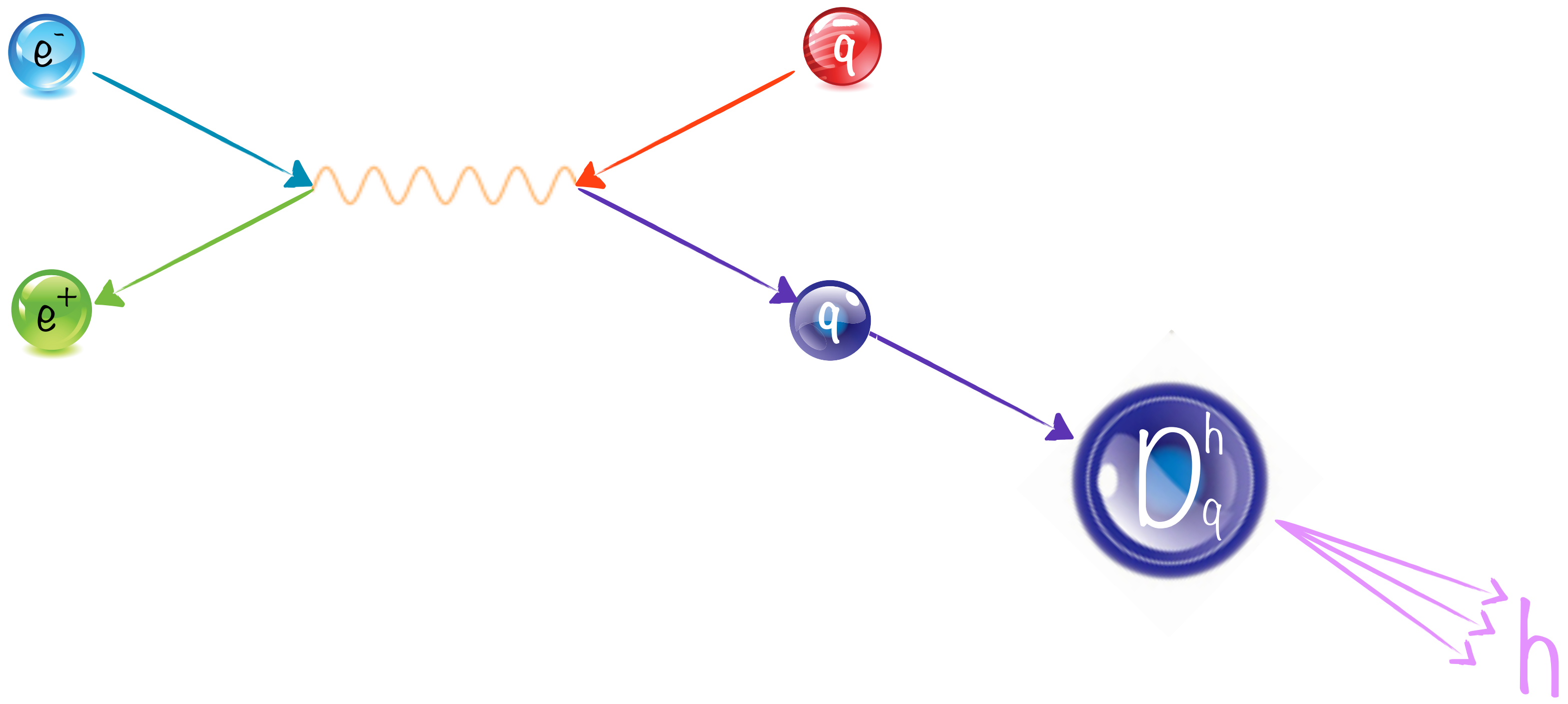
COLLINS MEASUREMENTS @ BELLE

Francesca Giordano for the BELLE Collaboration

Spin 2014, Beijing, China, October 22nd, 2014

Fragmentation process

or how do the hadrons get formed?

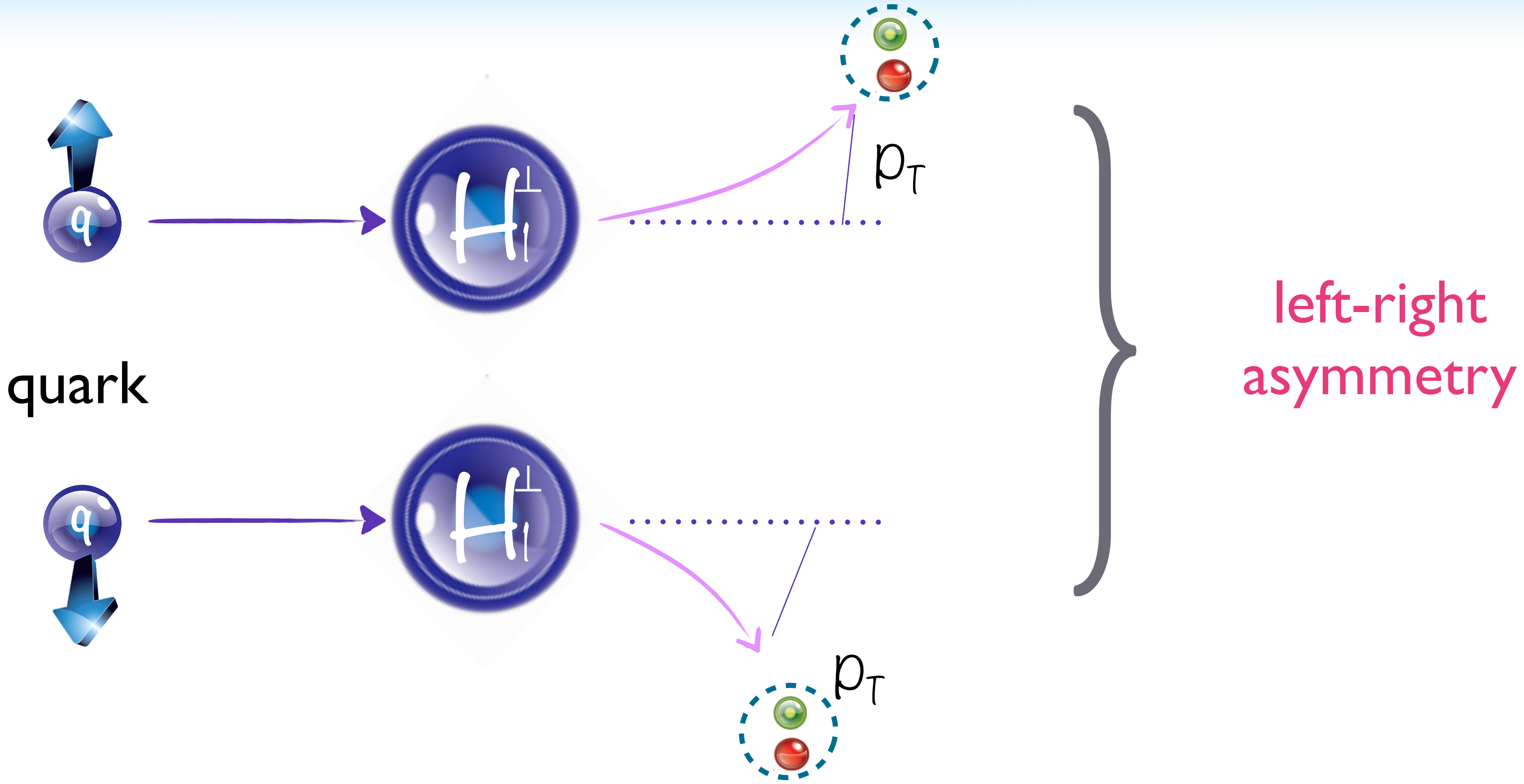


Cleanest way to access
FF is e+e- ➞ $q\bar{q}$

- Fragmentation function describes the process of hadronization of a parton
- Strictly related to quark confinement
- Universal: can be used to study the nucleon structure when combined with SIDIS and hadronic reactions data

$$A_{UT}^{h} = \frac{\boxed{\sigma^{\uparrow\Uparrow} - \sigma^{\uparrow\Downarrow}}}{\boxed{\sigma^{\uparrow\Uparrow} + \sigma^{\uparrow\Downarrow}}} \quad \begin{matrix} \propto H_{1q}^{\perp h} \\ \propto D_q^h \end{matrix}$$

Collins Fragmentation

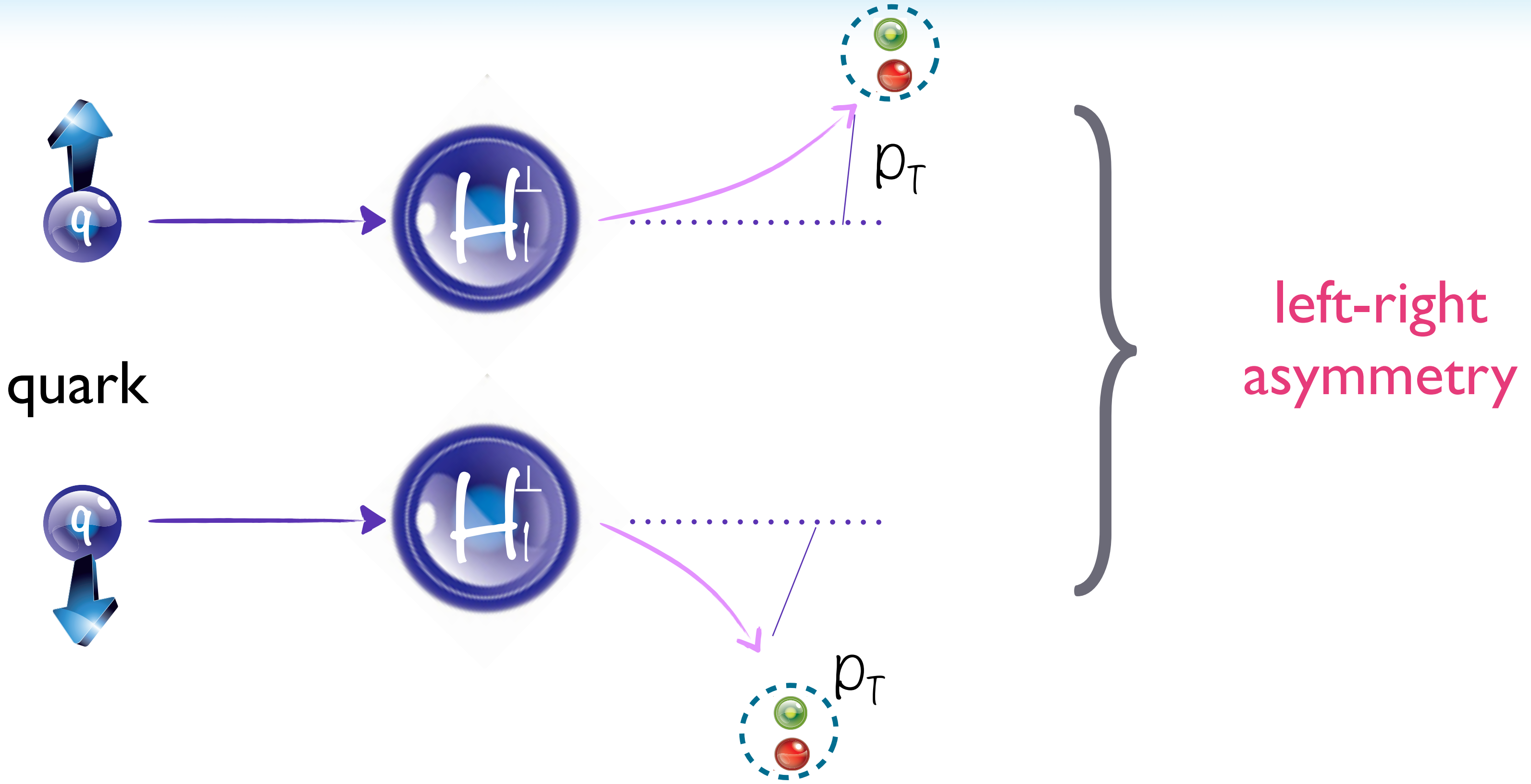


quark

left-right asymmetry

Collins mechanism: correlation between the parton transverse spin and the direction of final hadron

Collins Fragmentation

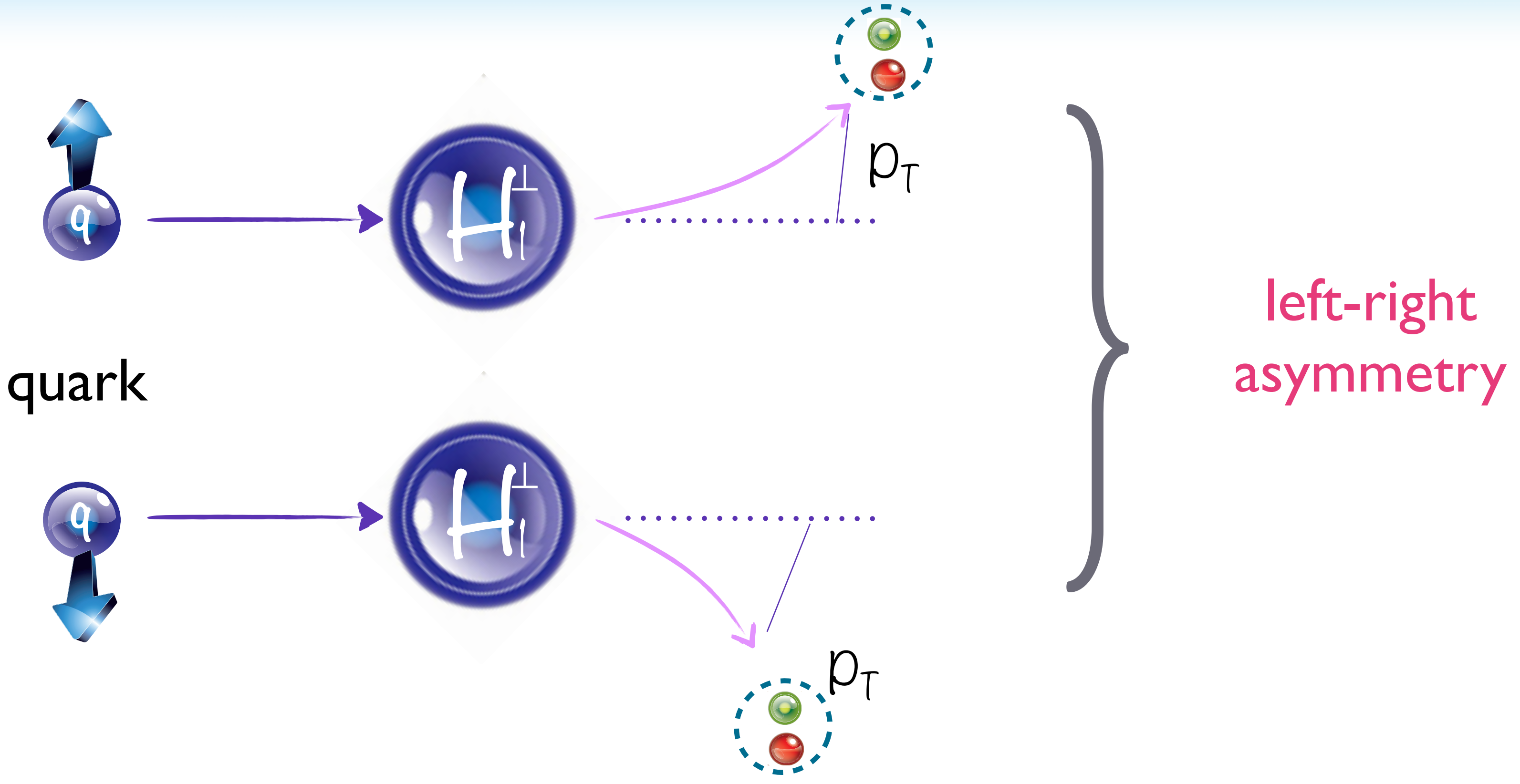


Collins mechanism: correlation between the parton transverse spin and the direction of final hadron

strictly related to the outgoing hadron tranverse momentum

TMD!

Collins Fragmentation



Collins mechanism: correlation between the parton transverse spin and the direction of final hadron

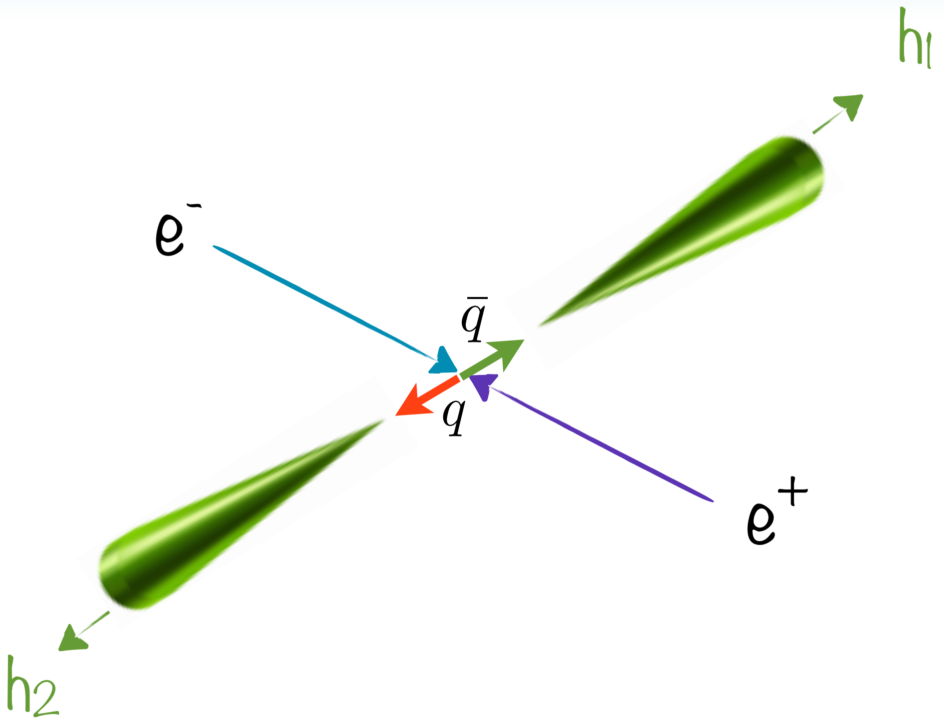
strictly related to the outgoing hadron tranverse momentum

TMD!

Chiral odd!

$$\underbrace{\underset{\text{chiral odd}}{X} \otimes \underset{\text{chiral odd}}{H_1^\perp}}_{\text{chiral even}}$$

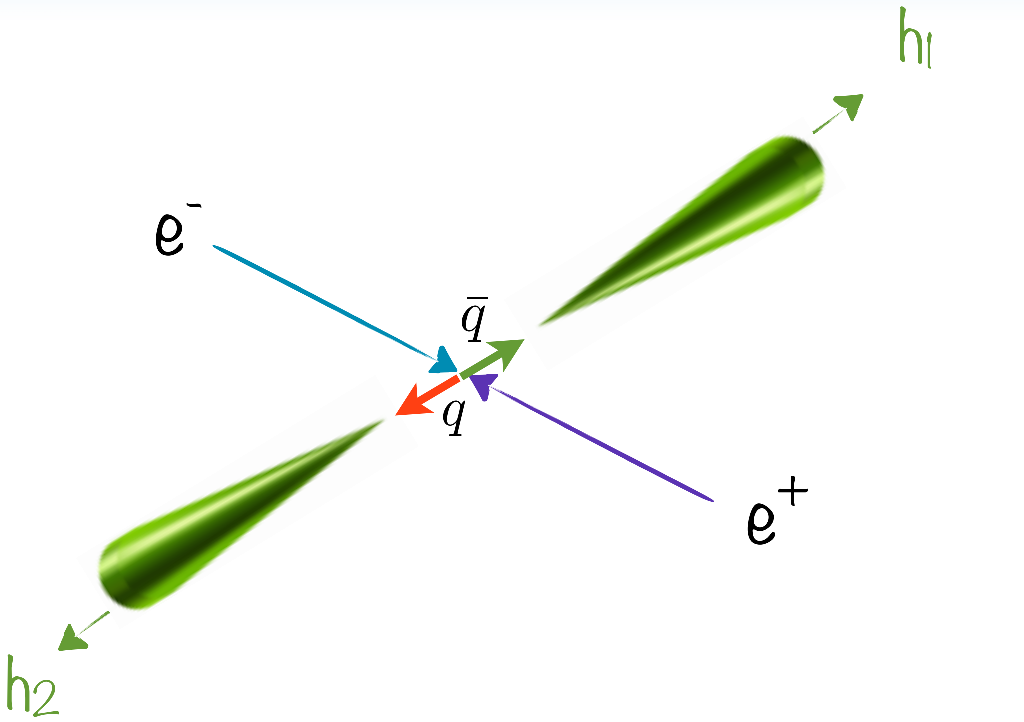
Collins Fragmentation



Back-to-Back jets

In e+e- reaction, there is no fixed transverse axis to define azimuthal angles to, and even if there were one the net quark polarization would be 0

Collins Fragmentation



Back-to-Back jets

In e+e- reaction, there is no fixed transverse axis to define azimuthal angles to, and even if there were one the net quark polarization would be 0

But if we look at the whole event, even though the q and q̄ spin directions are unknown, there is a known correlation between them

$$e^+e^- \to q\,\bar{q} \;\to h_1\,h_2\,X$$

$$h = \pi,\ K$$

Reference frames

$$z \equiv \frac{E_h}{E_q}$$

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ϕ_1+ϕ_2 method:
hadron azimuthal angles with respect to the q$\bar{\text{q}}$ axis proxy

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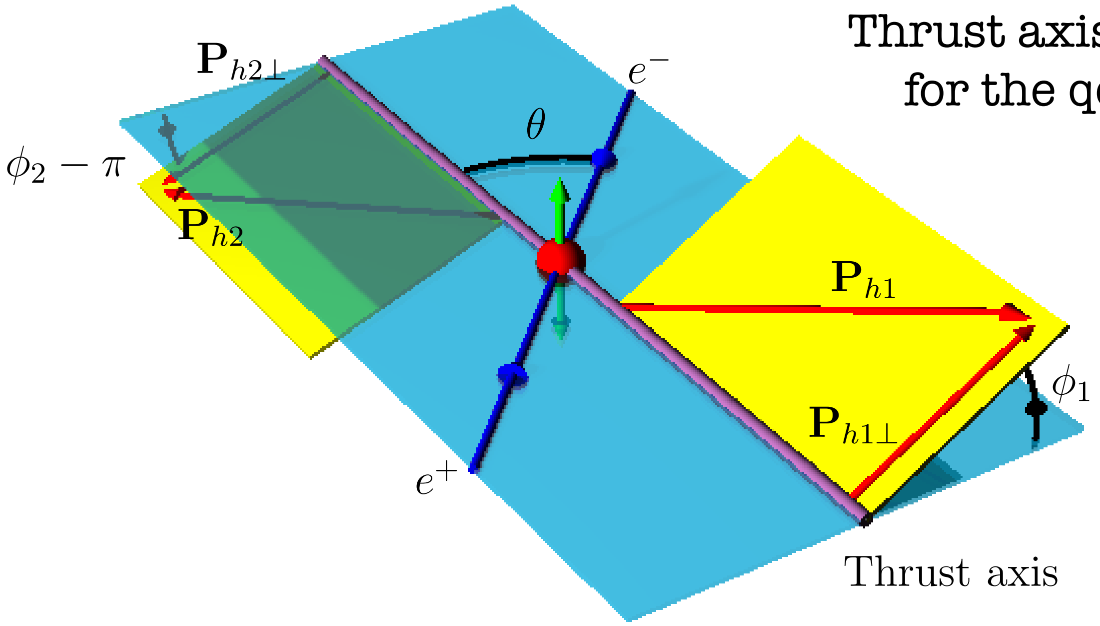
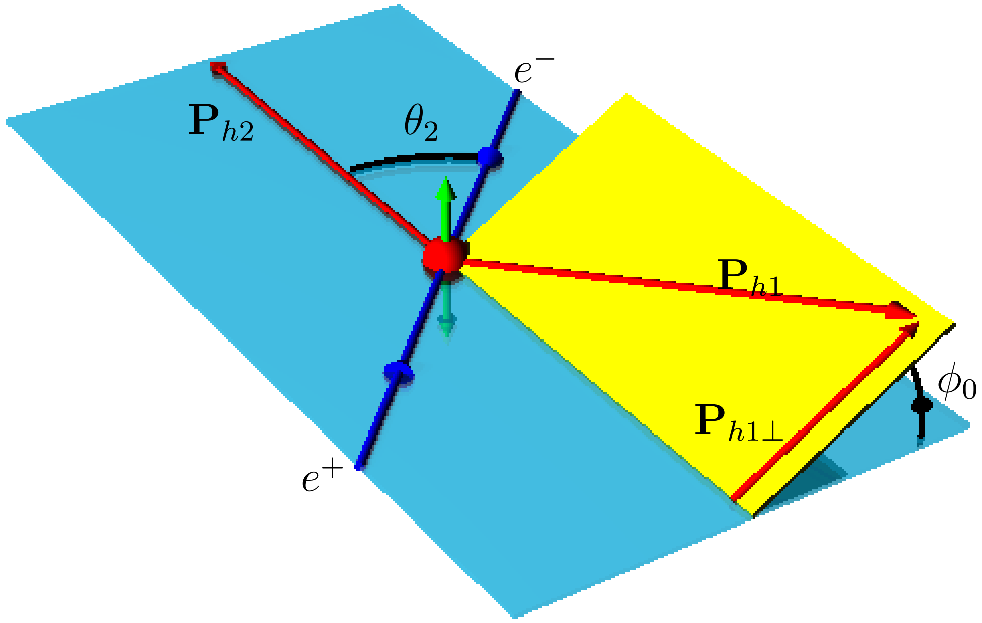


Thrust axis= proxy for the q$\bar{\text{q}}$ axis



$$\sigma \sim \mathcal{M}_{12}\Big(1 + \frac{\sin^2\theta_T}{1+\cos^2\theta_T}\, cos(\phi_1+\phi_2)\frac{H_1^{\perp[1]}(z_1)\bar{H}_1^{\perp[1]}(z_2)}{D_1^{[0]}(z_1)\bar{D}_1^{[0]}(z_2)}\Big)$$

$$\sigma \sim \mathcal{M}_0\Big(1 + \frac{\sin^2\theta_2}{1+\cos^2\theta_2}\, cos(2\phi_0)\mathcal{F}\Big[\frac{H_1^{\perp}(z_1)\bar{H}_1^{\perp}(z_2)}{D_1^{\perp}(z_1)\bar{D}_1^{\perp}(z_2)}\Big]\Big)$$

$$F^{[n]}(z_i) \equiv \int d|k_T|^2 \Big[\frac{|k_T|}{M_i}\Big]^{[n]} F(z_i, |k_T|^2)$$

$$\mathcal{F}[X] = \sum_{q\bar{q}} \int [2\hat{\mathbf{h}}\cdot\mathbf{k_{T1}}\hat{\mathbf{h}}\cdot\mathbf{k_{T2}} - \mathbf{k_{T1}}\cdot\mathbf{k_{T2}}]$$
$$d^2\mathbf{k_{T1}} d^2\mathbf{k_{T2}}\, \delta^2(\mathbf{k_{T1}} + \mathbf{k_{T2}} - \mathbf{q_T})X$$

D. Boer
Nucl.Phys.B806:23,2009

Reference frames

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hadron azimuthal angles with respect
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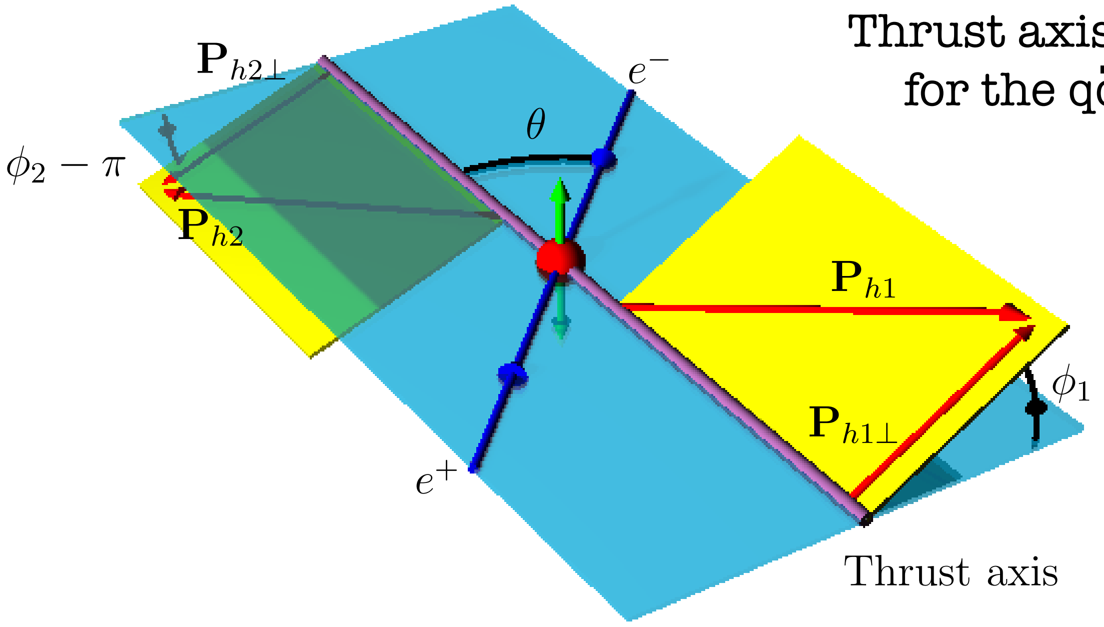

Thrust axis= proxy
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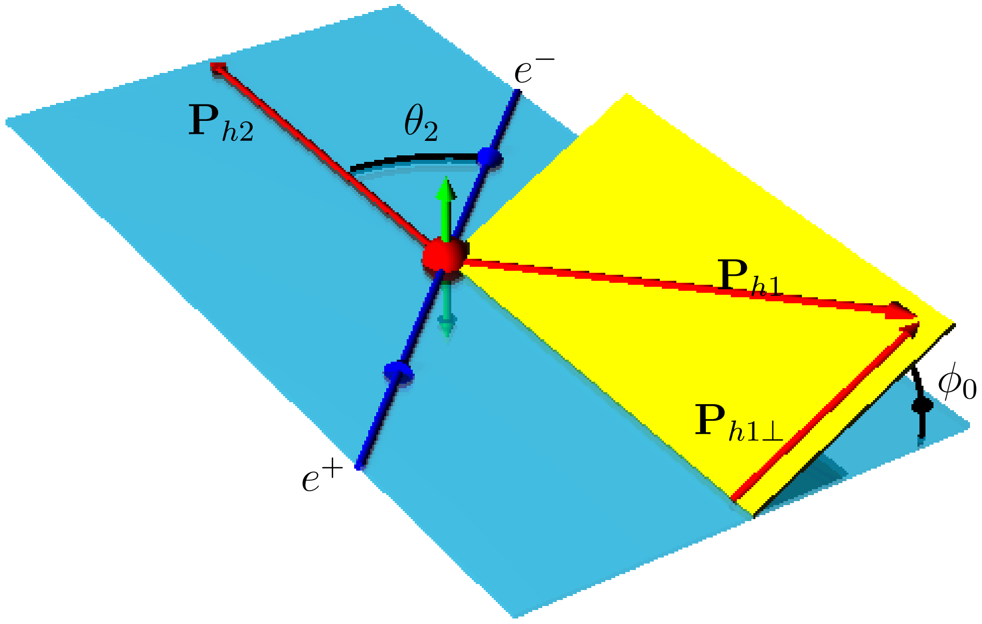
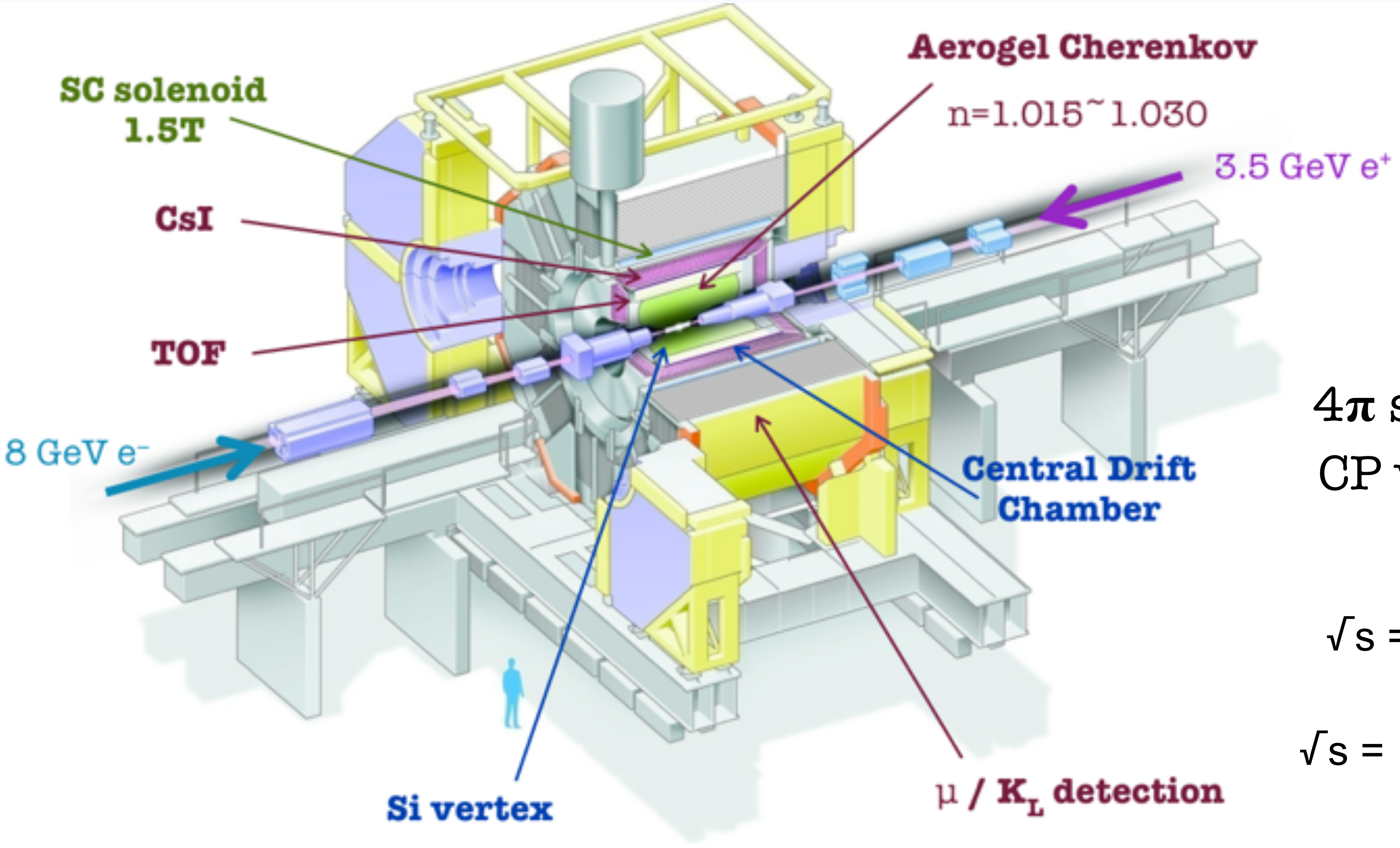


$$\mathcal{R}_{12} = \frac{N_{12}(\phi_1+\phi_2)}{\langle N_{12}\rangle}$$

$$\mathcal{R}_0 = \frac{N_0(\phi_0)}{\langle N_0\rangle}$$

BELLE @ KEKB



KEKB:

Asymmetric e+ e- collider
(3.5 / 8 GeV)

Belle spectrometer:

4π spectrometer optimized for CP violation in B-meson decay

On resonance:

$\sqrt{s}$ = 10.58 GeV (e+ e- → Y(4S) → $B\bar{B}$)

Off resonance

$\sqrt{s}$ = 10.52 GeV (e+ e- → $q\bar{q}$ (q=u,d,s,c))

Total Luminosity collected:
1000 fb^{-1}!!!

Good tracking Θ [17^0;150^0]
and vertex resolution

Good PID: $\varepsilon(\pi) \gtrsim 90\%$
$\varepsilon(K) \gtrsim 85\%$

Double-ratios

But! Acceptance and radiation effects also contribute to azimuthal asymmetries!

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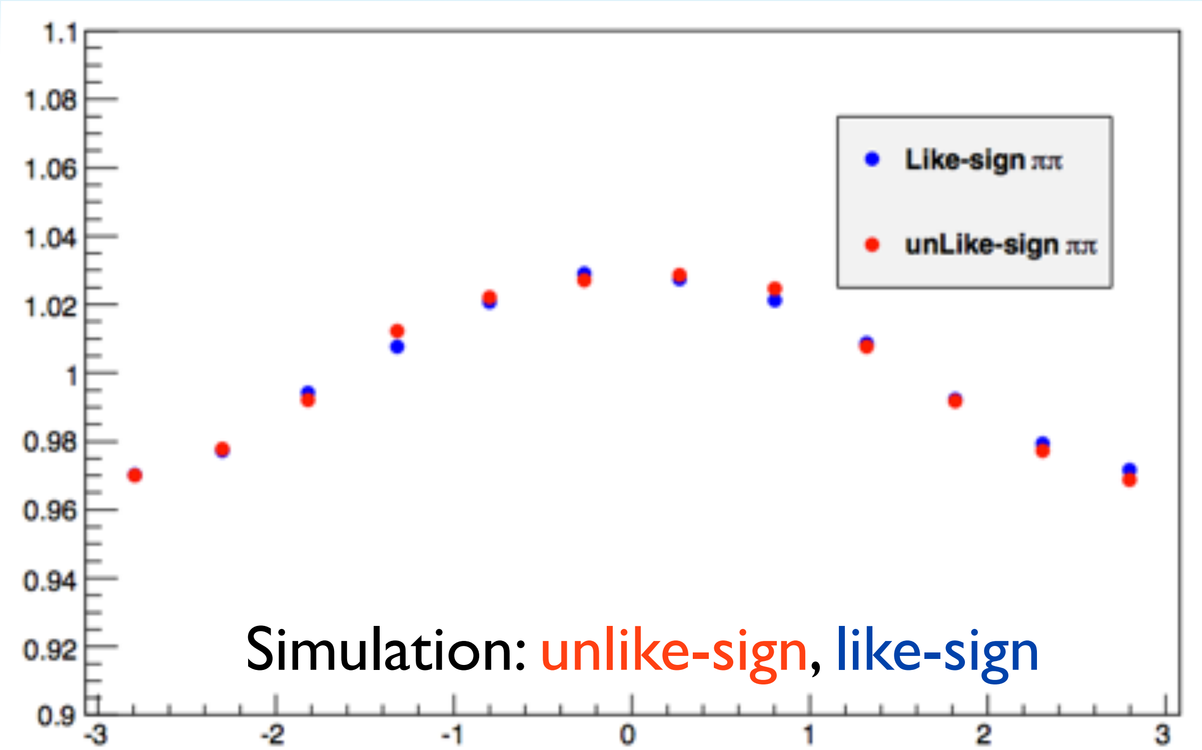
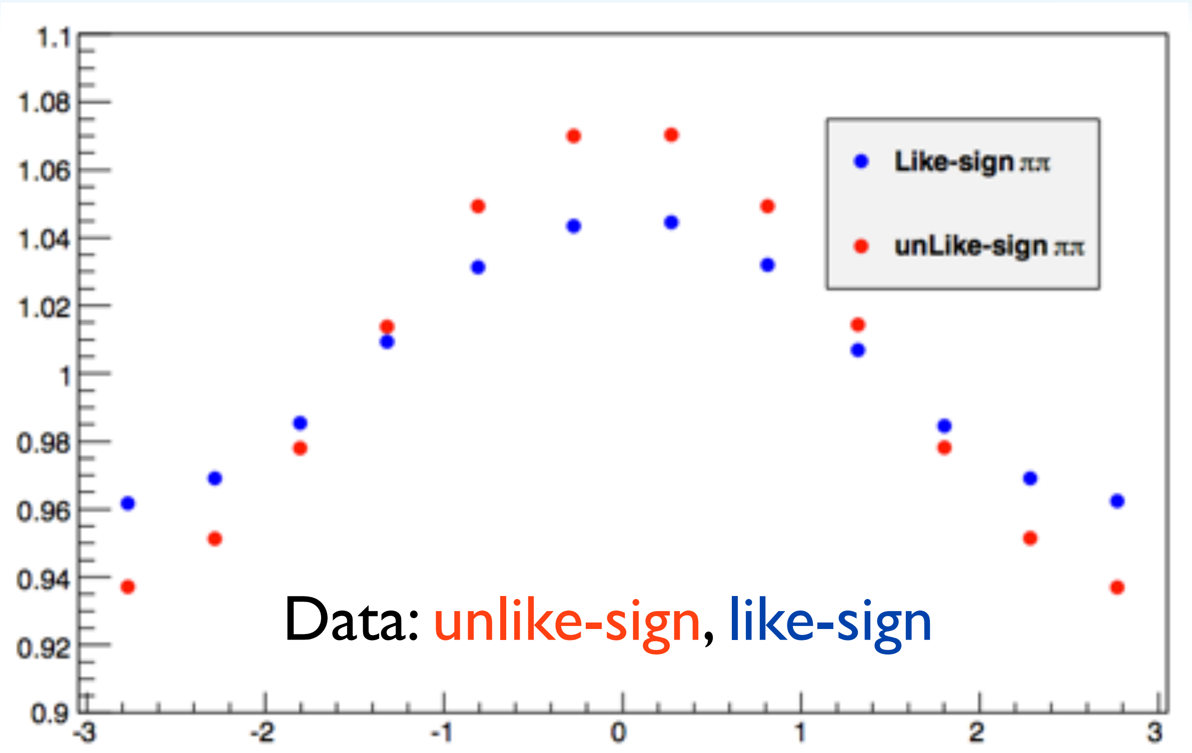
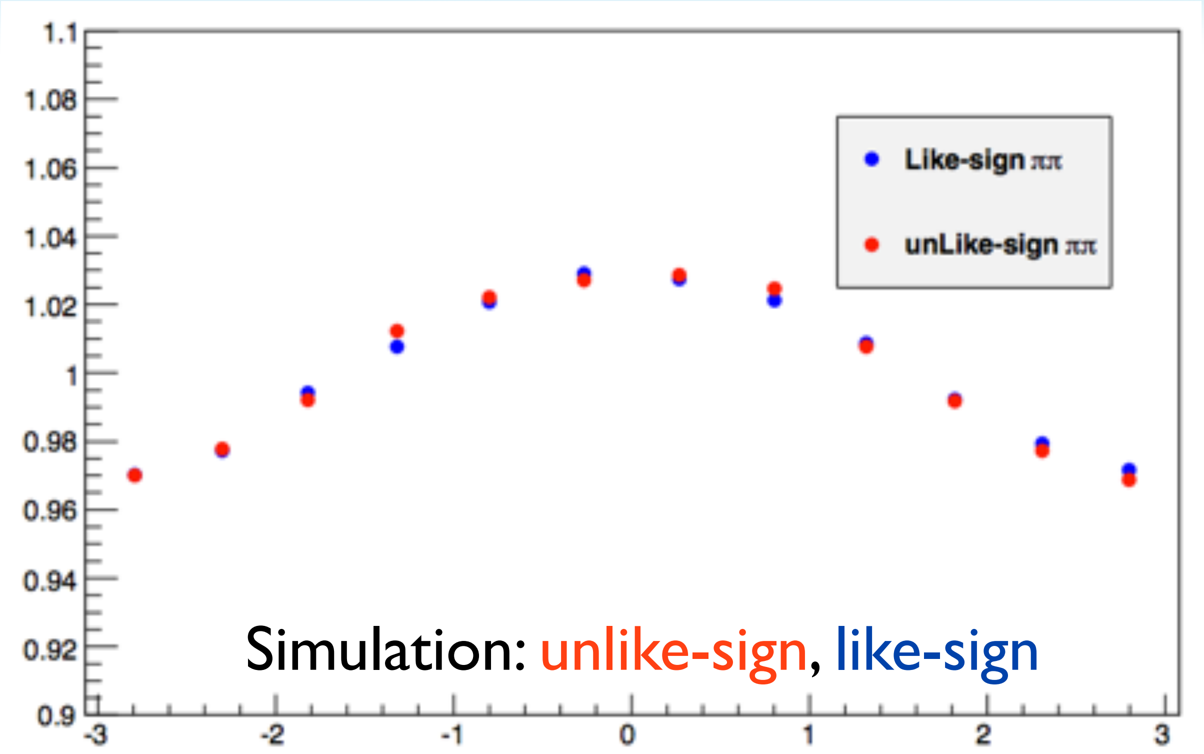


Simulation: unlike-sign, like-sign

Double-ratios



Data: unlike-sign, like-sign



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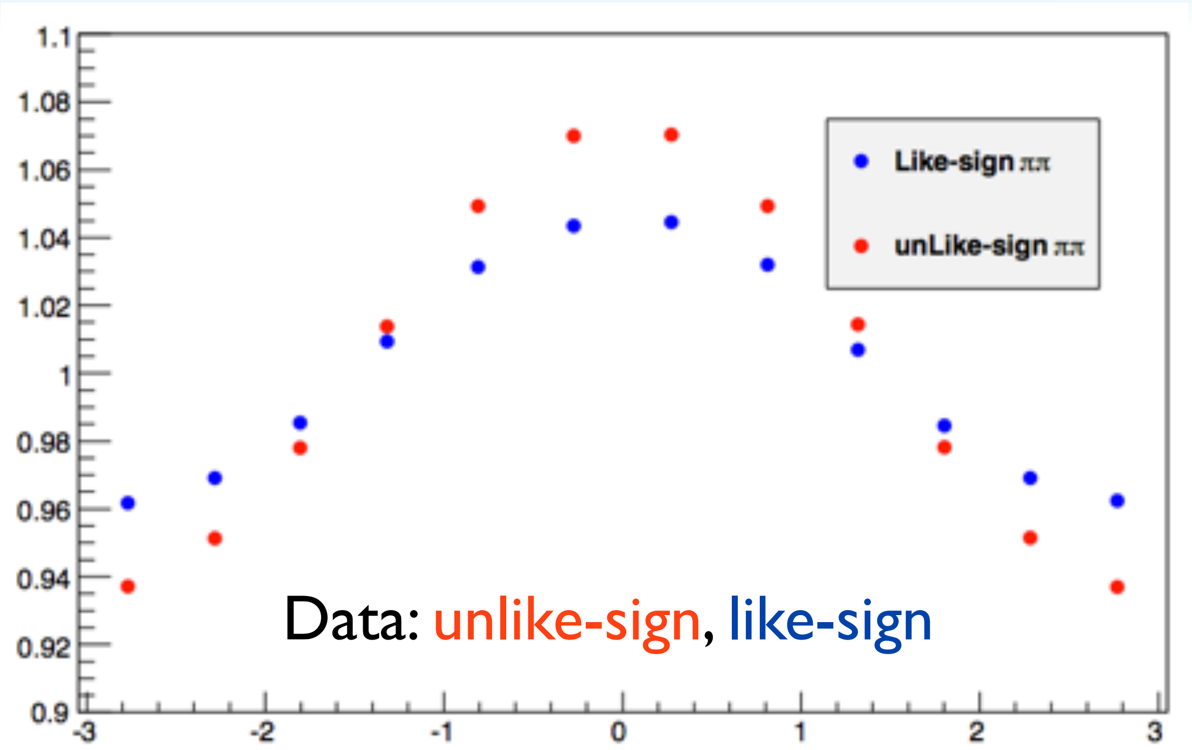

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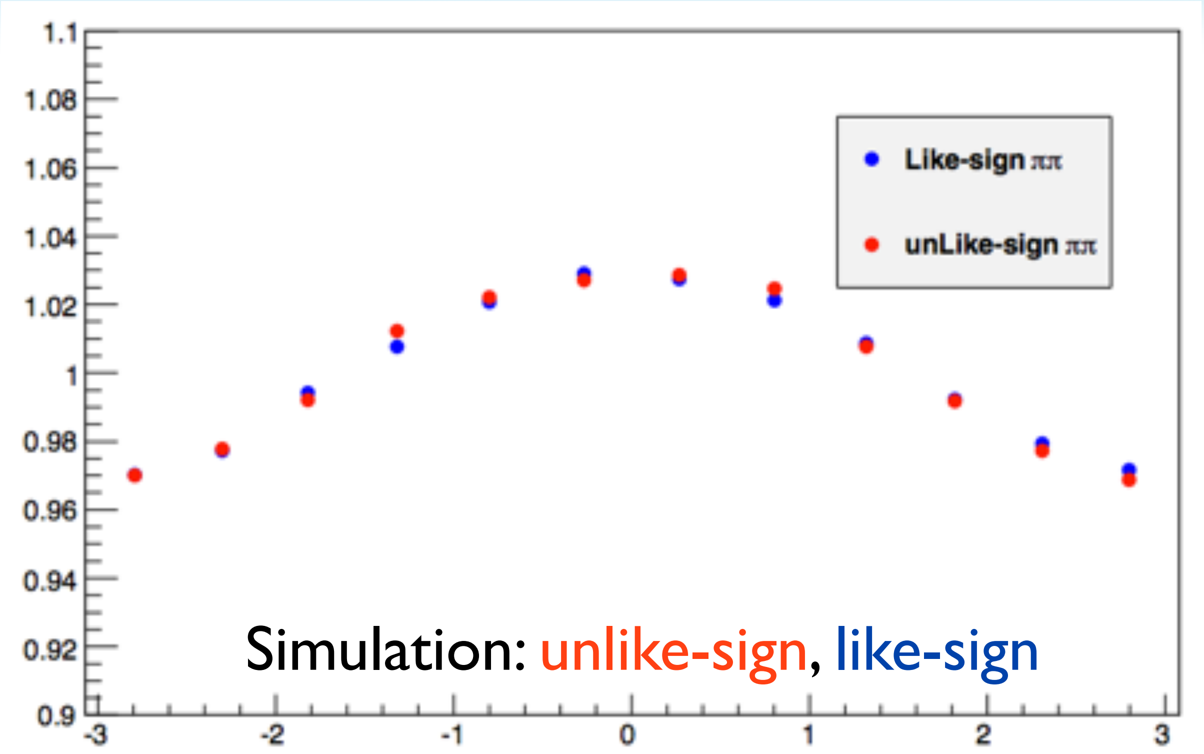

Simulation: unlike-sign, like-sign

To reduce such non-Collins effects:
divide the sample of hadron couples in unlike-sign and like-sign (or All-charges),
and extract the asymmetries of the super ratios between these 2 samples:

Unlike-sign couples / Like-sign couples

$$\mathcal{D}_{ul}^{h_1 h_2} = \mathcal{R}^U / \mathcal{R}^L$$

Unlike-sign couples / All charges

$$\mathcal{D}_{uc}^{h_1 h_2} = \mathcal{R}^U / \mathcal{R}^C$$

Reference frames

$$e^+e^- \to q\,\bar{q} \;\to h_1\,h_2\,X$$

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ϕ_1+ϕ_2 method:
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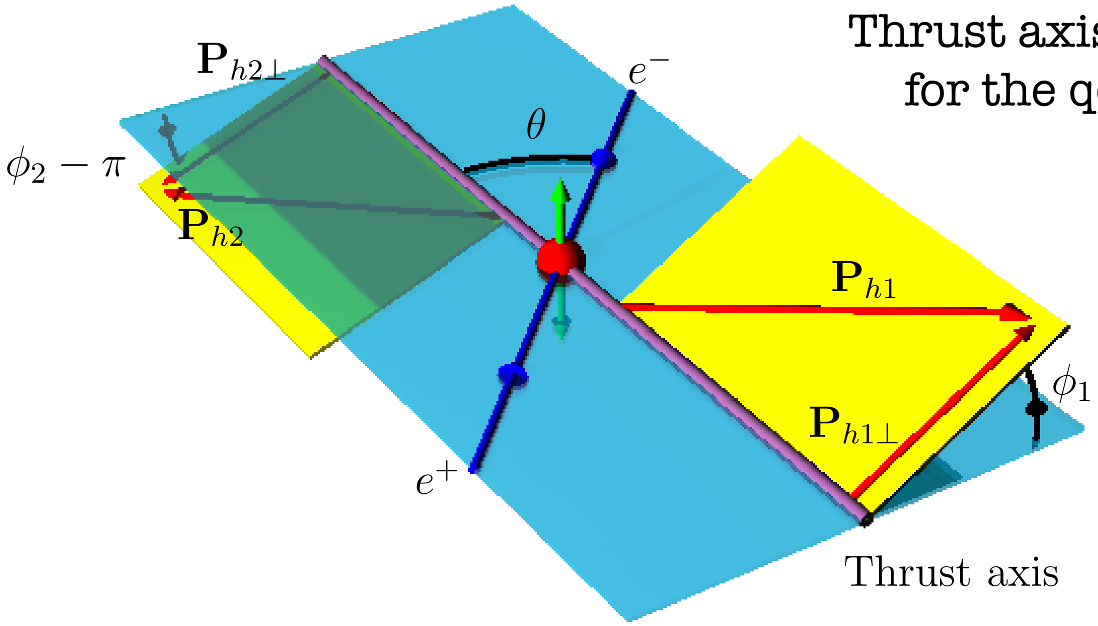

Thrust axis= proxy
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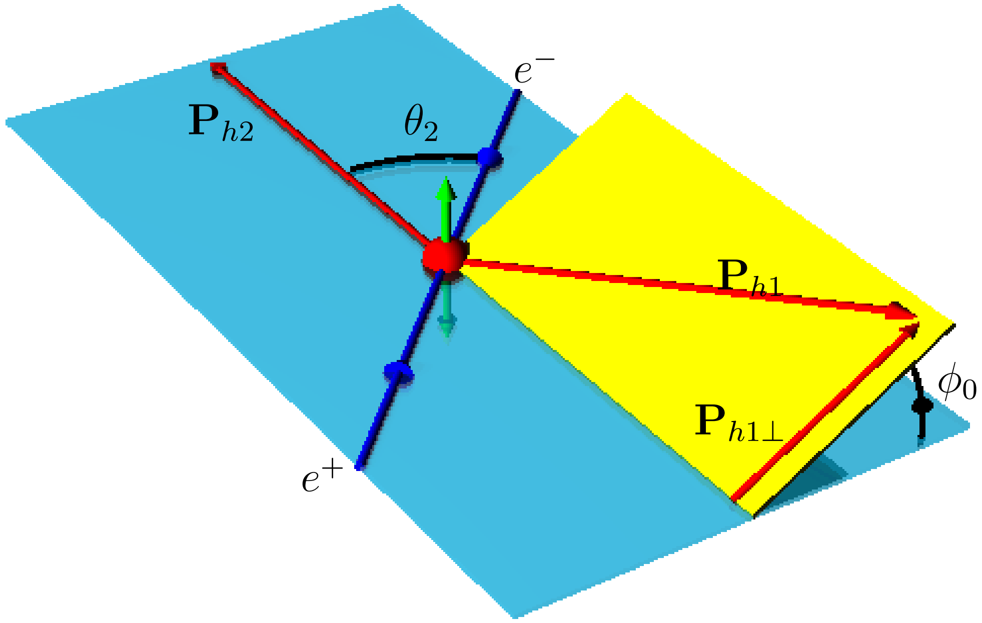

$$\mathcal{D}^{h_1h_2}_{ul} = \mathcal{R}^U/\mathcal{R}^L$$

Fitted by

$$\mathcal{D}^{h_1h_2}_{uc} = \mathcal{R}^U/\mathcal{R}^C$$

$$\mathcal{B}_{12}(1 + \mathcal{A}_{12}\cos(\phi_1 + \phi_2))$$

$$\mathcal{B}_0(1 + \mathcal{A}_0\cos(2\phi_0))$$

Reference frames

$$e^+e^- \rightarrow q\,\bar{q} \;\rightarrow h_1\,h_2\,X$$

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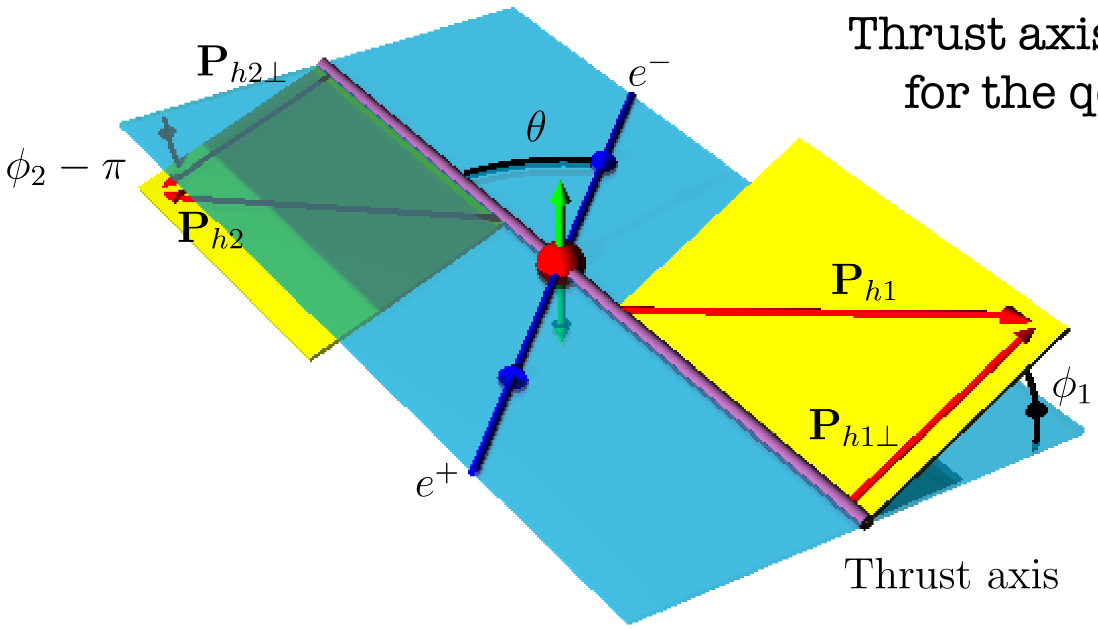

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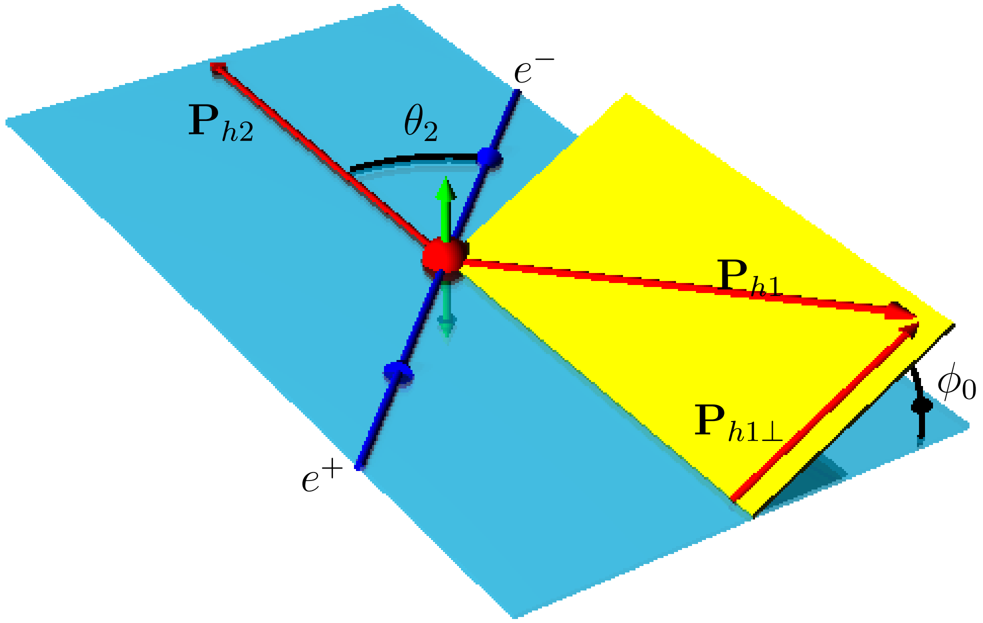

$$A_{12} = \frac{\sin^2\theta}{1+\cos^2\theta}\,\frac{H_1^{\perp[1]}(z_1)\bar{H}_1^{\perp[1]}(z_2)}{D_1^{[0]}(z_1)\bar{D}_1^{[0]}(z_2)}$$

$$A_0 = \frac{\sin^2\theta}{1+\cos^2\theta}\,\mathcal{F}\Big[\frac{H_1^{\perp}(z_1)\bar{H}_1^{\perp}(z_2)}{D_1^{\perp}(z_1)\bar{D}_1^{\perp}(z_2)}\Big]$$

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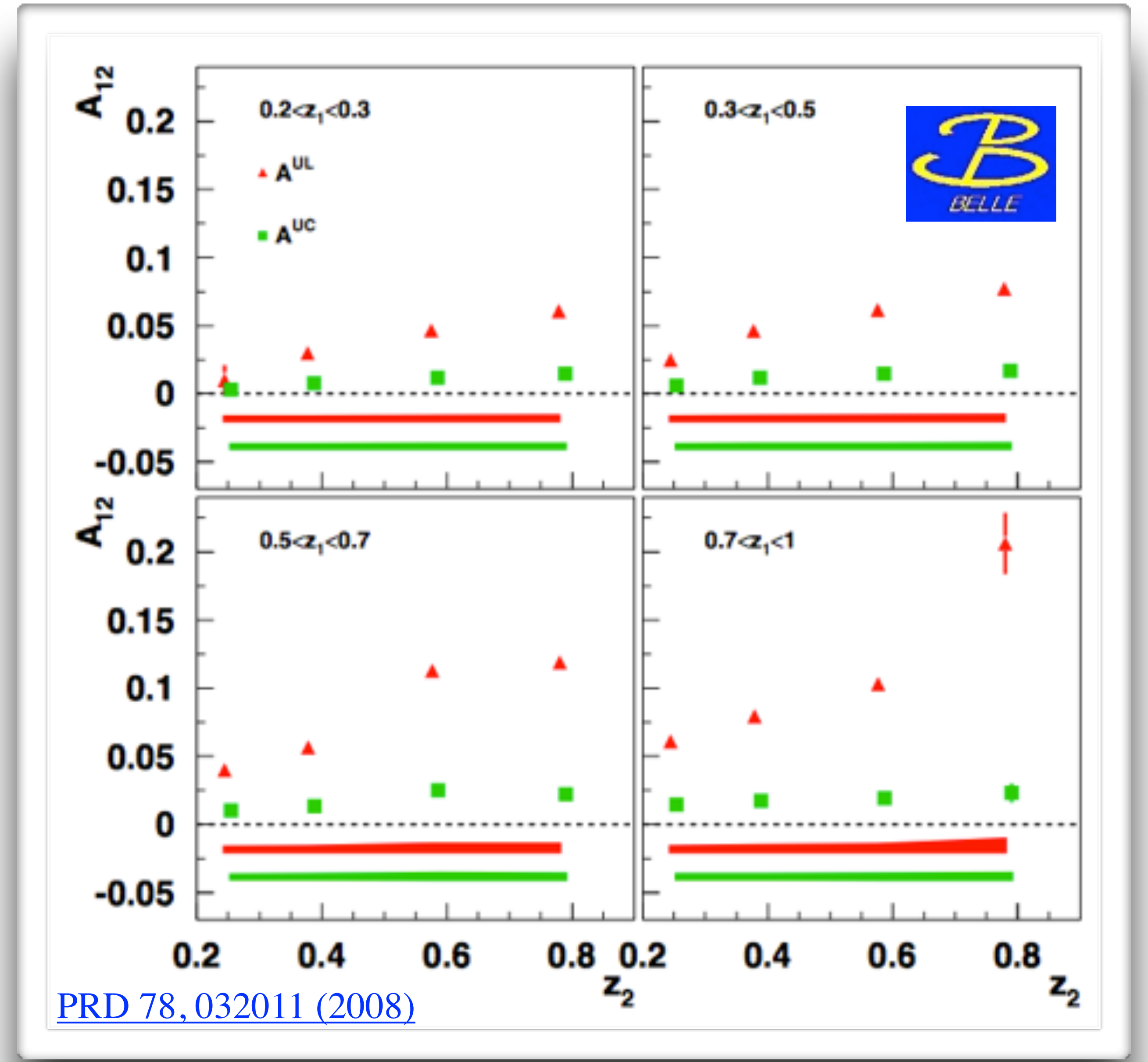
$$\mathcal{B}_0(1+\mathcal{A}_0\cos(2\phi_0))$$

Belle publications
PRL 96,232002, (2006)
PRD 78, 032011 (2008)

Published results: $\pi\pi$

$\phi_1+\phi_2$ method

ϕ_0 method



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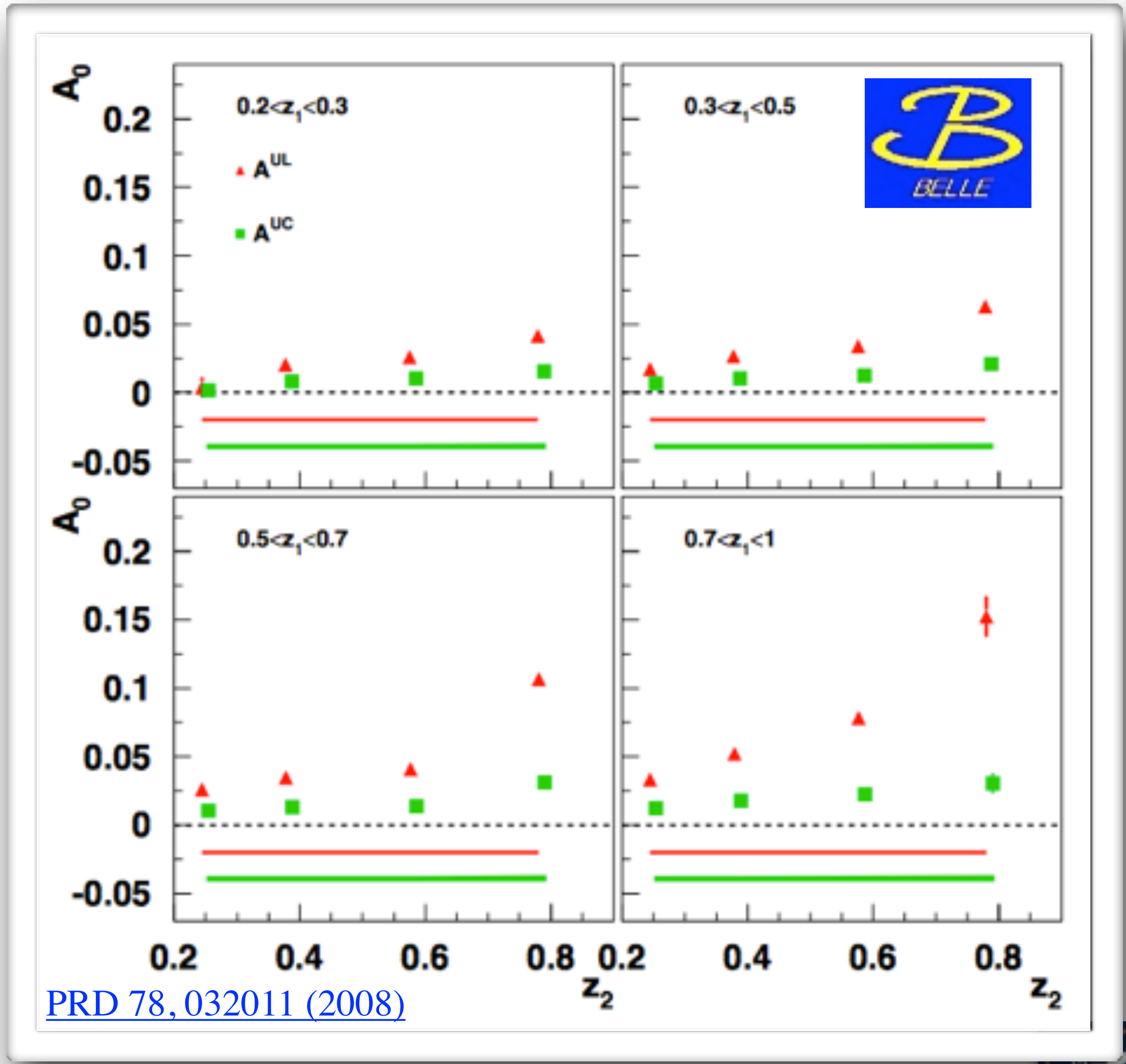
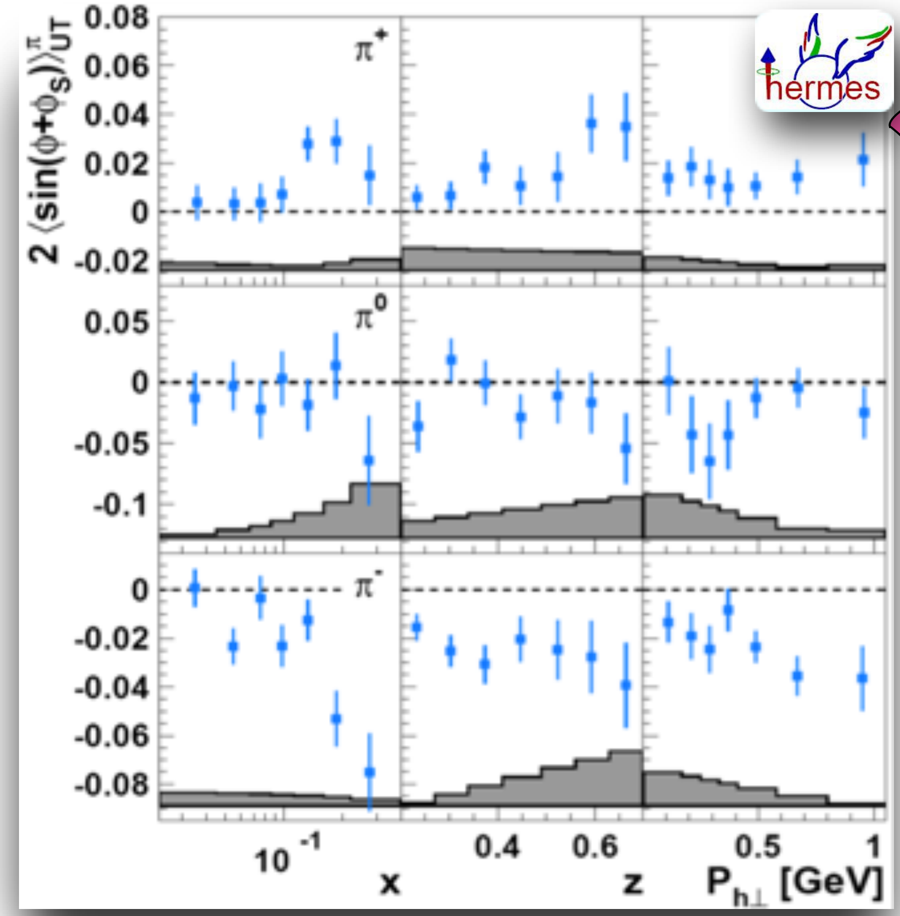


PRD 78, 032011 (2008)

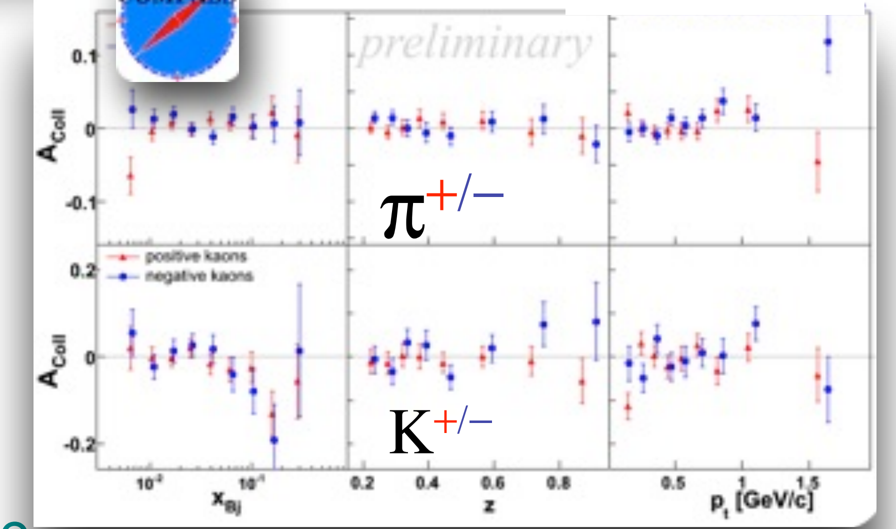
Collins amplitudes in SIDIS

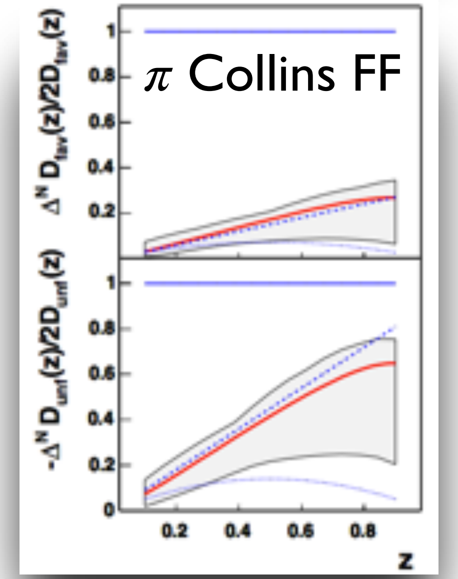
$$A_{UT} \propto h_1 \otimes H_1^{\perp}$$

[Airapetian *et al.*, Phys. Lett. B 693 (2010) 11-16]



Deuterium





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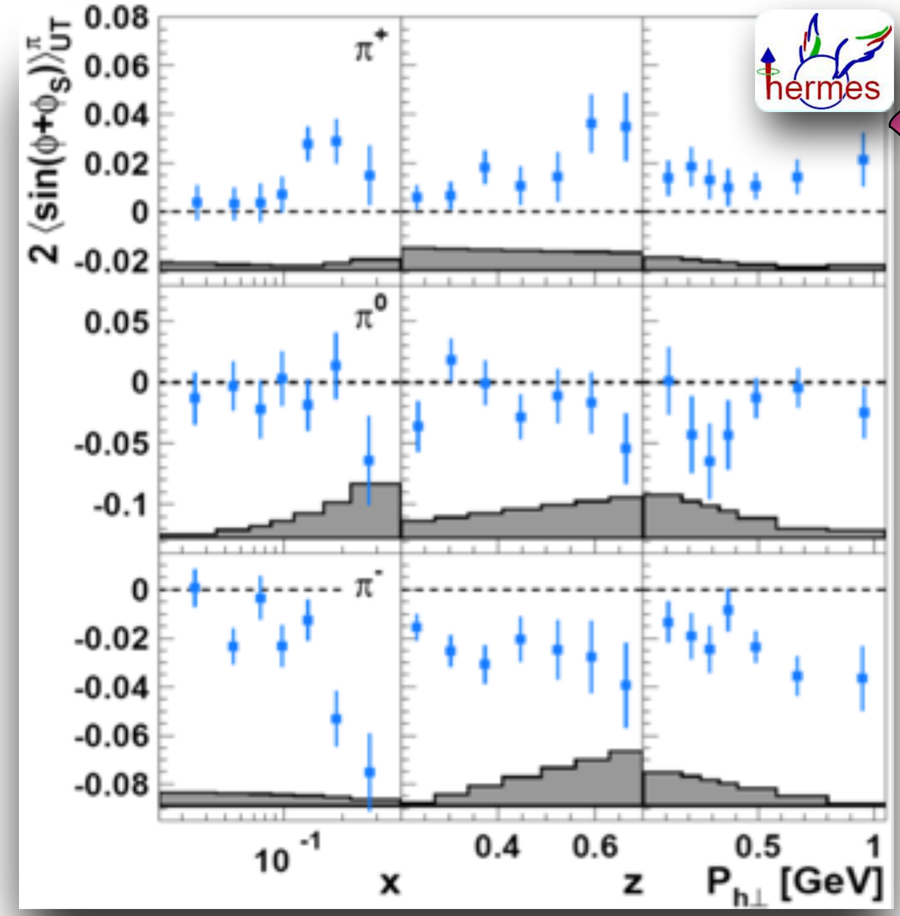

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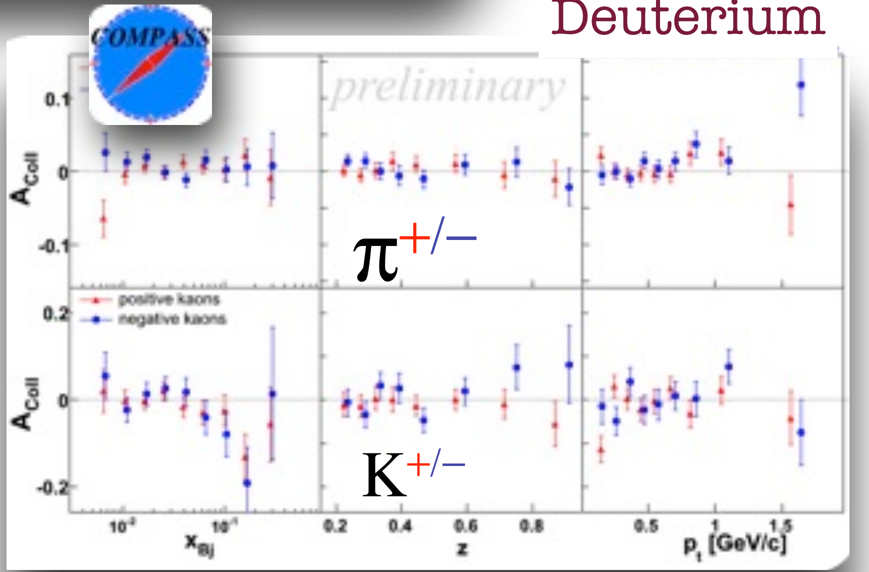
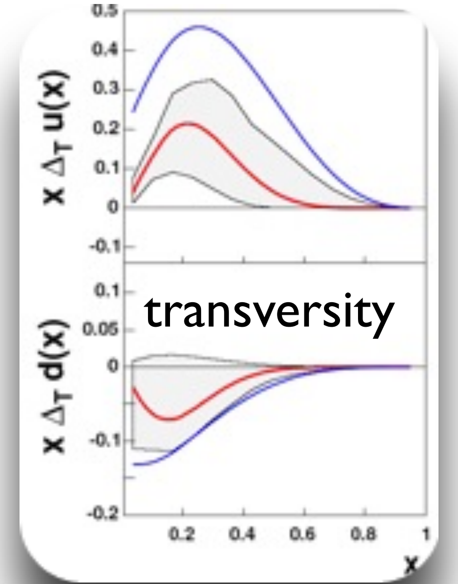
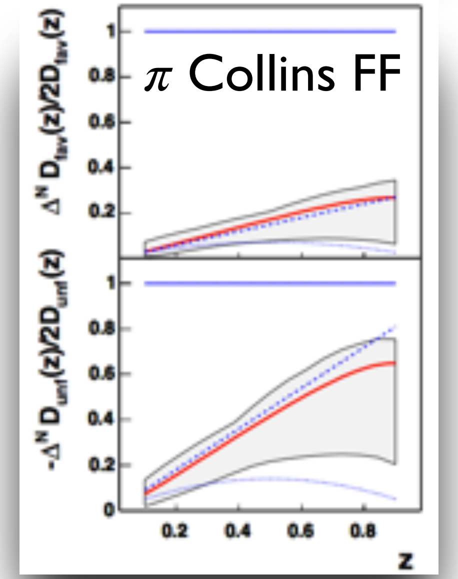


Anselmino et al.
Phys.Rev. D75 (2007)





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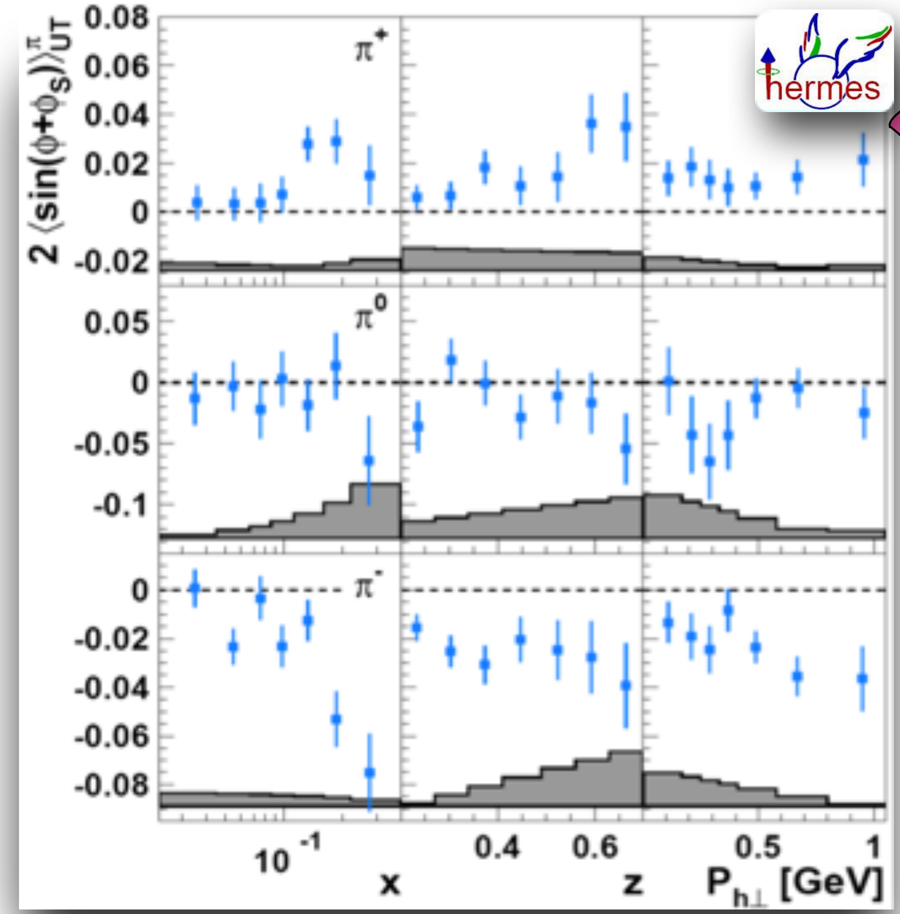

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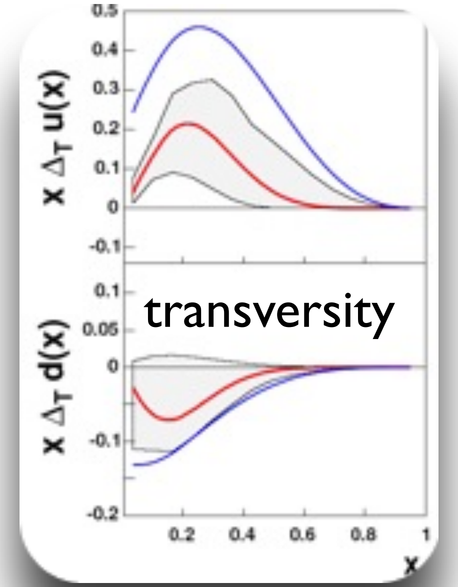

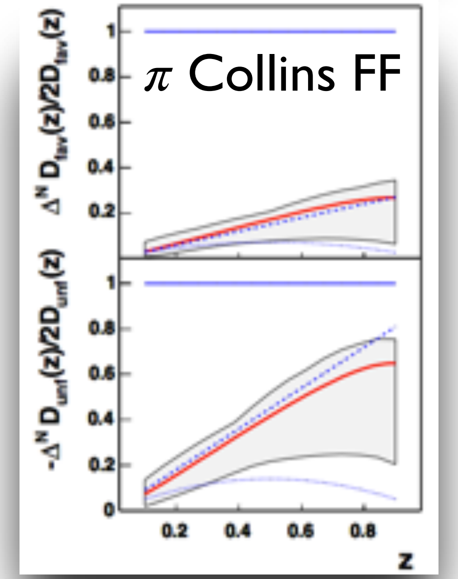

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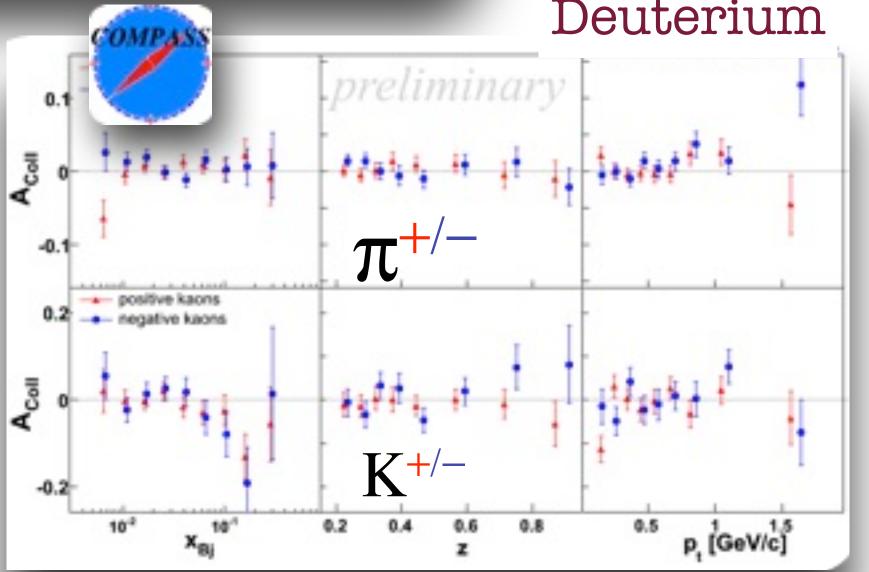
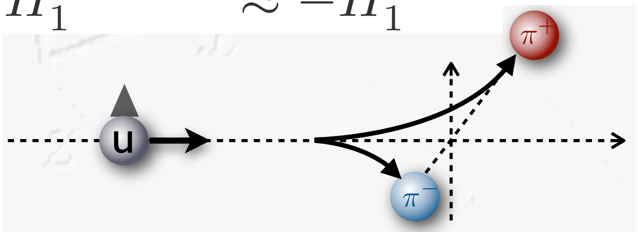
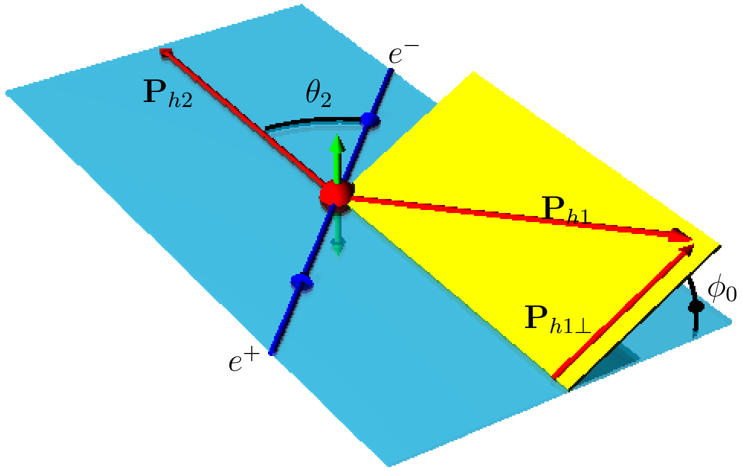


$$H_1^{\perp,unfav} \approx -H_1^{\perp,fav}$$



More recently...

ϕ_0 method



$$\sigma \sim \mathcal{M}_0\Big(1 + \frac{\sin^2\theta_2}{1+\cos^2\theta_2}\, cos(2\phi_0)\mathcal{F}\Big[\frac{H_1^\perp(z_1)\bar{H}_1^\perp(z_2)}{D_1^\perp(z_1)\bar{D}_1^\perp(z_2)}\Big]\Big)$$

Extraction of Collins asymmetries
for πK and KK couples!

ϕ_1+ϕ_2 method will
soon follow...

PID correction

$i = \pi, K$

$$N^{j,raw} = P_{ij} N^i$$

PID correction

$i = \pi, K$

$$N^{j,raw} = P_{ij} N^i$$

Perfect PID ⇨ $j = i$

$j = e, \mu, \pi, K, p$

$$P_{ij} = \begin{pmatrix} P_{e\to e} & P_{e\to \mu} & P_{e\to \pi} & P_{e\to K} & P_{e\to p} \\ P_{\mu\to e} & P_{\mu\to \mu} & P_{\mu\to \pi} & P_{\mu\to K} & P_{\mu\to p} \\ P_{\pi\to e} & P_{\pi\to \mu} & P_{\pi\to \pi} & P_{\pi\to K} & P_{\pi\to p} \\ P_{K\to e} & P_{K\to \mu} & P_{K\to \pi} & P_{K\to K} & P_{K\to p} \\ P_{p\to e} & P_{p\to mu} & P_{p\to \pi} & P_{p\to K} & P_{p\to p} \end{pmatrix}$$

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$\mathbf{p}_{\pi, K \to j}$ from D* decay

$\mathbf{p}_{\pi, p \to j}$ from Λ decay

$\mathbf{p}_{e, \mu \to j}$ from J/ψ decay

PID correction

$i = \pi, K$

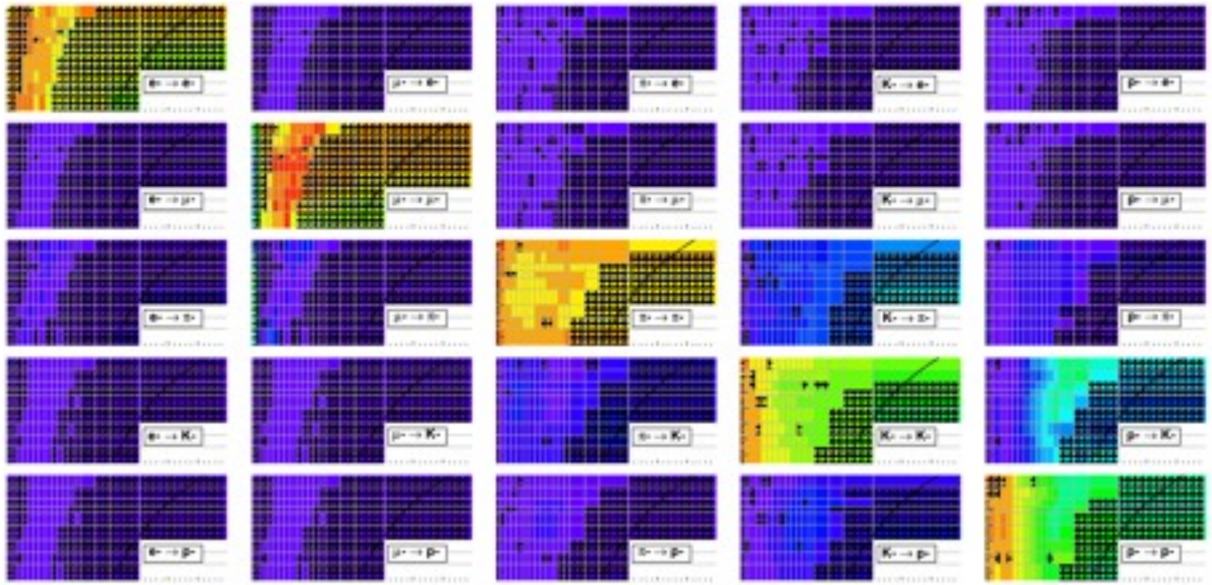
$$N^{j,raw} = P_{ij} N^i$$

Perfect PID ⇨ $j = i$

$j = e, \mu, \pi, K, p$

P_{ij} ⇨ $P_{ij}(p, \theta)$

$$P_{ij} = \begin{pmatrix} P_{e\to e} & P_{e\to \mu} & P_{e\to \pi} & P_{e\to K} & P_{e\to p} \\ P_{\mu\to e} & P_{\mu\to \mu} & P_{\mu\to \pi} & P_{\mu\to K} & P_{\mu\to p} \\ P_{\pi\to e} & P_{\pi\to \mu} & P_{\pi\to \pi} & P_{\pi\to K} & P_{\pi\to p} \\ P_{K\to e} & P_{K\to \mu} & P_{K\to \pi} & P_{K\to K} & P_{K\to p} \\ P_{p\to e} & P_{p\to mu} & P_{p\to \pi} & P_{p\to K} & P_{p\to p} \end{pmatrix}$$

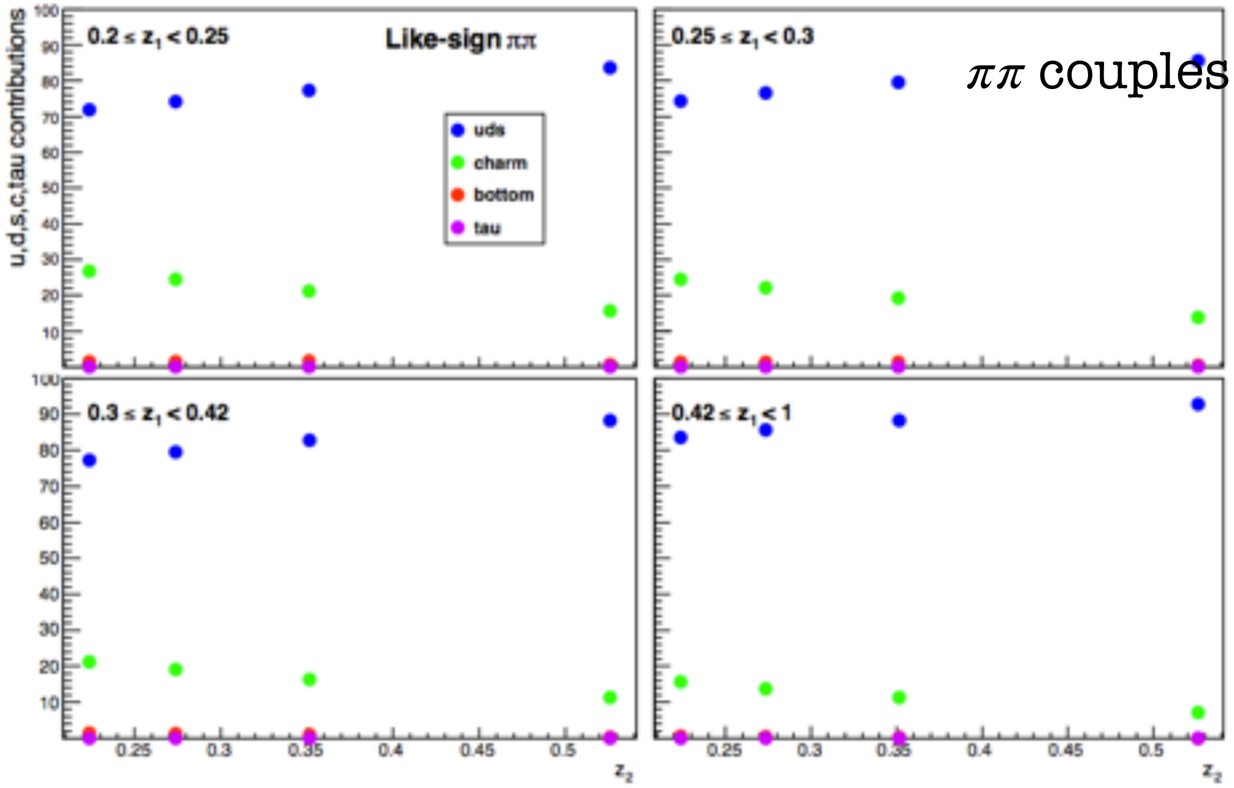


$\mathbf{p}_{\pi, K \text{->} j}$ from D* decay

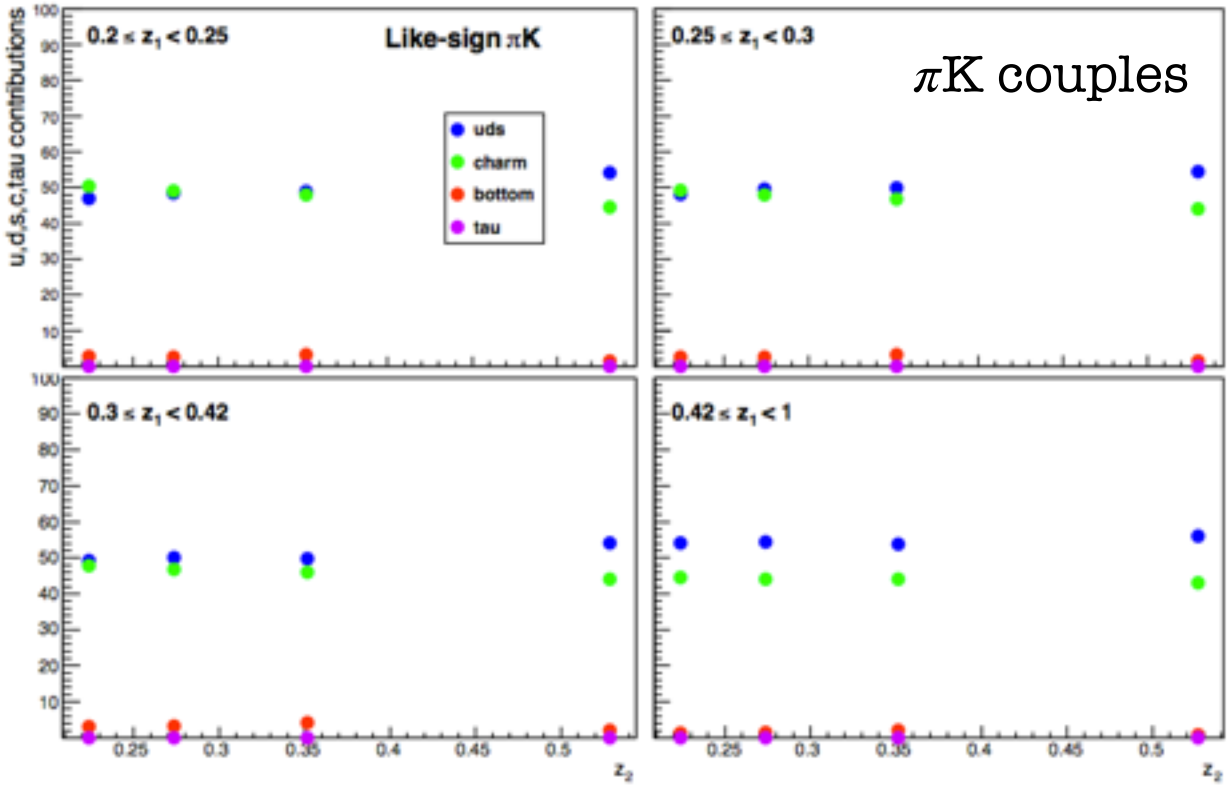
$\mathbf{p}_{\pi, p \text{->} j}$ from Λ decay

$\mathbf{p}_{e, \mu \text{->} j}$ from J/ψ decay

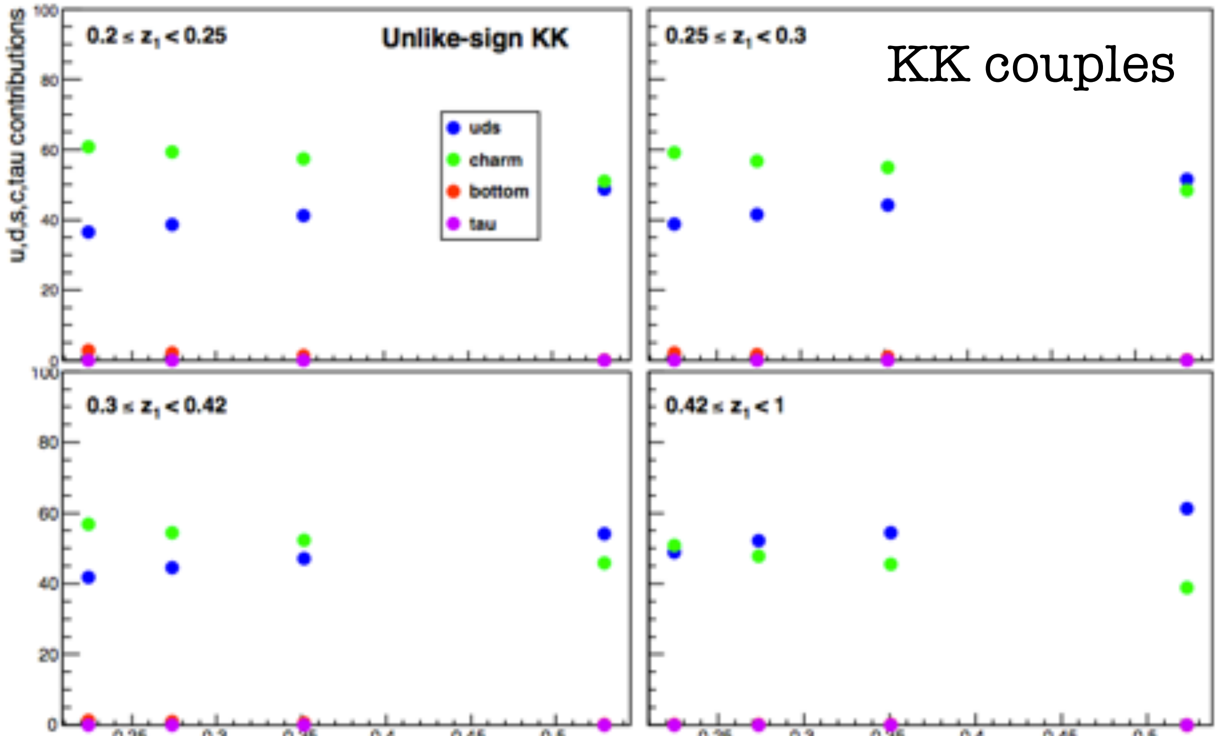
uds-charm-bottom-tau contributions



$\pi\pi$ couples

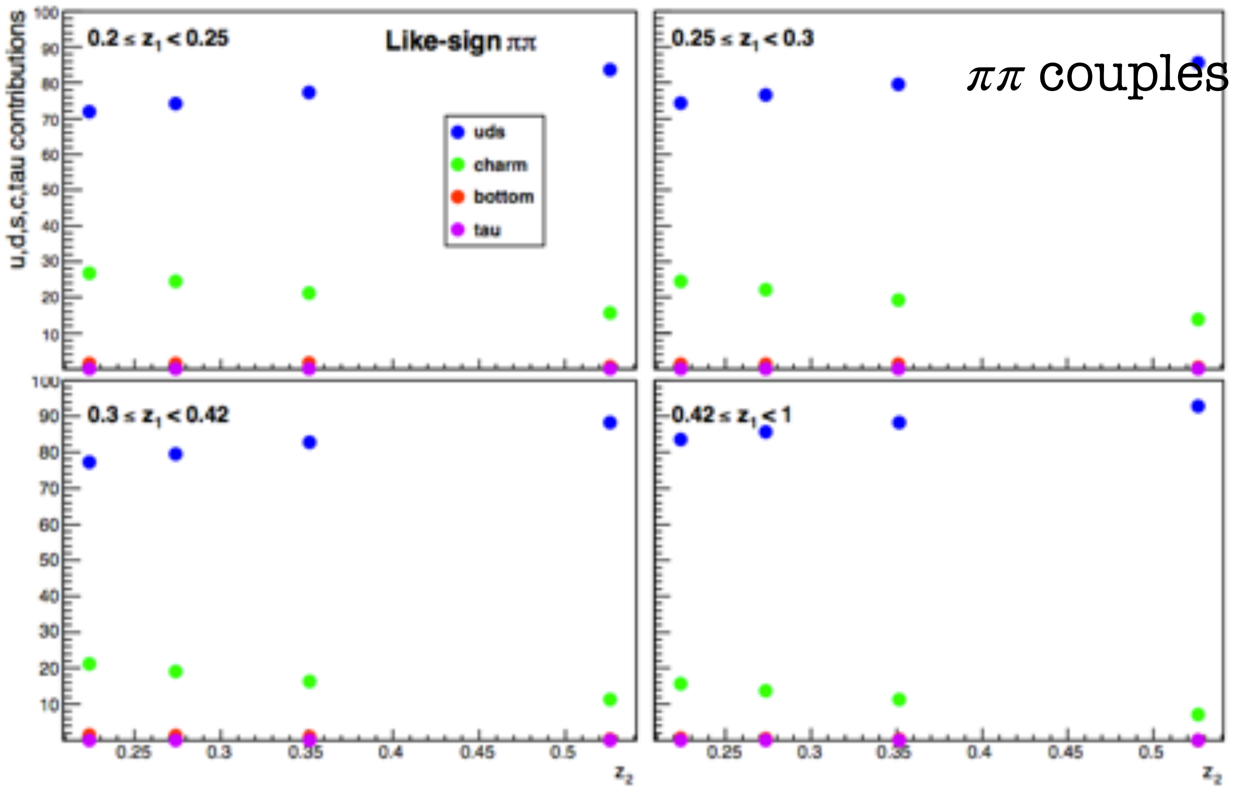


πK couples

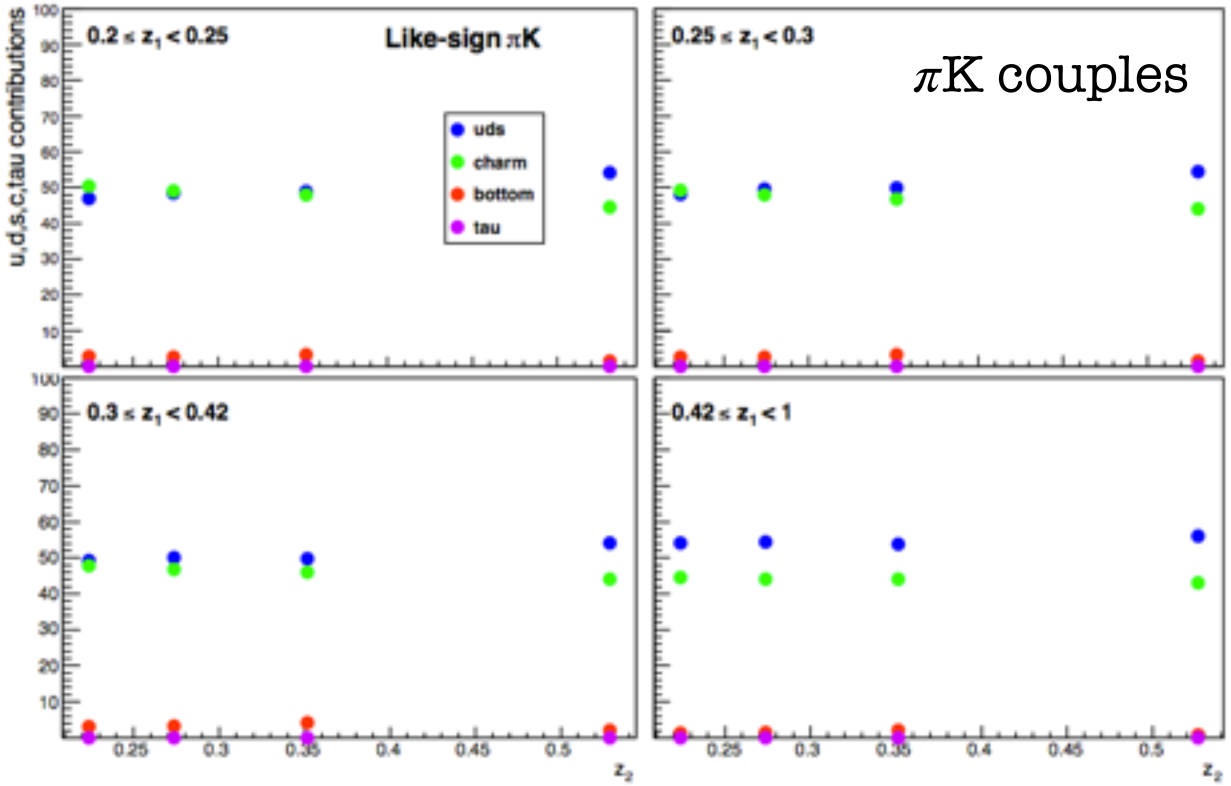


KK couples

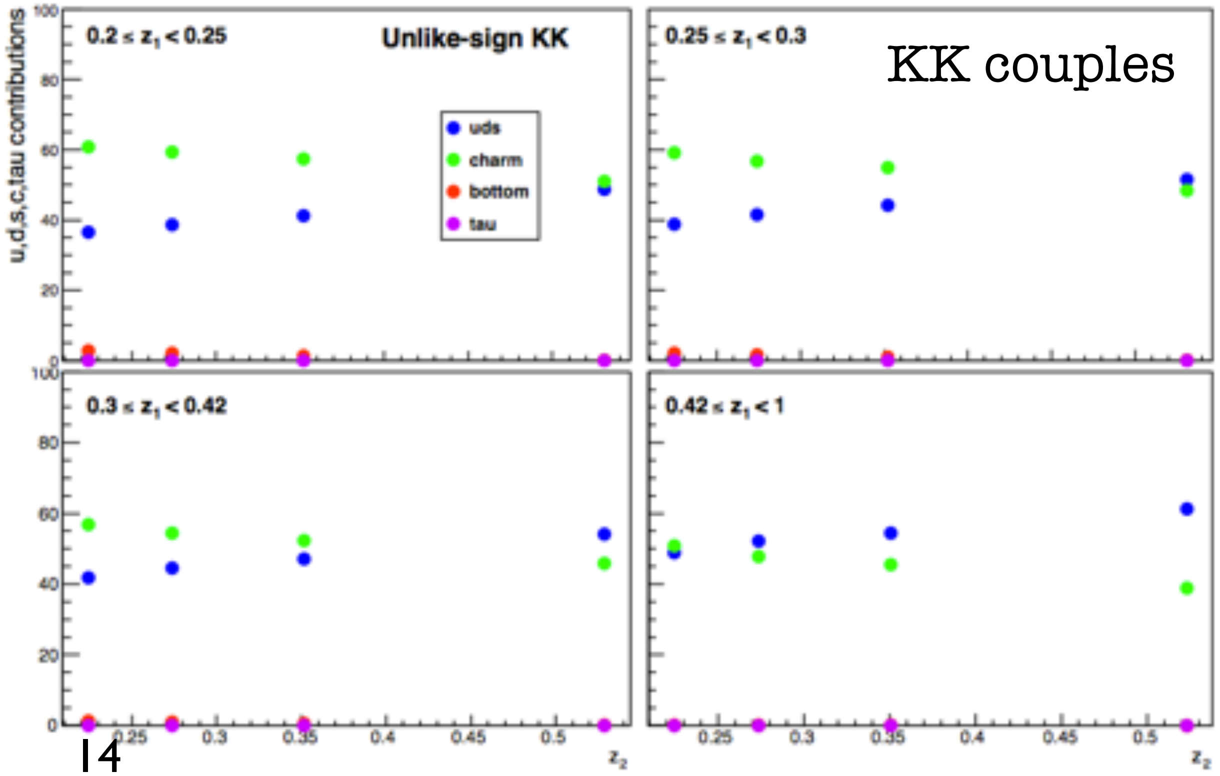
uds-charm-bottom-tau contributions



ππ couples



πK couples

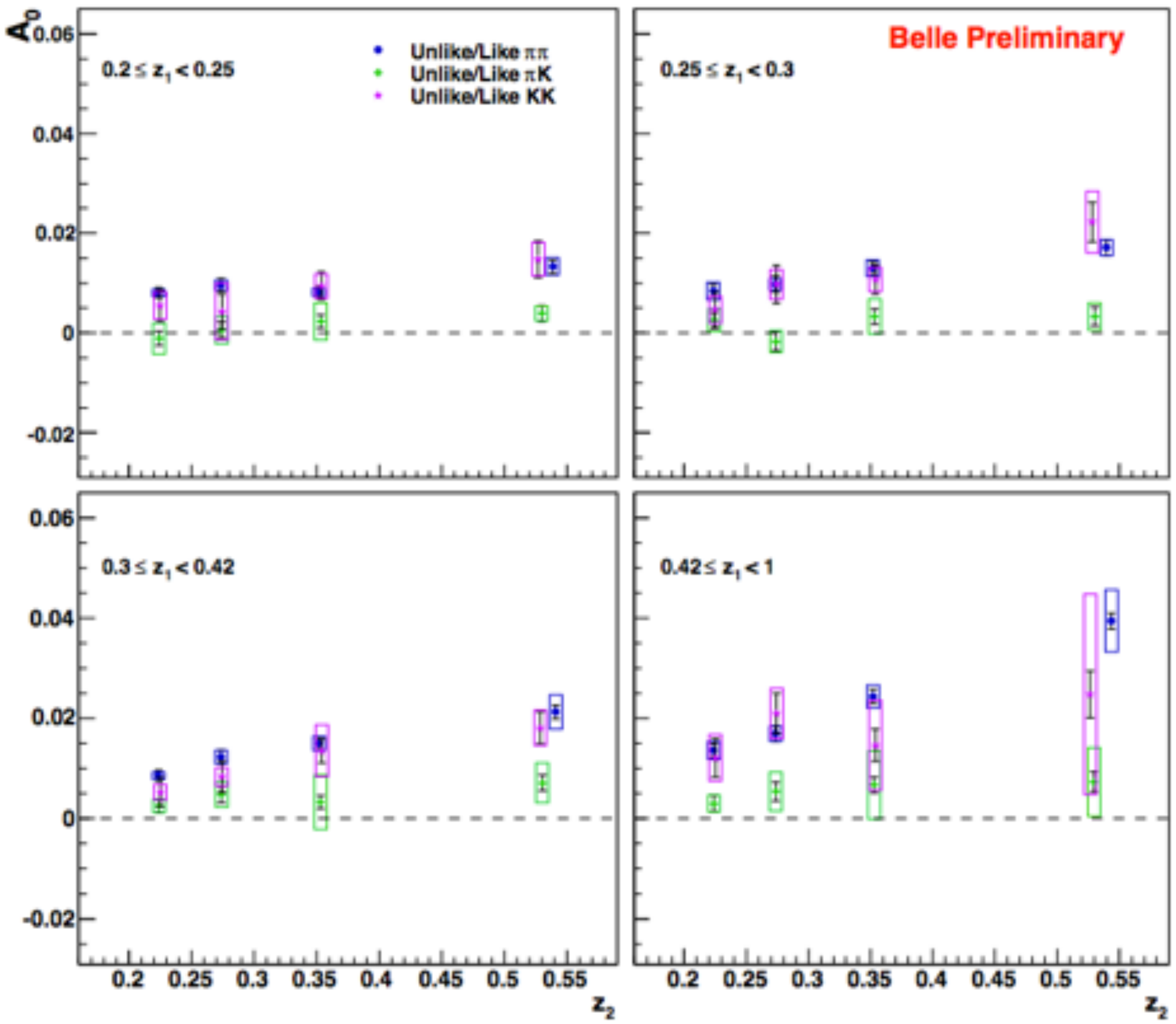


KK couples

For the moment charm contribution
is not being corrected out
in any of the samples (ππ, πK, KK)

Collins asymmetries

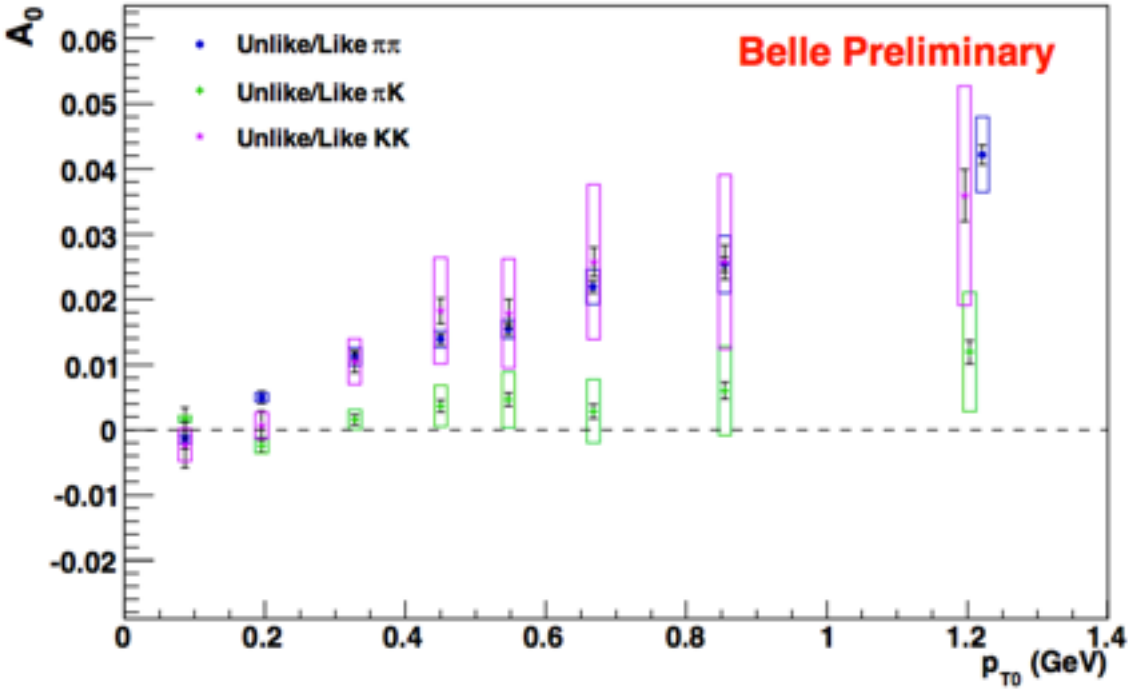
ϕ_0 asymmetries



$\pi\pi$ => non-zero asymmetries,
increase with z_1, z_2 and p_{T0}

πK => asymmetries compatible
with zero

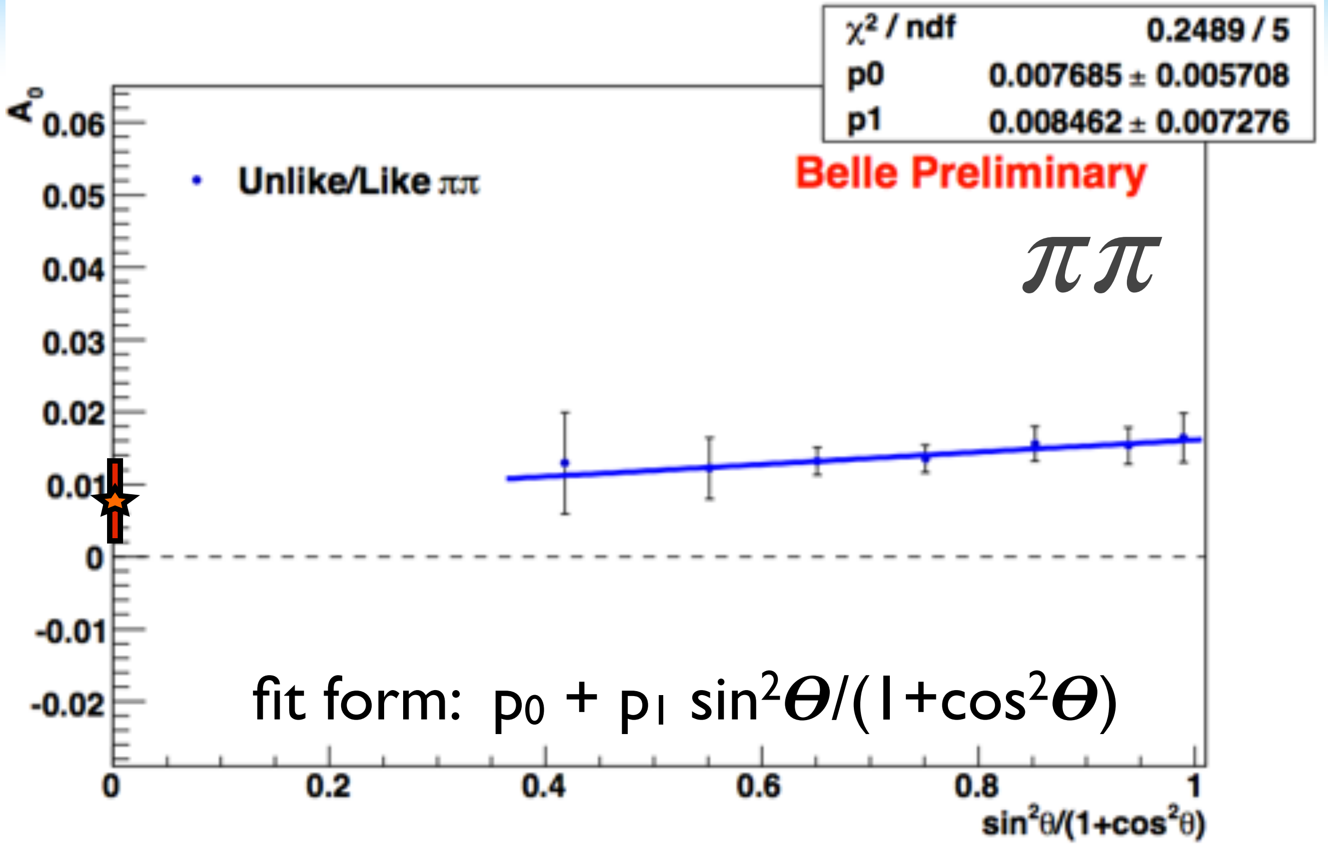
KK => non-zero asymmetries,
increase with z_1,z_2 and p_{T0}
similar size of pion-pion



But we must be careful!
charm have different contributions for
the different pairs

QCD test?

$\sin^2\theta/(1+\cos^2\theta)$ dependence

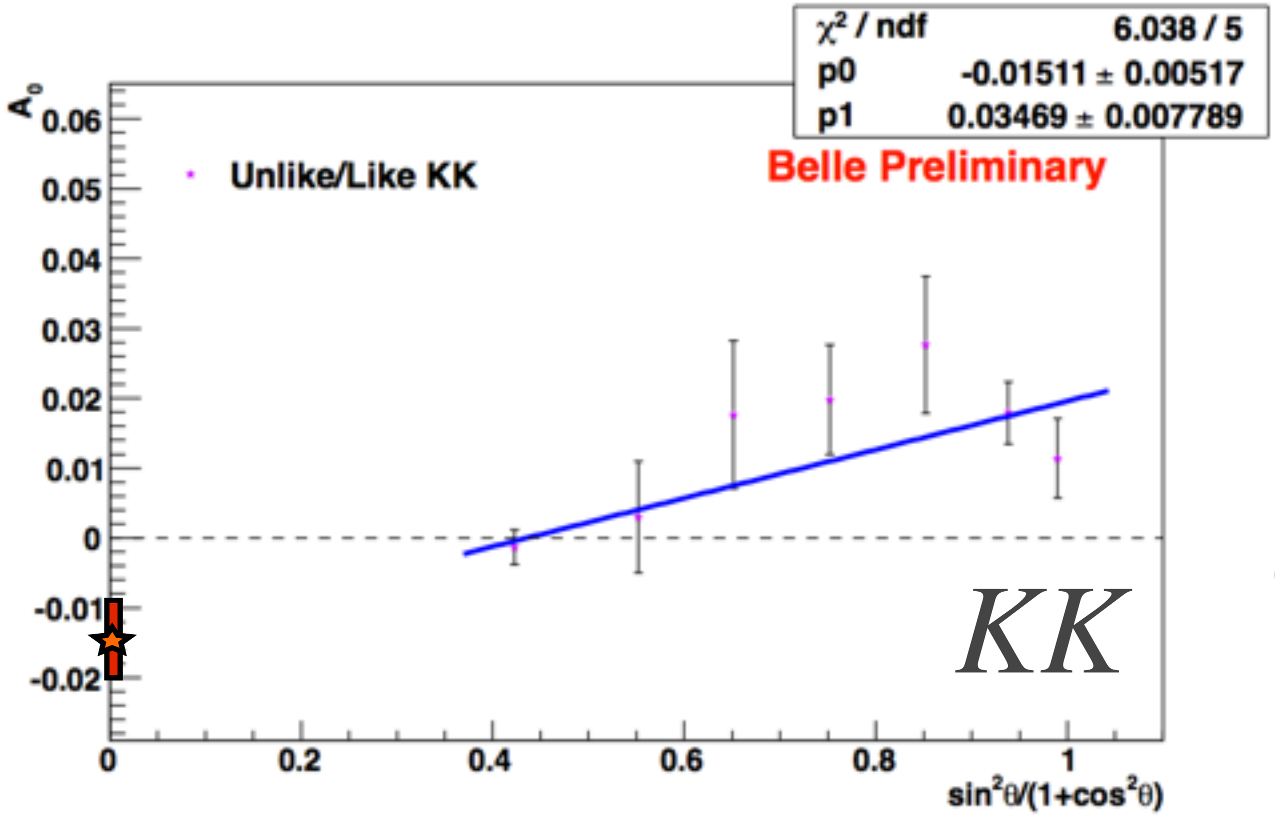


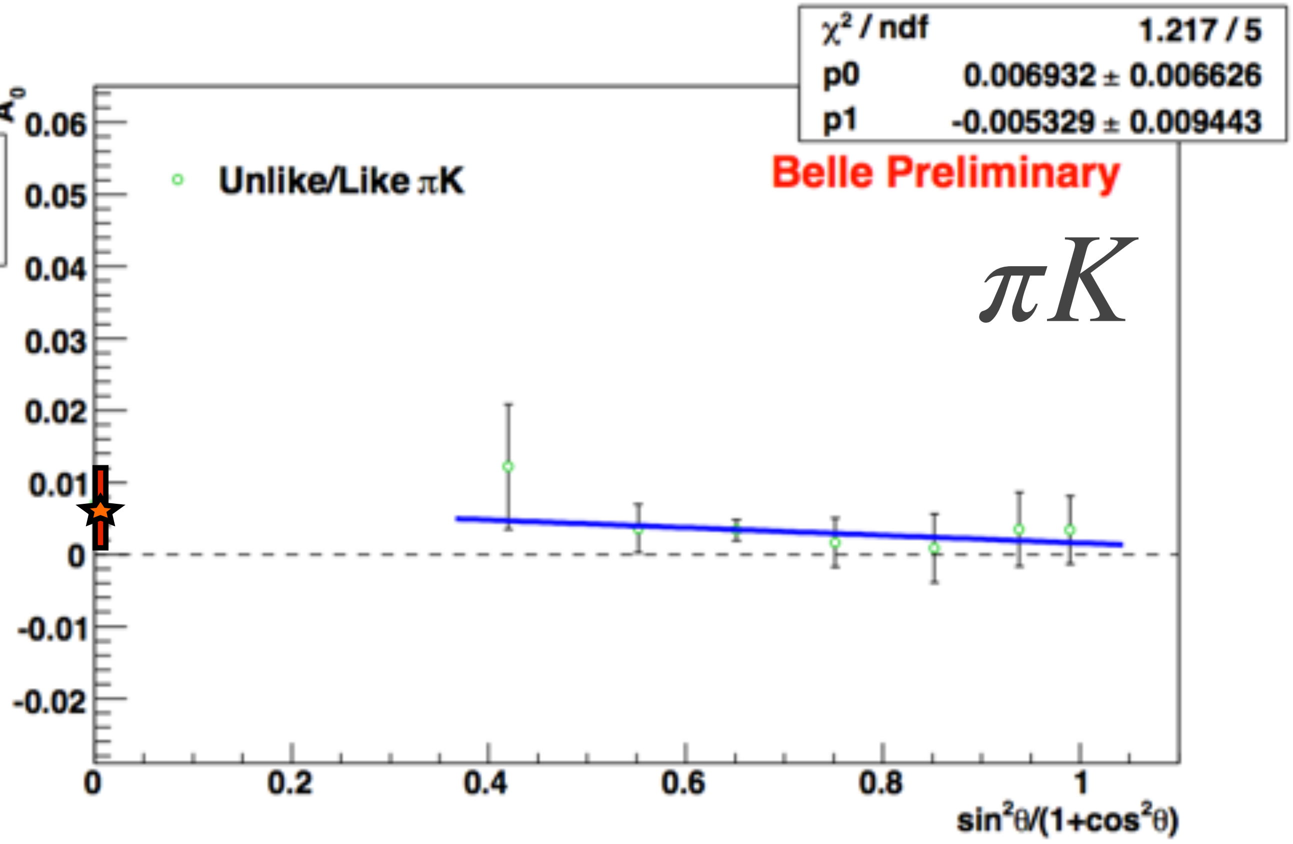
fit form: $p_0 + p_1 \sin^2\theta/(1+\cos^2\theta)$

$$A_0 = \frac{\sin^2\theta}{1+\cos^2\theta}\mathcal{F}\left[\frac{H_1^{\perp}(z_1)\bar{H}_1^{\perp}(z_2)}{D_1^{\perp}(z_1)\bar{D}_1^{\perp}(z_2)}\right]$$

linear in $\sin^2\theta/(1+\cos^2\theta)$,

go to 0 for $\sin^2\theta/(1+\cos^2\theta) \rightarrow 0$





Summary & outlook

- ϕ_0 asymmetries
  - present similar features for $\pi\pi$ and KK couples
  - very small/compatible with zero for πK couples
  - for $\pi\pi$ and πK the $\sin^2\Theta/(1+\cos^2\Theta)$ dependence of asymmetries are not inconsistent with a linear dependence going to zero
  - KK show a more convoluted $\sin^2\Theta/(1+\cos^2\Theta)$ dependence

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- study using jet algorithm instead of Thrust in progress

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- ϕ_{12} asymmetries with Thrust axis in progress
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Stay tuned!

Fragmentation contributions

$u,\ d \rightarrow \pi\ (u\bar{d},\ \bar{u}d)$

$$D^{fav} = D_u^{\pi^+} = D_d^{\pi^-} = D_{\bar{u}}^{\pi^-} = D_{\bar{d}}^{\pi^+}$$
$$D^{dis} = D_u^{\pi^-} = D_d^{\pi^+} = D_{\bar{u}}^{\pi^+} = D_{\bar{d}}^{\pi^-}$$

$s \rightarrow \pi\ (u\bar{d},\ \bar{u}d)$

$$D_{s\rightarrow\pi}^{dis} = D_s^{\pi^+} = D_s^{\pi^-} = D_{\bar{s}}^{\pi^+} = D_{\bar{s}}^{\pi^-}$$

$u,\ d \rightarrow K\ (u\bar{s},\ \bar{u}s)$

$$D_{u\rightarrow K}^{fav} = D_u^{K^+} = D_{\bar{u}}^{K^-}$$
$$D_{u,d\rightarrow K}^{dis} = D_u^{K^-} = D_{\bar{u}}^{K^+} = D_d^{K^+} = D_{\bar{d}}^{K^-} = D_d^{K^-} = D_{\bar{d}}^{K^+}$$

$s \rightarrow K\ (u\bar{s},\ \bar{u}s)$

$$D_{s\rightarrow K}^{fav} = D_s^{K^-} = D_{\bar{s}}^{K^+}$$
$$D_{s\rightarrow K}^{dis} = D_s^{K^+} = D_{\bar{s}}^{K^-}$$

In the end we are left with 7 possible fragmentation functions:

$$D^{fav},\ D^{dis},\ D_{s\rightarrow\pi}^{dis},\ D_{u\rightarrow K}^{fav},\ D_{u,d\rightarrow K}^{dis},\ D_{s\rightarrow K}^{fav},\ D_{s\rightarrow K}^{dis}$$

Assuming charm contribute
only as a dilution

Fragmentation contributions

For pion-pion couples:

$$D^{\frac{U_{\pi\pi}}{L_{\pi\pi}}} \propto 1 \quad + \cos 2\phi_0 \frac{\sin^2\theta}{1+cos^2\theta}\left(\frac{5H_1^{fav}H_2^{fav}+5H_1^{dis}H_2^{dis}+2H_{1\,s\to\pi}^{dis}H_{2\,s\to\pi}^{dis}}{5D_1^{fav}D_2^{fav}+5D_1^{dis}D_2^{dis}+2D_{1\,s\to\pi}^{dis}D_{2\,s\to\pi}^{dis}} - \frac{5H_1^{fav}H_2^{dis}+5H_1^{dis}H_2^{fav}+2H_{1\,s\to\pi}^{dis}H_{2\,s\to\pi}^{dis}}{5D_1^{fav}D_2^{dis}+5D_1^{dis}D_2^{fav}+2D_{1\,s\to\pi}^{dis}D_{2\,s\to\pi}^{dis}}\right)$$

For pion-Kaon couples:

$$D^{\frac{U_{\pi K}}{L_{\pi K}}} \propto 1 + \cos 2\phi_0 \frac{\sin^2\theta}{1+cos^2\theta} \times$$

$$\left(\frac{4H_1^{fav}H_{2\,K}^{fav}+H_{1K}^{dis}(5H_2^{dis}+H_2^{fav})+H_{2K}^{dis}(5H_1^{dis}+H_1^{fav})+4\,H_{1K}^{fav}H_2^{fav}+H_{1s\to\pi}^{dis}(H_{2s\to K}^{dis}+H_{2s\to K}^{fav})+H_{2s\to\pi}^{dis}(H_{1s\to K}^{fav}+H_{1s\to K}^{dis})}{4D_1^{fav}D_{2\,K}^{fav}+D_{1K}^{dis}(5D_2^{dis}+D_2^{fav})+D_{2K}^{dis}(5D_1^{dis}+D_1^{fav})+4\,D_{1K}^{fav}D_2^{fav}+D_{1s\to\pi}^{dis}(D_{2s\to K}^{dis}+D_{2s\to K}^{fav})+D_{2s\to\pi}^{dis}(D_{1s\to K}^{fav}+D_{1s\to K}^{dis})}\right.$$

$$\left.\frac{H_{2\,K}^{dis}(5H_1^{fav}+H_1^{dis})+4\,H_{1\,K}^{fav}H_2^{dis}+4H_1^{dis}H_{2K}^{fav}+H_{1K}^{dis}(5H_2^{fav}+H_2^{dis})+H_{1s\to\pi}^{dis}(H_{2s\to K}^{fav}+H_{2s\to K}^{dis})+(H_{1s\to K}^{dis}+H_{1s\to K}^{fav})H_{2s\to\pi}^{dis}}{D_{2\,K}^{dis}(5D_1^{fav}+D_1^{dis})+4\,D_{1\,K}^{fav}D_2^{dis}+4D_1^{dis}D_{2K}^{fav}+D_{1K}^{dis}(5D_2^{fav}+D_2^{dis})+D_{1s\to\pi}^{dis}(D_{2s\to K}^{fav}+D_{2s\to K}^{dis})+(D_{1s\to K}^{dis}+D_{1s\to K}^{fav})D_{2s\to\pi}^{dis}}\right)$$

For Kaon-Kaon couples:

$$D^{\frac{U_{KK}}{L_{KK}}} \propto 1 \quad + \cos 2\phi_0 \frac{\sin^2\theta}{1+cos^2\theta}\left(\frac{4H_{1K}^{fav}H_{2\,K}^{fav}+6H_{1\,K}^{dis}H_{2K}^{dis}+H_{1s\to K}^{dis}H_{2s\to K}^{dis}+H_{1s\to K}^{fav}H_{2s\to K}^{fav}}{4D_{1K}^{fav}D_{2\,K}^{fav}+6D_{1\,K}^{dis}D_{2K}^{dis}+D_{1s\to K}^{dis}D_{2s\to K}^{dis}+D_{1s\to K}^{fav}D_{2s\to K}^{fav}} - \frac{4H_{1K}^{fav}H_{2\,K}^{dis}+4H_{1\,K}^{dis}H_{2K}^{fav}+2H_{1K}^{dis}H_{2K}^{dis}+H_{1s\to K}^{dis}H_{2s\to K}^{fav}+H_{1s\to K}^{fav}H_{2s\to K}^{dis}}{4D_{1K}^{fav}D_{2\,K}^{dis}+4D_{1\,K}^{dis}D_{2K}^{fav}+2D_{1K}^{dis}D_{2K}^{dis}+D_{1s\to K}^{dis}D_{2s\to K}^{fav}+D_{1s\to K}^{fav}D_{2s\to K}^{dis}}\right)$$

Fragmentation contributions

For pion-pion couples:

$$D^{\frac{}{L_{\pi\pi}}} \propto 1 \quad + \cos 2\phi_0 \frac{\sin^2\theta}{1+cos^2\theta} \left(\frac{5H_1^{fav} H_2^{fav} + 5H_1^{dis} H_2^{dis} + 2H_{1\,s\to\pi}^{dis} H_{2\,s\to\pi}^{dis}}{5D_1^{fav} D_2^{fav} + 5D_1^{dis} D_2^{dis} + 2D_{1\,s\to\pi}^{dis} D_{2\,s\to\pi}^{dis}} - \frac{5H_1^{fav} H_2^{dis} + 5H_1^{dis} H_2^{fav} + 2H_{1\,s\to\pi}^{dis} H_{2\,s\to\pi}^{dis}}{5D_1^{fav} D_2^{dis} + 5D_1^{dis} D_2^{fav} + 2D_{1\,s\to\pi}^{dis} D_{2\,s\to\pi}^{dis}} \right)$$

For pion-Kaon couples:

$$D^{\frac{U_{\pi K}}{L_{\pi K}}} \propto 1 + \cos 2\phi_0 \frac{\sin^2\theta}{1+cos^2\theta} \times$$

$$\left(\frac{4H_1^{fav} H_{2\,K}^{fav} + H_{1K}^{dis}(5H_2^{dis}+H_2^{fav}) + H_{2K}^{dis}(5H_1^{dis}+H_1^{fav}) + 4\,H_{1K}^{fav} H_2^{fav} + H_{1s\to\pi}^{dis}(H_{2s\to K}^{dis}+H_{2s\to K}^{fav}) + H_{2s\to\pi}^{dis}(H_{1s\to K}^{fav}+H_{1s\to K}^{dis})}{4D_1^{fav} D_{2\,K}^{fav} + D_{1K}^{dis}(5D_2^{dis}+D_2^{fav}) + D_{2K}^{dis}(5D_1^{dis}+D_1^{fav}) + 4\,D_{1K}^{fav} D_2^{fav} + D_{1s\to\pi}^{dis}(D_{2s\to K}^{dis}+D_{2s\to K}^{fav}) + D_{2s\to\pi}^{dis}(D_{1s\to K}^{fav}+D_{1s\to K}^{dis})} \right.$$

$$\left. \frac{H_{2\,K}^{dis}(5H_1^{fav}+H_1^{dis}) + 4\,H_{1\,K}^{fav} H_2^{dis} + 4H_1^{dis} H_{2K}^{fav} + H_{1K}^{dis}(5H_2^{fav}+H_2^{dis}) + H_{1s\to\pi}^{dis}(H_{2s\to K}^{fav}+H_{2s\to K}^{dis}) + (H_{1s\to K}^{dis}+H_{1s\to K}^{fav})H_{2s\to\pi}^{dis}}{D_{2\,K}^{dis}(5D_1^{fav}+D_1^{dis}) + 4\,D_{1\,K}^{fav} D_2^{dis} + 4D_1^{dis} D_{2K}^{fav} + D_{1K}^{dis}(5D_2^{fav}+D_2^{dis}) + D_{1s\to\pi}^{dis}(D_{2s\to K}^{fav}+D_{2s\to K}^{dis}) + (D_{1s\to K}^{dis}+D_{1s\to K}^{fav})D_{2s\to\pi}^{dis}} \right)$$

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A full phenomenological study needed!

How to determine the P_{ij}?

How to determine the P_{ij}?

From data!

How to determine the P_{ij}?

$$\begin{tikzcd}
D^* \arrow[r] \arrow[dr] & D^0 \\
& \pi^+_{slow}
\end{tikzcd}$$

From data!

How to determine the P_{ij}?

$$\begin{tikzcd}
D^* \arrow[r] \arrow[dr] & D^0 \arrow[r] \arrow[dr] & K^- \\
 & \pi^+_{slow} & \pi^+_{fast}
\end{tikzcd}$$

From data!

How to determine the P_{ij}?

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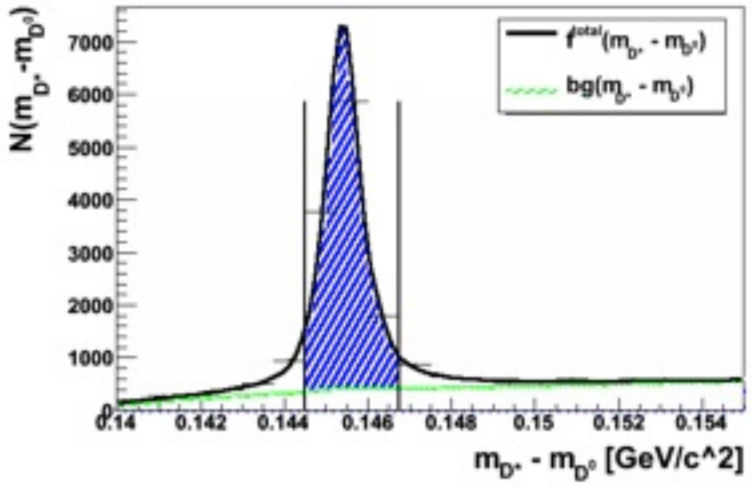
$$\begin{tikzcd} D^* \arrow[r] \arrow[dr] & D^0 \arrow[r] \arrow[dr] & K^- \\ & \pi^+_{slow} & \pi^+_{fast} \end{tikzcd}$$

$$m^*_D - m^0_D$$



Negative hadron = K^-
(no PID likelihood used)

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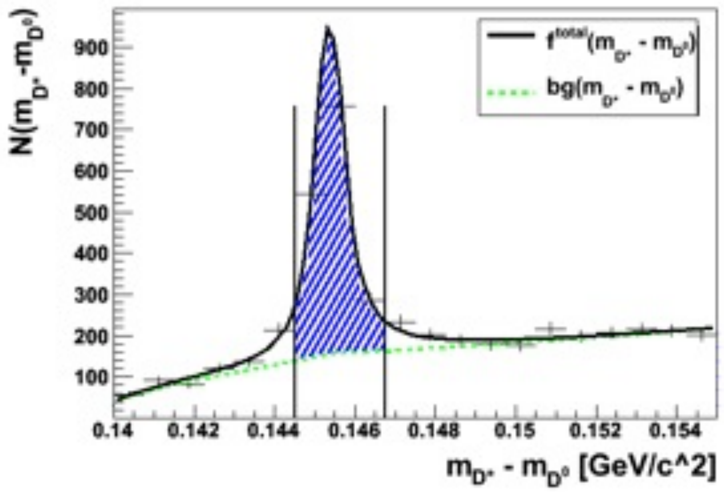
$$\begin{tikzcd} D^* \arrow[r] \arrow[dr] & D^0 \arrow[r] \arrow[dr] & K^- \\ & \pi^+_{slow} & \pi^+_{fast} \end{tikzcd}$$

$m_D^* - m_D^0$



Negative hadron identified as π^-

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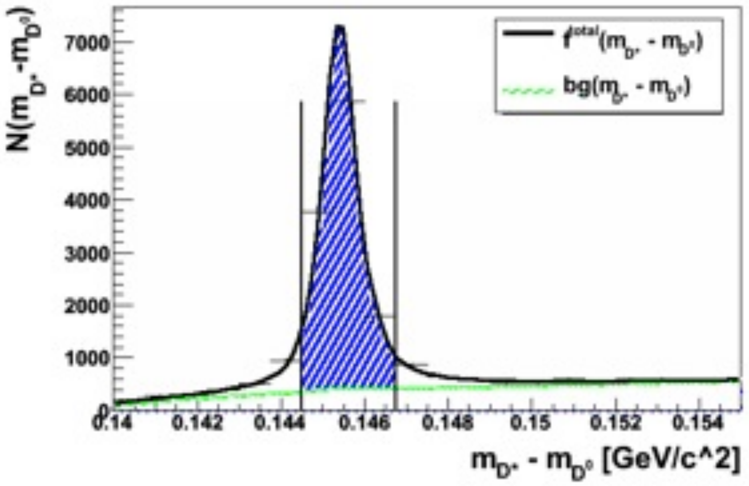

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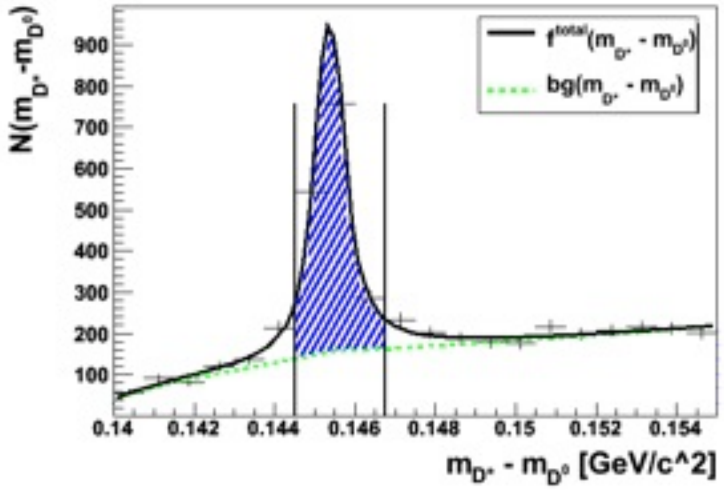
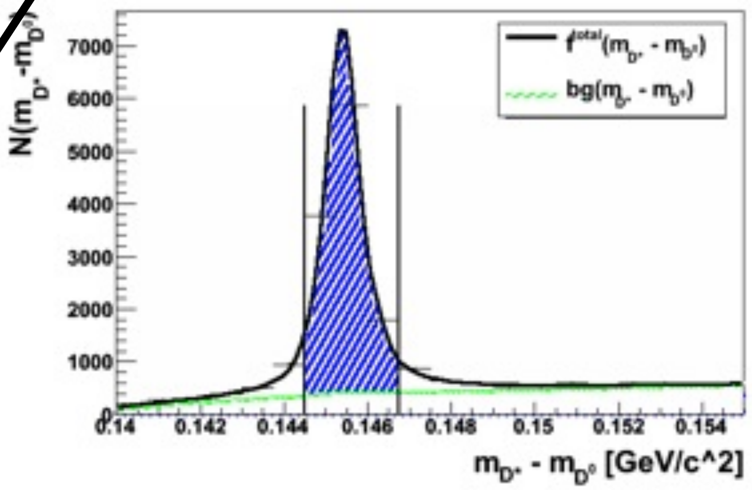


Negative hadron identified as π^-

$$P_{K^- \to \pi^-}$$

$$m_D^* - m_D^0$$



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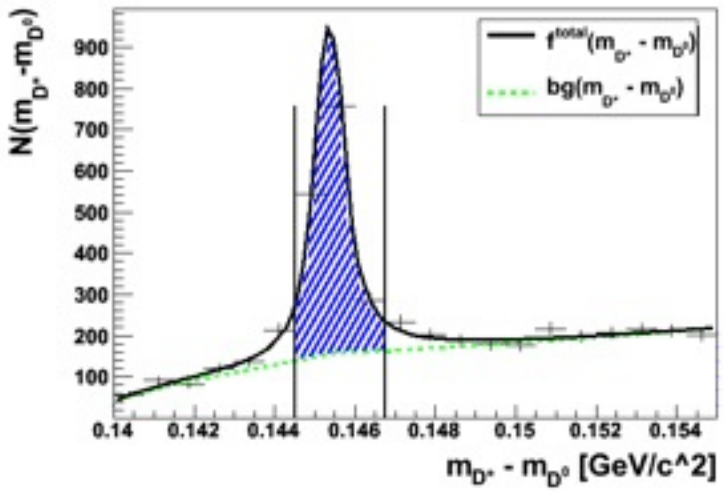

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K^-

$P_{K^- \to \pi^-}$

$P_{K^- \to K^-}$

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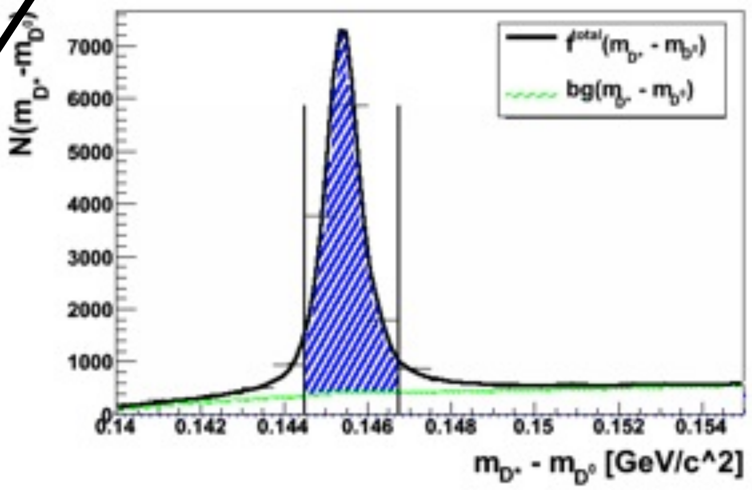

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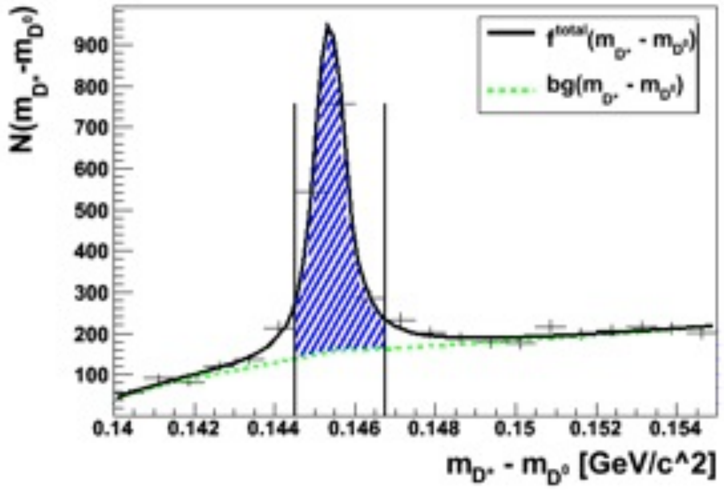
\end{tikzcd}$$

From data!

$m_D^* - m_D^0$



Negative hadron identified as π^-

K^-

$\bar{p}$

$P_{K^- \to \pi^-}$

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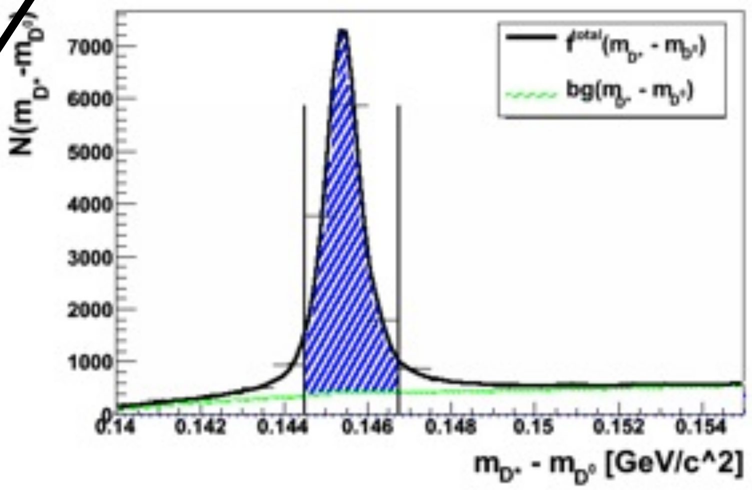

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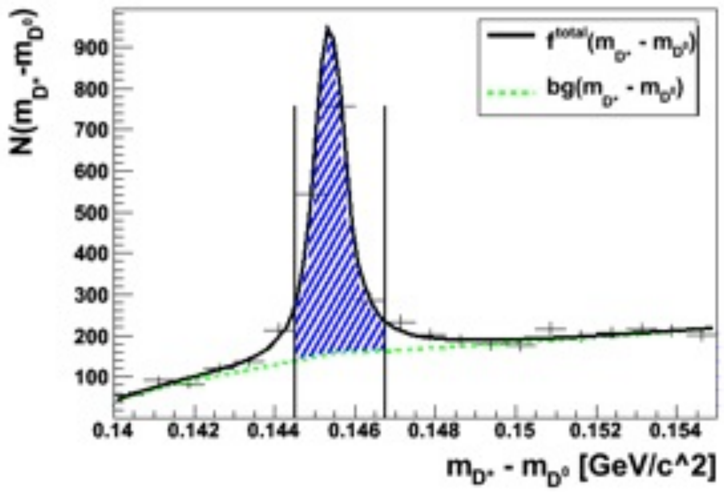
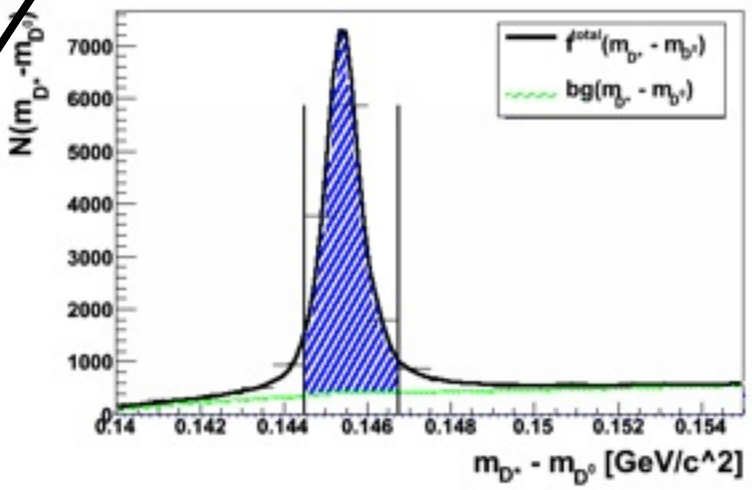


Negative hadron identified as π^-

K^- $\bar{p}$ μ^-

$m_D^* - m_D^0$



Negative hadron = K^-
(no PID likelihood used)

$P_{K^- \to \pi^-}$

$P_{K^- \to K^-}$

$P_{K^- \to \bar{p}}$

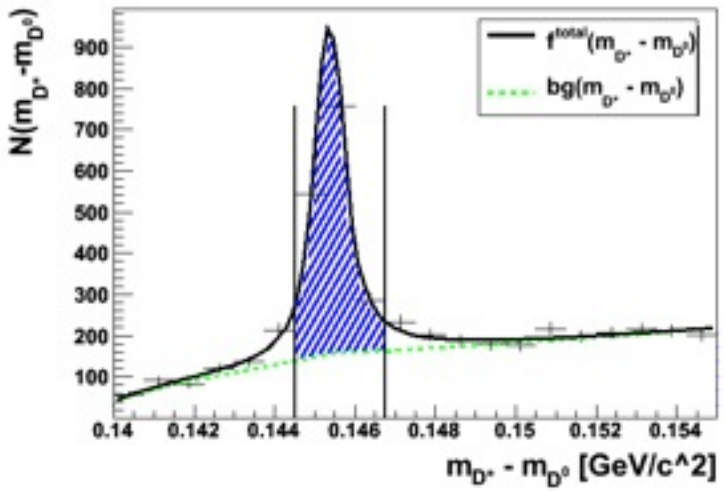
$P_{K^- \to \mu^-}$

How to determine the P_{ij}?

From data!

$$D^* \to D^0\,\pi^+_{slow}, \qquad D^0 \to K^-\,\pi^+_{fast}$$

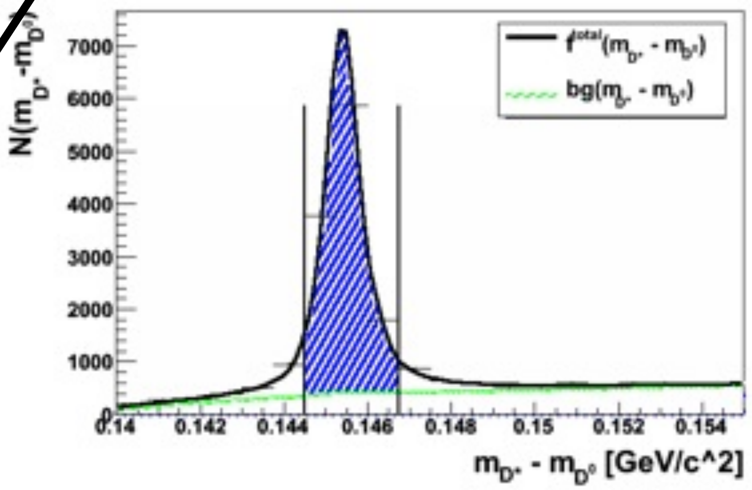
$m_D^* - m_D^0$



Negative hadron identified as π^-

$\pi^- \to K^-, \bar{p}, \mu^-, e^-$

$m_D^* - m_D^0$



Negative hadron = K^-
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$P_{K^- \to \pi^-}$

$P_{K^- \to K^-}$

$P_{K^- \to \bar{p}}$

$P_{K^- \to \mu^-}$

$P_{K^- \to e^-}$

Kinematic variables

$$z \equiv \frac{E_h}{E_p}$$

hadron energy fraction with respect to parton

$\boxed{z_1, z_2}$

p_T component of hadron momentum transverse to reference direction

1. $\phi_1+\phi_2$ method: the thrust axis $\boxed{p_{T1}, p_{T2}}$
2. ϕ_0 method: hadron 2 $\boxed{p_{T0}}$

q_T component of virtual photon momentum transverse to the h_1h_2 axis in the frame where h_1 and h_2 are back-to-back

z	0.2	0.25	0.3	0.42	1				
p_{T12}	0	0.13	0.3	0.5	3				
p_{T0}	0	0.13	0.25	0.4	0.5	0.6	0.75	1	3
q_T	0	0.5	1	1.25	1.5	1.75	2	2.25	2.5
$\sin^2\theta/(1+\cos^2\theta)$	0.4	0.45	0.5	0.6	0.7	0.8	0.9	0.97	1

Belle vs. Babar