Spin-Gravity Interactions and Equivalence Principle

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Oleg Teryaev
JINR, Dubna,
in collaboration with
Yu. Obukhov,
A.Silenko
Main Topics

- Description of spin-gravity interactions: Dirac eqn / EMT matrix elements
- Equivalence principle with spin and its tests
- Torsion and its manifestations
- Indirect tests of EP and its extension (validity separately for quarks and gluons) via sum rules for hadronic matrix elements of EMT
- Ultra non-inertial frame: rotation in heavy-ion collisions
Spin-gravity interactions

1. Dirac equation (Hehl and Ni)
   - Gauge structure of gravity manifested; limit of classical gravity - FW transformation

2. Matrix elements of Energy-Momentum Tensor
   - May be studied in non-gravitational experiments/theory
   - & separately for quarks (flavour by flavour) and gluons
   - Simple interpretation in comparison to EM field case
Gravitational Formfactors

\[ \langle p' | T_{q,g}^{\mu \nu} | p \rangle = \bar{u}(p') \left[ A_{q,g}(\Delta^2) \gamma^{(\mu} p^{\nu)} + B_{q,g}(\Delta^2) P^{(\mu} i \sigma^{\nu)} \alpha \Delta_{\alpha}/2M \right] u(p) \]

- Conservation laws - zero Anomalous Gravitomagnetic Moment: \( \mu_G = J \)  
  \( P_{q,g} = A_{q,g}(0) \quad A_q(0) + A_g(0) = 1 \)
  \( J_{q,g} = \frac{1}{2} [A_{q,g}(0) + B_{q,g}(0)] \quad A_q(0) + B_q(0) + A_g(0) + B_g(0) = 1 \)

- May be extracted from high-energy experiments/NPQCD calculations
- Describe the partition of angular momentum between quarks and gluons
- Describe interaction with both classical and TeV gravity; gravity smallness – only in coupling
Generalized Parton Distributions (related to matrix elements of non local operators) – models for both EM and Gravitational Formfactors (Selyugin, OT ’09)

- Smaller mass square radius (attraction vs repulsion!?)

\[
\rho(b) = \sum_q e_q \int dxq(x, b) = \int d^2q F_1(Q^2 = q^2) e^{i\vec{q} \cdot \vec{b}}
\]

\[
= \int_0^\infty \frac{qdq}{2\pi} J_0(qb) \frac{G_E(q^2) + \tau G_M(q^2)}{1 + \tau}
\]

\[
\rho_{0Gr}(b) = \frac{1}{2\pi} \int_0^\infty dq q J_0(qb)A(q^2)
\]

**FIG. 17:** Difference in the forms of charge density $F_1^P$ and "matter" density ($A$)
Electromagnetism vs Gravity

- Interaction – field vs metric deviation

\[ M = \langle P' | J^\mu_q | P \rangle A_\mu(q) \quad M = \frac{1}{2} \sum_{q,G} \langle P' | T^{\mu\nu}_{q,G} | P \rangle h_{\mu\nu}(q) \]

- Static limit

\[ \langle P | J^\mu_q | P \rangle = 2e_q P^\mu \]
\[ \sum_{q,G} \langle P | T^{\mu\nu}_{q,G} | P \rangle = 2P^\mu P^\nu \]
\[ h_{00} = 2\phi(x) \]

\[ M_0 = \langle P | J^\mu_q | P \rangle A_\mu = 2e_q M \phi(q) \quad M_0 = \frac{1}{2} \sum_{q,G} \langle P | T^{\mu\nu}_{q,G} | P \rangle h_{\mu\nu} = 2M \cdot M \phi(q) \]

- Mass as charge – equivalence principle
Gravitomagnetism

- Gravitomagnetic field (weak, except in gravity waves) — action on spin from
  \[ M = \frac{1}{2} \sum_{q,G} \langle P' | T^{\mu\nu}_{q,G} | P \rangle h_{\mu\nu}(q) \]
  \[ \vec{H}_J = \frac{1}{2} \text{rot} \vec{g}; \quad \vec{g}_i \equiv g_{0i}. \]
  spin dragging twice smaller than EM

- Lorentz force — similar to EM case: factor ½ cancelled with 2 from Larmor frequency same as EM
  \[ h_{00} = 2\phi(x) \]
  \[ \omega_J = \frac{\mu_G}{J} H_J = \frac{H_L}{2} = \omega_L \vec{H}_L = \text{rot} \vec{g} \]

- Orbital and Spin momenta dragging — the same - Equivalence principle
Experimental test of PNEP

- Reinterpretation of the data on G(EDM) search

- If (CP-odd!) GEDM=0 -> constraint for AGM (Silenko, OT’07) from Earth rotation – was considered as obvious (but it is just EP!) background

\[
\mathcal{H} = -g \mu_N B \cdot S - \zeta \hbar \omega \cdot S, \quad \zeta = 1 + \chi
\]

\[|\chi(^{201}\text{Hg}) + 0.369\chi(^{199}\text{Hg})| < 0.042 \quad (95\%\text{C.L.})\]
Equivalence principle for moving particles

- Compare gravity and acceleration: gravity provides EXTRA space components of metrics $h_{zz} = h_{xx} = h_{yy} = h_{00}$
- Matrix elements DIFFER $\mathcal{M}_g = (\epsilon^2 + p^2)h_{00}(q)$, $\mathcal{M}_a = \epsilon^2 h_{00}(q)$
- Ratio of accelerations: \[ R = \frac{\epsilon^2 + p^2}{\epsilon^2} \] confirmed by explicit solution of Dirac equation (Silenko, OT, ‘05)
- Arbitrary fields – Obukhov, Silenko, OT ‘09,’11,’13
Gravity vs accelerated frame for spin and helicity

- Spin precession – well known factor 3 (Probe B; spin at satellite – probe of PNEP!) – smallness of relativistic correction \( \sim P^2 \) is compensated by \( 1/P^2 \) in the momentum direction precession frequency

- Helicity flip – the same!

- No helicity flip in gravitomagnetic field – another formulation of PNEP (OT’99) and

- Flip by “gravitoelectric” field: relic neutrino? Black hole?

\[
\frac{d\sigma_{+-}}{d\sigma_{++}} = \frac{\tan^2(\phi/2)}{(2\gamma - \gamma^{-1})^2}
\]
Gyromagnetic and Gravigyromagnetic ratios

- Free particles – coincide
- \( <P+q|T^{mn}|P-q> = P^{m}<P+q|J^n|P-q>/e \) up to the terms linear in q
- Gravitomagnetic \( g=2 \) for any spin
- Special role of \( g=2 \) for ANY spin (asymptotic freedom for vector bosons)

- Should Einstein know about PNEP, the outcome of his and de Haas experiment would not be so surprising
- Recall also \( g=2 \) for Black Holes. Indication of “quantum” nature?!
Cosmological implications of PNEP

- Necessary condition for Mach’s Principle (in the spirit of Weinberg’s textbook) -
- Lense-Thirring inside massive rotating empty shell (=model of Universe)
- For flat “Universe” - precession frequency equal to that of shell rotation
- Simple observation-Must be the same for classical and quantum rotators – PNEP!
- More elaborate models - Tests for cosmology ?!
Torsion – acts only on spin

Hermitian Dirac Hamiltonian

$$e_i^0 = V \delta_i^0, \quad e_i^a = W^a_b \left( \delta_i^b - cK^b \delta_i^0 \right)$$

$$\mathcal{H} = \beta mc^2 V + q\Phi + \frac{c}{2} \left( \pi_b \mathcal{F}^b_a \alpha^a + \alpha^a \mathcal{F}^b_a \pi_b \right)$$

$$+ \frac{c}{2} \left( K \cdot \pi + \pi \cdot K \right) + \frac{\hbar c}{4} \left( \Xi \cdot \Sigma - \Upsilon \gamma_5 \right),$$

$$\mathcal{F}^b_a = VW^b_a, \quad \Upsilon = V \epsilon^{abc} \Gamma_{abc}, \quad \Xi^a = \frac{V}{c} \epsilon^{abc} \left( \Gamma_{abc} + \Gamma_{bca} + \Gamma_{cab} \right)$$

$$- \frac{\hbar c V}{4} \left( \Sigma \cdot \ddot{T} + \sigma \gamma_5 \ddot{T} \right)$$

Spin-torsion coupling

FW – semiclassical limit - precession

$$\Omega^{(T)} = -\frac{c}{2} \ddot{T} + \beta \frac{c^3}{8} \left\{ \frac{1}{e'}, \left\{ p, \ddot{T} \right\} \right\} + \frac{c}{8} \left\{ \frac{c^2}{e'(e' + mc^2)}, \left\{ p^2, \ddot{T} \right\} - \left\{ p, (p \cdot \ddot{T}) \right\} \right\}$$
Experimental bounds for torsion

- Magnetic field + rotation + torsion

\[ H = -g_N \frac{\mu_N}{\hbar} B \cdot s - \omega \cdot s - \frac{c}{2} \vec{T} \cdot s. \]

- Same ’92 EDM experiment

\[ \frac{\hbar c}{4} |\vec{T}| \cdot |\cos \Theta| < 2.2 \times 10^{-21} \text{eV}, \quad |\vec{T}| \cdot |\cos \Theta| < 4.3 \times 10^{-14} \text{m}^{-1} \]

- New (based on Gemmel et al ’10)

\[ \frac{\hbar c}{2} |\vec{T}| \cdot |(1 - G) \cos \Theta| < 4.1 \times 10^{-22} \text{eV}, \quad |\vec{T}| \cdot |\cos \Theta| < 2.4 \times 10^{-15} \text{m}^{-1}. \]

\[ G = g_{He}/g_{Xe} \]
Microworld: where is the fastest possible rotation?

- Non-central heavy ion collisions (~c/Compton wavelength) – “small Bang”

- Differential rotation – vorticity

- Calculation in quark - gluon string model (Baznat, Gudima, Sorin, OT, PRC’13)
Structure of velocity and vorticity fields (NICA@JINR-5 GeV/c)
Generalization of Equivalence principle

- Various arguments: AGM ≈ 0 separately for quarks and gluons – most clear from the lattice (LHPC/SESAM)
Recent lattice study (M. Deka et al. arXiv:1312.4816; plenary talk of K.F. Liu)

- Sum of u and d for Dirac (T1) and Pauli (T2) FFs
Extended Equivalence Principle=Exact EquiPartition

- In pQCD – violated
- Reason – in the case of ExEP- no smooth transition for zero fermion mass limit (Milton, 73)
- Conjecture (O.T., 2001 – prior to lattice data) – valid in NP QCD – zero quark mass limit is safe due to chiral symmetry breaking
- Gravityproof confinement? Nucleons do not break even by black holes?
Sum rules for EMT (and OAM)

- Integration of non-local operators matrix elements (related to integration over kinematical variables of some observables) in order to get EMT
- First seminal example: X. Ji’s sum rule (’96) (plenary talk) . Gravity counterpart for nucleons– OT’99
- Are there spin-dependent EMT related sum rule for inclusive processes (forward matrix elements of EMT)
- Burkardt sum rule – looks similar: can it be derived from EMT?
- Contribution of Sivers function (~gluonic pole) to EMT – seems to be identically equal zero BUT this changes in the case of pole prescription (OT’14)
Pole prescription and Burkardt SR

- Pole prescription (dynamics!) provides ("T-odd") symmetric part to the integrand!

\[ \sum \int dx_1 \int dx_2 \frac{T(x_1, x_2)}{x_1 - x_2 + i\varepsilon} = 0 \]

- SR: (but relation of gluon Sivers to twist 3 still not found – prediction!)

- Can it be valid separately for each quark flavour: nodes?

- Valid if structures forbidden for TOTAL EMT do not appear for each flavour

- Structure contains (besides S) gauge vector n: If GI separation of EMT – forbidden: SR valid separately!
Another manifestation of post-Newtonian (Ex)EP for spin 1 hadrons

- Tensor polarization - coupling of gravity to spin in forward matrix elements (OT’09)

\[ \langle P, S | \bar{\psi}(0) \gamma^{\nu} D^{\nu_1} ... D^{\nu_n} \psi(0) | P, S \rangle_{\mu^2} = i^{-n} M^2 S^{\nu \nu_1} P^{\nu_2} ... P^{\nu_n} \int_0^1 C_q^T(x)x^n dx \]

\[ \sum_q \langle P, S | T_i^{\mu \nu} | P, S \rangle_{\mu^2} = 2P^{\mu} P^{\nu} (1 - \delta(\mu^2)) + 2M^2 S^{\mu \nu} \delta_1(\mu^2) \]

\[ \langle P, S | T_g^{\mu \nu} | P, S \rangle_{\mu^2} = 2P^{\mu} P^{\nu} \delta(\mu^2) - 2M^2 S^{\mu \nu} \delta_1(\mu^2) \]

\[ \sum_q \int_0^1 C_i^T(x)x dx = \delta_1(\mu^2) \]

- Second moments of tensor distributions should sum to zero (EMT conservation: Efremov, OT’82,’93)
HERMES – data on tensor spin structure function

- Isoscalar target – proportional to the sum of u and d quarks – combination required by (Ex)EP
- Second moments – compatible to zero better than the first one (Close-Kumano SR): collective glue << sea – for valence:

\[ \int_0^1 C_i^T(x) \, dx = 0. \]
Are more accurate data possible?

- HERMES – unlikely

- Planned at JLab; SR study may provide information about collective sea and glue in deuteron and indirect new test of Equivalence Principle

- Complementary information from DY with deuteron beams: JPARC, NICA
CONCLUSIONS

- Spin-gravity interactions may be probed directly in gravitational (inertial) experiments and indirectly – studying EMT matrix element
- Torsion and EP may be tested as a byproduct of EDM experiments
- SR’s for deuteron tensor polarization indirectly probe EP and its extension separately for quarks and gluons – may be tested at JLab, JPARC and NICA
BACKUP SLIDES
EEP and AdS/QCD

- Recent development – calculation of Rho formfactors in Holographic QCD (Grigoryan, Radyushkin)
  - Provides g=2 identically!
  - Experimental test at time –like region possible