Spin-Gravity Interactions and Equivalence Principle

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Main Topics

- Description of spin-gravity interactions: Dirac eqn / EMT matrix elements
- Equivalence principle with spin and its tests
- Torsion and its manifestations
- Indirect tests of EP and it extension (validity separately for quarks and gluons) via sum rules for hadronic matrix elements of EMT
- Ultra non-inertial frame: rotation in heavy-ion collisions

Spin-gravity interactions

- 1. Dirac equation (Hehl and Ni)
- Gauge structure of gravity manifested; limit of classical gravity - FW transformation
- 2. Matrix elements of Energy- Momentum Tensor
- May be studied in non-gravitational experiments/theory
- ----------//--------- separately for quarks (flavour by flavour) and gluons
- Simple interpretation in comparison to EM field case

Gravitational Formfactors

$$\langle p'|T^{\mu\nu}_{q,g}|p\rangle = \bar{u}(p')\Big[A_{q,g}(\Delta^2)\gamma^{(\mu}p^{\nu)} + B_{q,g}(\Delta^2)P^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M]u(p)$$

- Conservation laws - zero Anomalous Gravitomagnetic Moment : $\mu_G = J$ (g=2)

$$P_{q,g} = A_{q,g}(0) \qquad A_q(0) + A_g(0) = 1$$

$$J_{q,g} = \frac{1}{2}\left[A_{q,g}(0) + B_{q,g}(0)\right] \qquad A_q(0) + B_q(0) + A_g(0) + B_g(0) = 1$$

- May be extracted from high-energy experiments/NPQCD calculations
- Describe the partition of angular momentum between quarks and gluons
- Describe interaction with both classical and TeV gravity; gravity smallness – only in coupling

Generalized Parton Diistributions (related to matrix elements of non local operators) – models for both EM and Gravitational Formfactors (Selyugin,OT ’09)

- Smaller mass square radius (attraction vs repulsion!?)

$$\rho(b) = \sum_q e_q \int dx q(x,b) \quad = \int d^2 q F_1(Q^2 = q^2) e^{i\vec{q}\,\vec{b}}$$

$$= \int_0^\infty \frac{qdq}{2\pi} J_0(qb) \frac{G_E(q^2) + \tau G_M(q^2)}{1+\tau}$$

$$\rho_0^{\rm Gr}(b) = \frac{1}{2\pi} \int_\infty^0 dq q J_0(qb) A(q^2)$$

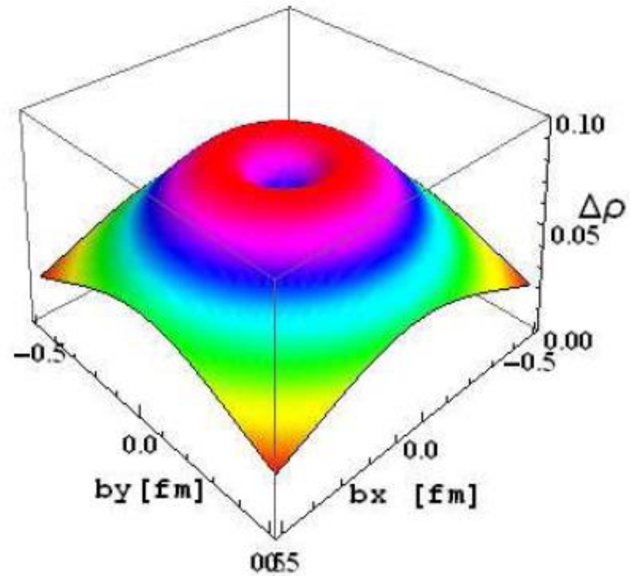

FIG. 17: Difference in the forms of charge density F_1^P and "matter" density (A)

Electromagnetism vs Gravity

- Interaction – field vs metric deviation

$$M = \langle P' | J_q^\mu | P \rangle A_\mu(q) \qquad M = \frac{1}{2} \sum_{q,G} \langle P' | T_{q,G}^{\mu\nu} | P \rangle h_{\mu\nu}(q)$$

- Static limit

$$\langle P | J_q^\mu | P \rangle = 2 e_q P^\mu \qquad \sum_{q,G} \langle P | T_i^{\mu\nu} | P \rangle = 2 P^\mu P^\nu$$

$$h_{00} = 2\phi(x)$$

$$M_0 = \langle P | J_q^\mu | P \rangle A_\mu = 2 e_q M \phi(q) \qquad M_0 = \frac{1}{2} \sum_{q,G} \langle P | T_i^{\mu\nu} | P \rangle h_{\mu\nu} = 2M \cdot M \phi(q)$$

- Mass as charge – equivalence principle

Gravitomagnetism

- Gravitomagnetic field (weak, except in gravity waves) – action on spin from $M = \frac{1}{2}\sum_{q,G}\langle P'|T^{\mu\nu}_{q,G}|P\rangle h_{\mu\nu}(q)$

$$\vec{H}_J = \frac{1}{2} rot\vec{g};\ \vec{g}_i \equiv g_{0i}$$

spin dragging twice smaller than EM

- Lorentz force – similar to EM case: factor ½ cancelled with 2 from $h_{00} = 2\phi(x)$ Larmor frequency same as EM

$$\omega_J = \frac{\mu_G}{J} H_J = \frac{H_L}{2} = \omega_L\ \vec{H}_L = rot\vec{g}$$

- Orbital and Spin momenta dragging – the same - Equivalence principle

Experimental test of PNEP

- Reinterpretation of the data on G(EDM) search

PHYSICAL REVIEW LETTERS

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Search for a Coupling of the Earth's Gravitational Field to Nuclear Spins in Atomic Mercury

B. J. Venema, P. K. Majumder, S. K. Lamoreaux, B. R. Heckel, and E. N. Fortson

Physics Department, FM-15, University of Washington, Seattle, Washington 98195

(Received 25 September 1991)

- If (CP-odd!) GEDM=0 -> constraint for AGM (Silenko, OT'07) from Earth rotation – was considered as obvious (but it is just EP!) background

$$\mathcal{H} = -g\mu_N \boldsymbol{B}\cdot\boldsymbol{S} - \zeta\hbar\boldsymbol{\omega}\cdot\boldsymbol{S}, \quad \zeta = 1+\chi$$

$$|\chi(^{201}\mathrm{Hg}) + 0.369\chi(^{199}\mathrm{Hg})| < 0.042 \quad (95\%\mathrm{C.L.})$$

Equivalence principle for moving particles

- Compare gravity and acceleration: gravity provides EXTRA space components of metrics $h_{zz} = h_{xx} = h_{yy} = h_{00}$
- Matrix elements DIFFER

$$\mathcal{M}_g = (\epsilon^2 + p^2) h_{00}(q), \qquad \mathcal{M}_a = \epsilon^2 h_{00}(q)$$

- Ratio of accelerations: $R = \frac{\epsilon^2 + p^2}{\epsilon^2}$ - confirmed by explicit solution of Dirac equation (Silenko, OT, ‘05)
- Arbitrary fields – Obukhov, Silenko, OT ’09,’11,’13

Gravity vs accelerated frame for spin and helicity

- Spin precession – well known factor 3 (Probe B; spin at satellite – probe of PNEP!) – smallness of relativistic correction (~$\mathbf{P}^2$) is compensated by 1/ $\mathbf{P}^2$ in the momentum direction precession frequency
- Helicity flip – the same!
- No helicity flip in gravitomagnetic field – another formulation of PNEP (OT'99) and
- Flip by "gravitoelectric" field: relic neutrino? Black hole?

$$\frac{d\sigma_{+-}}{d\sigma_{++}} = \frac{tg^2(\frac{\phi}{2})}{(2\gamma - \gamma^{-1})^2}$$

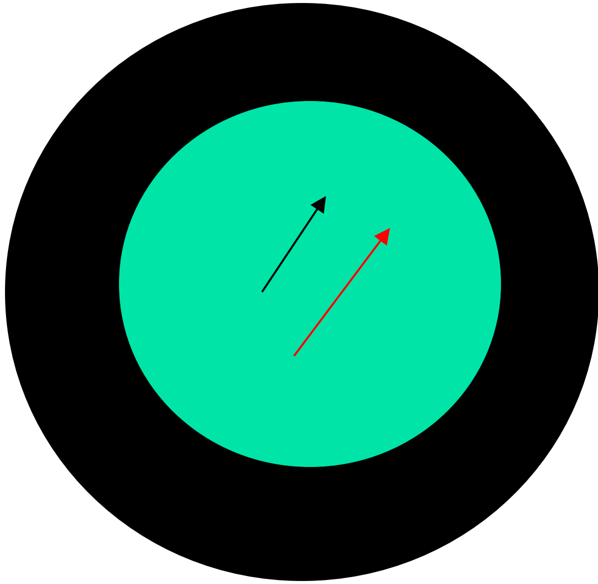
Gyromagnetic and Gravigyromagnetic ratios

- Free particles – coincide
- $<P+q|T^{mn}|P-q> = P^{\{m}<P+q|J^{n\}}|P-q>/e$ up to the terms linear in q
- Gravitomagnetic g=2 for any spin
- Special role of g=2 for ANY spin (asymptotic freedom for vector bosons)

- Should Einstein know about PNEP, the outcome of his and de Haas experiment would not be so surprising
- Recall also g=2 for Black Holes. Indication of "quantum" nature?!

Cosmological implications of PNEP

- Necessary condition for Mach's Principle (in the spirit of Weinberg's textbook) -
- Lense-Thirring inside massive rotating empty shell (=model of Universe)
- For flat "Universe" - precession frequency equal to that of shell rotation
- Simple observation-Must be the same for classical and quantum rotators – PNEP!
- More elaborate models - Tests for cosmology ?!



Torsion – acts only on spin

Dirac eq+FW transformation-Obukhov,Silenko,OT, in preparation

- Hermitian Dirac Hamiltonian

$$e_i^{\hat{0}} = V\,\delta_i^0, \qquad e_i^{\hat{a}} = W^{\hat{a}}{}_b\left(\delta_i^b - cK^b\,\delta_i^0\right)$$

$$ds^2 = V^2c^2dt^2 - \delta_{\hat{a}\hat{b}}W^{\hat{a}}{}_cW^{\hat{b}}{}_d\,(dx^c - K^c cdt)\,(dx^d - K^d cdt)$$

$$\mathcal{H} = \beta mc^2 V + q\Phi + \frac{c}{2}\left(\pi_b\,\mathcal{F}^b{}_a\alpha^a + \alpha^a\mathcal{F}^b{}_a\pi_b\right) + \frac{c}{2}\left(\boldsymbol{K}\cdot\boldsymbol{\pi} + \boldsymbol{\pi}\cdot\boldsymbol{K}\right) + \frac{\hbar c}{4}\left(\Xi\cdot\Sigma - \Upsilon\gamma_5\right),$$

$$\mathcal{F}^b{}_a = VW^b{}_{\hat{a}}, \qquad \Upsilon = V\epsilon^{\hat{a}\hat{b}\hat{c}}\Gamma_{\hat{a}\hat{b}\hat{c}}, \qquad \Xi^a = \frac{V}{c}\,\epsilon^{\hat{a}\hat{b}\hat{c}}\left(\Gamma_{\hat{0}\hat{b}\hat{c}} + \Gamma_{\hat{b}\hat{c}\hat{0}} + \Gamma_{\hat{c}\hat{0}\hat{b}}\right)$$

- Spin-torsion coupling

$$-\frac{\hbar cV}{4}\left(\boldsymbol{\Sigma}\cdot\check{\boldsymbol{T}} + c\gamma_5\check{T}^{\hat{0}}\right)$$

$$\check{T}^\alpha = -\frac{1}{2}\,\eta^{\alpha\mu\nu\lambda}T_{\mu\nu\lambda}$$

- FW – semiclassical limit - precession

$$\Omega^{(T)} = -\frac{c}{2}\check{\boldsymbol{T}} + \beta\frac{c^3}{8}\left\{\frac{1}{\epsilon'},\left\{\boldsymbol{p},\check{T}^{\hat{0}}\right\}\right\} + \frac{c}{8}\left\{\frac{c^2}{\epsilon'(\epsilon'+mc^2)},\left(\{\boldsymbol{p}^2,\check{\boldsymbol{T}}\} - \{\boldsymbol{p},(\boldsymbol{p}\cdot\check{\boldsymbol{T}})\}\right)\right\}$$

Experimental bounds for torsion

- Magnetic field+rotation+torsion

$$H = -g_N \frac{\mu_N}{\hbar} \boldsymbol{B} \cdot \boldsymbol{s} - \boldsymbol{\omega} \cdot \boldsymbol{s} - \frac{c}{2} \check{\boldsymbol{T}} \cdot \boldsymbol{s},$$

- Same '92 EDM experiment

$$\frac{\hbar c}{4} |\check{\boldsymbol{T}}| \cdot |\cos\Theta| < 2.2 \times 10^{-21}\,\mathrm{eV}, \qquad |\check{\boldsymbol{T}}| \cdot |\cos\Theta| < 4.3 \times 10^{-14}\,\mathrm{m}^{-1}$$

- New(based on Gemmel et al '10)

$$\frac{\hbar c}{2} |\check{\boldsymbol{T}}| \cdot |(1 - \mathcal{G}) \cos\Theta| < 4.1 \times 10^{-22}\,\mathrm{eV}, \qquad |\check{\boldsymbol{T}}| \cdot |\cos\Theta| < 2.4 \times 10^{-15}\,\mathrm{m}^{-1}.$$

$$\mathcal{G} = g_{He}/g_{Xe}$$

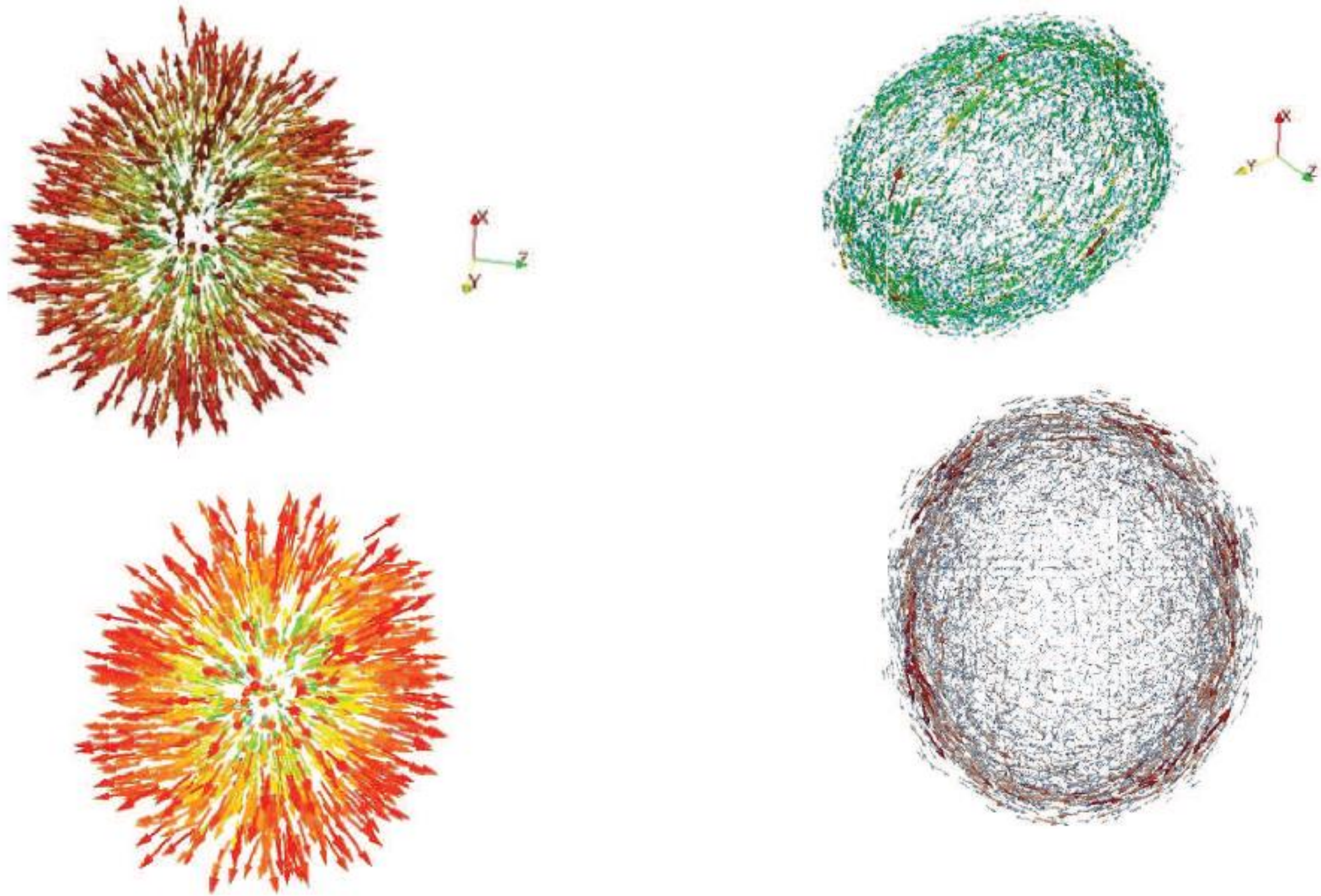
Microworld: where is the fastest possible rotation?

- Non-central heavy ion collisions (~c/Compton wavelength) – "small Bang"

- Differential rotation – vorticity

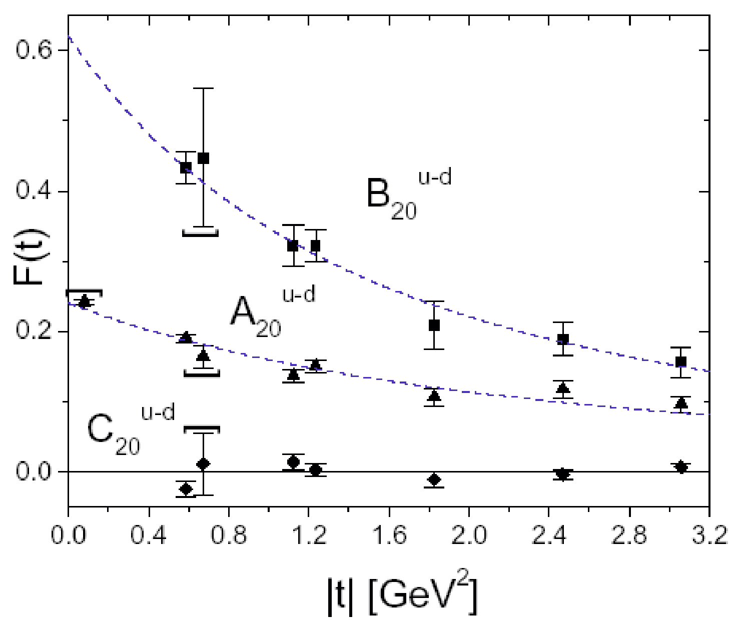
- Calculation in quark - gluon string model (Baznat, Gudima, Sorin, OT, PRC'13)

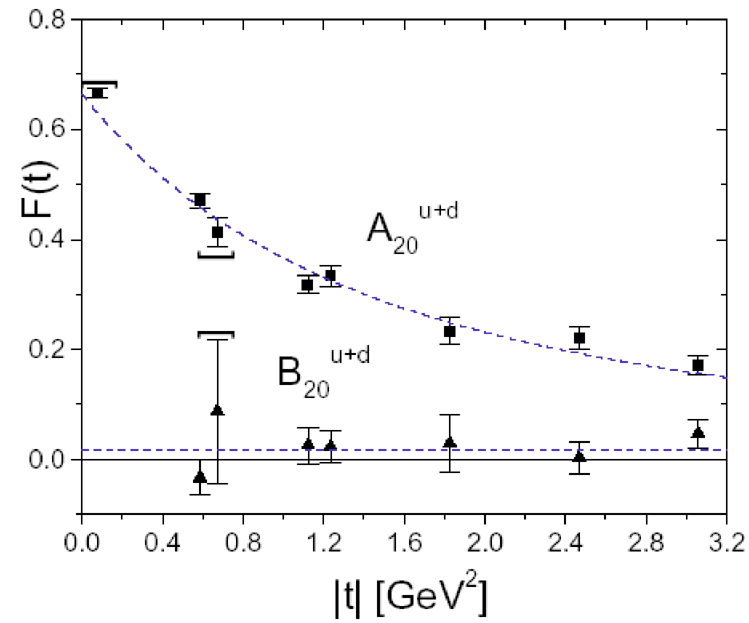
Structure of velocity and vorticity fields (NICA@JINR-5 GeV/c)



Generalization of Equivalence principle

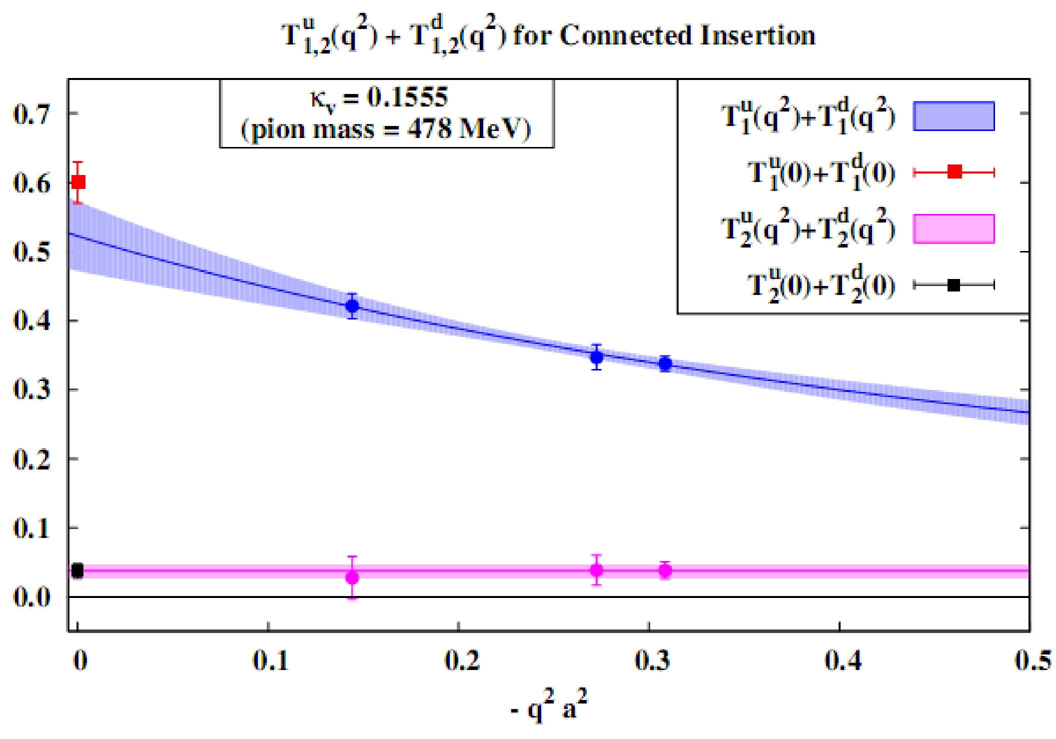
- Various arguments: AGM ≈ 0 separately for quarks and gluons – most clear from the lattice (LHPC/SESAM)





Recent lattice study (M. Deka et al. **arXiv:1312.4816**; **plenary talk of K.F. Liu**)

- Sum of u and d for Dirac (T1) and Pauli (T2) FFs



Extended Equivalence Principle=Exact EquiPartition

- In pQCD – violated
- Reason – in the case of ExEP- no smooth transition for zero fermion mass limit (Milton, 73)
- Conjecture (O.T., 2001 – prior to lattice data) – valid in NP QCD – zero quark mass limit is safe due to chiral symmetry breaking
- Gravityproof confinement? Nucleons do not break even by black holes?

Sum rules for EMT (and OAM)

- Integration of non-local operators matrix elements (related to integration over kinematical variables of some observables) in order to get EMT
- First seminal example: X. Ji's sum rule ('96) (plenary talk) . Gravity counterpart for nucleons– OT'99
- Are there spin-dependent EMT related sum rule for inclusive processes (forward matrix elements of EMT)
- Burkardt sum rule – looks similar: can it be derived from EMT?
- Contribution of Sivers function (~gluonic pole) to EMT – seems to be identically equal zero BUT this changes in the case of pole prescription (OT'14)

Pole prescription and Burkardt SR

- Pole prescription (dynamics!) provides ("T-odd") symmetric part to the integrand!
$$\sum \int \int dx_1 dx_2 \frac{T(x_1, x_2)}{x_1 - x_2 + i\varepsilon} = 0$$
- SR: $\sum \int dx T(x,x) = 0$ (but relation of gluon Sivers to twist 3 still not found – prediction!)
- Can it be valid separately for each quark flavour: nodes?
- Valid if structures forbidden for TOTAL EMT do not appear for each flavour
- Structure contains (besides S) gauge vector n: If GI separation of EMT – forbidden: SR valid separately!

Another manifestation of post-Newtonian (Ex)EP for spin 1 hadrons

- Tensor polarization - coupling of gravity to spin in forward matrix elements (OT'09)
- Second moments of tensor distributions should sum to zero (EMT conservation: Efremov, OT'82,'93)

$$\langle P,S|\bar{\psi}(0)\gamma^{\nu}D^{\nu_1}...D^{\nu_n}\psi(0)|P,S\rangle_{\mu^2} = i^{-n}M^2S^{\nu\nu_1}P^{\nu_2}...P\nu_n\int_0^1 C_q^T(x)x^n dx$$

$$\sum_q \langle P,S|T_i^{\mu\nu}|P,S\rangle_{\mu^2} = 2P^{\mu}P^{\nu}(1-\delta(\mu^2)) + 2M^2S^{\mu\nu}\delta_1(\mu^2)$$

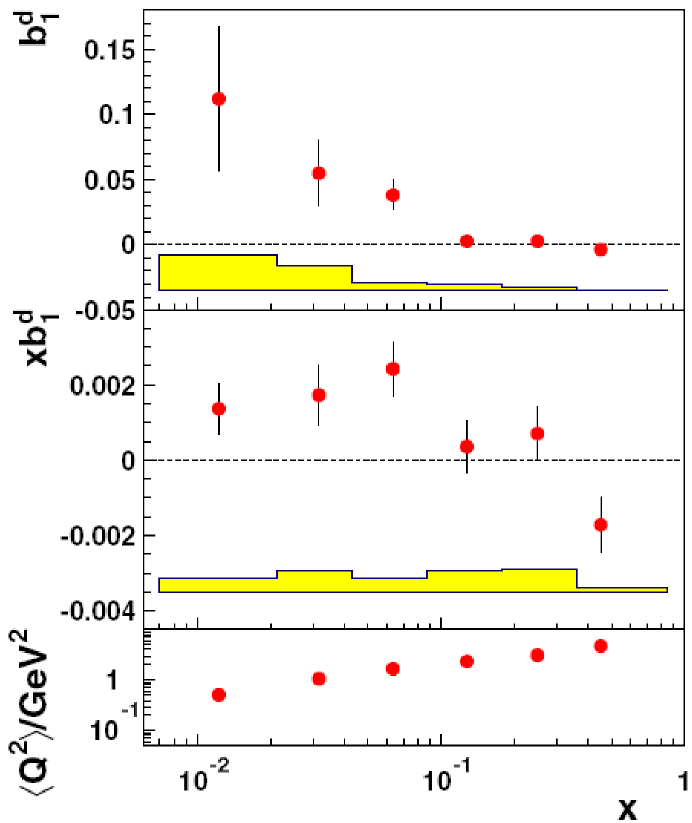
$$\langle P,S|T_g^{\mu\nu}|P,S\rangle_{\mu^2} = 2P^{\mu}P^{\nu}\delta(\mu^2) - 2M^2S^{\mu\nu}\delta_1(\mu^2)$$

$$\sum_q \int_0^1 C_i^T(x)xdx = \delta_1(\mu^2)$$

HERMES – data on tensor spin structure function

PRL 95, 242001 (2005)

- Isoscalar target – proportional to the sum of u and d quarks – combination required by (Ex)EP
- Second moments – compatible to zero better than the first one (Close-Kumano SR): collective glue << sea – for valence:



$$\int_0^1 C_i^T(x)dx = 0.$$

Are more accurate data possible?

- HERMES – unlikely
- Planned at JLab; SR study may provide information about collective sea and glue in deuteron and indirect new test of Equivalence Principle
- Complementary information from DY with deuteron beams: JPARC, NICA

CONCLUSIONS

- Spin-gravity interactions may be probed directly in gravitational (inertial) experiments and indirectly – studing EMT matrix element
- Torsion and EP may be tested as a byproduct of EDM experiments
- SR's for deuteron tensor polarization-indirectly probe EP and its extension separately for quarks and gluons – may be tested at JLab, JPARC and NICA

- BACKUP SLIDES

EEP and AdS/QCD

- Recent development – calculation of Rho formfactors in Holographic QCD (Grigoryan, Radyushkin)
- Provides g=2 identically!
- Experimental test at time –like region possible