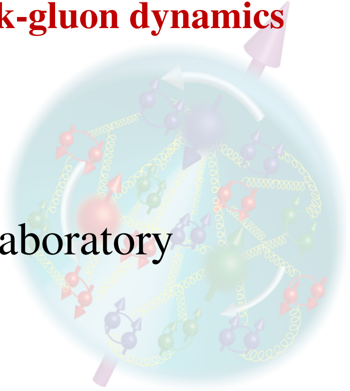
Nucleon Form Factors:

An incisive window into quark-gluon dynamics



Ian Cloët

Argonne National Laboratory

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Peking University, 20 – 24 October 2014

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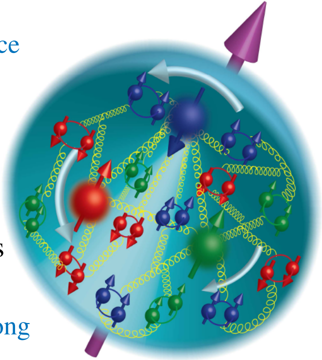
Roads to Discovery: 2014-2023

- Meson and baryon spectroscopy
  - the discovery of exotic or hybrid hadrons would force a dramatic reassessment of the distinction between the notions of matter fields and force fields



- Precision experimental study of the valence region, together with theoretical computation of distribution functions and distribution amplitudes
  - computation is critical – as data can only reveal limited information about the theory underlying strong interaction physics
- Exploit opportunities provided by new data on nucleon elastic and transition form factors
  - chart infrared evolution of QCD's coupling and dressed-masses using the DSEs
  - reveal correlations that are key to nucleon structure
  - expose the facts or fallacies in modern descriptions of nucleon structure

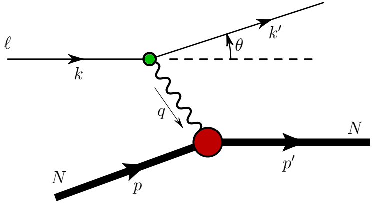
Discover the meaning of *confinement* and its relation to *dynamical chiral symmetry breaking*

– origin of visible mass –

- Nucleon electromagnetic current

$$\langle J^\mu \rangle = u(p') \left[\gamma^\mu F_1(Q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2\,M} F_2(Q^2) \right] u(p)$$

Dirac   Pauli

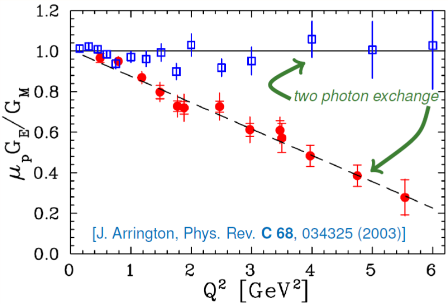


- *Provide vital information about the structure and composition of the most basic elements of nuclear physics*
  - elastic scattering – therefore form factors probe confinement at all energy scales
- Today accurate form factor measurements are creating a paradigm shift in our understanding of nucleon structure:
  - proton radius puzzle
  - $\mu_p\, G_{Ep}/G_{Mp}$ ratio and a possible zero-crossing
  - flavour decomposition and evidence for diquark correlations
  - meson-cloud effects
  - seeking verification of perturbative QCD scaling predictions & scaling violations

Nucleon Sachs Form Factors

- Experiment gives Sachs form factors: $G_E = F_1 - \frac{Q^2}{4\,M^2}\,F_2 \quad G_M = F_1 + F_2$

- Until the late 90s Rosenbluth separation experiments found that the $\mu_p\, G_{Ep}/G_{Mp}$ ratio was flat



- Polarization transfer experiments completely altered our picture of nucleon structure
  - distribution of charge and magnetization are not the same

- Proton charge radius puzzle $[5\sigma]$

$$\langle r_E^2 \rangle = -6\,\frac{\partial}{\partial Q^2}\,G_E(Q^2)\Big|_{Q^2=0}$$

$$r_{Ep} = 0.84184 \pm 0.00067 \text{ fm}$$

muonic hydrogen [Pohl *et al.* (2010)]

$$r_{Ep} = 0.8768 \pm 0.0069 \text{ fm}$$

ep elastic scattering [PDG]

  - one of the most interesting puzzles in hadron physics
  - so far defies explanation

Form Factors in Conformal Limit ($Q^2 \to \infty$)

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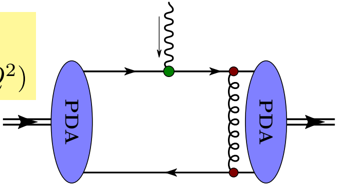
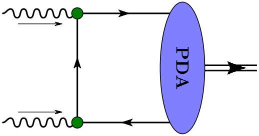
- At asymptotic energies hadron form factors factorize into *parton distribution amplitudes* (PDAs) and a hard scattering amplitude [Brodsky, Lepage 1980]
  - only the valence Fock state ($\bar{q}q$ or qqq) can contribute as $Q^2 \to \infty$
  - both confinement and asymptotic freedom in QCD are important in this limit

- Most is known about $\bar{q}q$ bound states, e.g., for the pion:

$$Q^2\, F_\pi(Q^2) \to 16\pi\, f_\pi^2\, \alpha_s(Q^2)$$



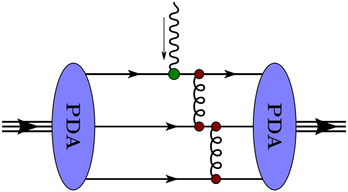
$$Q^2\, F_{\gamma^*\gamma\pi}(Q^2) \to 2\, f_\pi$$



- For nucleon normalization is unknown

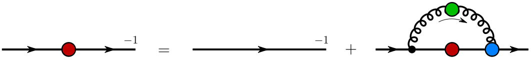
$$G_{E,M}(Q^2 \to \infty) \propto \alpha_s^2(Q^2)/Q^4$$

  - orbital angular momentum effects approach



QCD's Dyson-Schwinger Equations

- The equations of motion of QCD $\Longleftrightarrow$ QCD's Dyson–Schwinger equations
  - an infinite tower of coupled integral equations
  - tractability $\Longrightarrow$ must implement a symmetry preserving truncation
- The most important DSE is QCD's gap equation $\Longrightarrow$ quark propagator

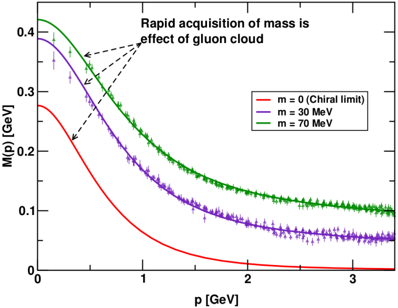


  - ingredients – dressed gluon propagator & dressed quark-gluon vertex

$$S(p) = \frac{Z(p^2)}{i\not p + M(p^2)}$$

- $S(p)$ has correct perturbative limit
- mass function, $M(p^2)$, exhibits dynamical mass generation
- complex conjugate poles
  - no real mass shell $\Longrightarrow$ confinement

[M. S. Bhagwat *et al.*, Phys. Rev. **C 68**, 015203 (2003)]



Pion's Parton Distribution Amplitude

- pion's PDA – $\varphi_\pi(x)$: *is a probability amplitude that describes the momentum distribution of a quark and antiquark in the bound-state's valence Fock state*
  - it's a function of the lightcone momentum fraction $x = \frac{k^+}{p^+}$ and the scale Q^2
- The pion's PDA is defined by

$$f_\pi \, \varphi_\pi(x) = Z_2 \int \frac{d^4k}{(2\pi)^2} \, \delta\left(k^+ - x\, p^+\right) \mathrm{Tr}\left[\gamma^+\gamma_5 \, S(k)\, \Gamma_\pi(k,p)\, S(k-p)\right]$$

- $S(k)\, \Gamma_\pi(k,p)\, S(k-p)$ is the pion's Bethe-Salpeter wave function
  - in the non-relativistic limit it corresponds to the Schrodinger wave function
- $\varphi_\pi(x)$: is the axial-vector projection of the pion's Bethe-Salpeter wave function onto the light-front [pseudo-scalar projection also non-zero]
- Pion PDA is an essentially nonperturbative quantity whose asymptotic form is known; in this regime governs, e.g., Q^2 dependence of pion form factor

$$Q^2\, F_\pi(Q^2) \overset{Q^2\to\infty}{\longrightarrow} 16\,\pi\, f_\pi^2\, \alpha_s(Q^2) \qquad \Longleftrightarrow \qquad \varphi_\pi^{\text{asy}}(x) = 6\,x\,(1-x)$$

QCD Evolution & Asymptotic PDA

- ERBL (Q^2) evolution for pion PDA [c.f. DGLAP equations for PDFs]

$$\mu \frac{d}{d\mu} \varphi(x,\mu) = \int_0^1 dy\, V(x,y)\, \varphi(y,\mu)$$

- This evolution equation has a solution of the form

$$\varphi_\pi(x,Q^2) = 6\,x\,(1-x) \left[1 + \sum_{n=2,\,4,\ldots} a_n^{3/2}(Q^2)\, C_n^{3/2}(2x-1)\right]$$

  - $\alpha = 3/2$ because in $Q^2 \to \infty$ limit QCD is invariant under the collinear conformal group $SL(2;\mathbb{R})$
  - Gegenbauer-$\alpha = 3/2$ polynomials are irreducible representations $SL(2;\mathbb{R})$

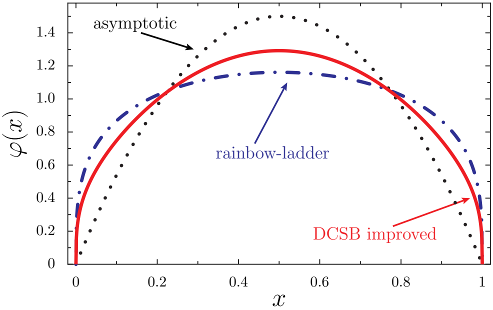
- The coefficients of the Gegenbauer polynomials, $a_n^{3/2}(Q^2)$, evolve logarithmically to zero as $Q^2 \to \infty$: $\varphi_\pi(x) \to \varphi_\pi^{\text{asy}}(x) = 6\,x\,(1-x)$

- At what scales is this a good approximation to the pion PDA?

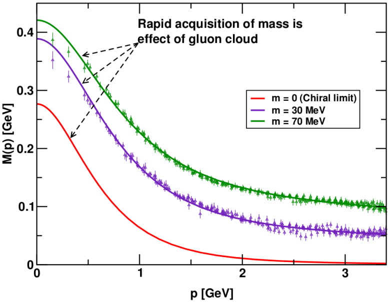
- E.g., AdS/QCD find $\varphi_\pi(x) \sim x^{1/2}\,(1-x)^{1/2}$ at $Q^2 = 1\,\text{GeV}^2$; expansion in terms of $C_n^{3/2}(2x-1)$ convergences slowly: $a_{32}^{3/2} / a_2^{3/2} \sim 10\,\%$

Pion PDA from the DSEs

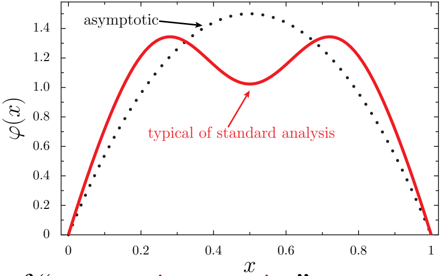
[L. Chang, ICC, *et al.*, Phys. Rev. Lett. **110**, 132001 (2013)]



[C.D. Roberts, Prog. Part. Nucl. Phys. 61 50 (2008)]



- Both DSE results, each using a different Bethe-Salpeter kernel, exhibit a pronounced broadening compared with the asymptotic pion PDA
  - scale of calculation is given by renormalization point $\zeta = 2\,\text{GeV}$
- Broading of the pion's PDA is directly linked to DCSB
- As we shall see the dilation of pion's PDA will influence the Q^2 evolution of the pion's electromagnetic form factor



- Lattice QCD can only determine one non-trivial moment

$$\int_0^1 dx\,(2\,x-1)^2 \varphi_\pi(x) = 0.27 \pm 0.04$$

[V. Braun *et al.*, Phys. Rev. D **74**, 074501 (2006)]

  - scale is $Q^2 = 4\,\text{GeV}^2$

- Standard practice to fit first coefficient of "*asymptotic expansion*" to moment

$$\varphi_\pi(x,Q^2) = 6\,x\,(1-x)\left[1 + \sum_{n=2,\,4,\ldots} a_n^{3/2}(Q^2)\, C_n^{3/2}(2x-1)\right]$$

  - however this expansion is guaranteed to converge rapidly only when $Q^2 \to \infty$
  - this procedure results in a *double-humped* pion PDA

- Advocate using a *generalized expansion*

$$\varphi_\pi(x,Q^2) = N_\alpha\, x^{\alpha-1/2}(1-x)^{\alpha-1/2}\left[1 + \sum_{n=2,\,4,\ldots} a_n^{\alpha}(Q^2)\, C_n^{\alpha}(2x-1)\right]$$

- Find $\varphi_\pi \simeq x^\alpha(1-x)^\alpha,\ \alpha = 0.35^{+0.32}_{-0.24}$; good agreement with DSE: $\alpha \simeq 0.30$

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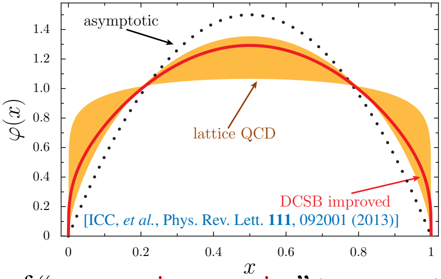

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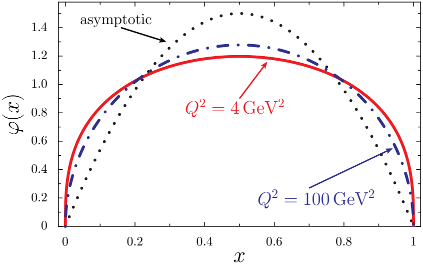
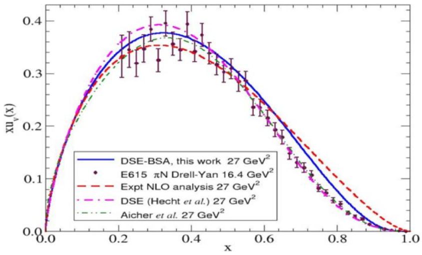
When is the Pion's PDA Asymptotic

[I. C. Cloët, *et al.*, Phys. Rev. Lett. **111**, 092001 (2013)]



[T. Nguyen, *et al.*, Phys. Rev. C **83**, 062201 (2011)]

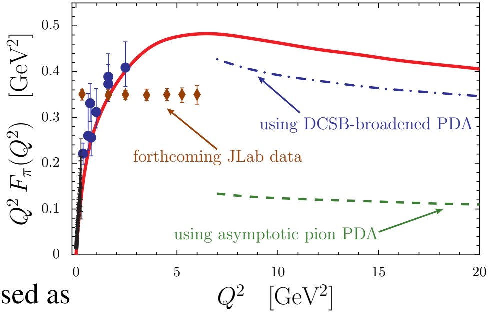


- Under leading order Q^2 evolution the pion PDA remains broad to well above $Q^2 > 100\,\mathrm{GeV}^2$, compared with $\varphi_\pi^{\rm asy}(x) = 6\,x\,(1-x)$
- *Consequently, the asymptotic form of the pion PDA is a poor approximation at all energy scales that are either currently accessible or foreseeable in experiments on pion elastic and transition form factors*
- Asymptotic pion PDA, $\varphi_\pi^{\rm asy}(x)$, only guaranteed be accurate approximation to $\varphi_\pi(x)$ when pion valence quark PDF satisfies: $q_v^\pi(x) \sim \delta(x)$
- This is far from valid at forseeable energy scales

Pion Elastic Form Factor

- Direct, symmetry-preserving computation of pion form factor predicts maximum in $Q^2 F_\pi(Q^2)$ at $Q^2 \approx 6\,\text{GeV}^2$
  - magnitude of this product is determined by strength of DCSB at all accessible scales

[L. Chang, ICC, *et al.*, Phys. Rev. Lett. **111**, 141802 (2013)]



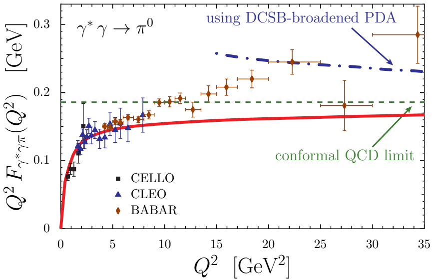
- The QCD prediction can be expressed as

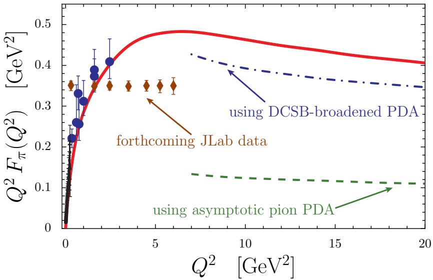
$$Q^2 F_\pi(Q^2) \overset{Q^2 \gg \Lambda^2_{\rm QCD}}{\sim} 16\,\pi\, f_\pi^2\, \alpha_s(Q^2)\, w_\pi^2; \qquad w_\pi = \frac{1}{3}\int_0^1 dx\, \frac{1}{x}\, \varphi_\pi(x)$$

- Continuum-QCD methods have provided a new picture of the pion form factor; with excellent agreement between direct calculation and the hard scattering formula – when all elements are computed self-consistently

- *Continuum-QCD predicts that QCD power law behaviour – with QCD's scaling law violations – sets in at* $Q^2 \sim 8\,\text{GeV}^2$

Pion Transition Form Factor



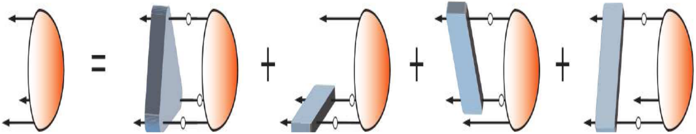


- At large Q^2 the hard gluon exchange in the $\gamma^* + \pi \to \pi$ form factor – needed to keep the pion intact – results in distinctly different behaviour to the pion transition form factor $\gamma^* + \pi \to \gamma$

$$Q^2 F_{\gamma^*\pi\gamma}(Q^2) \to 2\, f_\pi\, w_\pi^2 \qquad \text{c.f.} \quad Q^2 F_\pi(Q^2) \to 16\,\pi\, f_\pi^2\, \alpha_s(Q^2)\, w_\pi^2$$

- Therefore approach to asymptotic limit gives *inter alia* a unique window into quark-gluon confinement in QCD
- In full DSE calculation of $\gamma^*\pi \to \gamma$ conformal limit approached from below

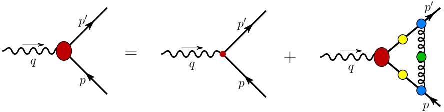
- The nucleon is a bound state of 3 dressed-quarks and in QCD appears as the lowest lying pole in a 6-point Green functions
- In DSEs wave function obtained from a Poincaré covariant Faddeev equation



  - sums all possible interactions between three dressed-quarks
  - strong diquark correlations a dynamical consequence of strong coupling in QCD
- A tractable Faddeev equation is based on the observation that an interaction which describes colour-singlet mesons also generates *non-pointlike* diquark correlations in the colour-$\bar{3}$ channel
  - scalar and axial-vector diquarks are most important for the nucleon
- Diquarks are directly related to DCSB, as this single mechanism produces both the (almost) massless pion and strong scalar diquark correlations

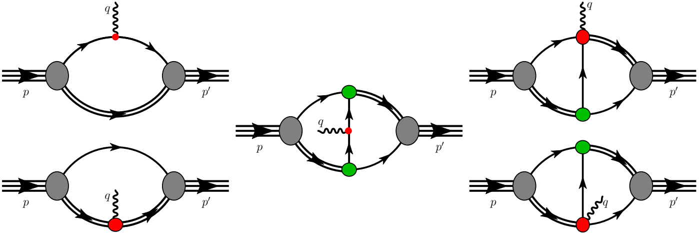
Nucleon EM Form Factors from DSEs

- A robust description of form factors is only possible if electromagnetic gauge invariance is respected; equivalently all relevant Ward-Takahashi identities (WTIs) must be satisfied
- For quark-photon vertex WTI implies:



$$q_\mu \, \Gamma^\mu_{\gamma qq}(p', p) = \hat{Q}_q \left[S_q^{-1}(p') - S_q^{-1}(p) \right]$$

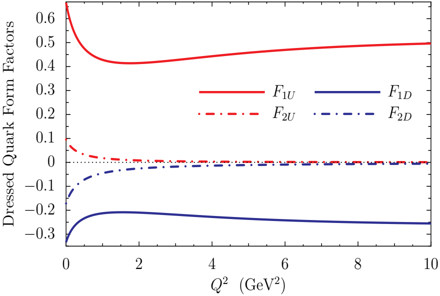
  - transverse structure unconstrained
- Diagrams needed for a gauge invariant nucleon EM current in DSEs



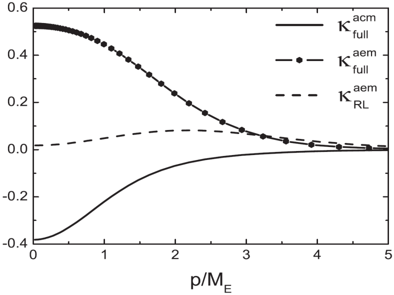
- Feedback with experiment can constrain elements of QCD via DSEs

Dressed Quarks are not Point Particles

[ICC, W. Bentz, A. W. Thomas, Phys. Rev. C **90**, 045202 (2014)]



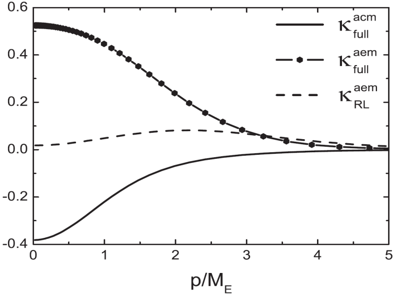
[L. Chang, Y. -X. Liu, C. D. Roberts, PRL **106**, 072001 (2011)]



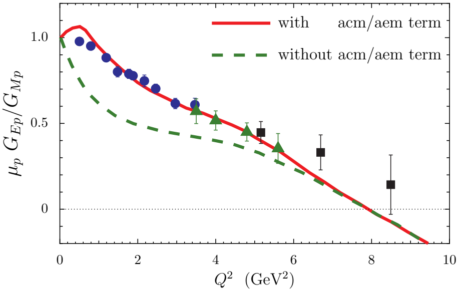
- EM properties of a spin-$\frac{1}{2}$ point particle are characterized by two quantities:
  - charge: e & magnetic moment: μ
- Strong gluon cloud dressing produces – from a massless quark – a dressed quark with $M \sim 400\,\text{MeV}$
  - expect gluon dressing to produce non-trivial EM structure for a dressed quark
  - analogous to pion dressing on nucleon giving large anomalous magnetic moment
- A large quark anomalous chromomagnetic moment in the quark-gluon vertex – *produces a large quark anomalous electromagnetic moment*

Proton G_E/G_M Ratio

[L. Chang, Y. -X. Liu, C. D. Roberts, Phys. Rev. Lett. **106**, 072001 (2011)]



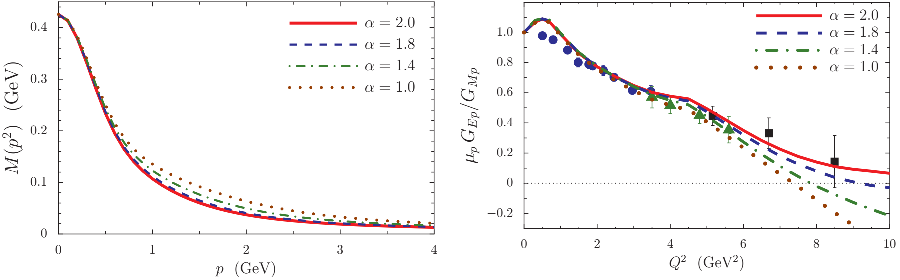
[ICC, C. D. Roberts, Prog. Part. Nucl. Phys. **77**, 1 (2014)]



- Latest results include effect from anomalous chromomagnetic moment term in the quark–gluon vertex
  - generates large anomalous electromagnetic term in quark–photon vertex
- Quark anomalous magnetic moment required for good agreement with data
  - important for low to moderate Q^2
- For massless quarks anomalous chromomagnetic moment is only possible via DSCB

Proton G_E form factor and DCSB

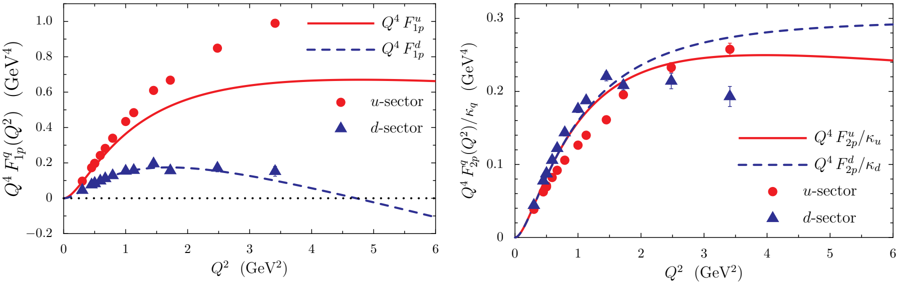
[I. C. Cloët, C. D. Roberts and A. W. Thomas, Phys. Rev. Lett. **111**, 101803 (2013)]



- Find that slight changes in $M(p)$ on the domain $1 \lesssim p \lesssim 3$ GeV have a striking effect on the G_E/G_M proton form factor ratio
  - *strong indication that position of a zero is very sensitive to underlying dynamics and the nature of the transition from nonperturbative to perturbative QCD*
- Zero in $G_E = F_1 - \frac{Q^2}{4\,M_N^2}\,F_2$ largely determined by evolution of $Q^2\,F_2$
  - F_2 is sensitive to DCSB through the dynamically generated quark anomalous electromagnetic moment – *vanishes in perturbative limit*
  - the quicker the perturbative regime is reached the quicker $F_2 \to 0$

Flavour separated proton form factors

[ICC, W. Bentz, A. W. Thomas, Phys. Rev. C **90**, 045202 (2014)]

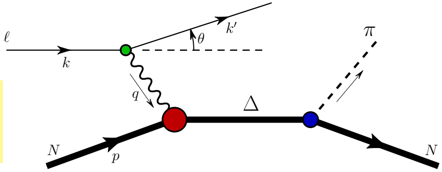


- Prima facie, these experimental results are remarkable
  - u and d quark sector form factors have very different scaling behaviour
- However, when viewed in context of diquark correlations results are straightforward to understand
  - e.g. in the proton the d quark is much more likely to be in a scalar diquark than the doubly-represented u quark; diquark $\implies 1/Q^2$ suppression
- Results for F_{2p}^q are influenced at low Q^2 by of magnetic moment enhancement from axial-vector diquarks and dressed quarks: $|\mu_d| \gg |\mu_u|$

Nucleon to Resonance Transitions

- Given the challenges posed by non-perturbative QCD it is insufficient to study hadron ground-states alone [see talks by Gothe, Mokeev]
- Nucleon to resonance transition form factors provide a critical extension to elastic form factors – providing many more windows and different perspectives on quark-gluon dynamics
  - e.g. nucleon resonances are more sensitive to long-range effects in QCD than the properties of ground states . . . analogous to exotic and hybrid mesons
- Important example is $N \to \Delta$ transition – parametrized by three form factors
  - $G_E^*(Q^2), \quad G_M^*(Q^2), \quad G_C^*(Q^2)$
  - if both N and Δ were purely S-wave then $G_E^*(Q^2) = 0 = G_C^*(Q^2)$
- When analyzing the $N \to \Delta$ transition it is common to construct the ratios:

$$R_{EM} = -\frac{G_E^*}{G_M^*}, \qquad R_{SM} = -\frac{|\boldsymbol{q}|}{2\,M_\Delta}\,\frac{G_C^*}{G_M^*}$$



$N \to \Delta$ form factor from the DSEs

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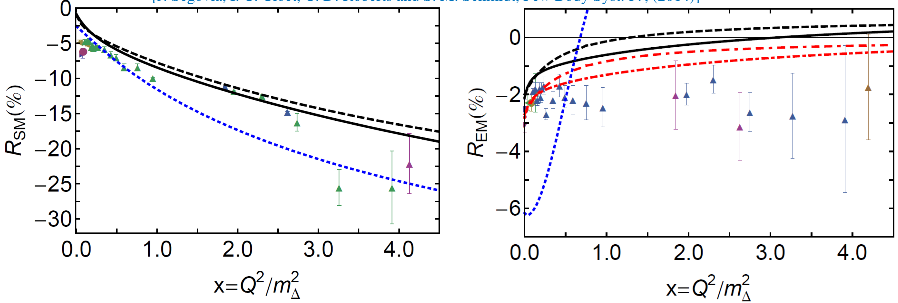
[J. Segovia, I. C. Cloët, C. D. Roberts and S. M. Schmidt, Few Body Syst. **57**, (2014)]



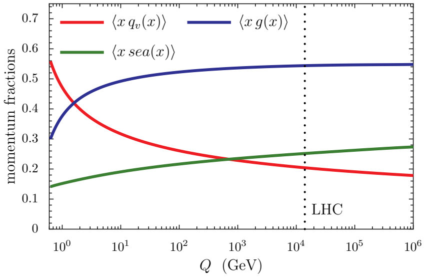
- Find that $R_{EM} = -\frac{G_E^*}{G_M^*}$ is a particular sensitive measure of *quark orbital angular momentum corrections* in the nucleon and Δ
- For $R_{SM} = -\frac{|\boldsymbol{q}|}{2\,M_\Delta}\,\frac{G_C^*}{G_M^*}$ DSEs reproduces rapid fall off with Q^2
- Perturbative QCD predictions are reproduced: $R_{EM} \to 1,\quad R_{SM} \to$ constant
  - however these asymptotic results are not reached until incredibility large Q^2; will not be accessible at any present or foreseeable facility
  - analogous to PDFs, where asymptotic valence PDFs are delta functions, however even at LHC energies this is far from the case

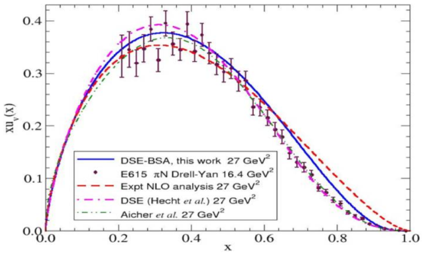
Conclusion

- QCD and therefore Hadron Physics is unique:
  - must confront a fundamental theory in which the elementary degrees-of-freedom are confined and only hadrons reach detectors
- QCD will only be solved by deploying a diverse array of experimental and theoretical methods:
  - must define and solve the problems of confinement and its relationship with DCSB
- These are two of the most important challenges in fundamental Science
- As an example, feedback with EM form factor measurements can constrain QCD's quark–gluon vertex within the DSE framework
  - knowledge of quark–gluon vertex provides $\alpha_s(Q^2)$ within DSEs $\Leftrightarrow$ confinement
- A robust understanding of the nucleon PDAs is an important near term goal
- *Experimental and theoretical study of the bound state problem in continuum QCD promises to provide many more insights*

Backup Slides

When is the Pion's Valence PDF Asymptotic





- LO QCD evolution of momentum fraction carried by valence quarks

$$\langle x\, q_v(x)\rangle (Q^2) = \left(\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right)^{\gamma_{qq}^{(0)2}/(2\beta_0)} \langle x\, q_v(x)\rangle (Q_0^2) \quad \text{where} \quad \frac{\gamma_{qq}^{(0)2}}{2\beta_0} > 0$$

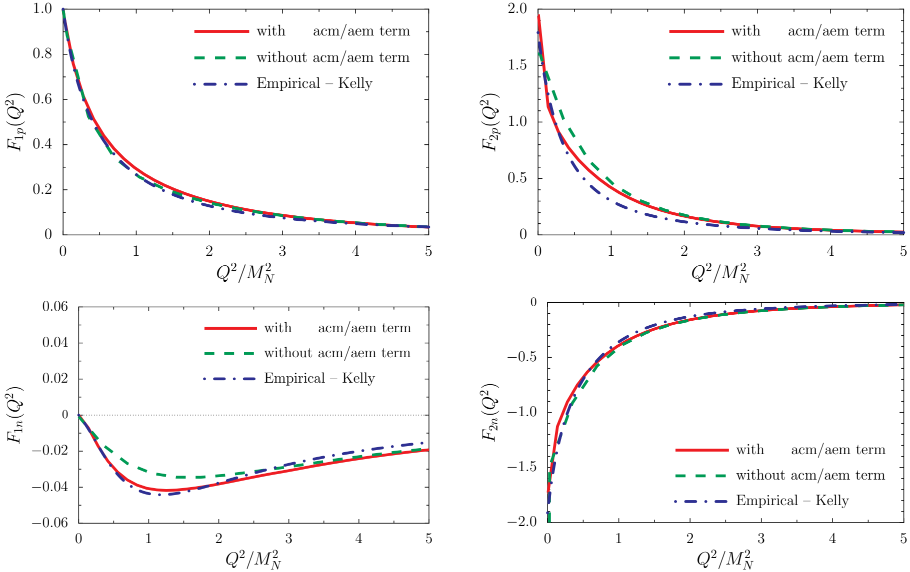
  - therefore, as $Q^2 \to \infty$ we have $\langle x\, q_v(x)\rangle \to 0$ implies $q_v(x) = \delta(x)$

- At LHC energies valence quarks still carry 20% of pion momentum
  - the gluon distribution saturates at $\langle x\, g(x)\rangle \sim 55\%$

- *Asymptotia is a long way away!*

Nucleon Dirac & Pauli form factors

[ICC, G. Eichmann, B. El-Bennich, T. Klahn and C. D. Roberts,, Few Body Syst. **46**, 1 (2009)]

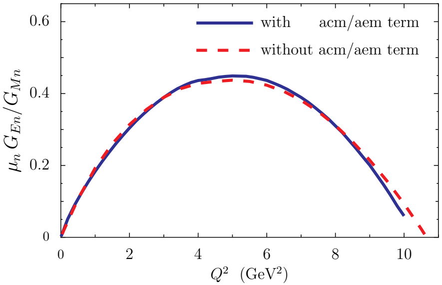


- quark aem term has important influence on Pauli form factors at low Q^2

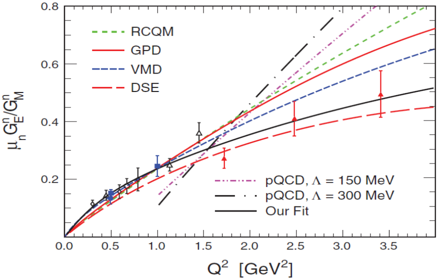
Neutron G_E/G_M Ratio

Argonne NATIONAL LABORATORY

[ICC, C. D. Roberts, Prog. Part. Nucl. Phys. **77**, 1 (2014)]



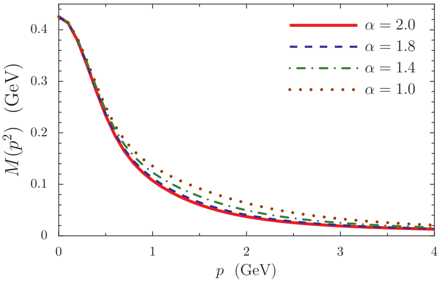
[S. Riordan *et al*, Phys. Rev. Lett. **105**, 262302 (2010)]

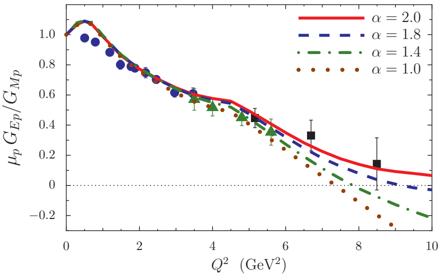


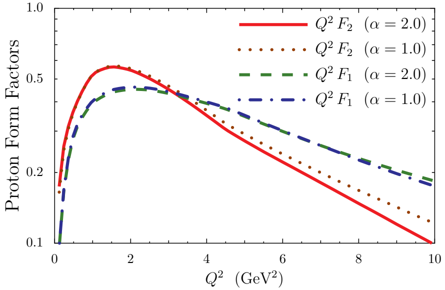
- Quark anomalous chromomagnetic moment – which drives the large anomalous electromagnetic moment – has only a minor impact on neutron Sachs form factor ratio
- Predict a zero-crossing in G_{En}/G_{Mn} at $Q^2 \sim 11\,\text{GeV}^2$
- DSE *predictions* were confirmed on domain $1.5 \lesssim Q^2 \lesssim 3.5\,\text{GeV}^2$

Proton G_E form factor and DCSB

[I. C. Cloët, C. D. Roberts and A. W. Thomas, Phys. Rev. Lett. **111**, 101803 (2013)]







- Recall: $G_E = F_1 - \frac{Q^2}{4 M_N^2} F_2$
- Only G_E is senitive to these small changes in the mass function
- *Accurate determination of zero crossing would put important contraints on quark-gluon dynamics within DSE framework*