Theory and Phenomenology of Generalized Parton Distributions

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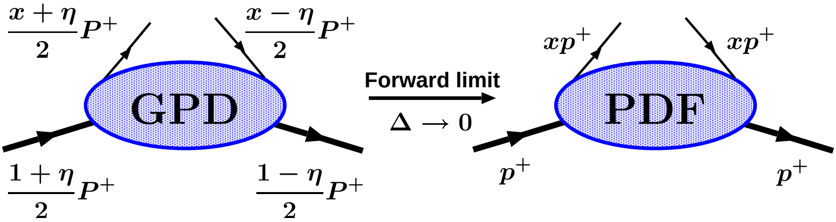
Outline

Attractiveness of GPDs (1/3)

- **1. Well-defined within the QCD**

  [Müller '92, et al. '94, Ji, Radyushkin '96]

$$F^q(x, \eta, t = \Delta^2) = \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle P_2 | \bar{q}(-z) \gamma^+ q(z) | P_1 \rangle \Big|_{z^+=0,\, \mathbf{z}_\perp = \mathbf{0}}$$



$$P = P_1 + P_2 \; ; \qquad \Delta = P_2 - P_1 \; ; \qquad \eta = -\frac{\Delta^+}{P^+} \;\; \text{(skewedness)}$$

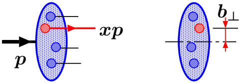
Attractiveness of GPDs (2/3)

- Decomposition into helicity conserving and non-conserving parts:

$$F^a = \frac{\bar{u}(P_2)\gamma^+ u(P_1)}{P^+} H^a + \frac{\bar{u}(P_2) i\sigma^{+\nu} u(P_1)\Delta_\nu}{2MP^+} E^a \qquad a = q, g$$

- **2. Close contact to 3D quark-gluon hadron structure**

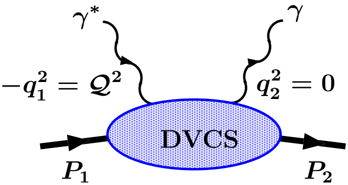
$$\frac{1}{2}\int_{-1}^{1} dx\, x\Big[H^q(x,\eta,0) + E^q(x,\eta,0)\Big] = J^q \qquad \text{[Ji '97]}$$



$$q(x, b_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{ib_\perp\cdot\Delta_\perp} H^q(x, 0, -\Delta_\perp^2) \qquad \text{[Burkardt '00]}$$

Attractiveness of GPDs (3/3)

- **3. Accessible to experiments**
- Deeply virtual Compton scattering (DVCS)



$$P = P_1 + P_2 \ , \qquad t = (P_2 - P_1)^2$$

$$q = (q_1 + q_2)/2$$

Generalized Bjorken limit:

$$-q^2 \simeq \mathcal{Q}^2/2 \to \infty$$

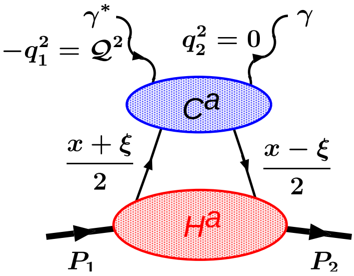
$$\xi = \frac{-q^2}{2P \cdot q} = \frac{x_B}{2 - x_B} \to \text{const}$$

- We work at leading order accuracy where cross-section can be expressed in terms of four Compton form factors (CFFs)

$$\mathcal{F} \in \{\mathcal{H}(\xi, t, \mathcal{Q}^2), \mathcal{E}(\xi, t, \mathcal{Q}^2), \tilde{\mathcal{H}}(\xi, t, \mathcal{Q}^2), \tilde{\mathcal{E}}(\xi, t, \mathcal{Q}^2)\}$$

Factorization of DVCS $\longrightarrow$ GPDs

- [Collins et al. '98]



$$P = P_1 + P_2\ , \qquad t = (P_2 - P_1)^2$$
$$q = (q_1 + q_2)/2$$

$$-q^2 \simeq \mathcal{Q}^2/2 \to \infty$$
$$\xi = \frac{-q^2}{2P\cdot q} \to \text{const}$$

- Compton form factor is a convolution:

$$^a\mathcal{H}(\xi, t, \mathcal{Q}^2) = \int \mathrm{d}x\ C^a(x, \xi, \mathcal{Q}^2/\mathcal{Q}_0^2)\ H^a(x, \eta = \xi, t, \mathcal{Q}_0^2)$$

a=NS,S(Σ,G)

Curse of dimensionality

- *It is relatively easy to find a coin lying somewhere on* 100 *meter string. It is very difficult to find it on a football field.*

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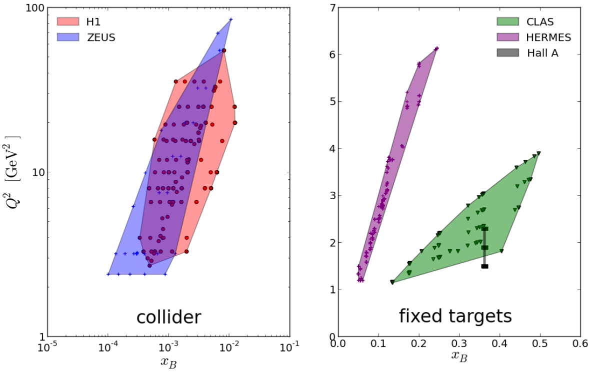
- *It is relatively easy to find a coin lying somewhere on* 100 *meter string. It is very difficult to find it on a football field.*
- When the dimensionality increases, the volume of the space increases so fast that the available data becomes sparse.
- Analogously, in contrast to *PDFs*(x), it is very difficult to perform truly model independent extraction of *GPDs*(x, ξ, t)
- Known GPD constraints don't decrease the dimensionality of the GPD domain space.
- As an intermediate step, one can attempt extraction of *CFFs*(x_B, t)
- (Dependence on additional variable, photon virtuality $\mathcal{Q}^2$, is in principle known — given by evolution equations.)

Fit types

	Fit type	Data used	pQCD order	Target
1.	Local fits	fixed target	LO	CFFs
2.	Global fits	collider/fixed target	((N)N)LO	CFFs/GPDs
3.	Neural nets	fixed target	LO	CFFs



Local fits to HERMES data

- Most complete set of asymmetries is measured in 12 bins:

$$d\sigma \propto |\mathcal{A}_{\rm BH}|^2 + |\mathcal{A}_{\rm DVCS}|^2(\mathcal{F}^2) + \mathcal{A}_{\rm Interference}(\mathcal{F})$$

$$A_C \equiv \frac{\mathrm{d}\sigma_{e^+} - \mathrm{d}\sigma_{e^-}}{\mathrm{d}\sigma_{e^+} + \mathrm{d}\sigma_{e^-}} = \frac{\mathcal{A}_{\rm Interference}(\mathcal{F})}{|\mathcal{A}_{\rm DVCS}|^2(\mathcal{F}^2) + |\mathcal{A}_{\rm BH}|^2}$$

$$A_{\rm C}^{\cos(1\phi)} \propto \left[F_1\,\mathfrak{Re}\,\mathcal{H} - \frac{t}{4M_p^2}F_2\,\mathfrak{Re}\,\mathcal{E} + \frac{x_{\rm B}}{2}(F_1+F_2)\,\mathfrak{Re}\,\widetilde{\mathcal{H}}\right]$$

- . . . and similarly for other observables $\longrightarrow$ almost linear relation between observables and CFFs
- (up to $|\mathcal{A}_{\rm DVCS}|^2$ contamination — taken into account)

Mapping

- Inverting these relations gives mapping from the set of observables ...

$$
\begin{array}{llll}
A_{\rm LU,I}^{\sin(1\phi)} & A_{\rm C}^{\cos(1\phi)} & A_{\rm C}^{\cos(0\phi)} & A_{\rm UL,+}^{\sin(1\phi)} \\
A_{\rm LL,+}^{\cos(1\phi)} & A_{\rm LL,+}^{\cos(0\phi)} & A_{\rm UT,I}^{\sin(\varphi)\cos(1\phi)} & A_{\rm UT,I}^{\cos(\varphi)\sin(1\phi)} \\
A_{\rm UT,DVCS}^{\sin(\varphi)\cos(0\phi)} & A_{\rm UT,I}^{\sin(\varphi)\cos(0\phi)} & A_{\rm LT,I}^{\sin(\varphi)\sin(1\phi)} & A_{\rm LT,I}^{\cos(\varphi)\cos(1\phi)} \\
A_{\rm LT,BH+DVCS}^{\cos(\varphi)\cos(0\phi)} & A_{\rm LT,I}^{\cos(\varphi)\cos(0\phi)} & &
\end{array}
$$

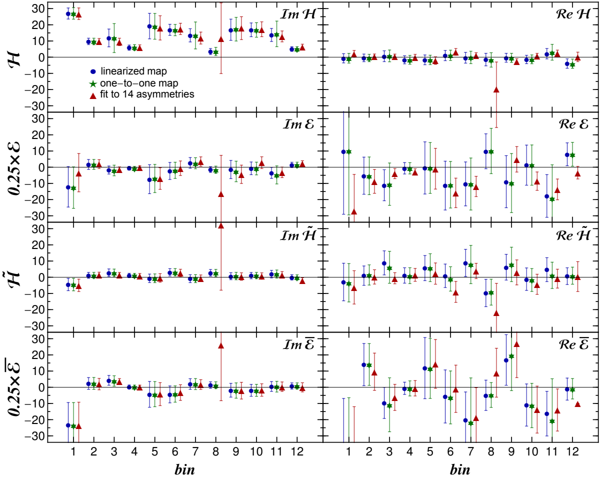
- ... to the set of eight real and imaginary parts of CFFs (sometimes called sub-CFFs or just CFFs) ...

$$
\boldsymbol{\mathcal{F}} = \left(\mathfrak{Im}\,\mathcal{H}, \mathfrak{Re}\,\mathcal{H}, \mathfrak{Im}\,\mathcal{E}, \mathfrak{Re}\,\mathcal{E}, \mathfrak{Im}\,\widetilde{\mathcal{H}}, \mathfrak{Re}\,\widetilde{\mathcal{H}}, \mathfrak{Im}\,\widetilde{\mathcal{E}}, \mathfrak{Re}\,\widetilde{\mathcal{E}}\right)
$$

- ... where error propagation is straightforward.

Mapping — results

- (Here compared also to standard local least-squares fit)



- Only $\mathfrak{Im}\,\mathcal{H}$, $\mathfrak{Re}\,\mathcal{H}$ and $\mathfrak{Im}\,\widetilde{\mathcal{H}}$ are reliably constrained.

Method of stepwise regression

- Constraints from data are too weak to constrain simultaneously all eight $\{\mathfrak{Im}\,\mathcal{F}, \mathfrak{Re}\,\mathcal{F}\}$ CFFs
- Let us take smaller number of CFFs, choosing only those which are reliably extracted. Stepwise regression algorithm:
  - Starting from zero, add, one by one, CFFs that best improve description of data until improvement becomes statistically insignificant.

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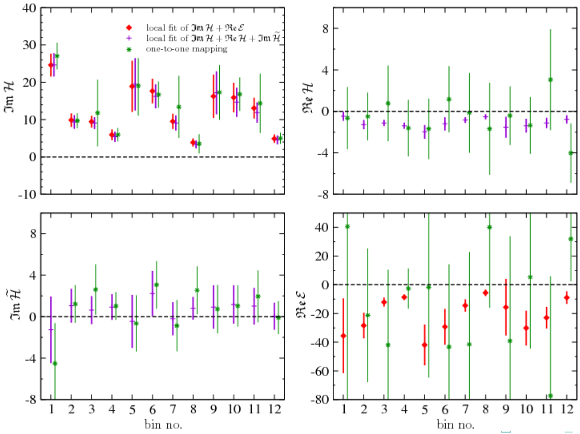
  - Starting from zero, add, one by one, CFFs that best improve description of data until improvement becomes statistically insignificant.
- Algorithm stops after just 2 CFFs, and there are two equally good pairs of CFFs:
  1. $(\mathfrak{Im}\,\mathcal{H},\ \mathfrak{Re}\,\mathcal{H})$ with $\chi^2/n_{\rm d.o.f.} = 102.3/120$, and
  2. $(\mathfrak{Im}\,\mathcal{H},\ \mathfrak{Re}\,\mathcal{E})$ with $\chi^2/n_{\rm d.o.f.} = 103.0/120$.

Stepwise regression — results

- Scenario 1: Fit of $\mathfrak{Im}\,\mathcal{H}$, $\mathfrak{Re}\,\mathcal{H}$ and $\mathfrak{Im}\,\widetilde{\mathcal{H}}$. $\chi^2/n_{\rm d.o.f.} = 148.8/144$. (In good agreement with [Guidal '10])
- Scenario 2: Fit of $\mathfrak{Im}\,\mathcal{H}$ and $\mathfrak{Re}\,\mathcal{E}$. $\chi^2/n_{\rm d.o.f.} = 134.2/144$.



Modelling GPDs in moment space

- Instead of considering momentum fraction dependence $H(x, \ldots)$
- ...it is convenient to make a transform into complementary space of conformal moments j:

$$H_j^q(\eta, \ldots) \equiv \frac{\Gamma(3/2)\Gamma(j+1)}{2^{j+1}\Gamma(j+3/2)} \int_{-1}^{1} \mathrm{d}x \; \eta^j \, C_j^{3/2}(x/\eta) \, H^q(x, \eta, \ldots)$$

- They are analogous to Mellin moments in DIS: $x^j \to C_j^{3/2}(x)$
- $C_j^{3/2}(x)$ — Gegenbauer polynomials

Advantages of conformal moments

1. The evolution equations are most simple: There is **no mixing** among moments at LO, and in special ($\overline{CS}$) scheme not even at NLO
2. Powerful analytic methods of **complex j** plane are available (similar to complex angular momentum of Regge theory)
3. Stable and fast **computer code** for evolution and fitting
4. Moments are equal to matrix elements of **local** operators and are thus directly accessible on the **lattice**

l-PW model — only leading SO(3) partial wave

- $$\mathbf{H}_j(\xi, t, \mu_0^2) = \begin{pmatrix} N'_\Sigma F_\Sigma(t)\, \mathrm{B}\big(1+j-\alpha_\Sigma(0), 8\big) \\ N'_G F_G(t)\, \mathrm{B}\big(1+j-\alpha_G(0), 6\big) \end{pmatrix}$$

$$\alpha_a(t) = \alpha_a(0) + 0.15t \qquad F_a(t) = \frac{j+1-\alpha(0)}{j+1-\alpha(t)} \left(1 - \frac{t}{M_0^{a2}}\right)^{-p_a}$$

...corresponding in forward case to PDFs of form

$$\Sigma(x) = N'_\Sigma\, x^{-\alpha_\Sigma(0)} (1-x)^7 \; ; \qquad G(x) = N'_G\, x^{-\alpha_G(0)} (1-x)^5$$

- $M_0^G = \sqrt{0.7}\,\mathrm{GeV}$ is fixed by the J/ψ production data
- Free parameter (for DVCS): M_0^Σ

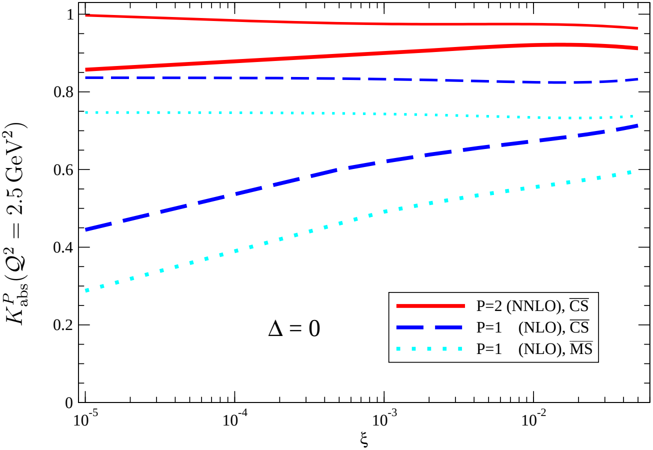
For small ξ (small x_{Bj}) valence quarks are less important $\Rightarrow \Sigma \approx$ sea

Inclusion of subleading PW — flexible models

$$\mathbf{H}_j(\eta, t) = \underbrace{\begin{pmatrix} N'_{\rm sea} F_{\rm sea}(t)\, \mathrm{B}\big(1+j-\alpha_{\rm sea}(0), 8\big) \\ N'_{\rm G} F_{\rm G}(t)\, \mathrm{B}\big(1+j-\alpha_{\rm G}(0), 6\big) \end{pmatrix}}_{\text{skewness } r\approx 1.6 \text{ (too large)}} + \underbrace{\begin{pmatrix} s_{\rm sea} \\ s_{\rm G} \end{pmatrix} \begin{pmatrix} \text{subleading partial waves, } \eta\text{-dependence!} \end{pmatrix}}_{\substack{<0 \\ \text{negative skewness}}}$$

- nl-PW — addition of second PW needed for good fits
- two new parameters: $s^{(2)}_{\rm sea}$ and $s^{(2)}_{\rm G}$
- nnl-PW — addition of third PW (doesn't improve fits much but makes possible positive gluon GPDs at small $\mathcal{Q}^2$) — $s^{(4)}_{\rm sea}$ and $s^{(4)}_{\rm G}$

(N)NLO corrections

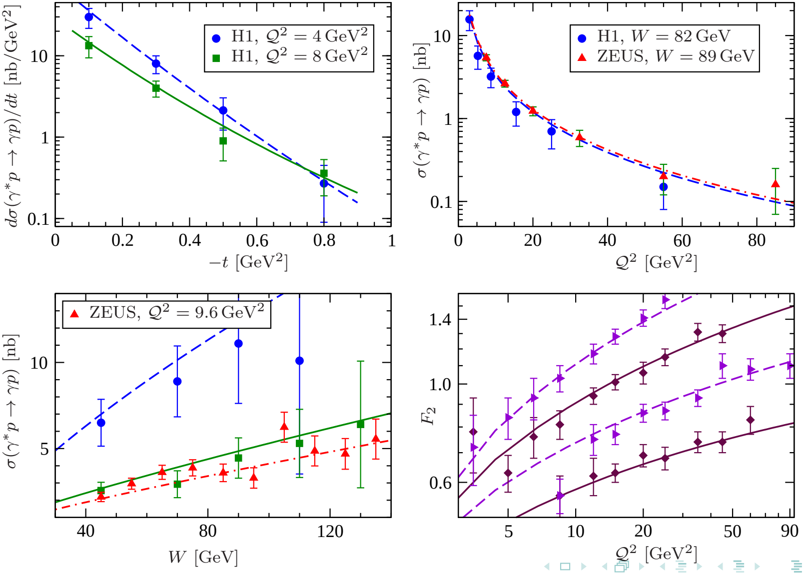


Thick lines:
"hard" gluon
$N_G = 0.4$
$\alpha_G(0){=}\alpha_\Sigma(0) + 0.05$

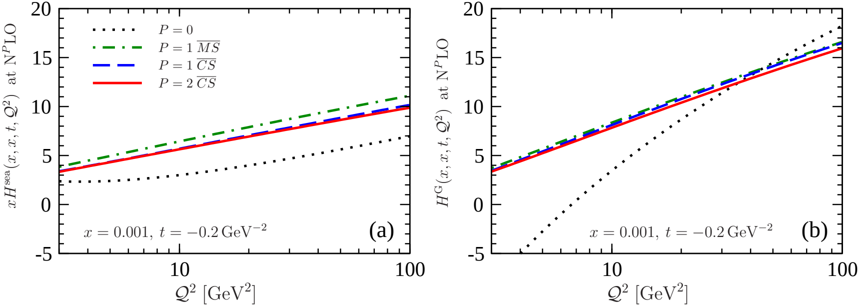
Thin lines:
"soft" gluon
$N_G = 0.3$
$\alpha_G(0){=}\alpha_\Sigma(0) - 0.02$

$$K^P_{\rm abs} \equiv \left| \frac{\mathcal{H}^{(P)}}{\mathcal{H}^{(P-1)}} \right|$$

Example of fit result



Resulting small-x $H(x, x, t)$



- P=0: LO; P=1: NLO; P=2: NNLO
- The whole procedure is extended to meson production [Müller, Lautenschlager, Passek-Kumerički, Schäfer '13]

Extending global analysis to fixed target data

- **Hybrid models** at LO (1st just for *unpolarized* target)
- Sea quarks and gluons modelled like just described (conformal moments + SO(3) partial wave expansion + $\mathcal{Q}^2$ evolution).
- Valence quarks model (ignoring $\mathcal{Q}^2$ evolution):

$$\mathfrak{Im}\,\mathcal{H}(\xi,t) = \pi\left[\frac{4}{9}H^{u_{\rm val}}(\xi,\xi,t) + \frac{1}{9}H^{d_{\rm val}}(\xi,\xi,t) + \frac{2}{9}H^{\rm sea}(\xi,\xi,t)\right]$$

$$H(x,x,t) = n\,r\,2^{\alpha}\left(\frac{2x}{1+x}\right)^{-\alpha(t)}\left(\frac{1-x}{1+x}\right)^{b}\frac{1}{\left(1-\frac{1-x}{1+x}\frac{t}{M^2}\right)^{p}}\,.$$

- Fixed: n (from PDFs), $\alpha(t)$ (eff. Regge), p (counting rules)

$$\alpha^{\rm val}(t) = 0.43 + 0.85\,t/{\rm GeV}^2 \quad (\rho,\,\omega)$$

- $\mathfrak{Re}\,\mathcal{H}$ determined by dispersion relations

$$\mathfrak{Re}\,\mathcal{H}(\xi,t) = \frac{1}{\pi}\mathrm{PV}\int_0^1 d\xi' \left(\frac{1}{\xi-\xi'} - \frac{1}{\xi+\xi'}\right) \mathfrak{Im}\,\mathcal{H}(\xi',t) - \frac{C}{\left(1-\frac{t}{M_C{}^2}\right)^2}$$

- Typical set of free parameters:

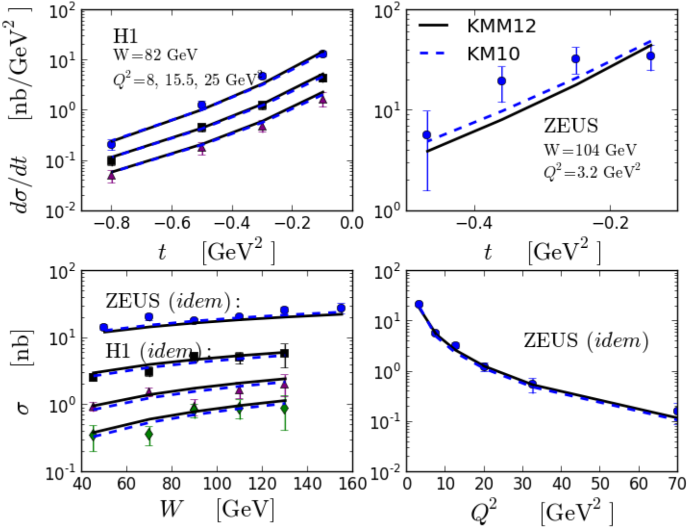
$M_0^{\rm sea}$, $s_{\rm sea}^{(2,4)}$, $s_G^{(2,4)}$	sea* quarks and gluons H
$r^{\rm val}$, $M^{\rm val}$, $b^{\rm val}$	valence H
$\tilde{r}^{\rm val}$, $\tilde{M}^{\rm val}$, $\tilde{b}^{\rm val}$	valence $\widetilde{H}$
C, M_C	subtraction constant (H, E)
r_π, M_π	"pion pole" $\widetilde{E}$

- Global fit to 175 data points turns out fine:

KM10 model: $\chi^2/d.o.f. = 135.9/160$.

* $s_{\rm sea,G}$ = strengths of subleading partial waves. LO evolution is included.

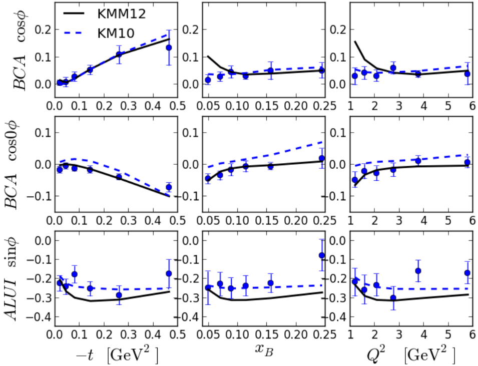
H1 (2007), ZEUS (2008)



HERMES (2008)

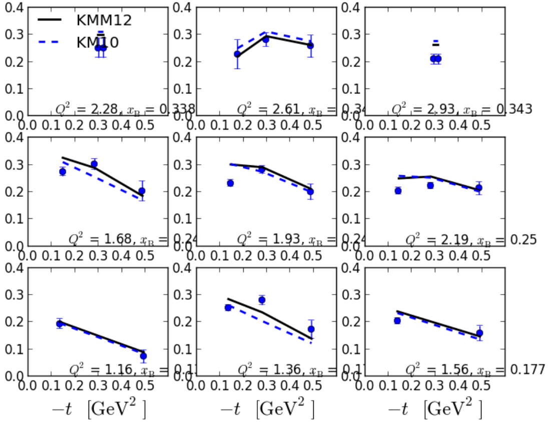
$$BCA \equiv \frac{d\sigma_{e^+} - d\sigma_{e^-}}{d\sigma_{e^+} + d\sigma_{e^-}} \sim A_C^{\cos 0\phi} + A_C^{\cos 1\phi} \cos\phi \sim \mathfrak{Re}\,\mathcal{H}$$

$$BSA \equiv \frac{d\sigma_{e\uparrow} - d\sigma_{e\downarrow}}{d\sigma_{e\uparrow} + d\sigma_{e\downarrow}} \sim A_{LU}^{\sin 1\phi} \sin\phi \sim \mathfrak{Im}\,\mathcal{H}$$



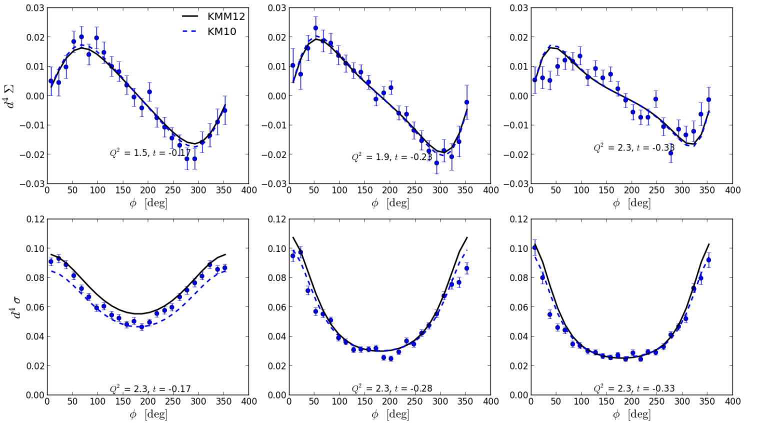
CLAS (2007)

- BSA. (Only data with $|t| \leq 0.3\,\mathrm{GeV}^2$ used for fits.)



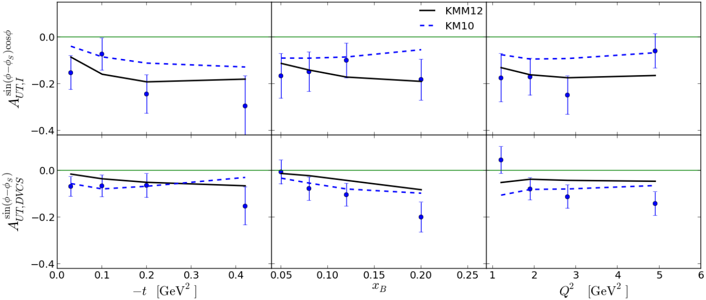
Hall A (2006)

- Fit to unpolarized cross section $d\sigma/(dx_B dQ^2 dt d\phi) \quad \sim \Re\mathfrak{e}\,\mathcal{H}$
- KM10 fit needs unusually large $\Re\mathfrak{e}\,\tilde{\mathcal{H}}$.

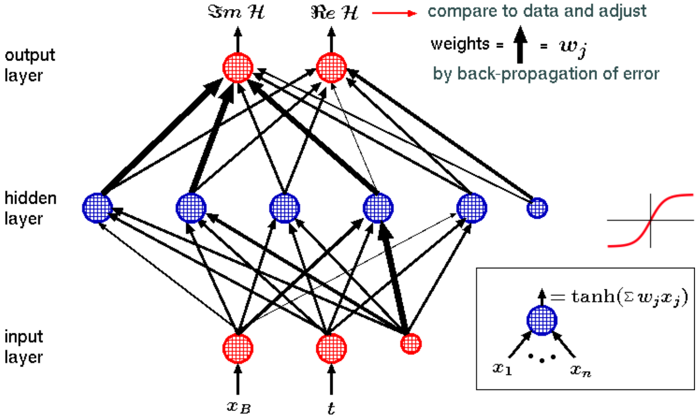


Including data with polarized target

- KMM12: $\chi^2/n_{\rm d.o.f.} = 124.1/80$, strictly speaking not a good fit, but best what we have at the moment



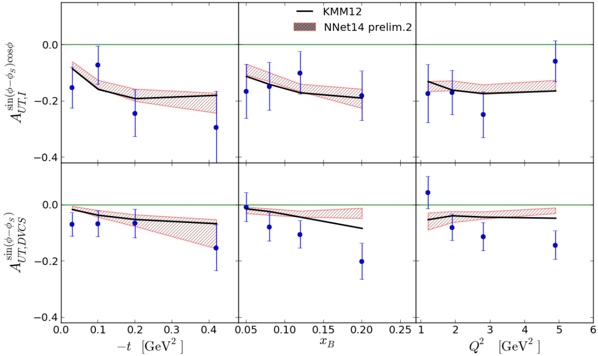
Multilayer perceptron



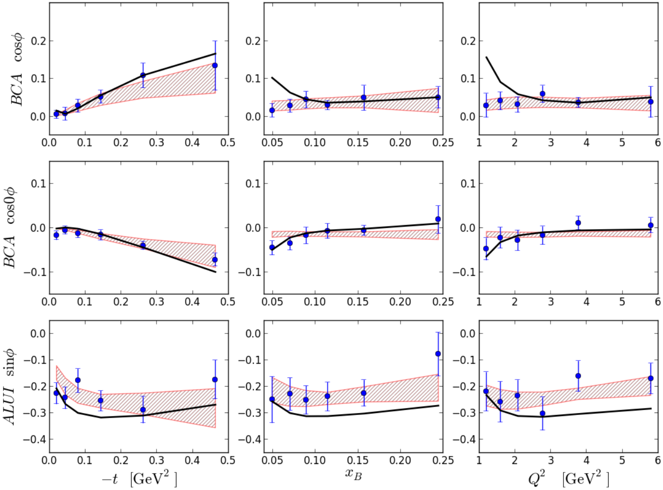
- Essentially a least-squares fit of a complicated many-parameter function. $f(x) = \tanh(\sum w_i \tanh(\sum w_j \cdots))$
  $\Rightarrow$ no theory bias

Preliminary neural Net HERMES fit

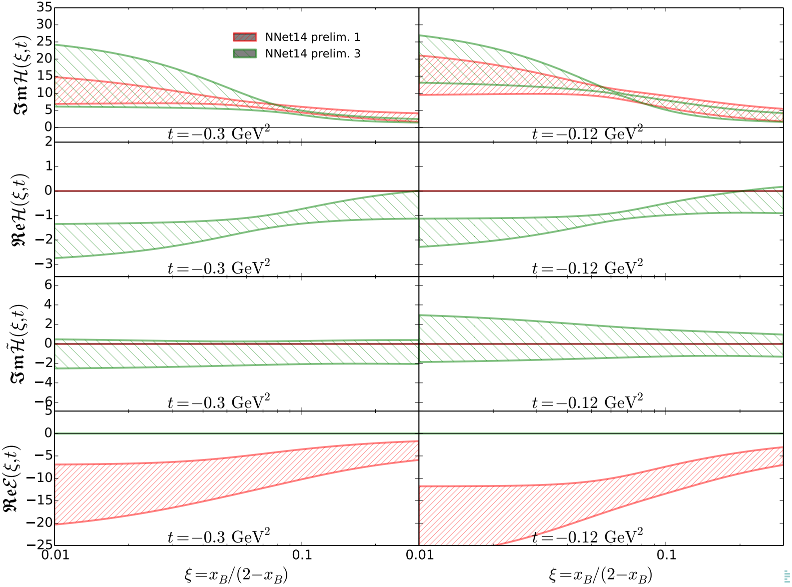
- Fit to all HERMES DVCS data with two types of neural nets
  - (x_B, t) – (7 neurons) – $(\mathfrak{Im}\,\mathcal{H},\ \mathfrak{Re}\,\mathcal{H},\ \mathfrak{Im}\,\tilde{\mathcal{H}})$: $\chi^2/n_{\rm pts} = 135.4/144$
  - (x_B, t) – (7 neurons) – $(\mathfrak{Im}\,\mathcal{H},\ \mathfrak{Re}\,\mathcal{E})$: $\chi^2/n_{\rm pts} = 120.2/144$



Neural Net HERMES fit - BSA/BCA

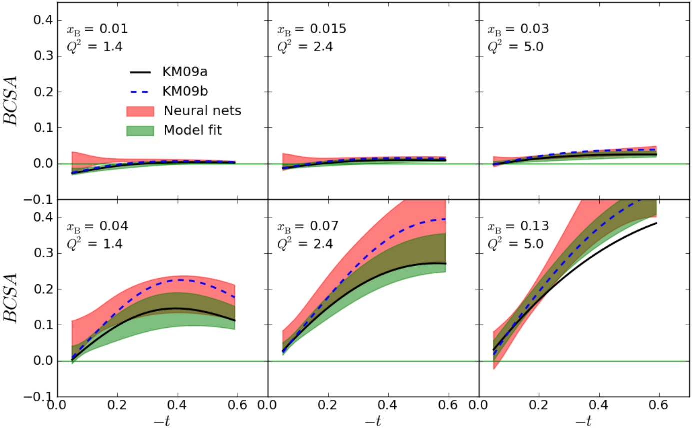


Neural Net HERMES fit - CFFs

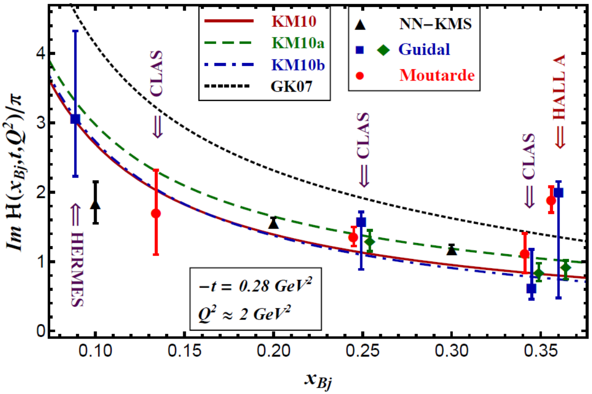


Prediction for COMPASS-II BCSA

$$BCSA = \frac{d\sigma_{\mu^{\downarrow +}} - d\sigma_{\mu^{\uparrow -}}}{d\sigma_{\mu^{\downarrow +}} + d\sigma_{\mu^{\uparrow -}}} \qquad (E_\mu = 160\,\mathrm{GeV})$$



Comparison to others



[Guidal '08, Guidal and Moutarde '09], seven CFF fit (blue squares), [Guidal '10] $\mathcal{H}$, $\tilde{\mathcal{H}}$ CFF fit (green diamonds), [Moutarde '09] H GPD fit (red circles). Also reasonable agreement with [Goloskokov and Kroll].

New directions

- Improved global LO fits with all unpolarized and polarized proton data.
- Adding deeply virtual meson production and going NLO [Müller, Lautenschlager, Schäfer '13]
- Including higher twists
- Global neural network fits

Function fitting by a neural net

- Theorem: Given enough neurons, any smooth function $f(x_1, x_2, \cdots)$ can be approximated to any desired accuracy. Single hidden layer is sufficient (but not always most efficient).

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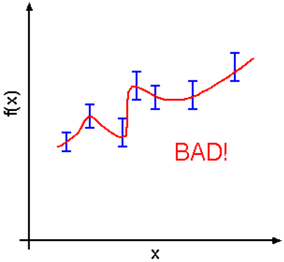

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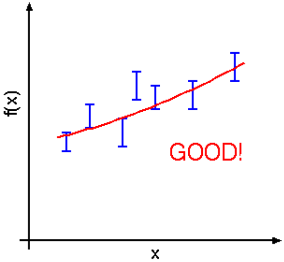

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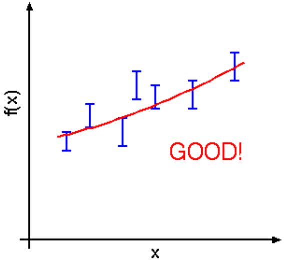
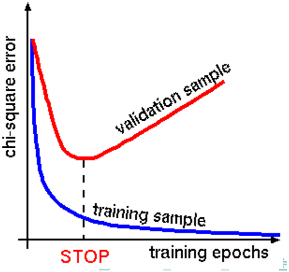
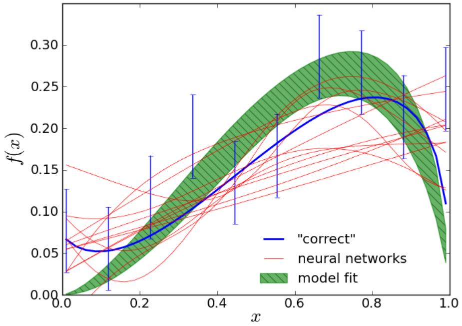
- With simple training of neural nets to data there is a danger of overfitting (a.k.a. overtraining)
- Solution: Divide data (randomly) into two sets: *training* sample and *validation sample*. Stop training when error of validation sample starts increasing.





Toy fitting example

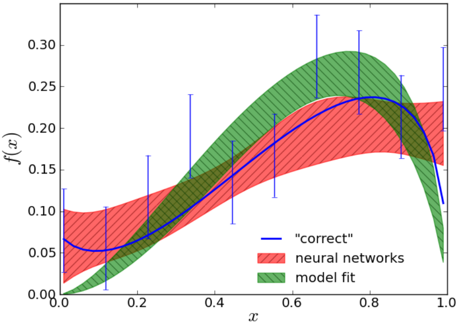
- Fit to data generated according to function (which we pretend not to know).



- Fit with
  1. Standard Minuit fit with ansatz $f(x) = x^a(1-x)^b$
  2. Neural network fit

Toy fitting example

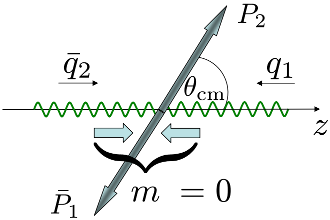
- Fit to data generated according to function (which we pretend not to know).



- Fit with
  1. Standard Minuit fit with ansatz $f(x) = x^a(1-x)^b$
  2. Neural network fit

Modelling conformal moments of GPDs (I)

- How to model η-dependence of GPD's $H_j(\eta, t)$?
- Idea: consider crossed t-channel process $\gamma^*\gamma \to p\bar{p}$



When crossing back to DVCS channel we have:

$$\cos\theta_{\rm cm} \to -\frac{1}{\eta}$$

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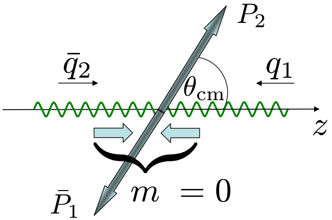


When crossing back to DVCS channel we have:

$$\cos\theta_{\rm cm} \to -\frac{1}{\eta}$$

- ... and dependence on $\theta_{\rm cm}$ in t-channel is given by SO(3) partial wave decomposition of $\gamma^*\gamma$ scattering

$$\mathcal{H}(\eta, \ldots) = \mathcal{H}^{(t)}(\cos\theta_{\rm cm} = -\frac{1}{\eta}, \ldots) = \sum_J (2J+1) f_J(\ldots) d^J_{0,\nu}(\cos\theta)$$

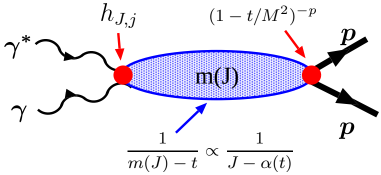
- $d^J_{0,\nu}$ — Wigner SO(3) functions (Legendre, Gegenbauer,...)
  $\nu = 0, \pm 1$ — depending on hadron helicities

Modelling conformal moments of GPDs (II)

- OPE expansion of both $\mathcal{H}$ and $\mathcal{H}^{(t)}$ leads to

$$H_j(\eta, t) = \eta^{j+1}\, H_j^{(t)}(\cos\theta = -\frac{1}{\eta}, s^{(t)} = t)$$

- and t-channel partial waves are modelled as:

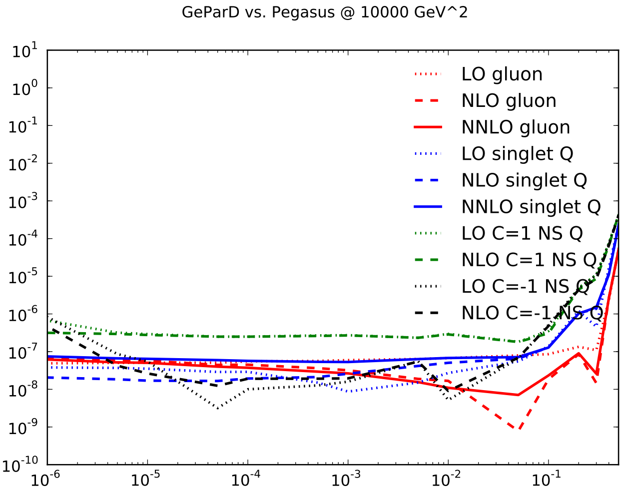


$$H_j(\eta, t) = \sum_J^{j+1} h_{J,j} \frac{1}{J - \alpha(t)} \frac{1}{\left(1 - \frac{t}{M^2(J)}\right)^p} \eta^{j+1-J} d^J_{0,\nu}$$

- Similar to "dual" parametrization [Polyakov, Shuvaev '02]

Checking the evolution code

- We checked agreement in the forward limit with QCD PEGASUS code for PDF evolution [Vogt '04]



Dispersion-relation access to GPDs at LO

[Teryaev '05; K.K., Müller and Passek-K. '07, '08; Diehl and Ivanov '07]

- LO perturbative prediction is "handbag" amplitude

$$\mathcal{H}(\xi, t, \mathcal{Q}^2) \stackrel{\text{LO}}{=} \int_{-1}^{1} dx \, \left(\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right) H(x, \xi, t, \mathcal{Q}^2)$$

- giving access to GPD on the "cross-over" line $\eta = x$

$$\frac{1}{\pi} \Im\mathfrak{m}\, \mathcal{H}(\xi = x, t, \mathcal{Q}^2) \stackrel{\text{LO}}{=} H(x, x, t, \mathcal{Q}^2) - H(-x, x, t, \mathcal{Q}^2)$$

- while dispersion relation connects it to $\Re\mathfrak{e}\, \mathcal{H}$

$$\Re\mathfrak{e}\, \mathcal{H}(\xi, t, \mathcal{Q}^2) =$$

$$\frac{1}{\pi} \mathrm{PV} \int_{0}^{1} d\xi' \left(\frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'} \right) \Im\mathfrak{m}\, \mathcal{H}(\xi', t, \mathcal{Q}^2) + \mathcal{C}_{\mathcal{H}}(t, \mathcal{Q}^2)$$

Fit results - LO

- For consistency, we don't take standard PDFs, but fit GPDs to DIS data. This determines $N_{\rm sea}$, N_G, $\alpha_{\rm sea}(0)$ and $\alpha_G(0)$, leaving only $M_0^{\rm sea}$, $s_{\rm sea}$ and $s_{\rm G}$ for DVCS data

- χ^2 values:

model	α_s	χ^2/d.o.f. DIS	χ^2/d.o.f. DVCS	χ^2_t/n.o.p.	χ^2_W/n.o.p.	$\chi^2_{Q^2}$/n.o.p.
l, dipole	LO	49.7/82	**280./100**	**181./56**	**63.6/29**	**36.2/16**
l, exp.	LO	49.7/82	**316./100**	**192./56**	**79./29**	**44.9/16**
nl, dipole	LO	49.7/82	95.9/98	53.2/56	27./29	15.8/16
nl, exp.	LO	49.7/82	97.9/98	49.1/56	31.2/29	17.7/16
Σ, dipole	LO	49.7/82	101./98	57.7/56	27.4/29	16./16
Σ, exp.	LO	49.7/82	102./98	51./56	32.3/29	18.6/16
l, dipole	LO	**321./182**		**189./56**	**51.1/29**	**27.9/16**

- Parameter values:

model	α_s	$N^{\rm sea}$	$\alpha^{\rm sea}(0)$	$(M^{\rm sea})^2$ [GeV^2]	$s^{\rm sea}$	$\alpha^{\rm G}(0)$	$s^{\rm G}$	$B^{\rm sea}$ [GeV^{-2}]	$b^{\rm eff}$ [GeV^{-2}]	BCA
l, dipole	LO	**0.152**	**1.158**	**0.062**		**1.247**		**33.**	**5.7**	**0.19**
l, exp.	LO	**0.152**	**1.158**			**1.247**		**29.**	**5.1**	**0.23**
nl, dipole	LO	0.152	1.158	0.48	-0.15	1.247	-0.81	4.8	5.5	0.13
nl, exp.	LO	0.152	1.158		-0.18	1.247	-0.86	3.1	5.8	0.14
Σ, dipole	LO	0.152	1.158	0.42	-11.	1.247	-32.	5.4	5.5	0.14
Σ, exp.	LO	0.152	1.158		-13.	1.247	-34.	3.1	5.8	0.15

(boldface numbers = bad fits)

Fit results - NLO

- χ^2 values:

model	α_s	χ^2/d.o.f DIS	χ^2/d.o.f DVCS	χ^2_t/n.o.p	χ^2_W/n.o.p	$\chi^2_{Q^2}$/n.o.p
l	NLO($\overline{\rm MS}$)	71.6/82	**148./100**	**77.6/56**	**36.8/29**	**33.9/16**
l	NLO($\overline{\rm CS}$)	71.6/82	105./100	62.9/56	25.1/29	17./16
nl	NLO($\overline{\rm MS}$)	71.6/82	102./98	60.2/56	23.9/29	17.5/16
nl	NLO($\overline{\rm CS}$)	71.6/82	104./98	61.4/56	24.9/29	18.1/16
Σ	NLO($\overline{\rm MS}$)	71.6/82	101./98	60./56	23.9/29	17.5/16
Σ	NLO($\overline{\rm CS}$)	71.6/82	104./98	61.5/56	24.9/29	18.1/16

- Parameter values:

model	α_s	$N^{\rm sea}$	$\alpha^{\rm sea}(0)$	$(M^{\rm sea})^2$	$s^{\rm sea}$	$\alpha^{\rm G}(0)$	$s^{\rm G}$	$B^{\rm sea}$	$b^{\rm eff}$	BCA
l	NLO($\overline{\rm MS}$)	**0.168**	**1.128**	**0.71**		**1.099**		**3.5**	**5.0**	**0.10**
l	NLO($\overline{\rm CS}$)	0.168	1.128	0.57		1.099		4.2	5.7	0.09
nl	NLO($\overline{\rm MS}$)	0.168	1.128	0.59	0.04	1.099	0.02	4.0	5.6	0.09
nl	NLO($\overline{\rm CS}$)	0.168	1.128	0.58	-0.01	1.099	-0.01	4.1	5.6	0.09
Σ	NLO($\overline{\rm MS}$)	0.168	1.128	0.60	3.10	1.099	1.10	4.0	5.7	0.09
Σ	NLO($\overline{\rm CS}$)	0.168	1.128	0.58	-0.42	1.099	-0.58	4.1	5.6	0.09

(boldface numbers = bad fits)

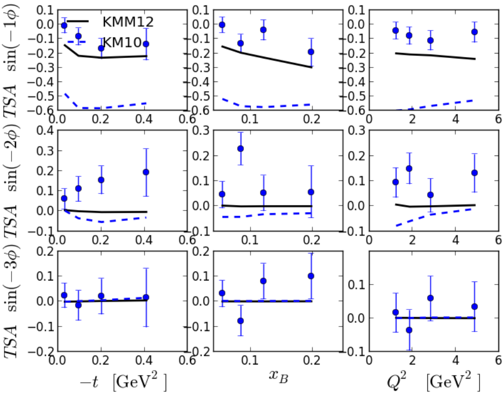
- $s^{\rm sea,G}$ small $\longrightarrow$ skewness ratio $r \sim 1.5$

Parameter values

	KMM12	KM10
Mv	0.951 +- 0.282	4.00 +- 3.33
rv	1.121 +- 0.099	0.62 +- 0.06
bv	0.400 +- 0.000	0.40 +- 0.67
C	1.003 +- 0.565	8.78 +- 0.98
MC	2.080 +- 3.754	0.97 +- 0.11
tMv	3.523 +- 13.17	0.88 +- 0.24
trv	1.302 +- 0.206	7.76 +- 1.39
tbv	0.400 +- 0.001	2.05 +- 0.40
rpi	3.837 +- 0.141	3.54 +- 1.77
Mpi	4.000 +- 0.036	0.73 +- 0.37
M02S	0.462 +- 0.032	0.51 +- 0.02
SECS	0.313 +- 0.039	0.28 +- 0.02
THIS	-0.138 +- 0.012	-0.13 +- 0.01
SECG	-2.771 +- 0.228	-2.79 +- 0.12
THIG	0.945 +- 0.107	0.90 +- 0.05

Polarized target (II)

- Surprisingly large $\sin(2\phi)$ harmonic of A_{UL} cannot be described within this leading twist framework



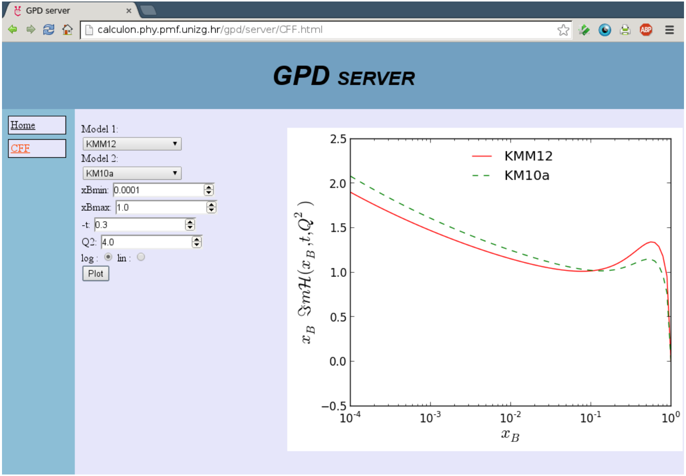
KM models are available at WWW

- Google for "gpd page" — get binary code for cross sections

```
% xs.exe

  xs.exe  ModelID  Charge  Polarization  Ee  Ep  xB  Q2  t  phi

returns cross section (in nb) for scattering of lepton of energy Ee
on unpolarized proton of energy Ep. Charge=-1 is for electron.

ModelID is one of
   0 debug, always returns 42,
   1 KM09a - arXiv:0904.0458 fit without Hall A,
   2 KM09b - arXiv:0904.0458 fit with Hall A,
   3 KM10  - preliminary hybrid fit with LO sea evolution, from Trento presentation,
   4 KM10a - preliminary hybrid fit with LO sea evolution, without Hall A data
   5 KM10b - preliminary hybrid fit with LO sea evolution, with Hall A data

xB Q2 t phi  -- usual kinematics (phi is in Trento convention)

% xs.exe 1 -1 1 27.6 0.938  0.111 3. -0.3 0

  0.18584386497251
```

GPD page and server

- Durham-like CFF/GPD server page



- Do we need "Les Houches Accord" CFF/GPD interface?