Spin superconductor and electric dipole superconductor

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Part I:

Spin current and spin superconductor

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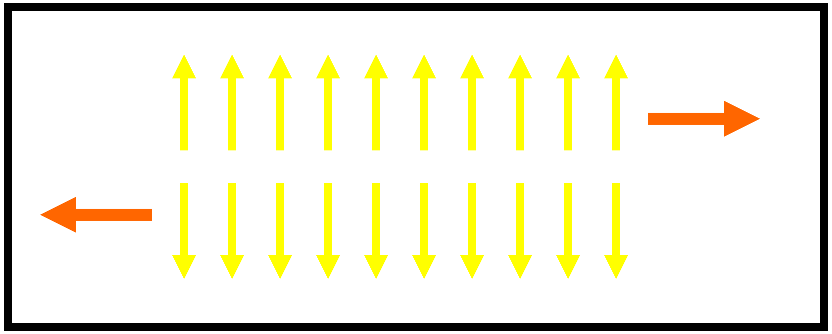
Outline

- **Basic Properties of Spin Current**
- **Spin-Superconductor (SSC) in a Ferromagnetic Graphene**
- **Results and Discussion**
- **Conclusion**

Spin Current

Spin-up electrons move in one way and same number of spin-down electrons move in the opposite way, thus charge current vanishes, but spin current is doubled.

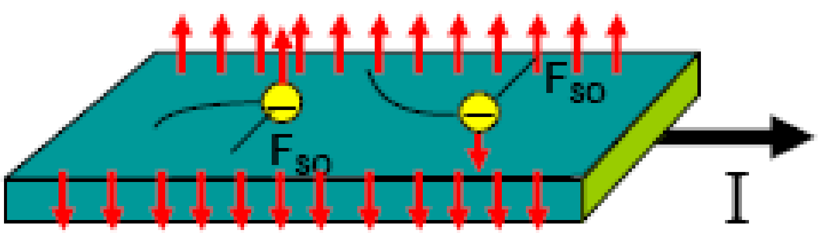


$$I_e = e(I_\uparrow + I_\downarrow) = 0$$

$$I_S = \frac{\hbar}{2}(I_\uparrow - I_\downarrow) \neq 0$$

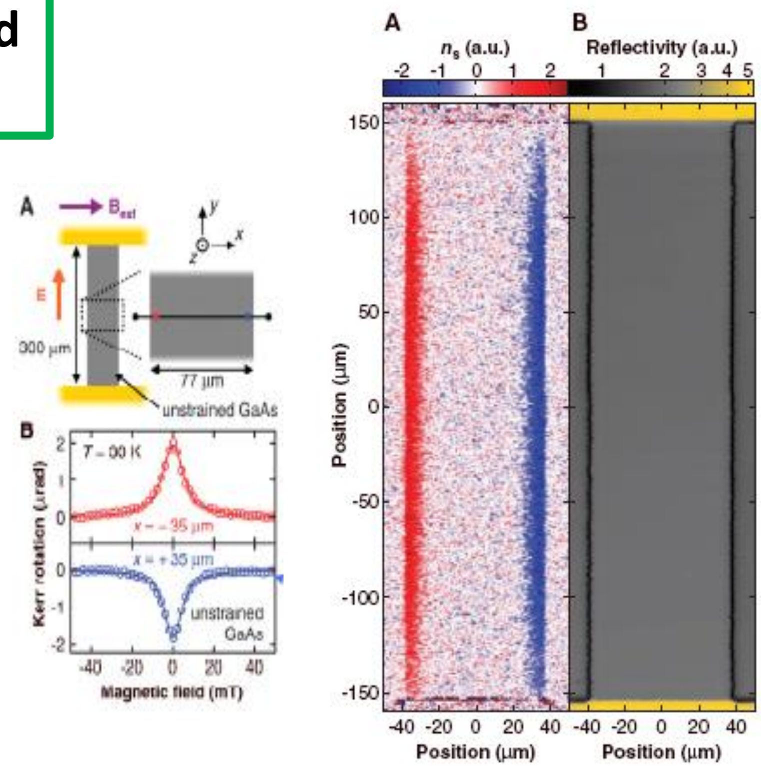
The spin Hall effect

Carriers with opposite spins are separated by spin-orbit coupling



$$J_j^{\,i} = \sigma_{spin}\varepsilon_{ijk}E_k$$

Murakmi et al, Science 31, 1248 (2003)



Y.K.Kato et al., Science, 306, 1910 (2004）

Spin current density and the spin continuity equation

1. charge continuity equation: $\frac{d}{dt}\rho(\mathbf{r},t) + \nabla \bullet \vec{j}^{e}(\mathbf{r},t) = 0.$

   definition of charge current density: $\vec{j}^{e}(\mathbf{r},t) = Re\Psi^{\dagger}(\mathbf{r},t)\vec{v}\Psi(\mathbf{r},t)$

   $\rho(\mathbf{r},t) = \Psi^{\dagger}\Psi$

2. Is there a spin continuity equation?
   How to define the spin current density?

   Similar as for the charge continuity equation,
   we calculate : $\frac{d}{dt}\vec{s}(\mathbf{r},t)$, where $\vec{s}(\mathbf{r},t) = \Psi^{\dagger}(\mathbf{r},t)\hat{\vec{s}}\Psi(\mathbf{r},t)$ is the local spin density. Then:

$$\begin{aligned}\frac{d}{dt}\vec{s}(\mathbf{r},t) &= \frac{\hbar}{2}\left\{\left[\frac{d}{dt}\Psi^{\dagger}\right]\hat{\vec{\sigma}}\Psi + \Psi^{\dagger}\hat{\vec{\sigma}}\frac{d}{dt}\Psi\right\} \\ &= \frac{1}{2i}[\Psi^{\dagger}\hat{\vec{\sigma}}H\Psi - (H\Psi)^{\dagger}\hat{\vec{\sigma}}\Psi]\end{aligned}$$

Take: $H = \frac{\vec{p}^2}{2m} + V(\mathbf{r}) + \hat{\vec{\sigma}} \bullet \vec{B} + \frac{\alpha}{\hbar} \hat{z} \bullet (\hat{\vec{\sigma}} \times \vec{p})$

We have:

$$\begin{aligned} \frac{d}{dt}\vec{s} &= -\frac{\hbar}{2}\nabla \bullet Re\Psi^{\dagger}[\frac{\vec{p}}{m} + \frac{\alpha}{\hbar}(\hat{z} \times \hat{\vec{\sigma}})]\hat{\vec{\sigma}}\Psi \\ &= Re\Psi^{\dagger}[\vec{B} + \frac{\alpha}{\hbar}\vec{p} \times \hat{z}] \times \hat{\vec{\sigma}}\Psi \end{aligned}$$

To introduce the tensor $\mathbf{j}_s(\mathbf{r}, t)$ and the vector $\vec{j}_\omega(\mathbf{r}, t)$:

$$\begin{aligned} \mathbf{j}_s(\mathbf{r}, t) &= Re\{\Psi^{\dagger}[\frac{\vec{p}}{m} + \frac{\alpha}{\hbar}(\hat{z} \times \hat{\vec{\sigma}})]\hat{\vec{s}}\Psi\} \\ \vec{j}_\omega(\mathbf{r}, t) &= Re\{\Psi^{\dagger}\frac{2}{\hbar}[\vec{B} + \frac{\alpha}{\hbar}(\vec{p} \times \hat{z})] \times \hat{\vec{s}}\Psi\} \end{aligned}$$

We have:

$$\frac{d}{dt}\vec{s}(\mathbf{r}, t) = -\nabla \bullet \mathbf{j}_s(\mathbf{r}, t) + \vec{j}_\omega(\mathbf{r}, t)$$

This is the spin continuity equation

Notice, due to

$$\hat{\vec{v}} = \frac{d}{dt}\mathbf{r} = \frac{\vec{p}}{m} + \frac{\alpha}{\hbar}(\hat{z} \times \hat{\vec{\sigma}})$$

$$\frac{d}{dt}\hat{\vec{\sigma}} = \frac{1}{i\hbar}[\hat{\vec{\sigma}}, H] = \frac{2}{\hbar}[\vec{B} + \frac{\alpha}{\hbar}\vec{p} \times \hat{z}] \times \hat{\vec{\sigma}}$$

$\mathbf{j}_s(\mathbf{r},t)$ and $\vec{j}_\omega(\mathbf{r},t)$ reduces into:

- $\mathbf{j}_s(\mathbf{r},t) \;=\; Re\{\Psi^\dagger(\mathbf{r},t)\hat{\vec{v}}\hat{\vec{s}}\Psi(\mathbf{r},t)\}$
- $\vec{j}_\omega(\mathbf{r},t) \;=\; Re\Psi^\dagger(d\hat{\vec{s}}/dt)\Psi = Re\Psi^\dagger\hat{\vec{\omega}} \times \hat{\vec{s}}\Psi$

The two quantities $\mathbf{j}_s(\mathbf{r},t)$ and $\vec{j}_\omega(\mathbf{r},t)$, respectively describe the translational and rotational motion of the spin at the space $\mathbf{r}$ and the time t. They will be named the linear and the angular spin current densities accordingly.

Both linear and angular spin currents can induce electric fields:

- $$\vec{E}_s = \frac{-\mu_0 g \mu_B}{4\pi} \nabla \times \int \mathbf{j}_s dV \bullet \frac{\mathbf{r}}{r^3},$$
- $$\vec{E}_\omega = \frac{-\mu_0 g \mu_B}{4\pi} \int \vec{j}_\omega dV \times \frac{\mathbf{r}}{r^3},$$

A conserved physical quantity:

Let us apply $\nabla\bullet$ acting on the equation :

$$\frac{d}{dt}\vec{s}(\mathbf{r},t) = -\nabla\bullet\mathbf{j}_s(\mathbf{r},t)+\vec{j}_\omega(\mathbf{r},t)$$

then:

$$\frac{d}{dt}(\nabla\bullet\vec{s})+\nabla\bullet(\nabla\bullet\mathbf{j}_s)-\nabla\bullet\vec{j}_\omega=0$$

$$\frac{d}{dt}(\nabla\bullet\vec{s})+\nabla\bullet(\nabla'\bullet\mathbf{j}_s-\vec{j}_\omega)=0$$

Introduce:

$$\vec{j}_{\nabla\bullet\vec{s}}\equiv\nabla'\bullet\mathbf{j}_s-\vec{j}_\omega$$

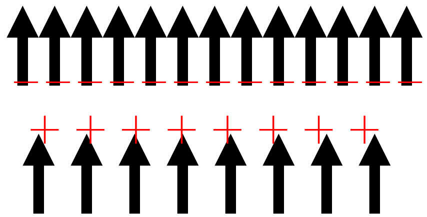
We have:

$$\frac{d}{dt}(\nabla\bullet\vec{s})+\nabla\bullet\vec{j}_{\nabla\bullet\vec{s}}=0$$

The current $\vec{j}_{\nabla\bullet\vec{s}}$ of the spin divergence is a conserved quantity ! ! !

The meaning of $\nabla \bullet \vec{s}$ is:

an equivalent magnetic charge.



The total electric field

can be represented as：

$$\vec{E} = \vec{E}_s + \vec{E}_\omega = \frac{\mu_0 g \mu_B}{4\pi} \int (\nabla' \bullet \mathbf{j}_s - \vec{j}_\omega) dV \times \frac{\mathbf{r}}{r^3}$$

$$\vec{E} = \frac{\mu_0 g \mu_B}{4\pi} \int \vec{j}_{\nabla \bullet \vec{s}} dV \times \frac{\mathbf{r}}{r^3}$$

Through the measurement $\vec{E}_T(\mathbf{r})$,

$\vec{j}_{\nabla \bullet \vec{s}}$ can be uniquely obtained ！！！

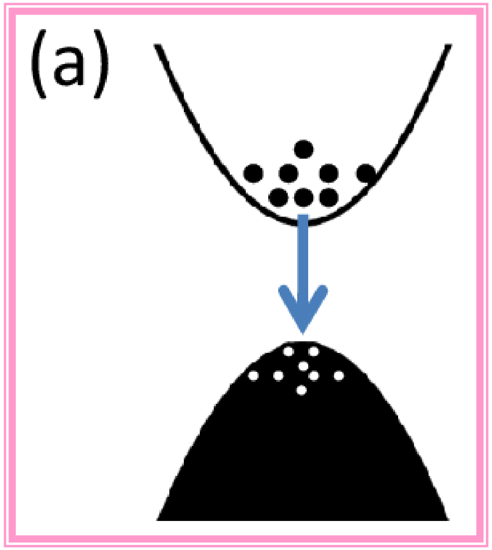
Spin superconductor in ferromagnetic graphene

- S-wave Cooper Pairs, 2e, spin singlet, BCS ground state
- Exciton, electron-hole pair, charge-neutral, spin singlet or triplet

A spin triplet exciton condensate is named as

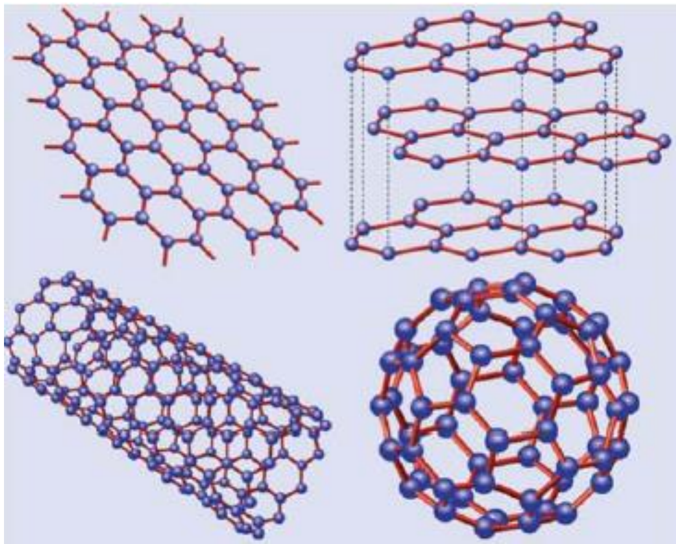
Spin-Superconductor (SSC)

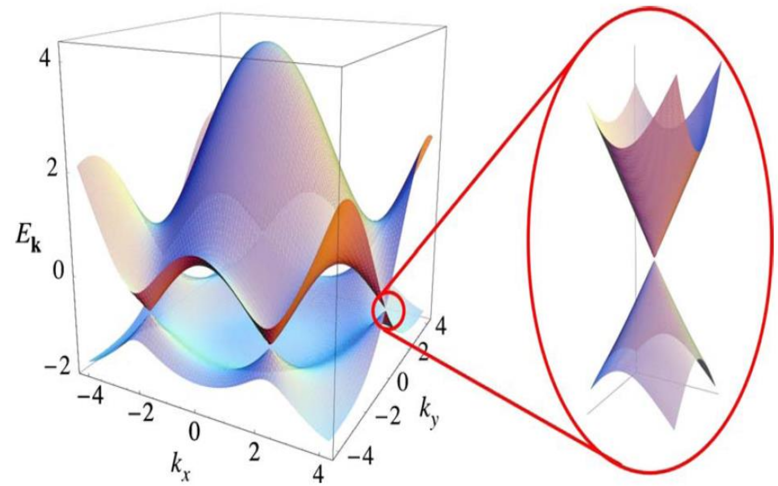
General drawback of the exciton condensate in many physical systems is its instability because of the electron-hole (e-h) recombination



Typical exciton lifetime: picosecond~microsecond, short

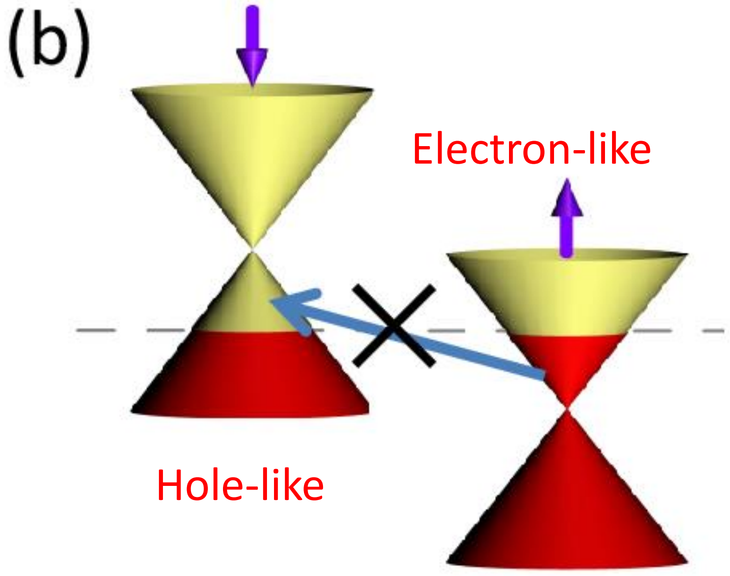
Exciton in normal graphene





Spin-singlet exciton excitation:
The charge carriers are spin-unpolarized in normal graphene

In the ferromagnetic (FM) graphene, the carriers are spin polarized.



Exciton:

These positive and negative carriers attract and form excitons that are stable against the e-h recombination due to the Coulomb interaction.

If a carrier jumps from the electron-like state to the hole-like one, the total energy of the system rises, which prevents the e-h recombination and means the exciton in the FM graphene is stable.

Spin-Superconductor (SSC)

- **Hamiltonian**

$$H = H_0 + U_C$$

with

$$H_0 = \sum_{\mathbf{k},\sigma}(a^\dagger_{\mathbf{k}\sigma}, b^\dagger_{\mathbf{k}\sigma})\begin{pmatrix} -\sigma M & k_x - ik_y \\ k_x + ik_y & -\sigma M \end{pmatrix}\begin{pmatrix} a_{\mathbf{k}\sigma} \\ b_{\mathbf{k}\sigma} \end{pmatrix},$$

$$U_C = \sum_{s,s';i,j;\sigma,\sigma'} U^{ss'}_{ij} n^s_{i\sigma} n^{s'}_{j\sigma'}. \qquad (1)$$

The total mean field Hamiltonian

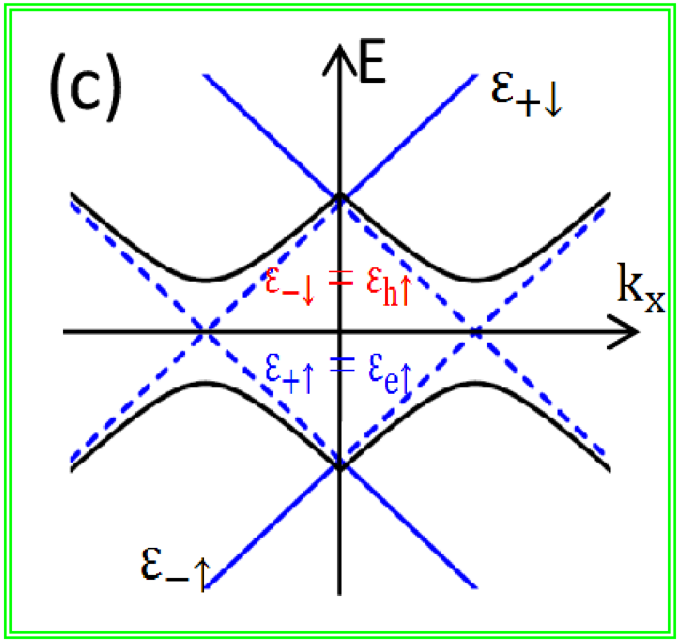
$$H_{MF} = \sum_{\mathbf{k}} (\alpha^{\dagger}_{\mathbf{k}e\uparrow}, \alpha_{\mathbf{k}h\uparrow}) \begin{pmatrix} \epsilon_{+\uparrow} & \Delta_{\mathbf{k}} \\ \Delta^{*}_{\mathbf{k}} & \epsilon_{-\downarrow} \end{pmatrix} \begin{pmatrix} \alpha_{\mathbf{k}e\uparrow} \\ \alpha^{\dagger}_{\mathbf{k}h\uparrow} \end{pmatrix}$$

The corresponding energy spectrum is shown by the solid curves.

An energy gap is opened.

$$|\Delta_{\mathbf{k}}|$$

This means the exciton condensed state of the e-h pairs is more stable than the unpaired state.



The ground state of the FM graphene is a neutral superconductor with spin ħ per pair ------SSC

The spin current is dissipationless and the spin resistance is zero.

The energy gap can be obtained self-consistently

$$\Delta_{\mathbf{k}} = \sum_{\mathbf{k}'} (U_{\mathbf{k}\mathbf{k}'} \Delta_{\mathbf{k}'} / 2A)\{f(-A) - f(A)\}$$

$$f(A) = 1/[\exp(A/k_B T) + 1]$$

$$A = \sqrt{(M - k')^2 + \Delta_{\mathbf{k}'}}$$

Self-consistent numerical results

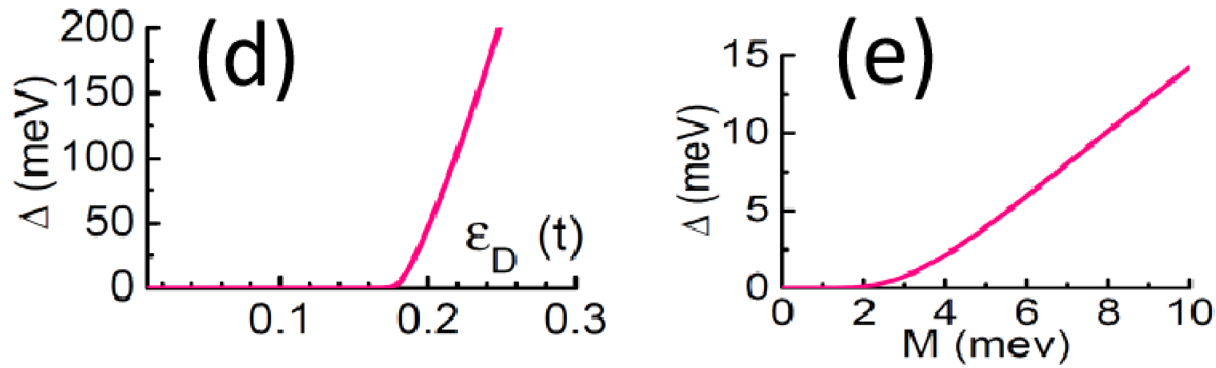


FIG. 1: (d) The gap Δ vs. ϵ_D at the FM magnetic moment $M = 5meV$ and (e) The gap vs. M at $\epsilon_D = 0.18t$.

We see that the gap Δ grows faster than exponentially with increase of ϵ_D. When $M = 5meV$ and $\epsilon_D = 0.18t$, $\Delta \approx$ 3meV. This yields the critical temperature of the transition from the SSF to the normal state is about several tens of Kelvin. Therefore, this SSF is observable as long as the realization of the FM graphene is achievable.

Result and Discussion-- 1. Meissner Effect

- The criterion that a superconductor differs from a perfect metal
- Described by the London equations.

$$\frac{\mathrm{d}\vec{J}_c}{\mathrm{d}t} = a\vec{E}$$

$$\nabla \times \vec{J}_c = b\vec{B}$$

Is there a Meissner-like effect for the SSC?

Consider a SSC with the superfluid carrier density n_s in an external electric field E and a magnetic field B

Magnetic force on a spin carrier $\mathbf{F} = (\mathbf{m} \cdot \nabla)\mathbf{B}$

$\mathbf{m} = g\mu_B\vec{\sigma} = (4\pi g\mu_B/h)\mathbf{s}$ is the magnetic moment of a carrier

The spin current density $\mathbb{J}_s = n_s\mathbf{v}\mathbf{s}$

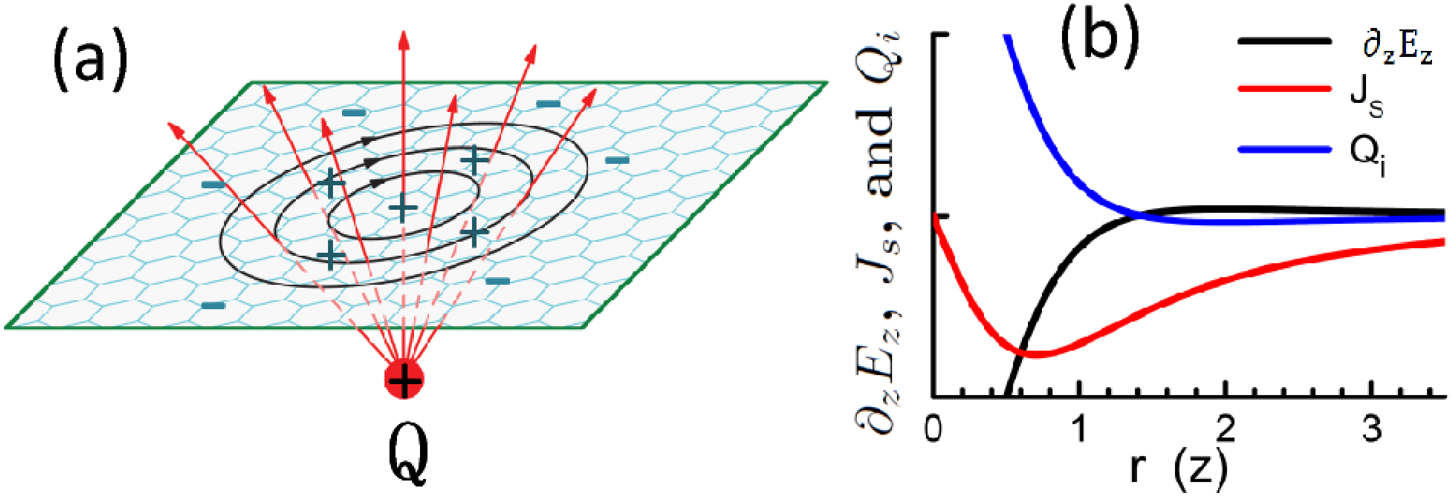
$$d\mathbb{J}_s/dt = a(\mathbf{s} \cdot \nabla)\mathbf{B}\mathbf{s} \qquad \text{(a)}$$

$$\nabla \times \mathbb{J}_s = \mu_0\epsilon_0 a(\mathbf{s} \cdot \nabla)\mathbf{E}\mathbf{s} \qquad \text{(b)}$$

Eqs. (a) and (b) for $\mathbb{J}_s$ play roles similar to the London equations in superconductor .

Example for this electric Meissner effect:

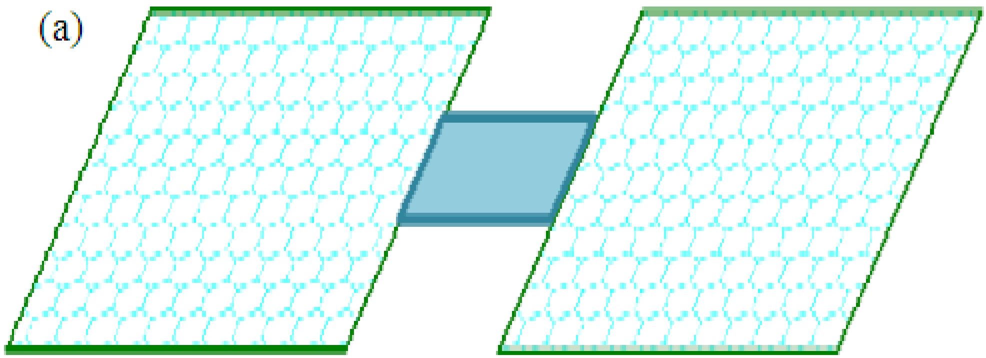
Consider a positive charge Q at the origin and an infinite FM graphene in the x-y plane at z = Z



$$(\mathrm{s}\cdot\nabla)E_z = \partial_z E_z = \frac{Q}{4\pi\epsilon_0}\frac{r^2-2z^2}{(z^2+r^2)^{5/2}}$$

$$J_s = -\frac{\mu_0 a Q}{4\pi}\frac{r}{(Z^2+r^2)^{3/2}}$$ **super-spin-current**

Result and Discussion-- 2. Josephson Effect



SSF/Normal Conductor/SSF junction

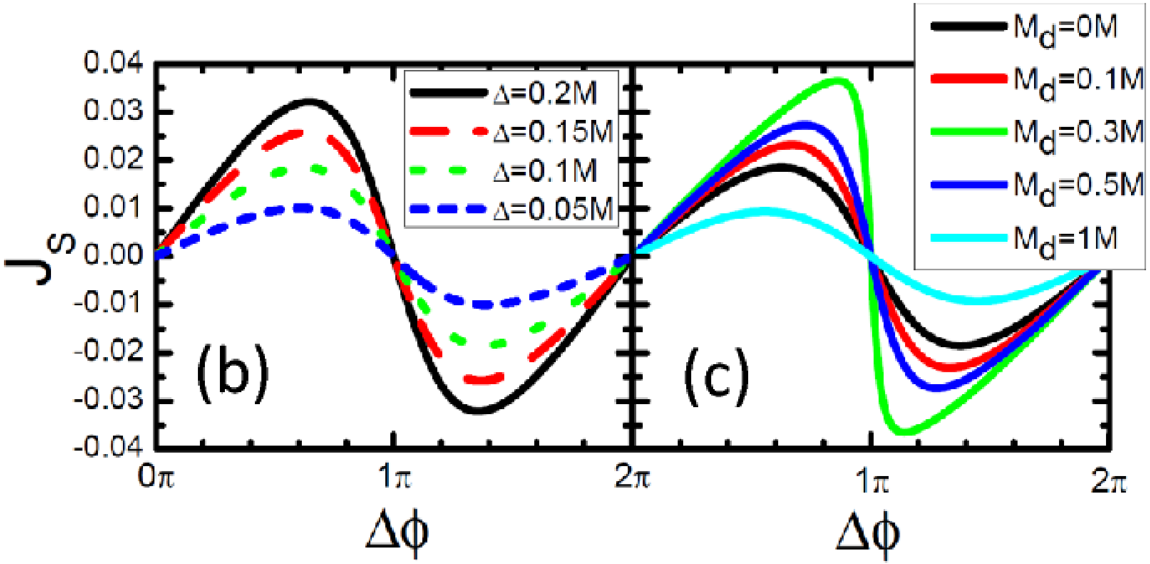


FIG. 3: (a) The schematic diagram of the weakly coupled SSF-normal conductor-SSF junction. (b) The spin current J_s vs. the phase difference $\Delta\phi$ for different Δ and $m_d = 0$. (c) The J_s-$\Delta\phi$ curves for different m_d and $\Delta = 0.1M$. Here $\epsilon_d = 0$ and $\Gamma \equiv 2\pi t_\beta^2 \rho_k = 0.1M$; ρ_k is the density of state of the FM graphene in momentum space.

There is a super-spin-current flowing through the junction that resembles the Josephson tunneling in a conventional superconductor junction.

Fig. 3(c) shows the super-spin-current Js can also be observed in non-zero md as far as $\Delta \neq 0$ and $\Delta\phi \neq 0, \pi$.

Ginzburg-Landau equations of the spin superconductor

Step 1: the form of the free energy

$$F_s = \int d^3 r f_s$$

$$f_s = f_n + \alpha(T)|\psi(\mathbf{r})|^2 + \frac{1}{2}\beta(T)|\psi(\mathbf{r})|^4 + \frac{1}{2m^*}|(-i\hbar\nabla + \alpha_0 \boldsymbol{\sigma} \times \nabla\varphi)\psi(\mathbf{r})|^2 + \frac{1}{2}\epsilon_0(\nabla\varphi)^2$$

Note: The fourth term is the kinetic energy.

electron charge in external electric and magneitic filed: $\mathbf{p} \longrightarrow \mathbf{p} - \frac{e}{c}\mathbf{A}$

magnetic moment in external electric and magneitic filed: $\mathbf{p} \longrightarrow \mathbf{p} - \frac{1}{c^2}\mathbf{m} \times \mathbf{E}$

$$H = \frac{1}{2m^*}(\mathbf{p} - \frac{1}{c^2}\mathbf{m} \times \mathbf{E})^2 - \mathbf{m} \cdot \mathbf{B} \xrightarrow[\text{equation}]{\text{canonical}} m^*\ddot{\mathbf{r}} = (\mathbf{m} \cdot \nabla)\mathbf{B} - \frac{1}{c^2}\mathbf{v} \times [(\mathbf{m} \cdot \nabla)\mathbf{E}]$$

Step 2: Derivate the Ginzburg-Landau equations by variational method

(a) Variate the free energy by ψ^*, we get the first Ginzburg-Landau equation:

$$\alpha\psi + \beta|\psi|^2\psi + \frac{1}{2m^*}[-i\hbar\nabla + \alpha_0(\boldsymbol{\sigma} \times \nabla\varphi)]^2\psi = 0$$

with the boundary condition:

$$[-i\hbar\nabla + \alpha_0(\boldsymbol{\sigma} \times \nabla\varphi)]_n\psi(\mathbf{r}) = 0$$

(b) Variate the free energy by φ , we we get the second Ginzburg-Landau equation:

$$\rho = \nabla \cdot \{[\frac{i\hbar\alpha_0}{2m^*}(\psi^*\nabla\psi - \psi\nabla\psi^*) - \frac{\alpha_0^2}{m^*}|\psi|^2(\boldsymbol{\sigma} \times \nabla\varphi)] \times \boldsymbol{\sigma}\}$$

If we definite $$\mathbf{j}_s = \frac{i\hbar\alpha_0}{2m^*}(\psi\nabla\psi^* - \psi^*\nabla\psi) + \frac{\alpha_0^2}{m^*}|\psi|^2(\boldsymbol{\sigma} \times \nabla\varphi)$$

we get $\rho = -\nabla \cdot (\mathbf{j}_s \times \boldsymbol{\sigma})$ → the second GL equation describes the equivalent charge induced by the super-spin-current.

Substitute $\psi(\mathbf{r}) = \sqrt{n_s(\mathbf{r})}e^{i\theta(\mathbf{r})}$, → **the generalized London equation**

we have $\mathbf{j}_s = \frac{n_s(\mathbf{r})\alpha_0}{m^*}[\hbar\nabla\theta + \alpha_0(\boldsymbol{\sigma} \times \nabla\varphi)]$

↓ if $n_s(\mathbf{r})$ is independent of $\mathbf{r}$

$$\nabla \times \mathbf{j}_s = -\frac{\alpha_0^2 n_s}{m^*}[(\nabla \cdot \mathbf{E})\boldsymbol{\sigma} - (\boldsymbol{\sigma} \cdot \nabla)\mathbf{E})]$$

↓ if $\nabla \cdot \mathbf{E} = 0$

$$\nabla \times \mathbf{j}_s = \frac{\alpha_0^2 n_s}{m^*}(\boldsymbol{\sigma} \cdot \nabla)\mathbf{E}.$$

→ **the second London equation**

Measurement of Spin Superconductor State

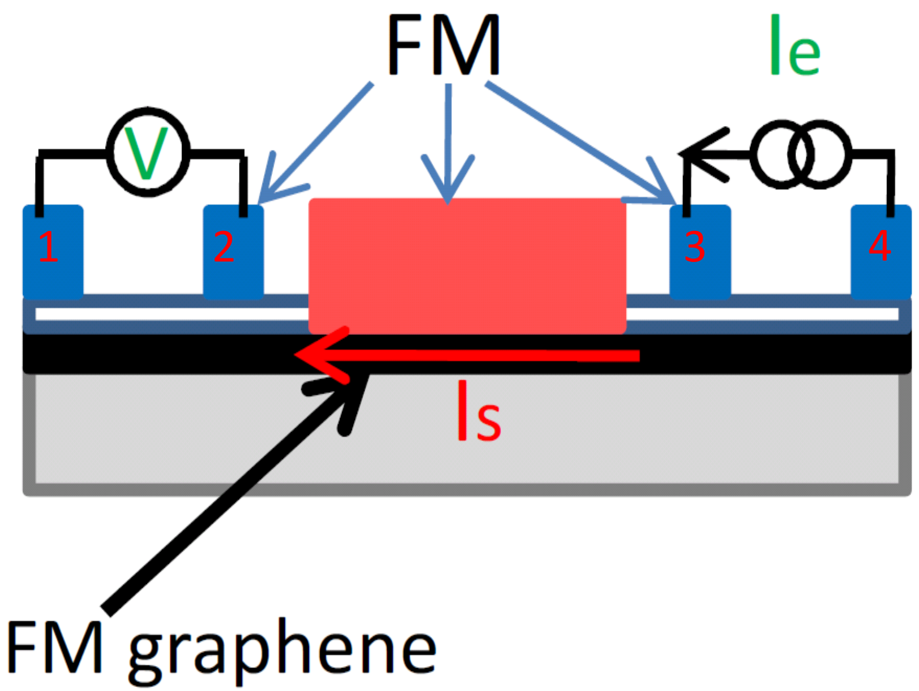
Scheme

zero spin resistance
spin supercurrent

Non-local resistance measurement

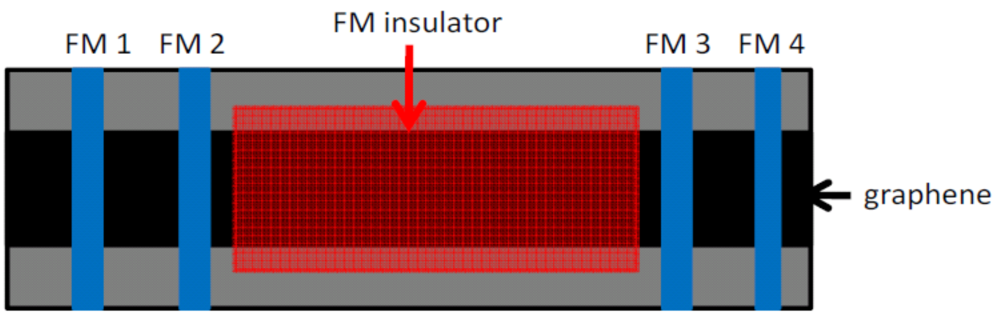
Apply a current Ie between two right electrodes 3 and 4;
Measure the bias V between two left electrodes 1 and 2.

Why is the bias V generated?

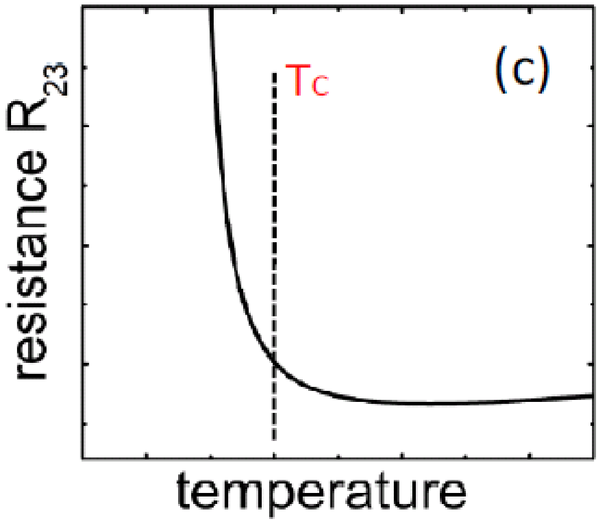


Four-terminal device used to measure the SSC state
(1,2,3,4 denote four terminals)

Based on the device in the paper of Nature 488, 571 (2007)



Top view of the four-terminal device



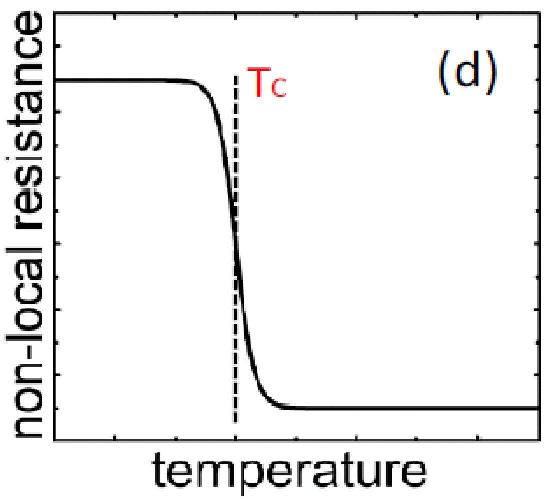
T>Tc, normal state, R23 is small

T<Tc, SSF state, R23 is large due to the gap

$R_{23} \sim \quad e^{\Delta/k_B T}$

Resistance versus the temperature

Non-local resistance



T>Tc, non-local resistance is small
because the normal state has the spin resistance

T<Tc, non-local resistance sharply increases

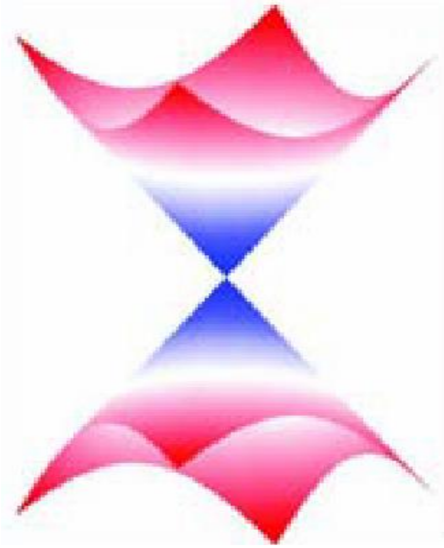
When graphene is in the SSC the non-local resistance is very large, because that the spin current can dissipationlessly flow through the super-spin-fluid region.

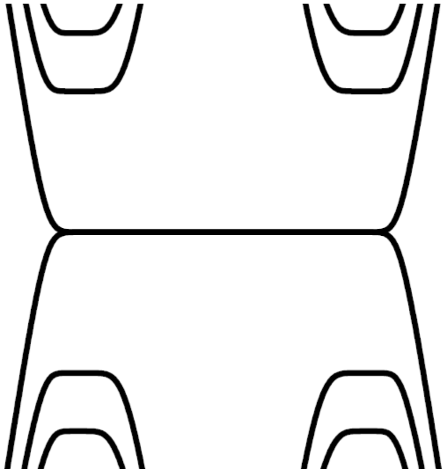
Here we emphasize that the changes of the normal resistance and non-local resistance are sharp, similar as the resistance change when a sample enters from a metal phase into a superconducting phase. Thus, these resistances can easily be measured in experiments.

In addition, for T < Tc the non-local resistance is independent of the length of the red strip, implying the zero spin resistance in the spin-superfluid state.

Graphene under a magnetic field

Display Landau level structure instead of linear dispersion.





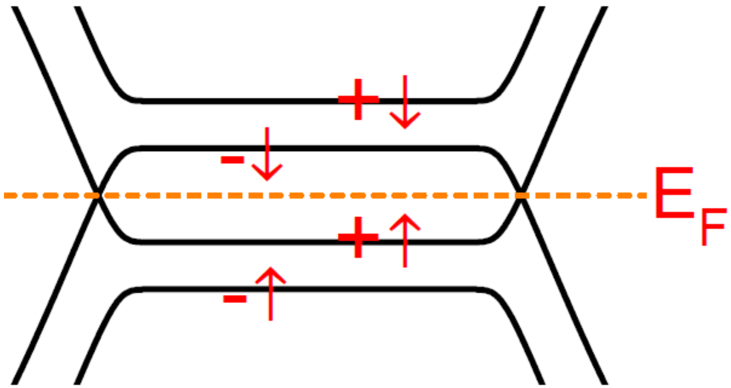
B=0

B≠0

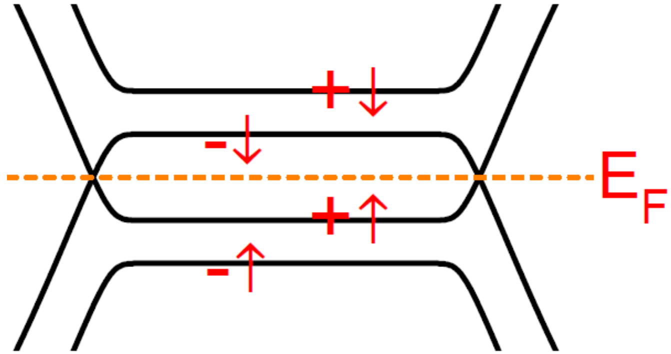
- **What is the effect of the formation of the LLs on the spin superconductor?**

Each LL is fourfold degenerate due to the spin and valley. The zeroth LL locates at the charge neutrality point and has the equal electron and hole compositions. The e-e interaction and Zeeman effect can lift the LL degeneracy.

Due to the spin split, now a +↑ LL is occupied by electrons and a −↓ LL is occupied by holes, there is a pair of counter-propagating edge states.

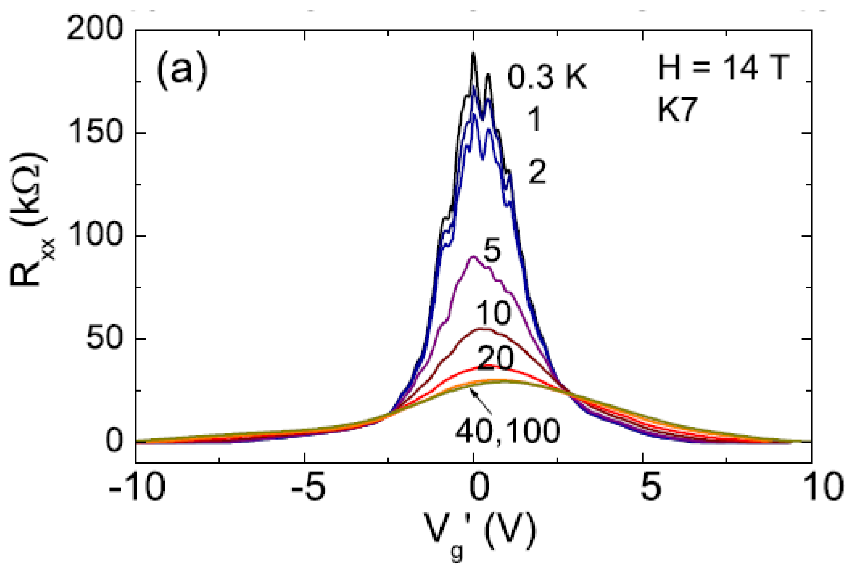


These edge states can carry both spin and charge currents. So the sample edge is metal.



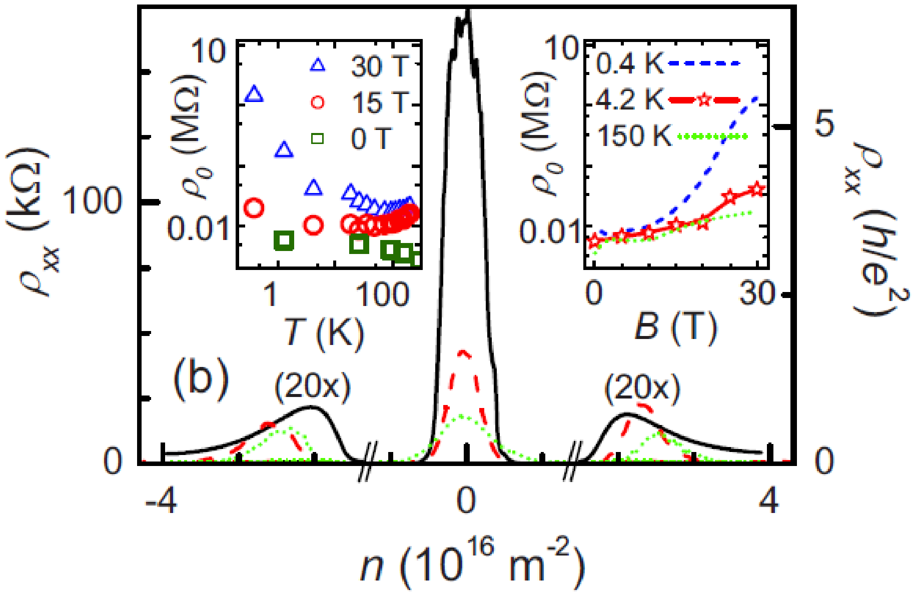
However, experiments have clearly shown an insulating behavior.

The Hall conductance has a plateau with the value zero, but the longitudinal resistance shows an insulating behavior, it increases quickly with decreasing temperature.



J.G. Gheckelsky, et.al. Phys. Rev. Lett. 100, 206801 (2008).

Many experiments show the same results.



A.J.M. Giesbers, et.al. Phys. Rev. B. 80, 201203(R) (2009).

also see: Phys. Rev. Lett. 107, 016803(2011);
Science 330, 812(2010);
Science 332, 328 (2011);
etc.

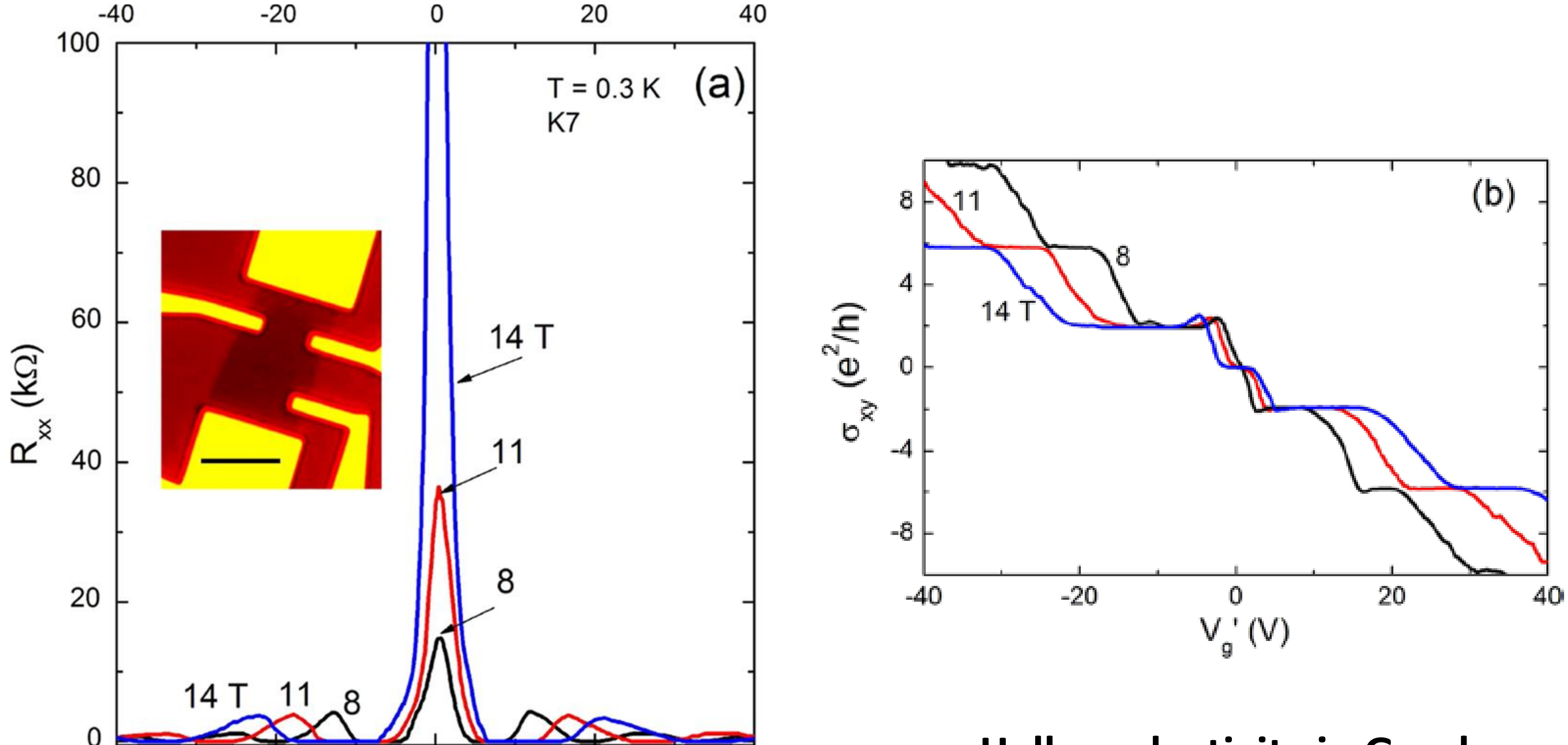
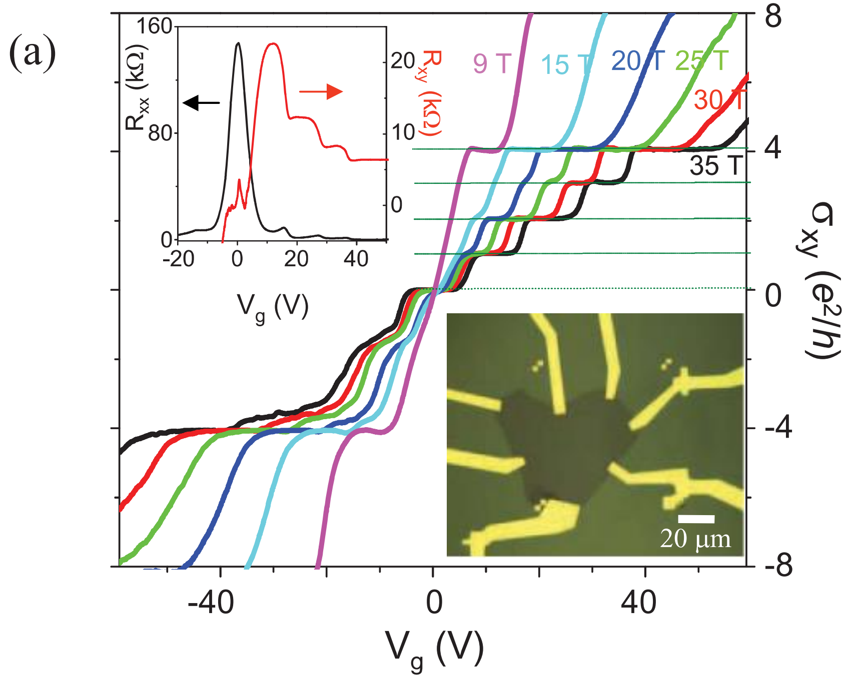


FIG. 1 (color online). The resistance R_{xx} (a) and Hall conductivity σ_{xy} (b) in Sample K7 versus (shifted) gate voltage $V_g' = V_g - V_0$ at 0.3 K with H fixed at 8, 11, and 14 T. Peaks of R_{xx} at finite V_g' correspond to the filling of the $n = 1$ and $n = 2$ LLs. At $V_g' = 0$, the peak in R_{xx} grows to 190 kΩ at 14 T. The inset shows sample K22 in false color (dark red) with Au leads deposited (yellow regions). The bar indicates 5 μm. Panel (b) shows the quantization of σ_{xy} at the values $(4e^2/h)(n + \frac{1}{2})$. At 0.3 K, $\sigma_{xy} = 0$ in a a 2-V interval around $V_g' = 0$.

Hall conductivity in Graphene，Zero-Energy Landau Level

PRL，100，206801 (2008)
N. P. Ong



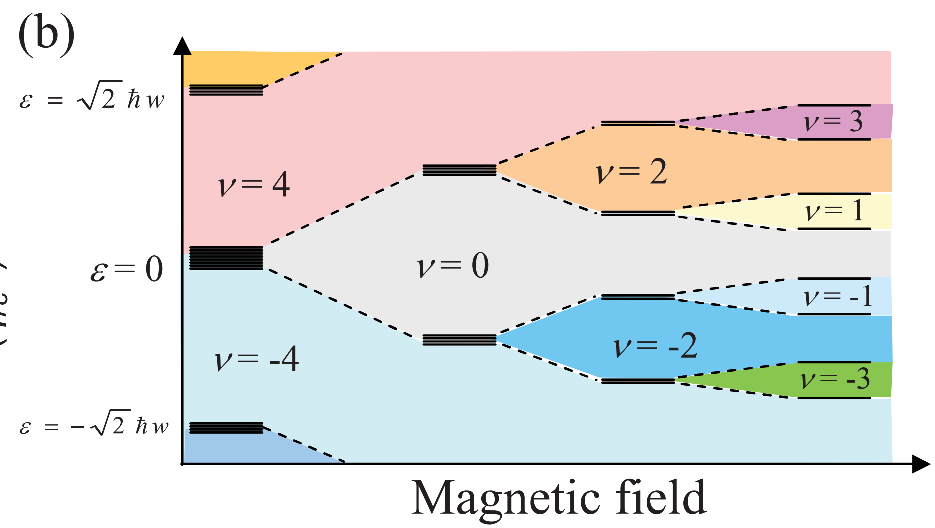
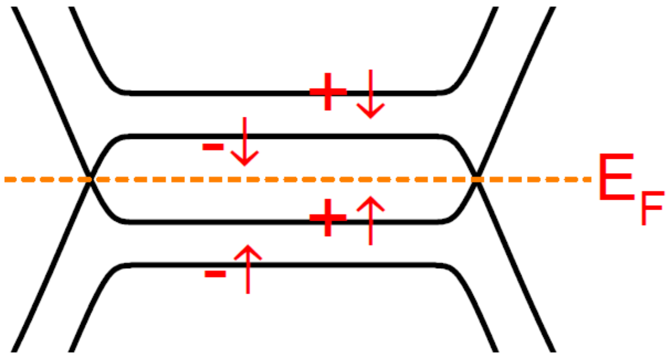


FIG. 1 (color online). (a) Hall conductivity σ_{xy}, as a function of gate voltage V_g at $T = 1.4$ K at different magnetic fields: 9, 15, 20, 25, 30, and 35 T. Upper inset: R_{xx} (in black) and R_{xy} [in gray (red)] as V_g varies at $B = 35$ T. Lower inset: Optical microscope image of a BLG device used in this experiment. (b) Schematic of the zero-energy LL hierarchy in bilayer graphene at high magnetic field.

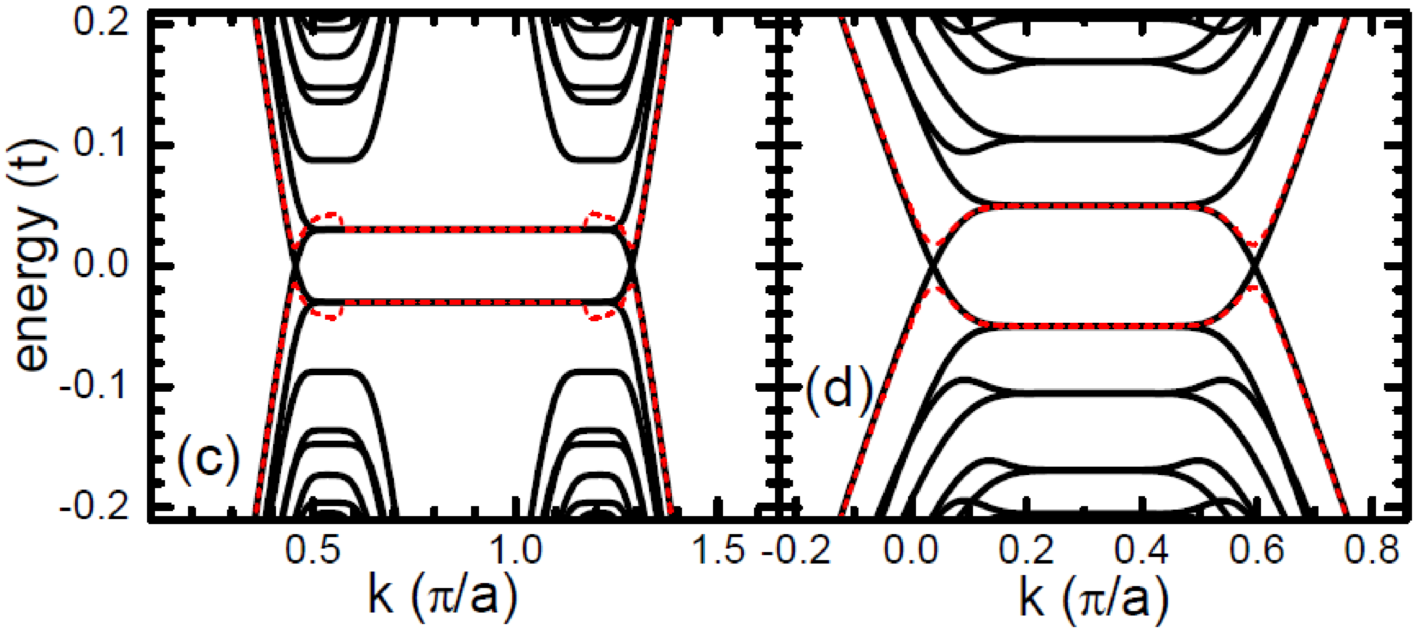
Hall conductivity in Bilayer Graphene，Zero-Energy Landau Level

PRL，104，066801 (2010)
P. Kim



* In present case, the carriers are both electrons and holes.

* Electrons and holes are both spin up. With an e-h attractive interaction, e and h may form an e-h pair and then condense into a spin superconductor at low T.

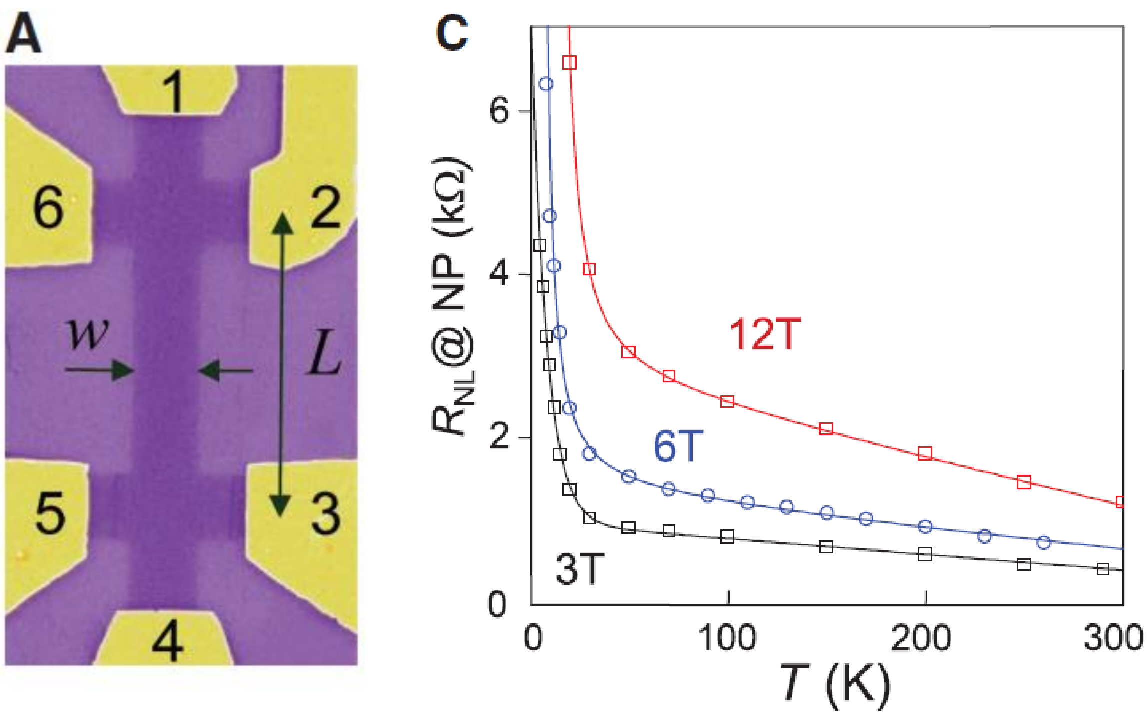


A gap opens in the edge bands. Now both edge and bulk bands have gaps, it is a charge insulator, consistent with the experimental results.

Resistance and nonlocal resistance:

Insulator at Dirac points at low T; the nonlocal resistance rapidly increases at low T, showing that the spin current can flow through the graphene.

These experimental results can be explained by a spin superconductor.



D.A. Abanin, et.al., Science 332, 328 (2011)

Summary

We predict a spin superconductor (SSC) state in the ferromagnetic graphene, as the counterpart to the (charge) superconductor. The SSC can carry the dissipationless super-spin-current at equilibrium.

BCS-type theory and Ginzburg-Landau theory for the SSC are presented, and an electrical 'Meissner effect' and a spin-current Josephson effect in SSC device are demonstrated.

Part II:

Theory for Electric Dipole Superconductor (EDS) with an application for bilayer excitons

In collaboration with:

Qing-feng Sun (Peking University)

Qing-dong Jiang (Peking University)

Zhi-qiang Bao (IOP, CAS)

Outline

- **Introduction**
  - **Pairing condensation**
  - **Exciton condensation in double-layer systems**
- **Theory for electric dipole superconductor**
- **Summary**

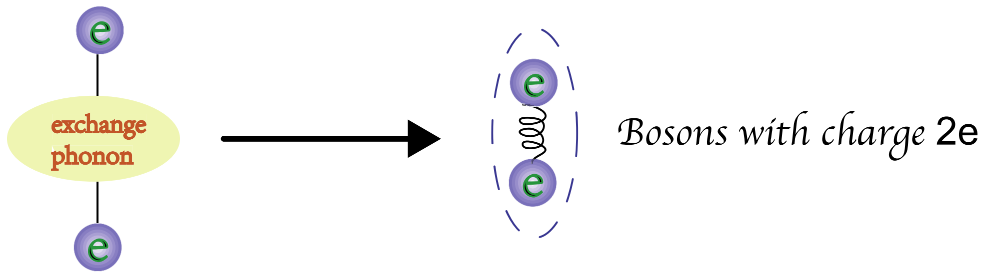
Pairing Condensation

Attractive interaction between fermions

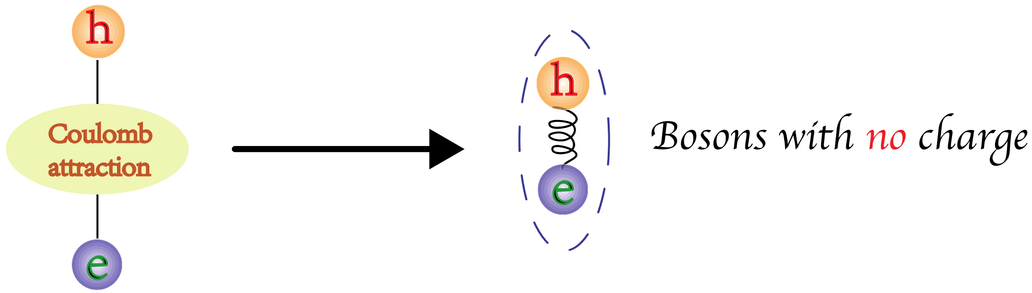
- Cooper problem (two-body problem) : bound states
- Normal fluid is unstable at low T

Examples

- Superconductor (e-e pair) : e-ph interaction

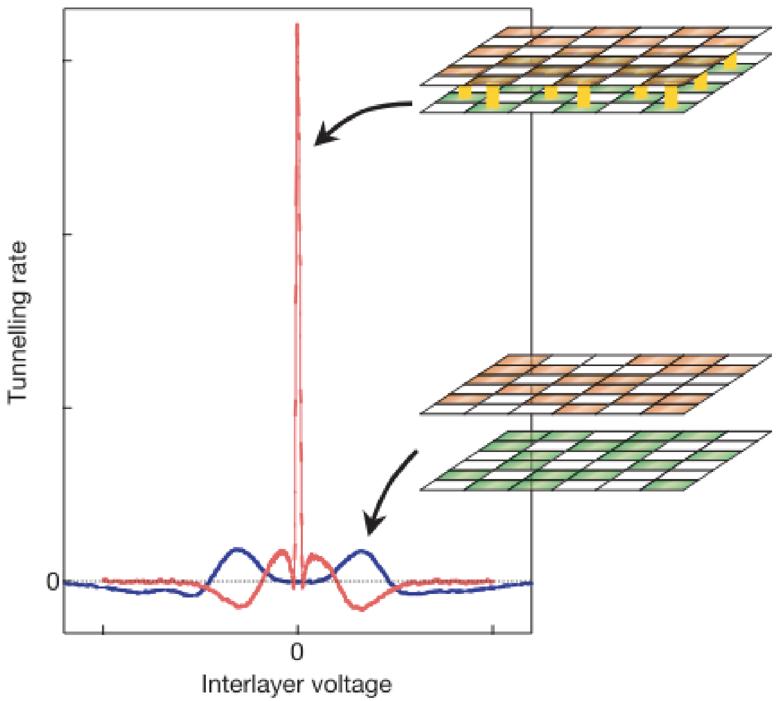


- Exciton (electron-hole pair) : Coulomb interaction



Exciton Condensation in Double-layer System

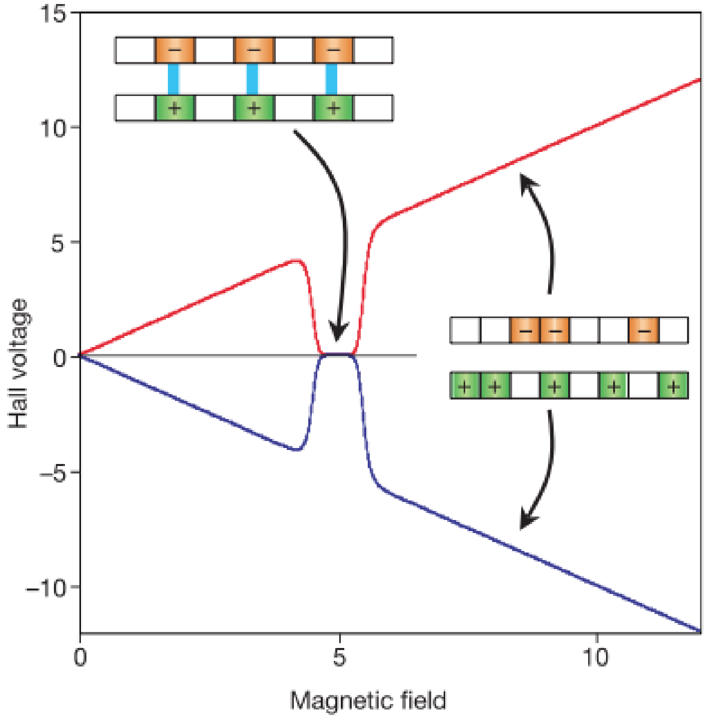
Tunneling



GaAs/AlGaAs d=9.9nm

Spielman et al, PRL (2000)

Vanished Hall voltage



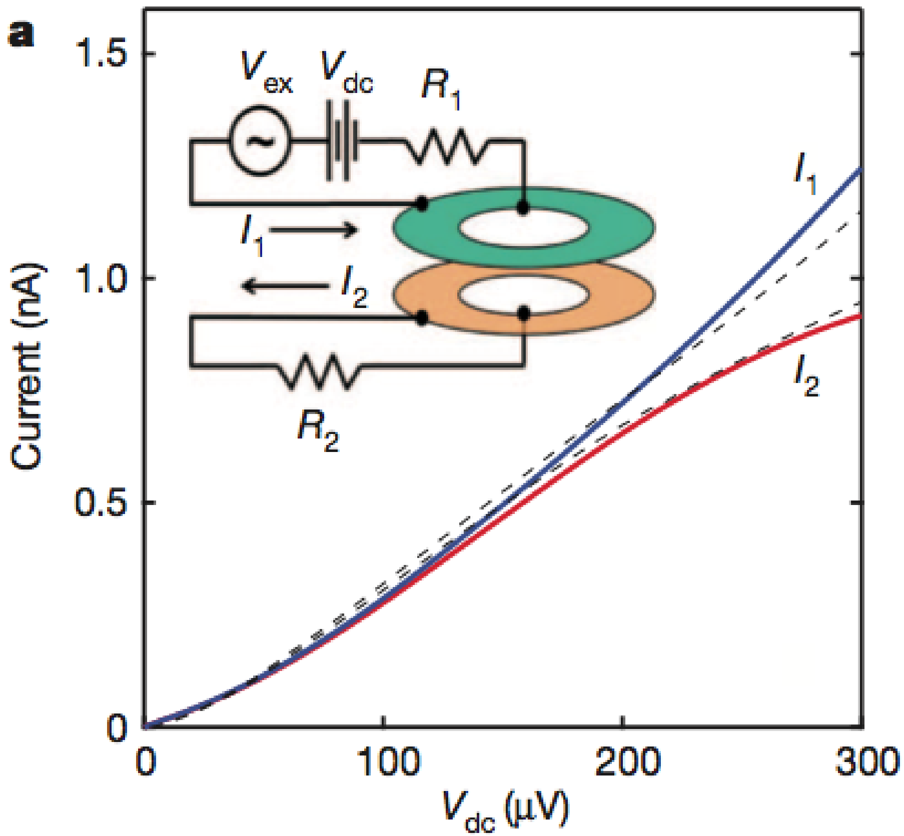
GaAs/AlAs d=7.5nm

Tutuc et al, PRL (2004)

Kellogg et al, PRL (2004)

Exciton Condensation in Double-layer System

Coulomb drag experiment



GaAs/AlGaAs d=10nm

D.Nandi et al, Nature (2012)

$\mathbf{I}_1$ is the drive current

$\mathbf{I}_2$ is the drag current

At small V_{dc}

$$\mathbf{I}_1 \approx \mathbf{I}_2$$

Perfect drag!

Exciton Condensation in Double-layer System

However, None of these measurement can
directly confirm the existence of exciton superfluid.

We need to discover
the basic characteristics of exciton superfluid.

We view the excitons in bilayer systems as **electric dipoles**.
Taking this point of veiw, we get

London equation

G-L equation

→

Meissner-type effect

Zero dipole resistance

Dipole Josephson effect

London-type equation for dipole superconductor

The force on an electric dipole $\mathbf{F} = (\mathbf{p}_0 \cdot \nabla)\mathbf{E} = m^* \frac{d\mathbf{v}}{dt}$

Define a super dipole current density $\mathbf{J}_p = n p_0 v$

Time derivative of $\mathbf{J}_p$ $\frac{d\mathbf{J_p}}{dt} = \beta(\hat{p}_0 \cdot \nabla)\mathbf{E}$

The curl of $\mathbf{J}_p$ $\nabla \times \mathbf{J}_p = -\beta(\hat{p}_0 \cdot \nabla)\mathbf{B}$

$$\beta = n p_0^2 / m^*$$

Ginzburg-Landau equation

Lagrangian

$$L = \frac{1}{2}m^*\mathbf{v}^2 + \mathbf{p}_0 \cdot \mathbf{E}^{eff}$$

$$\mathbf{E}^{eff} = \mathbf{v} \times \mathbf{B}$$

Hamiltonian

$$H = \hat{\mathbf{p}} \cdot \mathbf{v} - L = \frac{[\hat{\mathbf{p}} + \mathbf{p}_0 \times \mathbf{B}]^2}{2m^*}$$

Free energy density

$$f_s = f_n + \frac{|(\hat{\mathbf{p}} + \mathbf{p}_0 \times \mathbf{B})\psi(\mathbf{r})|^2}{2m^*} + \frac{|\mathbf{B}|^2}{2\mu_0} + \alpha(T)|\psi(\mathbf{r})|^2 + \frac{\beta(T)}{2}|\psi(\mathbf{r})|^4$$

Minimize the free energy with respect to $\psi(\mathbf{r})$ **and** $\mathbf{A}$

Ginzburg-Landau equation

First G-L type equation

$$\alpha(T)\psi(\mathbf{r}) + \beta(T)|\psi(\mathbf{r})|^2\psi(\mathbf{r}) + \frac{[\hat{\mathbf{p}} + \mathbf{p}_0 \times \mathbf{B}]^2\psi(\mathbf{r})}{2m^*} = 0 \quad (1)$$

Second G-L type equation

$$\nabla \times \mathbf{B} = -\mu_0 \nabla \times (\mathbf{J}_p \times \hat{p}_0) \quad (2)$$

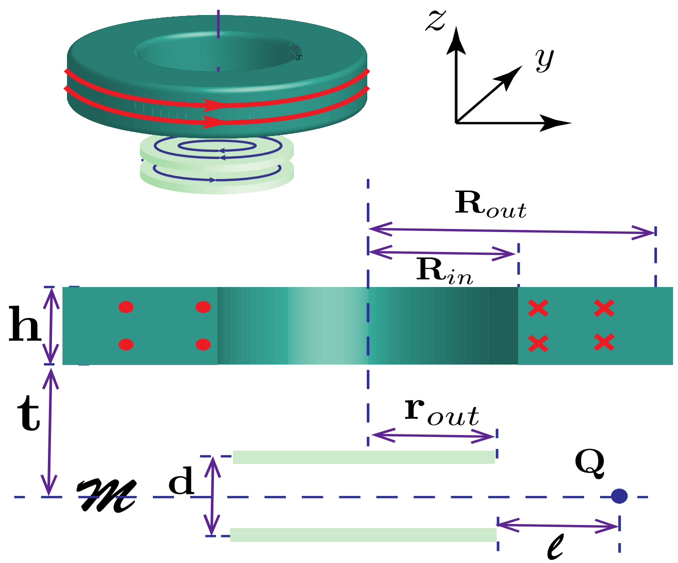
Super electric dipole current

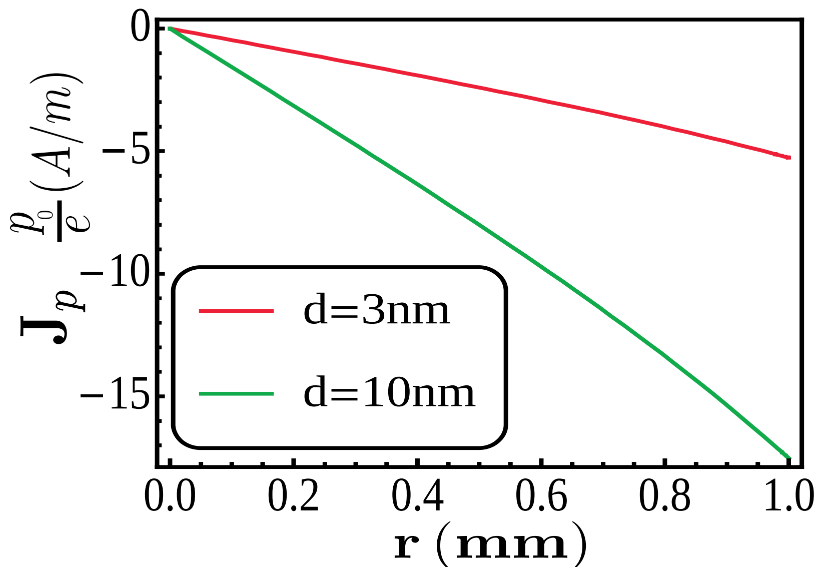
$$\mathbf{J}_p = Re(\psi^* \hat{\mathbf{v}} \psi) = \frac{p_0}{2m^*}[i\hbar(\psi \nabla \psi^* - \psi^* \nabla \psi) + 2\mathbf{p}_0 \times \mathbf{B}\,|\psi|^2]$$

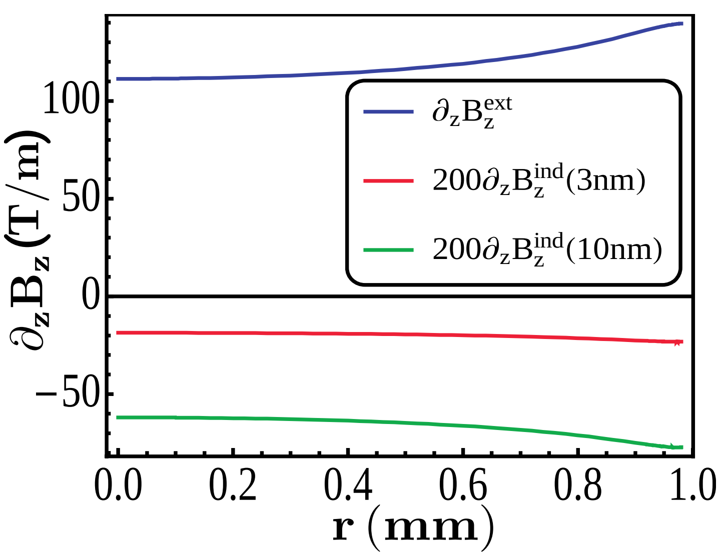
Assume $\psi(\mathbf{r}) = |\psi(\mathbf{r})|e^{\theta(\mathbf{r})}$

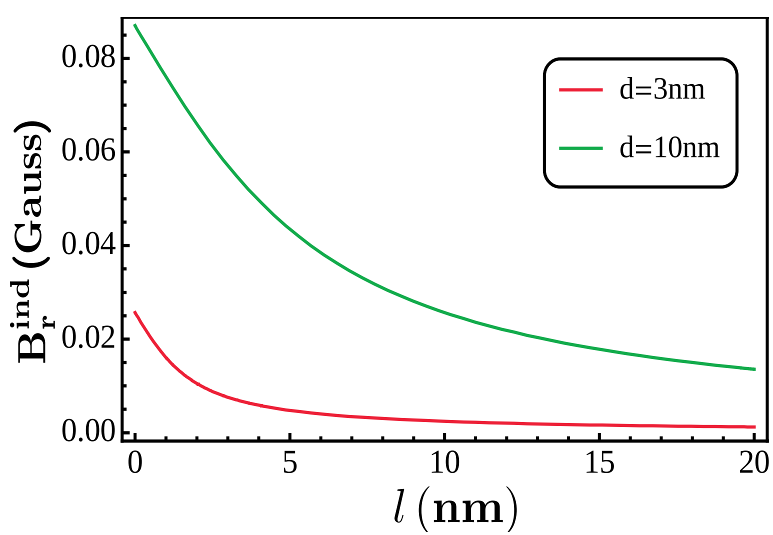
The London equation can be obtained

Meissner-type Effect:
Screen magnetic field gradient



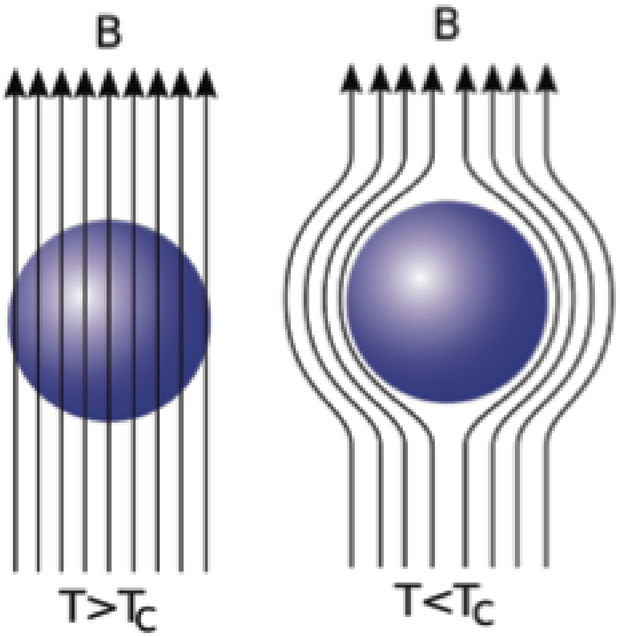




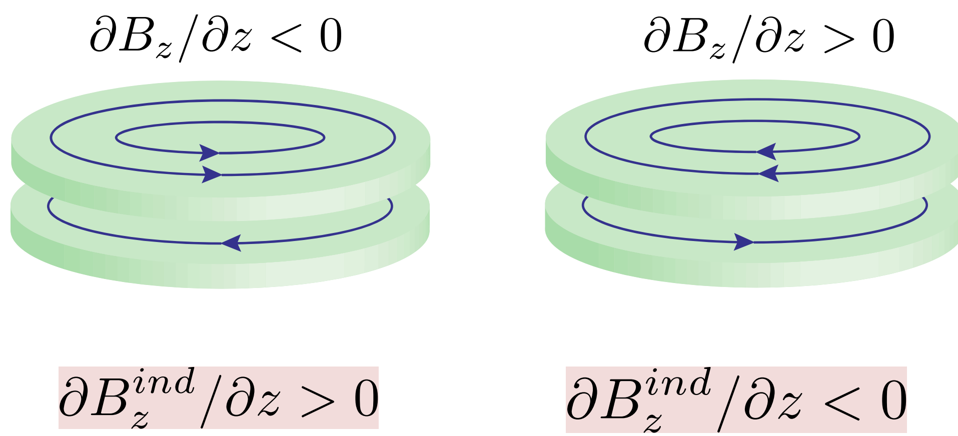


Comparison between SC and EDS

SC screen magnetic field B

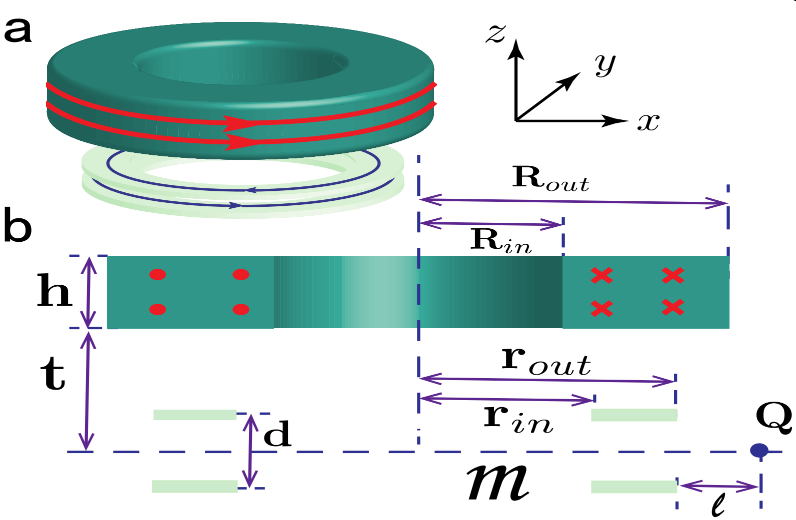


EDS screen magnetic field gradient $\partial_z \mathrm{B}$



Zero Dipole Resistance

Schematic figure



In the electric dipole superconductor region

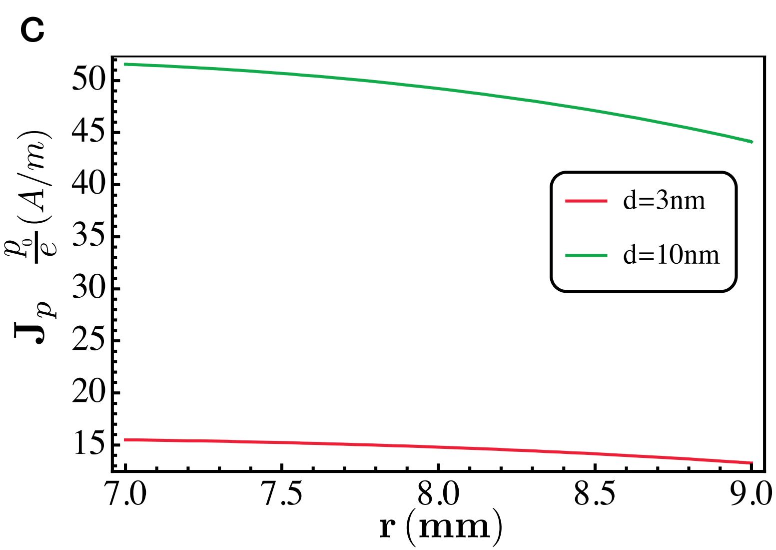
$\partial_z B_z \approx 0$ but $\partial_z A_\theta \neq 0$

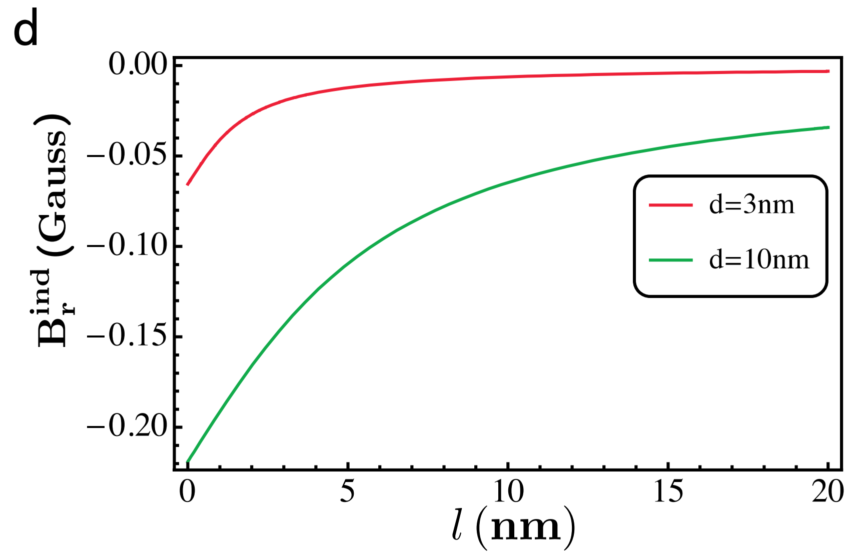
$$\nabla \times \mathbf{J}_p \propto \partial_z \mathbf{B} = \partial_z (\nabla \times \mathbf{A})$$

$$\Rightarrow \quad \mathbf{J}_p \propto \partial_z \mathbf{A}$$

Closing the current suddenly will induce a super electric dipole current

$$J_p = \beta \partial_z A_\theta^{ext}$$

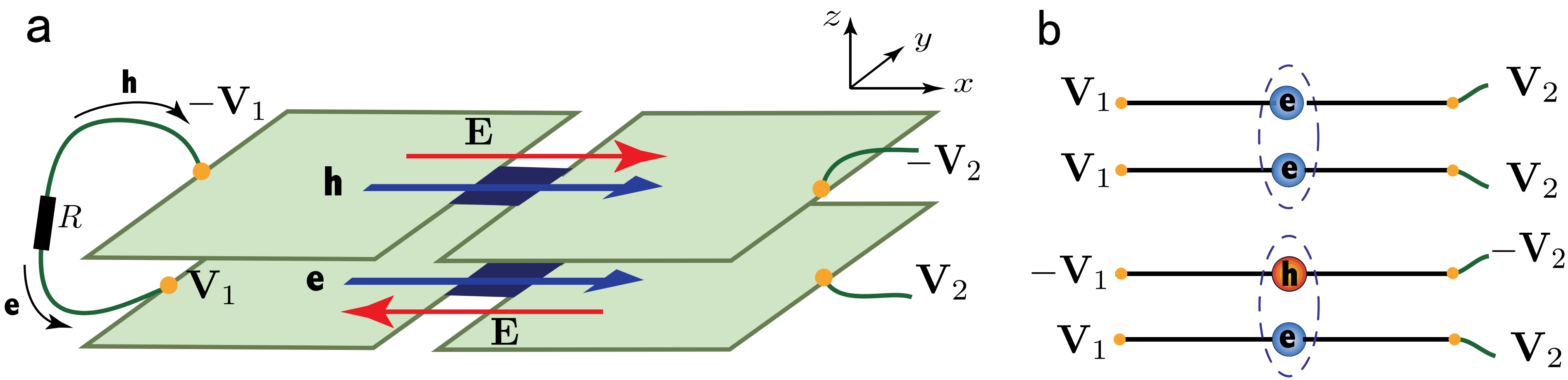




Electric dipole Current Josephson Effect

$$\gamma = \gamma_0 + \frac{1}{\hbar}\int_1^2 (\mathbf{p}_0 \times \mathbf{B}) \cdot \mathbf{e}_x dx = \gamma_0 - \omega_0 t$$

$$\omega_0 = -\frac{p_0}{\hbar}\int_1^2 \partial_z E_x dx$$



Analogy

$$\partial_z E_x = 2E_x/d$$

The frequency $\omega_0 = -\frac{2ed}{d\hbar}\int_1^2 E_x dx = \frac{2e}{\hbar}(V_2 - V_1)$

The same as superconductor!

Summary

1. We developed a general theory for electric dipole superconductor including London-type equation and Ginzburg-Landau equations.

2. View the bilayer excitons as electric dipoles, and we get three novel effects .

3. These effects are the characteristics of EDS, and can be used to justify the existence of exciton superfluid.

Thank you!