The nucleon spin and angular momentum

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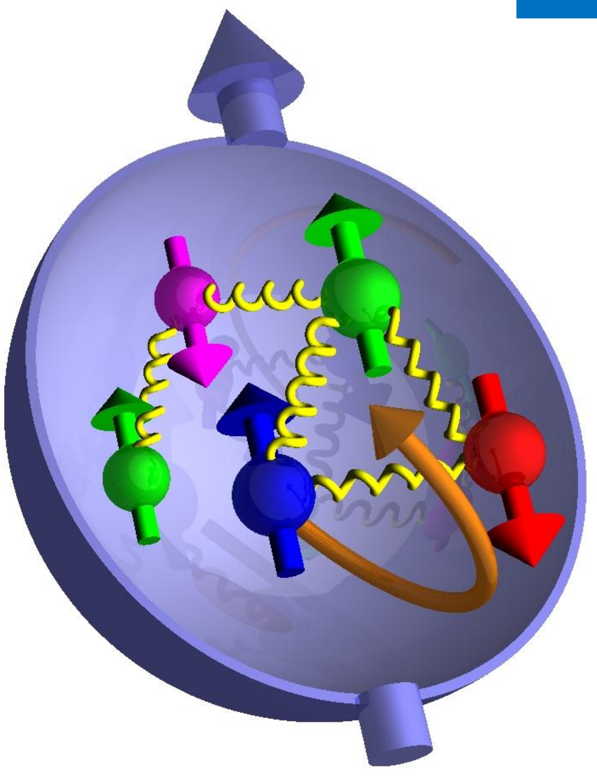
Outline

- Proton spin decomposition: Problems and resolution
- Orbital angular momentum
- Twist analysis
- Transverse polarization
- Method to compute ΔG on a lattice

1101.5989 (PRD)
1111.3547 (PLB)
1207.5332 (JHEP) with Shinsuke Yoshida
1211.2918 (JHEP) with Kazuhiro Tanaka and S. Yoshida
1310.4263 (PRD) with Xiangdong Ji and Yong Zhao

The proton spin problem

The proton has spin ½.
The proton is not an elementary particle.



$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_z$$

Quarks' helicity

Gluons' helicity

Orbital angular Momentum (OAM)

Quark model prediction: $\Delta\Sigma = 1$

$\Delta\Sigma \approx 0.7$ with relativistic effects

`Spin crisis'

In 1987, EMC (European Muon Collaboration) announced a very small value of the quark helicity contribution

$$\Delta\Sigma = 0.12 \pm 0.09 \pm 0.14 \quad !?$$

Latest result from NLO global analysis

$$\Delta\Sigma \approx 0.3 \qquad \int_{0.05}^{1} dx\, \Delta G(x) \approx 0.2$$

DeFlorian, Sassot, Stratmann, Vogelsang (2014)

QCD angular momentum tensor

QCD Lagrangian → Lorentz invariant $x^\mu \longrightarrow x^\mu + \omega^{\mu\nu} x_\nu$

→ Noether current $\partial_\mu M^{\mu\nu\lambda}_{can} = 0$

QCD angular momentum tensor

$$M^{\mu\nu\lambda}_{can} = x^\nu T^{\mu\lambda}_{can} - x^\lambda T^{\mu\nu}_{can} - \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}\bar{\psi}\gamma_5\gamma_\rho\psi + F^{\mu\lambda}A^\nu - F^{\mu\nu}A^\lambda$$

quark spin gluon spin

canonical energy momentum tensor

$$T^{\mu\nu}_{can} = \bar{\psi} i \gamma^\mu \overleftrightarrow{\partial}^\nu \psi - F^{\mu\alpha}\partial^\nu A^\alpha - g^{\mu\nu}\mathcal{L}$$

→ Quark OAM → Gluon OAM

Jaffe-Manohar decomposition (1990)

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q_{can} + L^g_{can}$$

Based on the canonical energy momentum tensor

Operators NOT gauge invariant.

Partonic interpretation in the light-cone gauge $A^+ = 0$

Ji decomposition (1997)

Improved (Belinfante) energy momentum tensor

$$\widetilde{T}^{\mu\nu} = T^{\mu\nu}_{can} + \partial_\rho G^{\rho\mu\nu}$$ ← One can add a total derivative.

$$= \bar{\psi} i \gamma^{(\mu} \overleftrightarrow{D}^{\nu)} \psi - F^{\mu\rho} F^{\nu}{}_{\rho} - g^{\mu\nu} \mathcal{L}$$

quark part gluon part

$$\frac{1}{2} = J_q + J_g$$

Further decomposition in the quark part

$$\bar{\psi} i \gamma^{(\mu} \overleftrightarrow{D}^{\nu)} \psi = \bar{\psi} i \gamma^{\mu} \overleftrightarrow{D}^{\nu} \psi - \frac{1}{4} \epsilon^{\mu\nu\rho\sigma} \partial_\rho (\bar{\psi} \gamma_5 \gamma_\sigma \psi)$$

$$J_q = \frac{1}{2} \Delta\Sigma + L_q$$

Generalized parton distributions (GPD)

Non-forward proton matrix element

$$\int d\lambda e^{i\lambda x} \frac{d\lambda}{2\pi} \langle P'S'|\bar{\psi}(0)\gamma^\mu \psi(\lambda)|PS\rangle$$

$$= \underline{H_q(x)}\bar{u}(P'S')\gamma^\mu u(PS) + \underline{E_q(x)}\bar{u}(P'S')\frac{i\sigma^{\mu\nu}\Delta_\nu}{2m}u(PS)$$

Twist-two GPDs

$$J^q = \frac{1}{2}\int dx x(H_q(x) + E_q(x)) \qquad J^g = \frac{1}{4}\int dx(H_g(x) + E_g(x))$$

Two spin communities divided

measured by PHENIX, STAR, COMPASS, HERMES

Jaffe-Manohar

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q_{can} + L^g_{can}$$

common and well-known

not measured yet
not even well-defined?

Define rigorously.
Must be related to GPD!

Ji

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + J_g$$

accessible from GPD at JLab, COMPASS, HERMES, J-PARC…
also calculated in lattice QCD

Complete decomposition

Chen, Lu, Sun, Wang, Goldman (2008)
Wakamatsu (2010)
Y.H. (2011)

$$M^{\mu\nu\lambda}_{\text{quark-spin}} = -\frac{1}{2}\epsilon^{\mu\nu\lambda\sigma}\bar{\psi}\gamma_5\gamma_\sigma\psi\,,$$

$$M^{\mu\nu\lambda}_{\text{quark-orbit}} = \bar{\psi}\gamma^\mu(x^\nu iD^\lambda_{\text{pure}} - x^\lambda iD^\nu_{\text{pure}})\psi\,,$$

$$M^{\mu\nu\lambda}_{\text{gluon-spin}} = F^{\mu\lambda}_a A^{\nu a}_{\text{phys}} - F^{\mu\nu}_a A^{\lambda a}_{\text{phys}}\,,$$

$$M^{\mu\nu\lambda}_{\text{gluon-orbit}} = F^{\mu\alpha}_a\left(x^\nu(D^\lambda_{\text{pure}}A^{\text{phys}}_\alpha)_a - x^\lambda(D^\nu_{\text{pure}}A^{\text{phys}}_\alpha)_a\right)$$

where $A^\mu_{phys} = \frac{1}{D^+}F^{+\mu}$ $\quad D^\mu_{pure} = D^\mu - iA^\mu_{phys}$

Gauge invariant completion of Jaffe-Manohar

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q_{can} + L^g_{can}$$

OAM from the Wigner distribution

Wigner distribution in QCD   Belitsky, Ji, Yuan (2003)

$$W(\vec{x},\vec{q}) = \int \frac{d^4 z}{(2\pi)^4} e^{iqz} \bar{\psi}\left(x - \frac{z}{2}\right)\gamma^\mu \psi\left(x + \frac{z}{2}\right)$$

position   momentum

Need a Wilson line !

Define $\vec{L} = \int dq\, \vec{x} \times \vec{q} \langle W(\vec{x},\vec{q})\rangle$ Lorce, Pasquini (2011)

Which OAM is this??

Canonical OAM from the light-cone Wilson line

YH (2011)

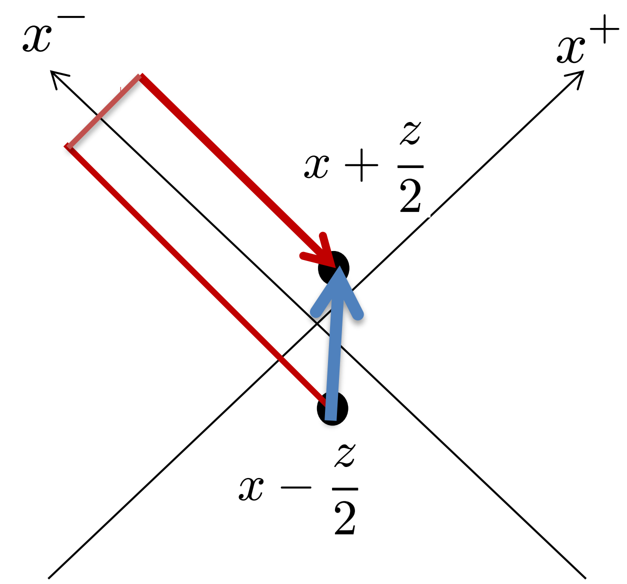
$$\int dq\, \vec{x} \times \vec{q} \langle W_{light-cone}(x,q)\rangle = \langle \bar{\psi}\gamma^\mu \vec{x} \times i \overleftrightarrow{D}_{pure}\psi\rangle$$

Kinetic OAM from the straight Wilson line

Ji, Xiong, Yuan (2012)

$$\int dq\, \vec{x} \times \vec{q} \langle W_{straight}(x,q)\rangle$$

$$= \langle \bar{\psi}\gamma^\mu \vec{x} \times i \overleftrightarrow{D}\psi\rangle$$



Difference between the two OAMs

$$L_{pot} = L - L_{can} = \vec{x} \times \int dx^- \vec{F}$$

` Potential' OAM

Torque acting on a quark

Burkardt (2012)

Twist analysis

YH, Yoshida (2012)

see, also, Ji, Xiong, Yuan (2012)

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q_{can} + L^g_{can}$$

Understand these relations at the density level

$$\Delta\Sigma = \sum_f \int dx \Delta q_f(x) \qquad \Delta G = \int dx \Delta G(x)$$

$$L^q_{can} = \int dx L^q_{can}(x) \qquad ??$$

c.f. $$\Delta q(x) = \frac{1}{4\pi S^+} \int dz^- e^{ixP^+z^-} \langle PS|\bar{\psi}(z^-)\gamma^+\gamma_5\psi(0)|PS\rangle$$

`Density' of OAM

Ji's OAM canonical OAM `potential OAM'

$$\langle \bar{\psi} x \times D \psi \rangle = \langle \bar{\psi} x \times D_{pure} \psi \rangle + ig \langle \bar{\psi} x \times A_{phys} \psi \rangle$$

$$A^{\mu}_{phys} = \frac{1}{D^+} F^{+\mu}$$

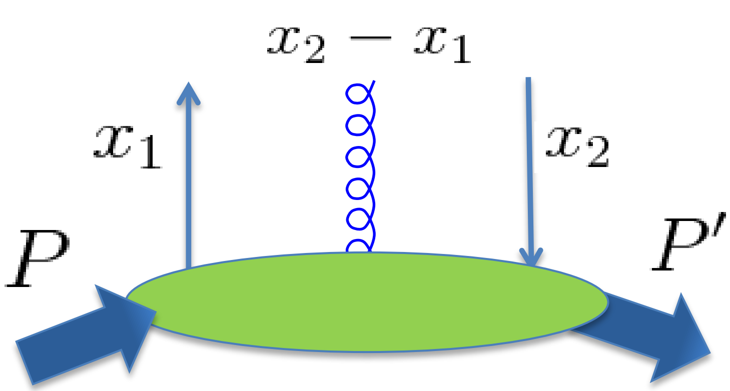
``F-type"

For a 3-body operator, it is natural to define the double density.

$$\int d\lambda d\mu e^{i\frac{\lambda}{2}(x_1+x_2)+i\mu(x_1-x_2)} \langle P'S' | \bar{\psi}(-\lambda/2) D^i(\mu) \psi(\lambda/2) | PS \rangle$$

$$\sim \epsilon^{ij} \Delta_j S^+ \Phi_D(x_1, x_2)$$

``D-type"



The D-type and F-type correlators are related.

Eguchi, Koike, Tanaka (2006)

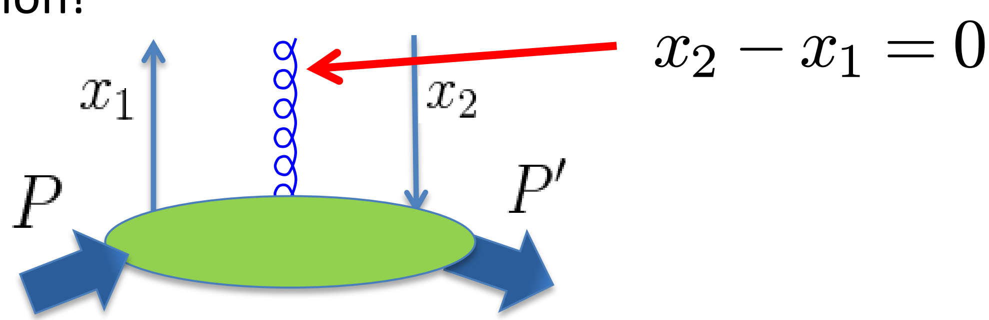
$$\langle \bar{\psi} x \times D\psi \rangle = \langle \bar{\psi} x \times D_{pure}\psi \rangle + ig\langle \bar{\psi} x \times A_{phys}\psi \rangle$$

doubly-unintegrate

$$\Phi_D(x_1, x_2) = \delta(x_1 - x_2) L^q_{can}(x_1) + \mathcal{P}\frac{1}{x_1 - x_2}\Phi_F(x_1, x_2)$$

Canonical OAM density

The gluon has zero energy
→ partonic interpretation!



Relation between $L^q_{can}(x)$ and twist-3 GPD

$$\int d\lambda e^{i\lambda x}\frac{d\lambda}{2\pi}\langle P'S'|\bar{\psi}(0)\gamma^\mu\psi(\lambda)|PS\rangle$$

$$= H_q(x)\bar{u}(P'S')\gamma^\mu u(PS) + E_q(x)\bar{u}(P'S')\frac{i\sigma^{\mu\nu}\Delta_\nu}{2m}u(PS)$$ twist-2

$$+\underline{G_3(x)}\bar{u}(P'S')\gamma^\mu_\perp u(PS) + \cdots$$ ← twist-3

From the equation of motion,

$$x(H_q(x) + E_q(x) + \underline{G_3(x)}) =$$

$$\Delta q(x) + \underline{L^q_{can}(x)} + \int dx'\mathcal{P}\frac{1}{x-x'}\left(\Phi_F(x,x') + \tilde{\Phi}_F(x,x')\right)$$

integrate ⇒ $$\int dx x G_3(x) = -L^q$$

Penttinen, Polyakov, Shuvaev, Strikman (2000)

Quark canonical OAM density

Wandzura-Wilczek part

$$L^q_{can}(x) = x\int_x^{\epsilon(x)} \frac{dx'}{x'}(H_q(x')+E_q(x')) - x\int_x^{\epsilon(x)} \frac{dx'}{x'^2}\Delta q(x')$$
$$-x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 \Phi_F(x_1,x_2)\mathcal{P}\frac{3x_1-x_2}{x_1^2(x_1-x_2)^2}$$
$$-x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 \tilde{\Phi}_F(x_1,x_2)\mathcal{P}\frac{1}{x_1^2(x_1-x_2)}\,.$$

genuine twist-three

First moment: $J^q = \frac{1}{2}\Delta\Sigma + L^q_{can} + L_{pot}$

The bridge between JM and Ji

Gluon canonical OAM density $L^g_{can}(x)$

$$\frac{1}{2}(H_g(x)+E_g(x)+\underline{F_g(x)})-\Delta G(x)+2\int dX\frac{\Phi_F(X,x)}{x}-\underline{2L^g_{can}(x)}$$

$$=-2\int dx'\,\mathcal{P}\frac{M_F(x,x')}{x(x-x')}-2\int dx'\,\mathcal{P}\frac{\tilde{M}_F(x,x')}{x(x-x')}$$

twist-three gluon GPD

$$L^g_{can}(x)=\frac{x}{2}\int_x^{\epsilon(x)}\frac{dx'}{x'^2}(H_g(x')+E_g(x'))-x\int_x^{\epsilon(x)}\frac{dx'}{x'^2}\Delta G(x')$$

WW part

$$+2x\int_x^{\epsilon(x)}\frac{dx'}{x'^3}\int dX\Phi_F(X,x')+2x\int_x^{\epsilon(x)}dx_1\int_{-1}^{1}dx_2\tilde{M}_F(x_1,x_2)\mathcal{P}\frac{1}{x_1^3(x_1-x_2)}$$

$$+2x\int_x^{\epsilon(x)}dx_1\int_{-1}^{1}dx_2M_F(x_1,x_2)\mathcal{P}\frac{2x_1-x_2}{x_1^3(x_1-x_2)^2}$$

genuine twist-three

first moment: $J^g+L_{pot}=\Delta G+L^g_{can}$

Complete transverse spin decomposition?

YH, Tanaka, Yoshida (2012)

Longitudinal

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q_{can} + L^g_{can}$$

same!

Transverse

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^{q+g}_{can}$$

cannot be separated in a frame-independent way

New movement:
Parton physics on a lattice

Question: Can one measure ΔG on a lattice?

$$\Delta G = \int dx \Delta G(x)$$
$$= \int dx \frac{i}{2xP^+} \int \frac{dy^-}{2\pi} e^{-ixP^+y^-} \langle PS | F^{+\mu}(y^-) W[y^-, 0] \tilde{F}_\mu{}^+(0) | PS \rangle$$

Nonlocal in the light-cone direction.
Real-time problem, not calculable in lattice QCD.
The same problem for PDF, TMD, GPD,…

??

$$\Delta \widetilde{G}(P_z) = \frac{1}{2P^0} \langle PS | \epsilon^{ij} F^{i0} A^j | PS \rangle$$

Instead find something similar
and calculable on a lattice.
Fix the gauge if necessary.
At large momentum $P_z \to \infty$,
there may be a relation. Ji, (2013)

Matching!

Ji, Zhang, Zhao (2013)
YH, Ji, Zhao (2013)

Compute $\Delta\widetilde{G}(P_z)$ in some gauge at large momentum, and do the matching.

$$\Delta\tilde{G}(P^z,\mu) = Z_{gg}(P^z/\mu)\Delta G(\mu) + Z_{gq}(P^z/\mu)\Delta\Sigma(\mu)$$

For instance, in Coulomb gauge, $Z_{gq} = \frac{C_F\alpha_s}{4\pi}\left(\frac{4}{3}\ln\frac{P_z^2}{\mu^2} - \frac{64}{9}\right)$

Caution: Not all gauges are allowed.

Coulomb $\vec{\nabla}\cdot\vec{A} = 0$ ✓

axial 1 $A^0 = 0$ ✓

axial 2 $A^z = 0$ ✓

covariant $\partial\cdot A = 0$ ✗

axial 3 $A^x = 0$ ✗

A bright future?

- $A^0 = 0$ gauge is particularly promising for ΔG

  Balitsky, Braun (1988); Wakamatsu (2013); YH, Ji, Zhao (2013)

- Generalization to the canonical OAMs.

  Ji, Zhang, Zhao (2014)

- x-dependence of PDFs, TMDs, GPDs, etc. may also be calculable. Ji, (2013)
- Matching to all-order? Ma, Qiu, (2014)

→ Talks by Y.Ma, S.Yoshida, J.Zhang, Y.Zhao

Summary

- Complete gauge invariant decomposition of nucleon spin now available in QCD, even at the density level.

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q_{can} + L^g_{can}$$

- Relation between the two decomposition schemes (JM vs Ji) fully revealed. The connection to twist-3 GPDs clarified.
- Progress towards calculating spin components on a lattice.