Meson and baryon spectroscopy

- the discovery of exotic or hybrid hadrons would force a dramatic reassessment of the distinction between the notions of matter fields and force fields

Precision experimental study of the valence region, together with theoretical computation of distribution functions and distribution amplitudes

- computation is critical – as data can only reveal limited information about the theory underlying strong interaction physics

Exploit opportunities provided by new data on nucleon elastic and transition form factors

- chart infrared evolution of QCD’s coupling and dressed-masses using the DSEs
- reveal correlations that are key to nucleon structure
- expose the facts or fallacies in modern descriptions of nucleon structure
Discover the meaning of confinement and its relation to dynamical chiral symmetry breaking
– origin of visible mass –
Nucleon Electromagnetic Form Factors

Nucleon electromagnetic current

\[ \langle J^\mu \rangle = u(p') \left[ \gamma^\mu F_1(Q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M} F_2(Q^2) \right] u(p) \]

Provide vital information about the structure and composition of the most basic elements of nuclear physics

- elastic scattering – therefore form factors probe confinement at all energy scales

Today accurate form factor measurements are creating a paradigm shift in our understanding of nucleon structure:

- proton radius puzzle
- \( \mu_p G_{Ep}/G_{Mp} \) ratio and a possible zero-crossing
- flavour decomposition and evidence for diquark correlations
- meson-cloud effects
- seeking verification of perturbative QCD scaling predictions & scaling violations
Nucleon Sachs Form Factors

- Experiment gives Sachs form factors:
  \[ G_E = F_1 - \frac{Q^2}{4M^2} F_2 \]
  \[ G_M = F_1 + F_2 \]

- Until the late 90s Rosenbluth separation experiments found that the \( \mu_p G_{Ep}/G_{Mp} \) ratio was flat

- Polarization transfer experiments completely altered our picture of nucleon structure
  - distribution of charge and magnetization are not the same

- Proton charge radius puzzle \([5\sigma]\)
  \[ r_{Ep} = 0.84184 \pm 0.00067 \text{ fm} \]

muonic hydrogen \([\text{Pohl et al. (2010)}]\)
  - one of the most interesting puzzles in hadron physics
  - so far defies explanation
At asymptotic energies hadron form factors factorize into *parton distribution amplitudes* (PDAs) and a hard scattering amplitude [Brodsky, Lepage 1980]

- only the valence Fock state ($\bar{q}q$ or $qqq$) can contribute as $Q^2 \to \infty$
- both confinement and asymptotic freedom in QCD are important in this limit

Most is known about $\bar{q}q$ bound states, e.g., for the pion:

$$Q^2 F_\pi(Q^2) \to 16\pi f_\pi^2 \alpha_s(Q^2)$$

For nucleon normalization is unknown

$$G_{E,M}(Q^2 \to \infty) \propto \alpha_s^2(Q^2)/Q^4$$

orbital angular momentum effects approach
QCD’s Dyson-Schwinger Equations

- The equations of motion of QCD ⇐⇒ QCD’s Dyson–Schwinger equations
  - an infinite tower of coupled integral equations
  - tractability ⇒ must implement a symmetry preserving truncation

- The most important DSE is QCD’s gap equation ⇒ quark propagator

\[
\begin{align*}
S(p) &= \frac{Z(p^2)}{\frac{i\not{p}}{\not{p} + M(p^2)}} \\
S(p) &= \frac{Z(p^2)}{\frac{i\not{p}}{\not{p} + M(p^2)}} - \frac{1}{\not{p} + M(p^2)} + \cdots
\end{align*}
\]

- ingredients – dressed gluon propagator & dressed quark-gluon vertex

\[ S(p) = \frac{Z(p^2)}{\frac{i\not{p}}{\not{p} + M(p^2)}} \]

- \( S(p) \) has correct perturbative limit
- mass function, \( M(p^2) \), exhibits dynamical mass generation
- complex conjugate poles
  - no real mass shell \( \Rightarrow \) confinement


[Graph showing mass function \( M(p) \) with different curves for \( m = 0 \) (Chiral limit), \( m = 30 \text{ MeV} \), and \( m = 70 \text{ MeV} \).]
**Pion’s Parton Distribution Amplitude**

- pion’s PDA – $\varphi_\pi(x)$: *is a probability amplitude that describes the momentum distribution of a quark and antiquark in the bound-state’s valence Fock state*
  - it’s a function of the lightcone momentum fraction $x = \frac{k^+}{p^+}$ and the scale $Q^2$

- The pion’s PDA is defined by

\[
f_\pi \varphi_\pi(x) = Z_2 \int \frac{d^4k}{(2\pi)^2} \delta\left(k^+ - xp^+\right) \text{Tr} \left[\gamma^+\gamma_5 S(k) \Gamma_\pi(k,p) S(k-p)\right]
\]

- $S(k) \Gamma_\pi(k,p) S(k-p)$ is the pion’s Bethe-Salpeter wave function
  - in the non-relativistic limit it corresponds to the Schrödinger wave function

- $\varphi_\pi(x)$: is the axial-vector projection of the pion’s Bethe-Salpeter wave function onto the light-front  
  - [pseudo-scalar projection also non-zero]

- Pion PDA is an essentially nonperturbative quantity whose asymptotic form is known; in this regime governs, e.g., $Q^2$ dependence of pion form factor

\[
Q^2 F_\pi(Q^2) \xrightarrow{Q^2 \to \infty} 16\pi f_\pi^2 \alpha_s(Q^2) \iff \varphi_\pi^{\text{asy}}(x) = 6x(1-x)
\]
ERBL ($Q^2$) evolution for pion PDA \[c.f.\] DGLAP equations for PDFs

$$\mu \frac{d}{d\mu} \varphi(x, \mu) = \int_0^1 dy \ V(x, y) \varphi(y, \mu)$$

This evolution equation has a solution of the form

$$\varphi_{\pi}(x, Q^2) = 6 \ x (1 - x) \left[1 + \sum_{n=2, 4, \ldots} a_n^{3/2}(Q^2) C_n^{3/2}(2x - 1)\right]$$

- $\alpha = 3/2$ because in $Q^2 \to \infty$ limit QCD is invariant under the collinear conformal group $SL(2; \mathbb{R})$
- Gegenbauer-$\alpha = 3/2$ polynomials are irreducible representations $SL(2; \mathbb{R})$

The coefficients of the Gegenbauer polynomials, $a_n^{3/2}(Q^2)$, evolve logarithmically to zero as $Q^2 \to \infty$: $\varphi_{\pi}(x) \to \varphi_{\pi}^{\text{asy}}(x) = 6 \ x (1 - x)$

At what scales is this a good approximation to the pion PDA?

E.g., AdS/QCD find $\varphi_{\pi}(x) \sim x^{1/2} (1 - x)^{1/2}$ at $Q^2 = 1 \text{ GeV}^2$; expansion in terms of $C_n^{3/2}(2x - 1)$ converges slowly: $a_{32}^{3/2} / a_2^{3/2} \sim 10\%$
Both DSE results, each using a different Bethe-Salpeter kernel, exhibit a pronounced broadening compared with the asymptotic pion PDA.

- Scale of calculation is given by renormalization point $\zeta = 2$ GeV

Broading of the pion’s PDA is directly linked to DCSB.

As we shall see the dilation of pion’s PDA will influence the $Q^2$ evolution of the pion’s electromagnetic form factor.
Lattice QCD can only determine one non-trivial moment
\[ \int_0^1 dx (2x - 1)^2 \varphi_\pi(x) = 0.27 \pm 0.04 \]

[V. Braun et al., Phys. Rev. D 74, 074501 (2006)]

scale is \( Q^2 = 4 \text{ GeV}^2 \)

Standard practice to fit first coefficient of “asymptotic expansion” to moment
\[ \varphi_\pi(x, Q^2) = 6x(1-x) \left[ 1 + \sum_{n=2, 4, \ldots} a_n^{3/2}(Q^2) C_n^{3/2}(2x - 1) \right] \]

however this expansion is guaranteed to converge rapidly only when \( Q^2 \to \infty \)
\[ \text{this procedure results in a double-humped pion PDA} \]

Advocate using a generalized expansion
\[ \varphi_\pi(x, Q^2) = N_\alpha x^{\alpha - 1/2}(1-x)^{\alpha - 1/2} \left[ 1 + \sum_{n=2, 4, \ldots} a_n^\alpha(Q^2) C_n^\alpha(2x - 1) \right] \]

Find \( \varphi_\pi \sim x^\alpha(1-x)^\alpha, \quad \alpha = 0.35^{+0.32}_{-0.24} \); good agreement with DSE: \( \alpha \simeq 0.30 \)
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Find \( \varphi_\pi \approx x^\alpha (1-x)^\alpha \), \( \alpha = 0.35^{+0.32}_{-0.24} \); good agreement with DSE: \( \alpha \approx 0.30 \)
Under leading order $Q^2$ evolution the pion PDA remains broad to well above $Q^2 > 100$ GeV$^2$, compared with $\varphi^\text{asy}_\pi(x) = 6x(1-x)$

Consequently, the asymptotic form of the pion PDA is a poor approximation at all energy scales that are either currently accessible or foreseeable in experiments on pion elastic and transition form factors.

Asymptotic pion PDA, $\varphi^\text{asy}_\pi(x)$, only guaranteed be accurate approximation to $\varphi_\pi(x)$ when pion valence quark PDF satisfies: $q^\text{v}_\pi(x) \sim \delta(x)$

This is far from valid at foreseeable energy scales.
Direct, symmetry-preserving computation of pion form factor predicts maximum in $Q^2 F_\pi(Q^2)$ at $Q^2 \approx 6$ GeV$^2$

- magnitude of this product is determined by strength of DCSB at all accessible scales

The QCD prediction can be expressed as

$$Q^2 F_\pi(Q^2) \propto \frac{1}{\Lambda_{\text{QCD}}^2} \quad Q^2 \gg \Lambda_{\text{QCD}}^2 \quad 16 \pi f_\pi^2 \alpha_s(Q^2) \omega_\pi^2; \quad \omega_\pi = \frac{1}{3} \int_0^1 dx \frac{1}{x} \varphi_\pi(x)$$

Continuum-QCD methods have provided a new picture of the pion form factor; with excellent agreement between direct calculation and the hard scattering formula – when all elements are computed self-consistently

**Continuum-QCD predicts that QCD power law behaviour – with QCD’s scaling law violations – sets in at** $Q^2 \sim 8$ GeV$^2$
At large $Q^2$ the hard gluon exchange in the $\gamma^* + \pi \rightarrow \pi$ form factor – needed to keep the pion intact – results in distinctly different behaviour to the pion transition form factor $\gamma^* + \pi \rightarrow \gamma$

$$Q^2 F_{\gamma^*\pi\gamma}(Q^2) \rightarrow 2 f_\pi \omega^2_\pi$$

c.f.

$$Q^2 F_{\pi}(Q^2) \rightarrow 16 \pi f_\pi^2 \alpha_s(Q^2) \omega^2_\pi$$

Therefore approach to asymptotic limit gives *inter alia* a unique window into quark-gluon confinement in QCD

In full DSE calculation of $\gamma^*\pi \rightarrow \gamma$ conformal limit approached from below
The nucleon is a bound state of 3 dressed-quarks and in QCD appears as the lowest lying pole in a 6-point Green functions.

In DSEs wave function obtained from a Poincaré covariant Faddeev equation sums all possible interactions between three dressed-quarks.

- strong diquark correlations a dynamical consequence of strong coupling in QCD
- A tractable Faddeev equation is based on the observation that an interaction which describes colour-singlet mesons also generates non-pointlike diquark correlations in the colour-\( \bar{3} \) channel.
- scalar and axial-vector diquarks are most important for the nucleon.
- Diquarks are directly related to DCSB, as this single mechanism produces both the (almost) massless pion and strong scalar diquark correlations.
A robust description of form factors is only possible if electromagnetic gauge invariance is respected; equivalently all relevant Ward-Takahashi identities (WTIs) must be satisfied.

For quark-photon vertex WTI implies:

\[ q_\mu \Gamma_{\gammaqq}^\mu(p', p) = \hat{Q}_q \left[ S_q^{-1}(p') - S_q^{-1}(p) \right] \]

transverse structure unconstrained

Diagrams needed for a gauge invariant nucleon EM current in DSEs

Feedback with experiment can constrain elements of QCD via DSEs
Dressed Quarks are not Point Particles

EM properties of a spin-$\frac{1}{2}$ point particle are characterized by two quantities:
- charge: $e$ & magnetic moment: $\mu$

Strong gluon cloud dressing produces – from a massless quark – a dressed quark with $M \sim 400\,\text{MeV}$
- expect gluon dressing to produce non-trivial EM structure for a dressed quark
- analogous to pion dressing on nucleon giving large anomalous magnetic moment

A large quark anomalous chromomagnetic moment in the quark-gluon vertex – *produces a large quark anomalous electromagnetic moment*
Latest results include effect from anomalous chromomagnetic moment term in the quark–gluon vertex

- generates large anomalous electromagnetic term in quark–photon vertex

- Quark anomalous magnetic moment required for good agreement with data
  - important for low to moderate $Q^2$

- For massless quarks anomalous chromomagnetic moment is only possible via DSCB


[ICC, C. D. Roberts, Prog. Part. Nucl. Phys. 77, 1 (2014)]
Find that slight changes in $M(p)$ on the domain $1 \lesssim p \lesssim 3$ GeV have a striking effect on the $G_E/G_M$ proton form factor ratio.

- strong indication that position of a zero is very sensitive to underlying dynamics and the nature of the transition from nonperturbative to perturbative QCD

Zero in $G_E = F_1 - \frac{Q^2}{4M_N^2} F_2$ largely determined by evolution of $Q^2 F_2$

- $F_2$ is sensitive to DCSB through the dynamically generated quark anomalous electromagnetic moment – vanishes in perturbative limit
- the quicker the perturbative regime is reached the quicker $F_2 \to 0$
Prima facie, these experimental results are remarkable
- $u$ and $d$ quark sector form factors have very different scaling behaviour

However, when viewed in context of diquark correlations results are straightforward to understand
- e.g. in the proton the $d$ quark is much more likely to be in a scalar diquark than the doubly-represented $u$ quark; diquark $\implies 1/Q^2$ suppression

Results for $F^q_{2p}$ are influenced at low $Q^2$ by of magnetic moment enhancement from axial-vector diquarks and dressed quarks: $|\mu_d| \gg |\mu_u|$
Given the challenges posed by non-perturbative QCD it is insufficient to study hadron ground-states alone [see talks by Gothe, Mokeev]

Nucleon to resonance transition form factors provide a critical extension to elastic form factors – providing many more windows and different perspectives on quark-gluon dynamics

- e.g. nucleon resonances are more sensitive to long-range effects in QCD than the properties of ground states . . . analogous to exotic and hybrid mesons

Important example is $N \rightarrow \Delta$ transition – parametrized by three form factors

- $G_E^*(Q^2)$, $G_M^*(Q^2)$, $G_C^*(Q^2)$
- if both $N$ and $\Delta$ were purely $S$-wave then $G_E^*(Q^2) = 0 = G_C^*(Q^2)$

When analyzing the $N \rightarrow \Delta$ transition it is common to construct the ratios:

$$R_{EM} = -\frac{G_E^*}{G_M^*}, \quad R_{SM} = -\frac{|q|}{2M\Delta} \frac{G_C^*}{G_M^*}$$
$^{N\rightarrow\Delta}$ form factor from the DSEs

Find that $R_{EM} = -\frac{G_E^*}{G_M^*}$ is a particular sensitive measure of quark orbital angular momentum corrections in the nucleon and $\Delta$.

For $R_{SM} = -\frac{|q|}{2M_\Delta} \frac{G_C^*}{G_M^*}$ DSEs reproduces rapid fall off with $Q^2$.

Perturbative QCD predictions are reproduced: $R_{EM} \rightarrow 1$, $R_{SM} \rightarrow$ constant.

- However these asymptotic results are not reached until incredibly large $Q^2$; will not be accessible at any present or foreseeable facility.
- Analogous to PDFs, where asymptotic valence PDFs are delta functions, however even at LHC energies this is far from the case.
QCD and therefore Hadron Physics is unique:
- must confront a fundamental theory in which the elementary degrees-of-freedom are confined and only hadrons reach detectors

QCD will only be solved by deploying a diverse array of experimental and theoretical methods:
- must define and solve the problems of confinement and its relationship with DCSB

These are two of the most important challenges in fundamental Science

As an example, feedback with EM form factor measurements can constrain QCD’s quark–gluon vertex within the DSE framework
- knowledge of quark–gluon vertex provides $\alpha_s(Q^2)$ within DSEs $\Leftrightarrow$ confinement

A robust understanding of the nucleon PDAs is an important near term goal

*Experimental and theoretical study of the bound state problem in continuum QCD promises to provide many more insights*
Backup Slides
LO QCD evolution of momentum fraction carried by valence quarks

\[
\langle x q_v(x) \rangle (Q^2) = \left( \frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)} \right) \frac{\gamma_{qq}^{(0)^2}}{2\beta_0} \langle x q_v(x) \rangle (Q_0^2) \quad \text{where} \quad \frac{\gamma_{qq}^{(0)^2}}{2\beta_0} > 0
\]

therefore, as \(Q^2 \rightarrow \infty\) we have \(\langle x q_v(x) \rangle \rightarrow 0\) implies \(q_v(x) = \delta(x)\)

At LHC energies valence quarks still carry 20% of pion momentum

- the gluon distribution saturates at \(\langle x g(x) \rangle \sim 55\%

Asymptotia is a long way away!
Nucleon Dirac & Pauli form factors


![Graphs showing Dirac and Pauli form factors](image)

- Quark aem term has important influence on Pauli form factors at low $Q^2$
Neutron $G_E/G_M$ Ratio

Quark anomalous chromomagnetic moment – which drives the large anomalous electromagnetic moment – has only a minor impact on neutron Sachs form factor ratio.

- Predict a zero-crossing in $G_{En}/G_{Mn}$ at $Q^2 \sim 11$ GeV$^2$
- DSE predictions were confirmed on domain $1.5 \lesssim Q^2 \lesssim 3.5$ GeV$^2$
Proton $G_E$ form factor and DCSB

Recall: $G_E = F_1 - \frac{Q^2}{4 M_N^2} F_2$

Only $G_E$ is sensitive to these small changes in the mass function

Accurate determination of zero crossing would put important constraints on quark-gluon dynamics within DSE framework