot\boldsymbol{k}_T}{2zMM_h}\times\left[xh_T H_1^\perp + xh_T^\perp H_1^\perp\right]\right\}$$

$$\begin{aligned}\frac{1}{2}\mathrm{Tr}[\Phi\gamma^\alpha] &= \frac{M}{P^+}\left[-\epsilon_T^{\alpha\rho}S_{T\rho}f_T' + \frac{(k_T\cdot S_T)\epsilon_T^{\alpha\rho}k_{T\rho}}{M^2}f_T^\perp\right]\\ &= \frac{M}{P^+}\left[-\epsilon_T^{\alpha\rho}S_{T\rho}f_T - \frac{(k_T^\alpha k_T^\rho - \frac{1}{2}k_T^2 g_T^{\alpha\rho})}{M^2}\epsilon_{T\rho\sigma}S_T^\sigma f_T^\perp\right],\end{aligned} \quad (4)$$

$$\frac{1}{2}\mathrm{Tr}[\Phi i\sigma^{+-}\gamma_5] = -\frac{M}{P^+}\left[\frac{k_T\cdot S_T}{M}h_T\right], \quad (5)$$

$$\frac{1}{2}\mathrm{Tr}[\Phi i\sigma^{\alpha\beta}\gamma_5] = \frac{M}{P^+}\left[\frac{S_T^\alpha k_T^\beta - k_T^\alpha S_T^\beta}{M}h_T^\perp\right]. \quad (6)$$

T-odd: $f_T(x,\boldsymbol{k_T^2})$, $f_T^\perp(x,\boldsymbol{k_T^2})$

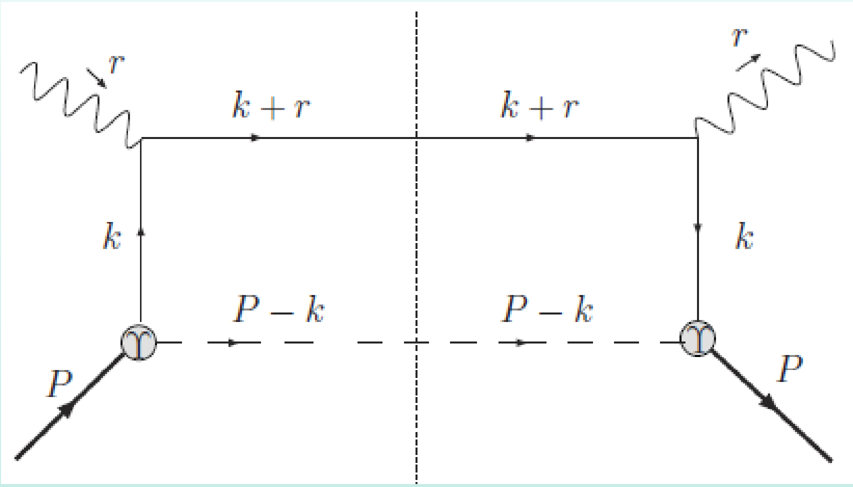
T-even: $h_T(x,\boldsymbol{k_T^2})$, $h_T^\perp(x,\boldsymbol{k_T^2})$

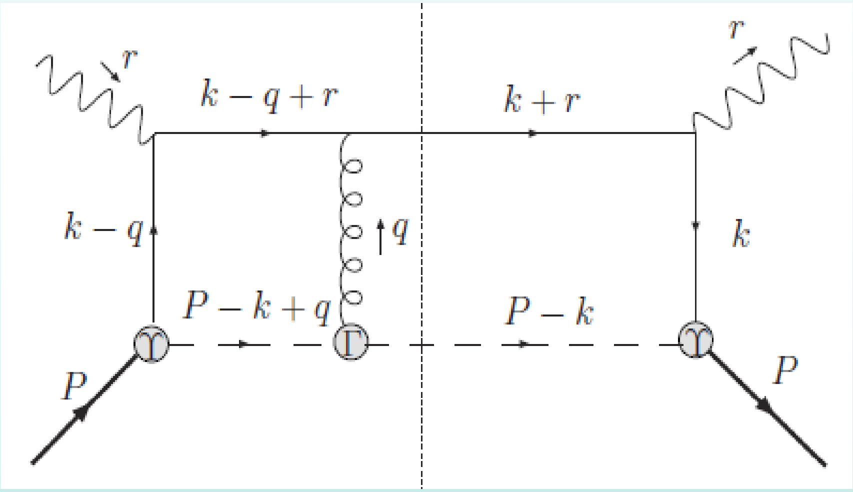
- The quark-quark corrector:

$$\Phi(x,k_T,S_T)\Big|_{\text{twist-3}} = \frac{M}{2P^+}\left\{-\epsilon_T^{\rho\sigma}\gamma_\rho S_{T\sigma}f_T' + \frac{(k_T\cdot S_T)\epsilon_T^{\rho\sigma}\gamma_\rho k_{T\sigma}}{M^2}f_T^\perp - \frac{k_T\cdot S_T}{M}\frac{[\not{n}_+,\not{n}_-]\gamma_5}{2}h_T + \frac{[\not{S}_T,\not{k}_T]\gamma_5}{2M}h_T^\perp + \cdots\right\}$$

Calculation of the TMD twist-3 distribution functions

- Cut diagrams for the spectator model calculation at tree level and one-loop level:





- Phenomenological approach to avoid the light-cone divergence—using dipolar form factor to replace the point-like:

$$\lambda \to \lambda(p^2) = \frac{N_X(p^2 - m^2)}{(p^2 - \Lambda^2)^2}$$

- Using a spectator model with axial-diquark , distinguish the axial-diquark by its isospin, $a(ud)$: isoscalar , $a'(uu)$: isovector

$$f^u = {c_s}^2 f^s + {c_a}^2 f^a, \qquad f^d = {c_{a'}}^2 f^{a'}$$

- Sum for axial-diquark polarizations (Brodsky *et al.*, NPB593, 311(2011))

$$d_{\mu\nu}(P-k) = -g_{\mu\nu} + \frac{(P-k)_\mu n_{-\nu} + (P-k)_\nu n_{-\mu}}{(P-k)\cdot n_-} - \frac{M_v^2}{\left[(P-k)\cdot n_-\right]^2} n_{-\mu} n_{-\nu}$$

Calculation of TMD distribution functions

- The distributions contributed by the scalar diquark:

$$h_T^s(x,\boldsymbol{k}_T^2) = \frac{{N_s}^2(1-x)^2}{16\pi^3}\frac{[(1-x)^2M^2-\boldsymbol{k}_T^2-M_s^2]}{(\boldsymbol{k}_T^2+L_s^2)^4},$$

$$h_T^{\perp s}(x,\boldsymbol{k}_T^2) = \frac{N_s^2(1-x)^2}{16\pi^3}\frac{1}{(\boldsymbol{k}_T^2+L_s^2)^4}\times\left[(1-x)(M^2+2mM+xM^2)-\boldsymbol{k}_T^2-M_s^2\right]$$

$$f_T^s(x,\boldsymbol{k}_T^2) = -\frac{{N_s}^2(1-x)^2}{32\pi^3}\frac{e_se_q}{4\pi}\frac{(x+\frac{m}{M})(L_s^2-\boldsymbol{k}_T^2)}{L_s^2(L_s^2+\boldsymbol{k}_T^2)^3}$$

$$f_T^{\perp s}(x,\boldsymbol{k}_T^2) = 0$$

- The distributions contributed by the axial-vector diquark:

$$h_T^v(x,\boldsymbol{k}_T^2) = \frac{N_v^2(1-x)}{16\pi^3}\frac{1}{(\boldsymbol{k}_T^2+L_v^2)^4}\times\left[(1-x)(m^2+2xmM+xM^2)+\boldsymbol{k}_T^2-xM_v^2\right]$$

$$h_T^{\perp v}(x,\boldsymbol{k}_T^2) = \frac{N_v^2(1-x)}{16\pi^3}\frac{1}{(\boldsymbol{k}_T^2+L_v^2)^4}\times\left[(1-x)(m^2-xM^2)-\boldsymbol{k}_T^2+xM_v^2\right]$$

$$f_T^v(x,\boldsymbol{k}_T^2) = 0$$

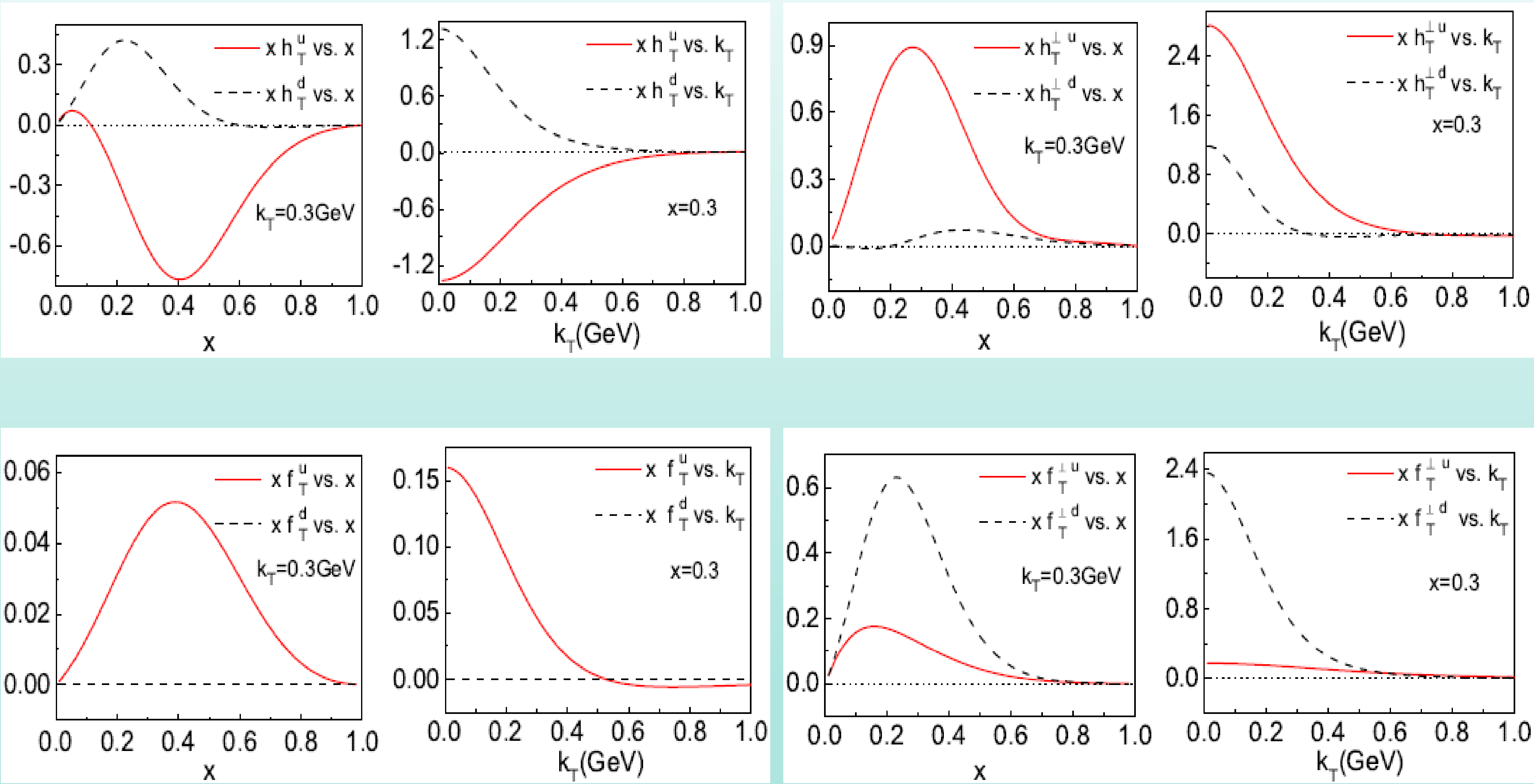
$$f_T^{\perp v}(x,\boldsymbol{k}_T^2) = -\frac{N_v^2(1-x)^2M(m+xM)}{16\pi^3(L_v^2+\boldsymbol{k}_T^2)^2\boldsymbol{k}_T^2}\frac{e_ve_q}{4\pi}\times\left[\frac{1}{\boldsymbol{k}_T^2}\ln\frac{\boldsymbol{k}_T^2+L_v^2}{L_v^2}+\frac{\boldsymbol{k}_T^2-L_v^2}{L_v^2(L_v^2+\boldsymbol{k}_T^2)}\right]$$

- Parameters from Bacchetta, Conti, Radici, PRD78, 074010(2008).

$$f^u = {c_s}^2 f^s + {c_a}^2 f^a, \qquad f^d = {c_{a'}}^2 f^{a'}$$

Calculation of TMD distribution functions

- Model results for the distribution functions:



Wenjuan Mao, Zhun Lu, Bo-Qiang Ma, Phys. Rev. D90, 014048 (2014).

- Formalism
- The calculation of the four involved twist-3 PDFs $f_T(x,\boldsymbol{k_T^2})$, $f_T^{\perp}(x,\boldsymbol{k_T^2})$, $h_T(x,\boldsymbol{k_T^2})$, and $h_T^{\perp}(x,\boldsymbol{k_T^2})$ in the spectator model
- Prediction on the transverse SSAs for pion production in semi-inclusive DIS
- Conclusion

Fragmentation functions and Constraints

- The involved fragmentation functions:

$$F_{UT}^{\sin\phi_S} \approx \frac{2M}{Q}\mathcal{C}\left\{ x f_T D_1 + \frac{\boldsymbol{p}_T\cdot\boldsymbol{k}_T}{2zMM_h}\left(x h_T H_1^\perp - x h_T^\perp H_1^\perp\right)\right\}$$

$$F_{UT}^{\sin(2\phi_h-\phi_S)} \approx \frac{2M}{Q}\mathcal{C}\left\{ \frac{2(\hat{\boldsymbol{P}}_T\cdot\boldsymbol{k}_T)^2-\boldsymbol{k}_T{}^2}{2M^2}\left(x f_T^\perp D_1\right) + \frac{2(\hat{\boldsymbol{P}}_T\cdot\boldsymbol{p}_T)(\hat{\boldsymbol{P}}_T\cdot\boldsymbol{k}_T)-\boldsymbol{p}_T\cdot\boldsymbol{k}_T}{2zMM_h}\times\left[x h_T H_1^\perp + x h_T^\perp H_1^\perp\right]\right\}$$

$$D_1^q\left(z,\boldsymbol{p}_T^2\right) = D_1^q(z)\,\frac{1}{\pi\langle \boldsymbol{p}_T^2\rangle}\,e^{-\boldsymbol{p}_T^2/\langle \boldsymbol{p}_T^2\rangle}$$

Leading-order set of the DSS parameterization (D. de Florian, R. Sassot, and M. Stratmann, Phys. Rev. D75, 114010 (2007).)

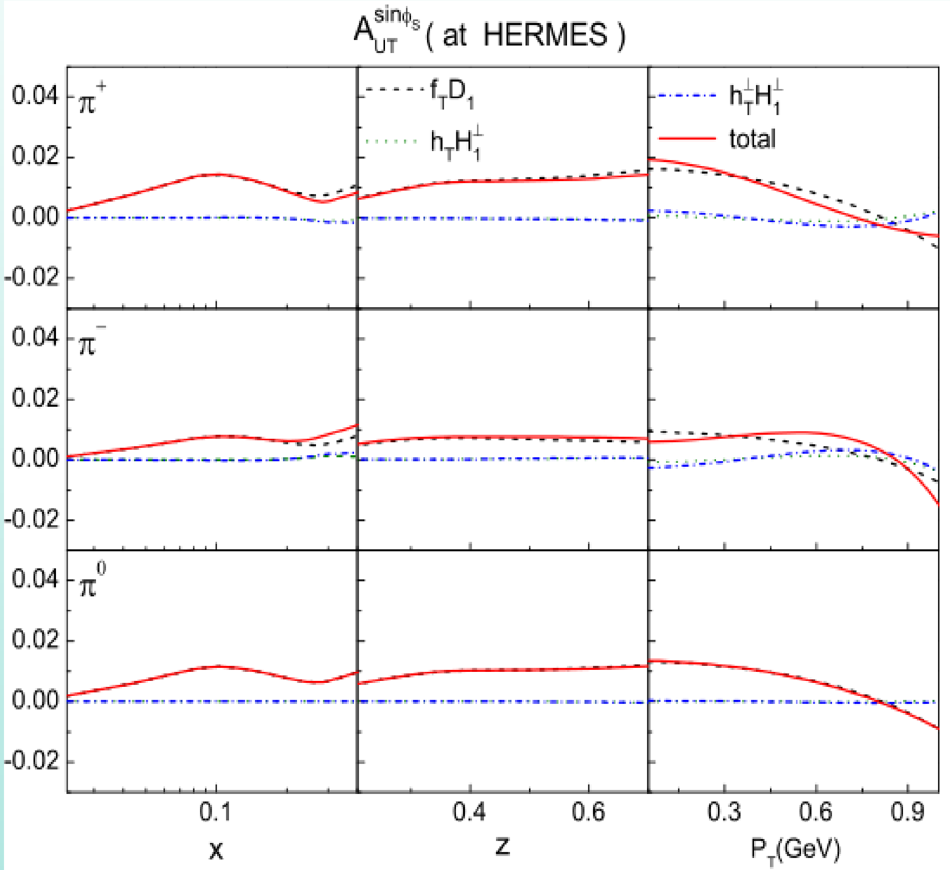
$$H_1^{\perp\pi^+/u} = H_1^{\perp\pi^-/d} \equiv H_{1fav}^\perp,$$
$$H_1^{\perp\pi^+/d} = H_1^{\perp\pi^-/u} \equiv H_{1unf}^\perp,$$
$$H_1^{\perp\pi^0/u} = H_1^{\perp\pi^0/d} \equiv \frac{1}{2}\left(H_{1fav}^\perp + H_{1unf}^\perp\right)$$

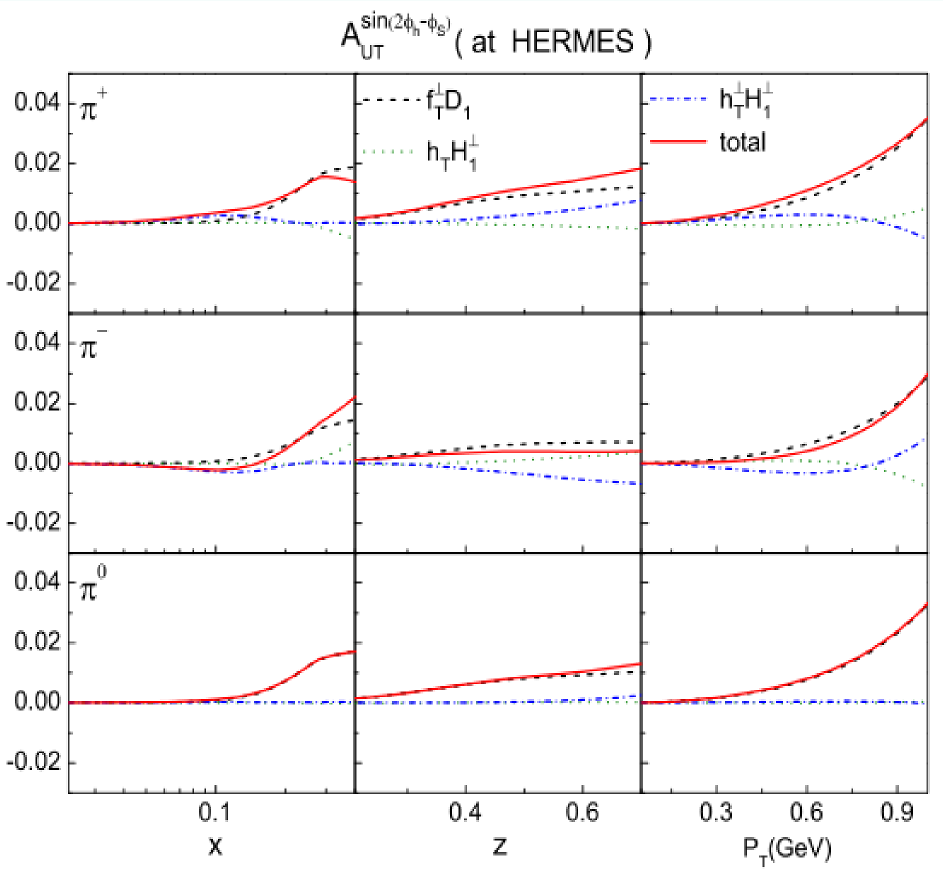
M. Anselmino, M. Boglione, U. D'Alesio, A. Kotzinian, F. Murgia, A. Prokudin, and S. Melis, Nucl. Phys. Proc. Suppl. 191, 98 (2009).

- Constraints for quark transverse momentum k_T (Boglione, Melis, Prokudin, Phy. Rev. D84, 034033(2011)) :

$$\begin{cases} k_T^2 \le (2-x)(1-x)Q^2, & \text{for } \; 0<x<1; \\ k_T^2 \le \frac{x(1-x)}{(1-2x)^2}\,Q^2, & \text{for } \; x<0.5. \end{cases}$$

Transverse SSAs at HERMES





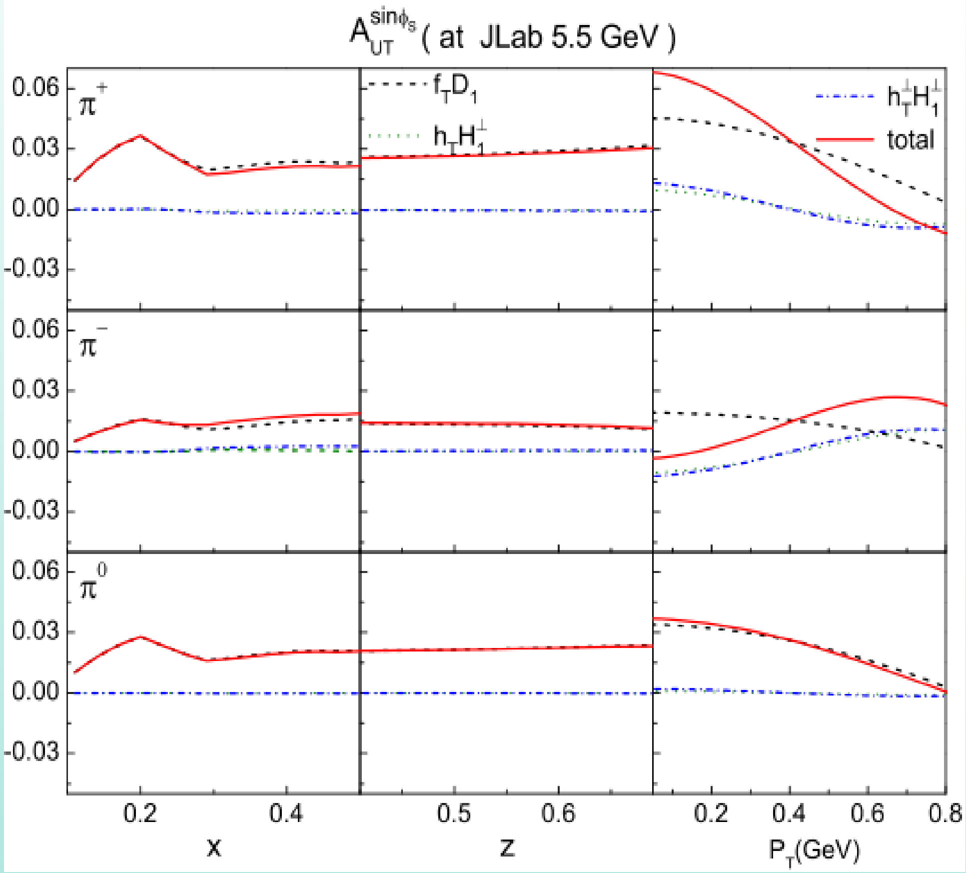
- Kinematics at HERMES (A. Airapetian *et al.,* Phys. Rev. Lett. 103, 152002 (2009)):

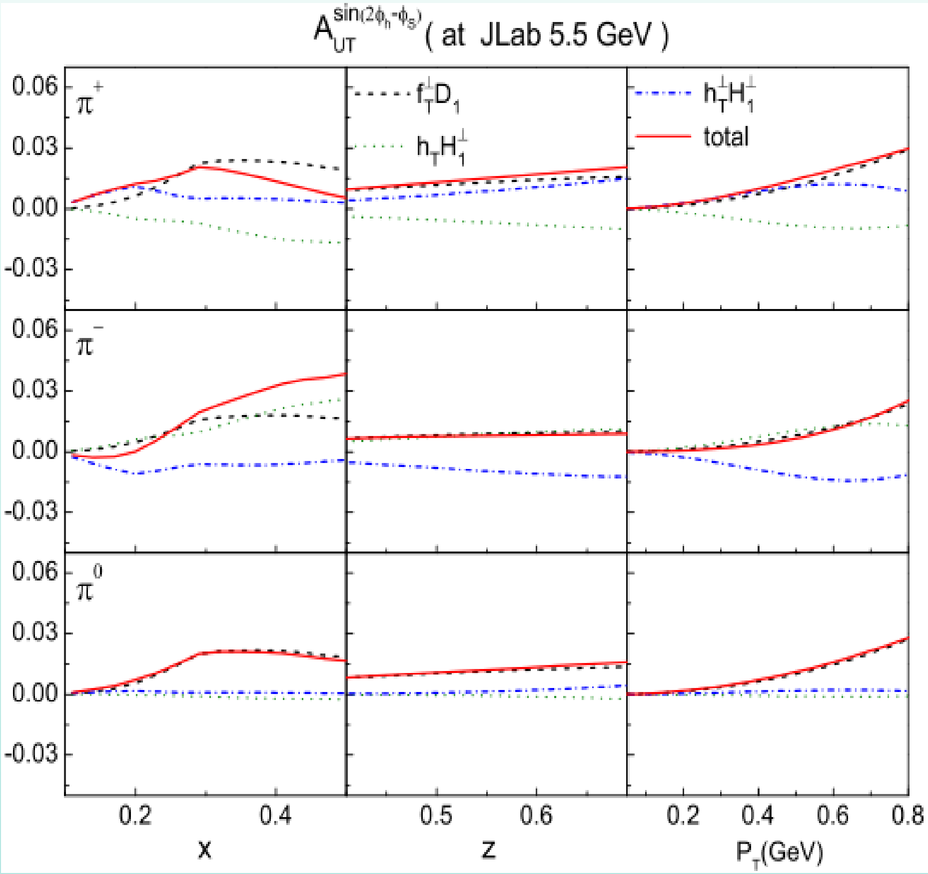
$$E_{beam} = 27.6\text{GeV}, \quad 0.023 < x < 0.4,$$

$$0 < y < 0.85, \quad 1\text{GeV}^2 < Q^2 < 15\,\text{GeV}^2,$$

$$W^2 > 4\,\text{GeV}^2, \quad 2\,\text{GeV} < P_h < 15\,\text{GeV}.$$

Transverse SSAs at JLab 5.5 GeV





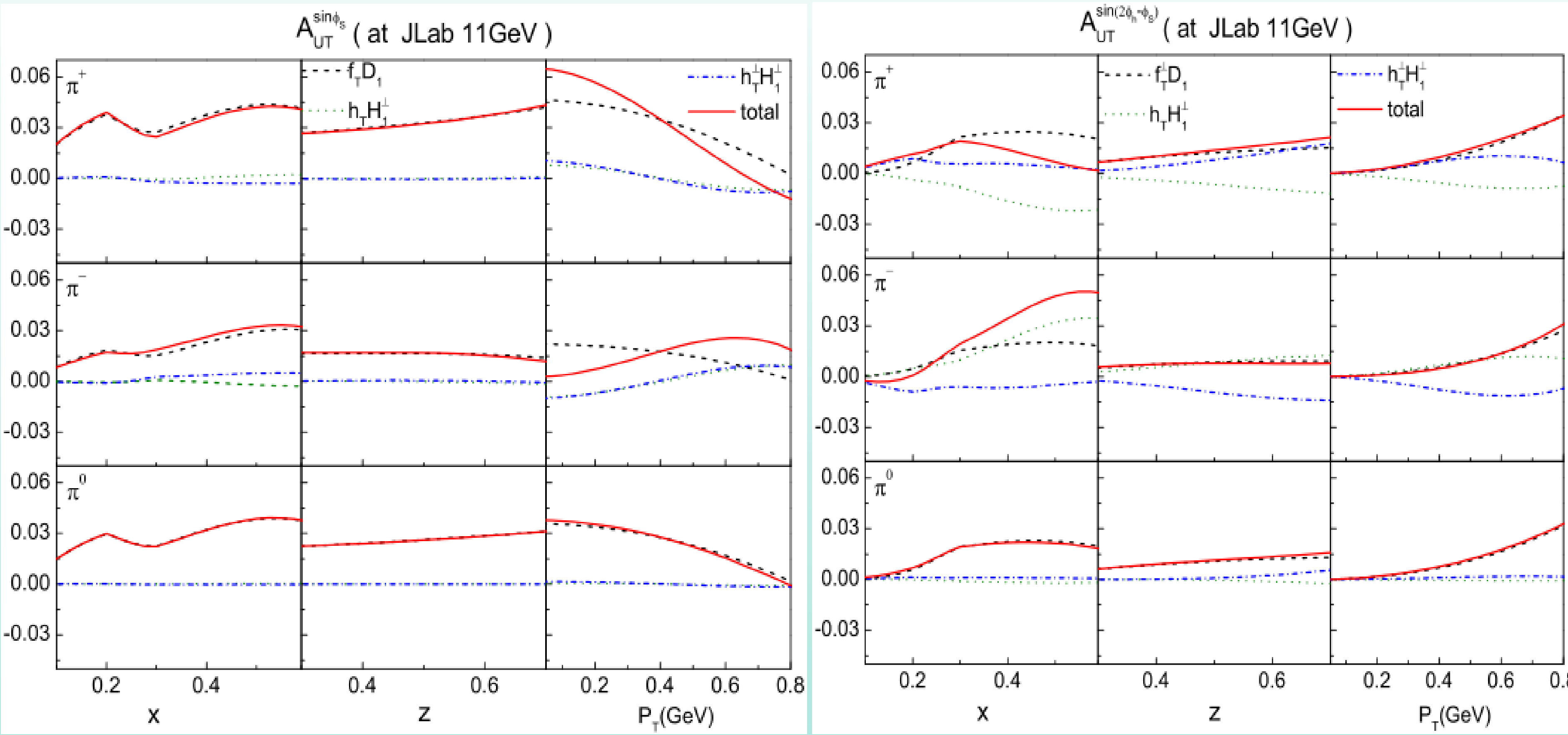
- Kinematics at JLab 5.5GeV (H. Avakian, Nuovo Cimento 36, 73 (2013)):

$$E_{beam} = 5.5\text{GeV}, \quad 0.1 < x < 0.6,$$

$$0.4 < z < 0.7, \quad Q^2 > 1\,\text{GeV}^2,$$

$$W^2 > 4\,\text{GeV}^2, \quad P_T > 0.05\,\text{GeV}.$$

Transverse SSAs at JLab 11 GeV



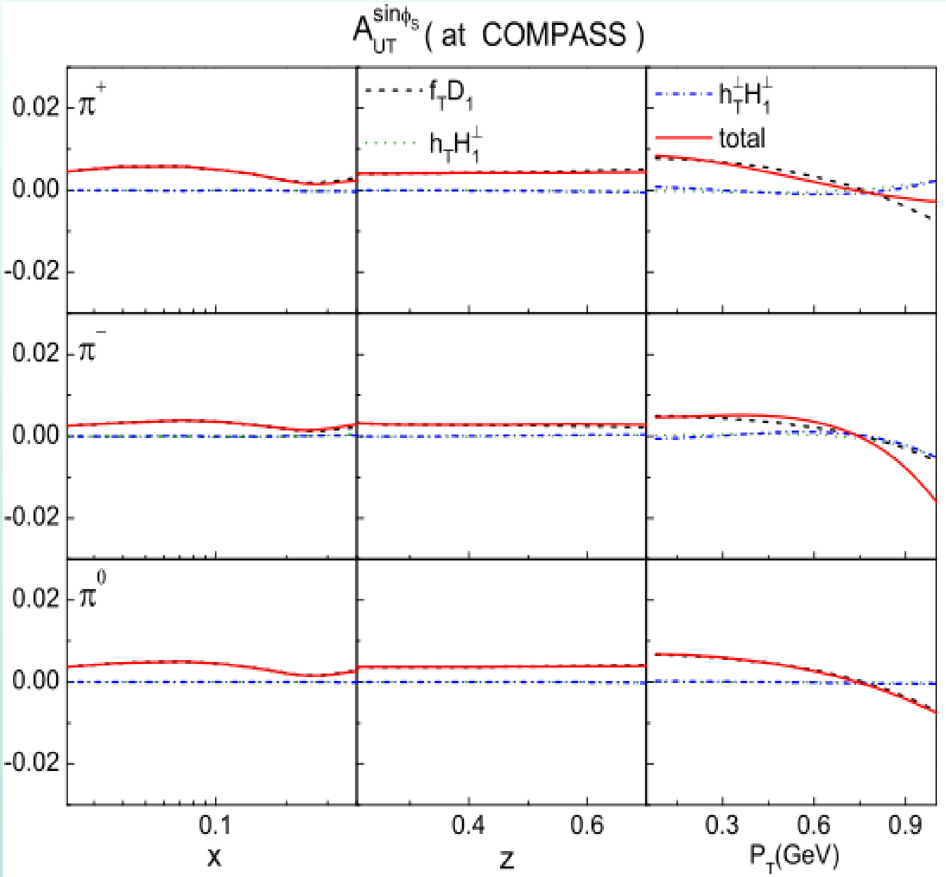
- Kinematics at JLab 11GeV (H. Avakian, AIP Conf. Proc. 1338, 464 (2011)):

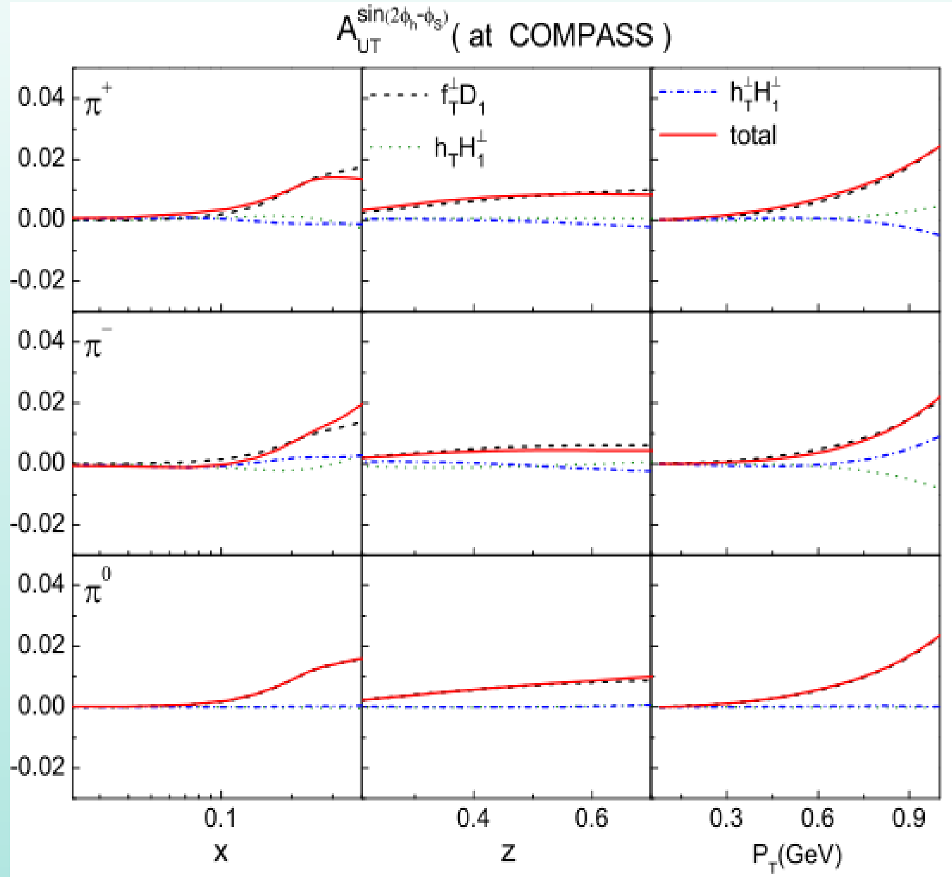
$$E_{beam} = 11\text{GeV}, \qquad 0.08 < x < 0.6,$$

$$0.3 < y < 0.8, \qquad Q^2 > 1\,\text{GeV}^2,$$

$$W^2 > 4\,\text{GeV}^2, \quad 0.05\,\text{GeV} < P_T < 0.8\,\text{GeV}.$$

Transverse SSAs at COMPASS





- Kinematics at COMPASS (M.G. Alekseev *et al.,* Phys. Lett. B 692, 240 (2010)):

$$E_{beam} = 160\text{GeV}, \qquad 0.004 < x < 0.7,$$

$$0.1 < y < 0.9, \qquad z > 0.2, \qquad P_T > 0.1\text{GeV},$$

$$Q^2 > 1\,\text{GeV}^2, \qquad W > 5\,\text{GeV}, \qquad E_h > 1.5\,\text{GeV}.$$

Conclusion

- Sizable transverse single spin asymmetries at subleading twist may be accessible at the kinematics of HERMES, JLab, and COMPASS.

- The measurements on the P_T dependence of the asymmetry $A_{UT}^{\sin\phi_S}$ may be employed to test the transverse momentum dependence of the twist-3 distribution function f_T (e.g., there is a node of f_T in k_T) .

- Measuring the transverse single spin asymmetries at subleading twist for π^0 are viable to provide clean probes on both of the twist-3 T-odd distribution functions f_T and $f_T^\perp$, since the contributions from the twist-3 T-even distribution functions h_T and $h_T^\perp$ are negligible.