Tensor Polarization Optimization and Measurement for Solid Spin 1 Targets

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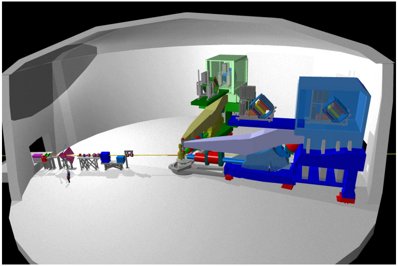
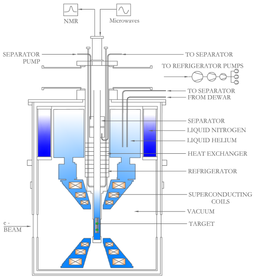
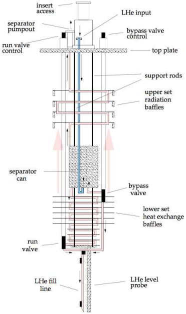


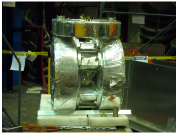
Solid Polarized Target Systems

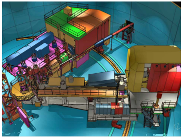




- (A) $\sim$100 nA
- (B) $10^{35}\mathrm{cm}^{-2}\mathrm{s}^{-1}$
- (C) Dilution factor f<50%
- (D) Cryogenic System 1.5 K

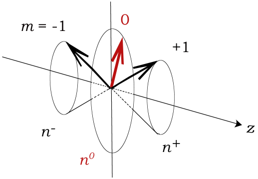
- (A) e^{-} $\sim$1 nA, γ
- (B) High Luminosity
- (C) Dilution factor f<15%
- (D) Cryogenic System down to 30 mK

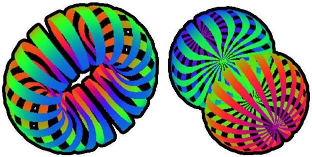




	Hermes	JLAB
P_{zz}	0.8	0.2
Dilution	0.9	0.30
$L(cm^{-2}s^{-1})$	10^{31}	10^{35}

- Alternative to Hermes atomic beam source with d-gas target
- VEPP-3 ABS provides d-gas jet up to 8×10^{16} atoms/s
- Upcoming: E12-13-011(b1), LOI12-14-002, Duke Tensor-HiFrost





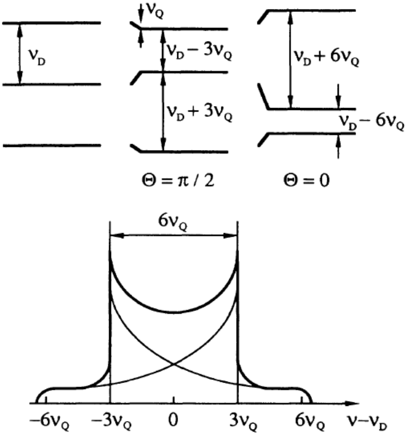
$$P = \frac{n_+ - n_-}{n_+ + n_- + n_0} \quad (-1 < P_z < 1)$$

$$P_{zz} = \frac{n_+ - 2n_0 + n_-}{n_+ + n_- + n_0} \quad (-2 < P_{zz} < 1)$$

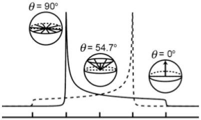
- (a) Tensor Structure Functions
- (b) Tensor Asymetries
- $T_{20}, T_{21}, T_{22}, A_T, b_1, b_2, \ldots$

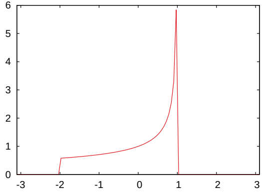
- Probe Spin 1 Observables use Spin 1 Target (Deuteron)
- Three Magnetic substates (+1,0,-1)
- Two Transitions ($+1 \rightarrow 0$) and ($0 \rightarrow -1$)
- Deuterons electric quadrupole moment eQ
- Interacts with electric field gradients within lattice

Spin 1



- ν_D: Larmor
- ν_Q: Quadrupole
- θ: $eq\angle H_0$





R

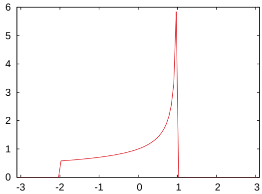
$$R = \frac{\omega - \omega_d}{3\omega_q}$$

$$\cos\theta = \sqrt{\frac{1 \pm R - \eta\cos 2\phi}{3 - \eta\cos 2\phi}}$$

- ω_d Larmor, ω_q quadrapole interaction
- θ polar angle between the D bond and H_0
- ϕ azimuthal angle V_{ij} not symmetric
- η symmetry parameter (peak position)

$$\Delta E_{\pm} = \hbar\omega_d \mp 3\hbar\omega_q([3 - \eta\cos 2\phi]\cos^2\theta - [1 - \eta\cos 2\phi])$$

Spin 1



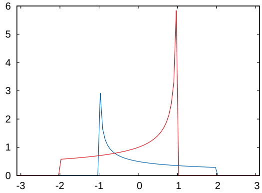
R

$$P \propto \frac{1}{(1-R)^{1/2}}$$

$$-2 \leq R < 1$$

- ω_d Larmor, ω_q quadrapole interaction
- θ polar angle between the D bond and H_0
- ϕ azimuthal angle V_{ij} not symmetric
- η symmetry parameter (peak position)

$$\Delta E_{\pm} = \hbar\omega_d \mp 3\hbar\omega_q([3 - \eta\cos 2\phi]\cos^2\theta - [1 - \eta\cos 2\phi])$$

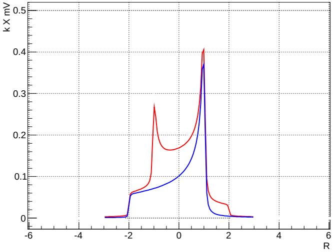


R

$$P \propto \frac{1}{(1-R)^{1/2}}$$

- $-2 \leq R \leq 2$
- $\Delta E_+ = E_0 - E_1$ with intensity I_+
- $\Delta E_- = E_1 - E_0$ with intensity I_-

$$\Delta E_\pm = \hbar\omega_d \mp 3\hbar\omega_q([3 - \eta\cos 2\phi]\cos^2\theta - [1 - \eta\cos 2\phi])$$



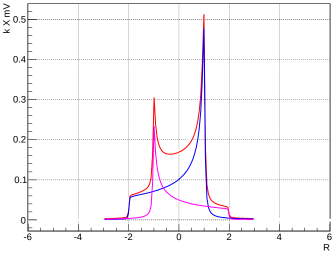
$$G(\omega) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(\omega-\omega_0)^2}{2\sigma^2}}$$

$$G(\omega) = \frac{\delta}{\pi} \frac{1}{\delta^2 + (\omega - \omega_0)^2}$$

Distribution of $\omega_d \sim$ Lorentzian $\rightarrow$ intensity spectrum is a convolution of the density of states with a Lorentzian function

$$f(R) = G(R) \otimes \frac{B}{(1-R)^{1/2}}$$

Homogenious Broadening

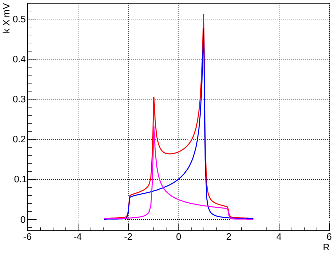


$$f_{\pm}(R, A, \eta \sim 0, \phi) = \frac{B}{2\pi\rho}\left[2\nu(R)\left(\arctan\frac{3-\rho^2}{2\sqrt{3}\rho\nu'(R)} + \frac{\pi}{2}\right) + \nu'(R)\ln\left(\frac{3+\rho^2+2\sqrt{3}\rho\nu(R)}{3+\rho^2-2\sqrt{3}\rho\nu(R)}\right)\right]$$

$$\rho = (A^2 + [1 \pm R])^{1/4}$$

$$\nu(R) = (1 + [1 \pm R]/\rho^2)^{1/4}/2$$

$$\nu'(R) = (1 - [1 \pm R]/\rho^2)^{1/4}/2$$



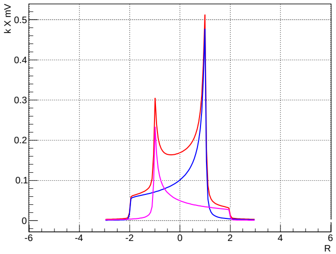
$$f_{\pm}(R, A, \eta \sim 0, \phi) = \frac{B}{2\pi\rho}\left[2\nu(R)\left(\arctan\frac{3-\rho^2}{2\sqrt{3}\rho\nu'(R)} + \frac{\pi}{2}\right) + \nu'(R)\ln\left(\frac{3+\rho^2+2\sqrt{3}\rho\nu(R)}{3+\rho^2-2\sqrt{3}\rho\nu(R)}\right)\right]$$

$$n_m \sim e^{-\beta E_m}$$

$$r^2 = e^{2\beta\hbar\omega_d} = n_+/n_-$$

$$P = \frac{n_+ - n_-}{n_+ + n_- + n_0} = \frac{r^2-1}{r^2+r+1} + \mathcal{O}((\beta\hbar\omega_q)^2)$$

DMR lineshape

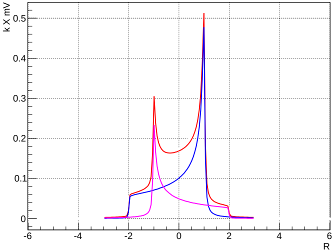


$$f_{\pm}(R, A, \eta \sim 0, \phi) = \frac{B}{2\pi\rho}\left[2\nu(R)\left(\arctan\frac{3-\rho^2}{2\sqrt{3}\rho\nu'(R)} + \frac{\pi}{2}\right) + \nu'(R)\ln\left(\frac{3+\rho^2+2\sqrt{3}\rho\nu(R)}{3+\rho^2-2\sqrt{3}\rho\nu(R)}\right)\right]$$

$$I_+ = \int_2^2 f_+(B, R)dR = C(a_+ - a_0)$$

$$I_- = \int_2^{-2} f_-(B, R)dR = C(a_0 - a_-)$$

$$P = \sum_m m a_m = (a_+ - a_0) + (a_0 - a_-) = \frac{1}{C}(I_+ + I_-)$$



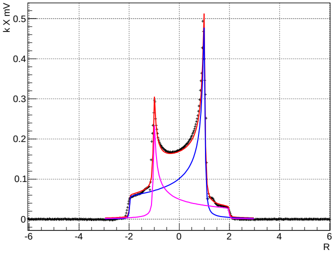
$$f_{\pm}(R, A, \eta \sim 0, \phi) = \frac{B}{2\pi\rho}\left[2\nu(R)\left(\arctan\frac{3-\rho^2}{2\sqrt{3}\rho\nu'(R)} + \frac{\pi}{2}\right) + \nu'(R)\ln\left(\frac{3+\rho^2+2\sqrt{3}\rho\nu(R)}{3+\rho^2-2\sqrt{3}\rho\nu(R)}\right)\right]$$

$$I_+ = \int_2^2 f_+(B, R)dR = C(a_+ - a_0)$$

$$I_- = \int_2^{-2} f_-(B, R)dR = C(a_0 - a_-)$$

$$A = (\sum_m 3m^2 a_m) - 2 = (a_+ - a_0) - (a_0 - a_-) = \frac{1}{C}(I_+ - I_-)$$

DMR line Fitting

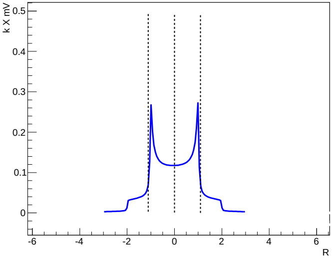


$$f_{\pm}(R, A, \eta \sim 0, \phi) = \frac{B}{2\pi\rho}\left[2\nu(R)\left(\arctan\frac{3-\rho^2}{2\sqrt{3}\rho\nu'(R)} + \frac{\pi}{2}\right) + \nu'(R)\ln\left(\frac{3+\rho^2+2\sqrt{3}\rho\nu(R)}{3+\rho^2-2\sqrt{3}\rho\nu(R)}\right)\right]$$

$$I_{+} = \int_{2}^{2} f_{+}(B, R)dR = C(a_{+} - a_0)$$

$$I_{-} = \int_{2}^{-2} f_{-}(B, R)dR = C(a_0 - a_{-})$$

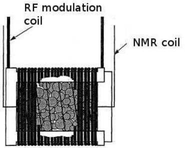
Fit to find $P = 42.3\%$ and $A = 13.1\%$



$$f(R) = G(R) \otimes \frac{B}{\sqrt{1-R}}$$

- (a) Adjacent Spins Dynamics
- (b) Different Resonant frequencies
- (c) Change in $\Gamma(\omega)$
  - Local $\Delta\Gamma(\omega)$
  - Spin Diffusion

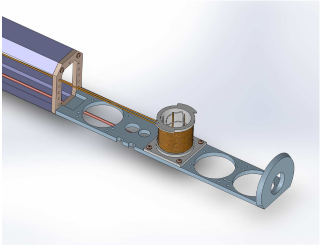
Deuteron RF Modulation



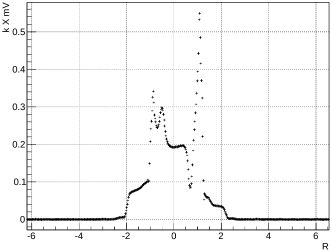
- Make RF induced excitations
- Cross-relaxation rate
- Change in $\Gamma(\omega)$

First Look

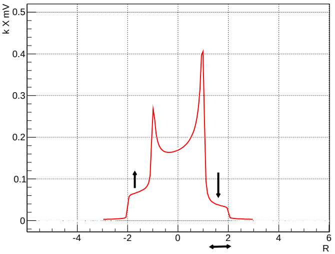
1. Secondary Coil (2 mT/A)
2. Translate NMR Area
3. RF power measure
4. Intermittent NMR



Interpretation of the DMR line

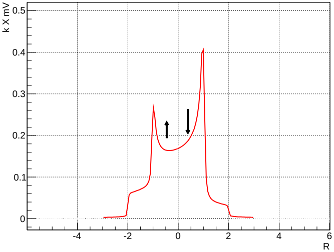


- (a) Understand change in $I_+(\omega)$, $I_-(\omega)$
- (b) Useful steady-states
- (c) Measure with uncertainty



- (a) Understand change in $I_+(\omega)$, $I_-(\omega)$
- (b) Useful steady-states
- (c) Measure with uncertainty

Interpretation of the DMR line



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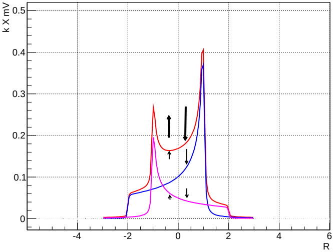
Interpretation of the DMR line



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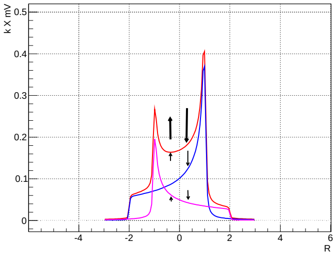
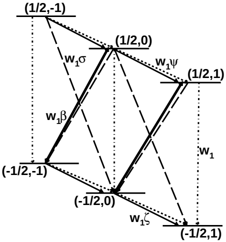




$$\frac{\partial\rho_+^S\partial\rho_-^I}{\partial t} = w_1(r\rho_-^S\rho_-^I - \rho_+^S\rho_-^I) + w_1\psi(r\rho_+^S\rho_0^I - \rho_+^S\rho_-^I) + w_1\sigma(r\rho_-^S\rho_0^I - \rho_+^S\rho_-^I) + w_1\zeta\rho_+^S\rho_0^I$$

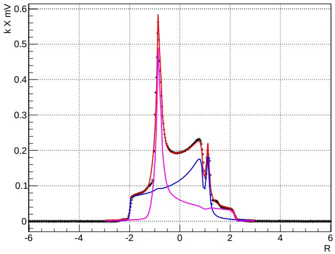
$$\frac{\partial\rho_+^S\partial\rho_0^I}{\partial t} = w_1(r\rho_-^S\rho_0^I - \rho_+^S\rho_0^I) + w_1\psi(r\rho_+^S\rho_+^I + \rho_+^S\rho_-^I - \rho_+^S\rho_0^I - r\rho_+^S\rho_0^I)$$

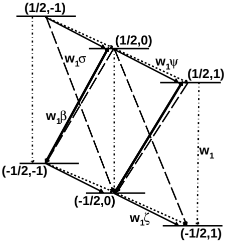
$$+w_1\sigma(r\rho_-^S\rho_+^I + r\rho_-^S\rho_-^I - 2\rho_+^S\rho_0^I) + w_1\beta(\rho_-^S\rho_-^I - \rho_+^S\rho_0^I) + w_1\zeta(\rho_+^S\rho_+^I - \rho_+^S\rho_0^I)$$

$$\frac{\partial\rho_+^S\partial\rho_+^I}{\partial t} = w_1(r\rho_-^S\rho_+^I - \rho_+^S\rho_+^I) + w_1\psi(\rho_+^S\rho_0^I - r\rho_+^S\rho_+^I)$$

$$+w_1\sigma(r\rho_-^S\rho_0^I - \rho_+^S\rho_+^I) + w_1\beta(\rho_-^S\rho_0^I - \rho_+^S\rho_+^I) - w_1\zeta\rho_+^S\rho_+^I$$

The Rate Equations





$$\frac{\partial \rho_+^S \partial \rho_-^I}{\partial t} = w_1(r\rho_-^S\rho_-^I - \rho_+^S\rho_-^I) + w_1\psi(r\rho_+^S\rho_0^I - \rho_+^S\rho_-^I) + w_1\sigma(r\rho_-^S\rho_0^I - \rho_+^S\rho_-^I) + w_1\zeta\rho_+^S\rho_0^I$$

$$\frac{\partial \rho_+^S \partial \rho_0^I}{\partial t} = w_1(r\rho_-^S\rho_0^I - \rho_+^S\rho_0^I) + w_1\psi(r\rho_+^S\rho_+^I + \rho_+^S\rho_-^I - \rho_+^S\rho_0^I - r\rho_+^S\rho_0^I)$$

$$+w_1\sigma(r\rho_-^S\rho_+^I + r\rho_-^S\rho_-^I - 2\rho_+^S\rho_0^I) + w_1\beta(\rho_-^S\rho_-^I - \rho_+^S\rho_0^I) + w_1\zeta(\rho_+^S\rho_+^I - \rho_+^S\rho_0^I)$$

$$\frac{\partial \rho_+^S \partial \rho_+^I}{\partial t} = w_1(r\rho_-^S\rho_+^I - \rho_+^S\rho_+^I) + w_1\psi(\rho_+^S\rho_0^I - r\rho_+^S\rho_+^I)$$

$$+w_1\sigma(r\rho_-^S\rho_0^I - \rho_+^S\rho_+^I) + w_1\beta(\rho_-^S\rho_0^I - \rho_+^S\rho_+^I) - w_1\zeta\rho_+^S\rho_+^I$$

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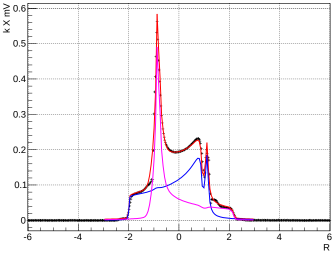


$$P = 0.476 \rightarrow 0.461 \quad P_{zz} = 0.178 \rightarrow 0.061$$

$$\frac{\partial\rho_+^S\partial\rho_-^I}{\partial t} = w_1(r\rho_-^S\rho_-^I - \rho_+^S\rho_-^I) + w_1\psi(r\rho_+^S\rho_0^I - \rho_+^S\rho_-^I) + w_1\sigma(r\rho_-^S\rho_0^I - \rho_+^S\rho_-^I) + w_1\zeta\rho_+^S\rho_0^I$$

$$\frac{\partial\rho_+^S\partial\rho_0^I}{\partial t} = w_1(r\rho_-^S\rho_0^I - \rho_+^S\rho_0^I) + w_1\psi(r\rho_+^S\rho_+^I + \rho_+^S\rho_-^I - \rho_+^S\rho_0^I - r\rho_+^S\rho_0^I)$$

$$+w_1\sigma(r\rho_-^S\rho_+^I + r\rho_-^S\rho_-^I - 2\rho_+^S\rho_0^I) + w_1\beta(\rho_-^S\rho_-^I - \rho_+^S\rho_0^I) + w_1\zeta(\rho_+^S\rho_+^I - \rho_+^S\rho_0^I)$$

$$\frac{\partial\rho_+^S\partial\rho_+^I}{\partial t} = w_1(r\rho_-^S\rho_+^I - \rho_+^S\rho_+^I) + w_1\psi(\rho_+^S\rho_0^I - r\rho_+^S\rho_+^I)$$

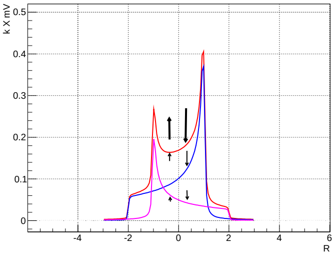
$$+w_1\sigma(r\rho_-^S\rho_0^I - \rho_+^S\rho_+^I) + w_1\beta(\rho_-^S\rho_0^I - \rho_+^S\rho_+^I) - w_1\zeta\rho_+^S\rho_+^I$$

Steady State Solutions

- $$w_1\zeta\rho_+^S\rho_0^I = -w_1(r\rho_-^S\rho_-^I - \rho_+^S\rho_-^I) - w_1\psi(r\rho_+^S\rho_0^I - \rho_+^S\rho_-^I) - w_1\sigma(r\rho_-^S\rho_0^I - \rho_+^S\rho_-^I)$$
- $$w_1\zeta(\rho_+^S\rho_+^I - \rho_+^S\rho_0^I) = w_1(r\rho_-^S\rho_0^I - \rho_+^S\rho_0^I) + w_1\psi(r\rho_+^S\rho_+^I + \rho_+^S\rho_-^I - \rho_+^S\rho_0^I - r\rho_+^S\rho_0^I)$$
  $$+w_1\sigma(r\rho_-^S\rho_+^I + r\rho_-^S\rho_-^I - 2\rho_+^S\rho_0^I) + w_1\beta(\rho_-^S\rho_-^I - \rho_+^S\rho_0^I)$$
- $$w_1\zeta\rho_+^S\rho_+^I = w_1(r\rho_-^S\rho_+^I - \rho_+^S\rho_+^I) + w_1\psi(\rho_+^S\rho_0^I - r\rho_+^S\rho_+^I) + w_1\sigma(r\rho_-^S\rho_0^I - \rho_+^S\rho_+^I)$$
  $$+w_1\beta(\rho_-^S\rho_0^I - \rho_+^S\rho_+^I) - w_1\zeta\rho_+^S\rho_+^I$$

- (a) $\zeta \ll \beta$ (Mininal variation)
- (b) $\zeta \gg \beta$ (Saturation)
- (c) $\zeta \sim \beta$ (Family of solutions)

Steady State Solutions



- (a) $\zeta \ll \beta$ (Mininal variation)
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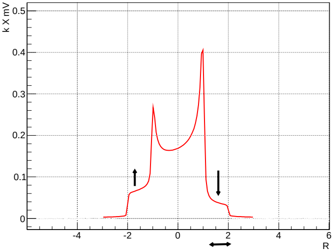
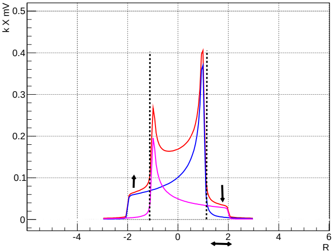


- (a) Modulated RF over region
- (b) Burn all the way down
- (c) Fit with piecewise function
- (d) χ^2 over V_{ij}

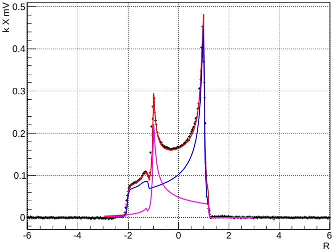
The Saturated Fitting



$$\mathfrak{F}^{+}(R^*, A_1^*, A_2^*, B_1^*, B_2^*, \eta, \phi) = \begin{cases} 0 & : 1 < R \\ f_1^{+}(R^*, A_1^*, B_1^*) & : -1 \leq R \leq 1 \\ \sum_i f_i^{+}(R^*, A_i^*, B_i^*) & : -3 \leq R < -1 \end{cases}$$

$$\mathfrak{F}^{-}(R^*, A_3^*, A_4^*, B_3^*, B_4^*, \eta, \phi) = \begin{cases} 0 & : 1 < R \\ f_3^{-}(R^*, A_3^*, B_3^*) & : -1 \leq R \leq 1 \\ \sum_i f_i^{-}(R^*, A_i^*, B_i^*) & : -3 \leq R < -1 \end{cases}$$

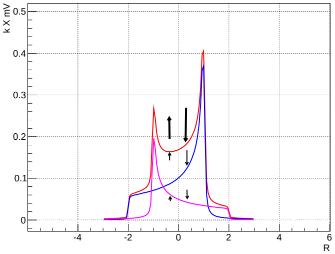
The Saturated Fitting



$$P = 0.414 \rightarrow 0.391 \quad P_{zz} = 0.128 \rightarrow 0.156$$

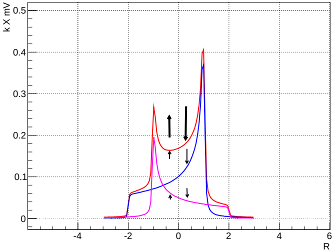
$$\mathfrak{F}^{+}(R^*, A_1^*, A_2^*, B_1^*, B_2^*, \eta, \phi) = \begin{cases} 0 & : 1 < R \\ f_1^{+}(R^*, A_1^*, B_1^*) & : -1 \leq R \leq 1 \\ \sum_i f_i^{+}(R^*, A_i^*, B_i^*) & : -3 \leq R < -1 \end{cases}$$

$$\mathfrak{F}^{-}(R^*, A_3^*, A_4^*, B_3^*, B_4^*, \eta, \phi) = \begin{cases} 0 & : 1 < R \\ f_3^{-}(R^*, A_3^*, B_3^*) & : -1 \leq R \leq 1 \\ \sum_i f_i^{-}(R^*, A_i^*, B_i^*) & : -3 \leq R < -1 \end{cases}$$



$$\zeta \sim \beta$$

The theoretical line-shape changes from the RF based on NMR response to the population deviation resulting from the RF power profile represented as a Gaussian with σ corresponding to the transverse power absorption profile and magnitude related to the effective amplitude of the RF signal where the neighboring spin orientation and frequency deviations are handled with a convolution with a Lorentzian.

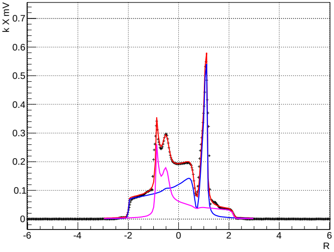


$$\hat{\mathfrak{F}}^{+}(R^*, A_1^*, B_1^*, \eta, \phi, h_1^*, \Gamma^*, k_1^*, \sigma^*, \delta) = f^{+}(R^*, A_1^*, B_1^*) - L(h_1^*, \Gamma^*, r) \otimes G(k_1^*, \sigma^*, \delta)$$

$$\hat{\mathfrak{F}}^{-}(R^*, A_2^*, B_2^*, \eta, \phi, h_2^*, \Gamma^*, k_2^*, \sigma^*, \delta) = f^{-}(R^*, A_2^*, B_2^*) + L(h_2^*, \Gamma^*, r) \otimes G(h_2^*, \sigma^*, \delta)$$

- Saturate ζ
- Use Power Profile to Gauge
- Semi-saturate to steady state

The Semi-saturated Fitting



$$P = 0.490 \rightarrow 0.472 \quad P_{zz} = 0.189 \rightarrow 0.128$$

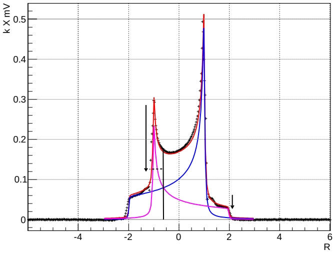
$$\hat{\mathfrak{F}}^{+}(R^*, A_1^*, B_1^*, \eta, \phi, h_1^*, \Gamma^*, k_1^*, \sigma^*, \delta) = f^{+}(R^*, A_1^*, B_1^*) - L(h_1^*, \Gamma^*, r) \otimes G(k_1^*, \sigma^*, \delta)$$

$$\hat{\mathfrak{F}}^{-}(R^*, A_2^*, B_2^*, \eta, \phi, h_2^*, \Gamma^*, k_2^*, \sigma^*, \delta) = f^{-}(R^*, A_2^*, B_2^*) + L(h_2^*, \Gamma^*, r) \otimes G(k_2^*, \sigma^*, \delta)$$

- Saturate ζ
- Use Power Profile to Gauge
- Semi-saturate to steady state

Enhancement Method

1. TE & $P_{zz} = (r^2 - 2r + 1)/(r^2 + r + 1)$
2. $P_{zz} = 1 - 3\frac{n_0}{N} = C(I_+ - I_-)$
3. Saturate Pedestal and Semi-saturate small peak
4. $$\hat{\mathfrak{F}}^- + \hat{\mathfrak{F}}^+ \wedge \mathfrak{F}^- + \mathfrak{F}^+$$



Error Estimates

- (a) Natural distribution (4-6%)
- (b) RF Saturation (5-7%)
- (c) Semi-saturation (6-10%)

Uncertainty in Tensor Polarization

Standard Uncertainty Contributions

$$\left(\frac{\delta C_{TE}}{C_{TE}}\right)^2 = \left(\frac{\delta P_{TE}}{P_{TE}}\right)^2 + \left(\frac{\delta A_{TE}}{A_{TE}}\right)^2$$

$$\frac{\delta P_E}{P_E} = \left[\left(\frac{\delta P_{TE}}{P_{TE}}\right)^2 + \left(\frac{\delta A_{TE}}{A_{TE}}\right)^2 + \left(\frac{\delta S_{TE}}{S_{TE}}\right)^2 + \left(\frac{\delta A_E}{A_E}\right)^2 + \left(\frac{\delta S_E}{S_E}\right)^2 + \left(\frac{\delta G}{G}\right)^2\right]^{1/2}$$

1. A_{TE} - Relative uncertainties in area acquired during TE
2. S_{TE} - Measurement limitation during TE
3. S_E - Systematic variation in enhanced signal
4. G - Error from gain

Additional Contributions (Steady-State)

$$\delta I_{\pm} = \sqrt{(\delta C)^2 + (\delta A_{\chi^2})^2 + (\delta A_{\partial t})}$$

- (δC) Standard Contributions from above
- (δA_{χ^2}) Variation in area over covariance matrix minimization
- $(\delta A_{\partial t})$ NMR measurement limitations with respects to relaxation rate

RF-Disturbed DMR line Measurement

- (a) Systematic Fitting Procedure
- (b) Expand Parameter Space
- (c) Use rates to add Constraints
- (d) Acquire Error Estimate

Tensor Optimization

- (a) Burn Pedestal and Semi-saturate peak
- (b) Fit result for P and P_{zz}
- (c) More Measurements and test to Come