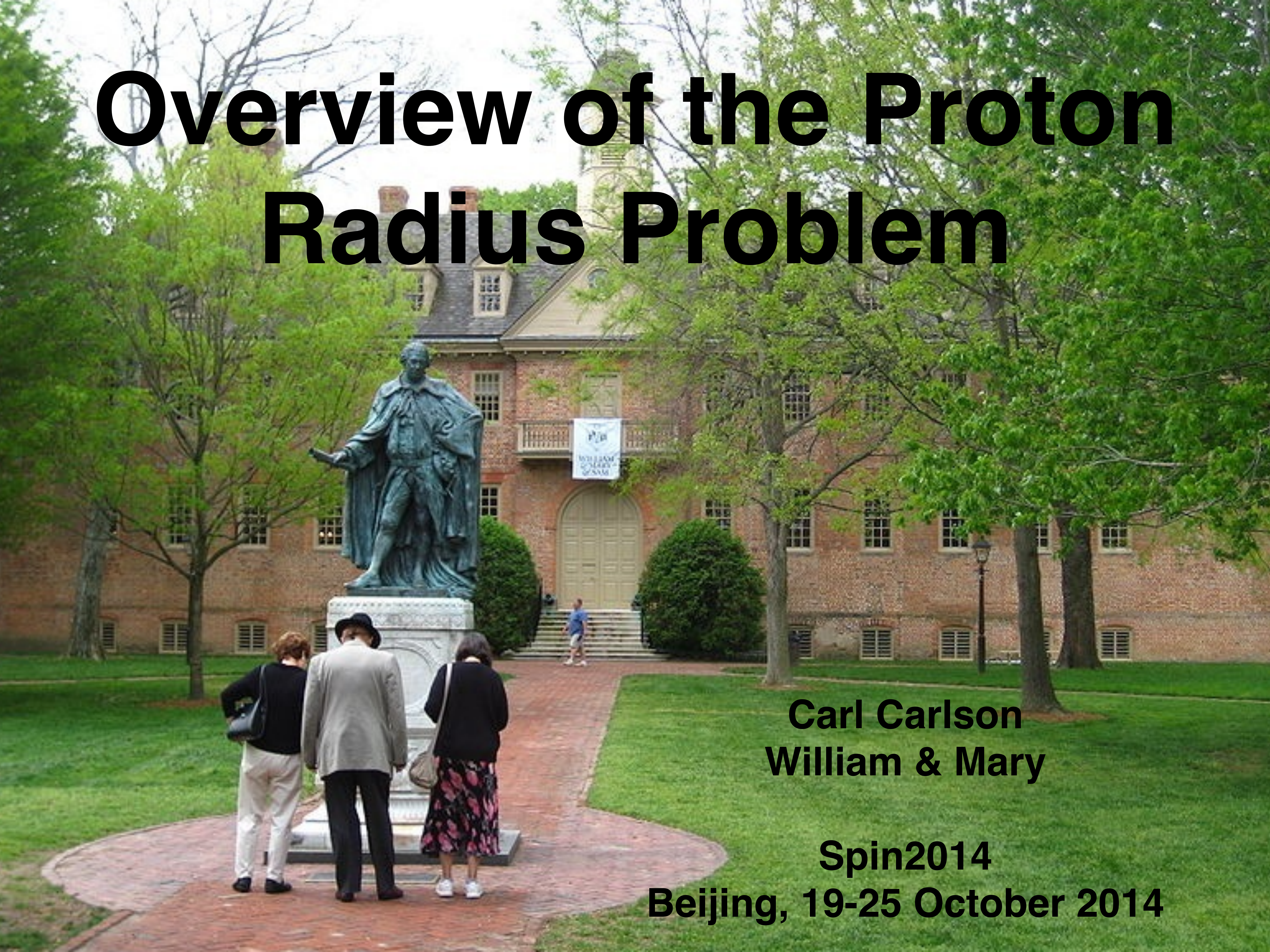


Overview of the Proton Radius Problem

Carl Carlson
William & Mary

Spin2014
Beijing, 19-25 October 2014

The Problem

- Measure charge radius of the proton different ways, get different answers
- Difference is 7 s.d.
  (was 5 s.d. when first announced, 2010)
- Why? Reason not yet known.

This talk

- The measurements:
  where the differences came from
- Suggested explanations
  - Humdrum explanations
    - Somebody screwed up
  - Exotic explanations
    - Physics Beyond the Standard Model (BSM)

The proton radius

- Measure radius by measuring form factors in e-p elastic scattering,

$$\frac{d\sigma}{d\Omega} = \left.\frac{d\sigma}{d\Omega}\right|_{NS} \times \frac{1}{(1+\tau)}\Big(G_E^2(Q^2) + \frac{\tau}{\epsilon}G_M^2(Q^2)\Big),$$

$$\tau = Q^2/(4m_p^2), \qquad \epsilon^{-1} = 1 + 2(1+\tau)\tan^2(\theta_e/2)$$

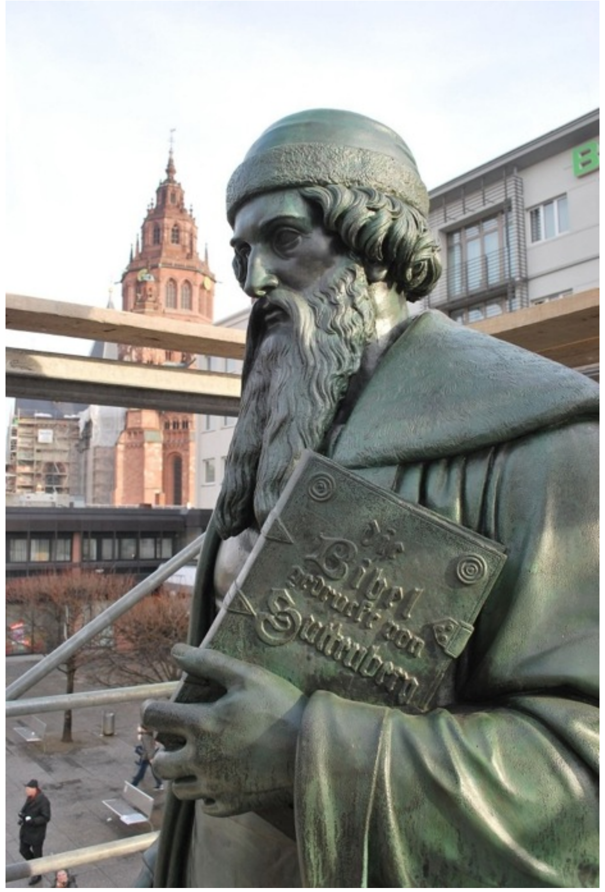
- Obtain charge radius from

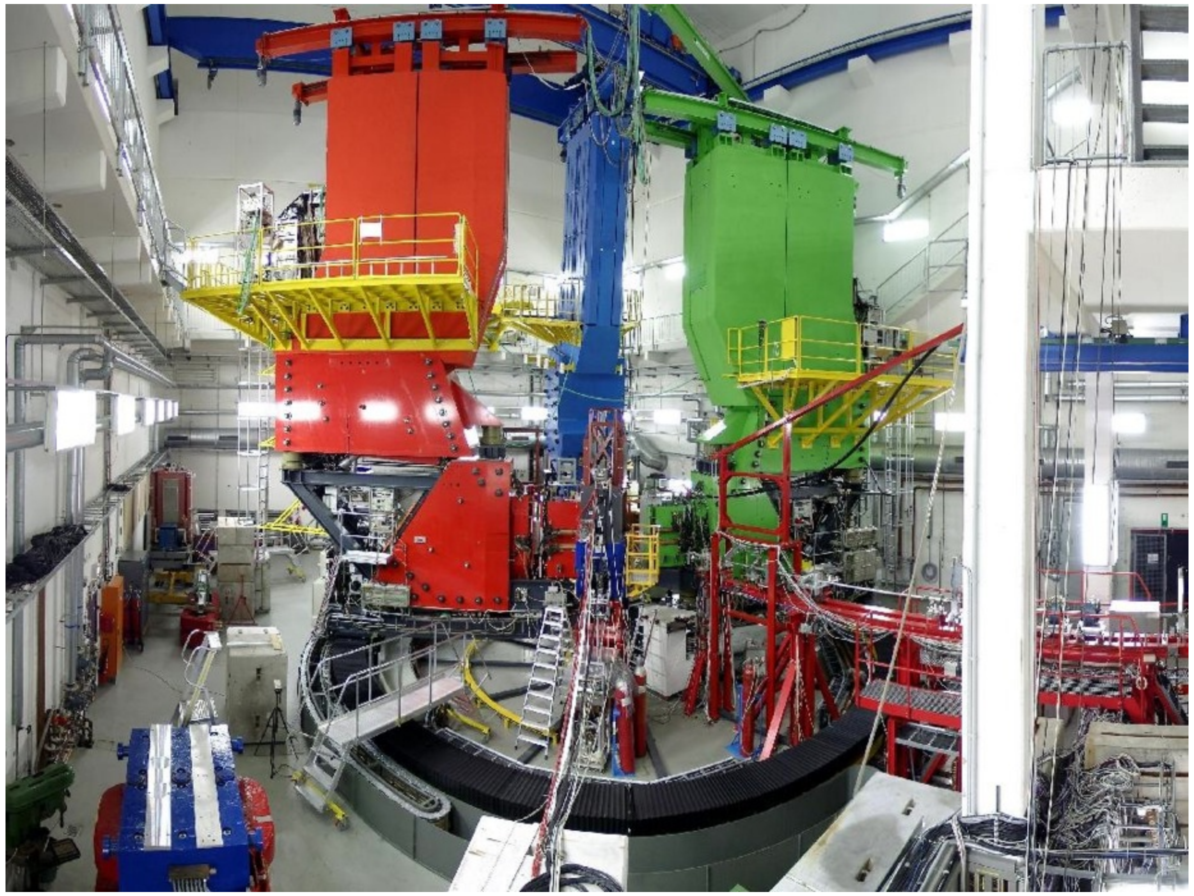
$$G_E(Q^2) = 1 - \frac{1}{6}R_E^2\,Q^2 + \dots$$

Best low-Q^2 scattering data

- Mainz, Jan Bernauer *et al.*, PRL 2010

- Mainz famous for Gutenberg and the Mainz electron accel.





- From their analysis,

$$R_E = 0.879(8)\ \text{fm}$$

Radius from atomic energy levels

- This is another method to measure proton radius.
- Schrödinger eq., H-atom, point protons

$$E = -\frac{Ryd}{n^2}$$

- where

$$Ryd = \frac{1}{2} m_e \alpha^2 \approx 13.6 \text{ eV}$$

Energy shift from proton size

- Since proton has finite size, energy perturbed upward a bit.

- Good HW problem for NR quantum course. From Karplus, Klein, Schwinger, 1952, for *nS* state

$$\Delta E_{\text{finite size}} = \frac{2\pi\alpha}{3}\phi_{nS}^2(0)\langle r^2\rangle$$

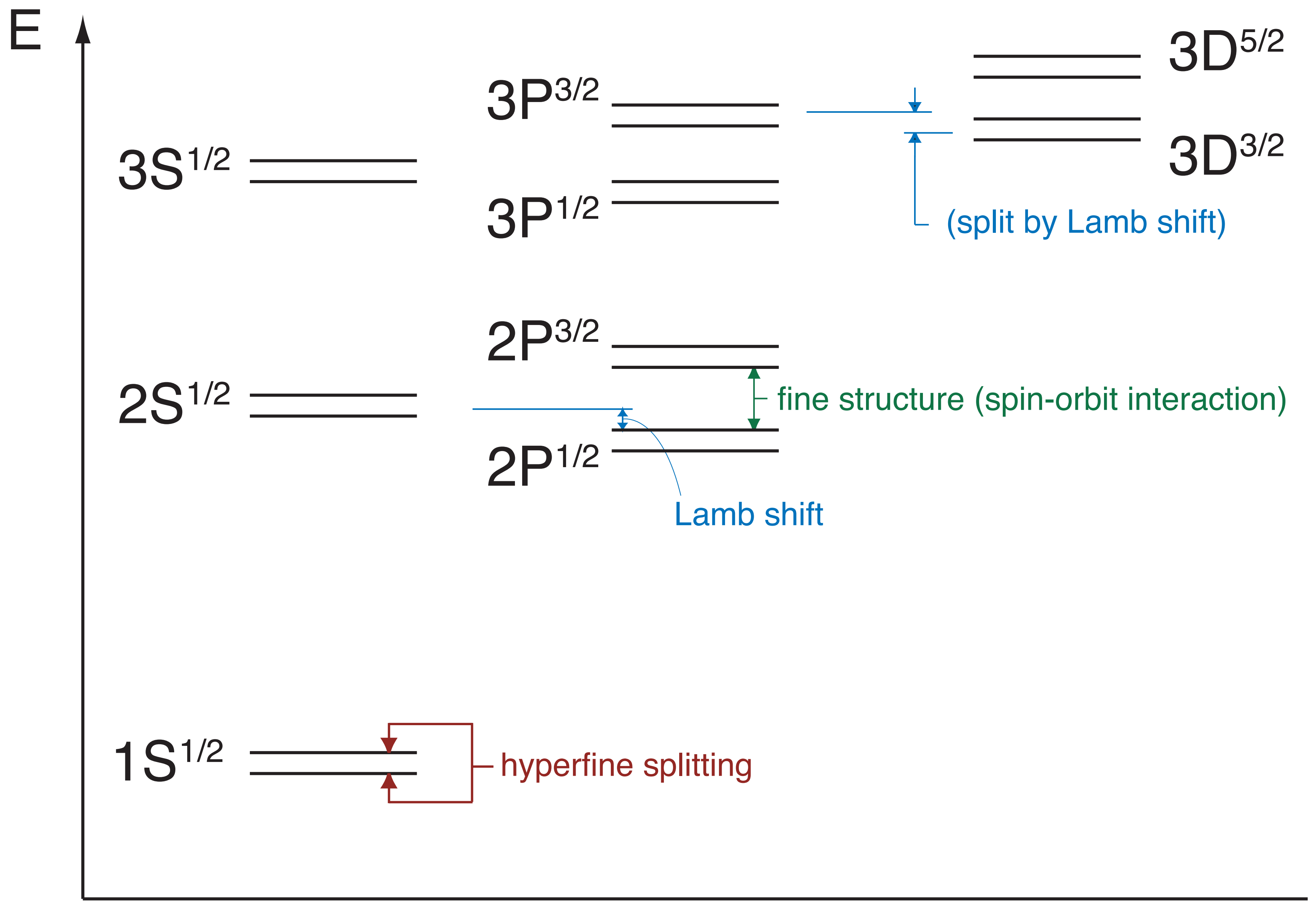
- Modernized and relativistic,

$$\Delta E_{\text{finite size}} = \frac{2\pi\alpha}{3}\phi_{nS}^2(0)R_E^2$$

w.f. at origin in coordinate space is $\phi_{nS}^2(0) = (m_r\alpha)^3/(n^3\pi)$

measure energy accurately
⇔ measure radius

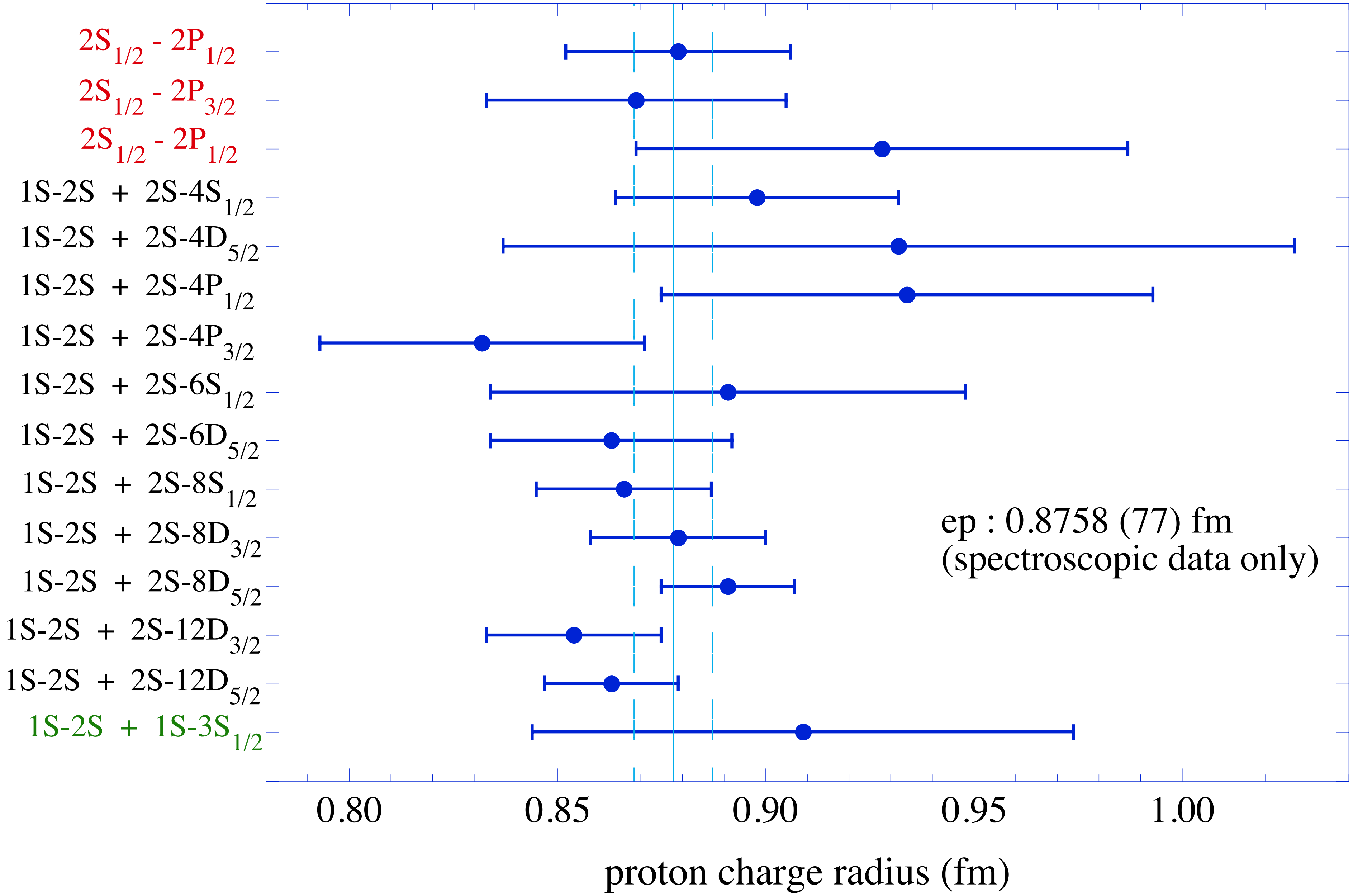
- More detail: hydrogen energy levels (not to scale)



Splitting measurements

- Measure small splitting, like Lamb shift, get R_E
- Or, measure large splitting, like $2S$-$4P$
- For large splittings, *Ryd* not known well enough from elsewhere to isolate proton radius effect
- But can combine with (say) $1S$-$2S$ splitting to obtain both *Ryd* and R_E
- Get proton radius to few %.

Atomic plot


$2S_{1/2}$ - $2P_{1/2}$
$2S_{1/2}$ - $2P_{3/2}$
$2S_{1/2}$ - $2P_{1/2}$
1S-2S + $2S\text{-}4S_{1/2}$
1S-2S + $2S\text{-}4D_{5/2}$
1S-2S + $2S\text{-}4P_{1/2}$
1S-2S + $2S\text{-}4P_{3/2}$
1S-2S + $2S\text{-}6S_{1/2}$
1S-2S + $2S\text{-}6D_{5/2}$
1S-2S + $2S\text{-}8S_{1/2}$
1S-2S + $2S\text{-}8D_{3/2}$
1S-2S + $2S\text{-}8D_{5/2}$
1S-2S + $2S\text{-}12D_{3/2}$
1S-2S + $2S\text{-}12D_{5/2}$
1S-2S + $1S\text{-}3S_{1/2}$
ep : 0.8758 (77) fm
(spectroscopic data only)
0.80
0.85
0.90
0.95
1.00
proton charge radius (fm)

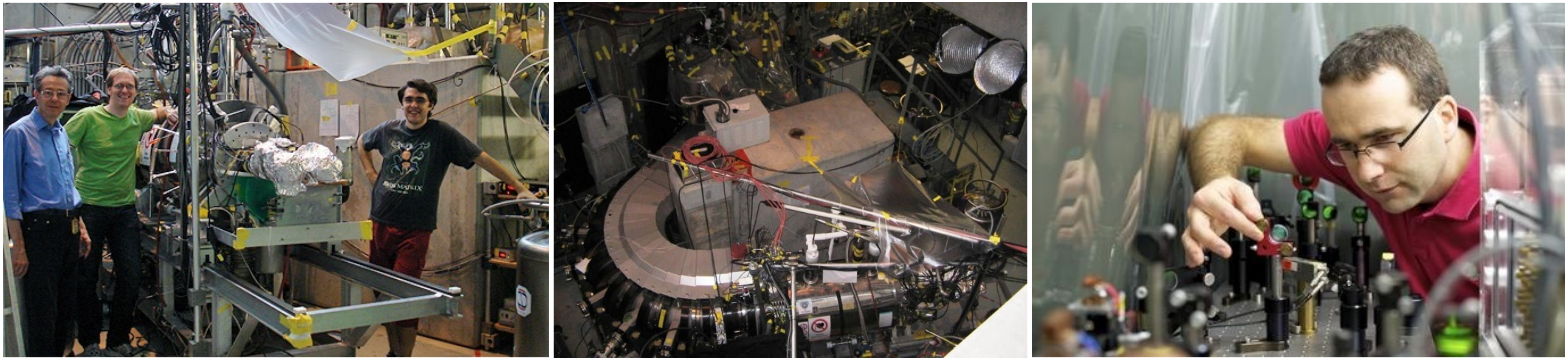
Combined electron results

- Spectroscopy collection gives uncertainty under 1%
- Consistent with scattering result
- Combined with scattering result by Committee on Data in Science and Technology (CODATA),

$$R_E = 0.8775(51)\ \mathrm{fm}$$

But…

- Along came CREMA = Charge Radius Experiment with Muonic Atoms



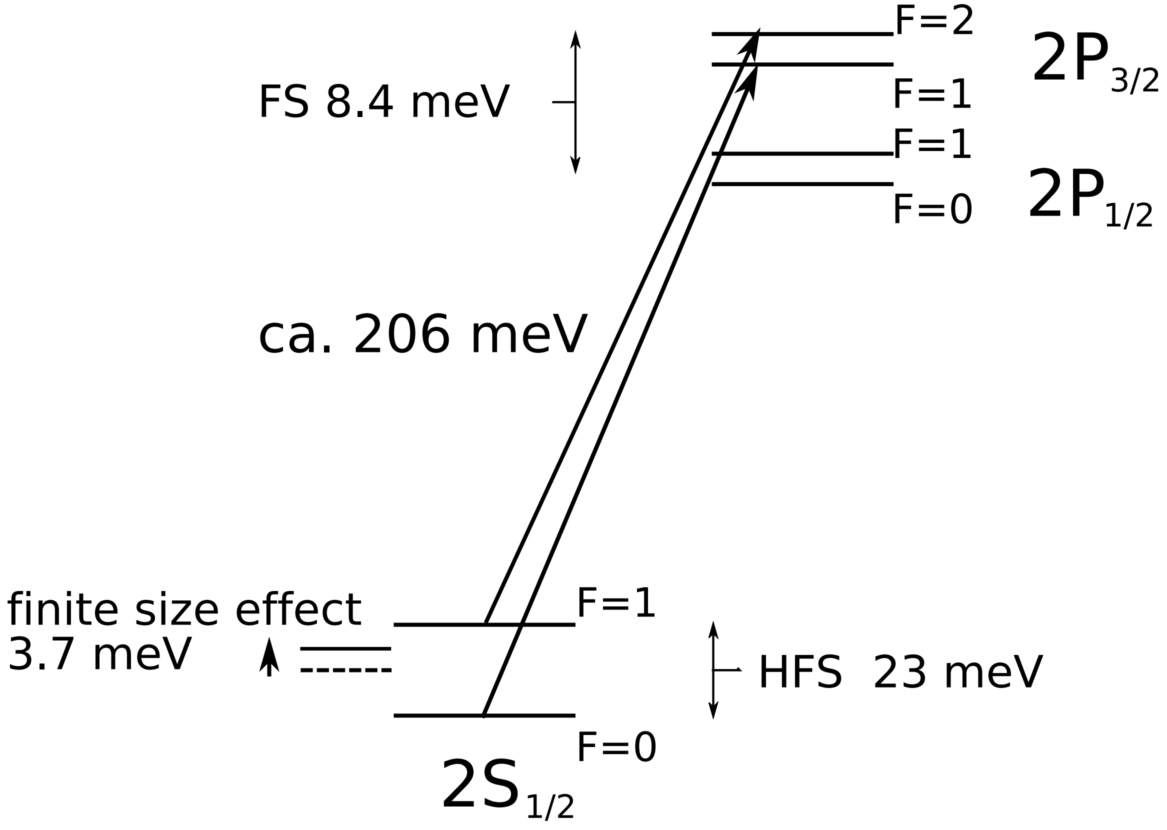
- Did atomic physics with muons (muon= electron, but weighs 200 times more).

- Orbits 200 times closer: proton looks 200 times bigger

- Goal: measure proton radius with factor 10 smaller uncertainty

More CREMA

- Detail of n = 2 states in μ-H



- 2S state metastable
- Laser induced two transitions.
  Pubs:
  Pohl *et al.*, Nature 2010
  Antognini *et al.*, Science 2013

- Interpreting finite size effect in terms of proton radius,

$$R_E = 0.84087(39)\,\mathrm{fm}$$

Recap

- electrons $R_E = 0.8775(51)\ \mathrm{fm}$

  muons (CREMA) $R_E = 0.84087(39)\ \mathrm{fm}$

- Met their uncertainty goal!

- But result 4% or 7σ small

- For later, also obtained HFS in 2S state,

$$\Delta E_{\mathrm{HFS}}^{\mathrm{exp}} = 22.8089(51)\ \mathrm{meV}$$

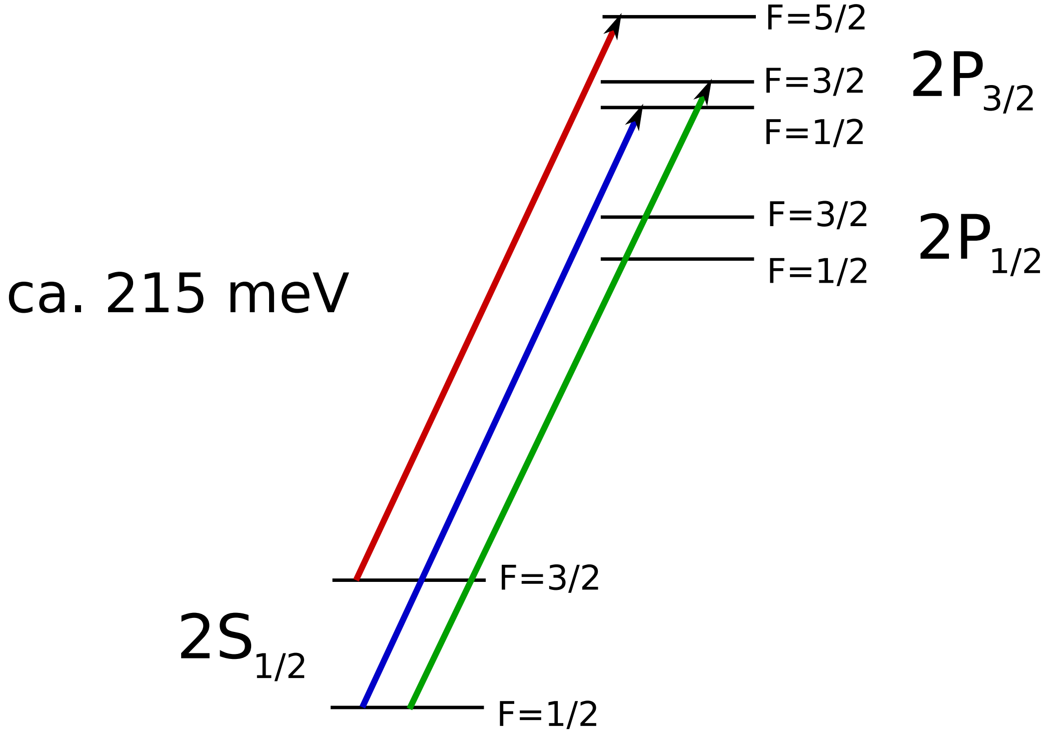
- This one agrees with best theory

$$\Delta E_{\mathrm{HFS}}^{\mathrm{thy}} = 22.8146(49)\ \mathrm{meV}$$

(Nazaryan, Griffioen, me)

Other data: μ-deuteron

- Conference reported 2013
- Measured three lines



- Quick summary: if proton radius is shrunken, this deuteron radius is also.

Other data: μ-Helium

- New 2013/2014 data
- μ-^{4}He at Mainz Proton Radius Workshop, 2014
- μ-^{3}He at Gordon Conference, N.H., 2014
- Quick summary: He radii from μ Lamb shift <u>in accord with</u> electron scattering radii.

Explanations?

- Hard to see problems with μ experiment
    - Hard to get working
    - But once working, easy to analyze
- Easier to see problems with analysis of electron experiments—but there are a lot of them
- Are BSM explanations possible?
    - If so, what further tests might there be?

Electron scattering data

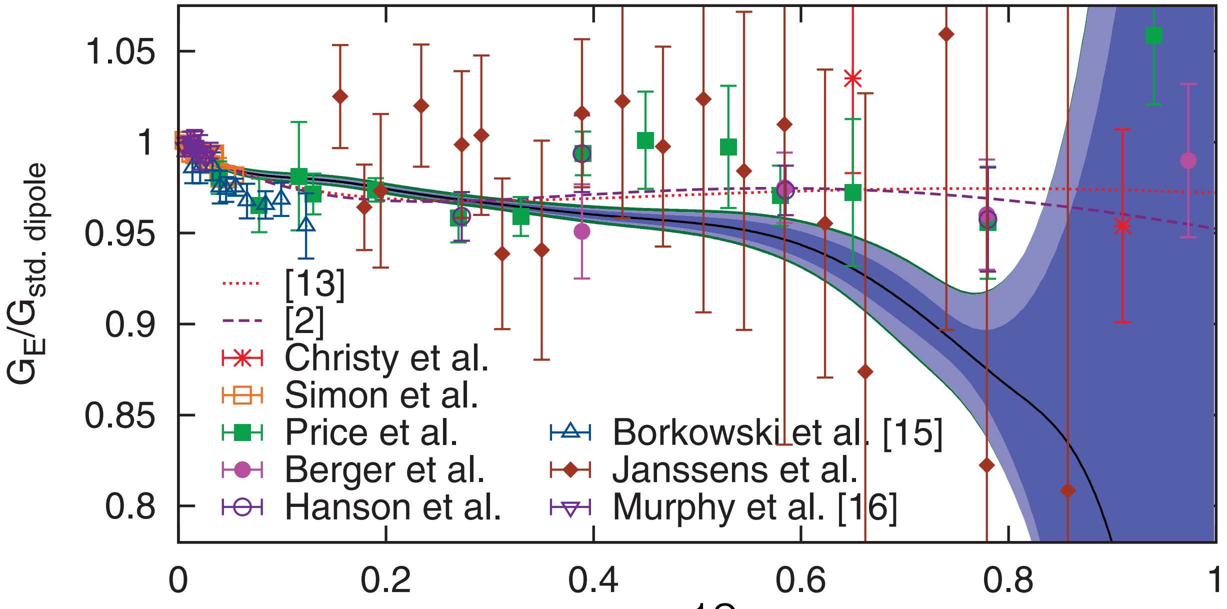
- Mainz 2010 measures differential cross section, has 1422 data points, about 0.3% relative error, about or below 2% absolute error.

- Want slope of G_E at $Q^2 = 0$. Cannot measure to $Q^2 = 0$, so extrapolate.

- Mainz data has $0.004 < Q^2 < 1$ GeV^2.

Mainz’s own fit

- The experimenters fit G_E and G_M to their data using polynomials or modified polynomials in Q^2.
- Results have small error limits compared to other data.
- Extrapolation to $Q^2 = 0$ gave “big” result quoted already.



On the other hand

- There is reason to believe polynomial expansion don't converge for Q^2 beyond $4m_\pi^2 \approx 0.08$ GeV^2.

- Lorenz and Meissner did a conformal transformation to a new variable in terms of which a polynomial expansion would be convergent.

- They fit the Mainz data and got

$$R_E = 0.84(1)\ \text{fm}$$

- hmm

But still

- Hill and Paz also did a fit over a wide range of Q^2 using the variable that should allow convergence.

$$R_E = 0.870(23)(12)\ \mathrm{fm}$$

- But they did not use the Mainz 2010 data, only a collection of older data.

And then there is

- A fit using only low Q^2 data, where convergence of a polynomial expansion should not be a problem.

- Low Q^2, but still a long enough range to well determine the charge radius upon extrapolation.

$$R_E = 0.840(5)\ \mathrm{fm}$$

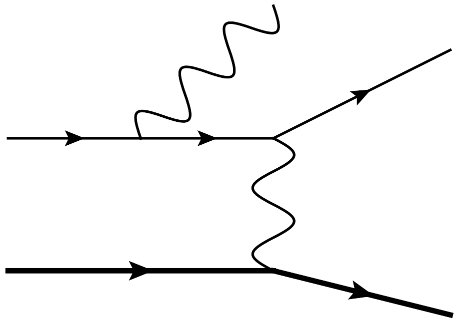
- Local product: Griffioen, Maddox, me.

- Conclusion: a bit up in the air.

Scattering future

- Further experiments with lower lowest Q^2. Reduces length of extrapolation to $Q^2 = 0$.

- PRad at JLab: Just target and detector screen, allowing very small scattering angles. Anticipate $Q^2|_{\text{low}} \approx 0.0002$ GeV^2. Hope running in 5 months.



- ISR (Initial State Radiation) at Mainz. Photon radiation takes energy out of electron, allowing lower Q at given scattering angle. Anticipate $Q^2|_{\text{low}} \approx 0.0001$ GeV^2. Data taken; under analysis.

MUSE

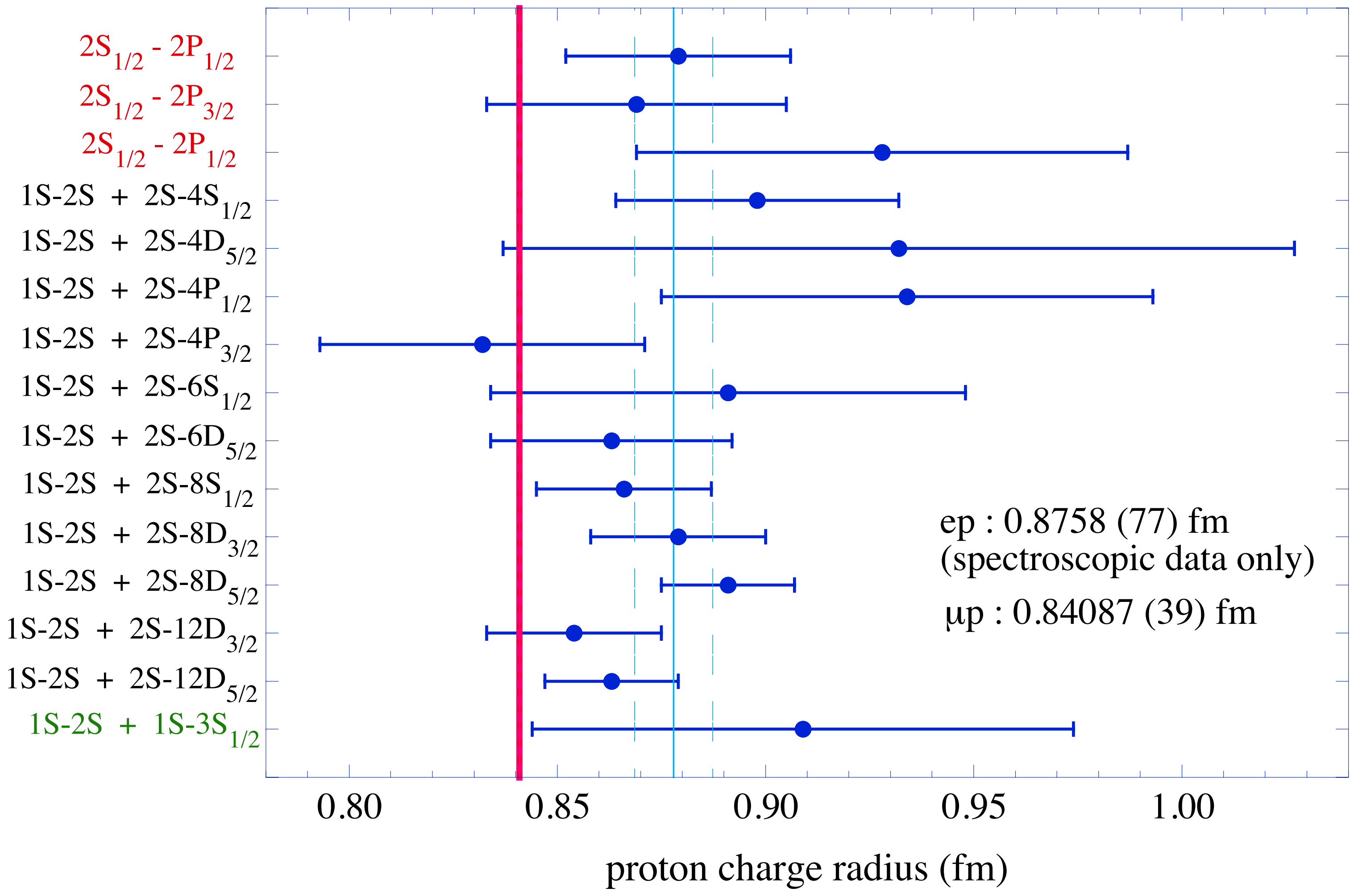
- Muon scattering experiment at the PSI.

- Proton radius measurement table

	atomic spectroscopy	**scattering**
electron	yes	yes
muon	yes	no

- MUSE will fill in table. Anticipate $Q^2|_{\rm low} \approx 0.002$ GeV^2. Production runs 2017/2018.

Back to atomic spectroscopy



- Same plot, but *μ*-H value added

- Possible: correlated systematic errors. There are more measurements than independent expt’l groups.

Short term future

- Independent groups are doing more precise experiments that will individually get the proton radius to under 1%.
- York University (Canada): Ordinary hydrogen 2*S*-2*P* Lamb shift
- MPI Quantum Optics (Garching): 2*S*-4*P* transition
- Laboratoire Kastler Brossel (Paris): 1*S*-3*S* transition
- All promise delivery before end of 2014

Numbers note

- Take 1*S*-3*S* as example (the LKB measurement)
- splitting about 2.9 x 10^{12} kHz
- difference due to CODATA vs. μ-H proton radii difference about 7.2 kHz
- ∴ need ppt accuracy. Wow.
- Already have (2010) measurement with 13 kHz error bar.

Exotic possibilities

- Consider breakdown of muon-electron universality. New particle coupling to muons and protons. Small or no coupling to other particles.

- References (positive side): Tucker-Smith & Yavin (2011), Batell, McKeen, & Pospelov (2011), Brax & Burrage (2011), Rislow & Carlson (2012, 2014)

- References (less positive): Barger, Chiang, Keung, Marfatia (2011, 2012), Karshenboim, McKeen, & Pospelov (2014)

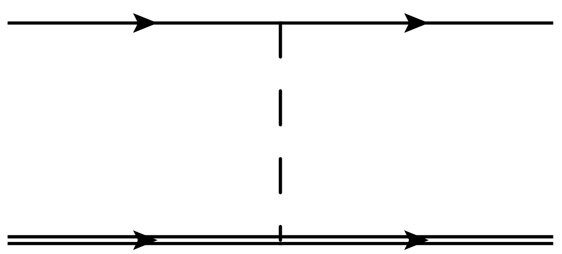
μ-H Lamb shift

- Idea: Experimenters do not directly measure proton radius. Measure energy deficit, 320 μeV. Interpret as proton radius deficit.

- Now: Proton radius unchanged. Energy deficit due to new force, carried by exchange of new particle.

- New particle is scalar or vector. Pseudoscalar or axial vector have little effect on Lamb shift for similar couplings.

Energy shift



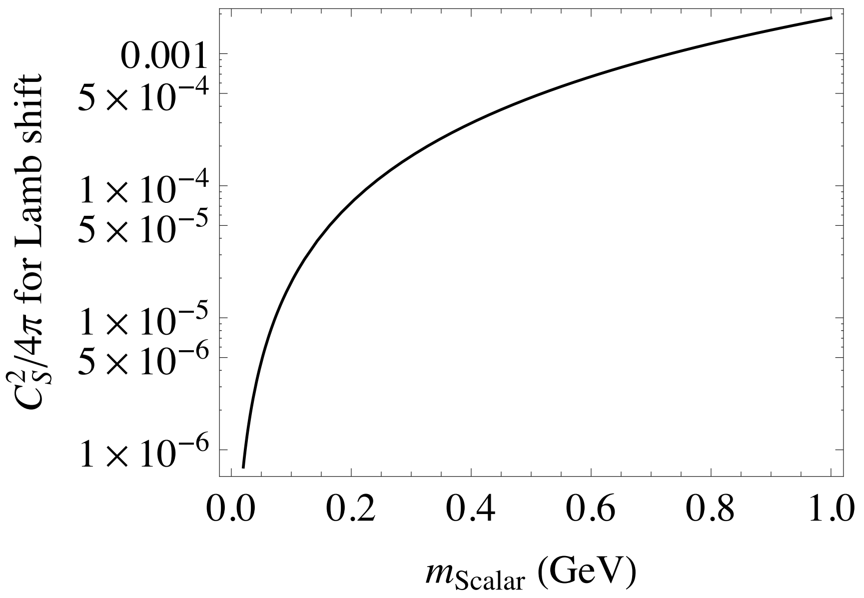
- e.g., scalar case

$$V(r) = -\frac{C_S^\mu C_S^p}{4\pi r} e^{-Mr}$$

$$\mathcal{L}_S = - C_S^\mu \phi \bar{\psi}_\mu \psi_\mu - C_S^p \phi \bar{\psi}_p \psi_p$$

$$\Delta E_{\text{new exch.}} = -\frac{C_S^\mu C_S^p}{4\pi} \frac{m_\phi^2 (m_r \alpha)^3}{2(m_\phi + m_r \alpha)^4}$$

- So far, easy. Pick $C_S^\mu C_S^p$ to give 320 μeV for given m_ϕ. (Plot for $C_S^\mu = C_S^p$.)



Muon couplings unavoidable

- Worry about other processes where new particle couples to muons. *E.g.:*
  - Loop corrections to μ magnetic moment
  - Radiative corrections to decays involving muons, like $K \rightarrow \mu\nu$ means $K \rightarrow \mu\nu\phi$ also allowed
  - Other Lamb shift related corrections, like the HFS and the μ-He Lamb shift non-corrections.

μ magnetic moment

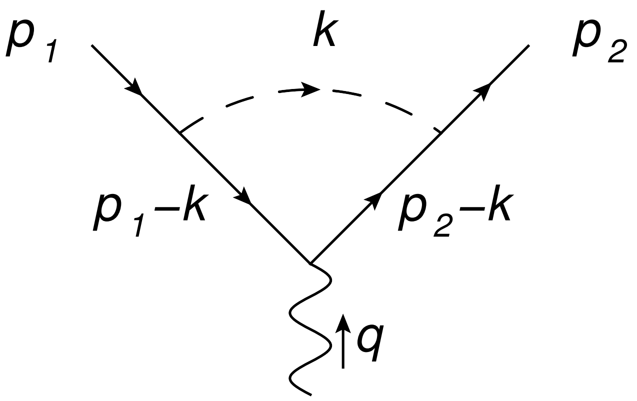
- Exists discrepancy between calculated and observed μ magnetic moment, phrased in terms of $a_\mu = (g\text{-}2)_\mu/2$, g = gyromagnetic ratio.

$$\begin{aligned} a_\mu(\text{data}) &= (116\,592\,089 \pm 63) \times 10^{-11} \quad [0.5\ \text{ppm}], \\ a_\mu(\text{thy.}) &= (116\,591\,840 \pm 59) \times 10^{-11} \quad [0.5\ \text{ppm}], \\ \delta a_\mu &= (249 \pm 87) \times 10^{-11} \quad [2.1\ \text{ppm} \pm 0.7\ \text{ppm}] \end{aligned}$$

- From Aoyama et al., who quoted theory from Hagiwara et al.

- From present viewpoint, 2 ppm discrepancy is good agreement.

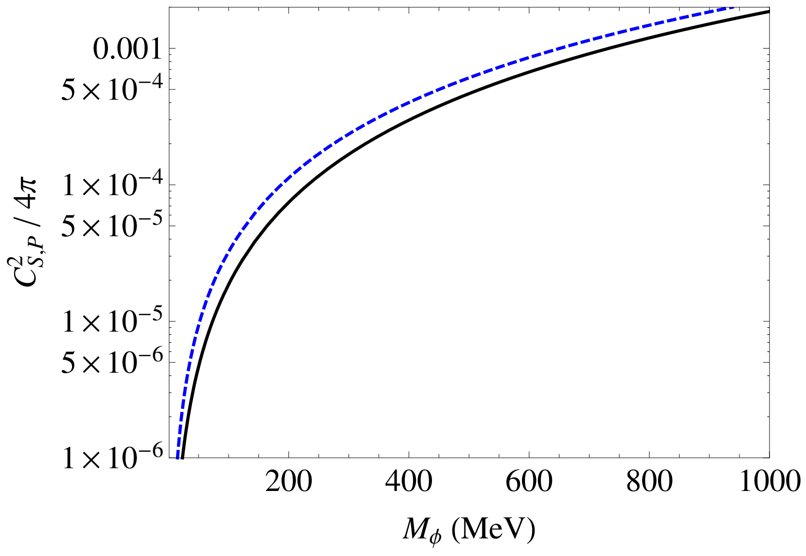
Fixing (g-2)$_\mu$

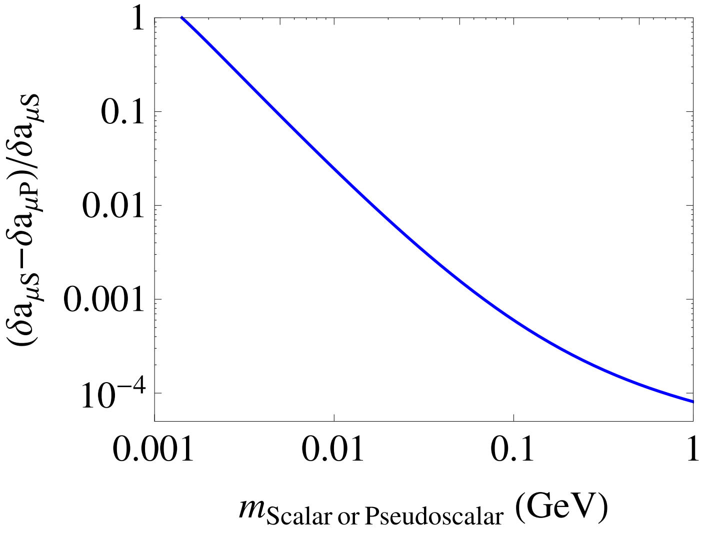


- Lucky break: corrections to (g-2) from regular vector and axial vector have opposite sign. Same is true of scalar and pseudoscalar.

- Add extra particle. Have new coupling, say C_P^i. Choose coupling to cancel in (g-2)$_\mu$. Won't much affect Lamb shift.

Fine tuning



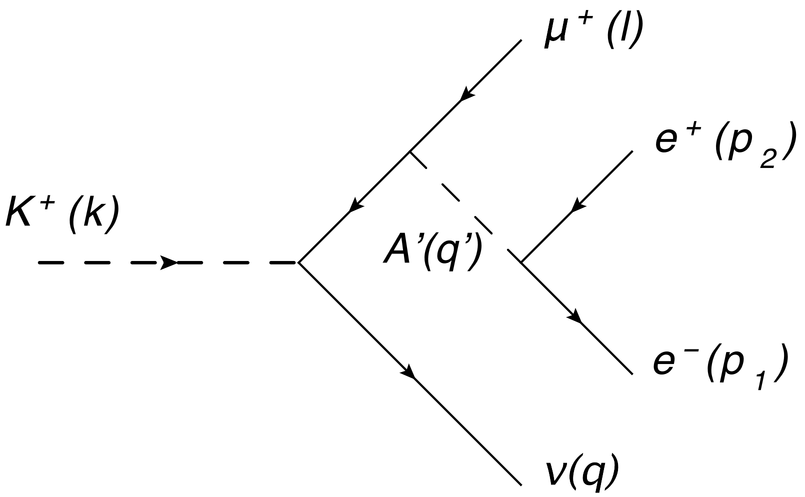


- (Above are for scalar-pseudoscalar)
- Low enough mass, cancellation not needed (TSY)
- Couplings now fixed, albeit mass sensitive.
- ∴ Predictions for other processes now fixed.

K decay

- One of the “other processes”

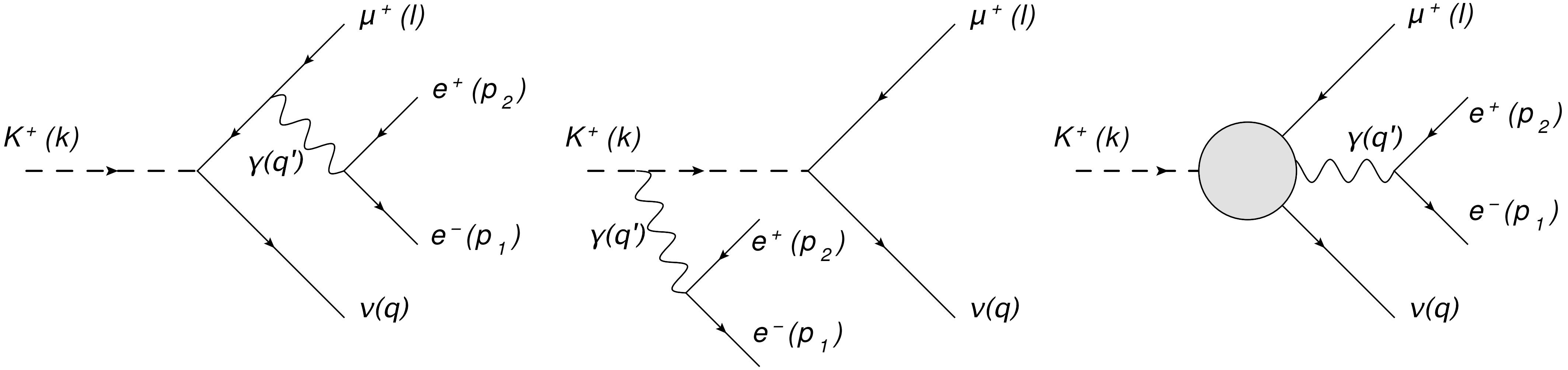
- If some (even small) coupling to electron, have



- (new particle here called *A*’.)

QED background

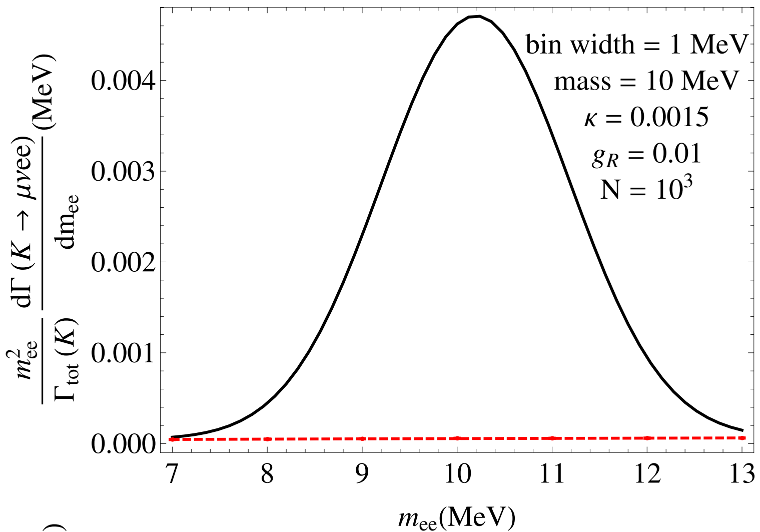
- QED gives same final state, with smooth spectrum of e^+e^-.



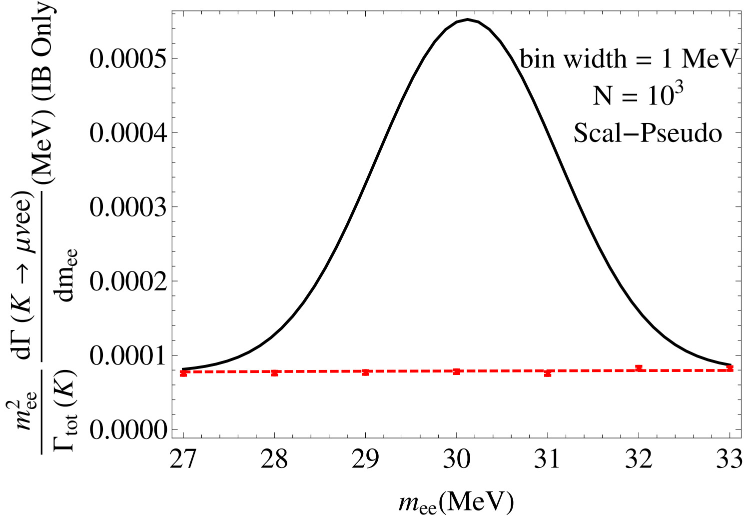
- Measured for m_{ee} > 140 MeV, otherwise calculated, notably by Bijnens *et al.* (1993).

A′ visible?

- A′ will give bump. Size calculable. Is it observable?



- Yes. (If it exists.)



- Note: TREK experiment (E36) at JPARC (Japan) will observe 10^{10} kaon decays, or about 200,000 $K \to \mu\nu e^+e^-$ events, about 1000 per MeV bin in the mass range we are considering. (Thanks to M. Kohl)

Plots from Rislow and me (2014)

μ-H HFS

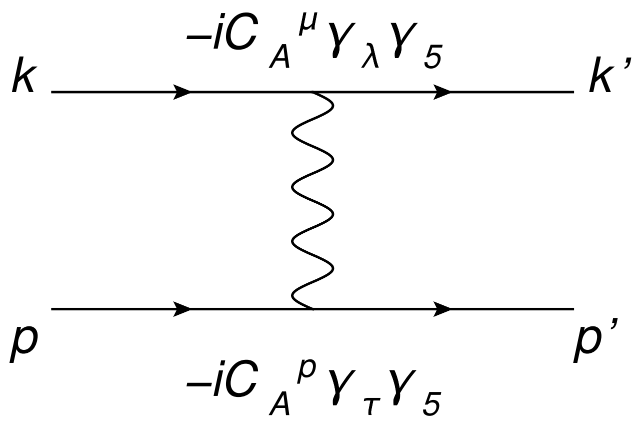
- Recall HFS in the 2*S* state of μ-H was measured as

$$\Delta E_{\mathrm{HFS}}^{\mathrm{exp}} = 22.8089(51)\ \mathrm{meV}$$

- and agreed with standard theory. Suggests HFS from exotics must be small, say below 5.1 μeV.

- Worst case is axial vector. Gives contribution to HFS in leading order in NR expansion.

HFS



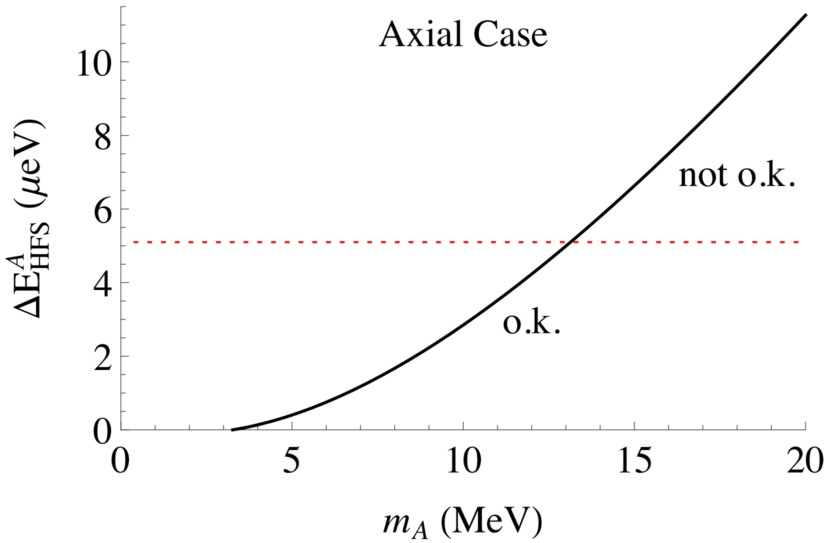
- As a scattering amplitude, above is

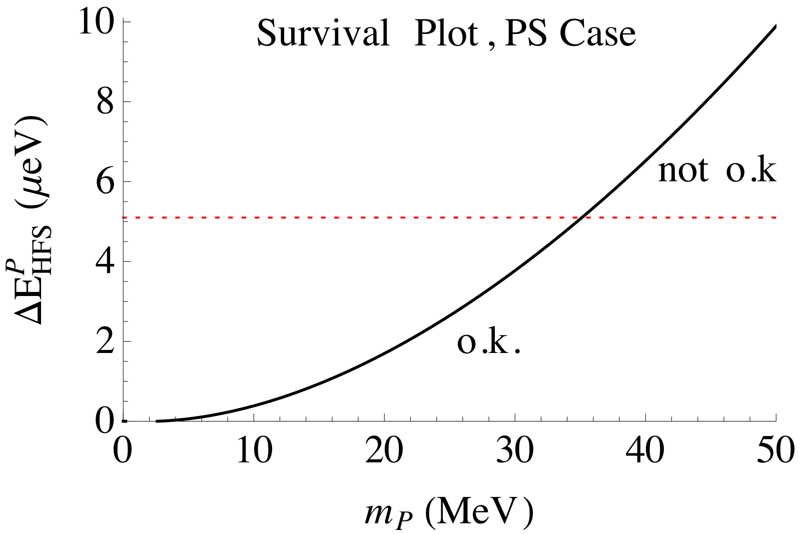
$$\mathcal{M}_A = 2m_\mu\, 2m_p \frac{C_A^\mu C_A^p}{\vec{q}^{\,2}+m_A^2}\, \vec{\sigma}_\mu\cdot\vec{\sigma}_p$$

- In terms of contribution to *S*=1 to *S*=0 HFS, for 2*S* state,

$$\Delta E^A_{\mathrm{HFS}} = -\frac{C_A^\mu C_A^p}{4\pi}\,\frac{2m_r^3\alpha^3}{m_A^2}\,\frac{m_A^2\left(m_A^2+\frac{1}{2}m_r^2\alpha^2\right)}{(m_A+m_r\alpha)^4}$$

HFS and mass limits





- Axial case (which is part of vector case by the previous fine tuning) is o.k. if mass below about 13 MeV.

- Analogous pseudoscalar/scalar case o.k. with mass limit of 35 MeV.

Helium Lamb shift

- He radii measured in electron scattering, to about 1/4%. This radius goes into prediction for Lamb shift in μ-He, with appropriate uncertainty limit.

- Preliminary data on μ-He Lamb shift agrees with prediction, to about 1σ. If due to heavy BSM particle exchange, should disagree by about 5σ.

- How does mass creep in?

Heavy atom Lamb shift

- Physics: Potential is like Yukawa potential, with range controlled by mass. Light mass, long range, like Coulomb potential, does not split S and P states.

- Application: Z=2 helium has orbital muons closer to nucleus than Z=1 hydrogen. What looks like long range to helium is short range to hydrogen, if mass chosen correctly.

Numbers for Z=2 Lamb shift

- BSM energy shift,

$$\Delta E^{A}_{\mathrm{BSM}} = -\frac{C^{\mu}_{\phi} C^{A}_{\phi}}{m^{2}_{\phi}} |\phi^{A}_{2\mathrm{S}}(0)|^{2} f(x)$$

- for $f(x) = x^4/(1+x)^4$ and $x = m_\phi\, a = m_\phi/(Z\, m_r\, \alpha)$
- Get suitable result for proton and small enough result for He if $m_\phi \approx 1$ MeV.

Lots of new data coming

- New CREMA measurements (out at conferences, 2013/14)
- MUSE (2017/2018)
- PRad (run 2015)
- ISR form factor meas. (data taken)
- Electron deuteron scattering (Griffioen *et al.*, Mainz) (data taken)

- High precision Lamb shift in e-H (York, 2014)
- $2P$-$4P$ e-H splitting at Garching (2014)
- $1S$-$3S$ e-H splitting at LKB, Paris (2014)
- TREK at JPARC
- Alternative measurements of the Rydberg (NIST, 2018)
- Trumuonium ($\mu^+\mu^-$) at JLab

End

- 4 years after the first announcement, the problem persists
- Interestingly little discussion of the correctness of the μ-H Lamb shift data
- More discussion of the extrapolations that obtain the charge radius from scattering data (not so long, but good precision is required). More data coming. But really need to settle the present discussion.
- Curiosity over systemic errors, but no real criticism of the atomic spectroscopy results. Nonetheless serious new experiments are in progress.
- Exotic or beyond the standard model explanations of the discrepancy face serious constraints, but windows are still open.
- Potential for immediate impact on other processes: the theory for $(g\text{-}2)_\mu$ cannot be considered settled until the proton radius problem is settled, and there may be striking corrections to decays that involve muons.

The end