Nucleon Structure from Lattice QCD

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The University of Adelaide

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Outline

- (Very) Brief introduction to Lattice QCD
  - Sources of systematic error
- Nucleon Axial Charge
- Nucleon Tensor Charge
- Electromagnetic Form Factors
  - strangeness + charge symmetry violation
- Spin content, including Δs [Also see K.-F. Liu (plenary Tue, 8:00)]
- Won't cover new method from X.Ji [PRL 110, 262002 (2013)] (plenary Mon, 8:30)

The Lattice

Ken Wilson (1974)

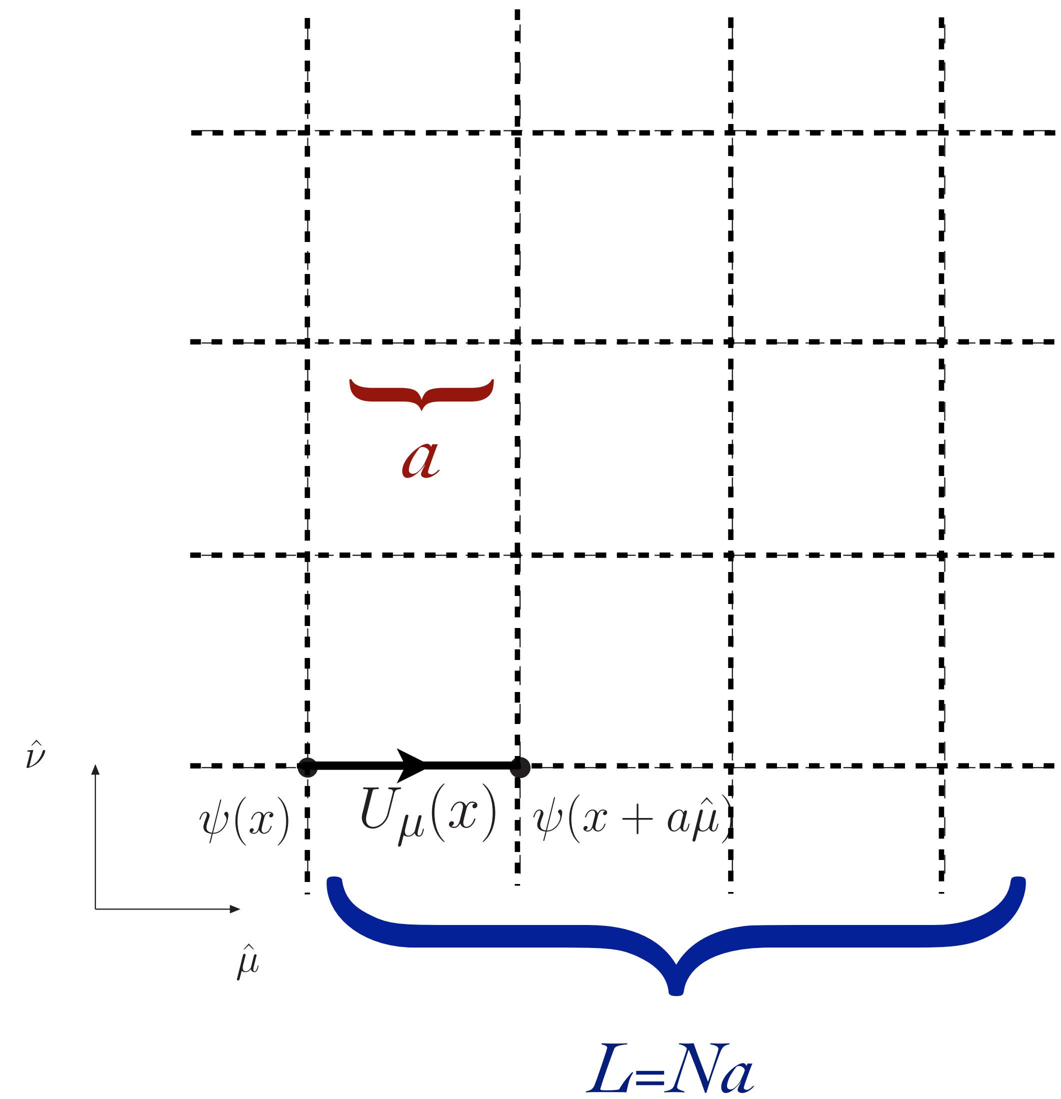
$t \to i\tau$

Lattice QCD provides a first principles, systematically improvable approach for QCD

Discretise space-time with lattice spacing *a* volume *L^3xT*

Quark fields reside on sites $\psi(x)$

Gauge fields on the links $U_\mu(x) = e^{-iagA_\mu(x)}$



Approximate the full QCD path integral by Monte Carlo methods

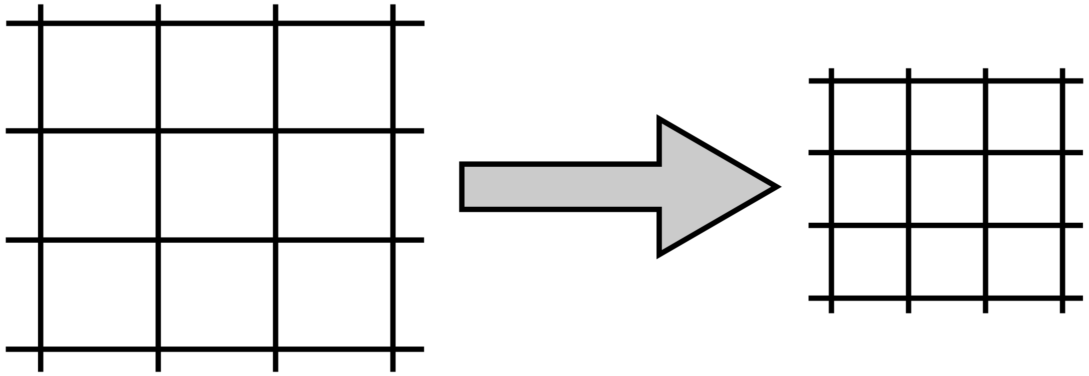
$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}A\mathcal{D}\bar{\psi}\mathcal{D}\psi\, \mathcal{O}[A,\bar{\psi},\psi]\, e^{-S[A,\bar{\psi},\psi]} \Rightarrow \langle \mathcal{O} \rangle \simeq \frac{1}{N_{\rm conf}} \sum_i^{N_{\rm conf}} \mathcal{O}([U^{[i]}])$$

With field configurations U_i distributed according to $e^{-S[U]}$

Put it on a supercomputer

Systematics of a Lattice Calculation

- Extrapolations:
  - Continuum
    - Unavoidable
    - Improved actions (errors O(a^2))
    - Finer lattice spacings



$$a \to 0$$

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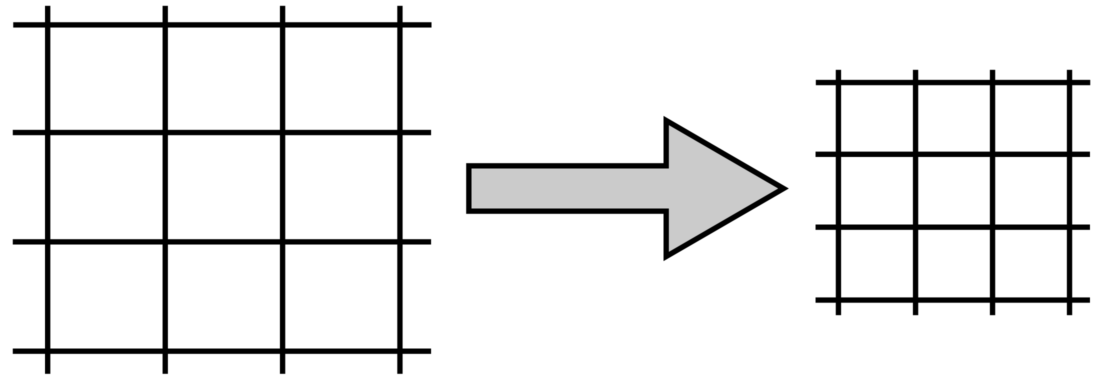
  - Finite volume
    - Large volumes so effects are exponentially suppressed



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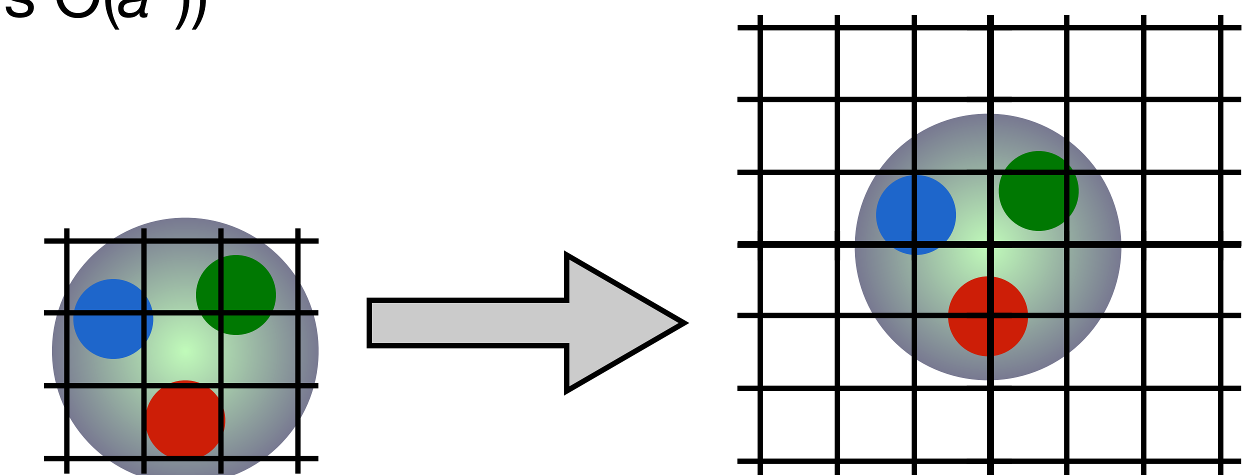

$$L \to \infty$$

Systematics of a Lattice Calculation

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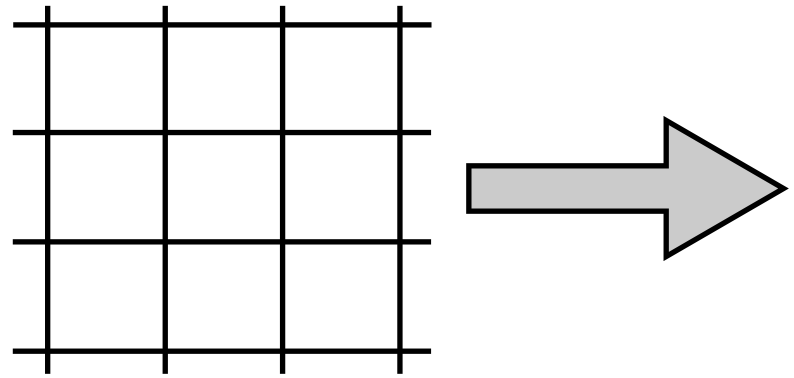
  - Finite volume
    - Large volumes so effects are exponentially suppressed
  - Chiral
    - Chiral perturbation theory
    - Simulate at physical quark masses



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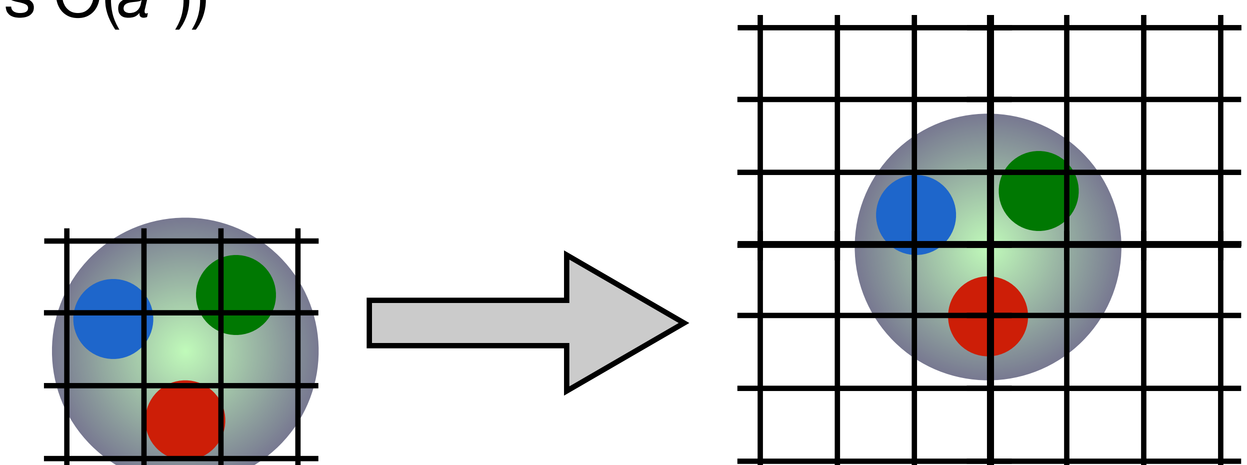

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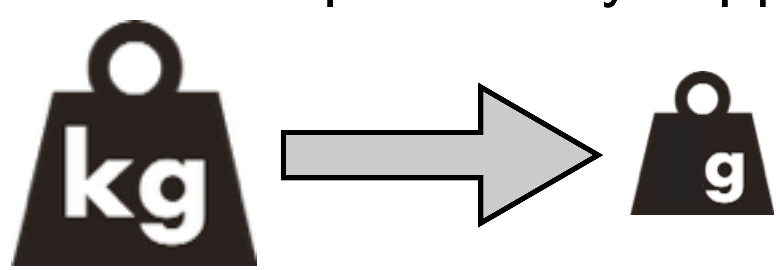
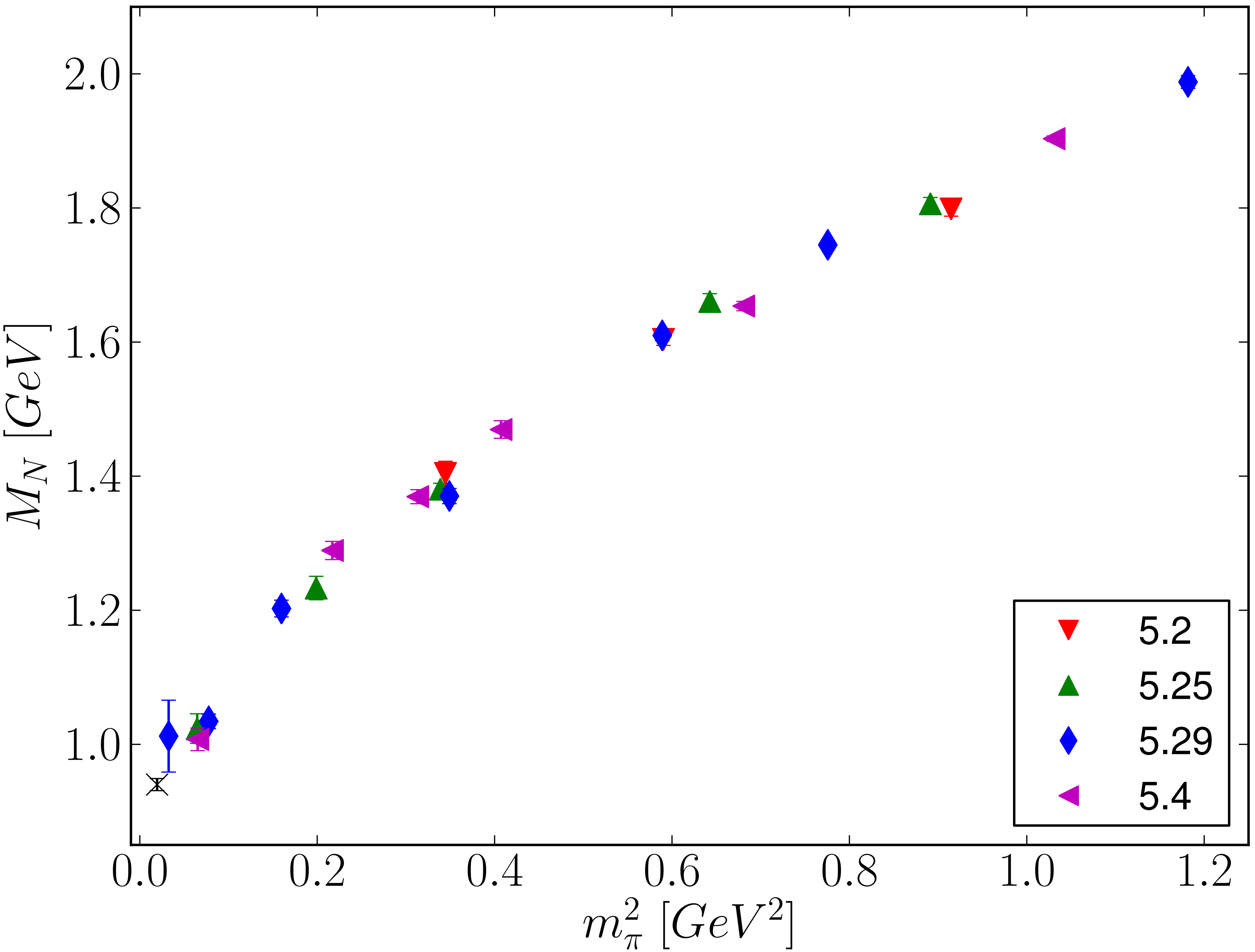


$$m_\pi \to 140 MeV$$

$$\text{GOR} \implies m_\pi^2 \propto m_q$$

Quark Mass Dependence

Nucleon Mass

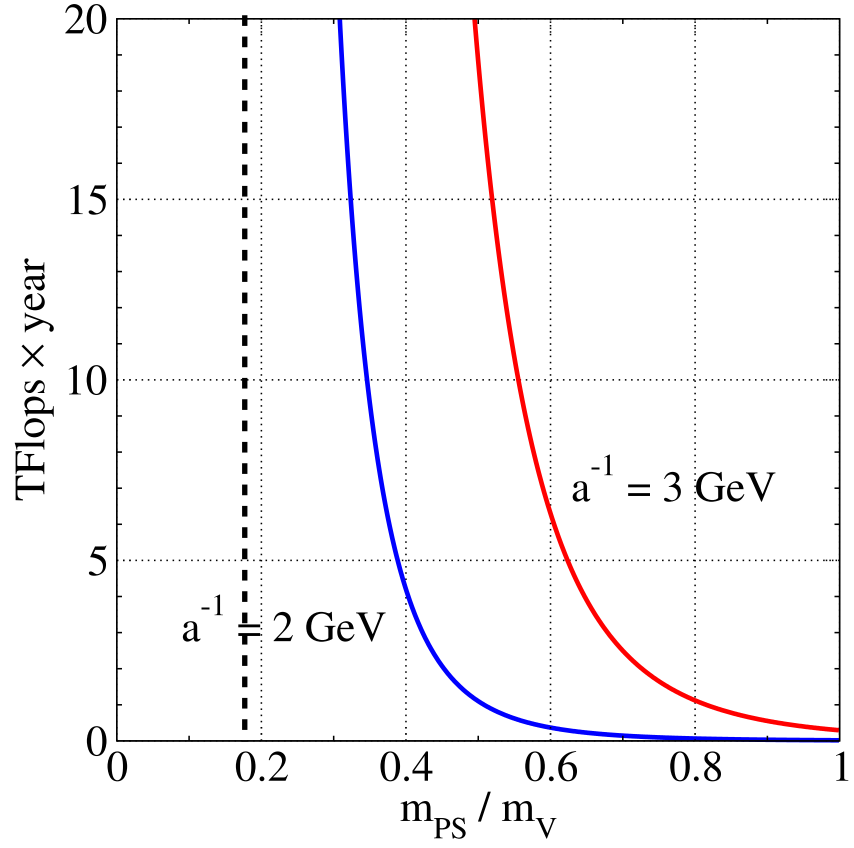


$m_\pi^2 \propto m_q$

Speed of a Lattice Calculation

1000 configurations with L=2fm

[Ukawa (Berlin, 2001)]



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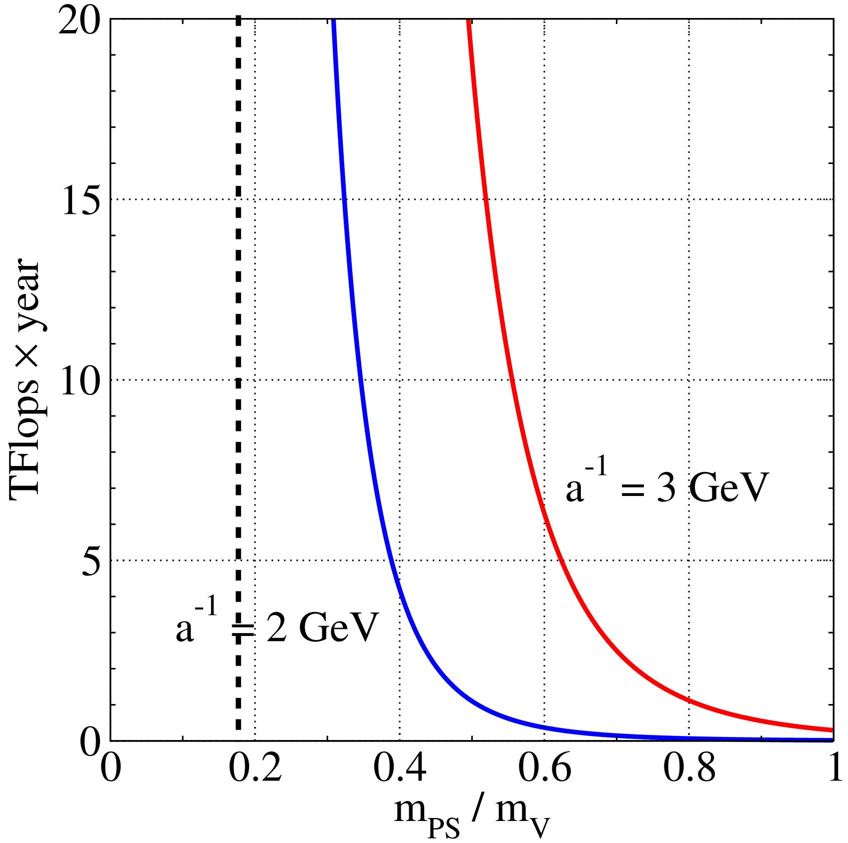
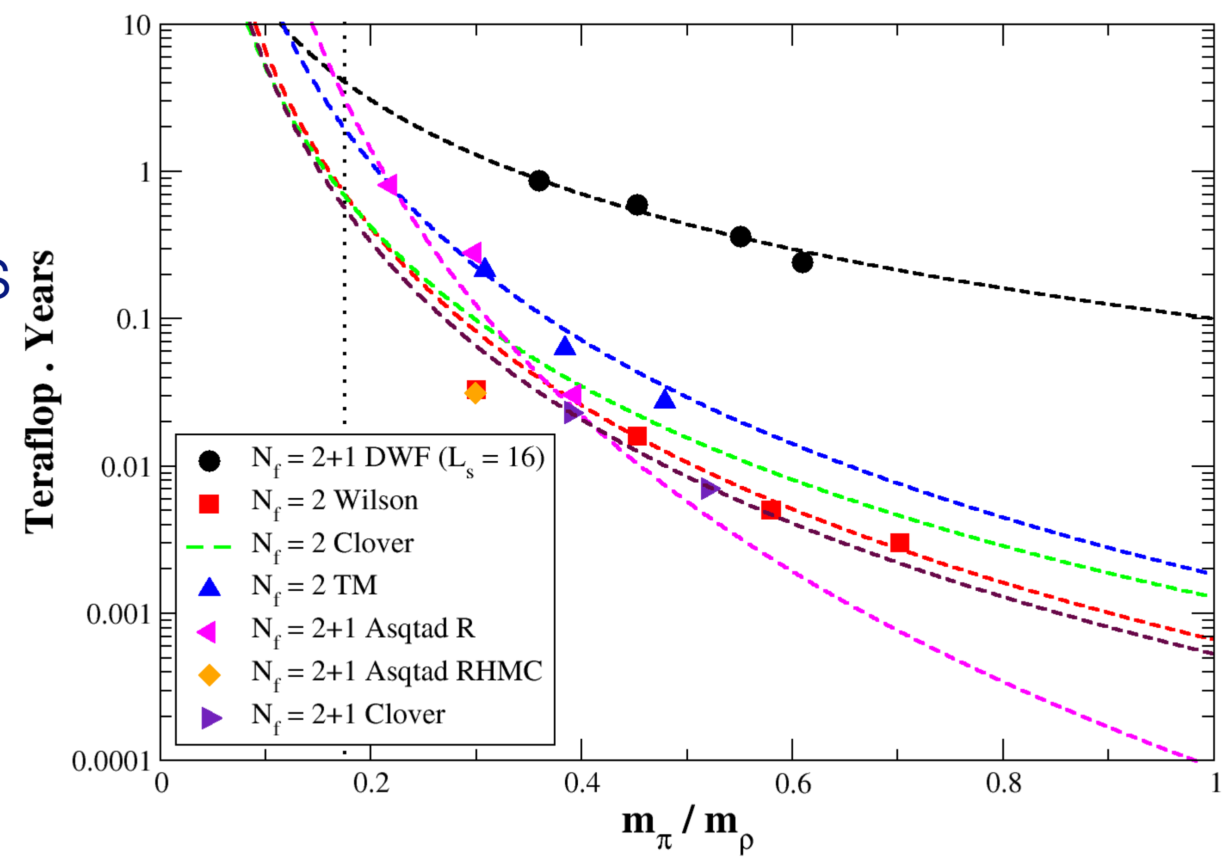


Algorithmic improvements

[Clark (Tucson, 2006)]



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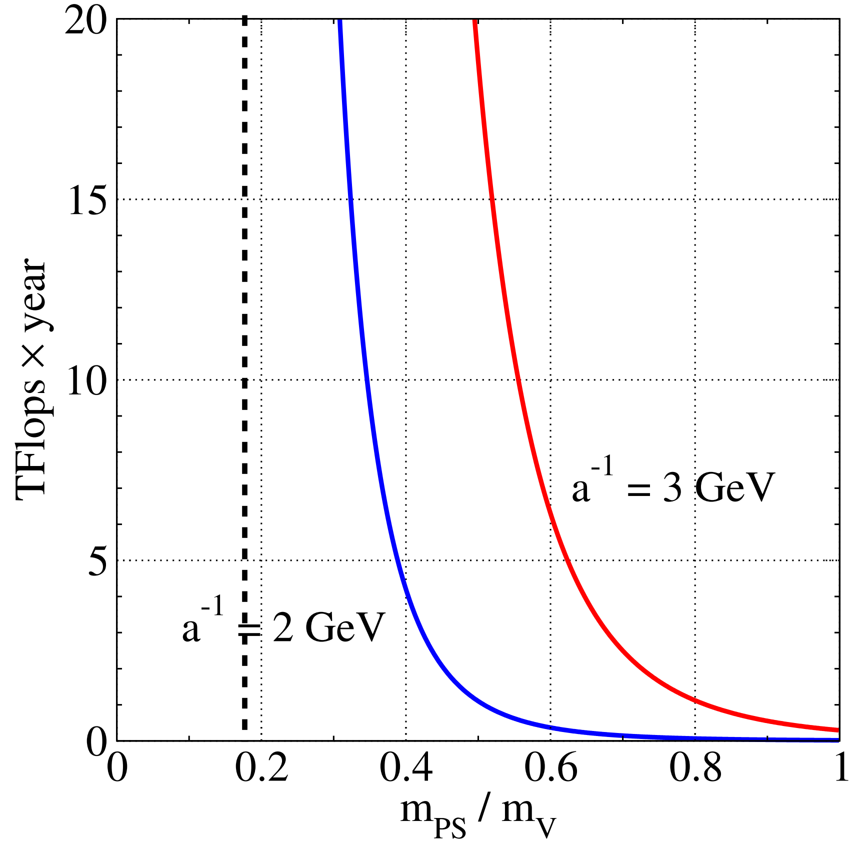

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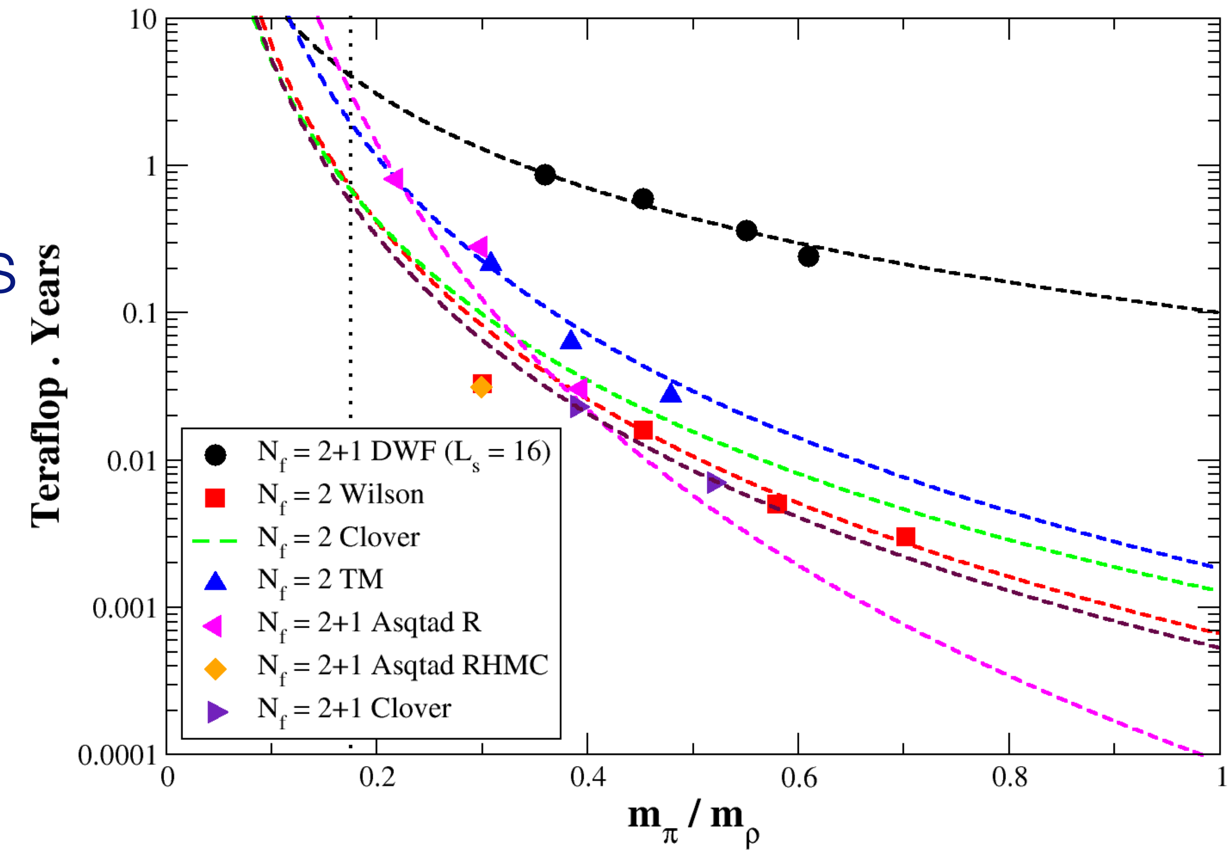

June 2004: Earth Simulator, 36 TFlops

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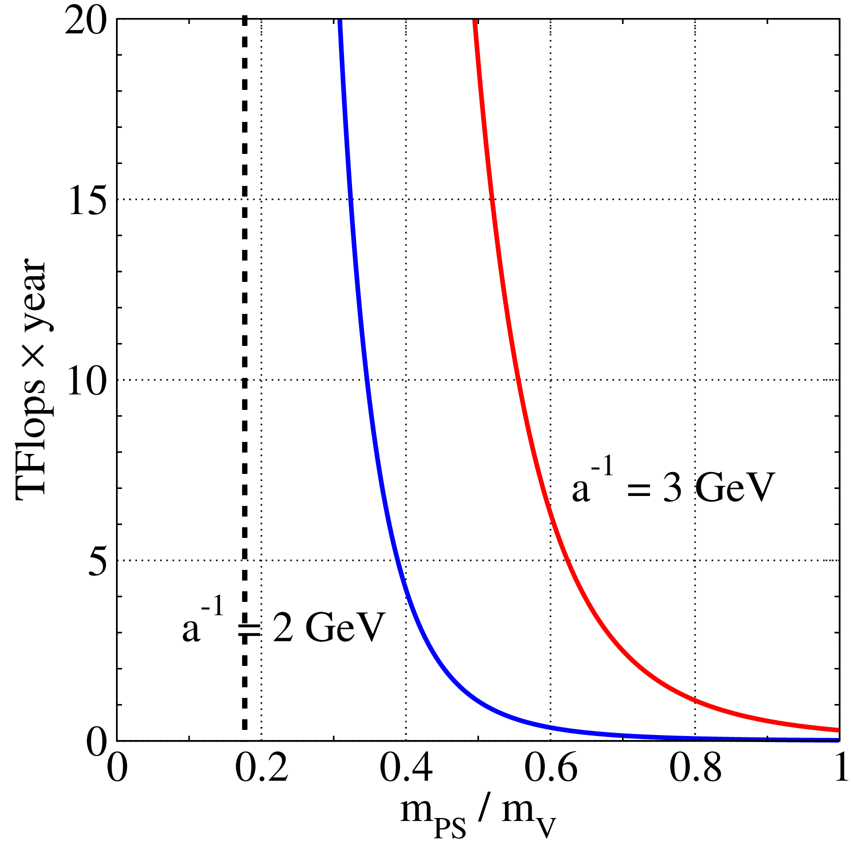

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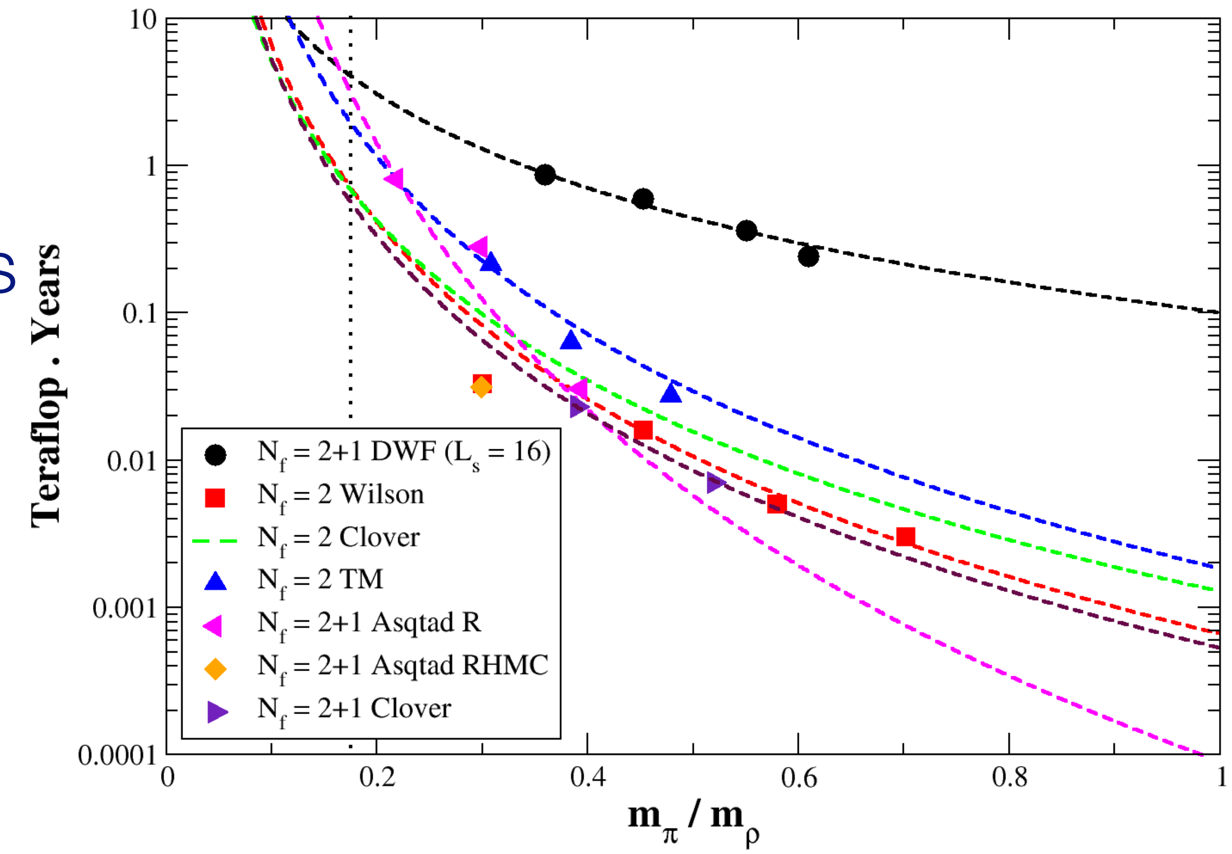
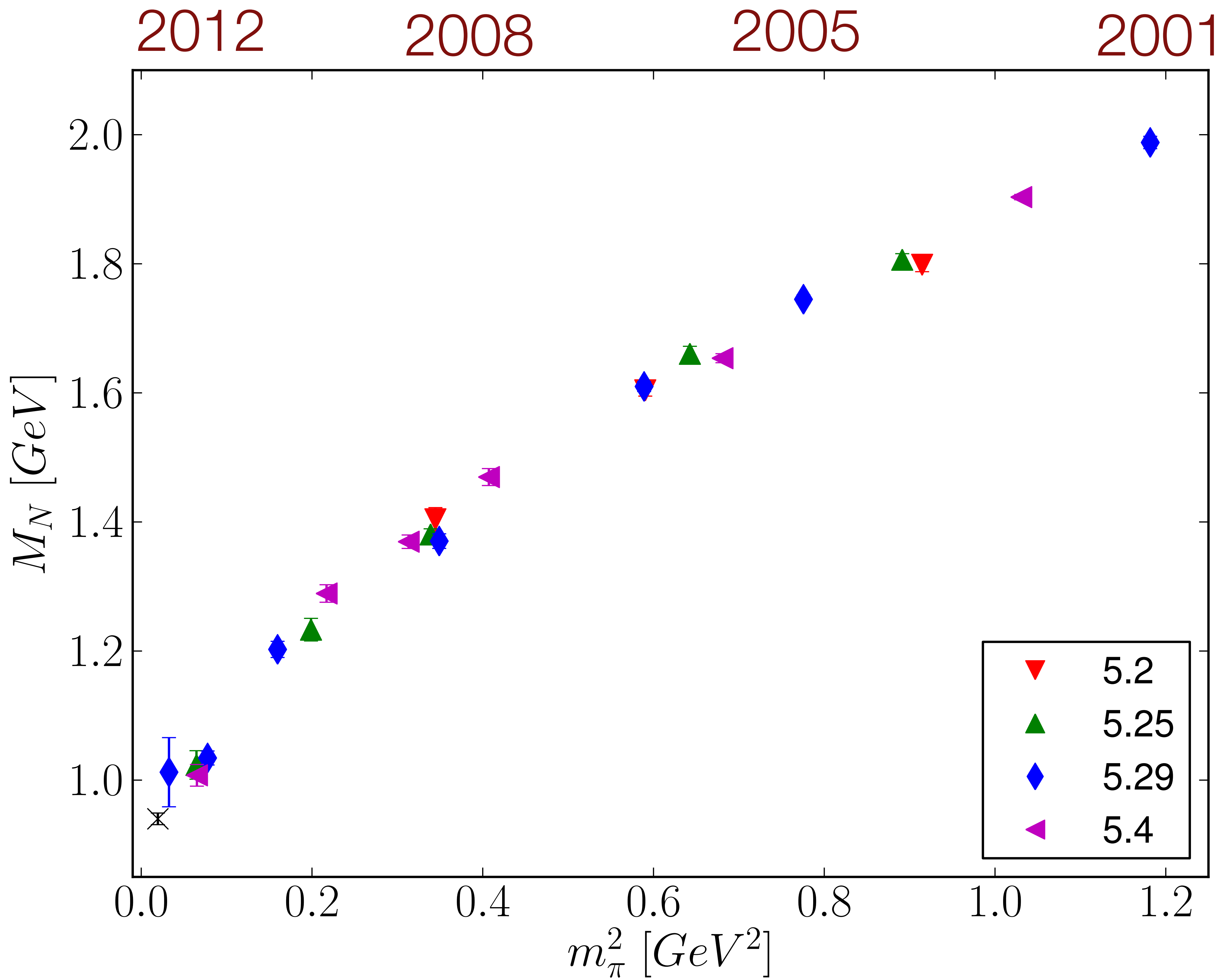


June 2004: Earth Simulator, 36 TFlops

June 2014: Tianhe-2, 34 PFlops

Real-Time Evolution of Lattice Results

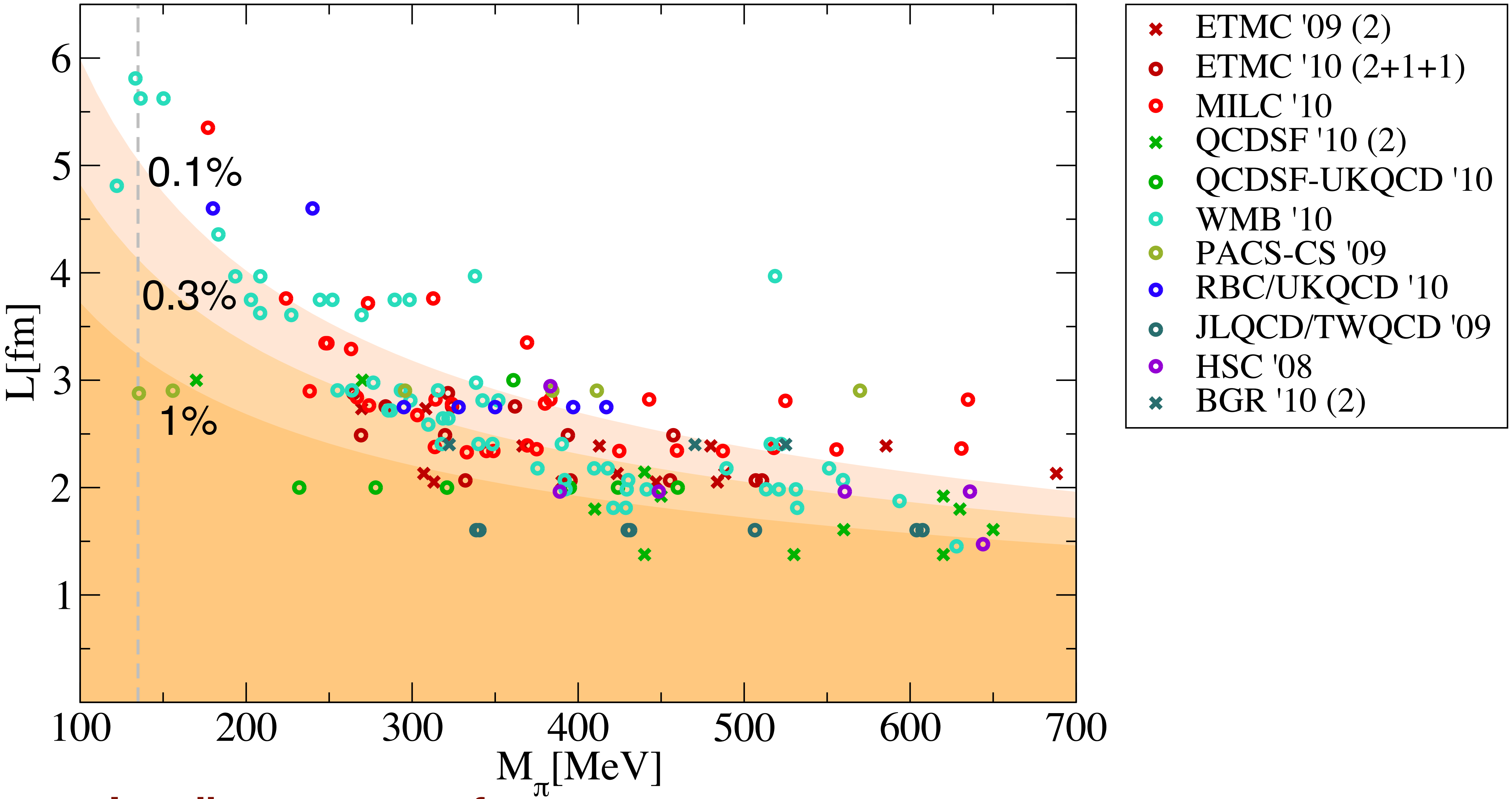
Nucleon Mass



$m_\pi^2 \propto m_q$

The Lattice Landscape

[Hoebling (Lattice 2010) 1102.0410]



- **Leading sources of error:**
    - **Unphysically large quark masses**
    - **Finite Volume**

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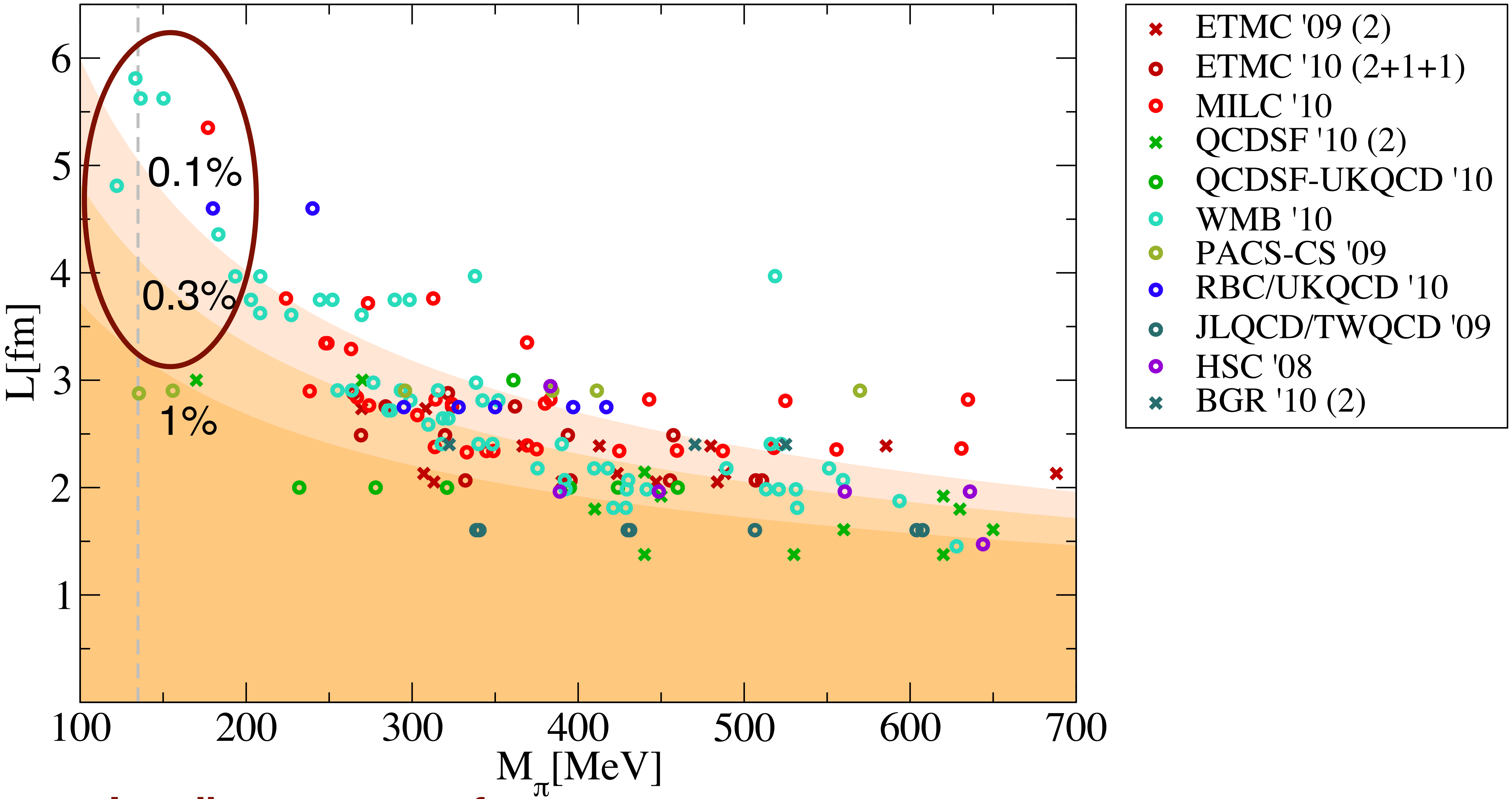

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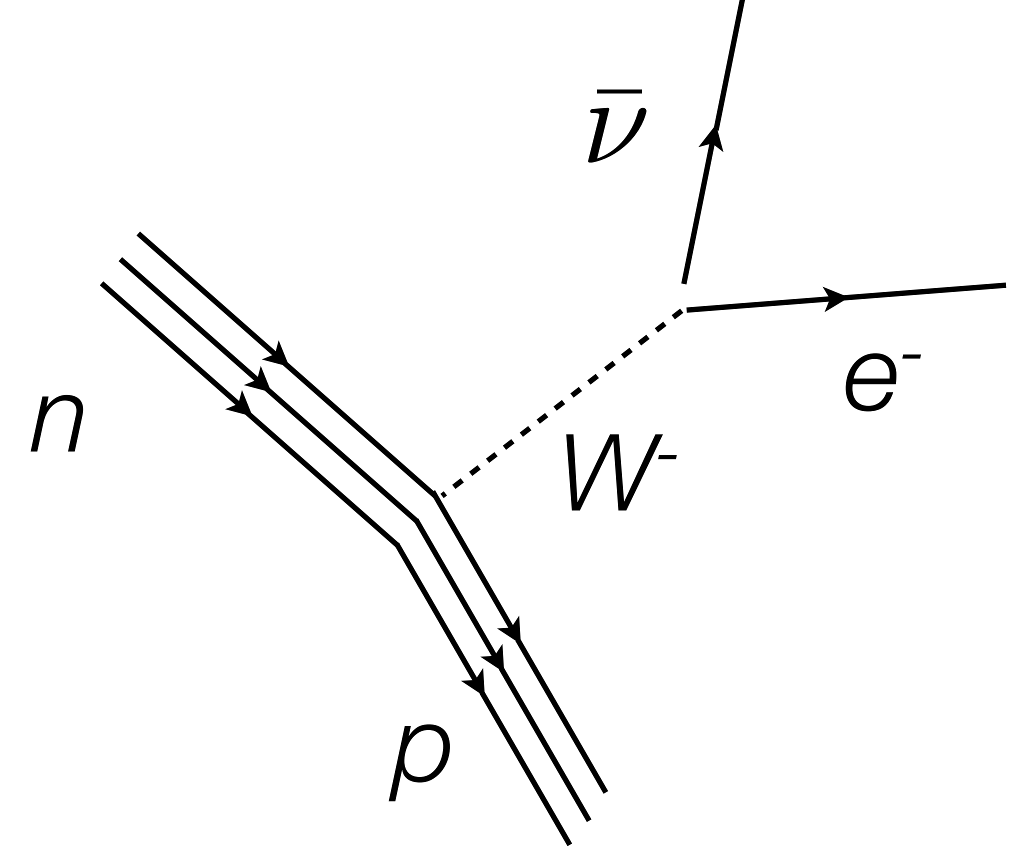
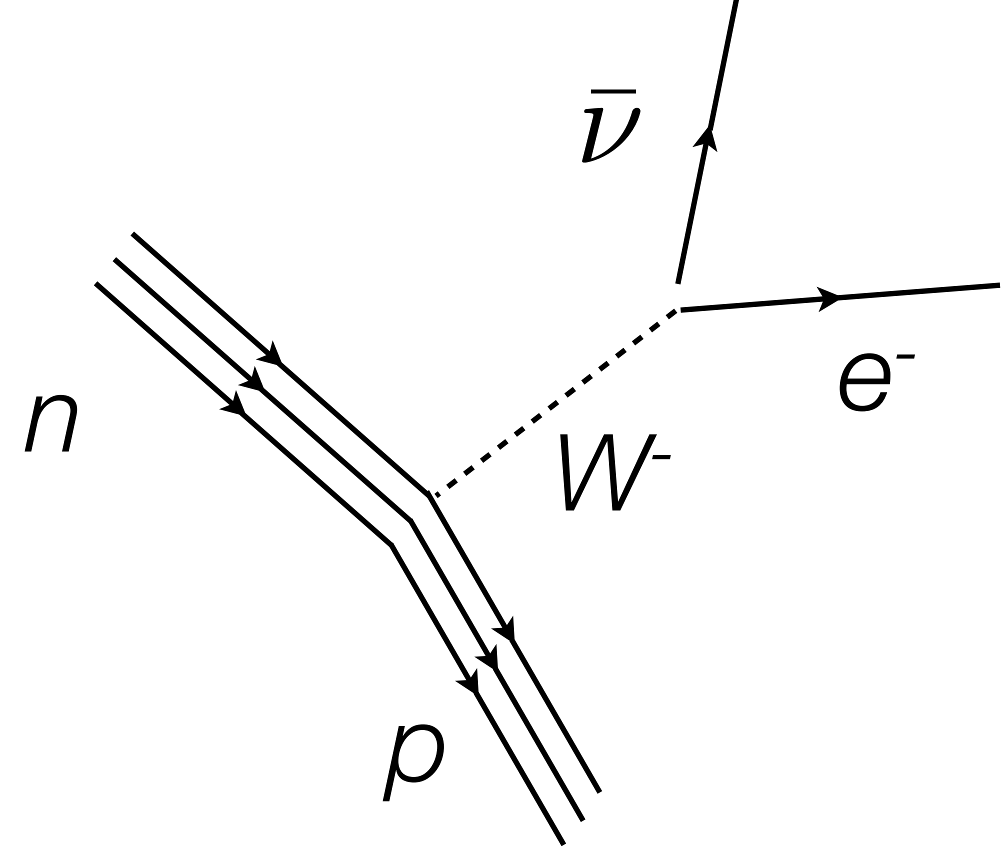


$$n \to p e^- \bar{\nu}_e$$

Neutron axial charge



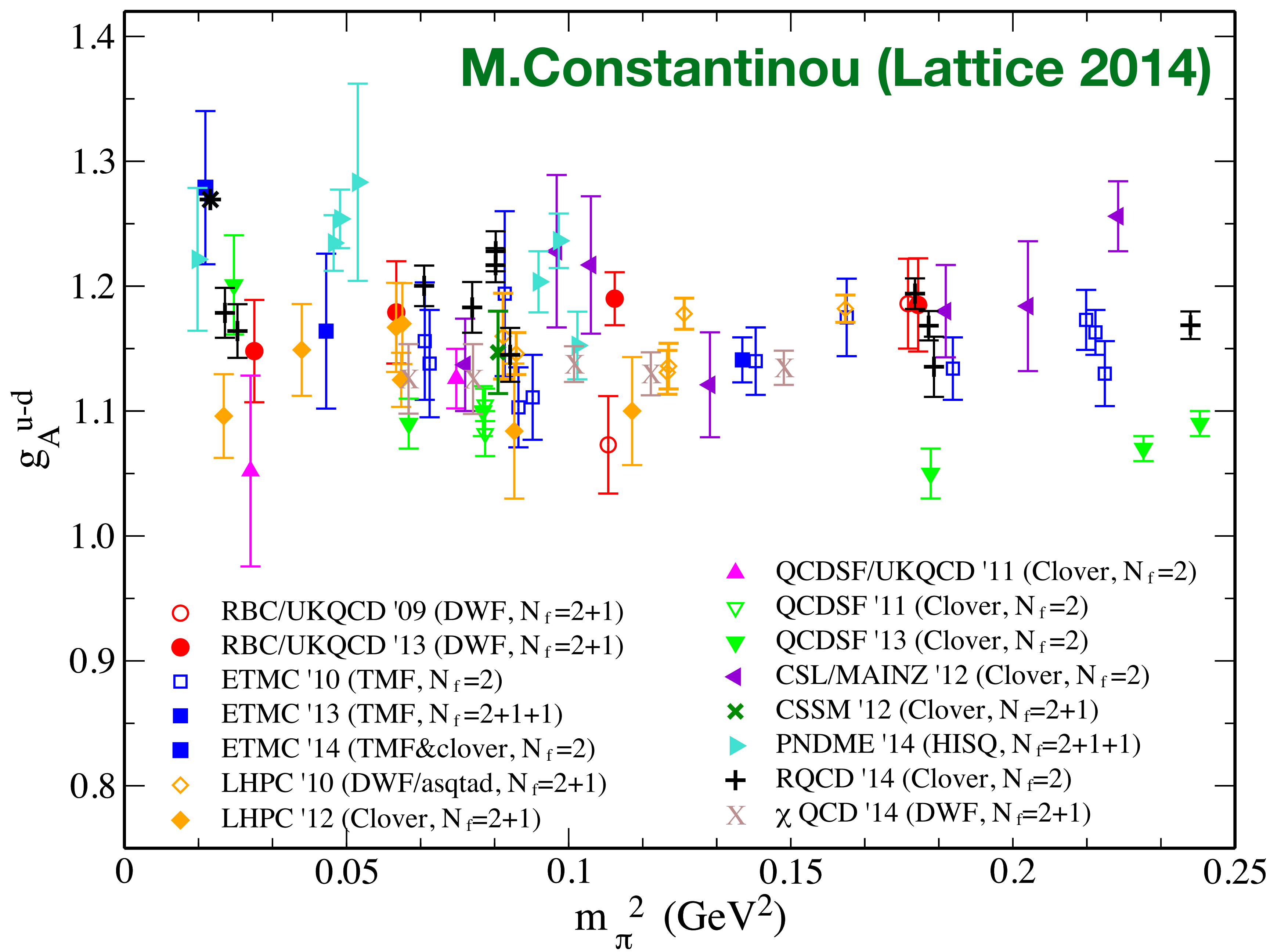
$$n \to p e^- \bar{\nu}_e$$

Neutron axial charge

Relatively simple to compute on the lattice

⇨ **Good benchmark for hadron structure (understanding systematic errors)**

Determination of g_A on the Lattice



Results systematically 10-20% below experiment

Large scatter in the results

Determination of g_A on the Lattice

What about lattice systematic errors?

Finite lattice spacing

Large quark masses

Finite volume

Contamination from excited states

Determination of g_A on the Lattice

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Contamination from excited states

ground state **1st excited state**

$$C(\vec{p},t) = A_0 e^{-E_0(\vec{p})t} + A_1 e^{-E_1(\vec{p})t} + \ldots$$

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ground state **1st excited state**

$$C(\vec{p},t) = A_0 e^{-E_0(\vec{p})t} + A_1 e^{-E_1(\vec{p})t} + \ldots$$

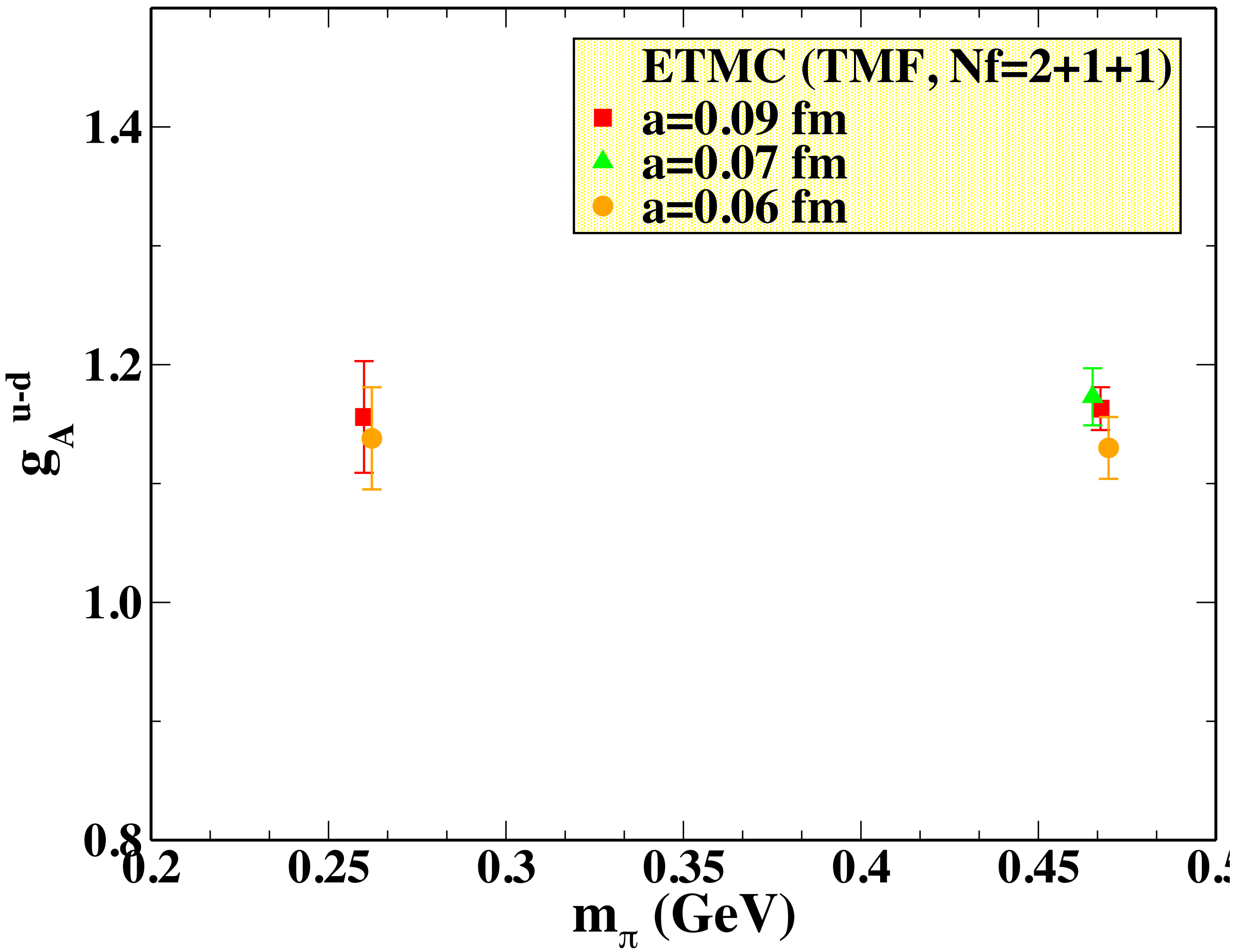
optimal operators ⇐ Maximise (A_0)

Minimise (A_1)

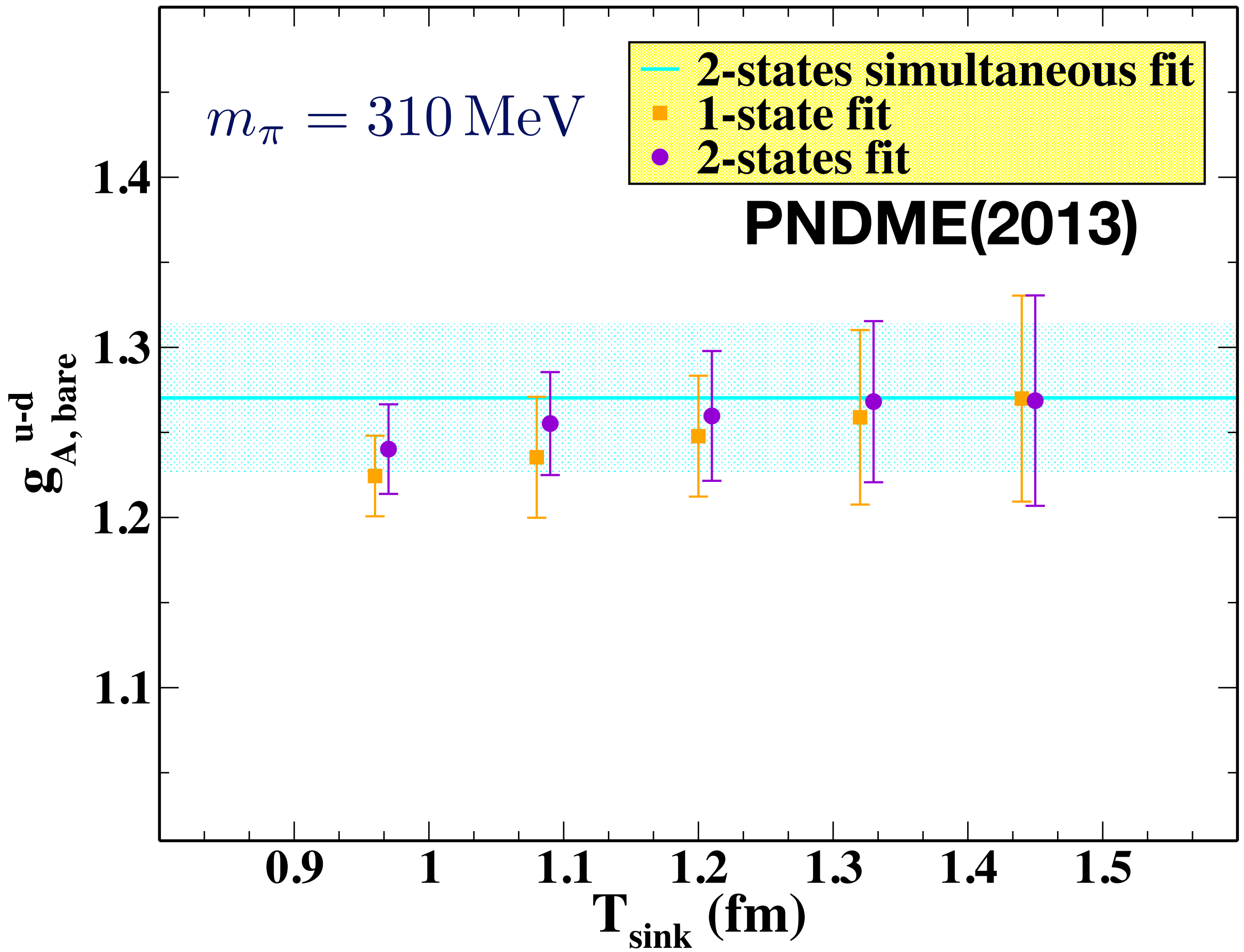
exponentially suppressed at large t ($e^{-E_1(\vec{p})t}$)

Determination of g_A on the Lattice

Lattice spacing dependence

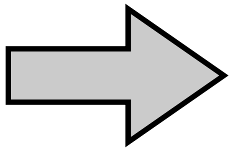


Excited state contamination



M.Constantinou (Lattice 2014)

Different colours correspond to different lattice spacings



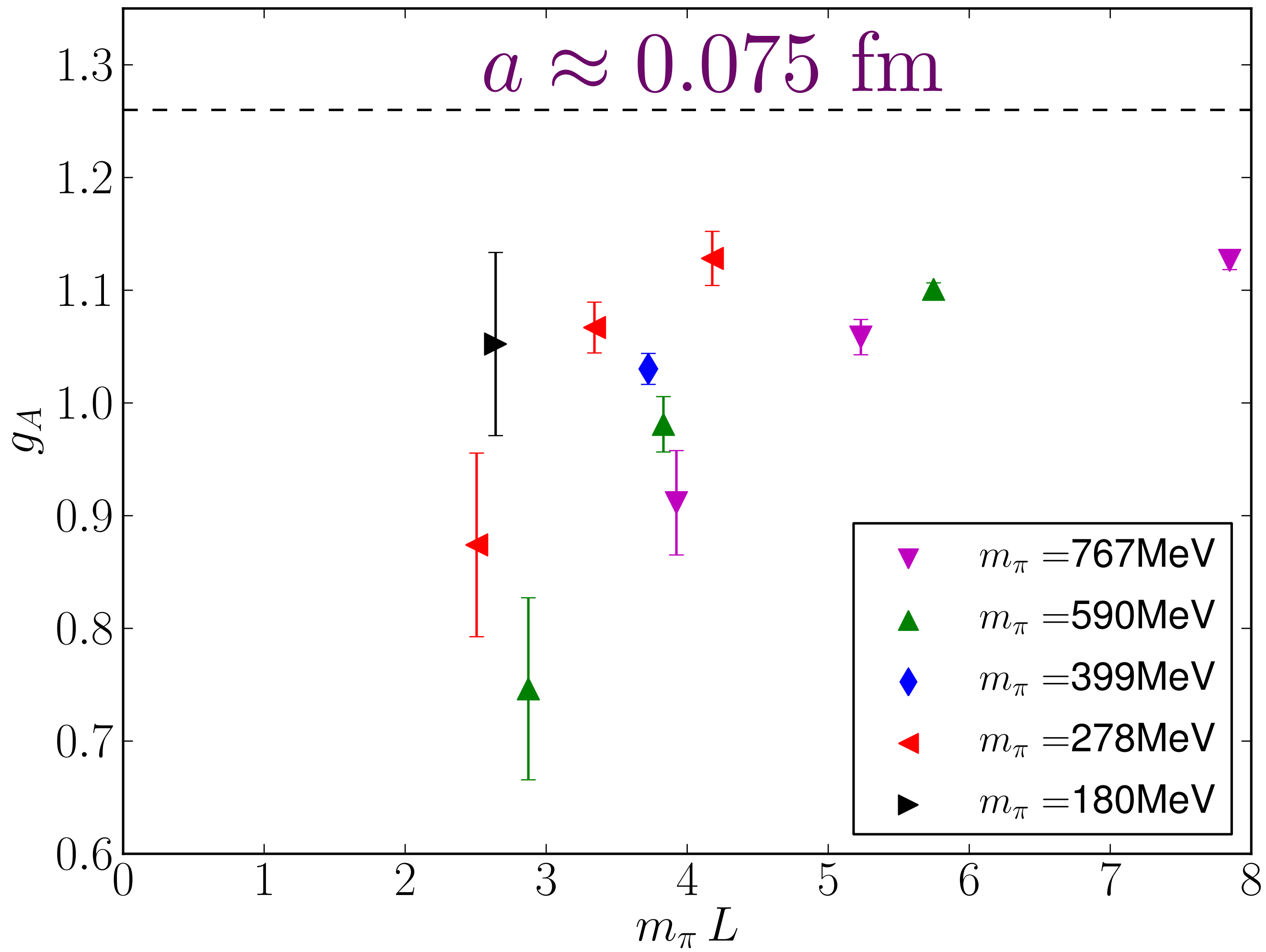
No obvious dependence on *a*

Varying the location of the sink

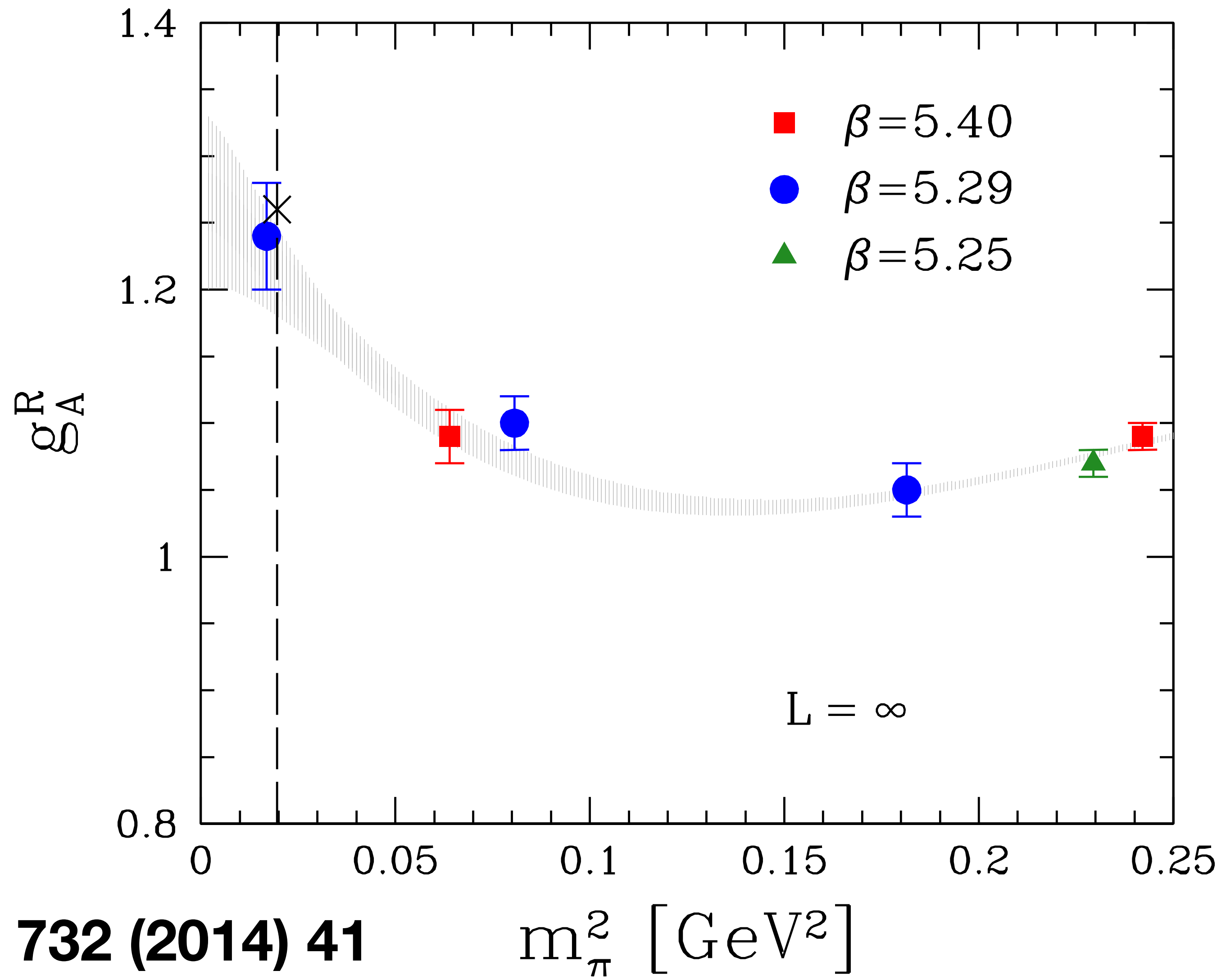
Small suppression of g_A for small times

Determination of g_A on the Lattice

Lattice volume dependence



Quark mass dependence



QCDSF: PLB 732 (2014) 41

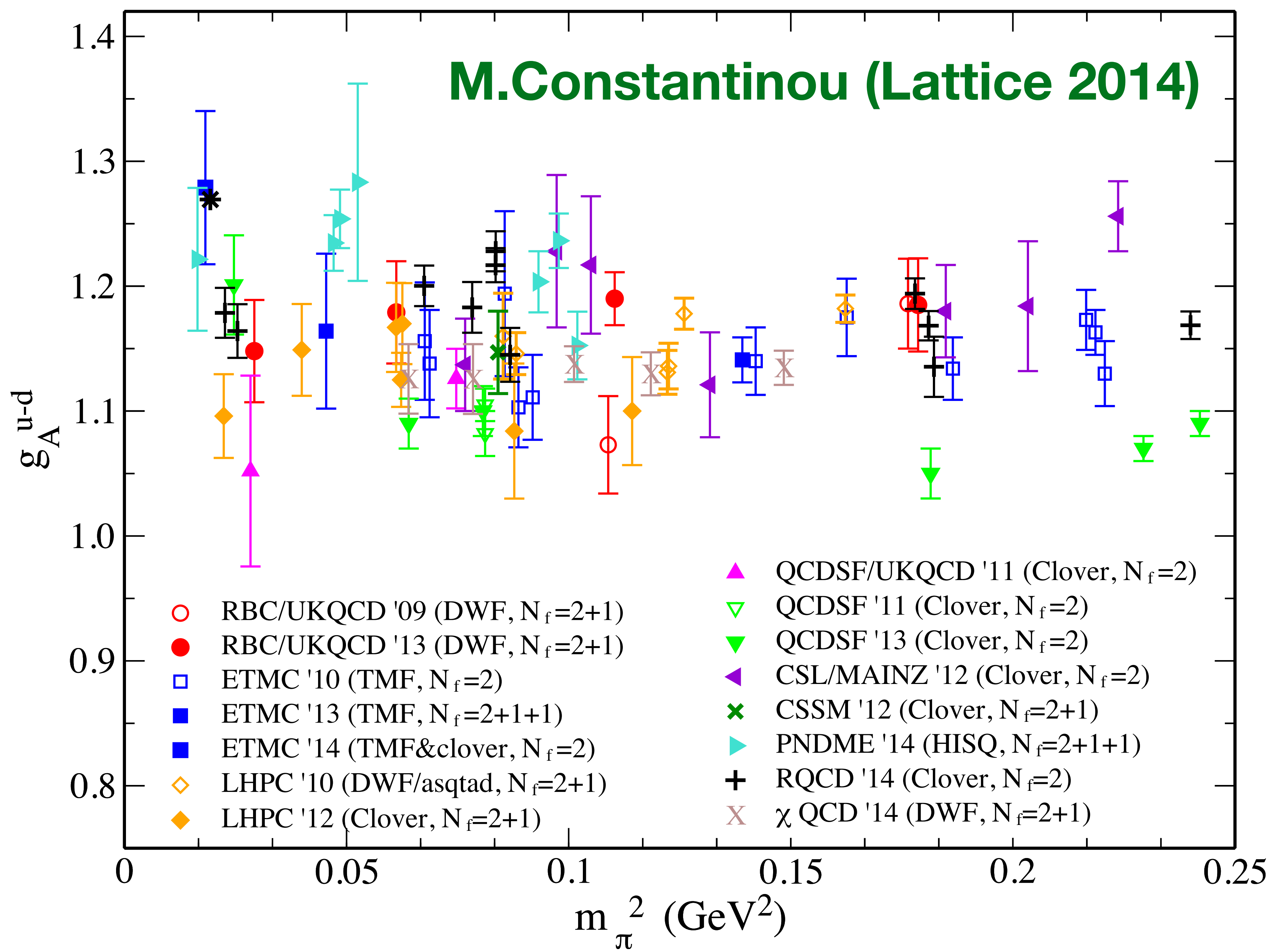
Substantial finite size effects

g_A suppressed on a finite volume

HBChPT suggests that enhancement is expected in the infinite volume at light quark masses

Sensitive to (interplay of Delta and N loops)

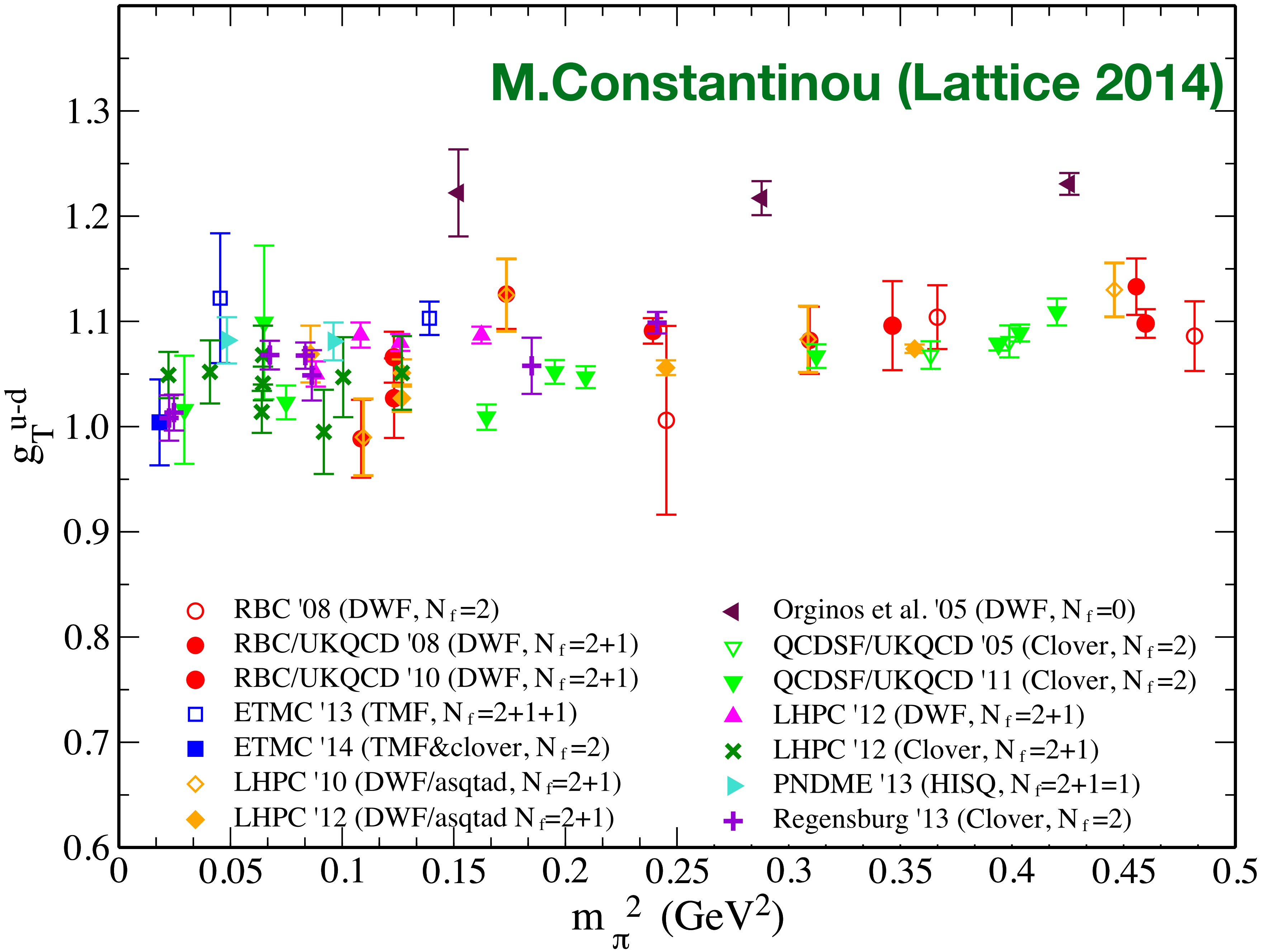
Determination of g_A on the Lattice



g_A appears to be very sensitive to Lattice systematics

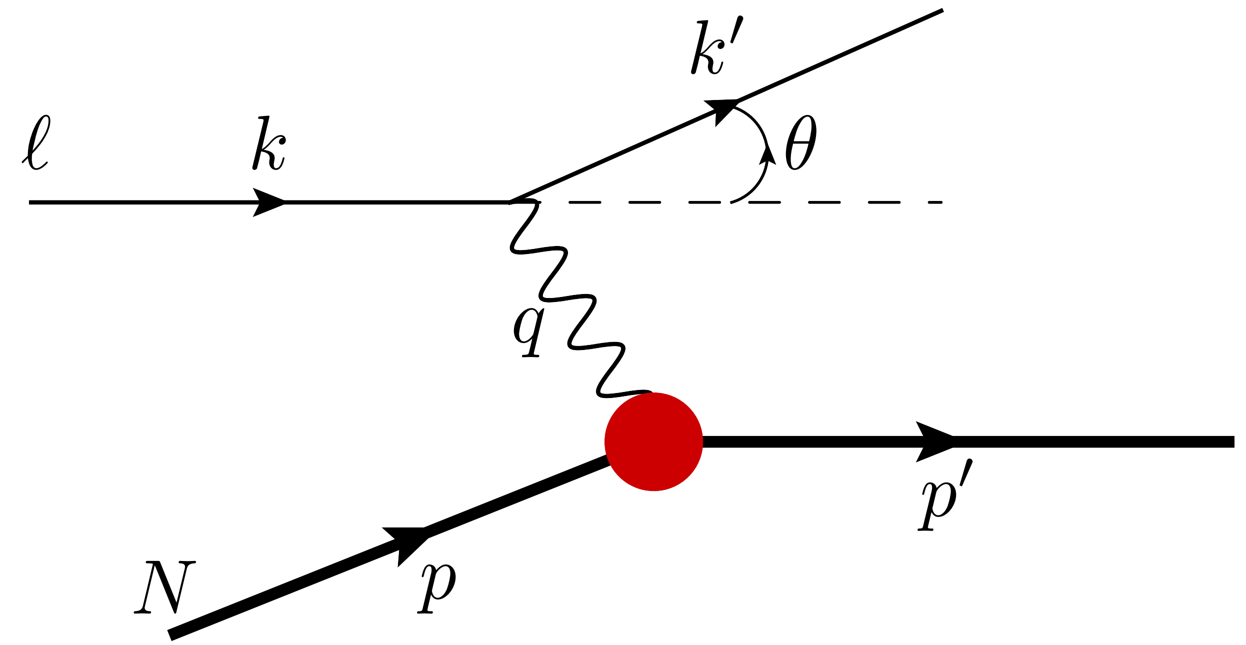
Lots of effort in reducing systematic errors ➡ **flow on for other quantities**

Tensor Charge, $g_T = \int dx\, [\delta u(x) - \delta d(x)]$



g_T appears to be well behaved

$g_T \approx 1$ **[c.f. M. Anselmino et al., 1303.3822: $g_T = 0.72^{+0.39}_{-0.18}$]** $\overline{\mathrm{MS}}\ \mu^2 = 4\,\mathrm{GeV}^2$

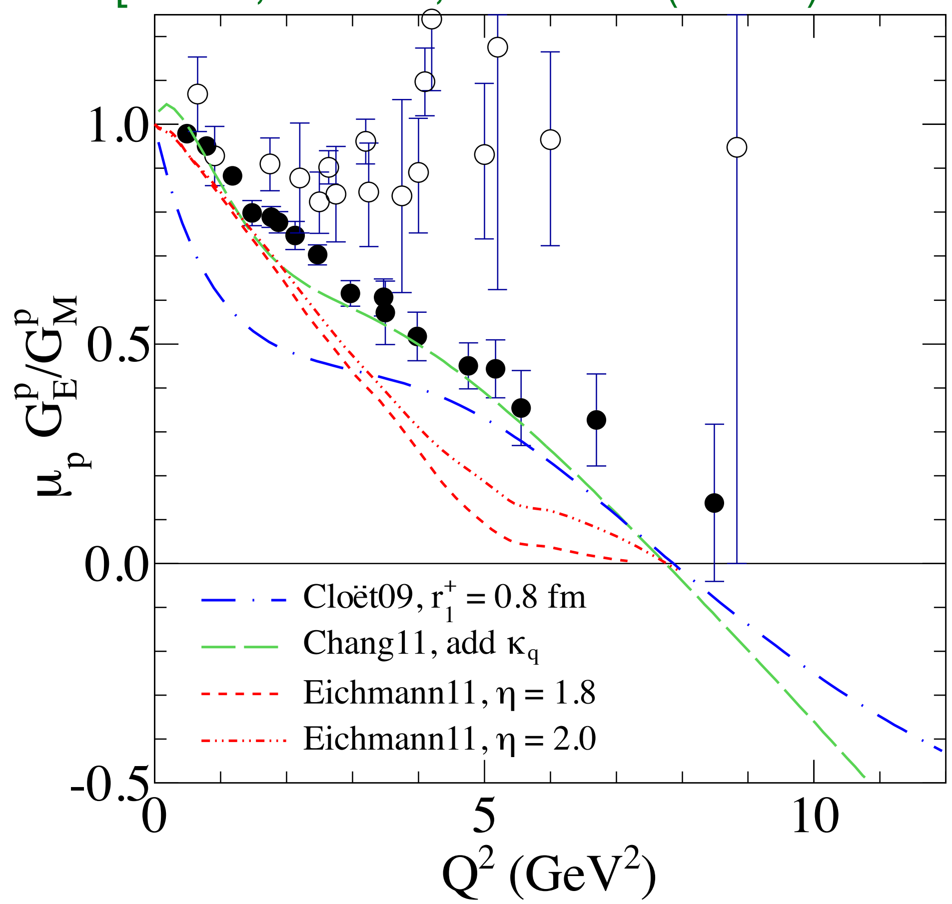


Electromagnetic Form Factors

Electromagnetic Form Factors

Several outstanding issues

[JLab, Hall A, PRC85 (2012) 045203]



Can some of these questions be addressed by a calculation from Lattice QCD?

Need to determine

$$\langle p',\, s'|J^\mu(\vec{q})|p,\, s\rangle = \bar{u}(p',\, s')\left[\gamma^\mu F_1(q^2) + i\sigma^{\mu\nu}\frac{q_\nu}{2m}F_2(q^2)\right]u(p,\, s)$$

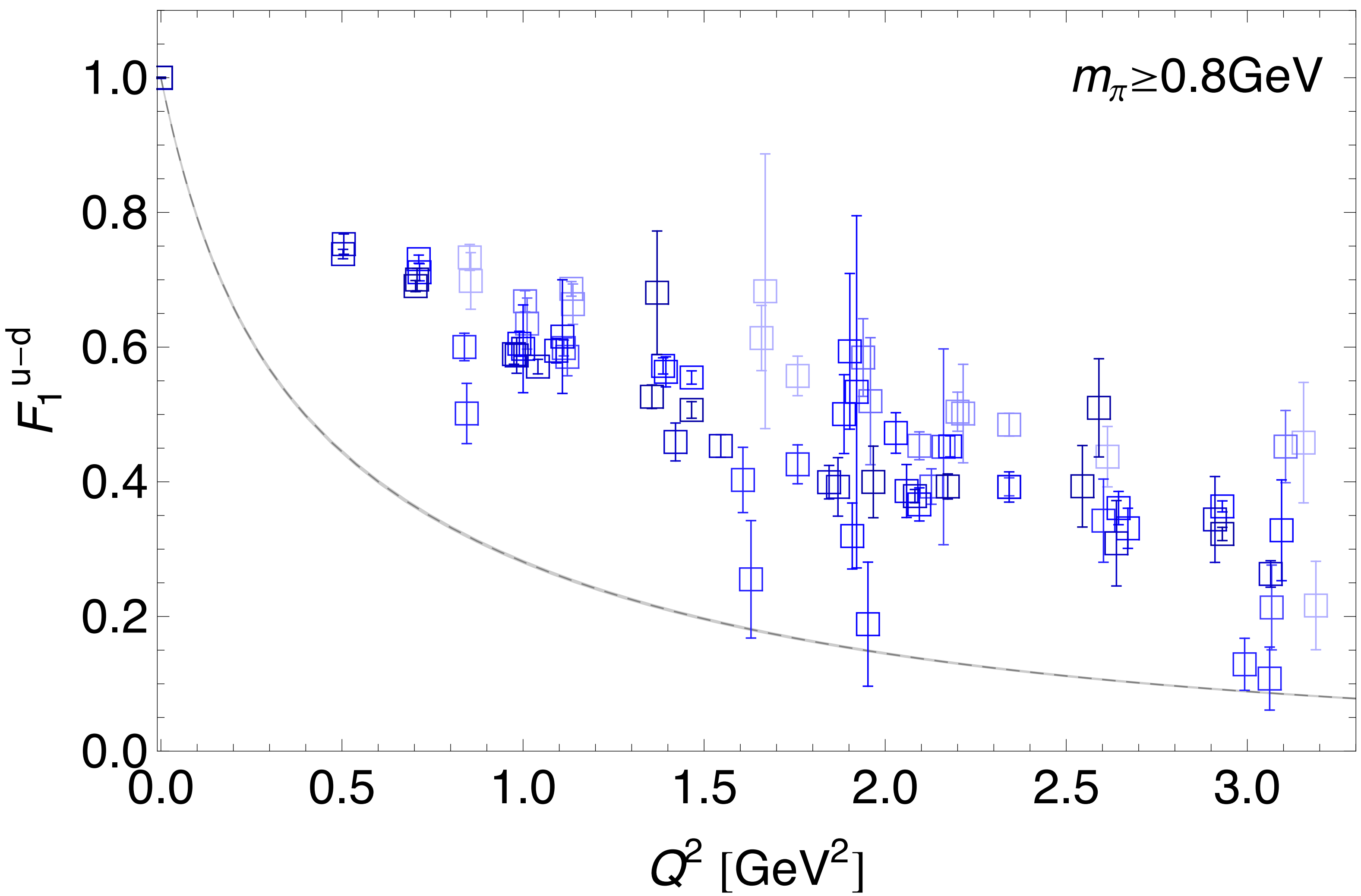
$$G_E(Q^2) = F_1(Q^2) - \tau F_2(Q^2)$$
$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

$$\tau = Q^2/(4M^2)$$

Electromagnetic Form Factors

S. Collins et al (QCDSF):PRD84 (2011) 074507

- **Strong dependence on quark mass**



$$p_i = \frac{2\pi}{L} n_i$$

- Darker colours ⇒ lighter masses
- Grey band ⇒ parameterisation of experimental data
- Lattice results lie above experiment with smaller slope

Electromagnetic Form Factors

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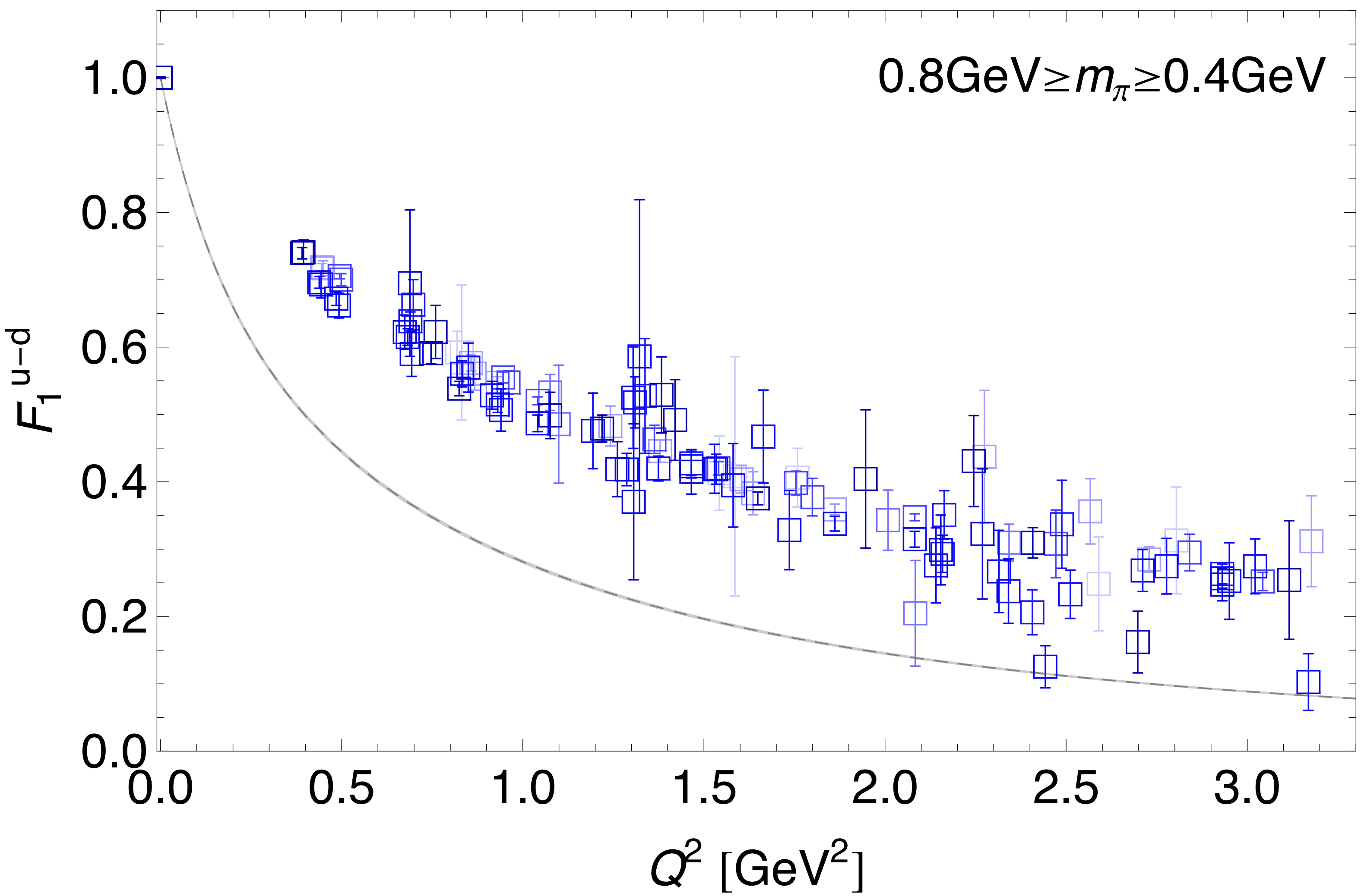

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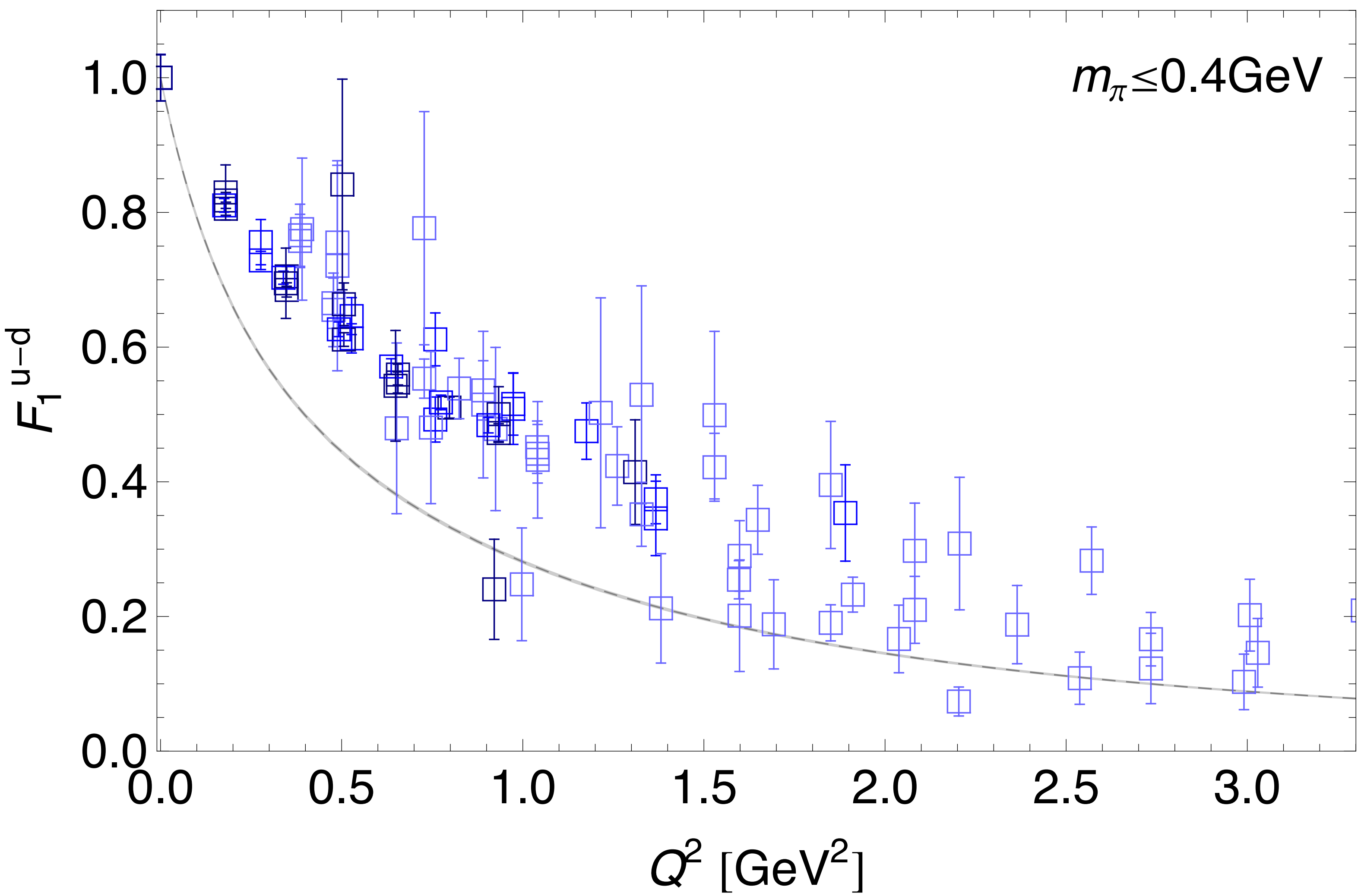

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Electromagnetic Form Factors

Traditional analysis method:

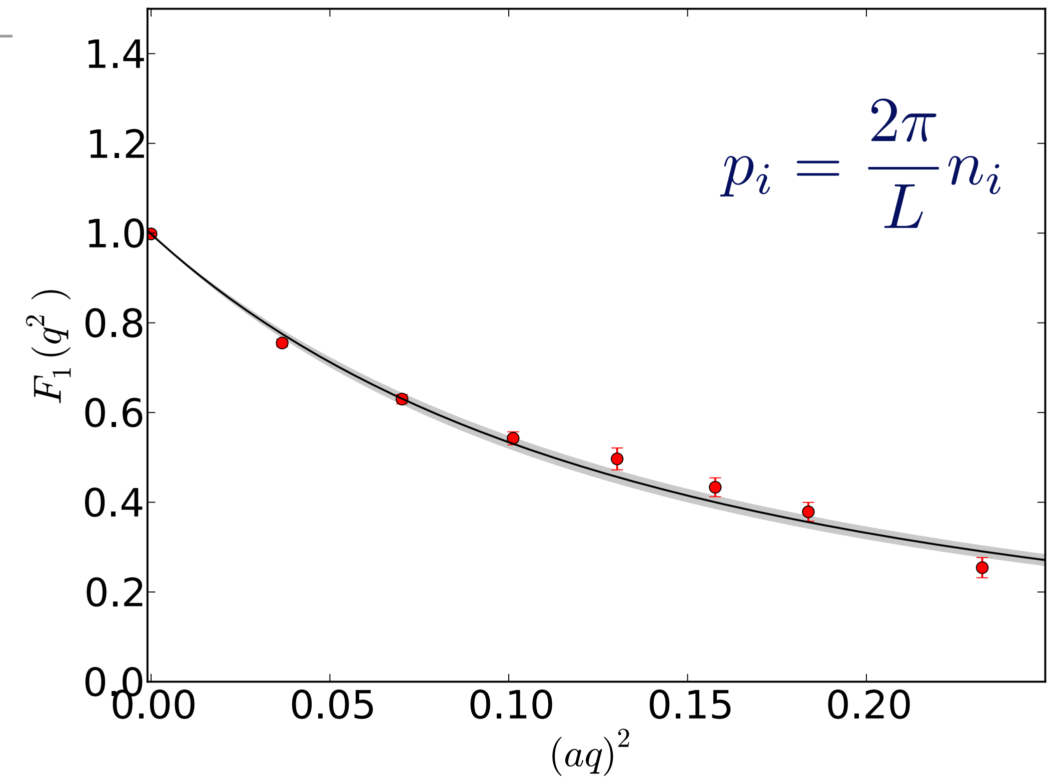
Fit with a dipole

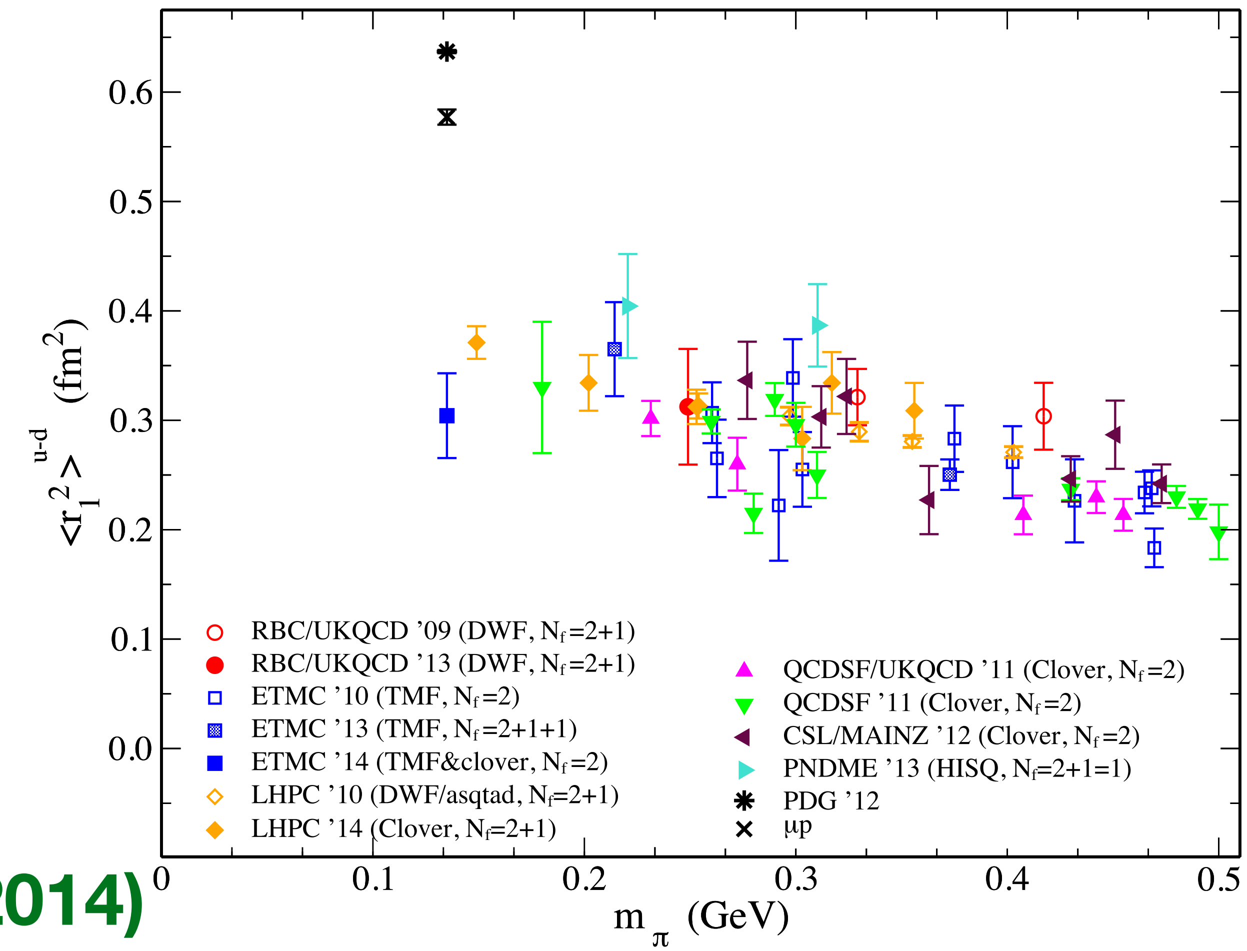
$$F(Q^2) = \frac{F(0)}{(1 - Q^2/M^2)^2}$$

Extract charge radius

$$\langle r^2 \rangle = \frac{12}{M^2}$$

Extrapolate to physical mass





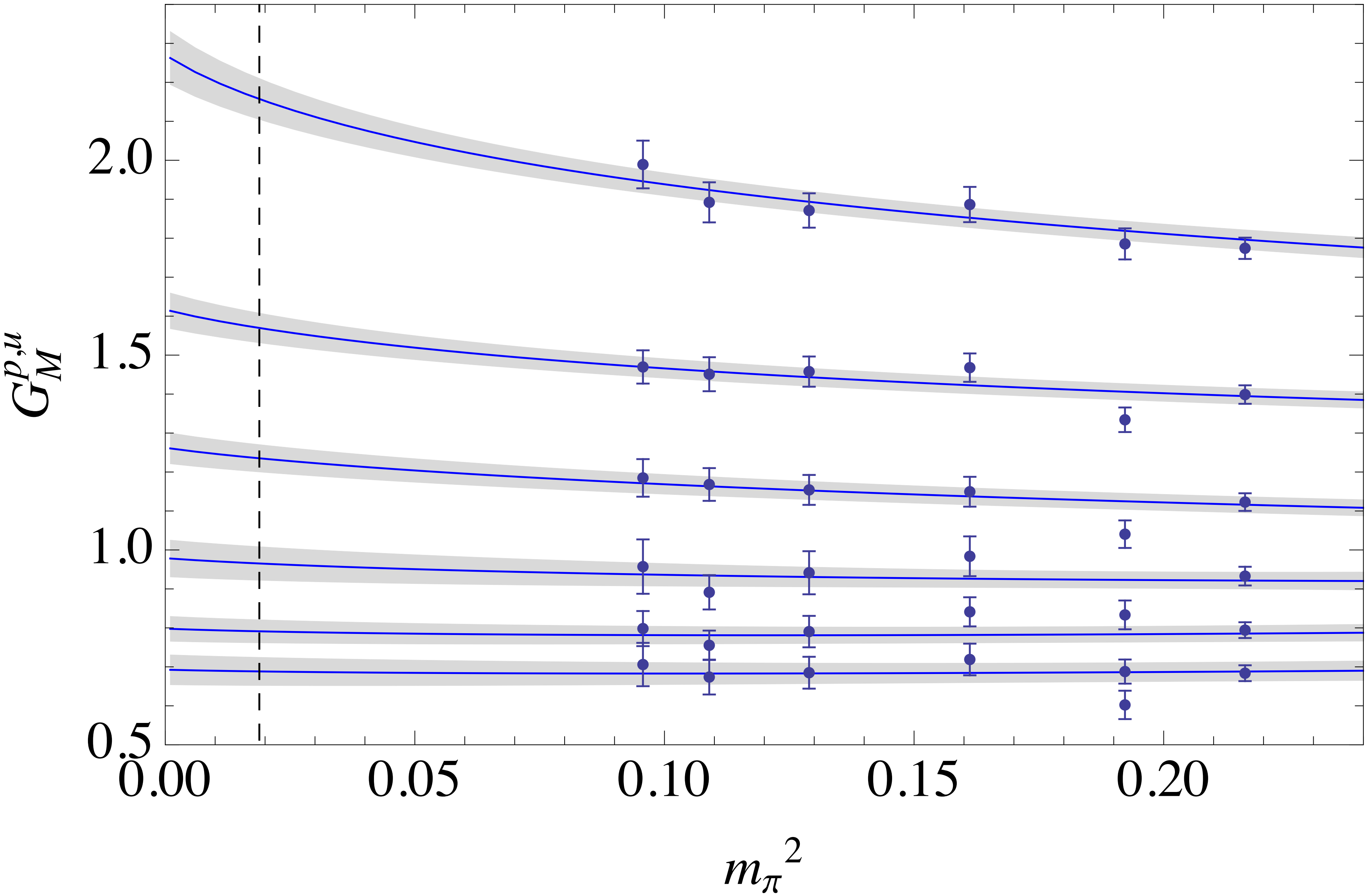
M.Constantinou (Lattice 2014)

Electromagnetic Form Factors

P.Shanahan et al (CSSM/QCDSF):
PRD89 (2014) 074511
PRD90 (2014) 034502
arXiv:1403.6537
in preparation

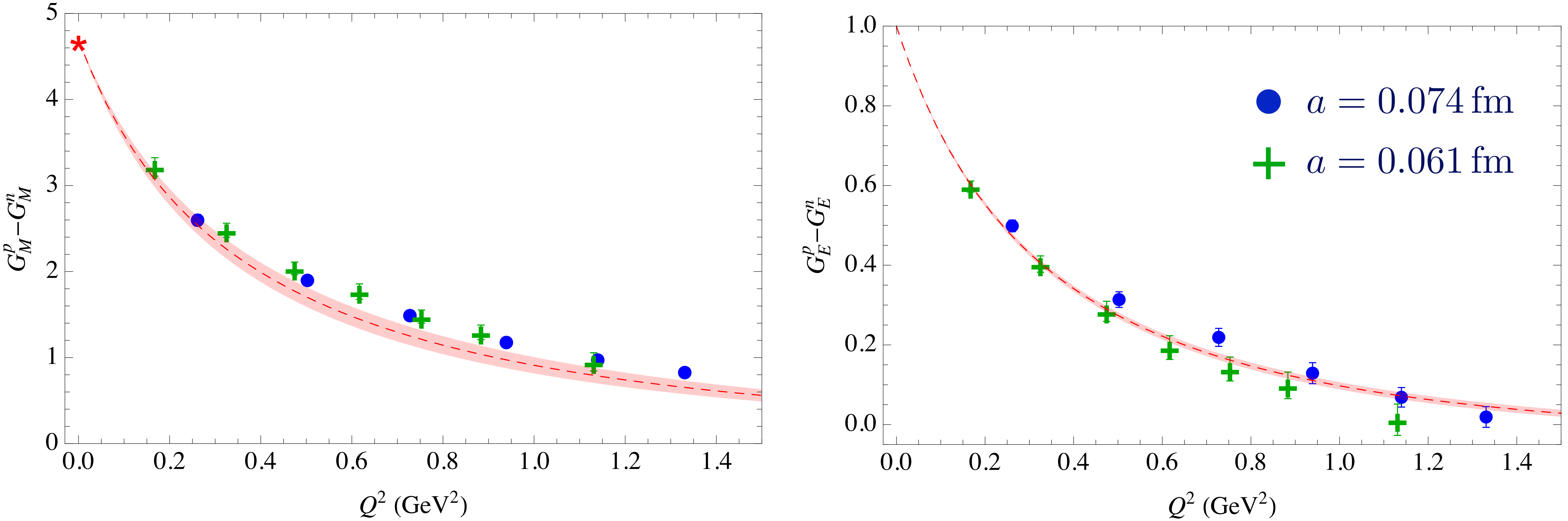
New method:

Perform chiral extrapolation at fixed-Q^2



Increasing Q^2

Electromagnetic Form Factors



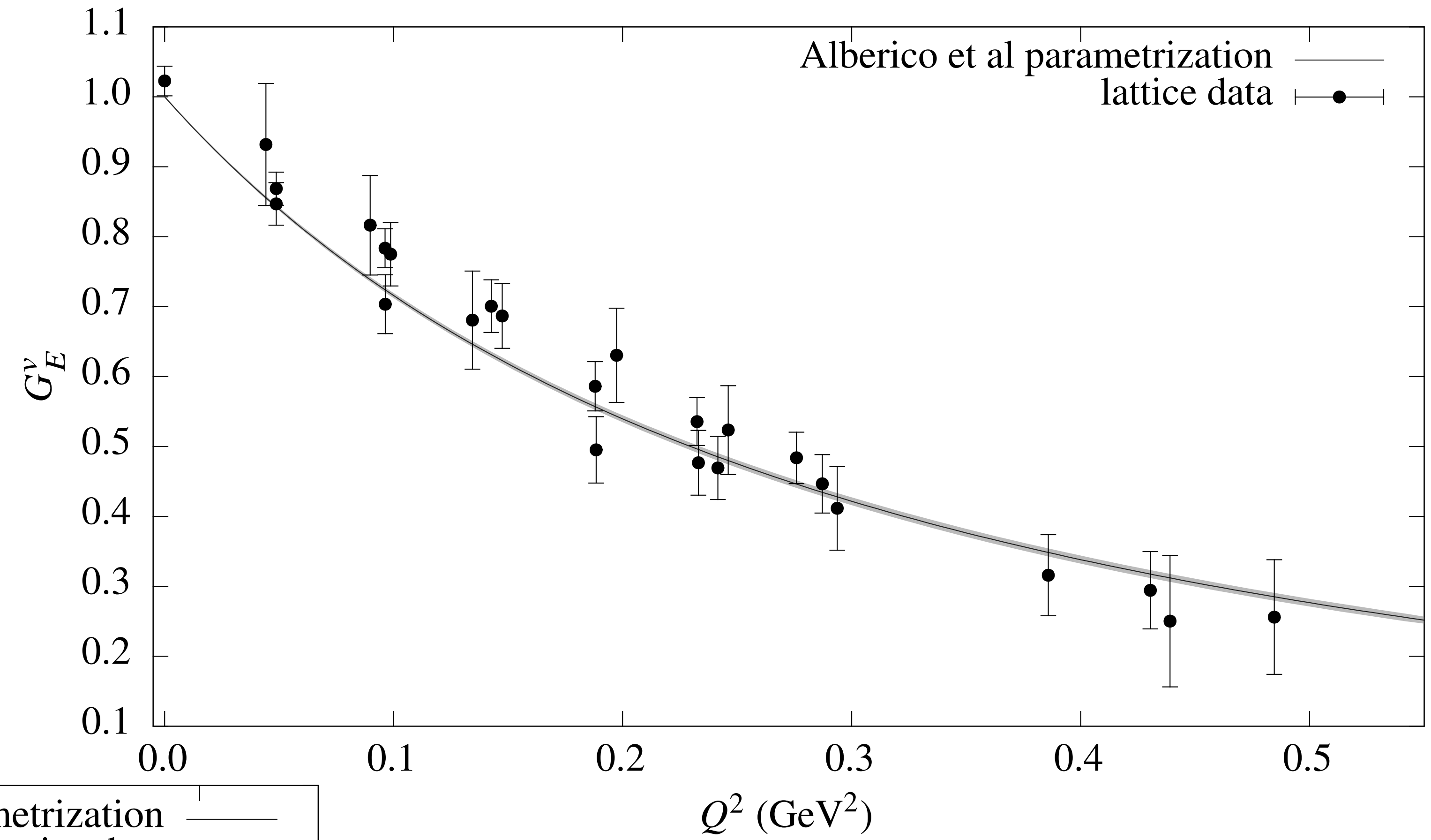
Good agreement with parameterisation of experimental data

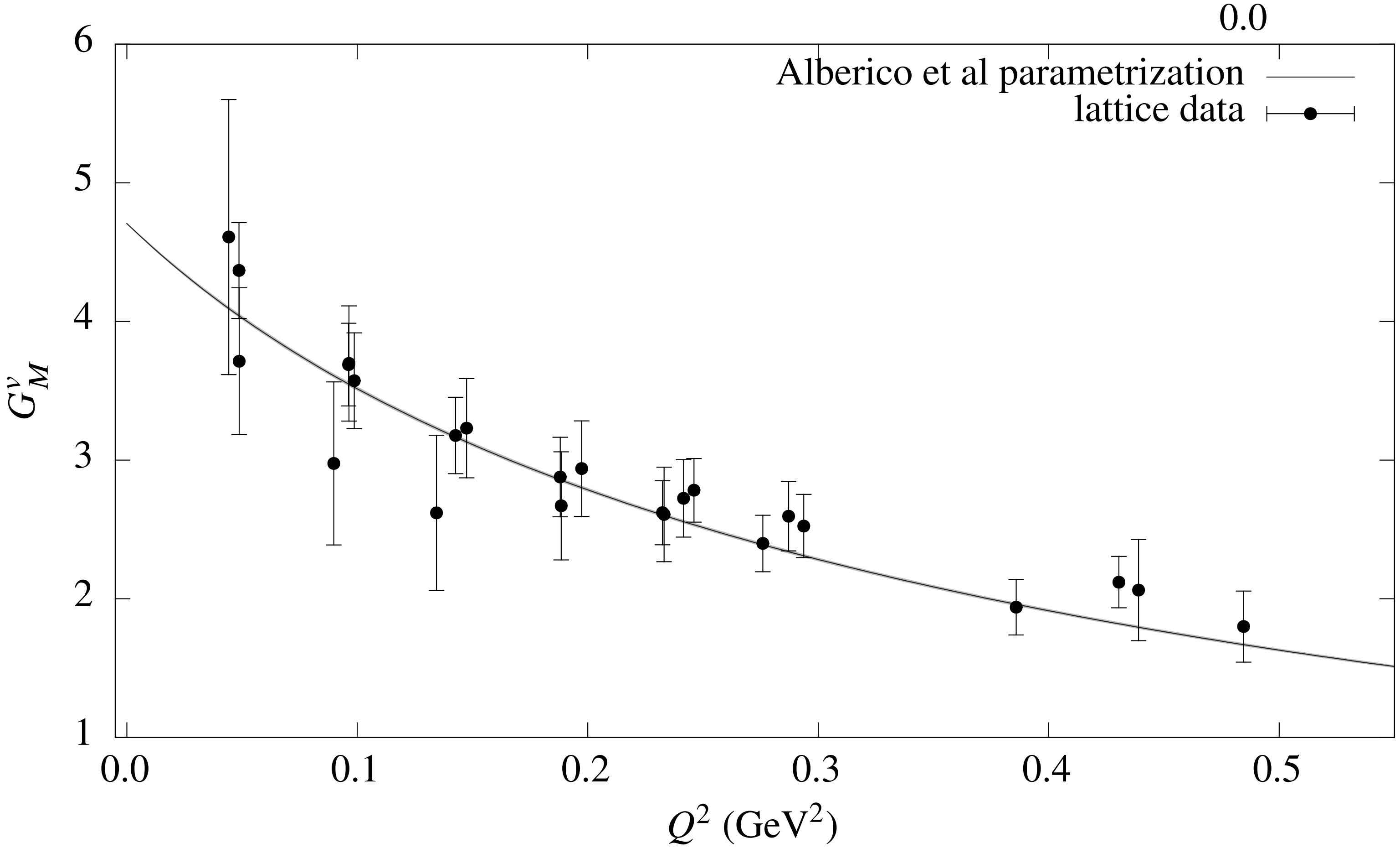
Electromagnetic Form Factors

J. Green et al. (LHPC): arXiv:1404.4029

Latest results close to the physical point

$$m_\pi \approx 149\,\mathrm{MeV}$$

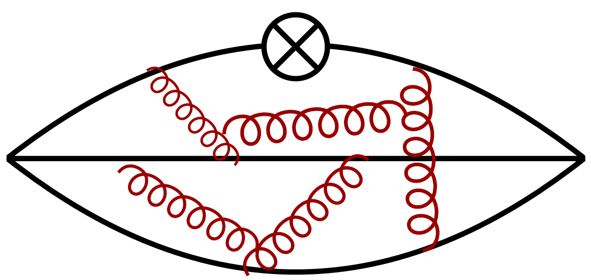




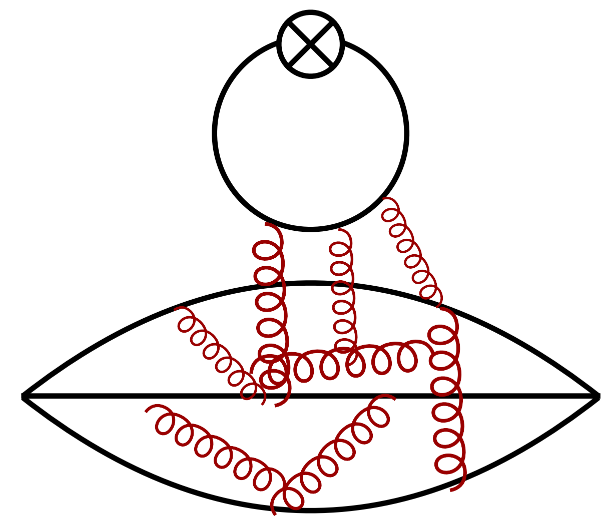
Strangeness Form Factor

Understanding hidden flavour - **A fundamental challenge of hadronic physics**

Contributions arise entirely through interactions with QCD vacuum



connected



disconnected

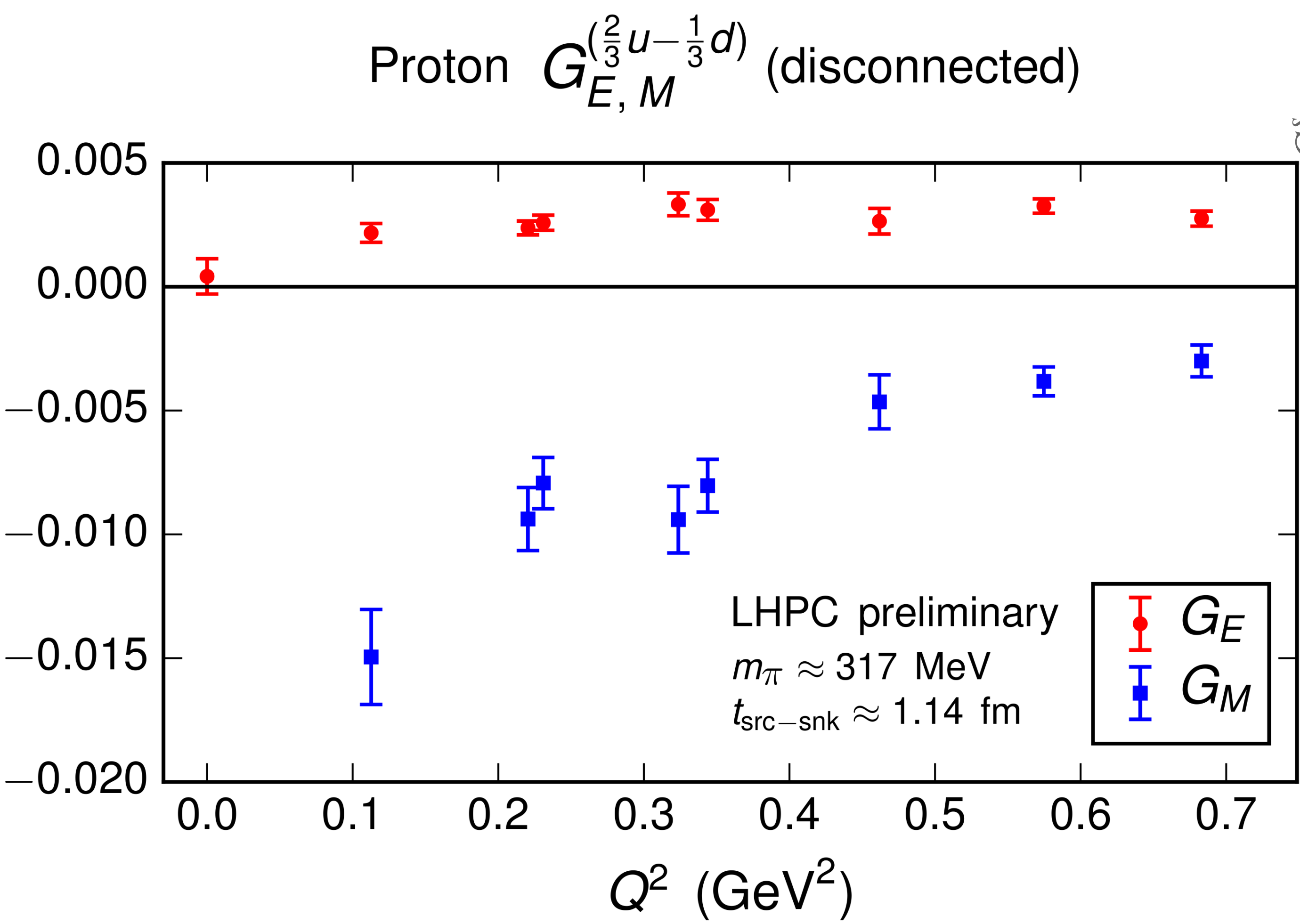
Expensive and noisy!

Vanish for isovector quantities (p-n)

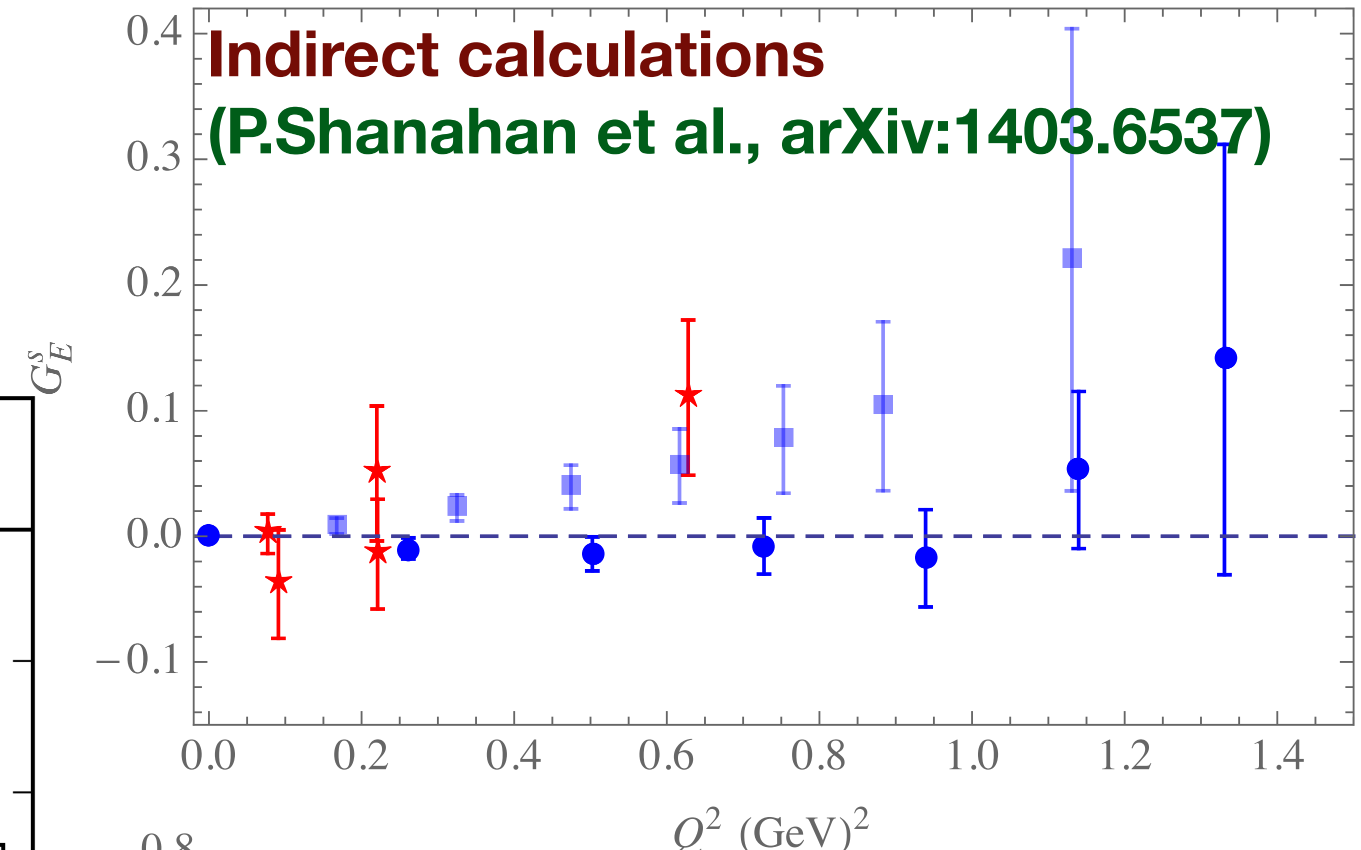
Essential for strangeness

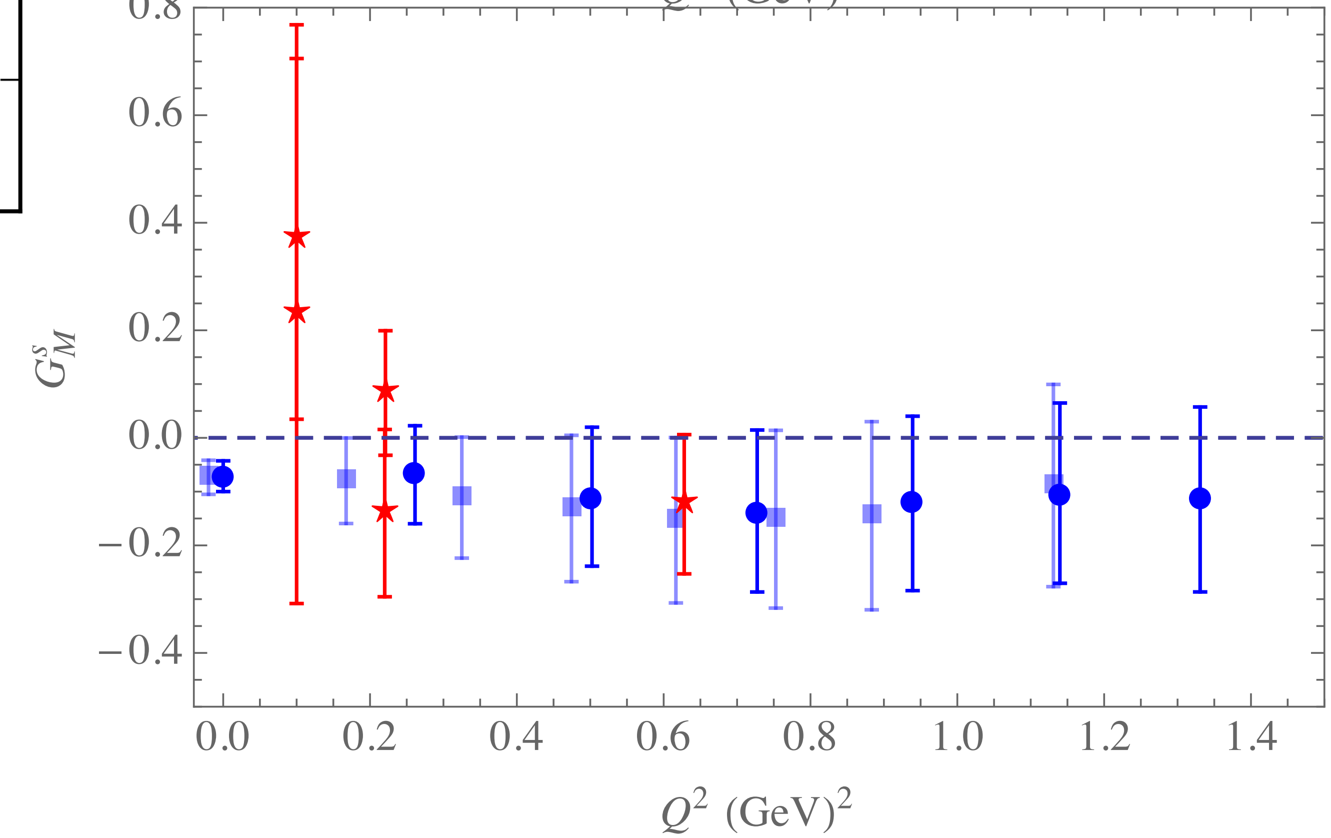
Strangeness Form Factor

Direct calculations - preliminary
(S. Meinel, Lattice 2014)



Indirect calculations
(P.Shanahan et al., arXiv:1403.6537)

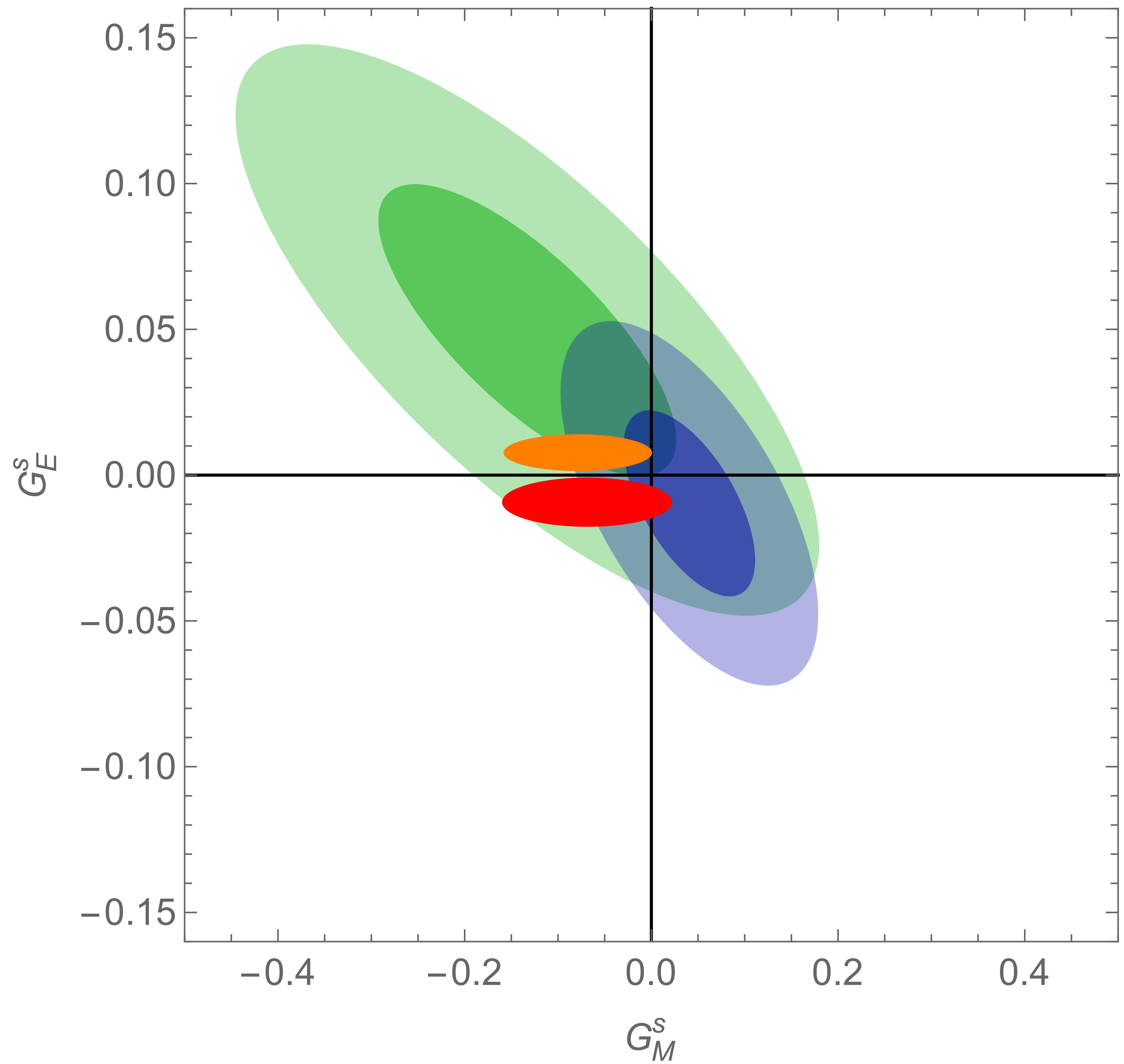




Strangeness Form Factor

P.Shanahan et al., arXiv:1403.6537

Comparison with experimental contraints



This work: 0.18 GeV²

This work: 0.26 GeV²

A4: 0.23 GeV²

G0: 0.23 GeV²

Dark: 1σ

Light: 2σ

Strangeness Form Factor

P.Shanahan et al., arXiv:1403.6537

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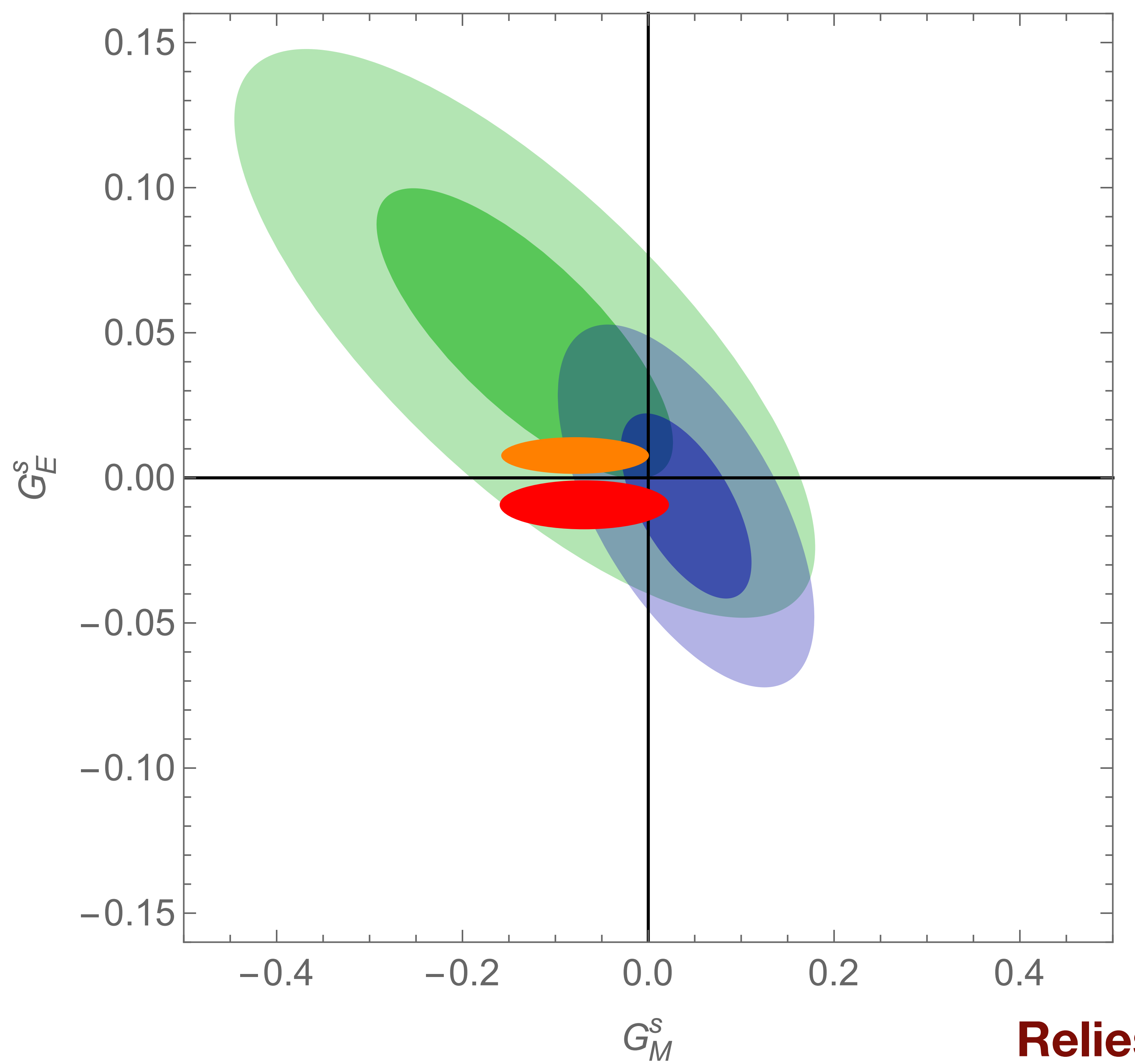

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Relies on assumptions of Charge Symmetry

$$u_p \equiv d_n$$

$$d_p \equiv u_n$$

Charge Symmetry

u quarks in the proton $\equiv$ *d* quarks in the neutron

We relied on charge symmetry in our determination of the strangeness form factors

So do the experiments!

EM and weak interactions give access to different combinations of $G^{p,(u/d/s)}$

$$G^{p,\gamma} = \frac{2}{3}G^{p,u} - \frac{1}{3}\left(G^{p,d} + G^{p,s}\right)$$
$$G^{p,Z} = \left(1 - \frac{8}{3}\sin^2\theta_W\right)G^{p,u} - \left(1 - \frac{4}{3}\sin^2\theta_W\right)\left(G^{p,d} + G^{p,s}\right)$$

Assume charge symmetry $\left(G^{p,u} = G^{n,d}, \quad G^{p,d} = G^{n,u}, \quad G^{p,s} = G^{n,s}\right)$

$$\Rightarrow \left(G^{p,s}_{E/M} = \left(1 - 4\sin^2\theta_W\right)G^{p,\gamma}_{E/M} - G^{n,\gamma}_{E/M} - G^{p,Z}_{E/M}\right.$$

Charge Symmetry Violation

P.Shanahan et al., in preparation

Determine the degree to which charge symmetry is violated in EM form factors by

Combining chiral perturbation theory fits to isospin-averaged hyperon FFs

Input m_u/m_d from experiment (or lattice)

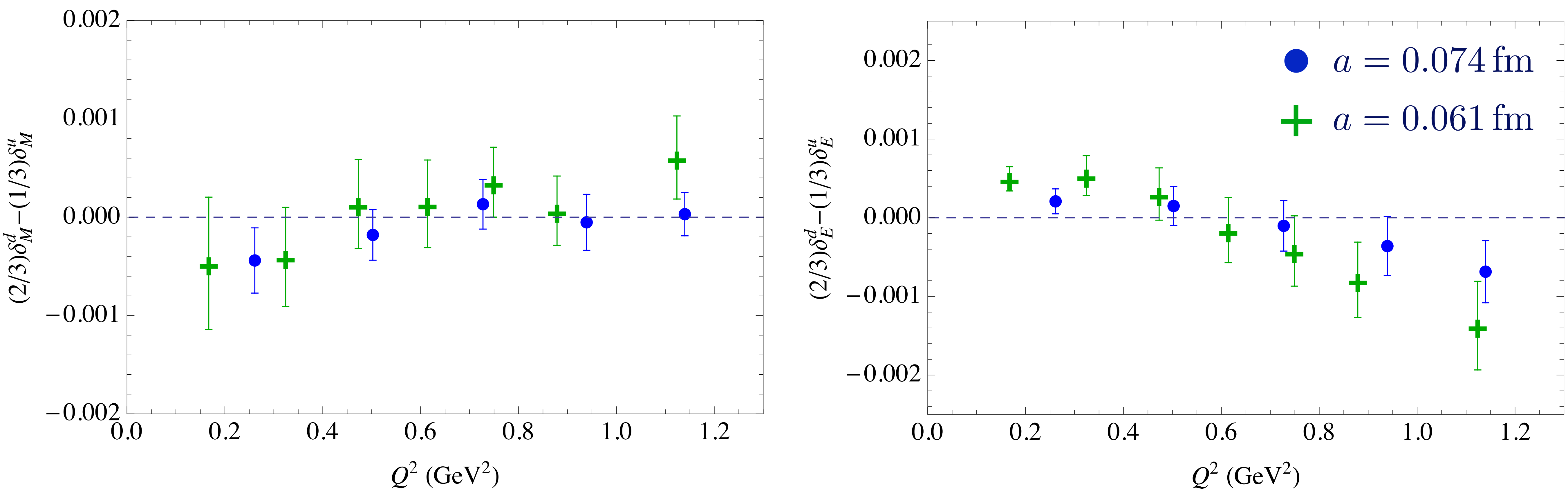
Charge Symmetry Violation

P.Shanahan et al., in preparation

Determine the degree to which charge symmetry is violated in EM form factors by

Combining chiral perturbation theory fits to isospin-averaged hyperon FFs

Input m_u/m_d from experiment (or lattice)



Charge Symmetry Violation

P.Shanahan et al., in preparation

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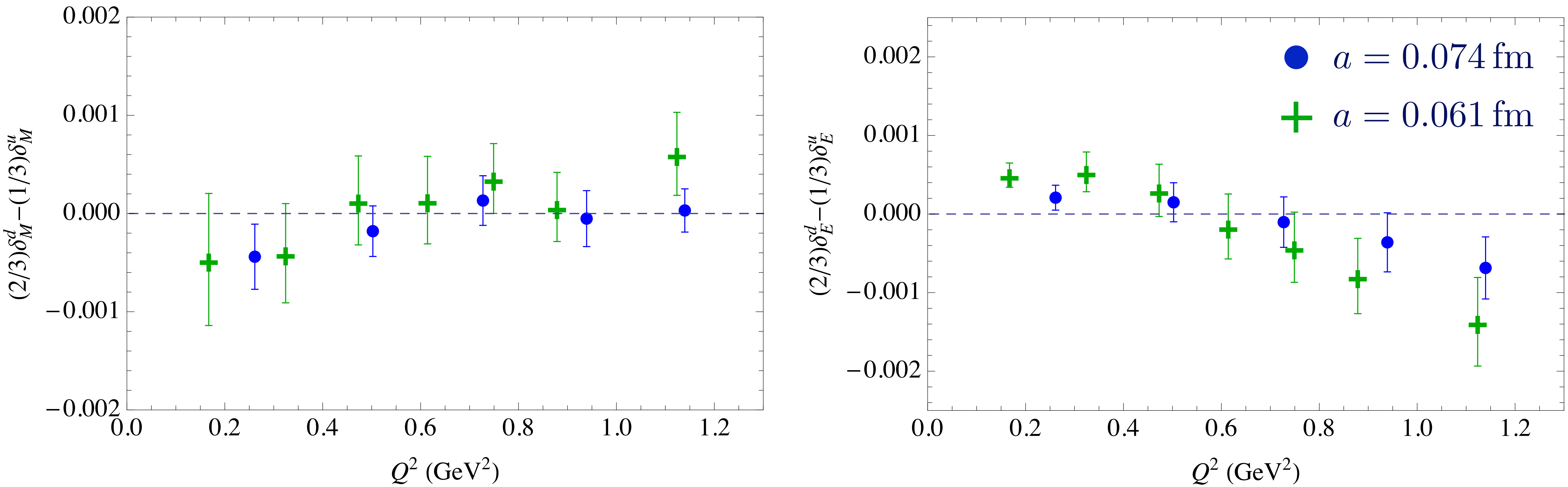


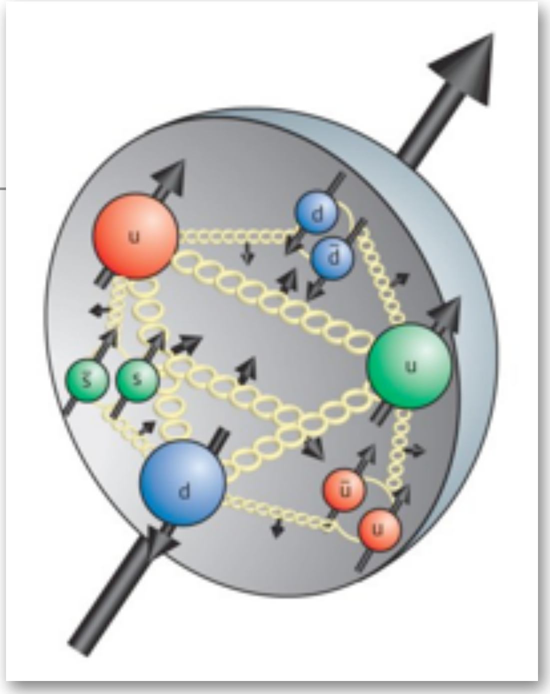
Charge symmetry satisfied to better than 0.2%

Spin of the Proton

[See also plenary by K.-F. Liu]

Spin of the Proton

How is the spin of the proton distributed between its constituents?



X. Ji (1997): $$\frac{1}{2} = \sum_q J_q(\mu^2) + J_g(\mu^2) \qquad J_q = \frac{1}{2}\Delta\Sigma_q + L_q$$

Express in terms of moments of Generalised Parton Distributions

$$J_{q/g} = \frac{1}{2}[A_{20}^{q/g}(\Delta^2 = 0) + B_{20}^{q/g}(\Delta^2 = 0)]$$

which are obtained from the matrix elements of the energy momentum tensor

$$\langle P'|T^{\mu\nu}|P\rangle = \overline{U}(P')\left\{\gamma^\mu \overline{P}^\nu A_{20}(\Delta^2) + \frac{i\sigma^{\mu\rho}\Delta_\rho \overline{P}^\nu}{2m_N} B_{20}(\Delta^2) + \frac{\Delta^\mu \Delta^\nu}{m_N} C_{20}(\Delta^2)\right\} U(P)$$

Momentum conservation:

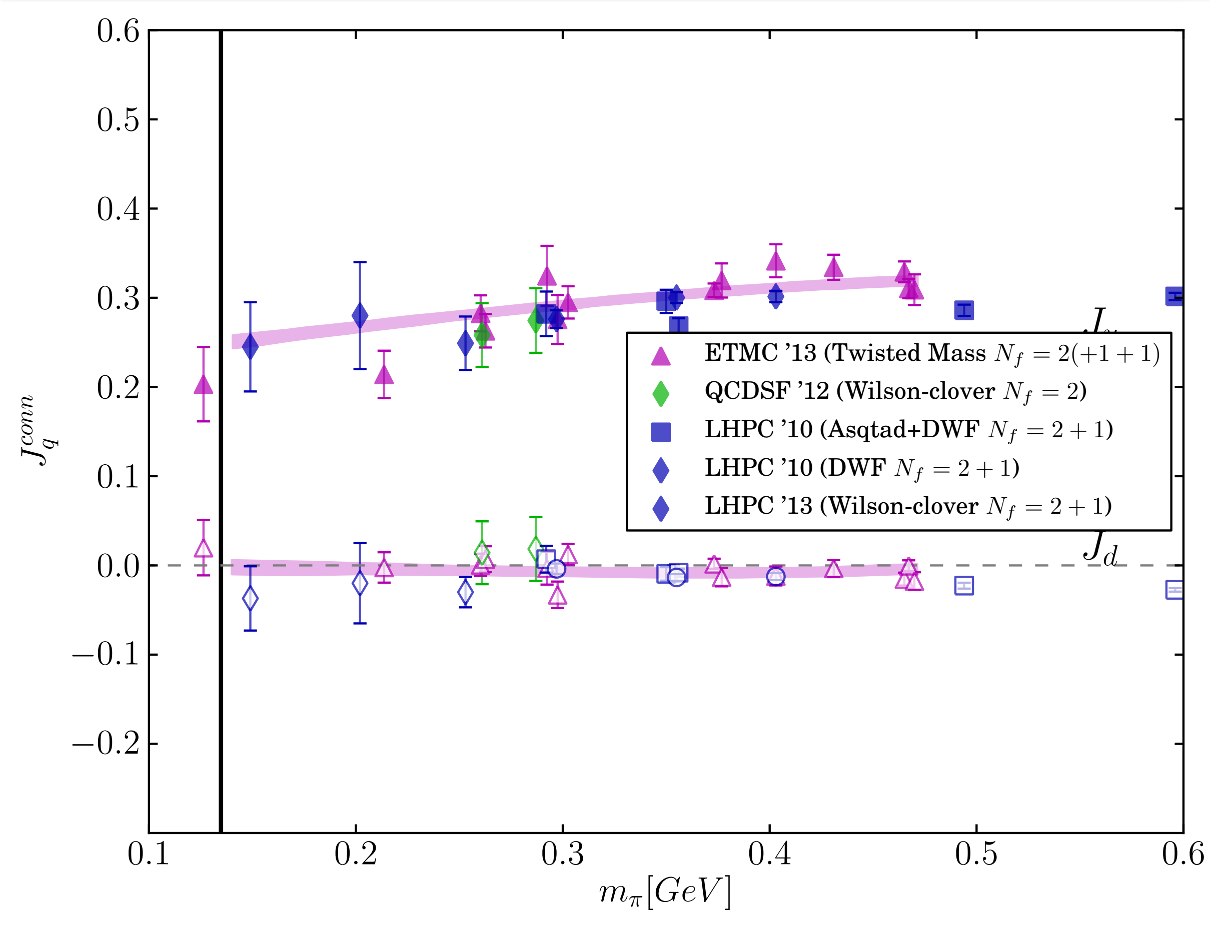
$$1 = \sum_q A_{20}^q(0) + A_{20}^g(0) = \sum_q \langle x\rangle_q + \langle x\rangle_g$$

Anomalous gravitomagnetic moment

$$0 = \sum_q B_{20}^q(0) + B_{20}^g(0)$$

(Review by S.Syritsyn, Lattice 2013)

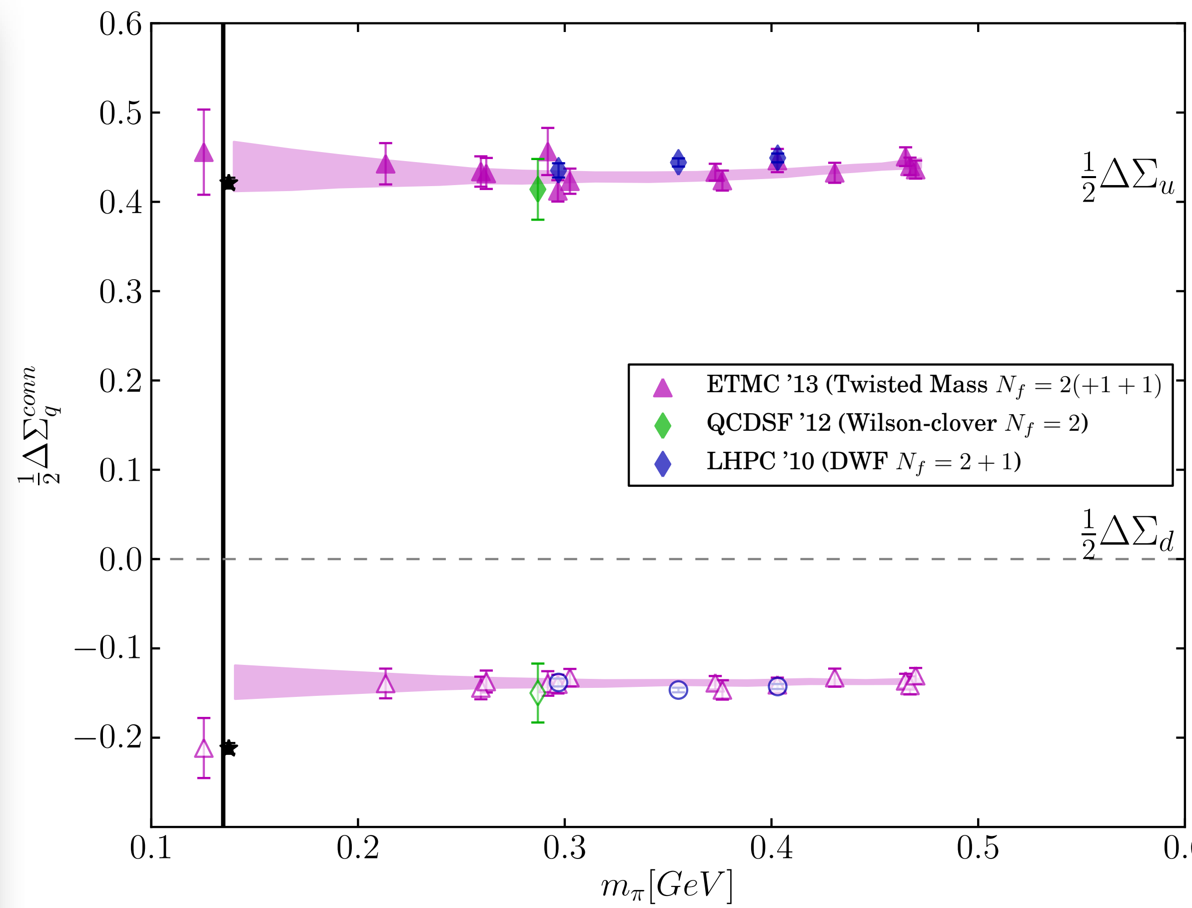
Quark Angular Momentum and Spin (Connected)

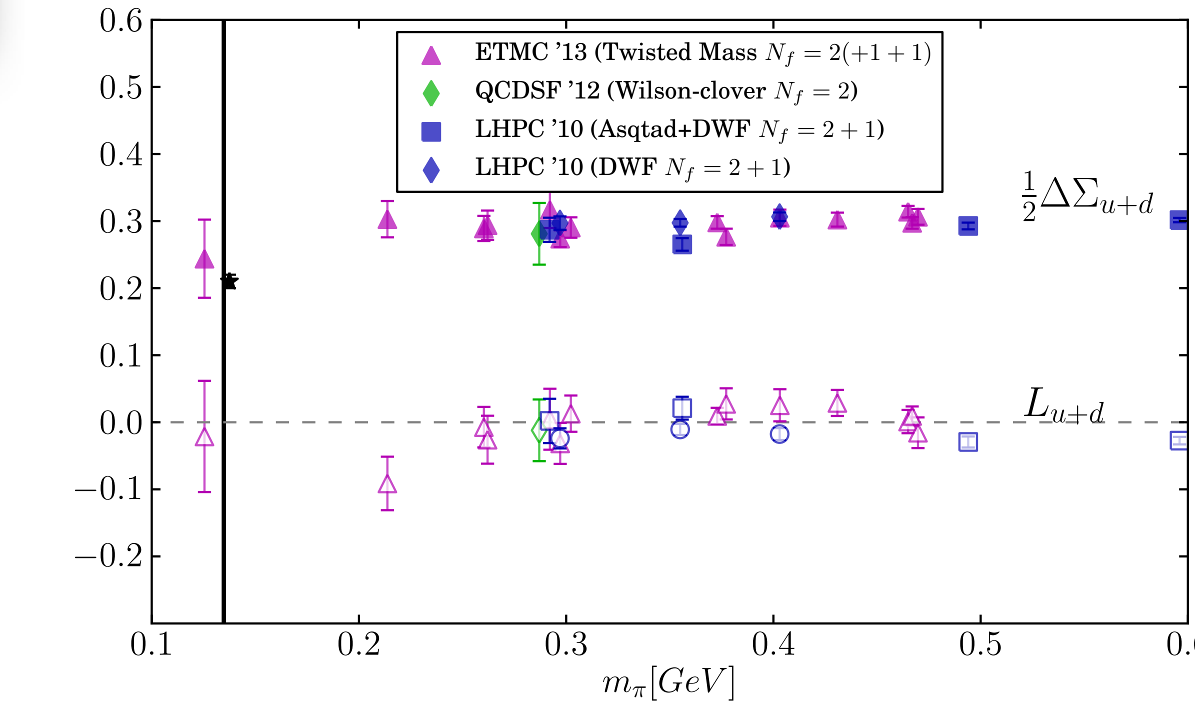


$$J_u \approx 40 - 50\% \quad *$$
$$|J_d| \lesssim 10\% \quad *$$

$$|L_{u+d}| \ll \frac{1}{2}\Delta\Sigma_{u+d} \quad *$$

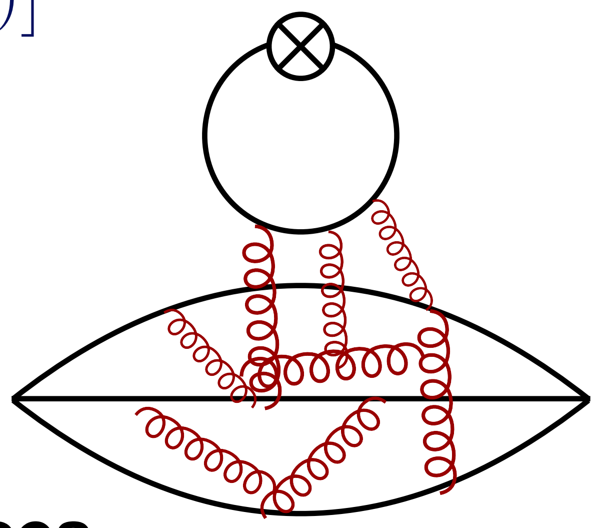
(*) not including disconnected diagrams!





Spin of the Proton

$$\Delta s = \int_0^1 dx \left[\Delta s(x) + \Delta \bar{s}(x)\right]$$



Δs **is a purely quark-line disconnected contribution**

A challenge on the lattice

Standard procedure: Use stochastic (random noise) sources

e.g. **PRL108, 222001 (arXiv:1112.3354)**

$m_\pi = 285\,\mathrm{MeV}$

$\overline{\mathrm{MS}} \quad \mu = \sqrt{7.4}\,\mathrm{GeV}$

$$\Delta s = -0.020(10)(4)$$

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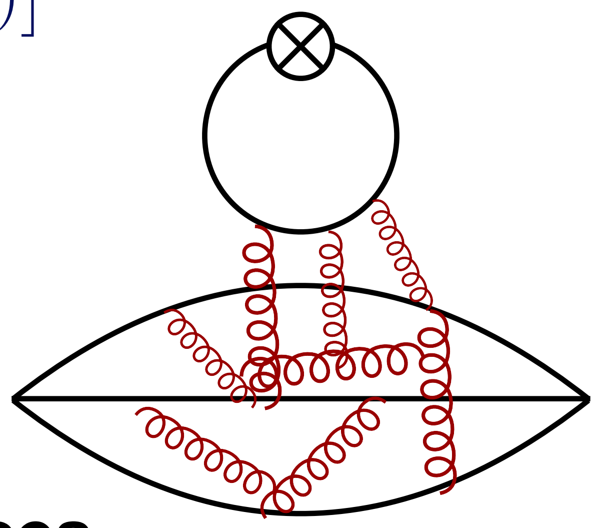

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A new alternative: **[A. Chambers et al. (CSSM/QCDSF) PRD90 (2014) 014510]**

Apply the Feynman-Hellmann method to lattice Green's functions

$$\frac{\partial E_H(\lambda)}{\partial\lambda} = \frac{1}{2E_H(\lambda)}\left\langle H\left|\frac{\partial S(\lambda)}{\partial\lambda}\right|H\right\rangle$$

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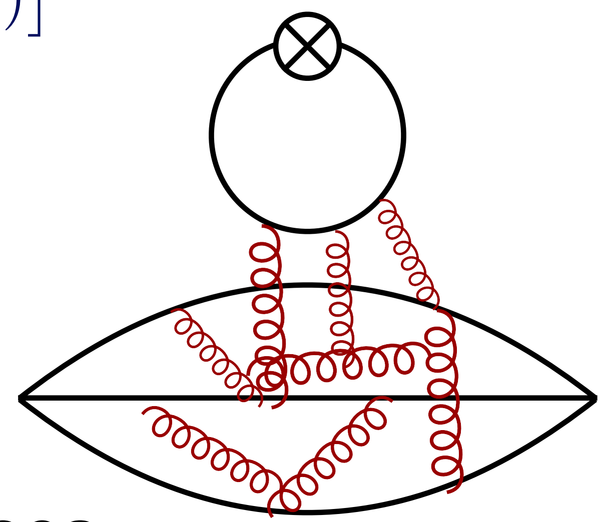

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Also allows for full nonperturbative determination of singlet renormalisation constants

$$g_A = Z_A^{\mathrm{NS}} g_A^{\mathrm{latt}}$$

[CSSM/QCDSF, arXiv:1410.3078]

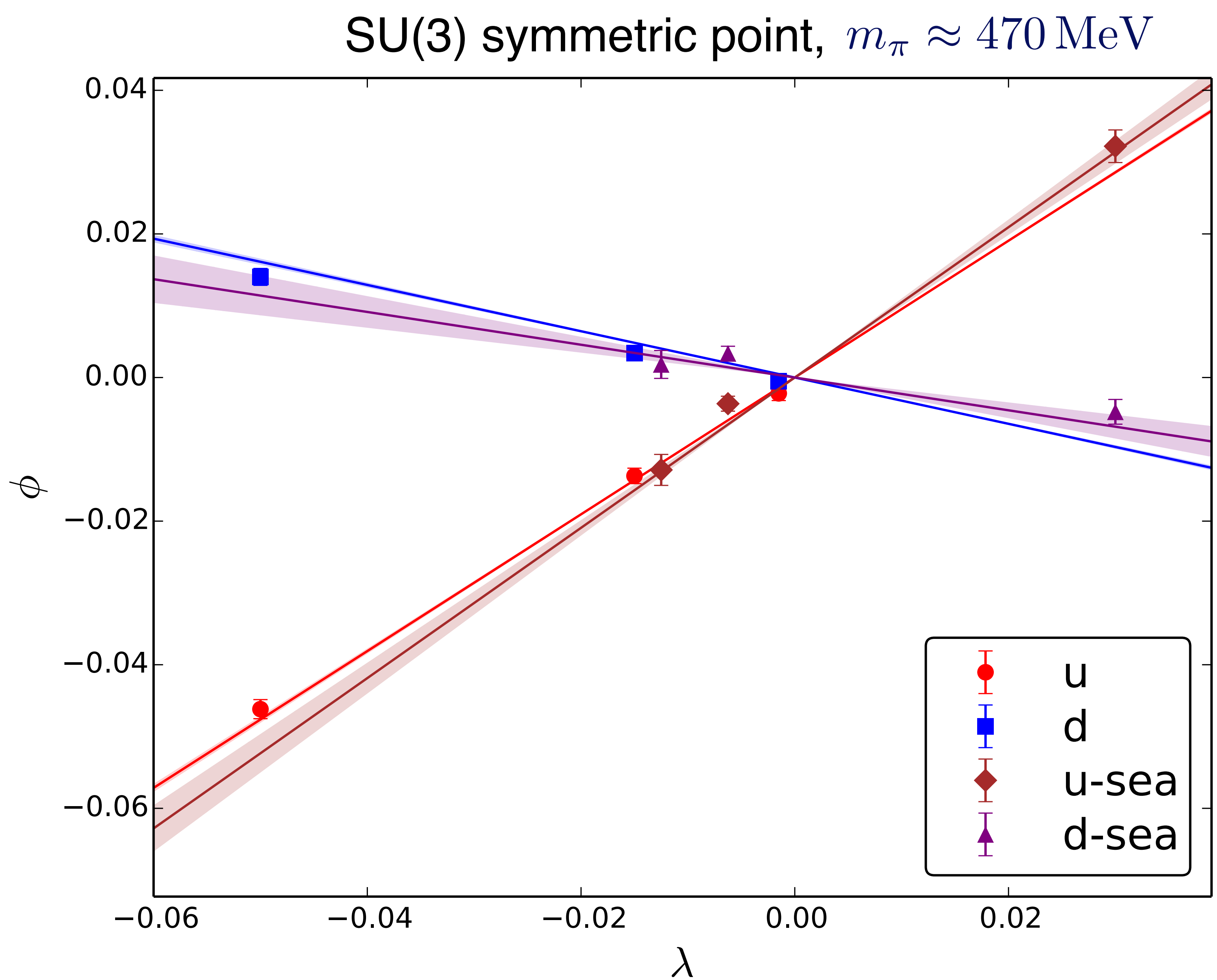
$$\Delta\Sigma = Z_A^{\mathrm{S}} \Delta\Sigma^{\mathrm{latt}}$$

[Alternative method using AWI: K.-F. Liu]

Disconnected Spin Contributions

Preliminary!

[A.Chambers]



$\overline{\mathrm{MS}}\ \mu^2 = 4\,\mathrm{GeV}^2$

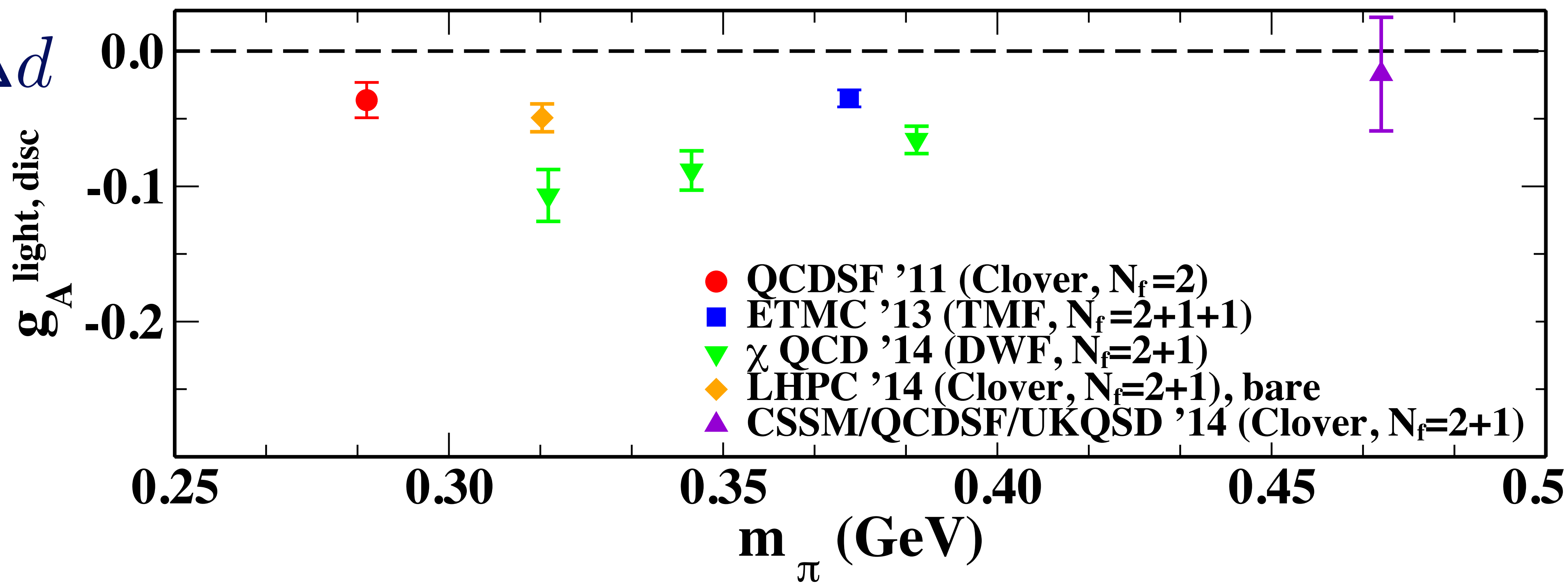
$$\Delta\Sigma_{\mathrm{disc}} = 3\Delta q_{\mathrm{disc}} = -0.081(46)$$

$$\Delta u_{\mathrm{disc}} = \Delta d_{\mathrm{disc}} = \Delta s_{\mathrm{disc}} = -0.025(15)$$

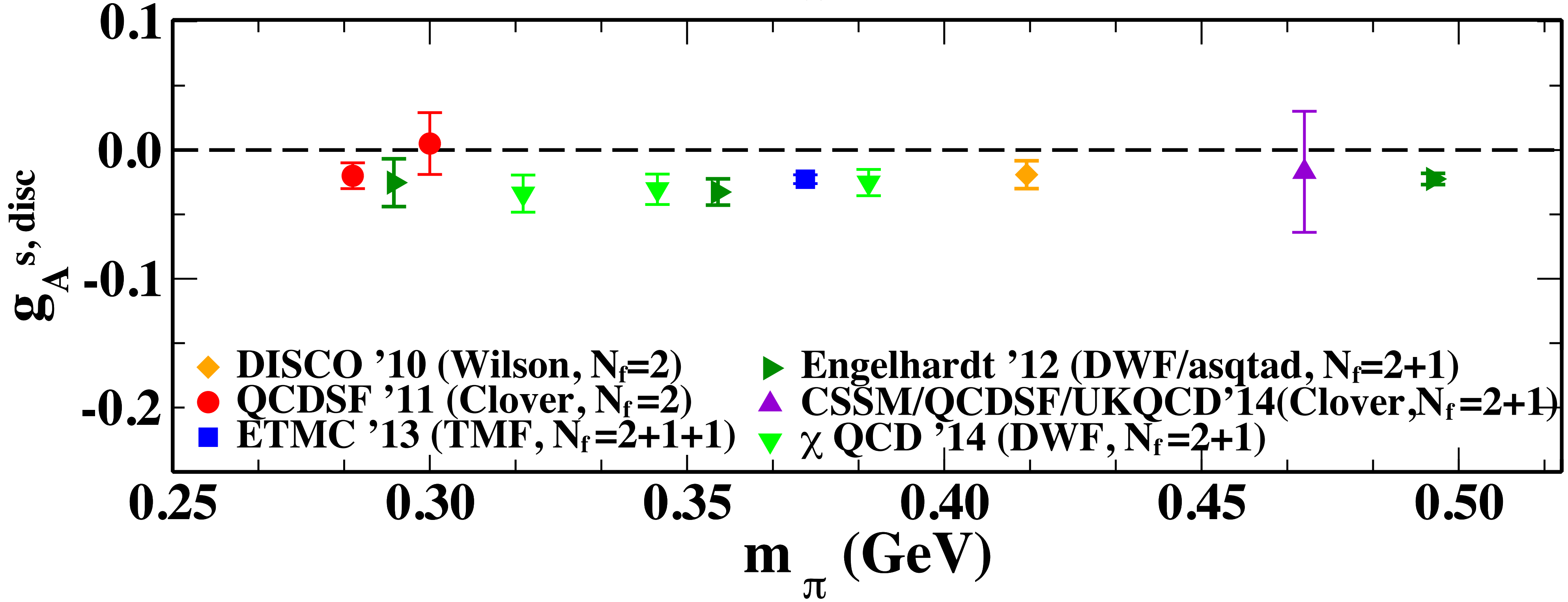
Disconnected Spin Contributions

M.Constantinou (Lattice 2014)

$\Delta u = \Delta d$



Δs



Other Observables

Moments of Parton Distribution Functions $\langle x \rangle_{q,g}$

Moments of Generalised Parton Distribution Functions

Sigma terms $\sigma_{\pi N}, \ \sigma_s$

Moments of TMDs

[M.Engelhardt et al. PRD 85 (2012) 094510, Lattice 2014]

Other hadrons $(\pi, \Sigma, \Lambda, \ldots)$

- Review: Ph. Hägler, 0912.5483
- Lattice review talks
  - 2013: S.Syritsyn 1403.4686
  - 2012: H.-W. Lin 1212.6849
  - 2011: H. Wittig 1201.4774

Summary

Lattice hadron structure simulations now approaching the physical point

Lots of effort in understanding systematic errors (e.g. in g_A)

Form factors are in good shape

- **Small strangeness form factors**

- **Small charge symmetry violation**

Excellent progress in decomposing nucleon spin

$$\Delta s \sim -5\%$$

$$L^{u+d}_{\mathrm{conn}} \sim 0$$

$$L^{q}_{\mathrm{disc}}\ ?$$

[Plenary by K.-F. Liu]