# Nucleon Tomography:

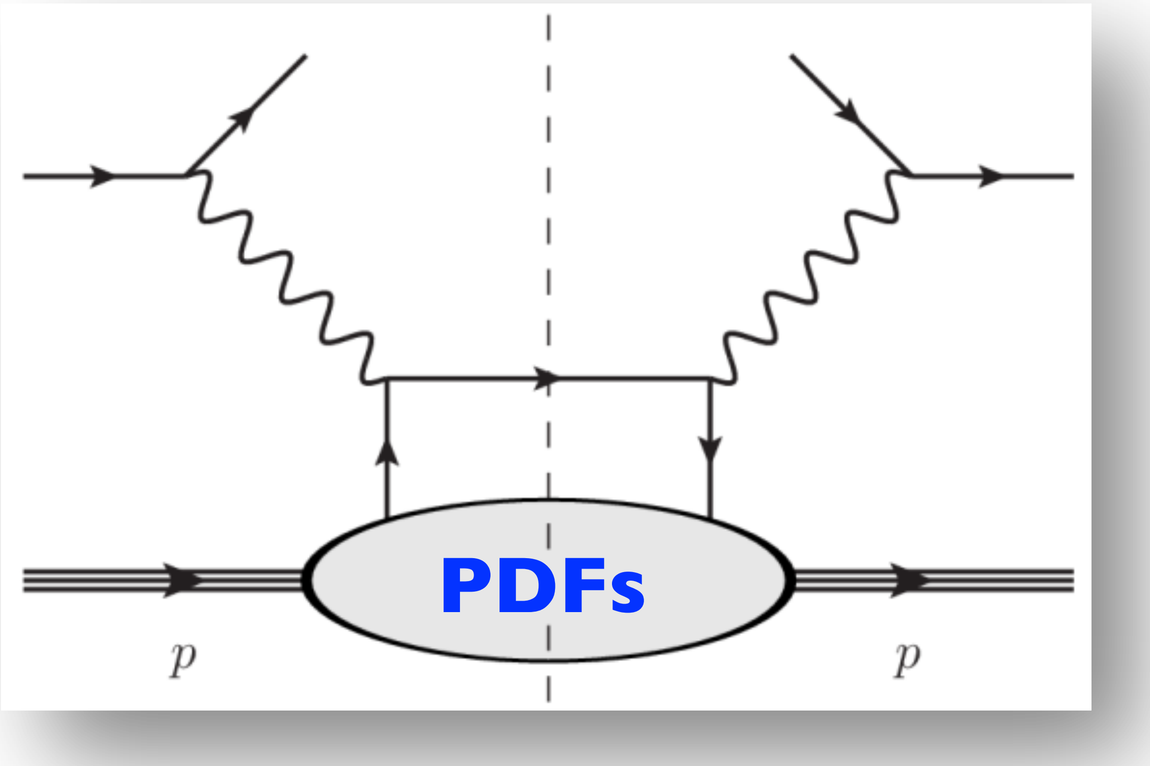
# Wigner Distributions

Barbara Pasquini

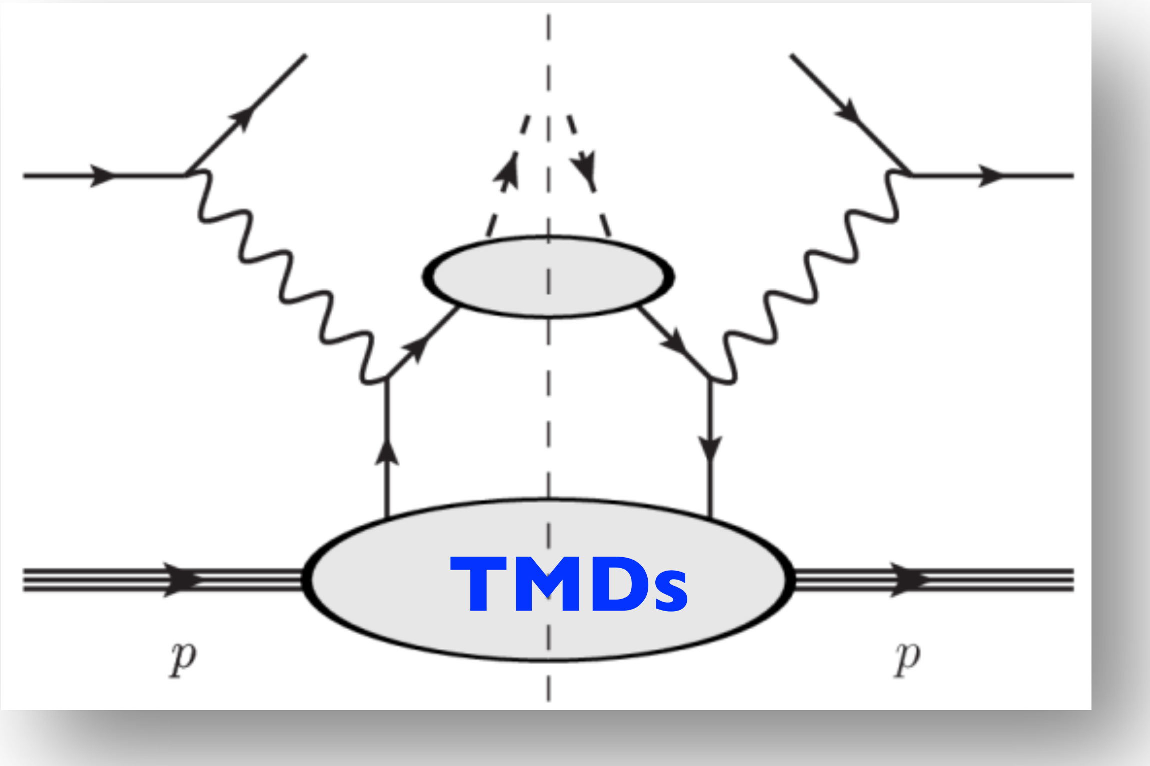
Università di Pavia & INFN

# Goal: understanding the partonic structure of the nucleon

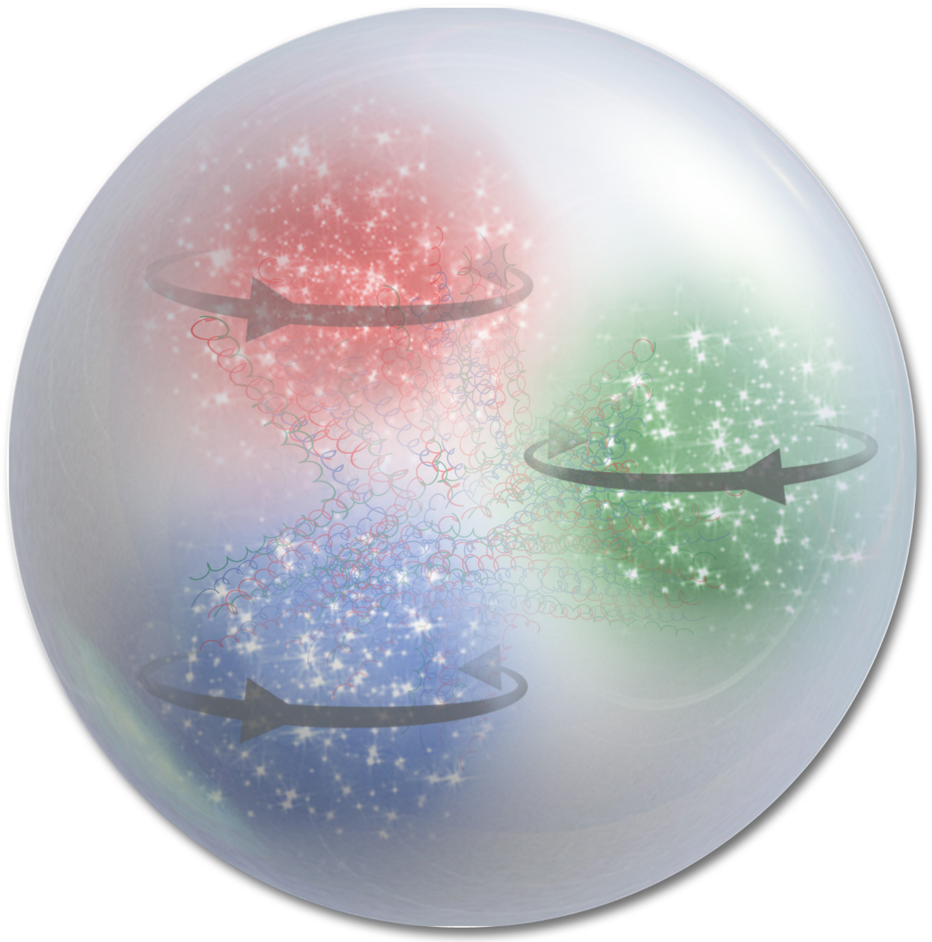
Deep Inelastic Scattering



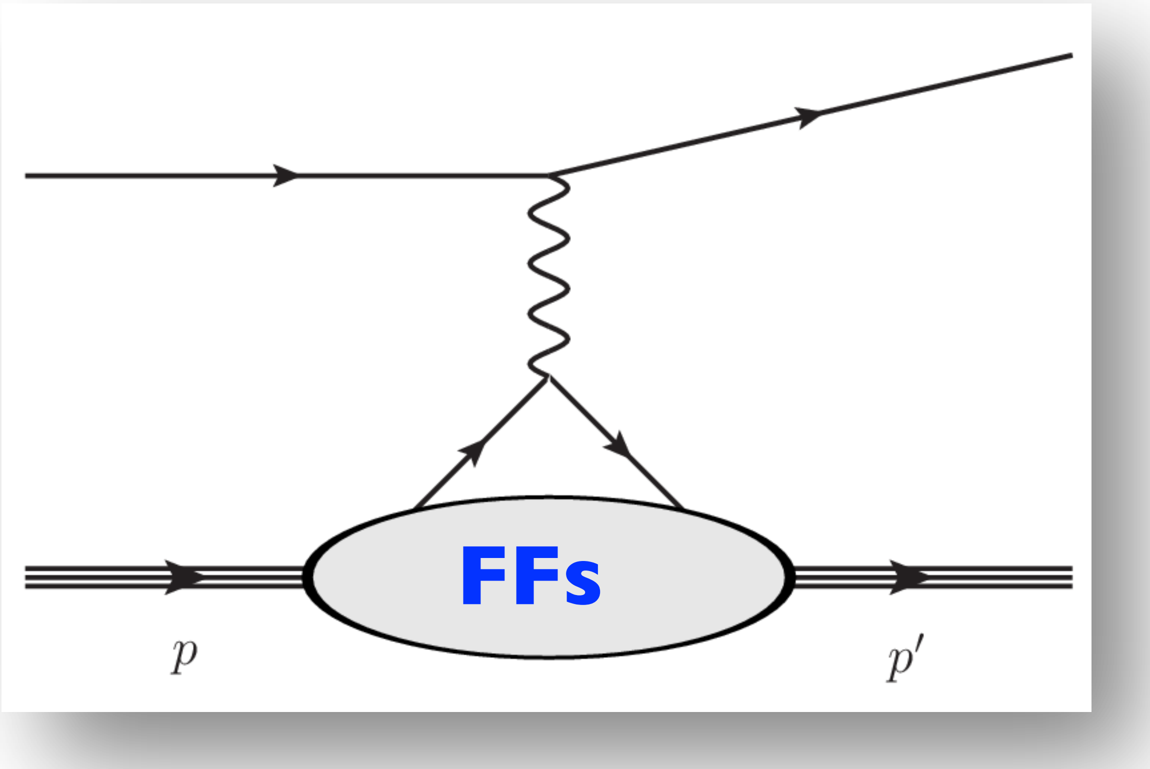
Semi-Inclusive
Deep Inelastic Scattering



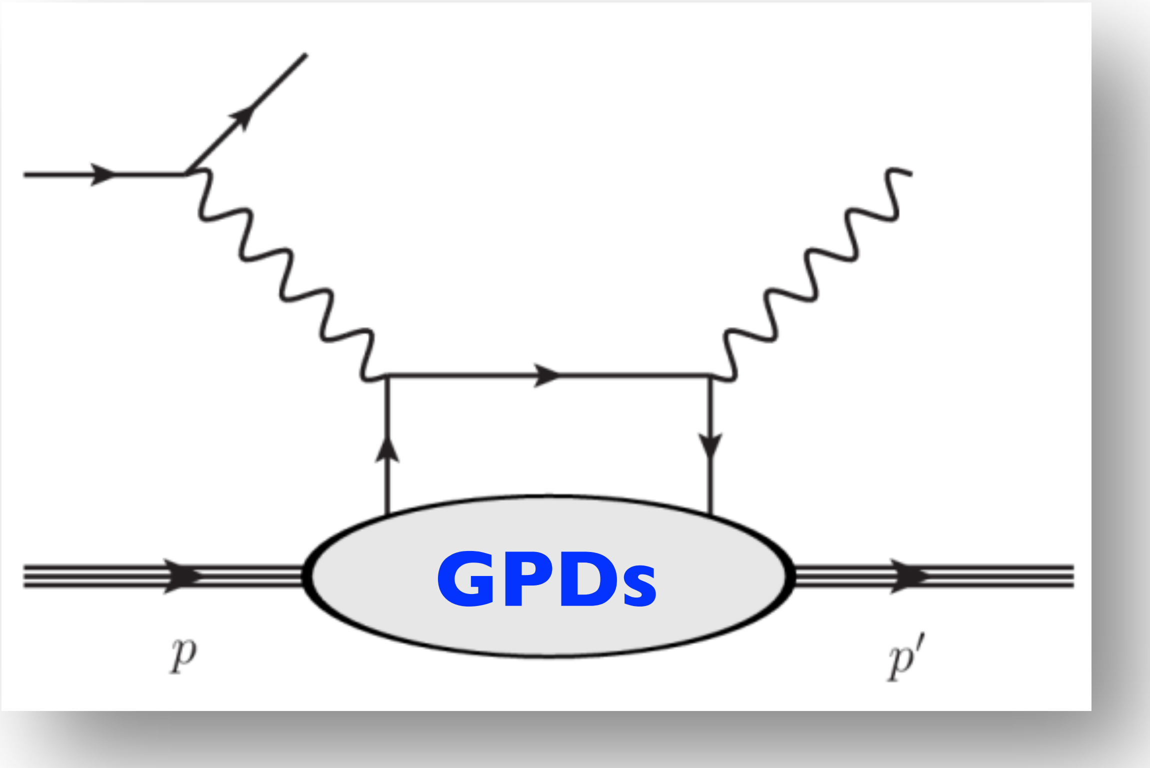
**Momentum Space**



Elastic Scattering



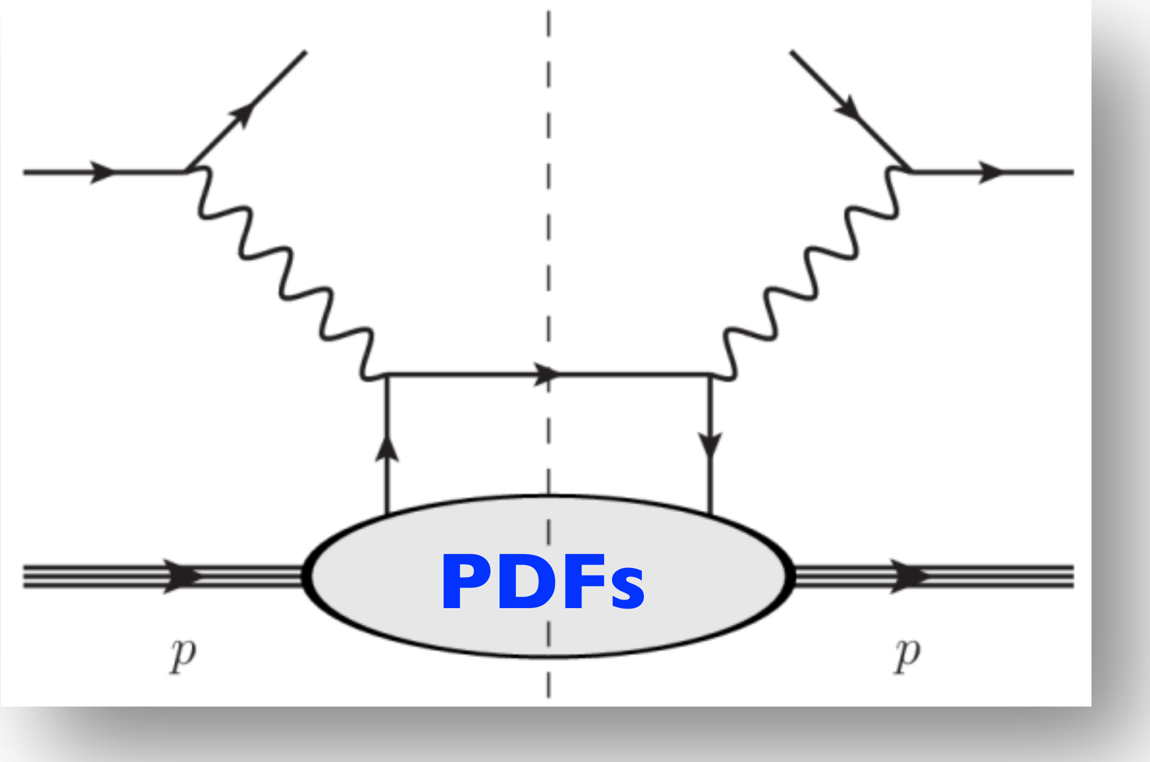
Deeply Virtual Compton
Scattering



**Transverse Coordinate Space**

# Goal: understanding the partonic structure of the nucleon

Deep Inelastic Scattering



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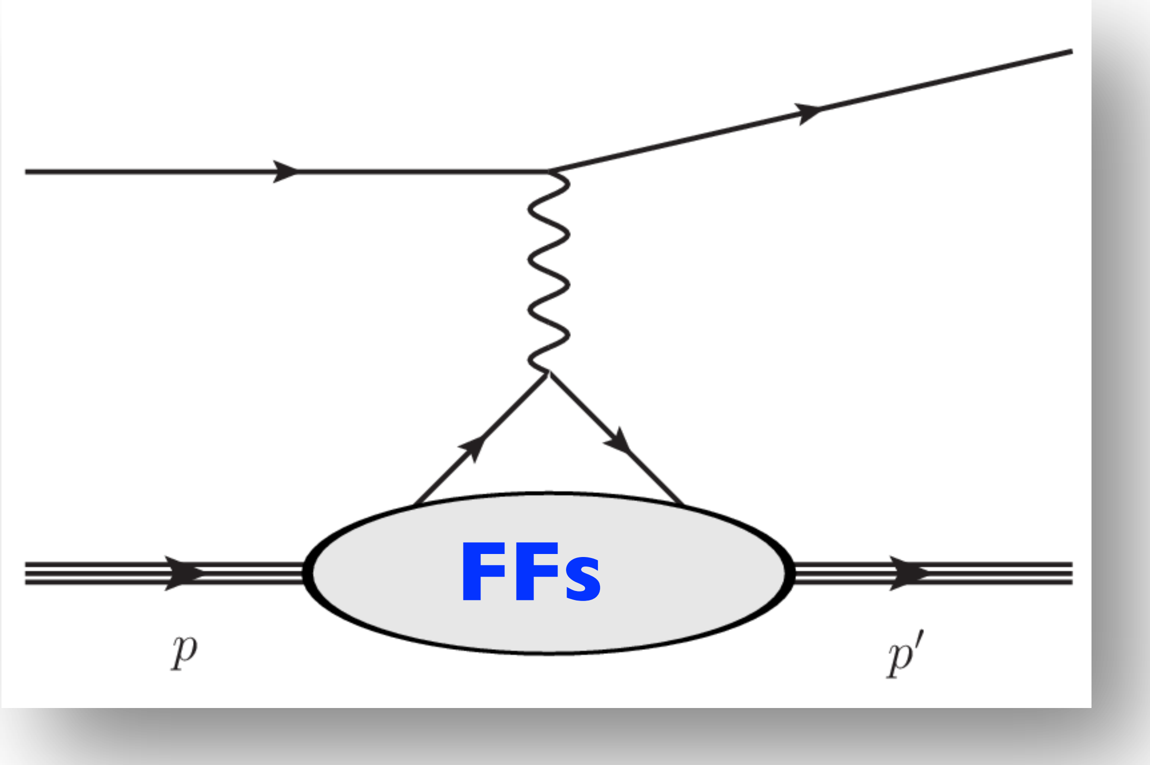

Semi-Inclusive
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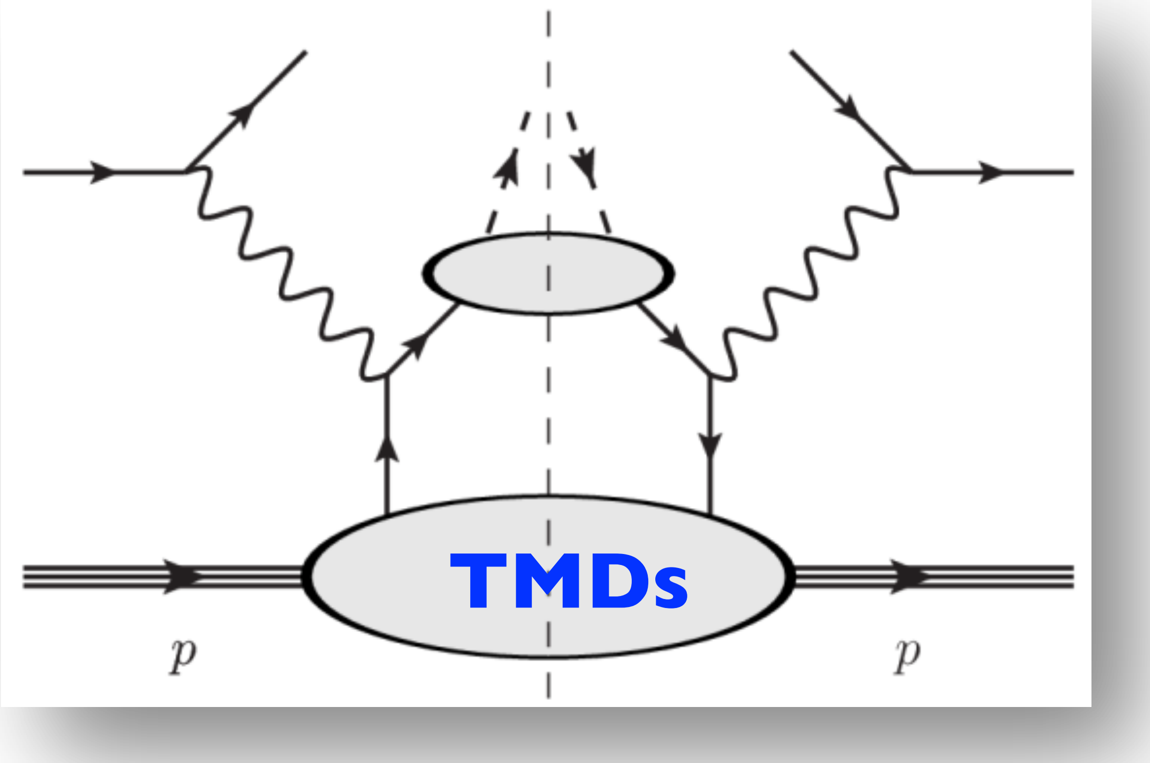

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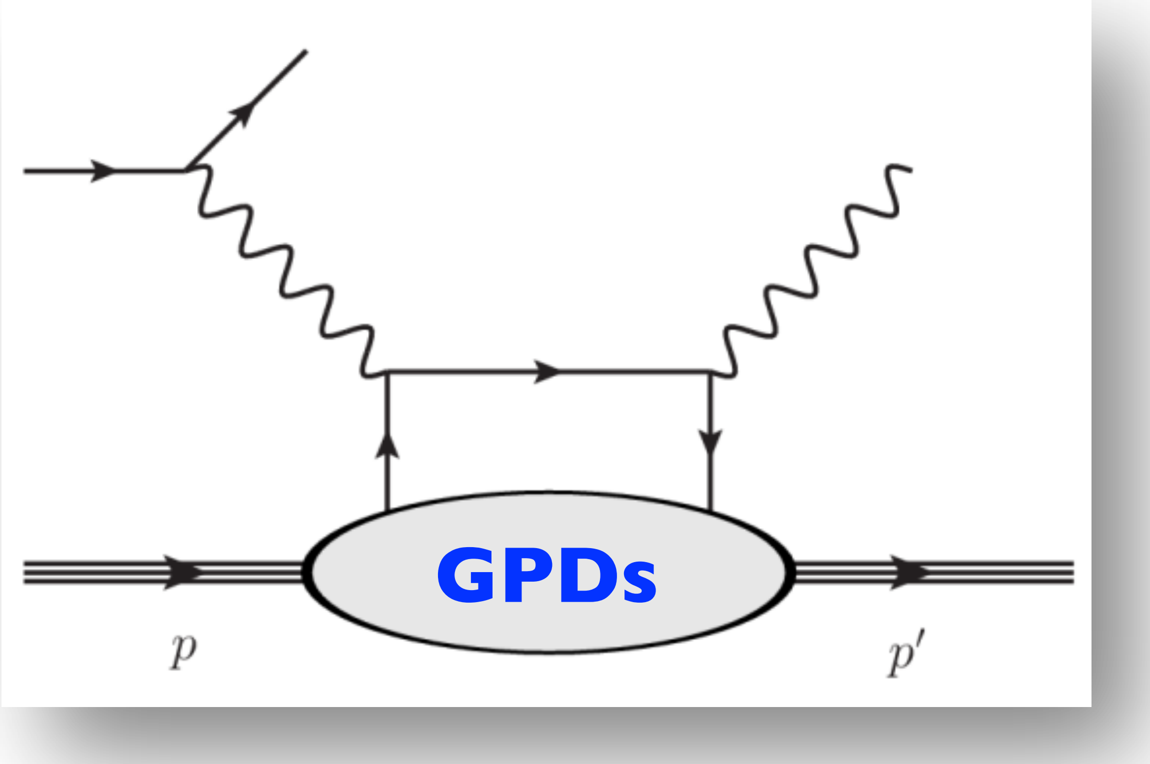


Complete Proton Tomography
in 3+2 D
from phase-space distributions

**GTMDs ⟷ Wigner distr.**

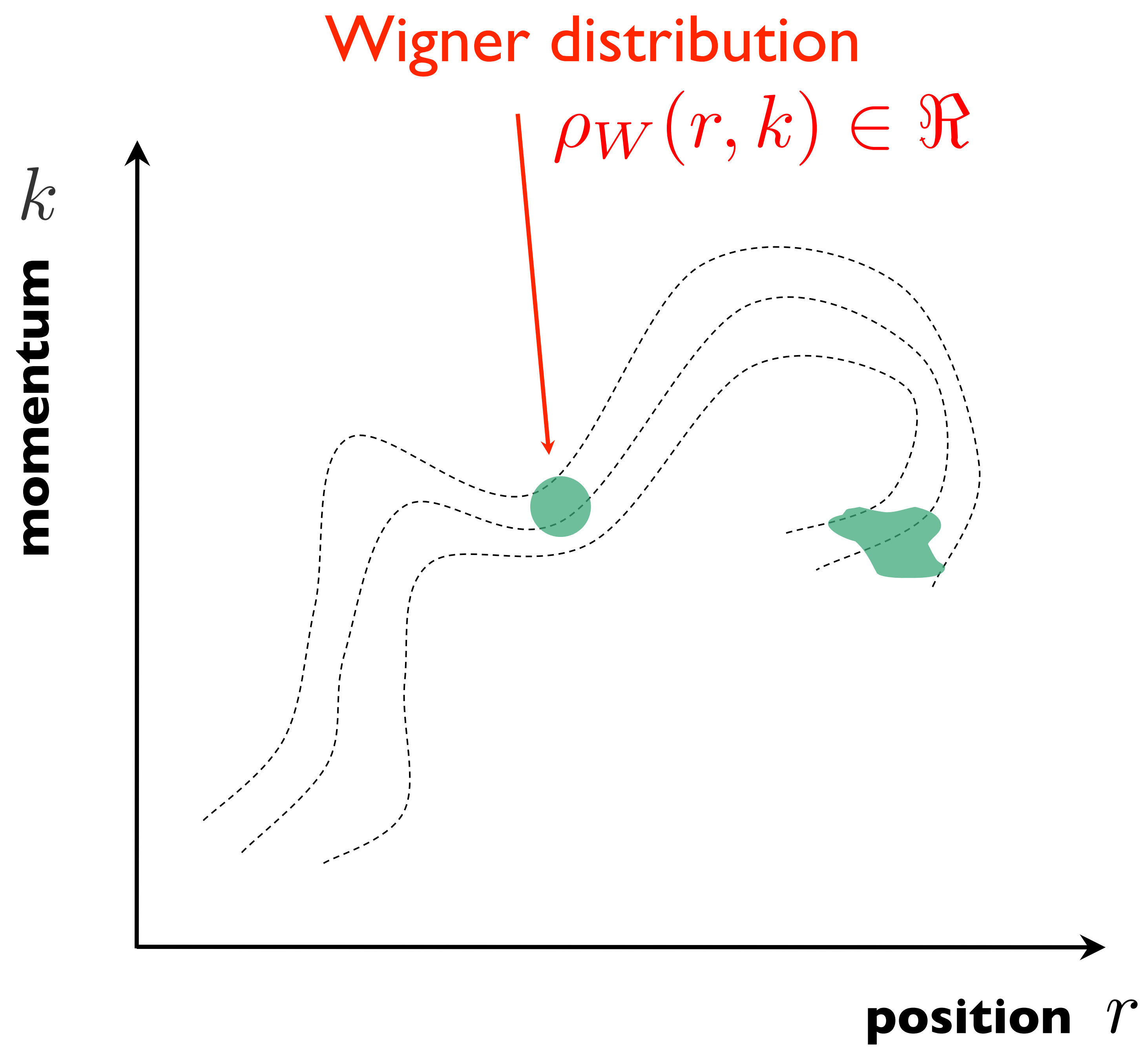
**Momentum Space**

**Transverse Coordinate Space**

# Phase-space distribution in QM

Quantum Mechanics

[Wigner (1932)]
[Moyal (1949)]



$$\rho_W(r,k) = \int \frac{\mathrm{d}z}{2\pi} e^{-ikz} \psi^*(r-\frac{z}{2})\psi(r+\frac{z}{2})$$
$$= \int \frac{\mathrm{d}\Delta}{2\pi} e^{-i\Delta r} \phi^*(k+\frac{\Delta}{2})\phi(k-\frac{\Delta}{2})$$

Position-space density

$$|\psi(r)|^2 = \int \mathrm{d}k\, \rho_W(r,k)$$

Momentum-space density

$$|\phi(k)|^2 = 2\pi \int \mathrm{d}r\, \rho_W(r,k)$$

Quantum average

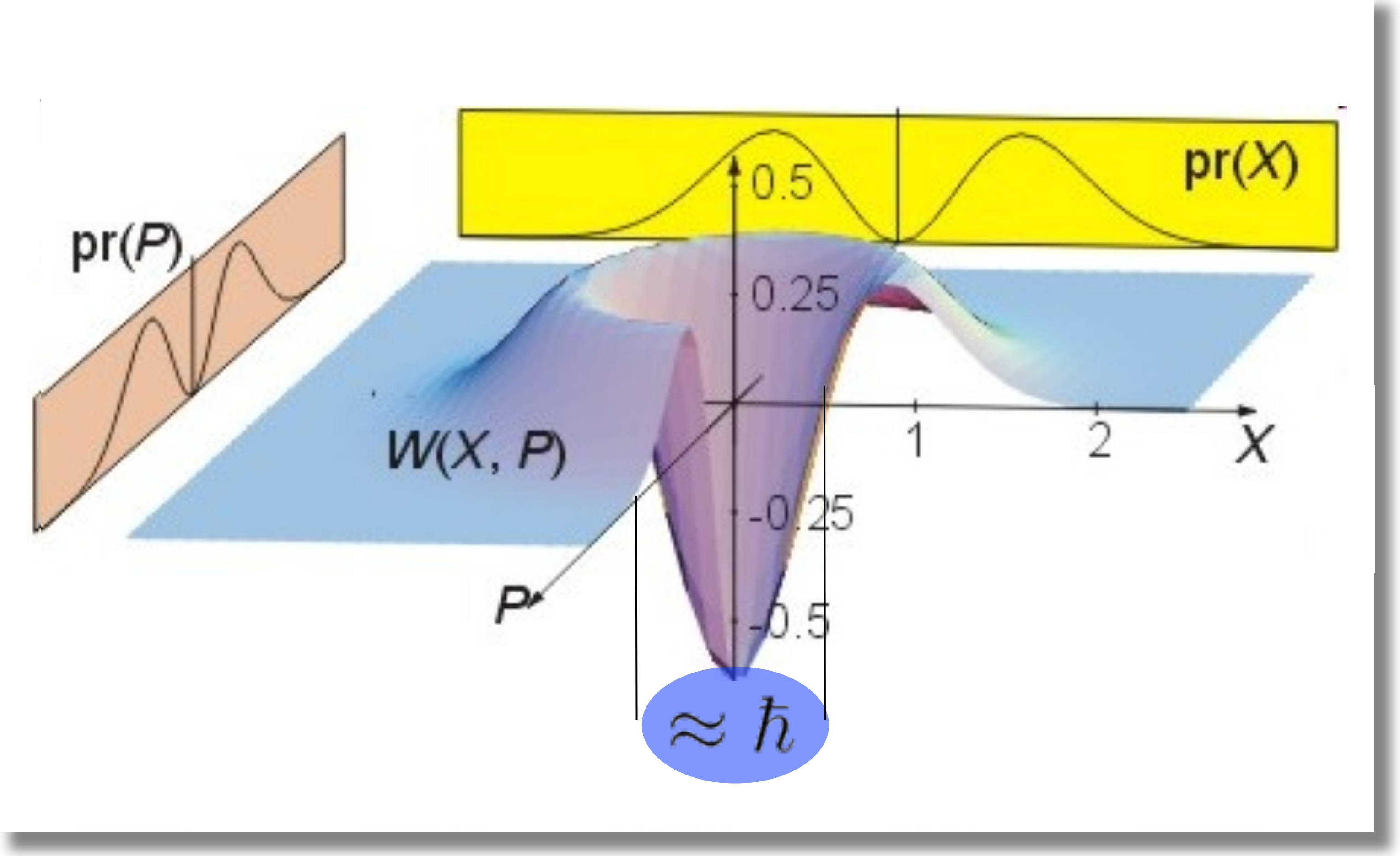
$$\langle \hat{O} \rangle = \int \mathrm{d}r\, \mathrm{d}k\, O(r,k)\, \rho_W(r,k)$$

# Phase-space quasi-distribution

## Wigner distribution

Numerous applications in

- Nuclear physics
- Quantum chemistry
- Quantum molecular dynamics
- Quantum information
- Quantum optics
- Classical optics
- Signal analysis
- Image processing
- Heavy ion collisions
- …



[Antonov et al. (1980-1989)]

Heisenberg's uncertainty relation

→ Quasi-probabilistic interpretation $\xrightarrow{\hbar \to 0}$ classical density

# Wigner Distributions in QFT

Quark Wigner operator

$$\widehat{W}^{[\Gamma]}(\vec{r},k)=\int\frac{\mathrm{d}^4z}{(2\pi)^4}e^{ik\cdot z}\bar{\psi}(\vec{r}-\frac{z}{2})\,\overbrace{\Gamma}^{\text{Dirac matrix }\sim\text{ quark polarization}}\,\underbrace{\mathcal{W}}_{\text{Wilson line}}\,\psi(\vec{r}+\frac{z}{2})$$

# Wigner Distributions in QFT

Dirac matrix

$\sim$ quark polarization

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Wilson line

Fixed light-front time

$$z^+=0 \quad\Longleftrightarrow\quad \int\mathrm{d}k^-$$

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Dirac matrix
~ quark polarization

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Wigner distributions
in the Breit frame

$$\rho^{[\Gamma]}_{\Lambda'\Lambda}(\vec{r},k^+,\vec{k}_\perp)=\frac{1}{2}\int\frac{\mathrm{d}^3\vec{\Delta}}{(2\pi)^3}e^{-i\vec{\Delta}\cdot\vec{r}}\langle\frac{\vec{\Delta}}{2},\Lambda'|\widehat{W}^{[\Gamma]}(0,k^+,\vec{k}_\perp)|-\frac{\vec{\Delta}}{2},\Lambda\rangle$$

**3+3 D**

no semi-classical interpretation

[Ji (2003)]
[Belitsky, Ji, Yuan (2004)]

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Wigner distributions
in the Drell-Yan frame
$(\Delta^+=0)$

$$\rho^{[\Gamma]}_{\Lambda'\Lambda}(\vec{b}_\perp,k^+,\vec{k}_\perp)=\frac{1}{2}\int\frac{\mathrm{d}^2\vec{\Delta}_\perp}{(2\pi)^2}e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp}\langle p^+,\frac{\vec{\Delta}_\perp}{2},\Lambda'|\widehat{W}^{[\Gamma]}(0,k^+,\vec{k}_\perp)|p^+,-\frac{\vec{\Delta}_\perp}{2},\Lambda\rangle$$

**2+3 D**

semi-classical interpretation

[Lorcè, BP (2011)]
Lorcè, BP, Xiong, Yuan (2012)]

# Wigner Distributions in QFT

Quark Wigner operator

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$\sim$ quark polarization

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Fixed light-front time

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**2+3 D**

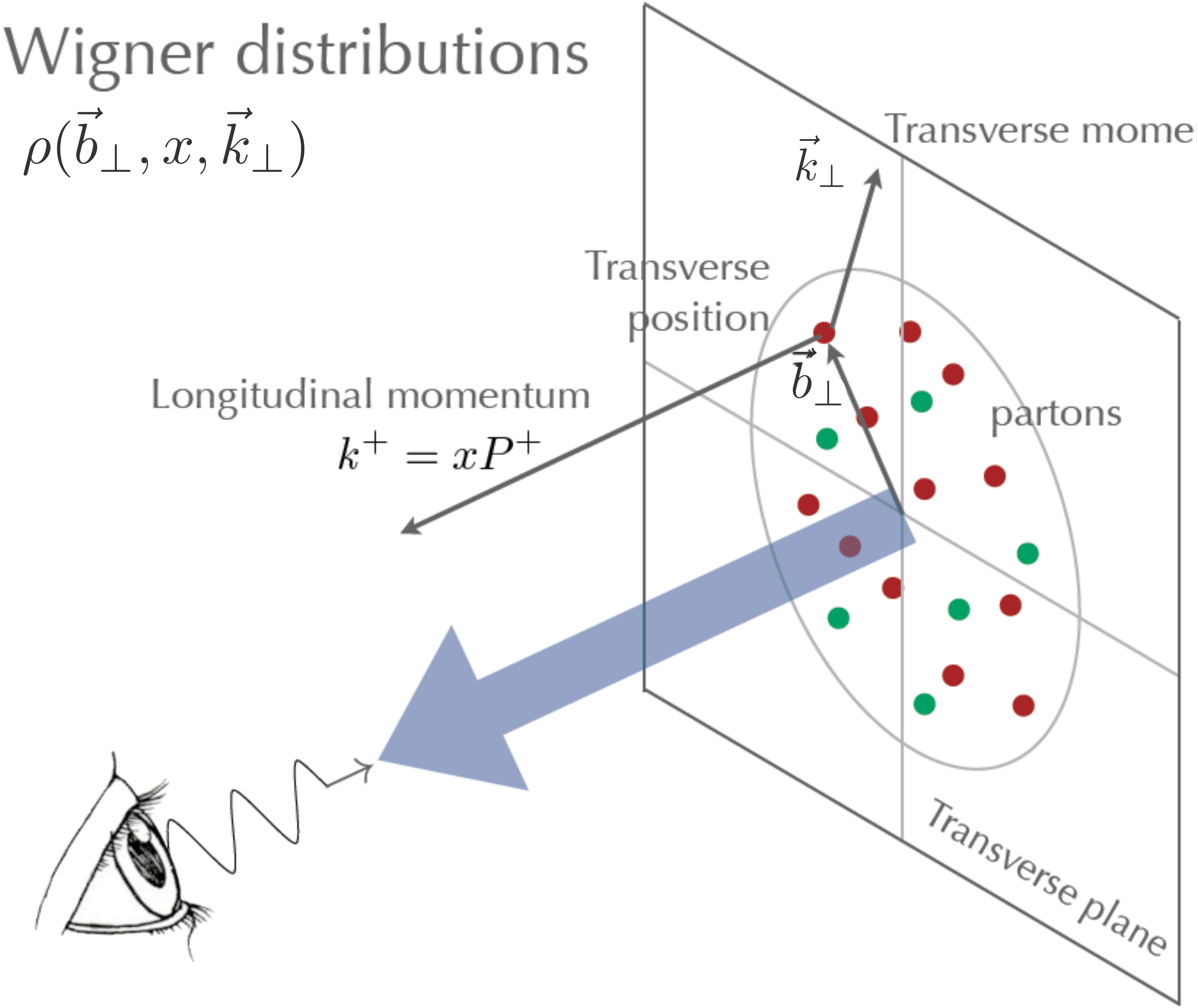
semi-classical interpretation

Generalized Transverse Momentum Dependent

[Meissner, Metz, Schlegel (2007)]

# Wigner distributions

$$\rho(\vec{b}_\perp, x, \vec{k}_\perp)$$



Transverse momentum

$\vec{k}_\perp$

Transverse position

$\vec{b}_\perp$

Longitudinal momentum

$k^+ = xP^+$

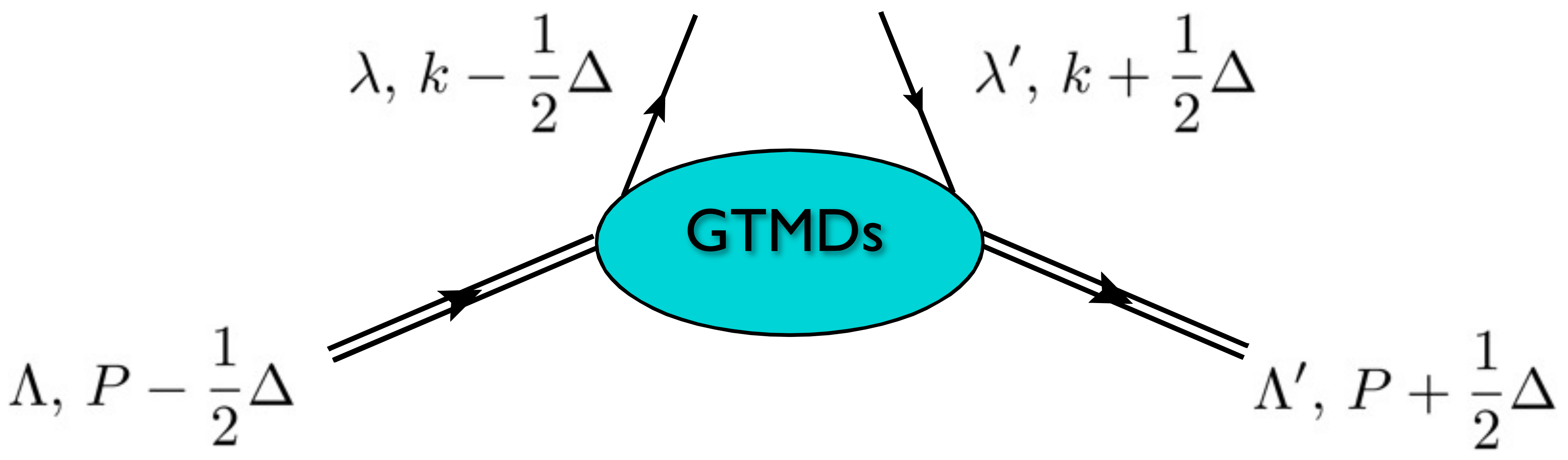
partons

Transverse plane

# Generalized TMDs and Wigner Distributions

[Meißner, Metz, Schlegel (2009)]

[Gluon GTMDs: Lorcé, BP, (2014)]



Quark polarization

$$\boxed{W^{[\Gamma]}_{\Lambda'\Lambda} = \frac{1}{2}\int \frac{\mathrm{d}z^-\mathrm{d}^2 z_\perp}{(2\pi)^3}\langle P',\Lambda'|\bar{\psi}^q_{\lambda'}(-\frac{z}{2})\,\Gamma\,\mathcal{W}(-\frac{z}{2},\frac{z}{2}|n)\psi^q_{\lambda}(\frac{z}{2})|P,\Lambda\rangle e^{i(xP^+z^- - \vec{k}_\perp\cdot\vec{z}_\perp)}}$$

Nucleon polarization

4 X 4 =16 polarizations ↔ 16 complex GTMDs (at twist-2)

$$W^{\Gamma}_{\Lambda',\Lambda}(x,\xi,\vec{k}_\perp,\vec{\Delta}_\perp)$$

# Generalized TMDs and Wigner Distributions

[Meißner, Metz, Schlegel (2009)]

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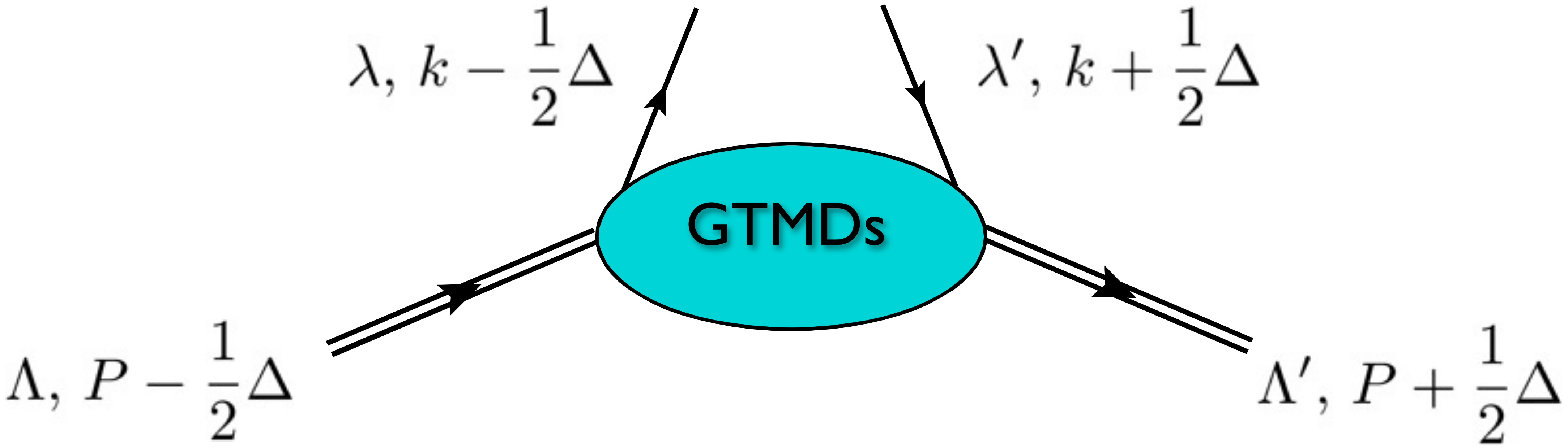


Quark polarization

$$W^{[\Gamma]}_{\Lambda'\Lambda} = \frac{1}{2}\int \frac{\mathrm{d}z^- \mathrm{d}^2 z_\perp}{(2\pi)^3} \langle P', \Lambda' | \bar{\psi}^q_{\lambda'}(-\frac{z}{2})\,\Gamma\,\mathcal{W}(-\frac{z}{2}, \frac{z}{2}|n)\psi^q_{\lambda}(\frac{z}{2})|P, \Lambda\rangle e^{i(xP^+ z^- - \vec{k}_\perp \cdot \vec{z}_\perp)}$$

Nucleon polarization

4 X 4 =16 polarizations ⟷ 16 complex GTMDs (at twist-2)

$$W^{\Gamma}_{\Lambda',\Lambda}(x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp)$$

x: average fraction of quark longitudinal momentum

ξ: fraction of longitudinal momentum transfer

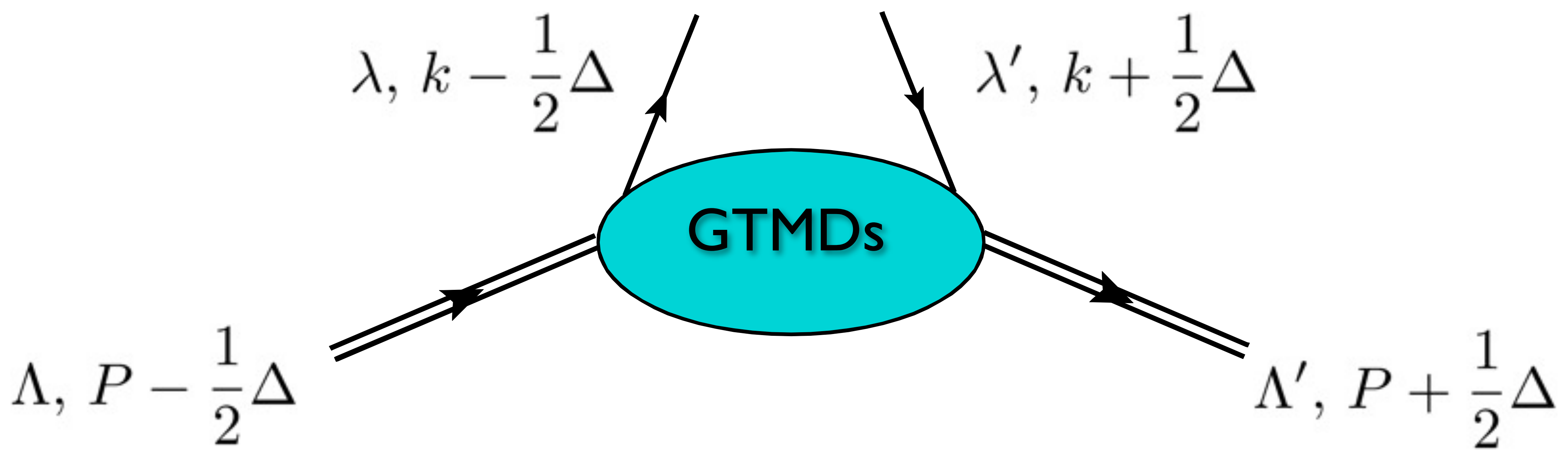
$\vec{k}_\perp$: average quark transverse momentum

$\vec{\Delta}_\perp$ : nucleon transverse-momentum transfer

# Generalized TMDs and Wigner Distributions

[Meißner, Metz, Schlegel (2009)]

[Gluon GTMDs: Lorcé, BP, (2014)]



Quark polarization

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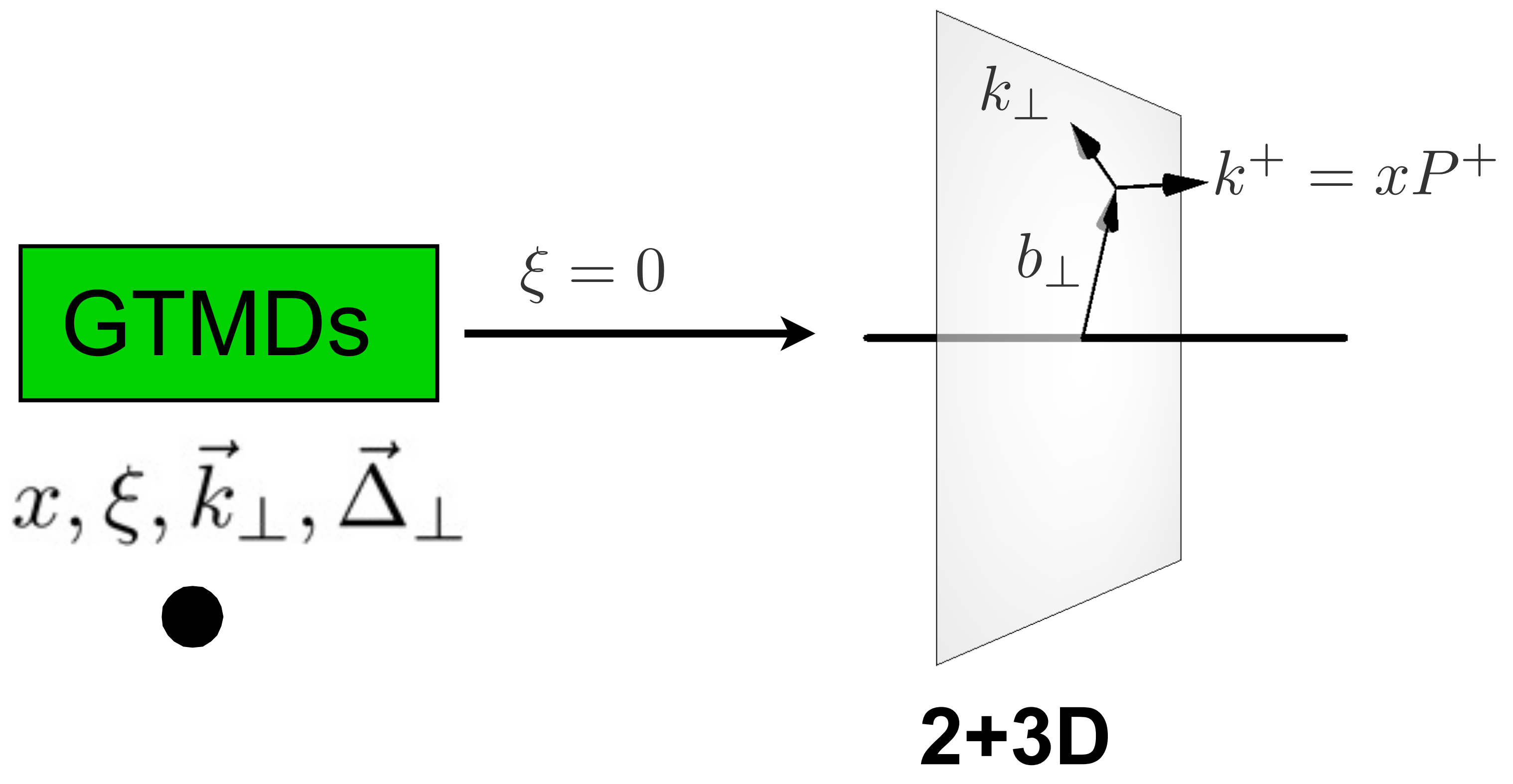
Nucleon polarization

4 X 4 =16 polarizations ⟷ 16 complex GTMDs (at twist-2)

$W^{\Gamma}_{\Lambda',\Lambda}(x,\xi,\vec{k}_\perp,\vec{\Delta}_\perp)$

$\downarrow$ Fourier transform $\vec{\Delta}_\perp\leftrightarrow\vec{b}_\perp$

$\tilde{W}^{\Gamma}_{\Lambda',\Lambda}(x,\xi,\vec{k}_\perp,\vec{b}_\perp)$ 16 real Wigner distributions



**GTMDs**

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

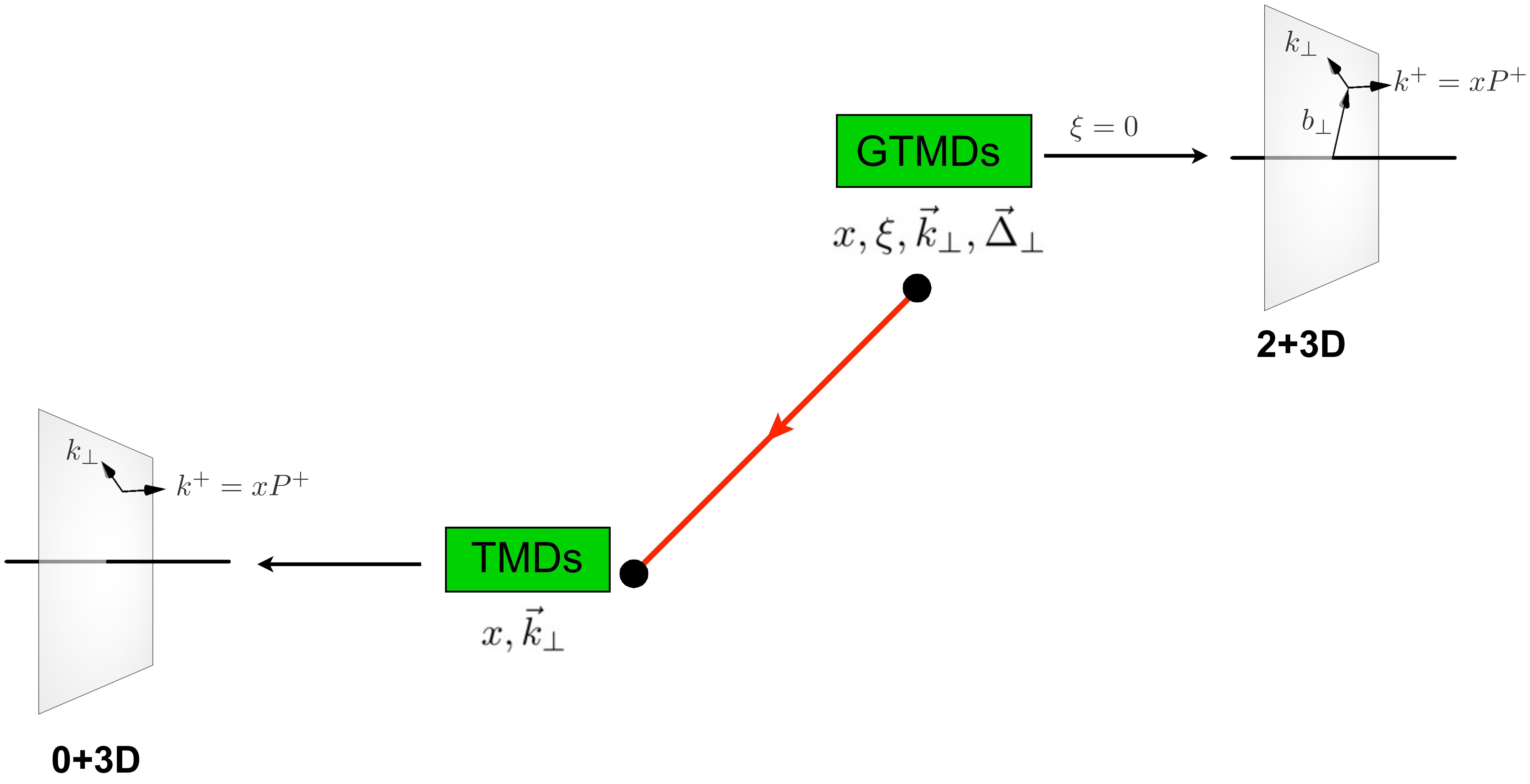
●

→ $\vec{\Delta} = 0$

→ $\int dk_\perp$

→ $\int dx$

[ Lorcé, BP, Vanderhaeghen, JHEP05 (2011)]



GTMDs

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

$\xi = 0$

**2+3D**

TMDs

$x, \vec{k}_\perp$

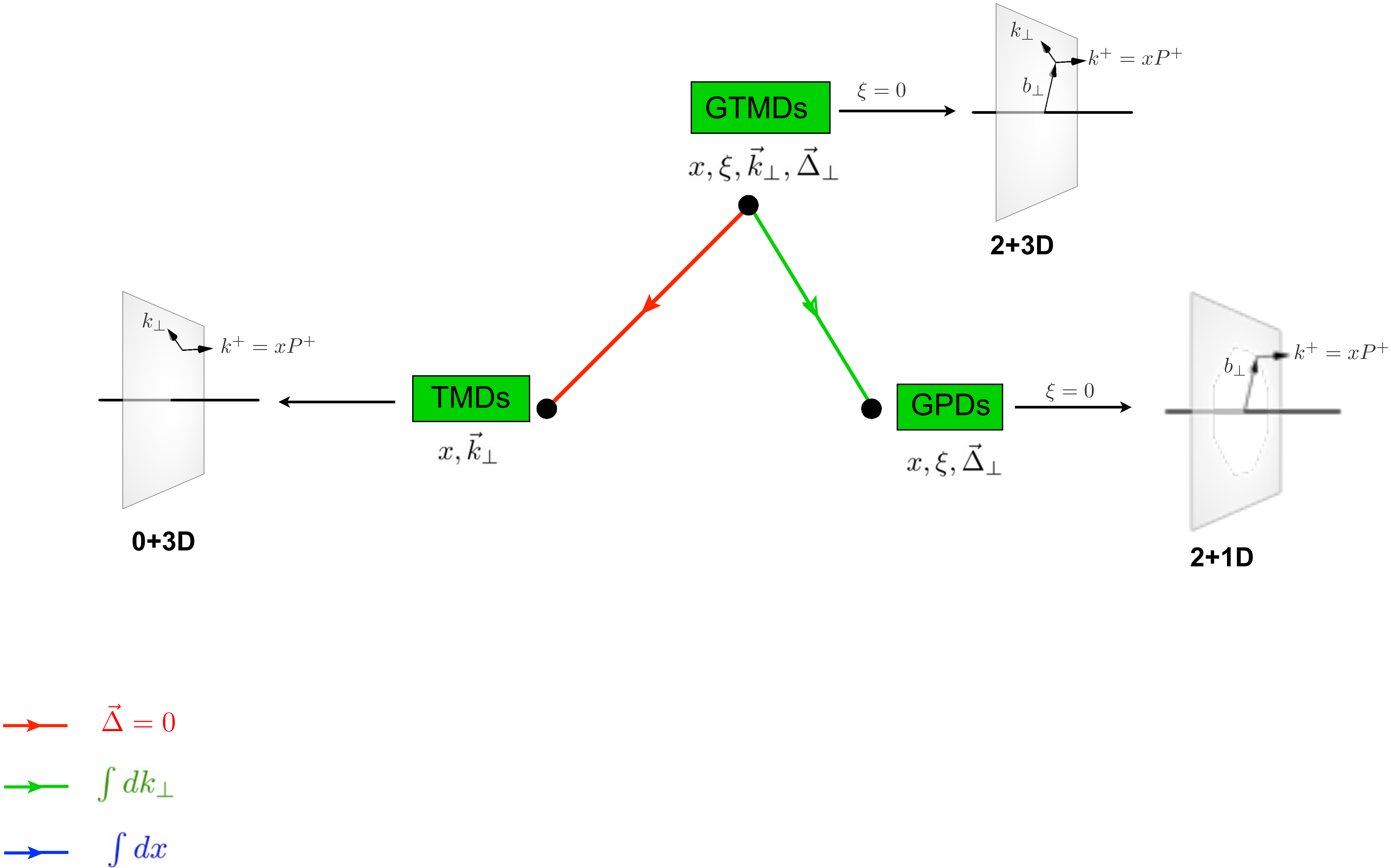
**0+3D**

→ $\vec{\Delta} = 0$

→ $\int dk_\perp$

→ $\int dx$

[ Lorcé, BP, Vanderhaeghen, JHEP05 (2011)]



**GTMDs**

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

**TMDs**

$x, \vec{k}_\perp$

**GPDs**

$x, \xi, \vec{\Delta}_\perp$

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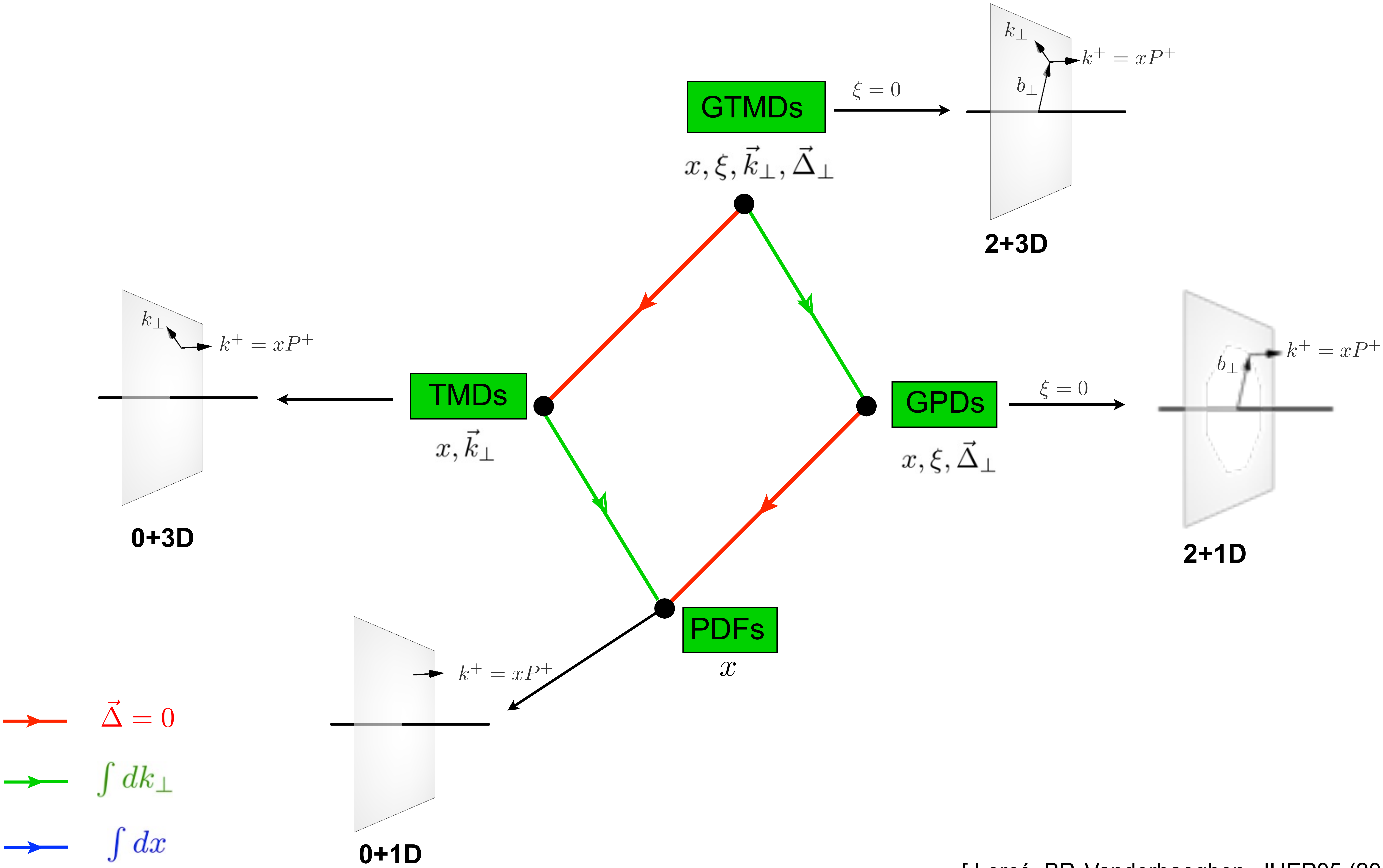


GTMDs

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

$\xi = 0$

$k_\perp$

$b_\perp$

$k^+ = xP^+$

**2+3D**

TMDs

$x, \vec{k}_\perp$

$k_\perp$

$k^+ = xP^+$

**0+3D**

GPDs

$x, \xi, \vec{\Delta}_\perp$

$\xi = 0$

$b_\perp$

$k^+ = xP^+$

**2+1D**

PDFs

$x$

$k^+ = xP^+$

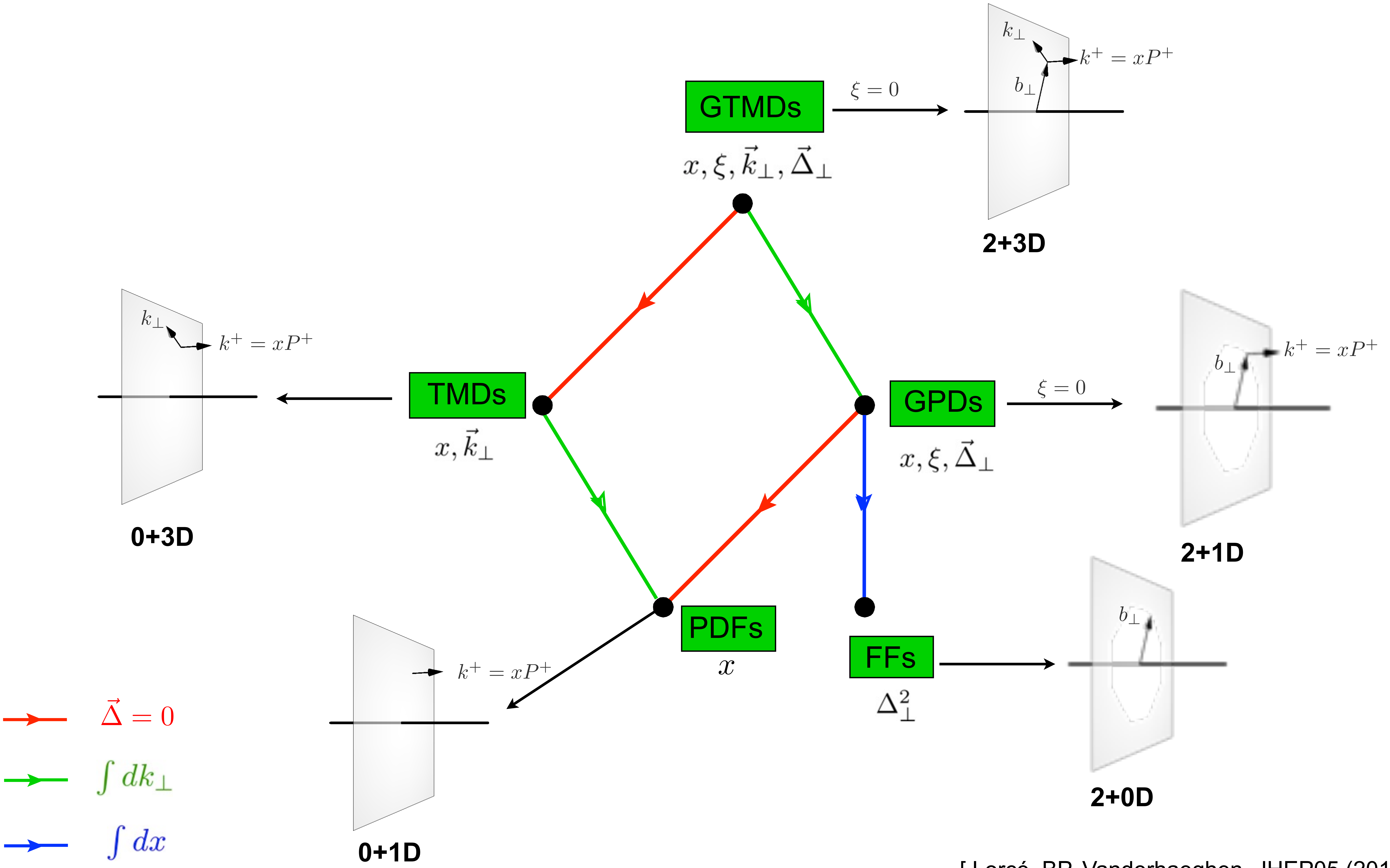
**0+1D**

🟥 $\vec{\Delta} = 0$

🟩 $\int dk_\perp$

🟦 $\int dx$

[ Lorcé, BP, Vanderhaeghen, JHEP05 (2011)]



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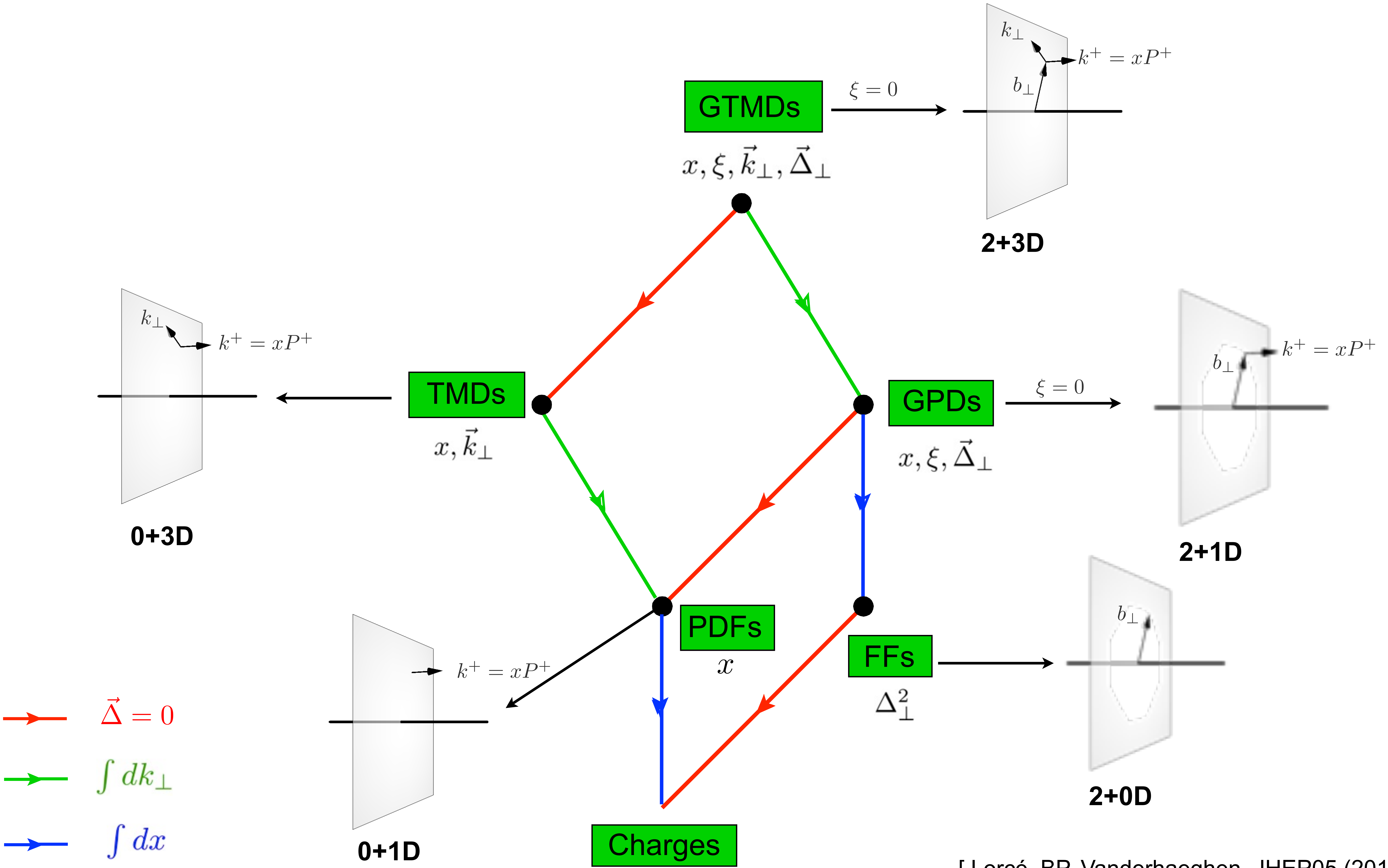


**GTMDs**

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

$\xi = 0$

**2+3D**

**TMDs**

$x, \vec{k}_\perp$

**0+3D**

**GPDs**

$x, \xi, \vec{\Delta}_\perp$

$\xi = 0$

**2+1D**

**PDFs**

$x$

**0+1D**

**FFs**

$\Delta_\perp^2$

**2+0D**

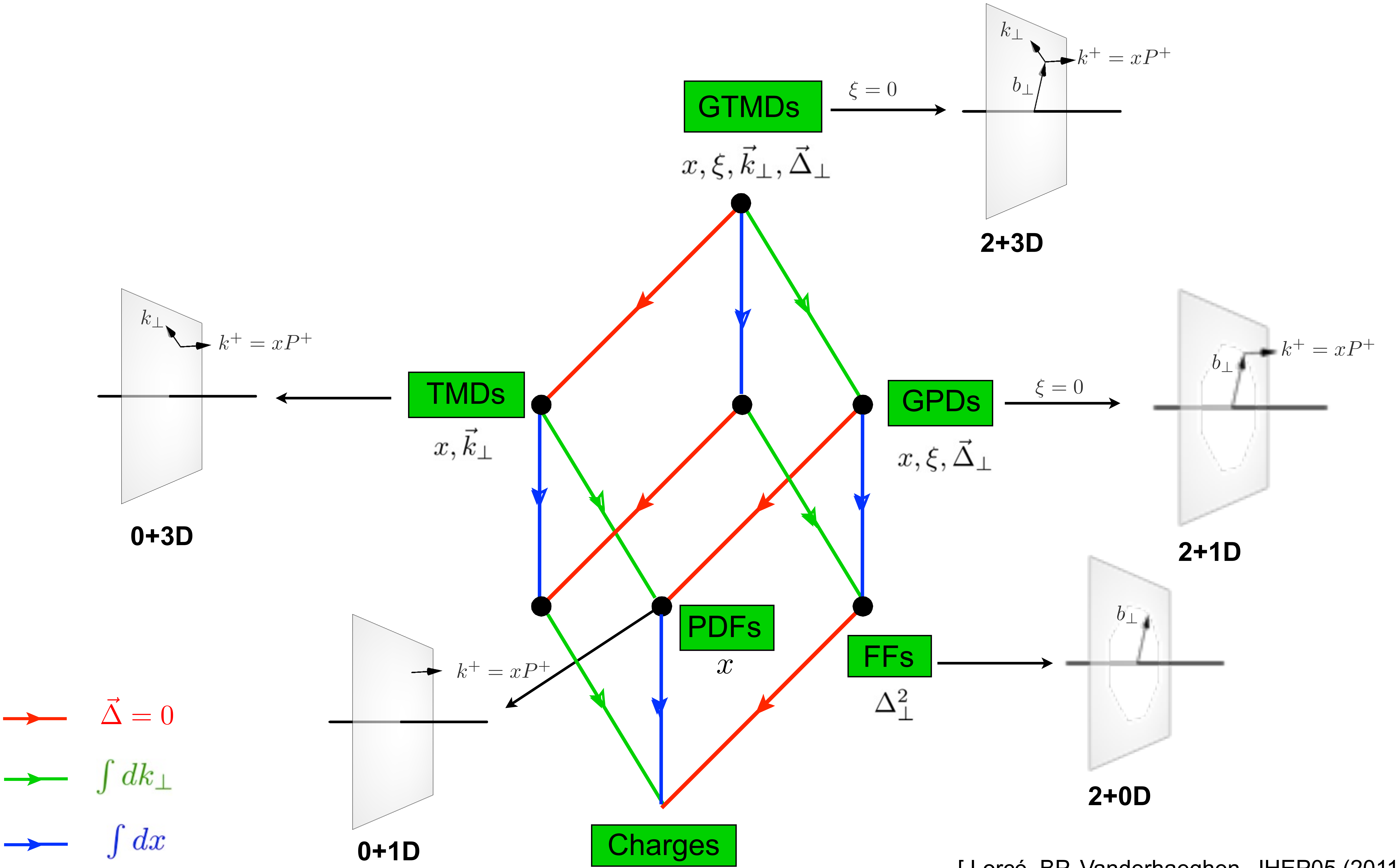
**Charges**

$\vec{\Delta} = 0$

$\int dk_\perp$

$\int dx$

[ Lorcé, BP, Vanderhaeghen, JHEP05 (2011)]



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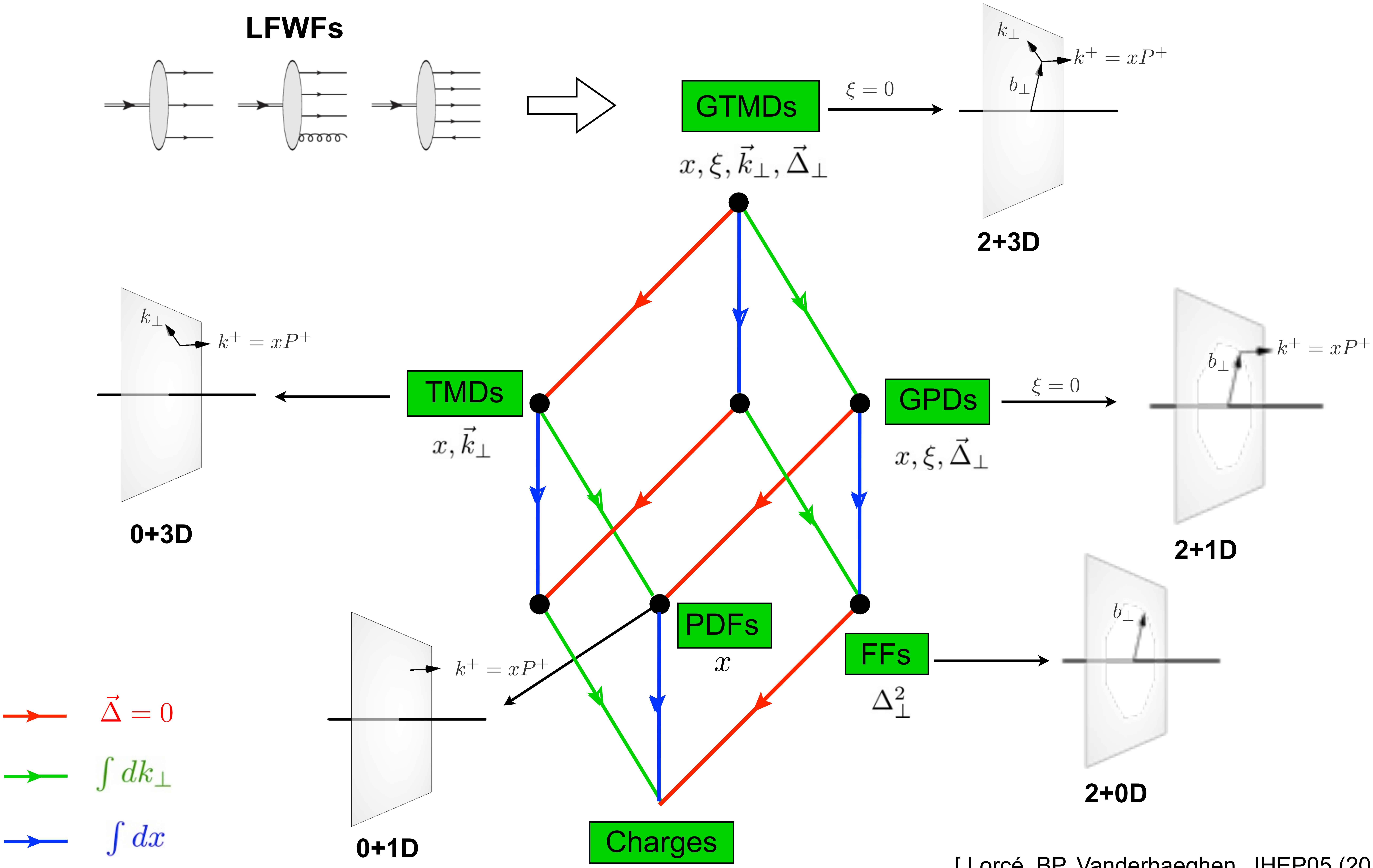


**LFWFs**

GTMDs

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

$\xi = 0$

$k_\perp$

$b_\perp$

$k^+ = xP^+$

**2+3D**

TMDs

$x, \vec{k}_\perp$

$k_\perp$

$k^+ = xP^+$

**0+3D**

GPDs

$x, \xi, \vec{\Delta}_\perp$

$\xi = 0$

$b_\perp$

$k^+ = xP^+$

**2+1D**

PDFs

$x$

$k^+ = xP^+$

**0+1D**

FFs

$\Delta_\perp^2$

$b_\perp$

**2+0D**

Charges

$\vec{\Delta} = 0$

$\int dk_\perp$

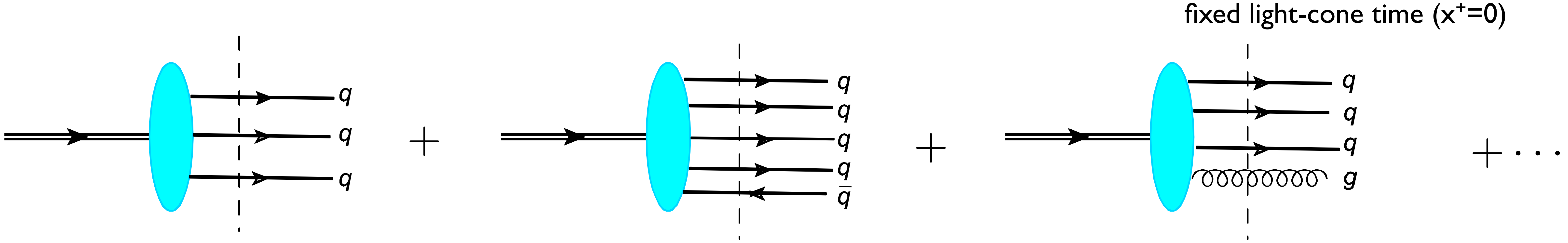
$\int dx$

[ Lorcé, BP, Vanderhaeghen, JHEP05 (2011)]

# Light-Front Wave Functions (LFWFs)

- ✦ Fock expansion of Nucleon state:

$$|N\rangle = \Psi_{3q}|qqq\rangle + \Psi_{3q\,q\bar{q}}|3q\,q\bar{q}\rangle + \Psi_{3q\,g}|qqqg\rangle + \cdots$$



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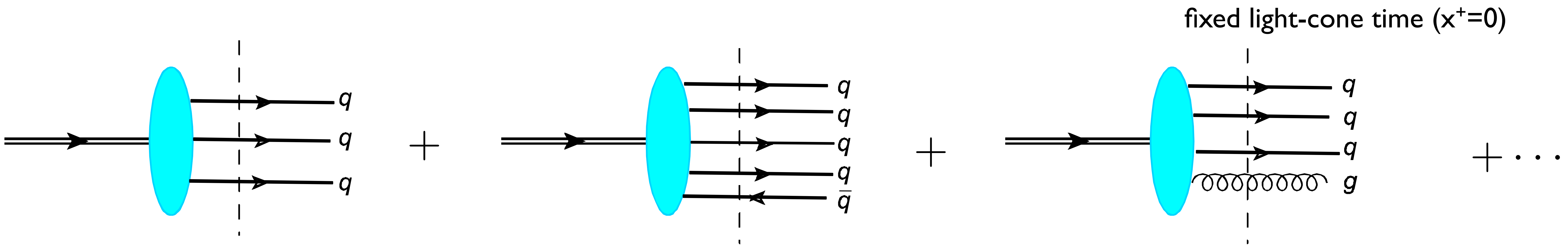

- ✦ Probability to find N partons in the nucleon

$$\rho^{\Lambda}_{N,\beta} = \int [dx]_N [d^2k_\perp]_N |\Psi^{\Lambda}_{\lambda_1\cdots\lambda_N}|^2$$

normalization $\sum_{N,\beta} \rho^{\Lambda}_{N,\beta} = 1$

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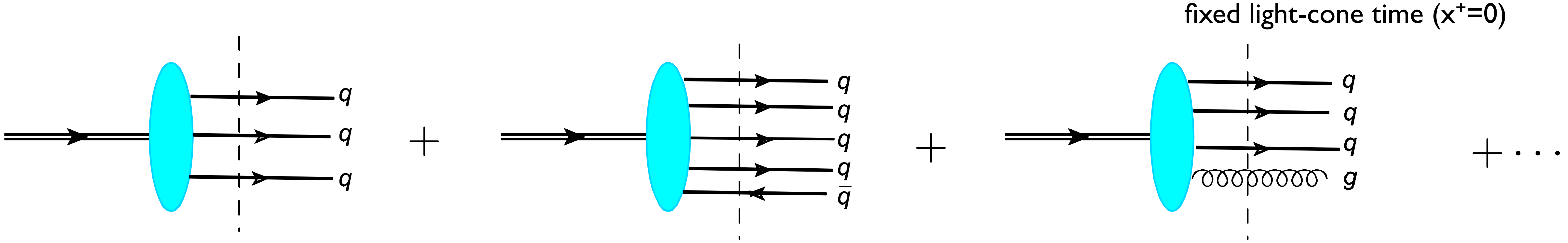

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✦ Eigenstates of momentum $P^+ = \sum_{i=1}^{N} k_i^+$ $\vec{P}_\perp = \sum_{i=1}^{N} \vec{k}_{i\,\perp} = \vec{0}_\perp$

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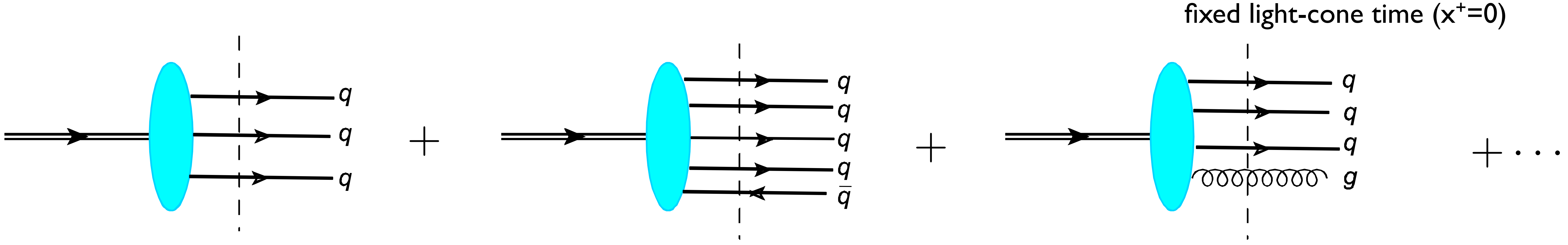

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✦ Eigenstates of parton light-front helicity $\hat{S}_{iz}\,\Psi^{\Lambda}_{\lambda_1\cdots\lambda_N} = \lambda_i\,\Psi^{\Lambda}_{\lambda_1\lambda_2\cdots\lambda_N}$

$$\boxed{\Lambda = \sum_{i=1}^{N} \lambda_i + l_z}$$

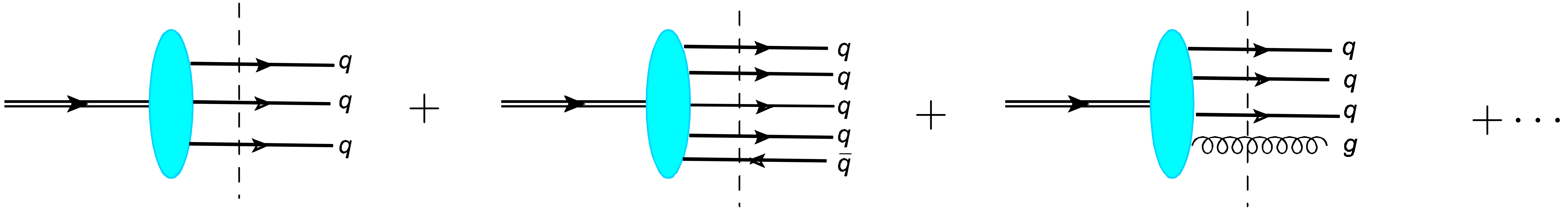
✦ Eigenstates of total orbital angular momentum $\hat{L}_z\,\Psi^{\Lambda}_{\lambda_1\cdots\lambda_N} = l_z\,\Psi^{\Lambda}_{\lambda_1\lambda_2\cdots\lambda_N}$

⚠ $A^+ = 0$ gauge

# Light-Front Wave Functions (LFWFs)

- Fock expansion of Nucleon state:

$$|N\rangle = \Psi_{3q}|qqq\rangle + \Psi_{3q\,q\bar{q}}|3q\,q\bar{q}\rangle + \Psi_{3q\,g}|qqqg\rangle + \cdots$$



fixed light-cone time ($x^+$=0)

- Probability to find N partons in the nucleon $\rho^{\Lambda}_{N,\beta} = \int [dx]_N [d^2k_\perp]_N |\Psi^{\Lambda}_{\lambda_1\cdots\lambda_N}|^2$

normalization $\sum_{N,\beta}\rho^{\Lambda}_{N,\beta} = 1$

- Eigenstates of momentum $P^+ = \sum_{i=1}^N k_i^+ \qquad \vec{P}_\perp = \sum_{i=1}^N \vec{k}_{i\,\perp} = \vec{0}_\perp$

- Eigenstates of parton light-front helicity $\hat{S}_{iz}\,\Psi^{\Lambda}_{\lambda_1\cdots\lambda_N} = \lambda_i\,\Psi^{\Lambda}_{\lambda_1\lambda_2\cdots\lambda_N}$

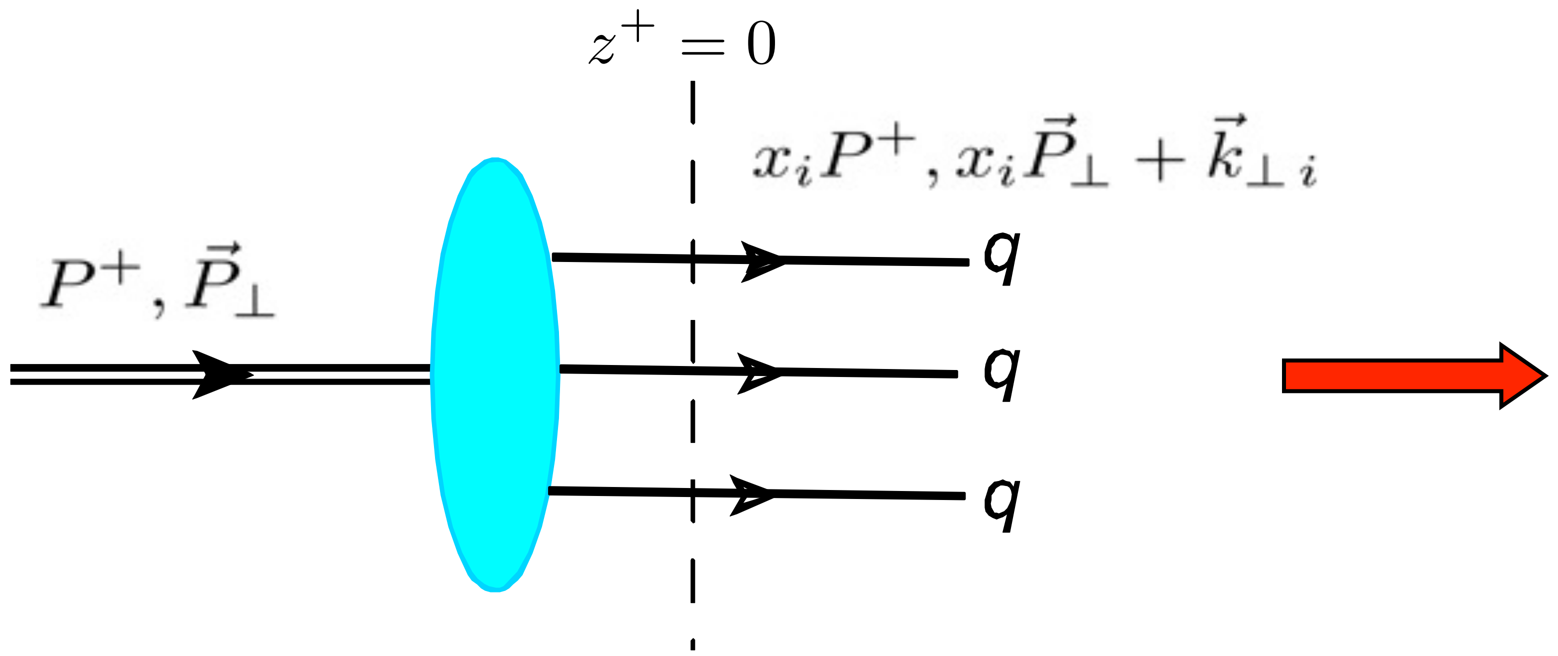
- Eigenstates of total orbital angular momentum $\hat{L}_z\,\Psi^{\Lambda}_{\lambda_1\cdots\lambda_N} = l_z\,\Psi^{\Lambda}_{\lambda_1\lambda_2\cdots\lambda_N}$

$$\boxed{\Lambda = \sum_{i=1}^N \lambda_i + l_z}$$

⚠ $A^+ = 0$ gauge

| total helicity | total OAM | nucleon helicity |
|---|---|---|
| $s_z = \langle\hat{S}_z\rangle = \sum_{N,\beta}\sum_{i=1}^N \lambda_i\,\rho^{\Lambda}_{N,\beta}$ | $\ell_z = \langle\hat{L}_z\rangle = \sum_{N,\beta}\sum_{i=1}^N l_z\,\rho^{\Lambda}_{N,\beta}$ | $\Lambda = s_z + \ell_z$ |

# LFWF Overlap Representation



$\Psi^{\Lambda;q_1q_2q_3}_{\lambda_1\lambda_2\lambda_3}(x_i, \vec{k}_{\perp,i})$

invariant under boost, independent of $P^\mu$

internal variables: $\sum_{i=1}^{3} x_i = 1, \sum_{i=1}^{3} \vec{k}_{\perp\, i} = \vec{0}_\perp$

[Brodsky, Pauli, Pinsky, '98]

[ Lorcé, BP,Vanderhaeghen, JHEP05 (2011)]

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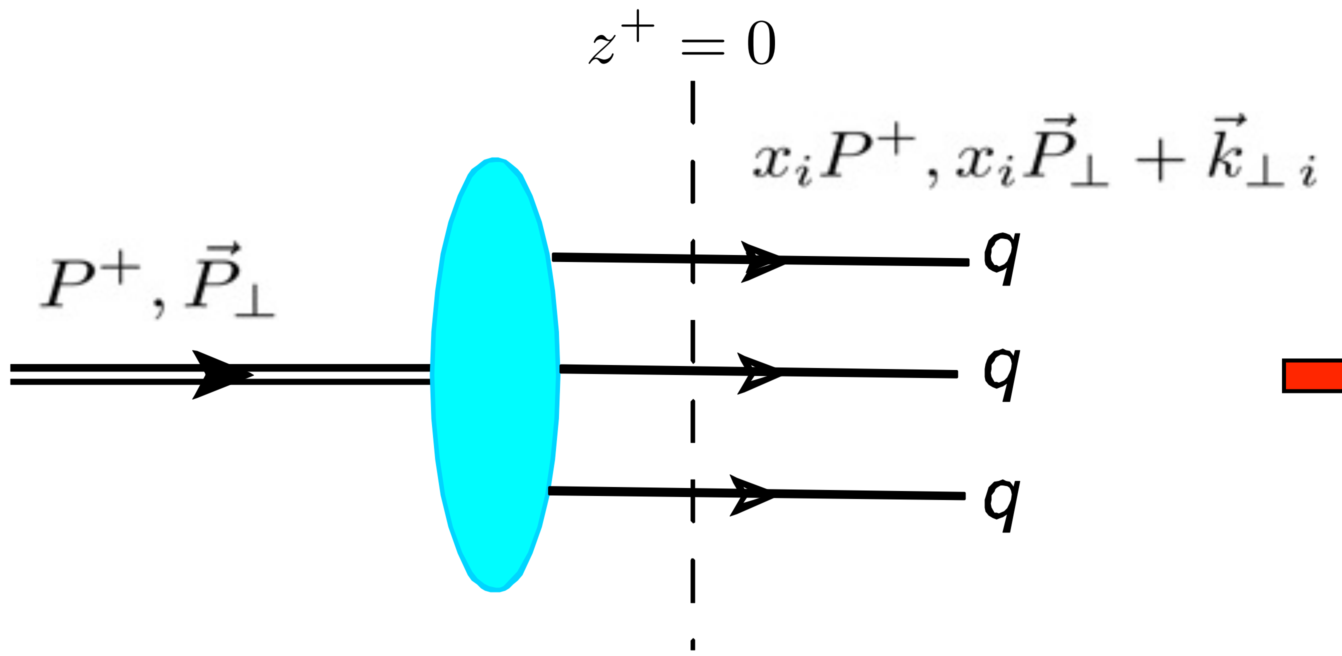

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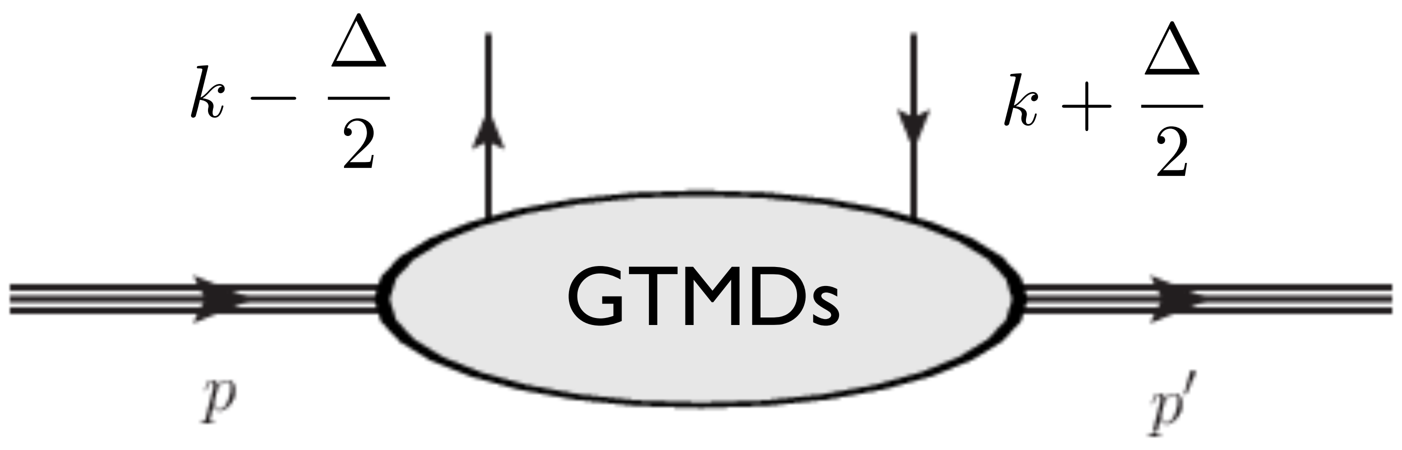
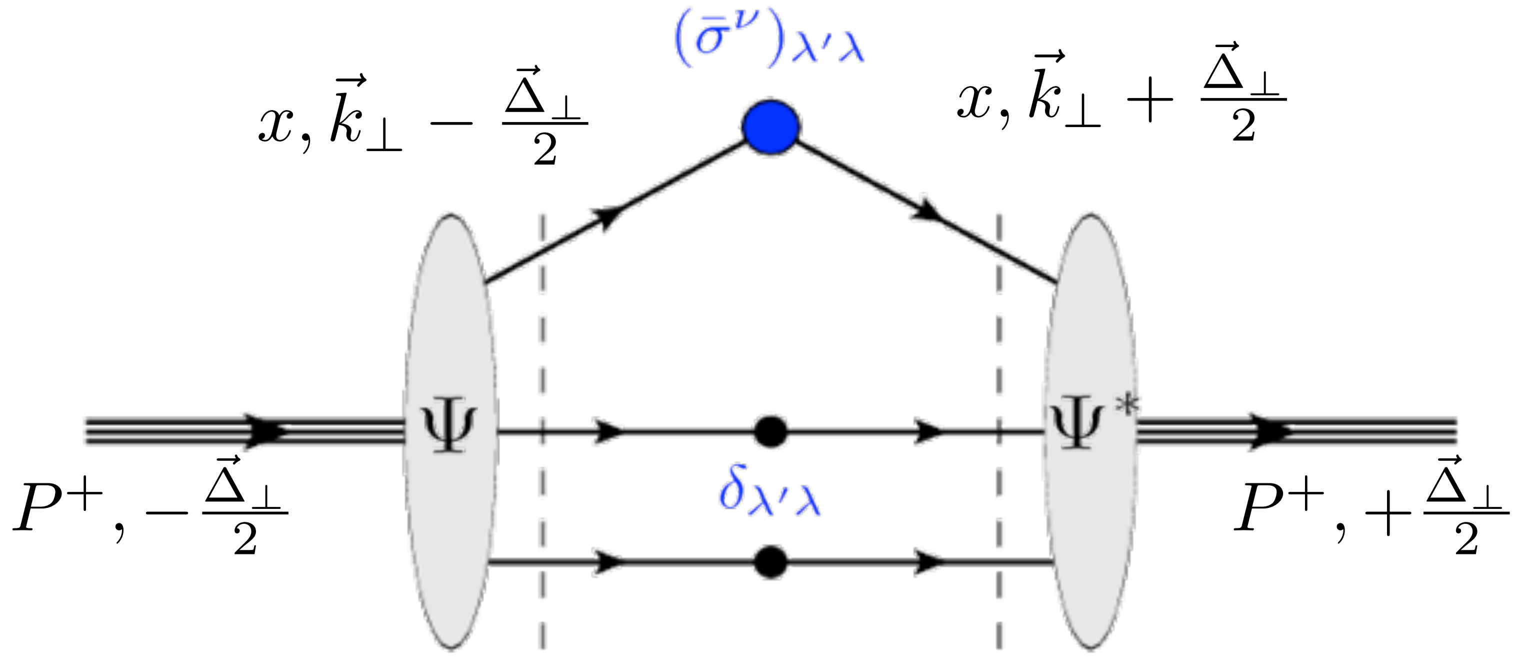


($\Delta^+$ =0)



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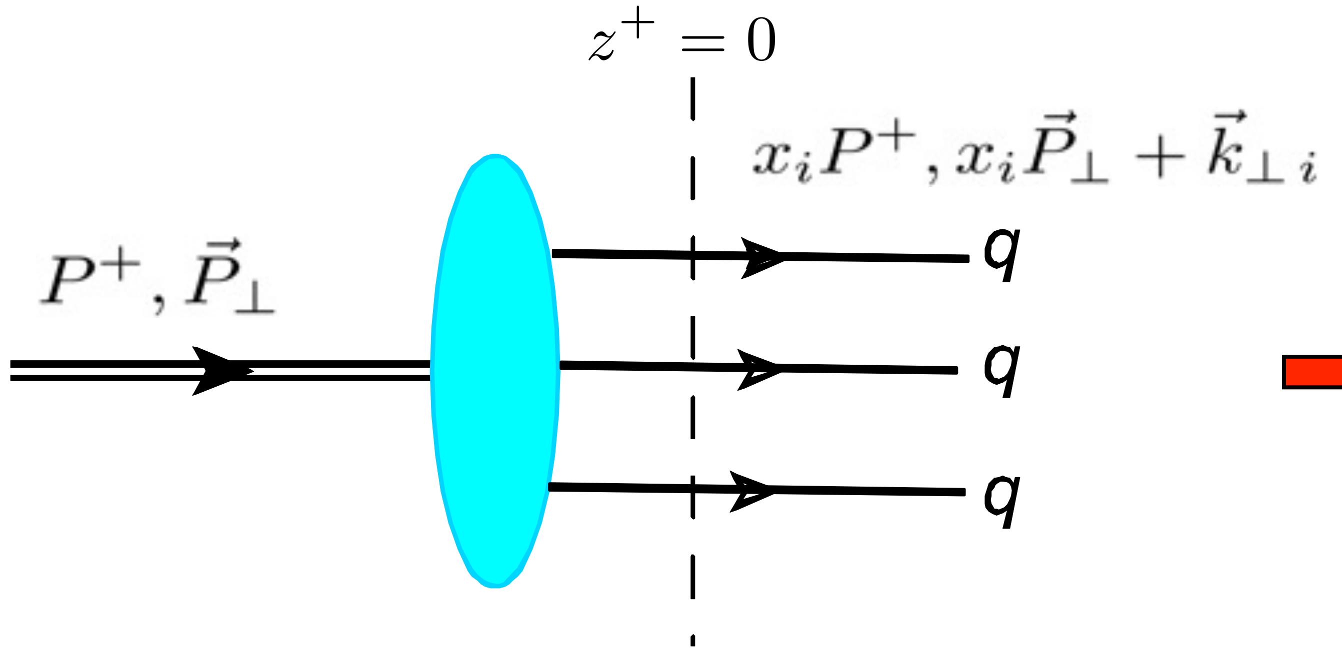

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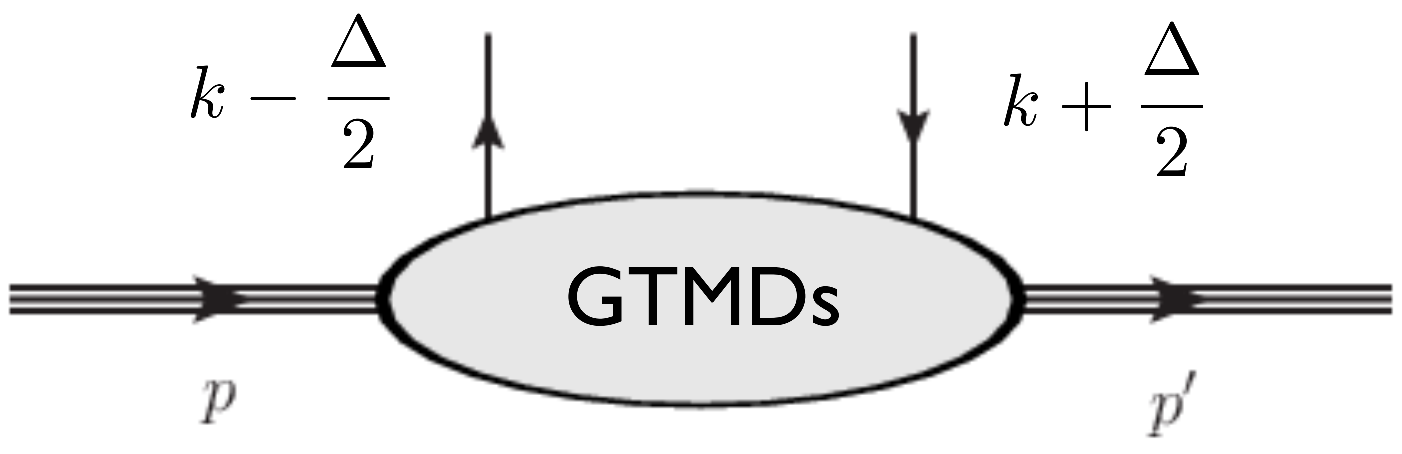

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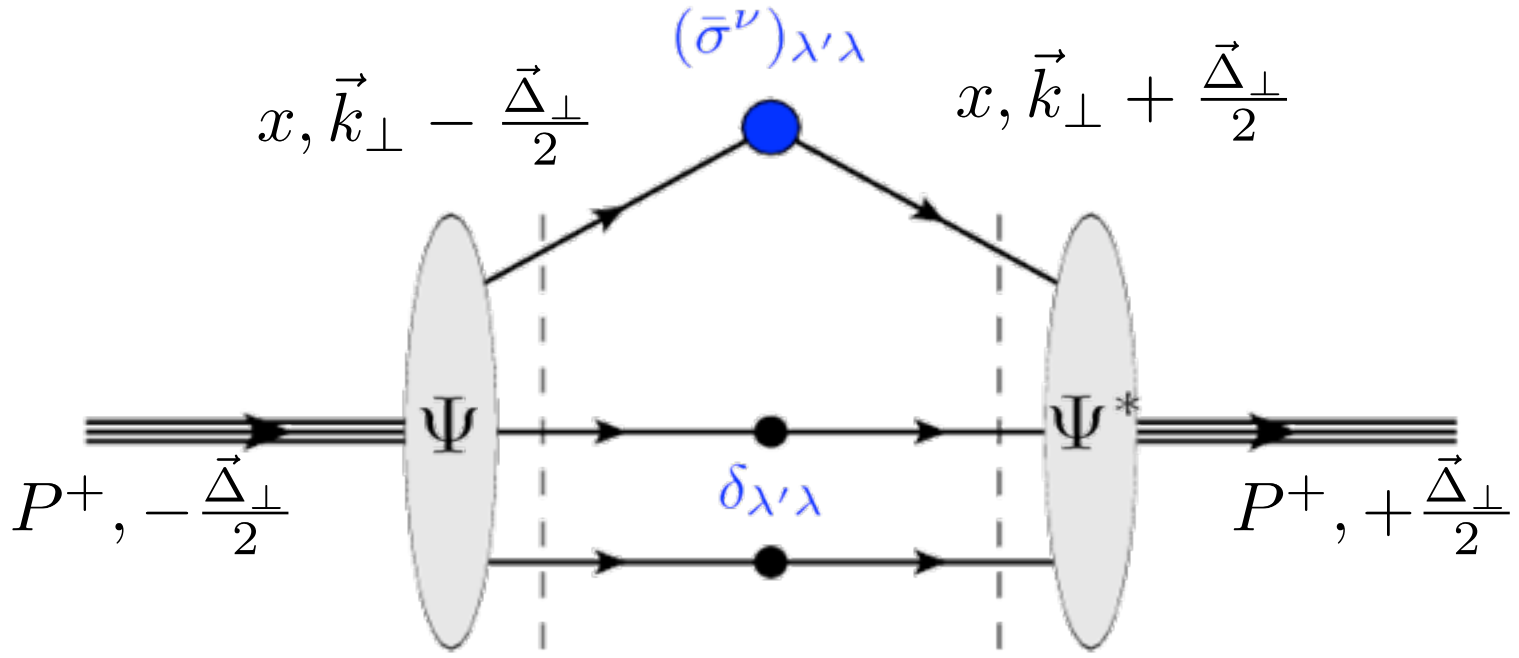

$$\Psi^{\Lambda;q_1q_2q_3}_{\lambda_1\lambda_2\lambda_3}(x_i, \vec{k}_{\perp,i}) = \sum_{s_i} \underbrace{\phi(x_i, \vec{k}_{\perp,i})}_{\text{momentum wf}} \underbrace{\Phi^{\Lambda;q_1q_2q_3}_{s_1s_2s_3}}_{\text{spin-flavor wf}} \prod_i \underbrace{D^{1/2*}_{s_i\lambda_i}(R_{cf})}_{\text{rotation from canonical spin to light-front spin}}$$

**General formalism valid for**

**Bag Model, LFχQSM, LFCQM, Quark-Diquark, Covariant Parton Models**

[ Lorcé, BP, Vanderhaeghen, JHEP05 (2011)]

# Light-Front Constituent Quark Model

# Light-Front Constituent Quark Model

- **momentum-space wf**

  [Schlumpf, Ph.D.Thesis, hep-ph/921155]

$$\Psi(k_i) = \frac{N}{(M_0^2+\beta^2)^\gamma}$$

$N$ : normalization constant

$$M_0 = \sum_i^3 \sqrt{m_i^2 + \vec{k}_i^2}$$

$\beta,\ \gamma$ parameters fitted to anomalous magnetic moments of the nucleon

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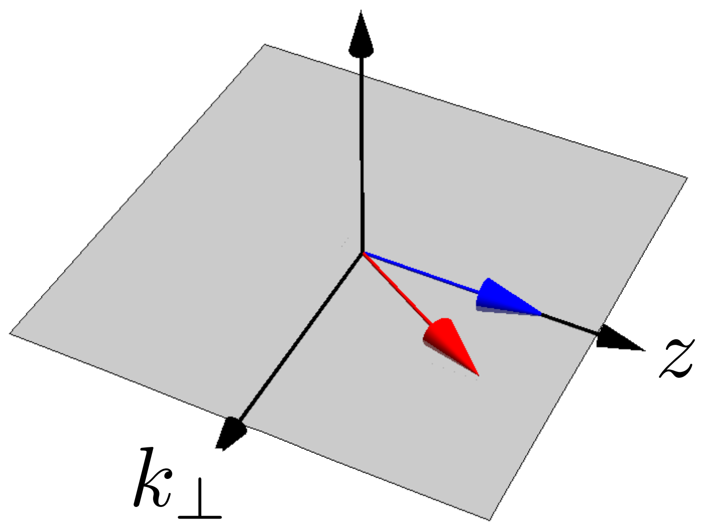
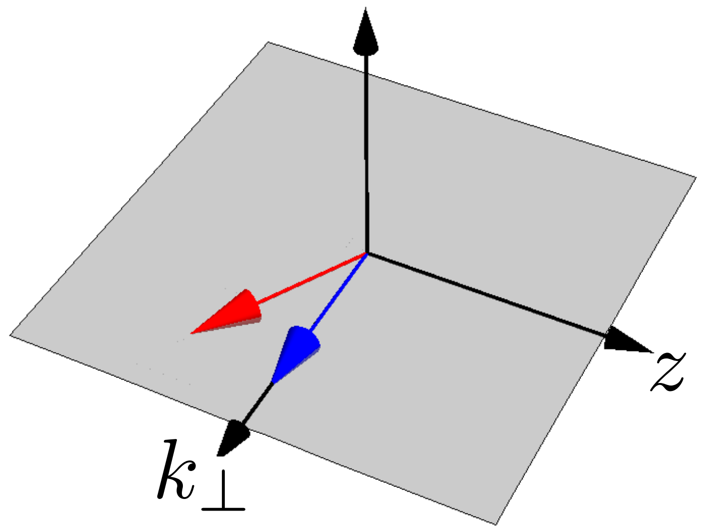
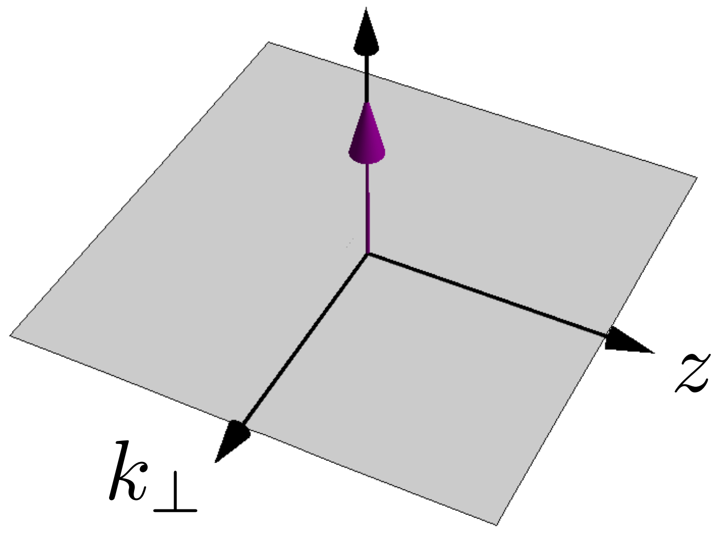
$$q_\lambda^{LC}(k) = D_{\lambda s}^{(1/2)*}\, q_s^C(k) \qquad D_{\lambda s}^{(1/2)*}(k) = \frac{1}{|\vec{K}|}\begin{pmatrix} K_z & K_L \\ -K_R & K_z \end{pmatrix}$$







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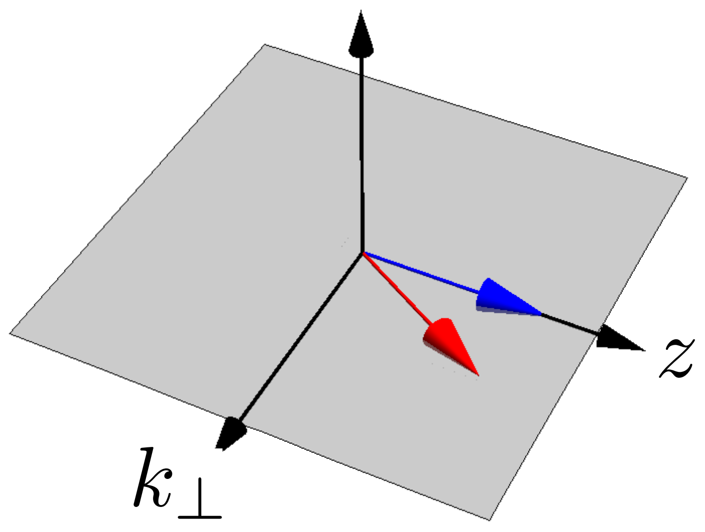
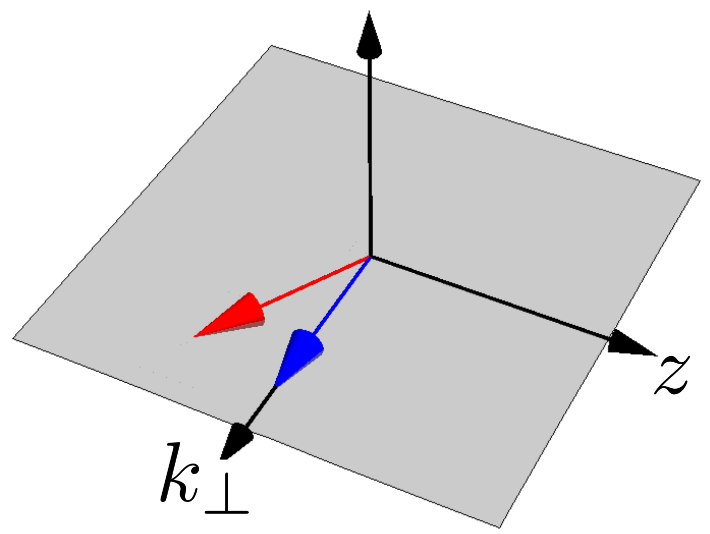
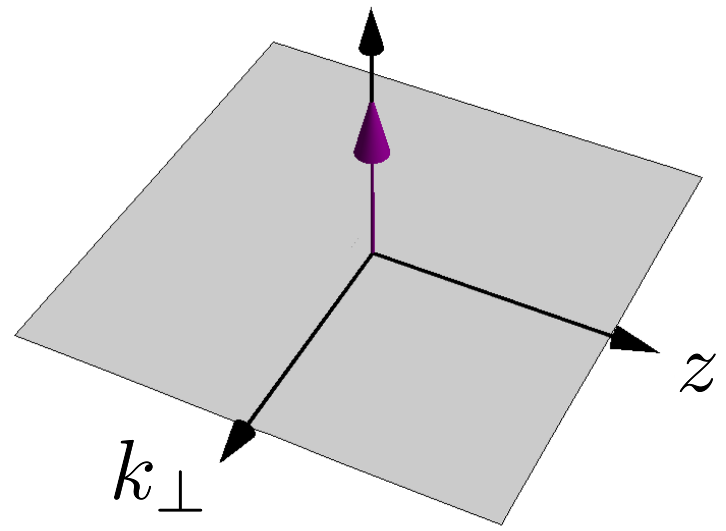
free quarks → $K_z = m + x\mathcal{M}_0 \quad \vec{K}_\perp = \vec{k}_\perp$ (Melosh rotation)







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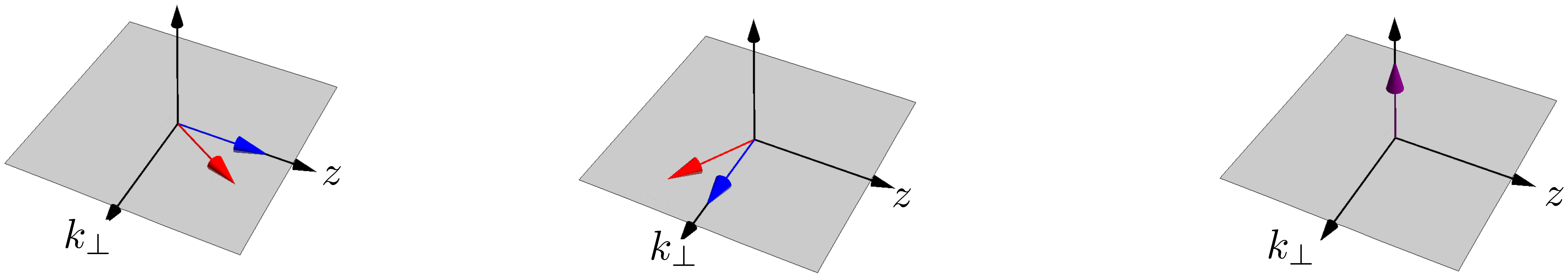

- SU(6) symmetry

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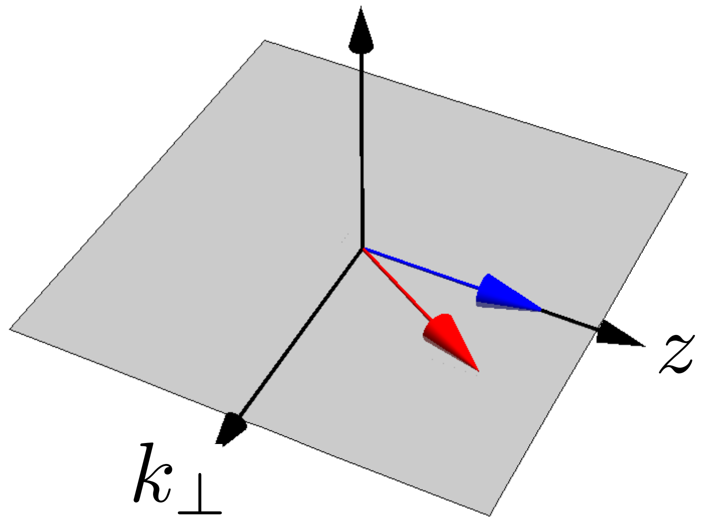
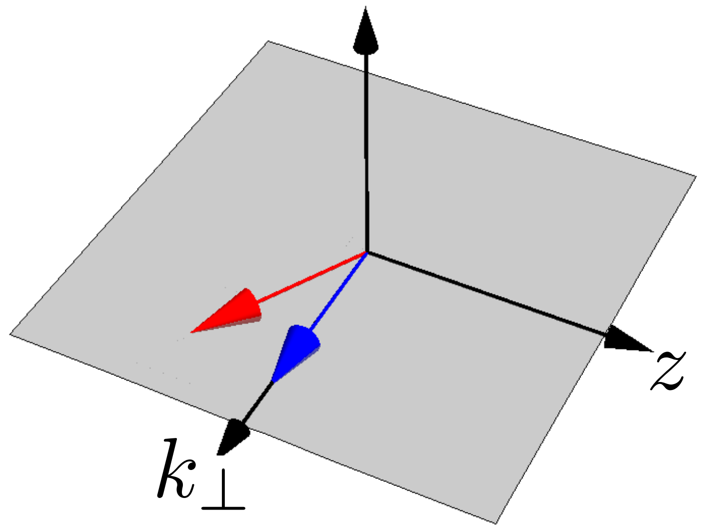
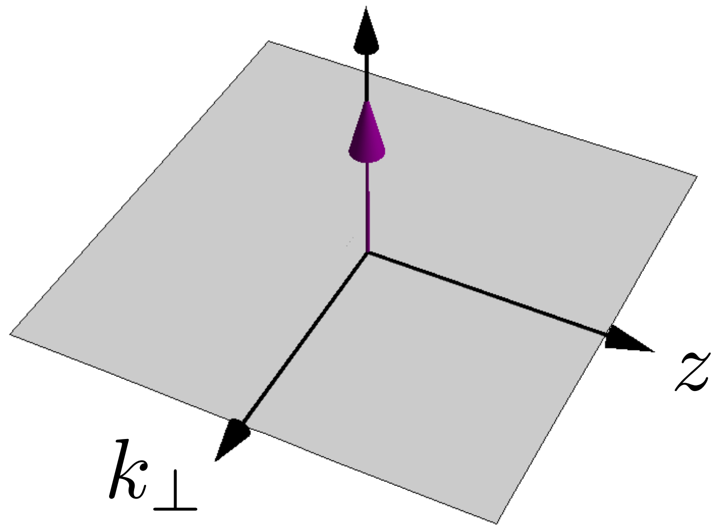






- **SU(6) symmetry**

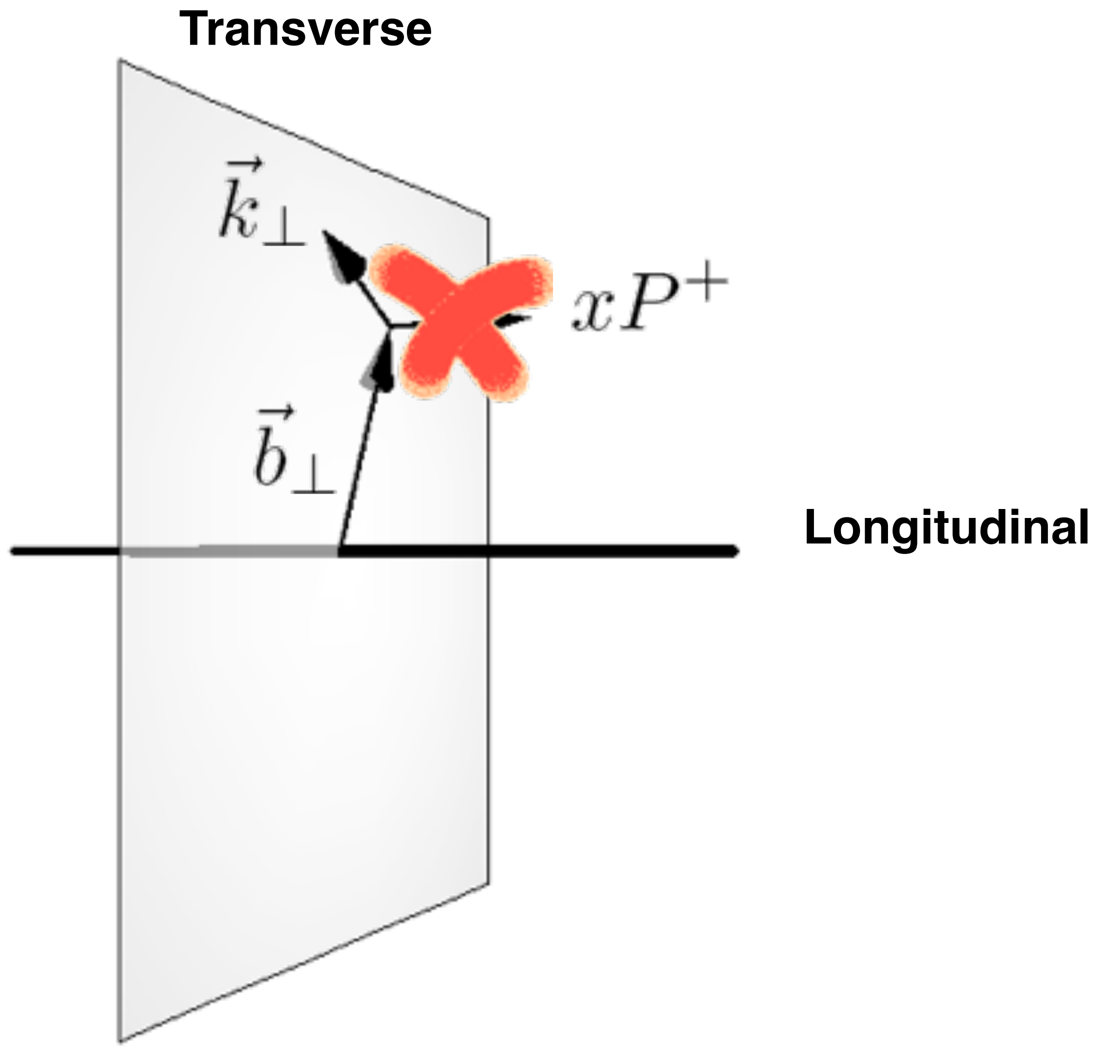
Applications of the model to:

**GPDs and Form Factors:** BP, Boffi, Traini (2003)-(2005);

**TMDs:** BP, Cazzaniga, Boffi (2008); BP, Yuan (2010); Lorcè, BP, Vanderhaeghen (2011)

**Azimuthal Asymmetries:** Schweitzer, BP, Boffi, Efremov (2009)

# Quark Wigner Distributions



$$\rho(\vec{k}_\perp, \vec{b}_\perp) = \int \mathrm{d}x \, \rho(x, \vec{k}_\perp, \vec{b}_\perp)$$ **2+2D**

at fixed $\vec{k}_\perp$ → two-dimensional distributions in impact-parameter space

★ Twist-2 ~ LO in P$^+$ $\boxed{\Gamma_{\text{twist}-2} = \gamma^+, \, \gamma^+\gamma_5, \, i\sigma^{j+}\gamma_5}$

quark polarization → **U** **L** **T**

★ Nucleon polarization: **U** **L** **T**

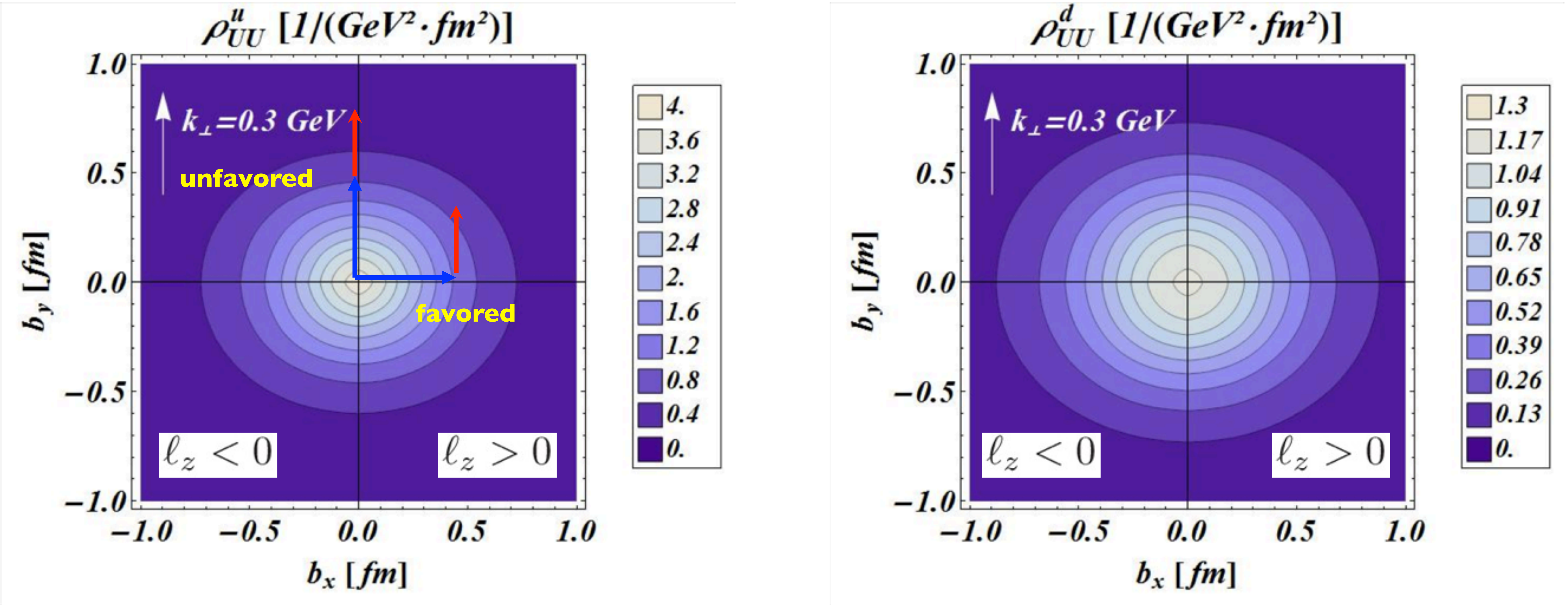
→ 16 independent Wigner distributions

# Unpol. quarks in Unpol. Proton

fixed $\vec{k}_\perp$: ↑

up quark

down quark



Distortion due to correlations between $\vec{k}_\perp$ and $\vec{b}_\perp$

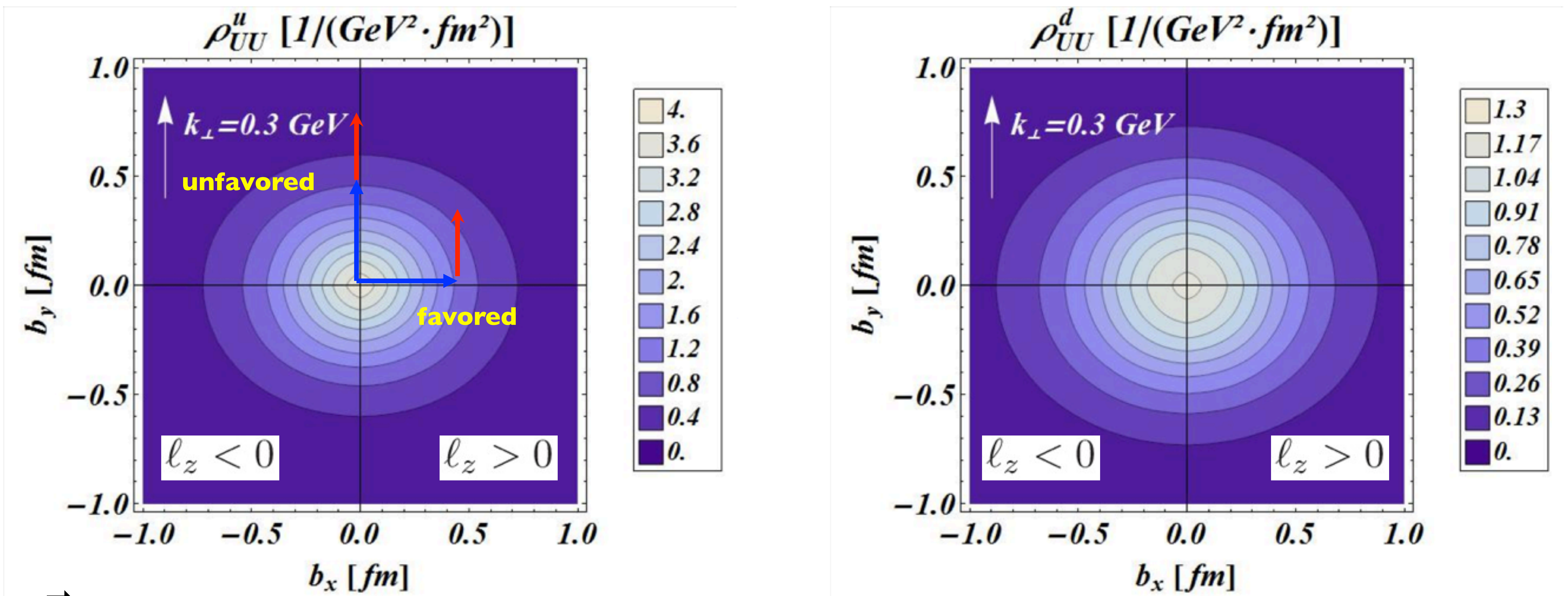
↳ absent in GPD and TMD !

Left-right symmetry → no net quark OAM

[Lorcé, Pasquini (2011)]

# Unpol. quarks in Unpol. Proton

fixed $\vec{k}_\perp$: ↑



up quark

down quark

✦ integrating over $\vec{b}_\perp$ ➡ transverse-momentum density

$$f_1^q(k_\perp^2) = \int \mathrm{d}x\, f_1^q(x, k_\perp^2)$$

✦ integrating over $\vec{k}_\perp$ ➡ charge density in the transverse plane $\vec{b}_\perp$

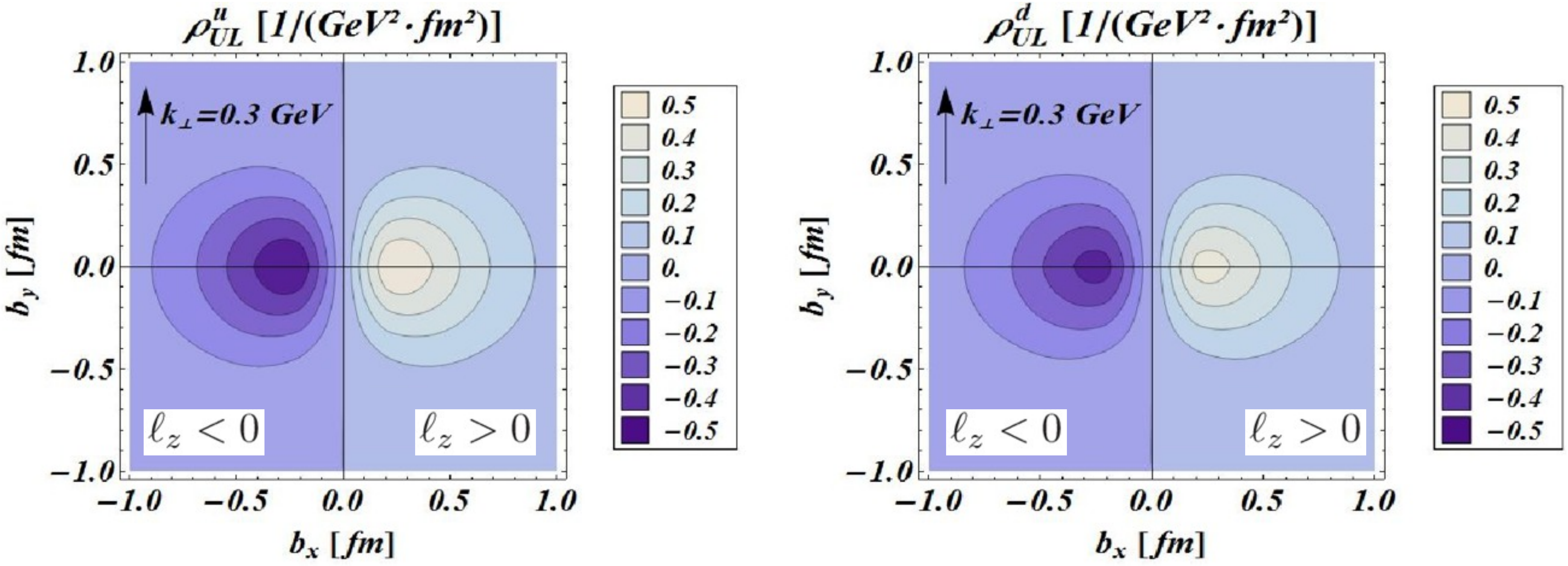
$$\rho^q(b_\perp^2) = e^q \int \mathrm{d}^2\Delta_\perp e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} F_1^q(\Delta_\perp^2)$$

**Monopole Distributions**

[Miller (2007); Burkardt (2007)]

# Long. pol. quark in Unpol. Proton

fixed $\vec{k}_\perp \uparrow$



up quark

down quark

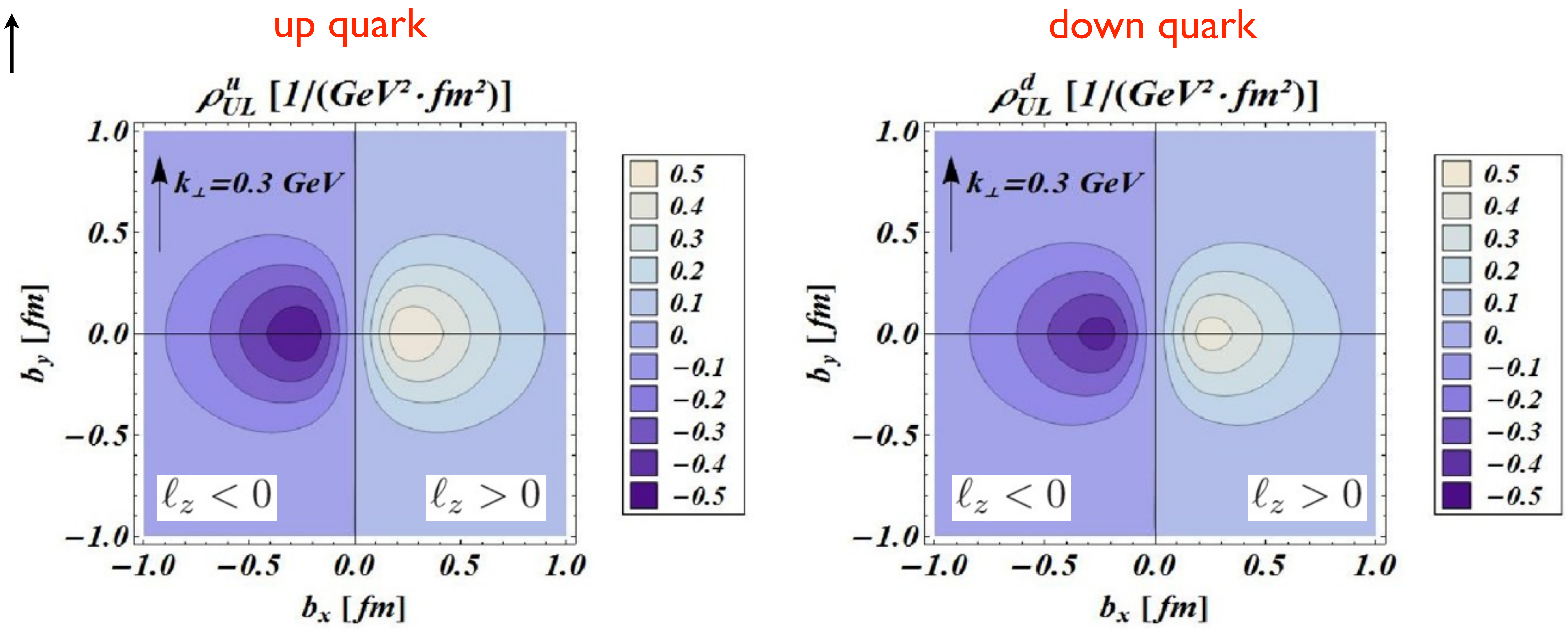
- ✦ projection to GPD and TMD is vanishing

$\Rightarrow$ unique information on OAM from Wigner distributions

[Lorcé, Pasquini (2011)]
[Lorcé, (2014)]

# Long. pol. quark in Unpol. Proton

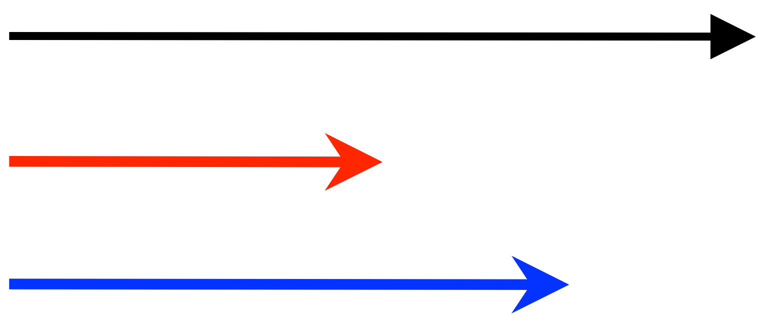
fixed $\vec{k}_\perp \uparrow$



correlation between quark spin and quark OAM

$$\boxed{C^q_z = \int \mathrm{d}x\, \mathrm{d}\vec{k}_\perp\, \mathrm{d}\vec{b}_\perp \left(\vec{b}_\perp \times \vec{k}_\perp\right) \rho^q_{UL}(x, \vec{k}_\perp, \vec{b}_\perp)}$$

| | u-quark | d-quark |
|---|---|---|
| $C^q_z$ | 0.23 | 0.19 |



**Quark spin**

u-quark OAM

d-quark OAM

[Lorcé, Pasquini (2011)]
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# Unpol. quark in Long. pol. Proton

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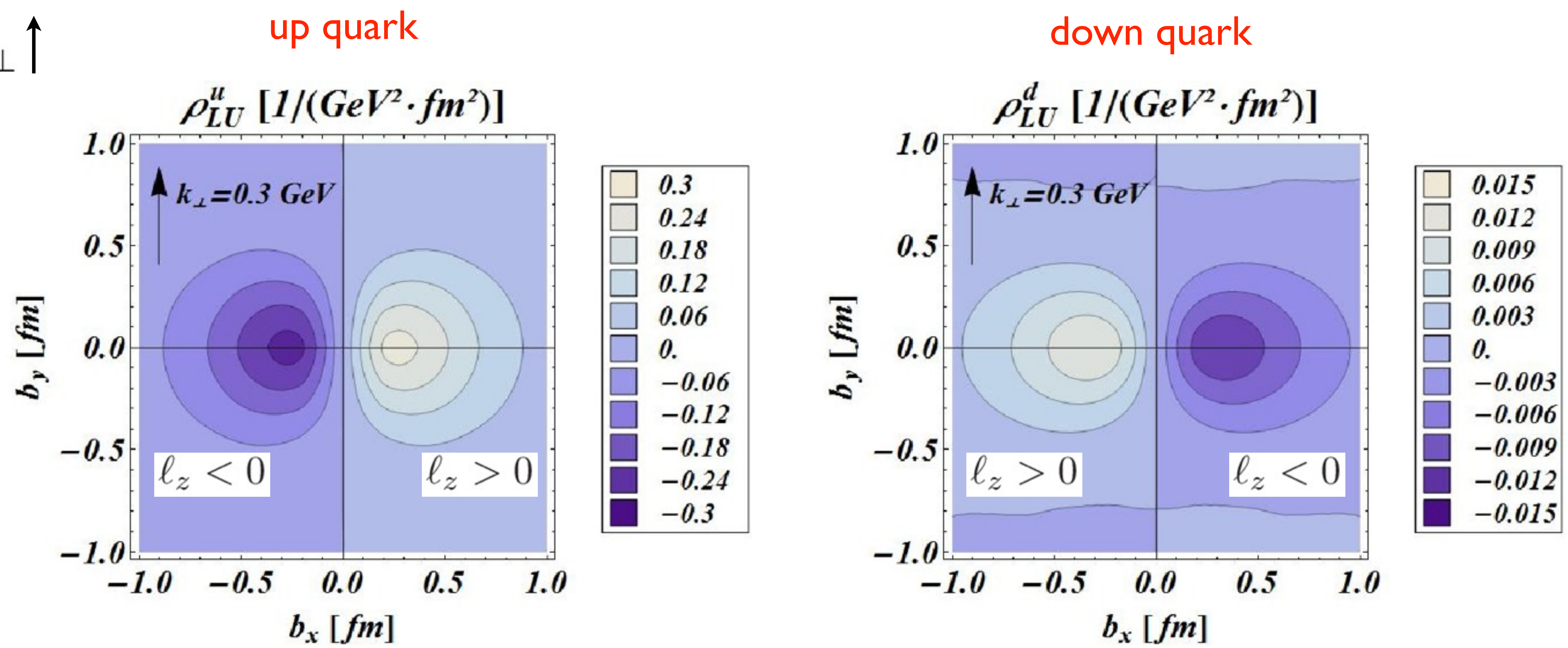

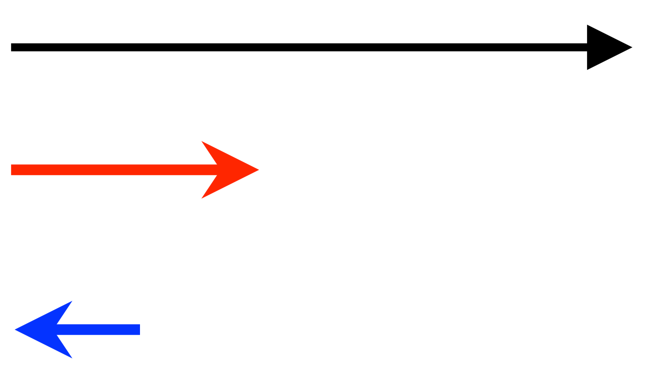

**Proton spin**

u-quark OAM

d-quark OAM

★ projection to GPD and TMD is vanishing

→ unique information on OAM from Wigner distributions

[Lorcé, Pasquini (2011)]

# Quark Orbital Angular Momentum

$$\ell_z^q = \int \mathrm{d}x\, \mathrm{d}^2\vec{k}_\perp \mathrm{d}^2\vec{b}_\perp (\vec{b}_\perp \times \vec{k}_\perp)\rho^q_{LU}(\vec{b}_\perp, \vec{k}_\perp, x) = -\int \mathrm{d}x \mathrm{d}^2\vec{k}_\perp \frac{\vec{k}_\perp^2}{M^2} F^q_{14}(x, 0, \vec{k}_\perp, \vec{0}_\perp)$$

⬇

Wigner distribution
for Unpolarized quark in a Longitudinally pol. nucleon

[Lorcé, BP (11)
Hatta (12)
Ji, Xiong, Yuan (12)
Kanazawa et al., (2014)]

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$$= \int \mathrm{d}^2\vec{b}_\perp \vec{b}_\perp \times \langle \vec{k}^q_\perp \rangle \longrightarrow \langle \vec{k}^q_\perp \rangle = \int \mathrm{d}x\, \mathrm{d}\vec{k}_\perp\, \vec{k}_\perp \rho^q_{LU}(\vec{b}_\perp, \vec{k}_\perp, x)$$

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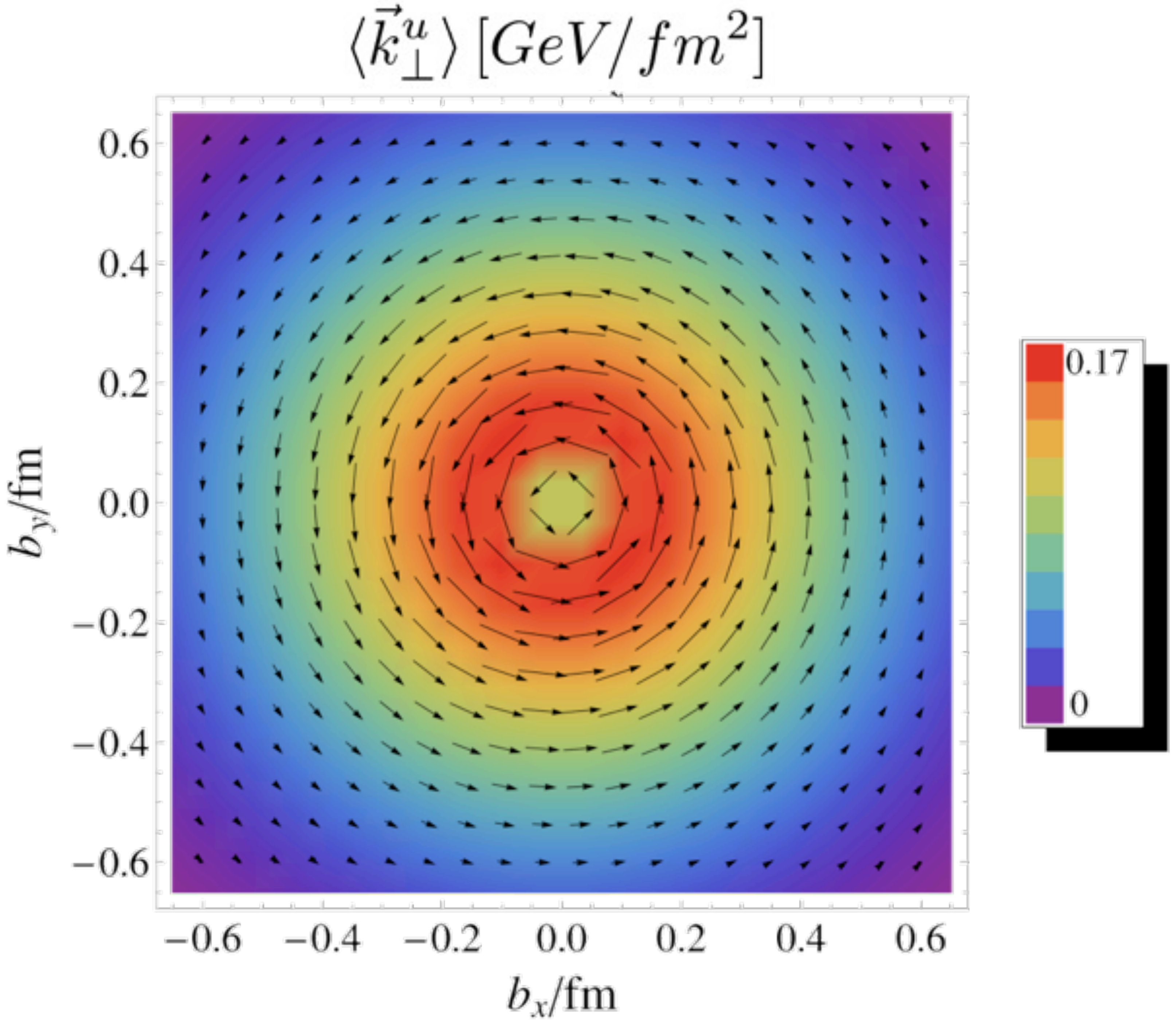

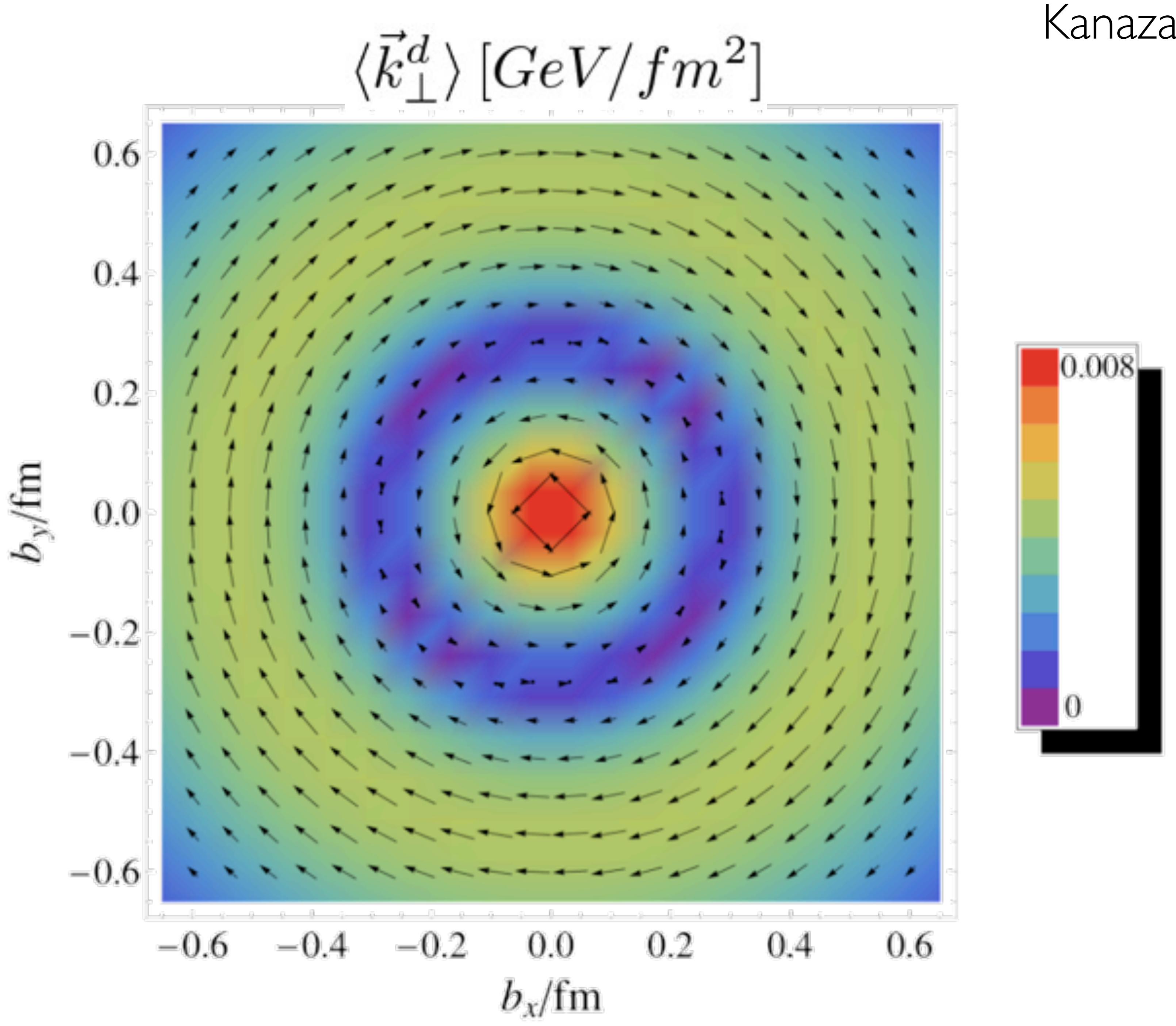

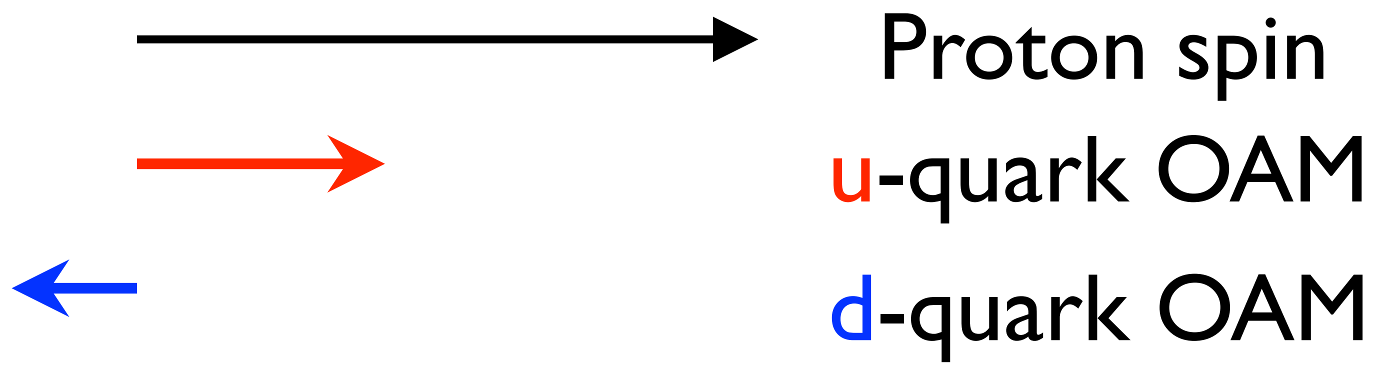
Proton spin

u-quark OAM

d-quark OAM

Results in a light-front constituent quark model:
Lorcé, BP, Xiong, Yuan, PRD85 (2012)

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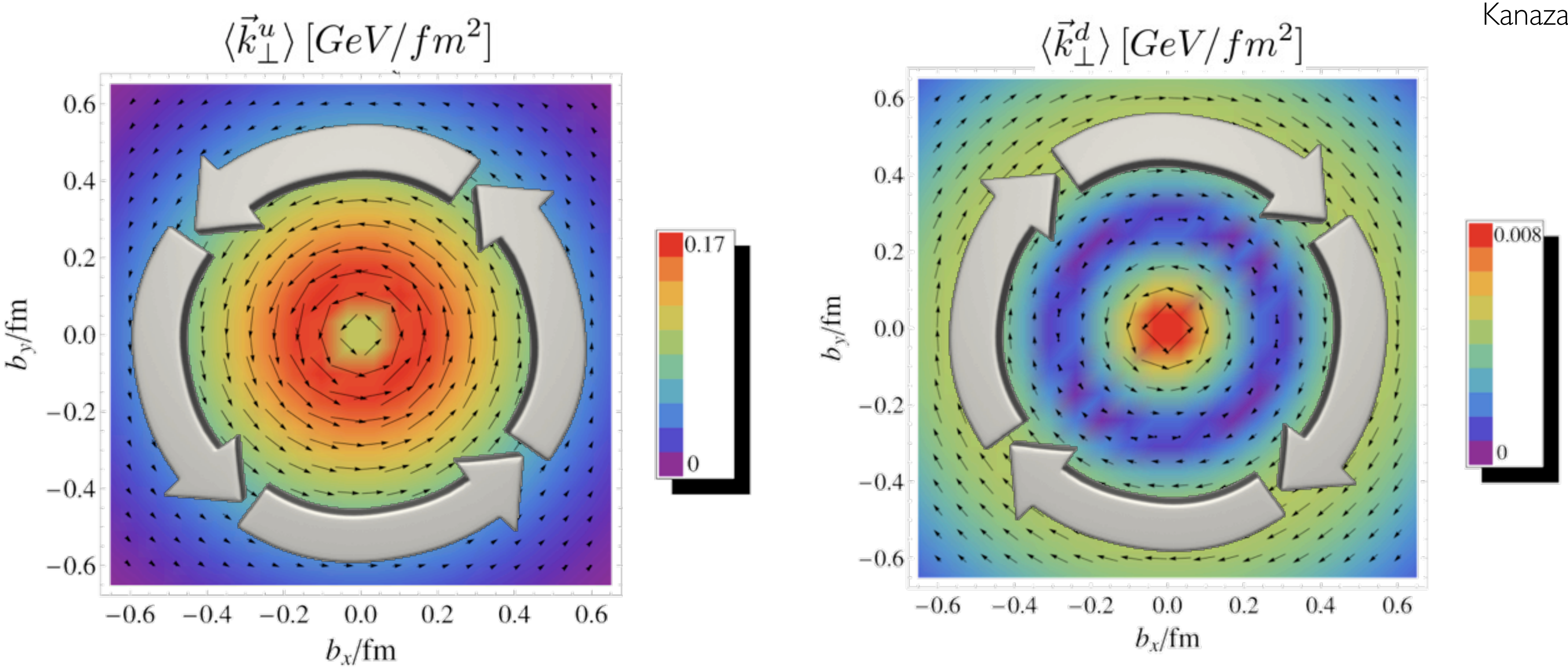

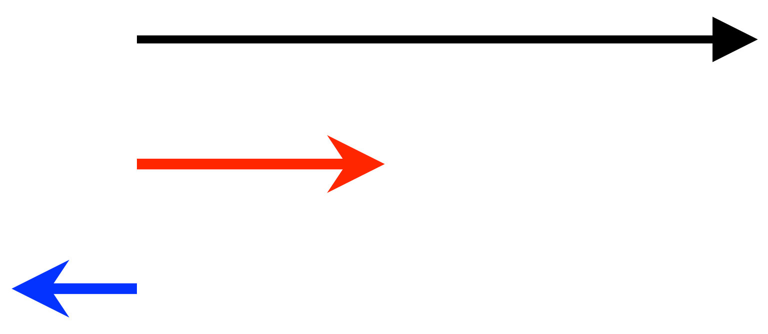
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$$\boxed{\ell_z^q = \int \mathrm{d}x\mathrm{d}^2\vec{k}_\perp\mathrm{d}^2\vec{b}_\perp(\vec{b}_\perp \times \vec{k}_\perp)\rho_{LU}(\vec{b}_\perp,\vec{k}_\perp,x)}$$

Light-cone gauge $A^+ = 0$

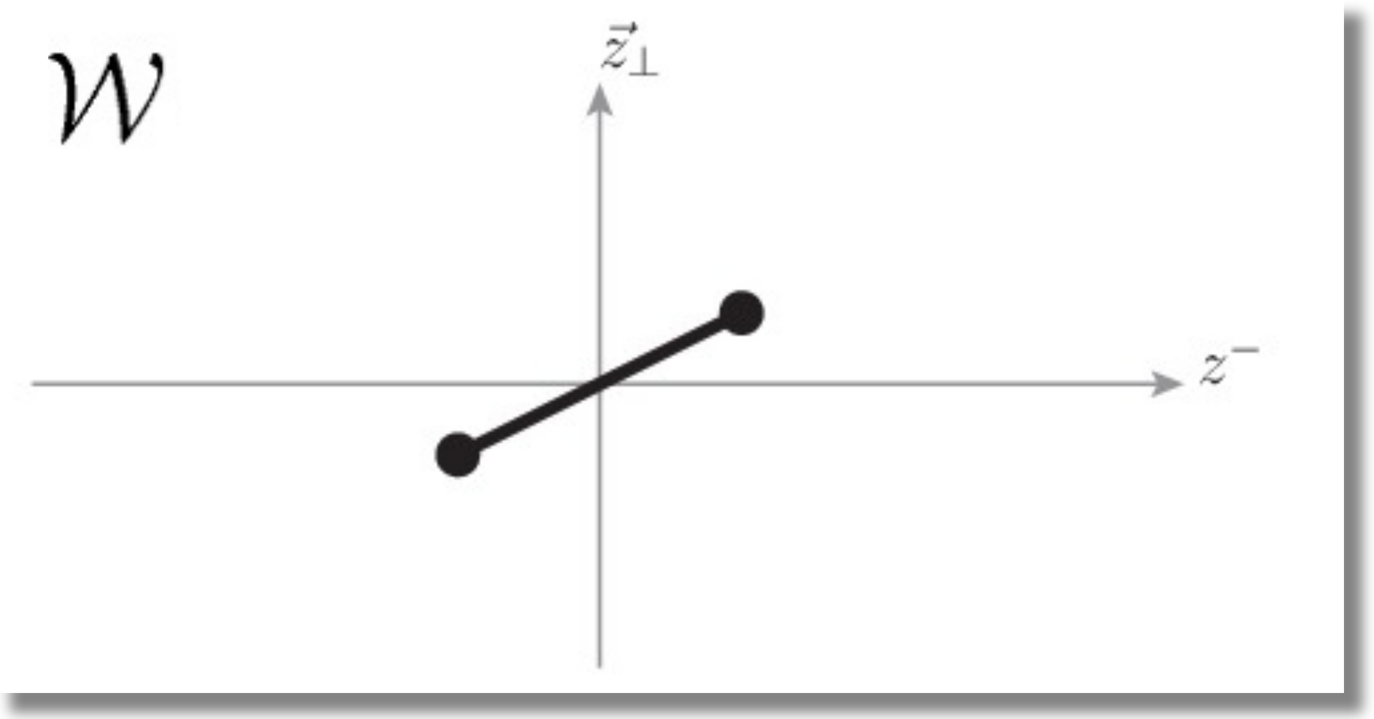
not gauge invariant, but with simple partonic interpretation

Gauge-invariant extension

$$\rho_{LU} \to \rho_{LU}^{\overset{\mathcal{W}}{}} \longrightarrow \text{Wilson line}$$

**Kinetic**

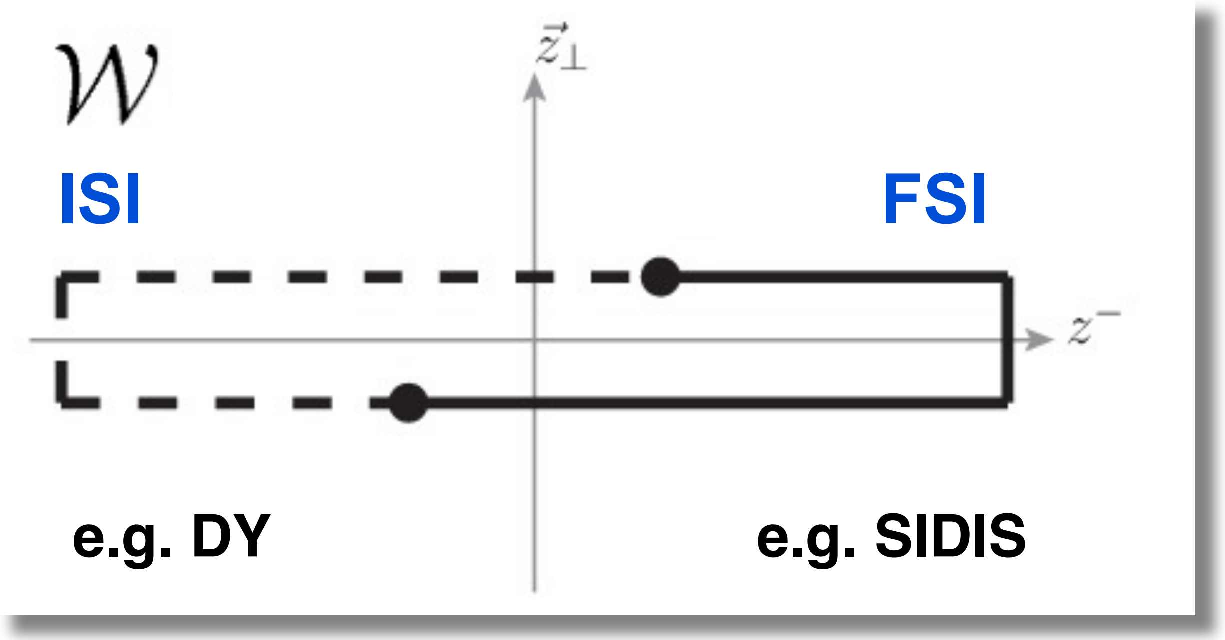
[Ji (1997)]



[Ji, Xiong, Yuan (2012)]
[Burkardt (2012)]

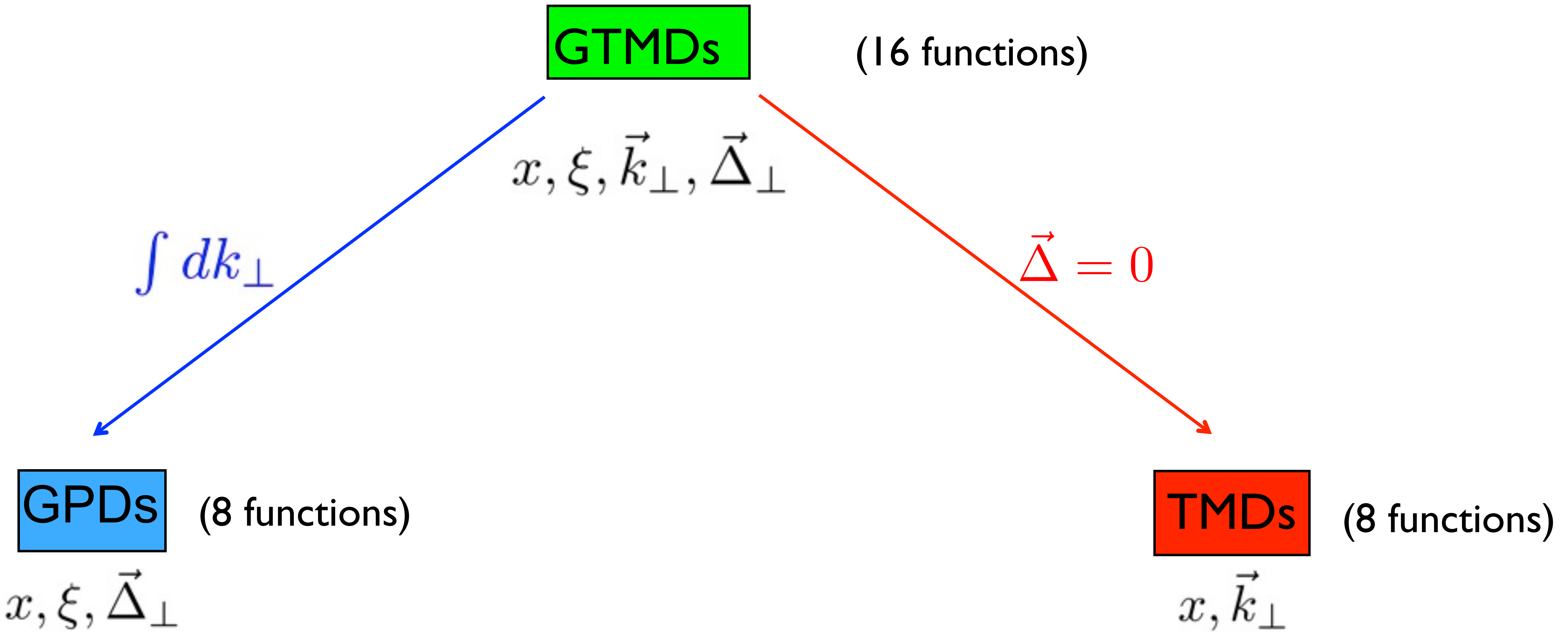
**Canonical**

[Jaffe, Manohar (1990)]



$$\ell_z^{\mathrm{DY}} = \ell_z^{\mathrm{SIDIS}}$$

[Hatta (2012)]



GTMDs (16 functions)

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

$\int dk_\perp$

$\vec{\Delta} = 0$

GPDs (8 functions)

$x, \xi, \vec{\Delta}_\perp$

TMDs (8 functions)

$x, \vec{k}_\perp$

Quark polarization

Nucleon polarization

| | $U$ | $T$ | $L$ |
|---|---|---|---|
| $U$ | $H$ | $\mathcal{E}_T$ | |
| $T$ | $E$ | $H_T, \tilde{H}_T$ | $\tilde{E}$ |
| $L$ | | $\tilde{E}_T$ | $\tilde{H}$ |

Quark polarization

Nucleon polarization

| | $U$ | $T$ | $L$ |
|---|---|---|---|
| $U$ | $f_1$ | $h_1^\perp$ | |
| $T$ | $f_{1T}^\perp$ | $h_1,\ h_{1T}^\perp$ | $g_{1T}$ |
| $L$ | | $h_{1L}^\perp$ | $g_{1L}$ |

✦ almost all distributions (in red) vanish if there is no quark orbital angular momentum

✦ quark GPDs (at ξ=0) and TMDs given by the same overlap of LFWFs but in different kinematics
⇒ each distribution contains unique information

⇒ no model-independent relations between GPDs and TMDs

# Quark OAM from Pretzelosity

$h_{1T}^{\perp} =$  -  “pretzelosity”

model-dependent relation

$$\mathcal{L}_z = -\int \mathrm{d}x \mathrm{d}^2\vec{k}_\perp \frac{k_\perp^2}{2M^2}\, h_{1T}^{\perp}(x, k_\perp^2)$$

first derived in LF-diquark model and bag model

[She, Zhu, Ma, 2009; Avakian, Efremov, Schweitzer, Yuan, 2010]

# Quark OAM from Pretzelosity

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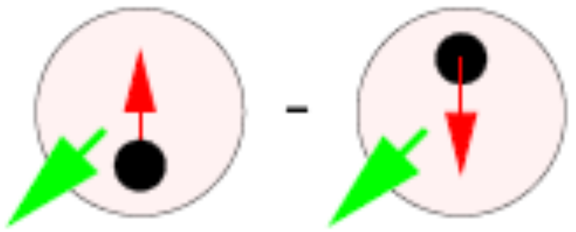

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[She, Zhu, Ma, 2009; Avakian, Efremov, Schweitzer, Yuan, 2010]

$\mathcal{L}_z$

chiral even and charge even

$\Delta L_z = 0$

$h_{1T}^{\perp}$

chiral odd and charge odd

$|\Delta L_z| = 2$

no operator identity
relation at level of matrix elements of operators

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$h_{1T}^{\perp}$ =  -  "pretzelosity"

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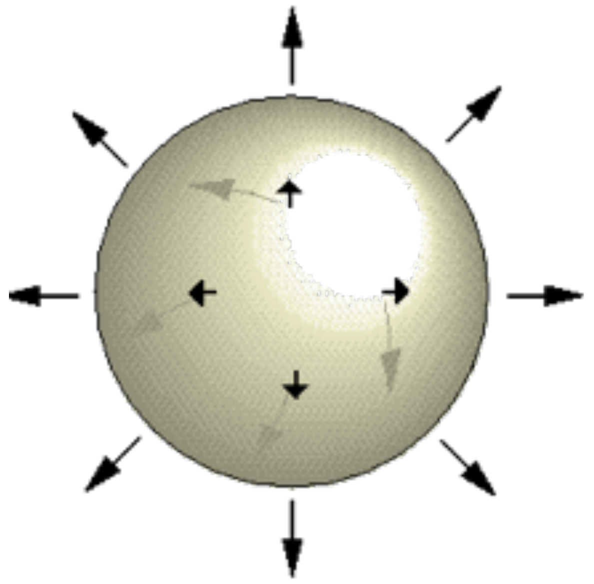
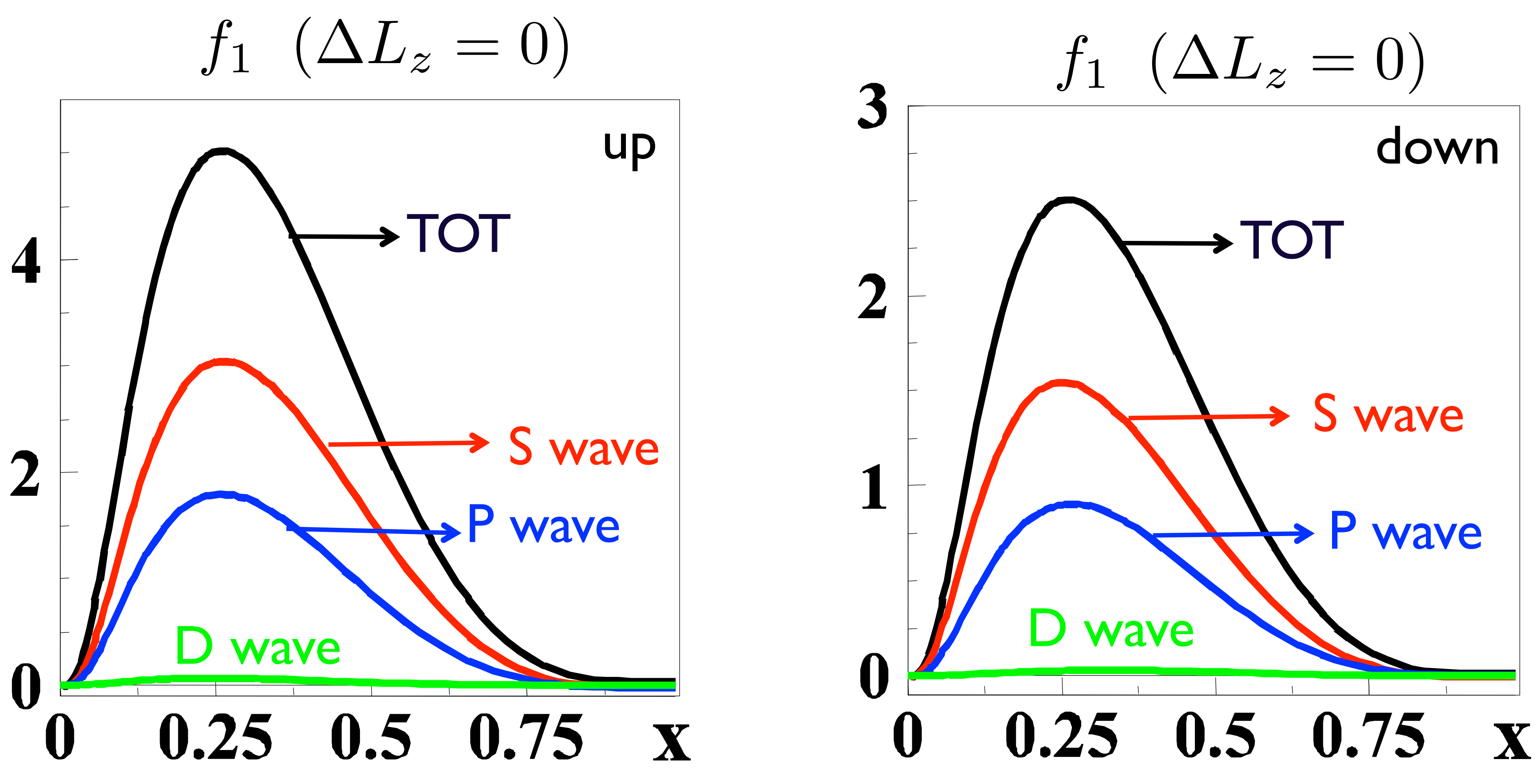
⇩

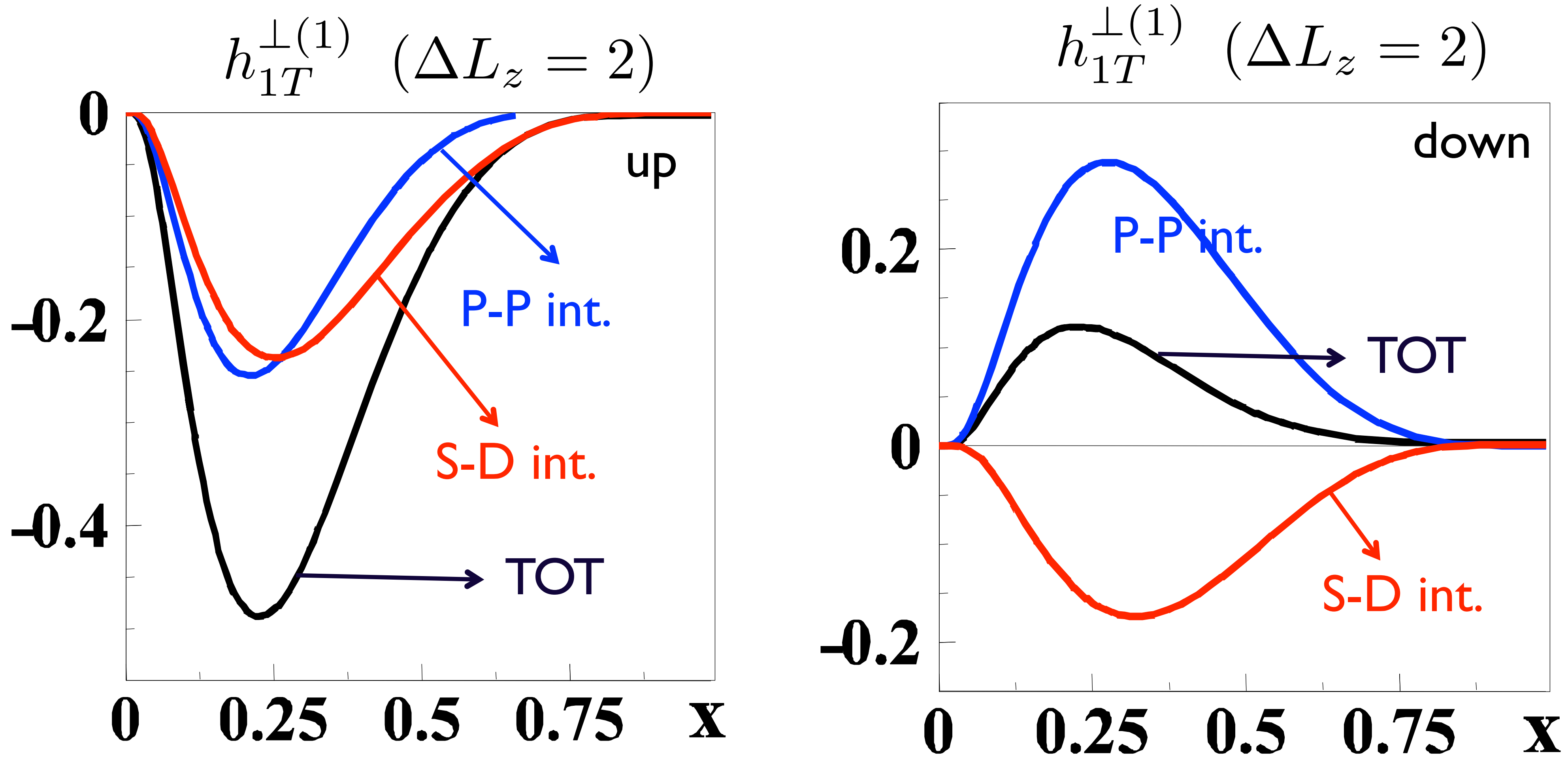


valid in all quark models with spherical symmetry in the rest frame

[Lorcé, BP, PLB (2012)]

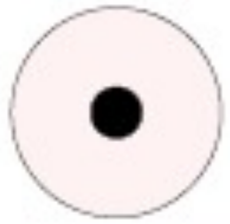
## ✦ Orbital angular momentum content of TMDs (light-front constituent quark model)



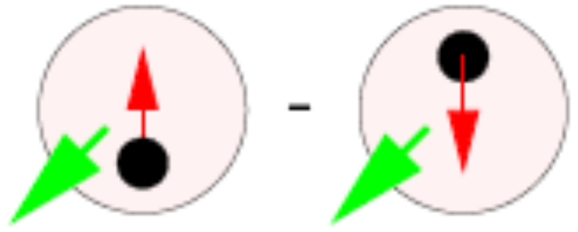


“pretzelosity”

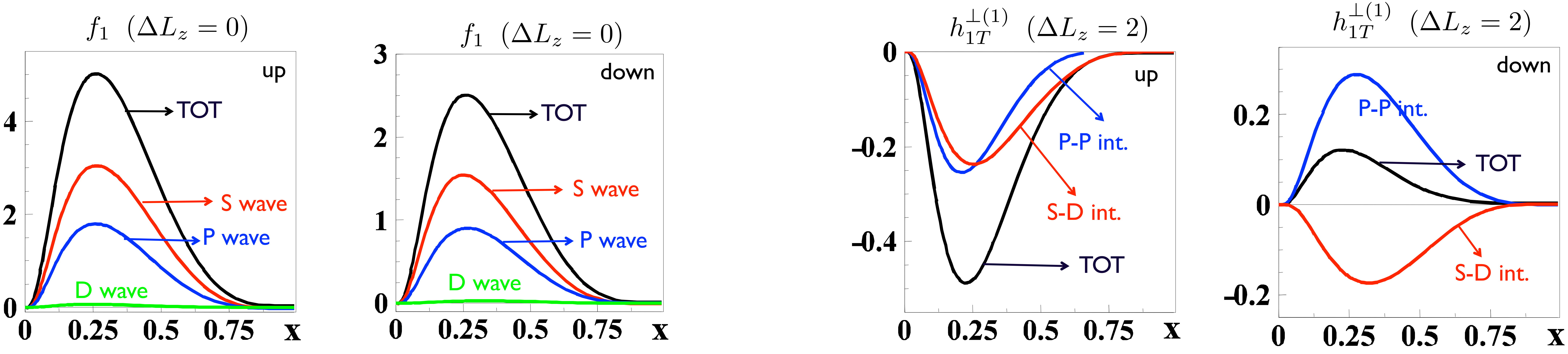
$f_1 =$



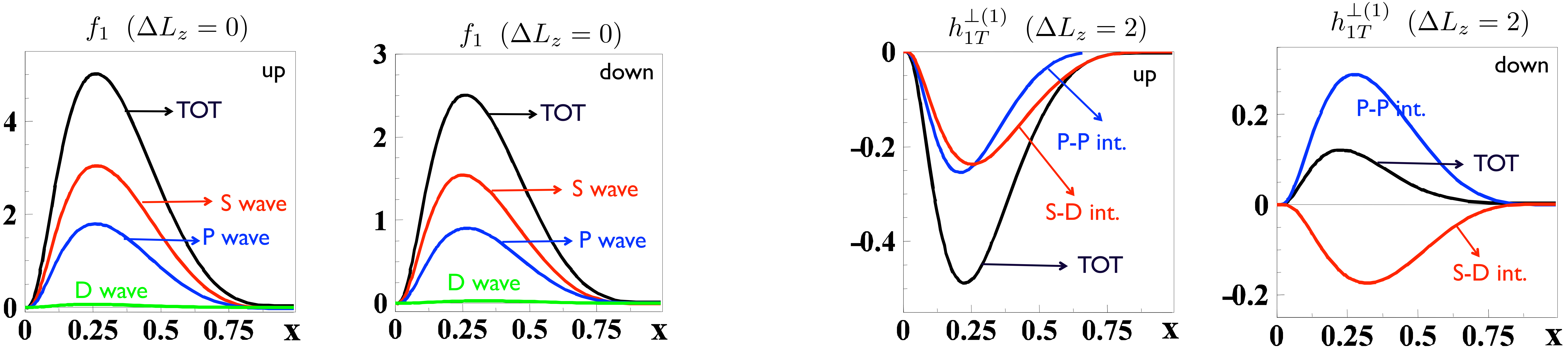
$h_{1T}^{\perp} =$



# ✦ Orbital angular momentum content of TMDs (light-front constituent quark model)

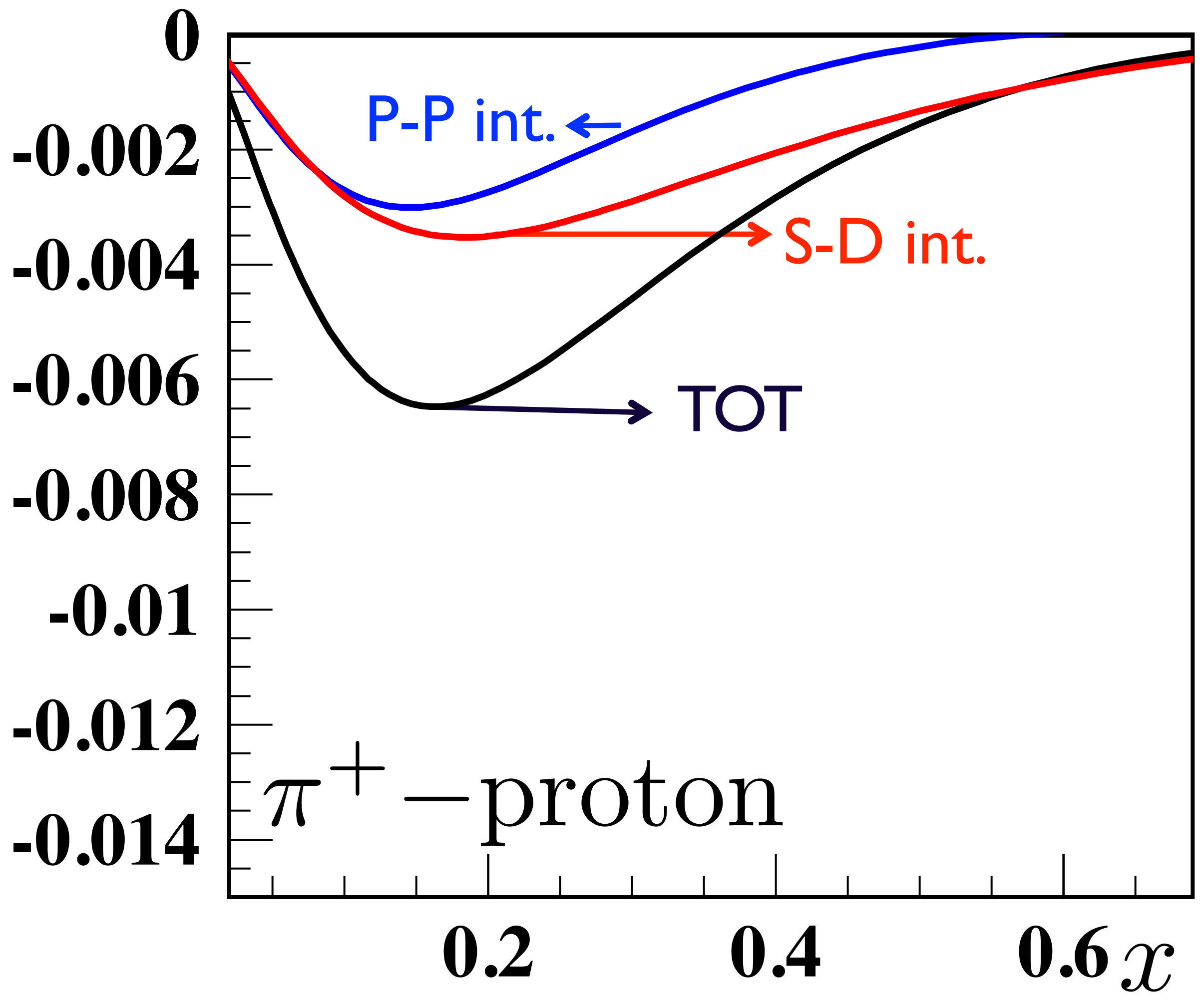


# ✦ Orbital angular momentum content of TMDs (light-front constituent quark model)

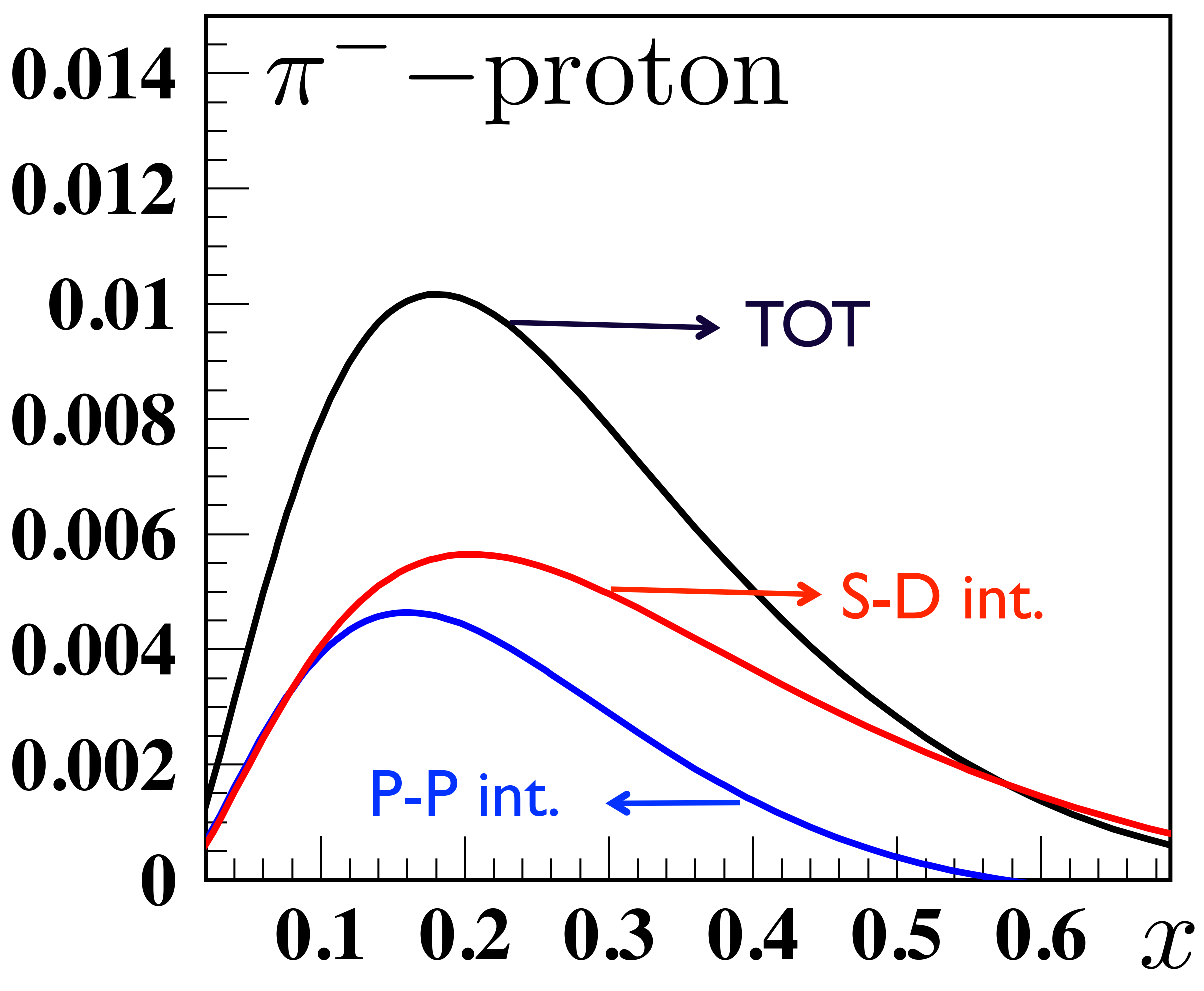


## ✦ Effects on SIDIS observables

$$A_{UT}^{\sin(3\phi-\phi_S)} \sim \frac{h_{1T}^{\perp} \otimes H_1}{f_1 \otimes D_1}$$



$\langle Q^2 \rangle = 2.5 \ \mathrm{GeV}^2$



Boffi, Efremov, BP, Schweitzer, PRD79(2009); White paper of SOLID Coll., arXiv:1409.7741

# Quark spin and OAM

## GTMDs

### Quark spin (from DIS)

$$S_z^q = \frac{1}{2}\int \mathrm{d}x\,\mathrm{d}^2k_\perp\, G_{14}^q(x,0,\vec{k}_\perp,\vec{0}_\perp)$$

**polarized PDF**
**inclusive DIS**

$$\ell_z^q = -\int \mathrm{d}x\,\mathrm{d}^2k_\perp\,\frac{\vec{k}_\perp^2}{M^2}\, F_{14}^q(x,0,\vec{k}_\perp,\vec{0}_\perp)$$

[Lorcé, BP (2011)]
[Hatta (2011)]
[Lorce',BP, et al. (2012)]

$$\ell_{iz}^{\text{int}} = \vec{b}_{i\perp}\times\vec{k}_{i\perp}$$

## TMDs

### Quark spin (from DIS)

$$S_z^q = \frac{1}{2}\int \mathrm{d}x\,\mathrm{d}^2k_\perp\, g_{1L}^q(x,\vec{k}_\perp)$$

**polarized PDF**
**inclusive DIS**

$$\mathcal{L}_z^q(x,\vec{k}_\perp) = -\frac{\vec{k}_\perp^2}{2M^2}\, h_{1T}^{\perp q}(x,\vec{k}_\perp^2)$$

[Burkardt (2007)]
[Efremov et al. (2008,2010)]
[She, Zhu, Ma (2009)]
[Avakian et al. (2010)]
[Lorcé, BP (2011)]

- **Model-dependent**
- **Not intrinsic!**

⚠

$$\mathcal{L}_{iz} = \vec{r}_{i\perp}\times\vec{k}_{i\perp}$$

## GPDs

### Quark spin (from DIS)

$$S_z^q = \frac{1}{2}\int \mathrm{d}x\,\tilde{H}^q(x,0,0)$$

**polarized PDF**
**inclusive DIS**

### Ji's relation

$$J^q = \tfrac{1}{2}\int \mathrm{d}x\,x\,[H^q(x,0,0)+E^q(x,0,0)]$$

$$L^q = J^q - S_z^q$$

[Ji (1997)]

### Twist-3

$$L_z^q = -\int \mathrm{d}x\,x\,G_2^q(x,0,0)$$

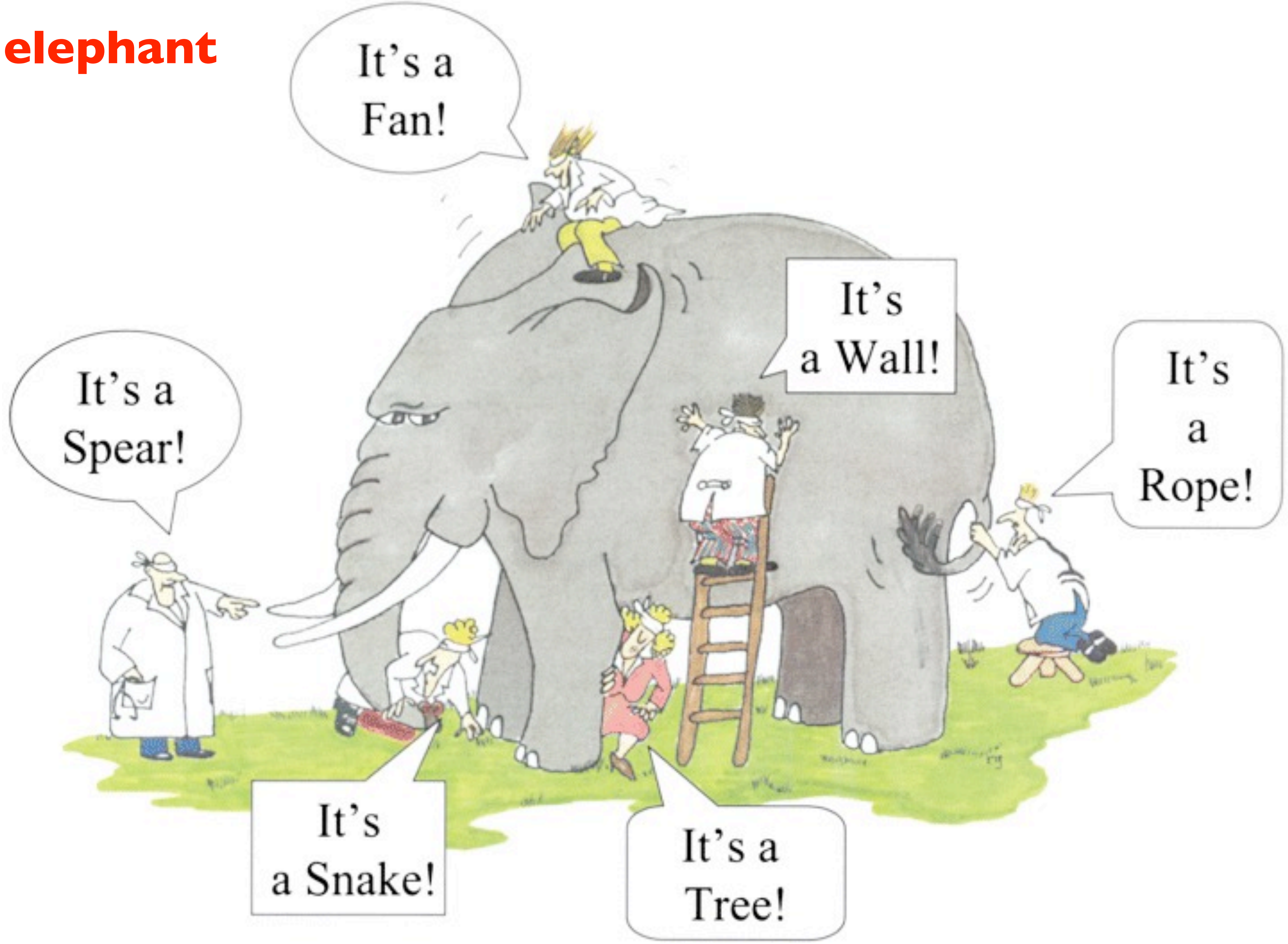
**Pure twist-3!**

[Penttinen et al. (2000)]

for a review: Lorcé, Leader, Phys.Rep. 541 (2014)

# The blind men and the elephant

from H. Avakian



- TMDs and GPDs provide different and complementary 3D pictures of the nucleon
- TMDs and GPDs are projections of the phase-space/Wigner distributions
- Full phase-space/Wigner distributions are not yet accessible from experiments
- Models constrained by data on GPDs and TMDs give access to Wigner distributions