Calculation of coupling spin resonance harmonics with Snakes

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ASPIRRIN code features

- Calculates the set of spin-orbital functions F_i, which can be used to:
  - Evaluate equilibrium polarization and depolarizing resonances in electron rings with SR.
  - Evaluate depolarization due to different diffusion mechanisms.
  - Evaluate strength of the imperfection resonances, as well as the resonances produced by the spin-flippers, using response function technique.

Algorithm description: "Spin response formalism in circular accelerators", V. Ptitsyn, Yu.M. Shatunov, S. Mane, NIM, A608 (2009), p.225.

- Capability was added to the ASPIRRIN code to calculate the intrinsic spin resonance harmonics in an accelerator with an arbitrary configuration of the stable spin on the design orbit.
- Lately the capability to calculate first order resonance harmonics for resonances caused by betatron coupling has been developed.

General definitions

$$\vec{S}' = \left(\vec{W}_0 + \vec{w}\right) \times \vec{S}$$

On reference orbit — Precession perturbation

$\vec{n}_0$ -Periodical spin solution on the reference orbit

ν -Spin tune on the reference orbit

$\vec{n}_0, \vec{l}_0, \vec{m}_0$ - spin related right-handed system formed by the spin solutions on the reference orbit($\mathbf{W_0}$);

$$\vec{k}_0 = \vec{l}_0 + i\vec{m}_0$$

$\vec{n}_0, \vec{l}, \vec{m}$ -periodical right-handed system, convenient for analyzing the perturbation of spin motion

$$\vec{k} = \vec{l} + i\vec{m}; \qquad \vec{k} = \vec{k}_0 e^{i\nu\theta}$$

Spin presentation, using complex variable α:

$$\mathbf{n} = \sqrt{1-|\alpha|^2}\mathbf{n_0} + \Re(\alpha\mathbf{k}^*)$$

$$\frac{d\alpha}{d\theta} = i\left(\nu + \mathbf{w}\cdot\mathbf{n_0}\right)\alpha - i\mathbf{w}\cdot\mathbf{k}\sqrt{1-|\alpha|^2}$$

Spin resonance harmonics

Definition of (first-order) spin resonance harmonics for the case of arbitrary configuration of stable spin direction on the design orbit:

$$\mathbf{w} \cdot \mathbf{k} = \sum_{r,p} \epsilon_{rp} e^{i(p+Q_r)\theta} = \sum_{r,p} A_r \tilde{v}_{rp} e^{i(p+Q_r)\theta}$$

Calculated as the sum over lattice elements:

$$\tilde{v}_{rp} = \sum_j \tilde{v}_{rp}^j$$

Complex vector **k** is defined by the spin motion on the design closed orbit (Snakes, spin rotators, partial snake, Figure-8)

Orbital tune vector:
$Q_r = (Q_x, -Q_x, Q_y, -Q_y)$

Components of spin perturbation vector **w** in linear approximation:

$$w_x = (1+\nu_0)y'' + (1+G)K_s x'$$

$$w_s = (1+G)(K_y' y + \Delta K_s - K_s \tfrac{\Delta p}{p}) - (\nu_0 - G)K_y y'$$

$$w_y = -(1+\nu_0)x'' + \left(\nu_0 + \tfrac{G}{\gamma}\right)K_y \tfrac{\Delta p}{p} + (1+G)K_s y'$$

General presentation of orbital motion:

$$X_i = f_{ik} A_k + X_i^{co} + D_i \tfrac{\Delta p}{p}$$

Column matrix of orbital eigenvectors (f_{ik})

Vector of orbital amplitudes (A_k)

Rolled quadrupole term in the resonance harmonic

<u>For a quadrupole with gradient g and roll angle ϕ :</u>

$$\tilde{v}_{rp}^{quad} = Y[k_{0x}(\delta_{rp}^2 \sin(2\phi)C_1 + (g - \delta_{rp}^2 \cos(2\phi))C_3) - k_{0y}((g + \delta_{rp}^2 \cos(2\phi))C_1 + \delta_{rp}^2 \sin(2\phi)C_3)]$$

where

$$Y = g(1 + \nu_0)/(g^2 - \delta_{rp}^4)$$

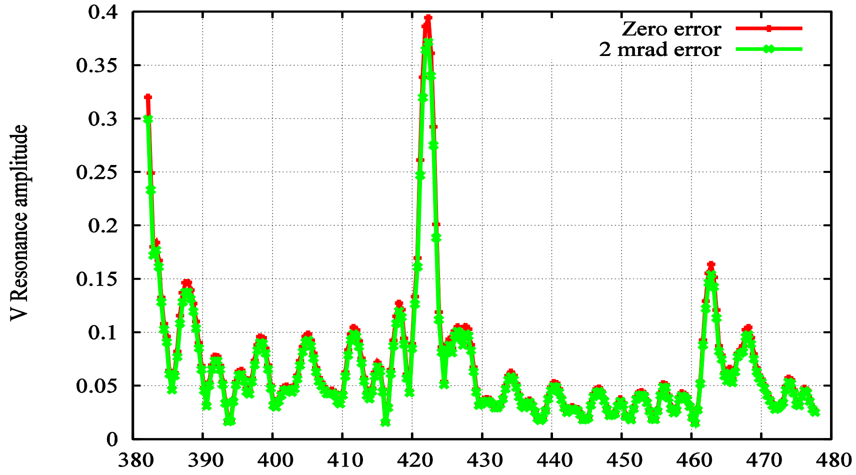
$$C_1 = \left[\left(f_{1r}' - i\delta_{rp} f_{1r} \right) e^{i\delta_{rp}\theta} \right]_{\theta_1}^{\theta_2}$$

$$C_3 = \left[\left(f_{3r}' - i\delta_{rp} f_{3r} \right) e^{i\delta_{rp}\theta} \right]_{\theta_1}^{\theta_2}$$

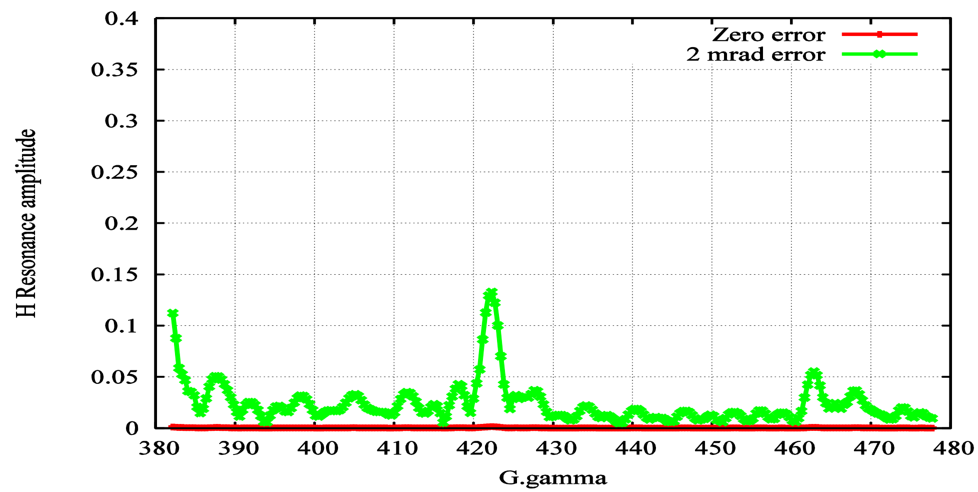
$$\delta_{rp} = \nu_{sp} - (p + Q_r)$$

Example with a quadrupole roll. Intrinsic resonance modifications.

Vertical resonance



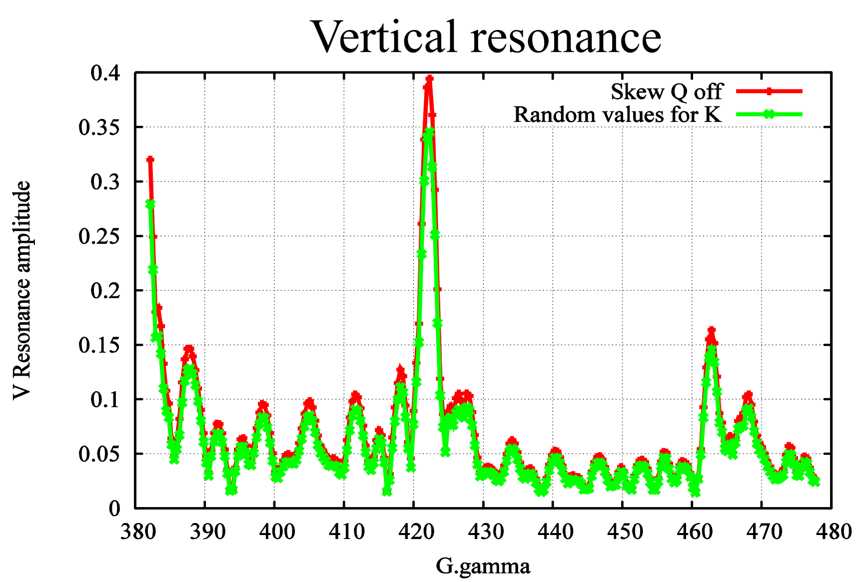
Horizontal resonance

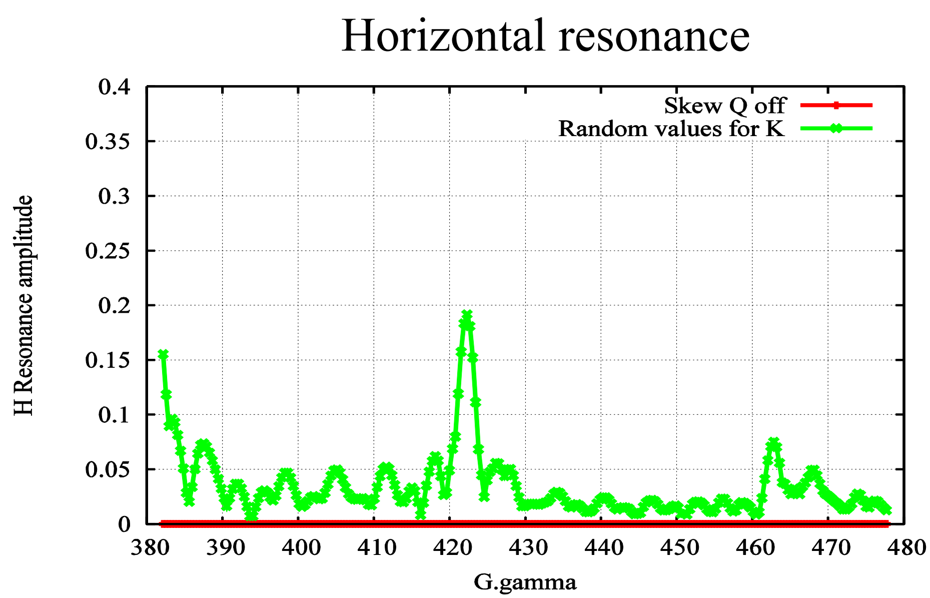


RHIC IR quad Q1O6 2mrad roll error $\Delta Q_{min} = 0.017$

Example with skew quadrupole correctors. Intrinsic resonance modifications.

Random gradient values of RHIC 12 skew quadrupole correctors in RHIC interaction region. Integrated gradient range: 0-3, 10^{-3} 1/m .

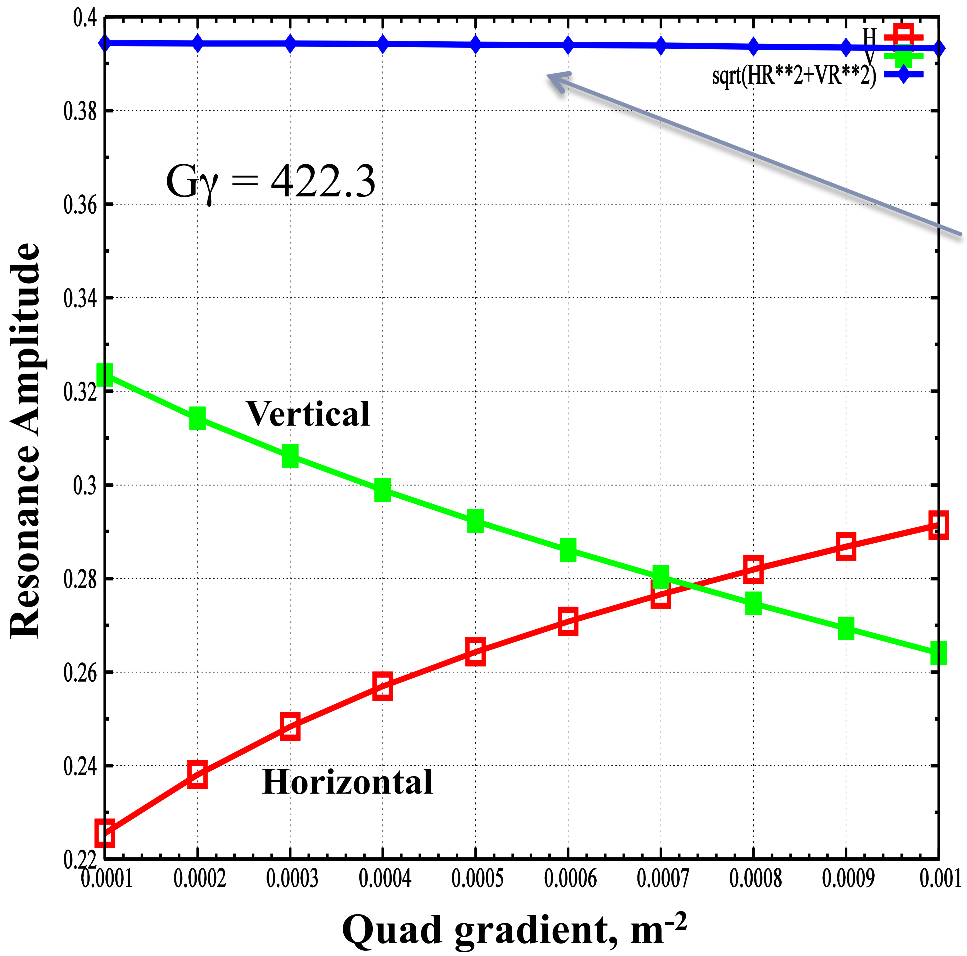




Normalized betatron amplitudes: $A_x = A_y = 10$ *mm*mrad*

$\Delta Q_{min} = 0.031$

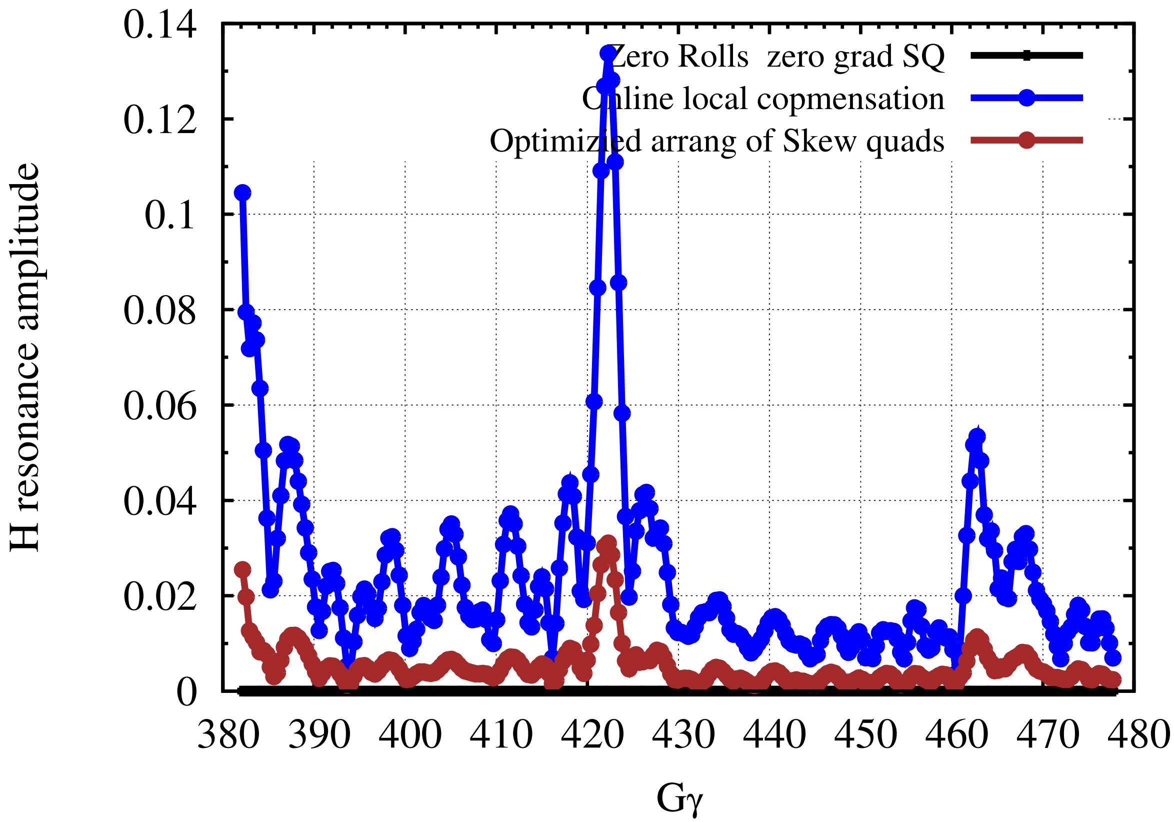
Intrinsic resonance harmonic amplitude modification law



$$\varepsilon_h^2 + \varepsilon_v^2 = const$$

A skew quadrupole strength variation

Horizontal resonance amplitude with present RHIC IR skew errors and correction



Triplet quad roll errors, mrad

Sector	Triplet	Blue Ring
5	Q1	1.35
	Q2	1.33
	Q3	1.37
6	Q1	1.59
	Q2	-1.63
	Q3	-3.69
7	Q1	-0.89
	Q2	1.23
	Q3	-1.32
8	Q1	4.67
	Q2	-2.10
	Q3	0.17
1	Q1	4.72
	Q2	2.40
	Q3	1.38
2	Q1	1.26
	Q2	3.21
	Q3	-0.87

Skew quad corrector strength, 10^{-3} 1/m	
bi5qs3	0
bo6qs3	0.1
bo7qs3	0.9
bi8qs3	-1.4
bi9qs3	-0.35
bo10qs3	-0.65
bo11qs3	-0.5
bi12qs3	0.32
bi1qs3	0.2
bo2qs3	-1.2
bo3qs3	-0.32
bi4qs3	-0.32

Solenoid magnet term in the resonance harmonic

For a solenoid magnet with normalized field $K_s = H_s / \langle B_y \rangle$:

$$\tilde{v}_{rp}^{sol} = \frac{G - \nu_0}{\delta_{rp}^2 - (GK_s)^2} \Big(-i\delta_{rp} \Big[e^{i\delta_{rp}\theta} \Big(\big(f_{2r} + \tfrac{1}{2} K_s f_{3r} \big) k_{0x} + \big(f_{4r} - \tfrac{1}{2} K_s f_{1r} \big) k_{0y} \Big) \Big]_{\theta_1}^{\theta_2} +$$

$$+ GK_s \Big[e^{i\delta_{rp}\theta} \Big(\big(f_{2r} + \tfrac{1}{2} K_s f_{3r} \big) k_{0y} - \big(f_{4r} - \tfrac{1}{2} K_s f_{1r} \big) k_{0x} \Big) \Big]_{\theta_1}^{\theta_2} \Big) +$$

$$+ \frac{(1 + \nu_0) K_s}{2} \Big[e^{i\delta_{rp}\theta} \big(f_{1r} k_{0x} + f_{3r} k_{0y} \big) \Big]_{\theta_1}^{\theta_2}$$

Contribution from the solenoid ends

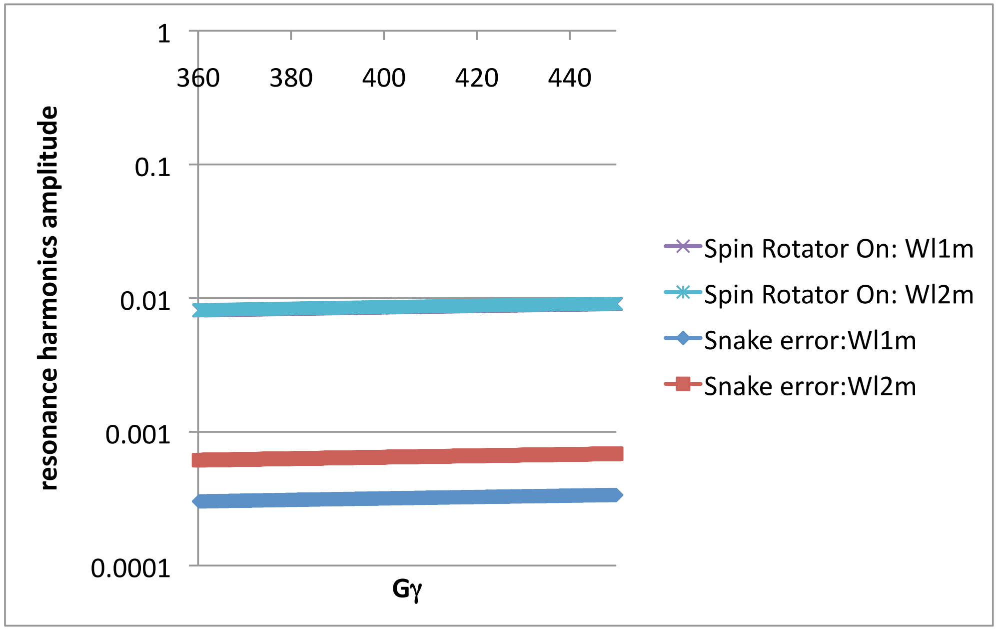
Summary

- The ASPIRRIN contains now the full 4-D calculation of the first –order resonance harmonics caused by the betatron coupling, including effects from rolled quads, skew quadrupoles and solenoid magnets.
- The conversation law for the sum of squares of the horizontal and vertical resonance harmonic amplitudes was observed.
- Further work plan:
  - Horizontal resonances in AGS: interplay of partial snake and skew correctors
  - High-order resonance harmonics calculation following perturbation theory.

    (S.R. Mane, NIM A 680, p.35, 2012)

Backup Slides

Example of horizontal resonance harmonics



Horizontal resonance harmonics can appear if:

- •vector n_0 on closed orbit deviates from the vertical (due to imperfection resonances)
- •betatron coupling

No secondary linear harmonics: $w_{xl1s}=w_{xl2s}=0$

The resonance harmonics value ~ Gγ

Two examples shown for the non-vertical n_0:

1) IP6 spin rotators turned on (with energy independent settings)

2) Second case: 5° spin rotation angle error of in 9 o'clock snake