

Working with Wilson Lines

Spin 2014, Beijing

Frederik F. Van der Veken[†]

[†]Universiteit Antwerpen

October 24, 2014

Universiteit **Antwerpen**





Outline

- 1 Wilson Line Exponentials
- 2 Linear Wilson Lines
- 3 Piecewise Linear Wilson Lines: Methodology
- 4 Example Calculations

Path-Ordered Exponentials

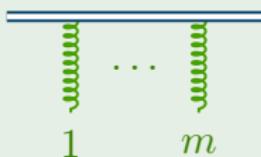
Wilson Line

$$\mathcal{U}[\mathcal{C}] = \mathcal{P} \exp \left(ig \int_a^b d\lambda \ (z^\mu)' A_\mu(\lambda) \right)$$

Path-Ordered Exponentials

Wilson Line

$$\mathcal{U}[\mathcal{C}] = \mathcal{P} \exp \left(ig \int_a^b d\lambda \ (z^\mu)' A_\mu(\lambda) \right)$$



$$A(\lambda_m) \cdots A(\lambda_1)$$

Path-Ordering

Path-Ordering for Linear Lines

$$z^\mu = r^\mu + \hat{n}^\mu \lambda \quad \lambda = a \dots b \quad \lambda_m \geq \dots \geq \lambda_1$$

$$\begin{aligned} \mathcal{P} \int_c \cdots \int_c dz_1 \cdots dz_m &= m! \int_a^b \int_a^b \cdots \int_a^b d\lambda_1 \cdots d\lambda_m \\ &= m! \int_a^b \int_a^{\lambda_m} \cdots \int_a^{\lambda_2} d\lambda_m \cdots d\lambda_1 \end{aligned}$$

Motivation

Interest in Wilson Lines

- Singularity structure of TMD governed by underlying Wilson line structure
- T-odd non-universality effects due to different Wilson line structures
- Investigation of Wilson loops, to describe geometric evolution of TMDs



Outline

- 1 Wilson Line Exponentials
- 2 Linear Wilson Lines
- 3 Piecewise Linear Wilson Lines: Methodology
- 4 Example Calculations

Feynman Rules

Wilson Line Bounded From Below

$$\begin{aligned} \mathcal{U}_{(+\infty; r)} = & \sum_{m=0}^{\infty} (\mathrm{i}g)^m \int \left(\frac{\mathrm{d}^4 k_i}{16\pi^4} \right)^m \hat{n} \cdot A(-k_m) \cdots \hat{n} \cdot A(-k_1) \times \cdots \\ & \times \int\limits_0^{\infty} \int\limits_{\lambda_1}^{\infty} \cdots \int\limits_{\lambda_{n-1}}^{\infty} \mathrm{d}\lambda_1 \cdots \mathrm{d}\lambda_m \, e^{\mathrm{i}(r + \hat{n} \lambda_1) \cdot k_1} \cdots e^{\mathrm{i}(r + \hat{n} \lambda_m) \cdot k_m} \end{aligned}$$

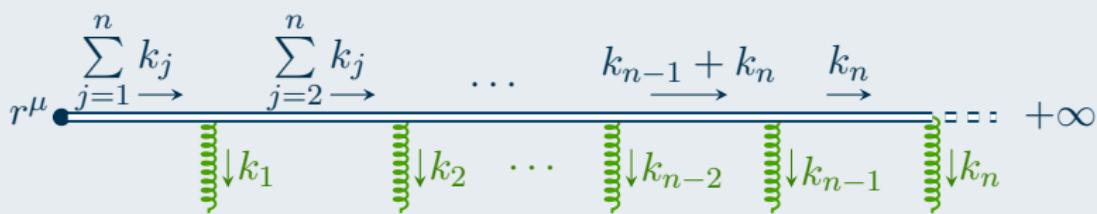
Feynman Rules

Wilson Line Bounded From Below

$$\mathcal{U}_{(+\infty; r)} = \sum_{m=0}^{\infty} (\mathrm{i}g)^m \int \left(\frac{\mathrm{d}^4 k_i}{16\pi^4} \right)^m \hat{n} \cdot A(-k_m) \cdots \hat{n} \cdot A(-k_1) \times \cdots$$

$$K^\downarrow(j) = \sum_{l=j}^n k_l \quad \cdots \times e^{\mathrm{i}r \cdot K} \prod_{j=1}^m \frac{\mathrm{i}}{\hat{n} \cdot K^\downarrow(j) + \mathrm{i}\eta}$$

Feynman Diagram



Feynman Rules

Feynman Rules for Linear Wilson Lines

1) Wilson line propagator:

$$\overbrace{\text{---}}^k = \frac{i}{\hat{n} \cdot k + i\eta}$$

2) external point:

$$r^\mu \bullet \overbrace{\text{---}}^k = e^{ir \cdot k}$$

3) infinite point:

$$\overbrace{\dots\dots\dots}^{+\infty} = 1 \quad (k=0)$$

4) Wilson vertex:

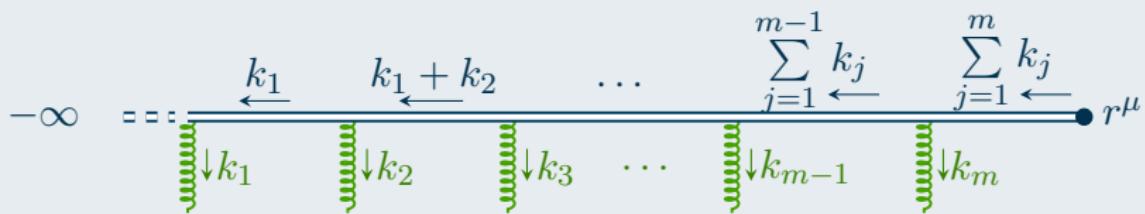
$$j \overbrace{\text{---}}^{\mu, a} i = ig \hat{n}^\mu (t^a)_{ij}$$

Feynman Rules

Wilson Line Bounded From Above

$$\begin{aligned} \mathcal{U}_{(r; -\infty)} = & \sum_{m=0}^{\infty} (\mathrm{i}g)^m \int \left(\frac{\mathrm{d}^4 k_i}{16\pi^4} \right)^m \hat{n} \cdot A(-k_m) \cdots \hat{n} \cdot A(-k_1) \times \\ K^\uparrow(j) = & \sum_{l=1}^j k_l \quad \times e^{\mathrm{i}r \cdot K} \prod_{j=1}^m \frac{-\mathrm{i}}{\hat{n} \cdot K^\uparrow(j) - \mathrm{i}\eta} \end{aligned}$$

Feynman Diagram



Different Types

Semi-Infinite Lines and Path Reversals

$$\rightarrow \quad (\mathrm{i}g)^m A_m \cdots A_1 e^{\mathrm{i}r \cdot K} \prod_j^m \frac{\mathrm{i}}{\hat{n} \cdot K^\downarrow(j) + \mathrm{i}\eta} \stackrel{\text{N}}{=} A^m(r, \hat{n})$$

$$\leftarrow \quad (-\mathrm{i}g)^m A_m \cdots A_1 e^{\mathrm{i}r \cdot K} \prod_j^m \frac{\mathrm{i}}{\hat{n} \cdot K^\uparrow(j) + \mathrm{i}\eta} \stackrel{\text{N}}{=} B^m(r, \hat{n})$$

$$\rightarrowtail \quad (-\mathrm{i}g)^m A_m \cdots A_1 e^{\mathrm{i}r \cdot K} \prod_j^m \frac{-\mathrm{i}}{\hat{n} \cdot K^\downarrow(j) - \mathrm{i}\eta} = A^m(r, -\hat{n})$$

$$\leftarrowtail \quad (\mathrm{i}g)^m A_m \cdots A_1 e^{\mathrm{i}r \cdot K} \prod_j^m \frac{-\mathrm{i}}{\hat{n} \cdot K^\uparrow(j) - \mathrm{i}\eta} = B^m(r, -\hat{n})$$

More Types

Finite Line

$$\mathcal{U}_{(b; a)} = \sum_{n=0}^{\infty} (\mathrm{i}g)^m \int \left(\frac{\mathrm{d}^4 k_i}{16\pi^4} \right)^m \hat{n} \cdot A(-k_m) \cdots \hat{n} \cdot A(-k_1) \times \cdots \\ \cdots \times \sum_{l=0}^m \mathrm{e}^{-\mathrm{i}a \cdot K(l)} \mathrm{e}^{-\mathrm{i}b \cdot K(m-l)} \prod_{j=1}^l \frac{-\mathrm{i}}{\hat{n} \cdot \tilde{K}(j)} \prod_{j=l+1}^m \frac{\mathrm{i}}{n \cdot K(j)}$$

Hermitian Conjugate

$$\mathcal{U}_{(r; -\infty)}^\dagger = \sum_{n=0}^{\infty} (-\mathrm{i}g)^m \int \left(\frac{\mathrm{d}^4 k_i}{16\pi^4} \right)^m \hat{n} \cdot A(-k_m) \cdots \hat{n} \cdot A(-k_1) \times \cdots \\ \cdots \times \mathrm{e}^{\mathrm{i}b \cdot K} \prod_{j=1}^m \frac{-\mathrm{i}}{\hat{n} \cdot K^\downarrow(j) - \mathrm{i}\eta}$$

More Types

Finite Lines and Hermitian Conjugates

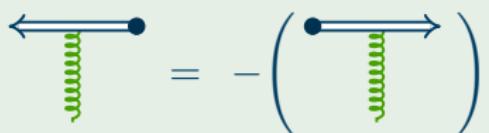
$$\begin{array}{c} a^\mu \\ \bullet \longrightarrow \bullet \end{array} = \bullet \longrightarrow \otimes \longleftarrow \bullet$$

$$(\longleftarrow \bullet)^\dagger = \bullet \longleftarrow \quad (\bullet \longrightarrow)^\dagger = \longleftarrow \bullet$$

Relation Between A^m and B^m

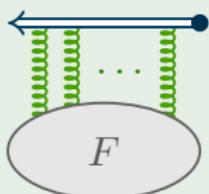
Relation Between A^m and B^m

$$B^m(r, \hat{n}) = (-)^m A^m(r, \hat{n}) \Big|_{(k_1, \dots, k_m) \rightarrow (k_m, \dots, k_1)}$$



Relation Between A^m and B^m Relation Between A^m and B^m

$$B^m(r, \hat{n}) = (-)^m A^m(r, \hat{n}) \Big|_{(k_1, \dots, k_m) \rightarrow (k_m, \dots, k_1)}$$



$$= (-)^m \int \left(\frac{dk_i}{16\pi^4} \right)^m A^m(r, \hat{n}) F_{a_1 \dots a_m}^{\mu_1 \dots \mu_m}(k_m, \dots, k_1)$$

(absorb gluon propagators in F)

Relation Between A^m and B^m

After Symmetrising the Blob

$$\text{Diagram of a blob } F \text{ with a horizontal double-headed arrow above it.} = (-)^m \text{Diagram of a blob } F|_{\overline{\{a_i\}}} \text{ with a horizontal double-headed arrow below it.}$$

Relation Between A^m and B^m

After Symmetrising the Blob

$$\begin{array}{c} \text{↔} \\ \text{↔} \\ \text{↔} \\ \cdots \\ \text{↔} \\ \text{↔} \end{array} = (-)^m \begin{array}{c} \text{↔} \\ \text{↔} \\ \text{↔} \\ \cdots \\ \text{↔} \\ \text{↔} \end{array}$$

F $F|_{\overline{\{a_i\}}}$

Easy Blob Example: 3-Gluon Vertex

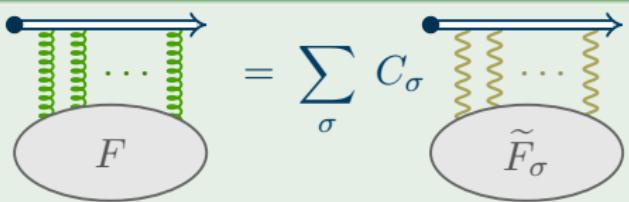
$$\begin{array}{c} \text{↔} \\ \text{↔} \\ \text{↔} \\ \text{↔} \\ \text{↔} \end{array} = \begin{array}{c} \text{↔} \\ \text{↔} \\ \text{↔} \\ \text{↔} \\ \text{↔} \end{array}$$

Non-Trivial Colour Structure

Blob With Non-Trivial Colour Structure

$$F_{\mu_1 \dots \mu_m}^{a_1 \dots a_m}(k_1, \dots, k_m) = \sum_{\text{perm}} C^{\sigma(a_1 \dots a_m)} \tilde{F}_{\sigma(\mu_1 \dots \mu_m)} (\sigma(k_1, \dots, k_m))$$

Factorise Out Colour



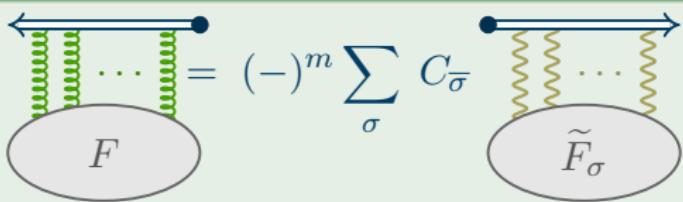
$$C_{\sigma} = t^{a_m} \dots t^{a_1} C^{\sigma(a_1 \dots a_m)}$$

Non-Trivial Colour Structure

Blob With Non-Trivial Colour Structure

$$F_{\mu_1 \dots \mu_m}^{a_1 \dots a_m}(k_1, \dots, k_m) = \sum_{\text{perm}} C^{\sigma(a_1 \dots a_m)} \tilde{F}_{\sigma(\mu_1 \dots \mu_m)} (\sigma(k_1, \dots, k_m))$$

Factorise Out Colour



$$C_{\bar{\sigma}} = t^{a_1} \dots t^{a_m} C^{\sigma(a_1 \dots a_m)}$$

Non-Trivial Colour Structure

 $m = 4$

$$\begin{aligned} \text{Diagram: } &= \sum_{\sigma} f^{a_1 a_2 x} f^x a_3 a_4 \tilde{F}_{\mu_1 \dots \mu_4}(k_1, \dots, k_4) \\ \text{Diagram: } &= C_{4321} \text{ (green)} + C_{4231} \text{ (yellow)} + C_{4132} \text{ (yellow)} + C_{4123} \text{ (yellow)} \\ \text{Diagrams: } &\quad F \quad \tilde{F} \quad F \quad \tilde{F} \end{aligned}$$

$$C_{ijkl} = t^{a_i} t^{a_j} t^{a_k} t^{a_l} f^{a_1 a_2 x} f^x a_3 a_4$$

Non-Trivial Colour Structure

 $m = 4$

$$\begin{aligned} \text{Diagram: } & \text{A green wavy line connecting two vertices, labeled } F \text{ at the bottom left and } \tilde{F} \text{ at the top right.} \\ & = \sum_{\sigma} f^{a_1 a_2 x} f^x a_3 a_4 \tilde{F}_{\mu_1 \dots \mu_4}(k_1, \dots, k_4) \\ \text{Diagram: } & \text{A horizontal line with a double-headed arrow above it, connected to a vertical stack of four green wavy lines. Below it is an oval labeled } F. \\ & = C_{\overline{4321}} + C_{\overline{4231}} + C_{\overline{4132}} + C_{\overline{4123}} \end{aligned}$$

$$C_{\overline{ijkl}} = C_{lkji} = t^{a_l} t^{a_k} t^{a_j} t^{a_i} f^{a_1 a_2 x} f^x a_3 a_4 = C_{ijkl}$$

Non-Trivial Colour Structure

 $m = 4$ 

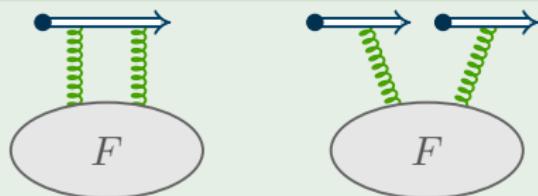


Outline

- 1 Wilson Line Exponentials
- 2 Linear Wilson Lines
- 3 Piecewise Linear Wilson Lines: Methodology
- 4 Example Calculations

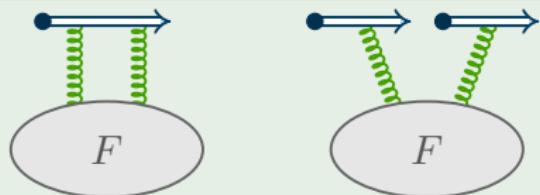
Basic Diagrams

$m = 2$



Basic Diagrams

$m = 2$

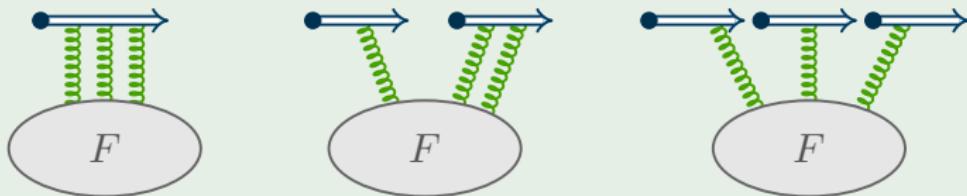


$$\sum_{J=1}^M \text{Diagram } J + \sum_{K=2}^M \sum_{J=1}^{K-1} \text{Diagram } K \text{ and } J$$

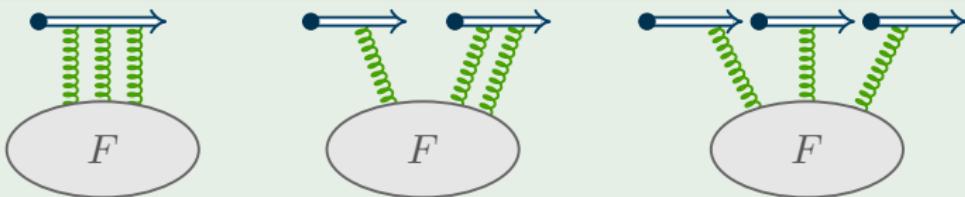
The first term is a sum over J from 1 to M , where the diagram consists of a single horizontal line with two vertices connected by a double-headed green spring, with an oval labeled F below it. The second term is a sum over K from 2 to M and a sum over J from 1 to $K-1$, where the diagram consists of two horizontal lines with two vertices each, connected by two parallel green springs, with an oval labeled F below it.

Basic Diagrams

$m = 3$



Basic Diagrams

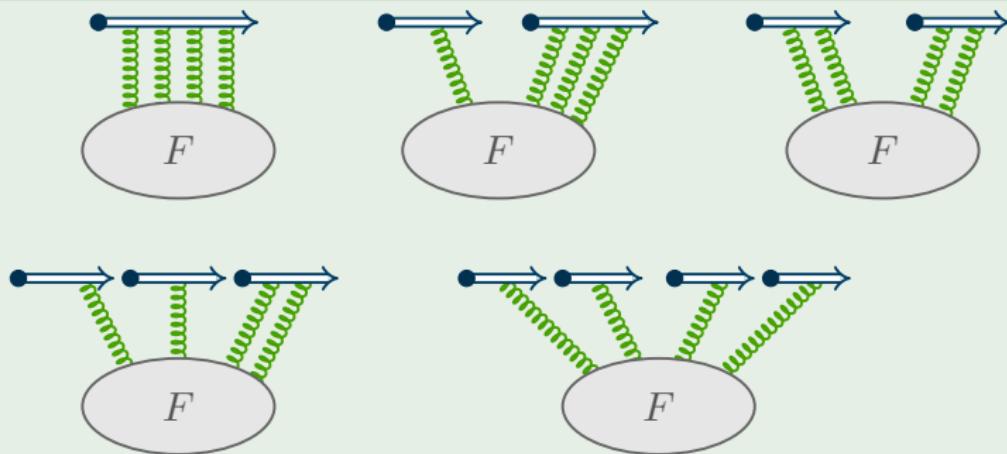
 $m = 3$ 

$$\sum_{J=1}^M \text{Diagram } J + 2 \sum_{K=2}^M \sum_{J=1}^{K-1} \text{Diagram } (K, J) + \sum_{L=3}^M \sum_{K=2}^{L-1} \sum_{J=1}^{K-1} \text{Diagram } (L, K, J)$$

The equation shows the expansion of a sum of Wilson line contributions. The first term is a sum over J from 1 to M of a diagram with a single vertical segment. The second term is a sum over K from 2 to M and a sum over J from 1 to $K-1$ of a diagram with two vertical segments. The third term is a sum over L from 3 to M , a sum over K from 2 to $L-1$, and a sum over J from 1 to $K-1$ of a diagram with three vertical segments.

Basic Diagrams

$m = 4$



Basic Diagrams

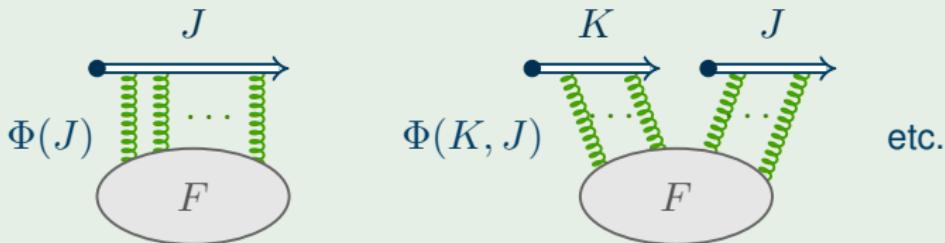
$m = 4$

$$\begin{aligned} & \sum_{J=1}^M \text{Diagram } J + \sum_{K=2}^M \sum_{J=1}^{K-1} \left(2 \text{Diagram } (K, J) + \text{Diagram } K \text{ with } J \right) \\ & + 3 \sum_{L=3}^M \sum_{K=2}^{L-1} \sum_{J=1}^{K-1} \text{Diagram } (L, K, J) + \sum_{O=4}^M \sum_{L=3}^{O-1} \sum_{K=2}^{L-1} \sum_{J=1}^{K-1} \text{Diagram } (O, L, K, J) \end{aligned}$$

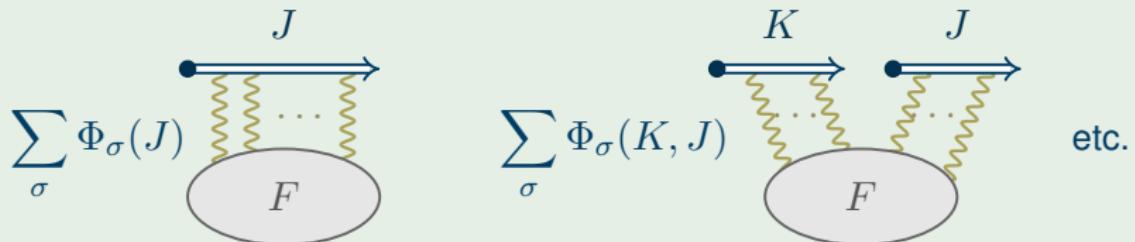
The diagrams consist of a horizontal line with arrows pointing right, ending at a black dot, which then connects to a green wavy line. The wavy line then connects to a pink oval. The first diagram has one wavy line and is labeled J . Subsequent diagrams have multiple wavy lines and are labeled with pairs of indices, such as (K, J) or (L, K, J) .

Path Constants

For a Blob With Trivial Colour Structure



For a Blob With Non-Trivial Colour Structure



Blob Examples

$m = 2$

$$\text{Diagram: A green wavy line connecting two vertices.} = \delta^{ab} \delta^{(4)}(k_1 - k_2) D_{\mu\nu}(k_1)$$

$$\text{Diagram: A green wavy line connecting two vertices with a blue double-headed arrow below it.} = \text{Diagram: A green wavy line connecting two vertices with a blue double-headed arrow above it.} \Rightarrow \Phi(J) = +1$$

$$\text{Diagram: Two blue double-headed arrows with a green wavy line connecting them.} = \text{Diagram: A green wavy line connecting two vertices with two blue double-headed arrows below it.} = - \text{Diagram: A green wavy line connecting two vertices with two blue double-headed arrows above it.}$$

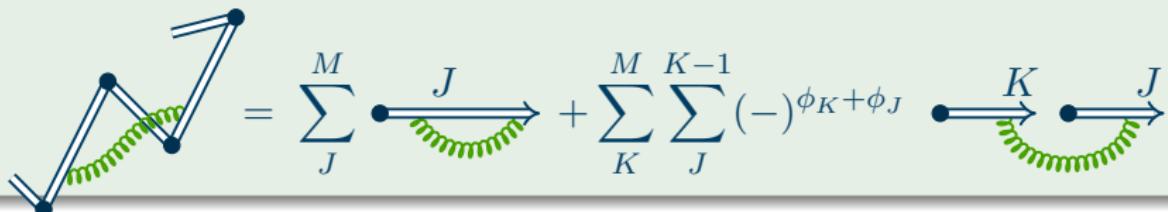
$$\Rightarrow \Phi(K, J) = (-)^{\Phi_K + \Phi_J}$$

$$\phi_J = \begin{cases} 0 & \xrightarrow{\quad} \\ 1 & \xleftarrow{\quad} \end{cases}$$

Blob Examples

 $m = 2$

$$\mathcal{U}_2 = \sum_{J=1}^M \mathcal{U}_2^J + \sum_{K=2}^M \sum_{J=1}^{K-1} \mathcal{U}_1^K \mathcal{U}_1^J$$



Blob Examples

 $m = 3$ 

$$= g f^{abc} D_{\mu_1 \nu_1}^{k_1} D_{\mu_2 \nu_2}^{k_2} D_{\mu_3 \nu_3}^{k_3} g^{\nu_1 \nu_2} (k_1 - k_2)^{\nu_3} + \text{cross.}$$



$$= \quad \leftarrow \quad \rightarrow \quad \Rightarrow \Phi(J) = +1$$



$$= - \quad \leftarrow \quad \rightarrow \quad \Phi(K, J) = (-)^{\phi_K + \phi_J}$$

Blob Examples

 $m = 3$

$$\mathcal{U}_3 = \sum_{J=1}^M \mathcal{U}_3^J + \sum_{K=2}^M \sum_{J=1}^{K-1} [\mathcal{U}_1^K \mathcal{U}_2^J + \mathcal{U}_2^K \mathcal{U}_1^J] + \sum_{L=3}^M \sum_{K=2}^{L-1} \sum_{J=1}^{K-1} \mathcal{U}_1^L \mathcal{U}_1^K \mathcal{U}_1^J$$

$$= \sum_J^M \text{---} \overset{J}{\longrightarrow} + 2 \sum_K^M \sum_{J=1}^{K-1} (-)^{\phi_K + \phi_J} \text{---} \overset{(K)}{\longrightarrow} \text{---} \overset{J}{\longrightarrow} + \sum_{L=3}^M \sum_{K=2}^{L-1} \sum_{J=1}^{K-1} (-)^{\phi_L + \phi_K + \phi_J} \text{---} \overset{L}{\longrightarrow} \text{---} \overset{K}{\longrightarrow} \text{---} \overset{J}{\longrightarrow}$$

Blob Example With Non-Trivial Colour Structure

 $m = 4$

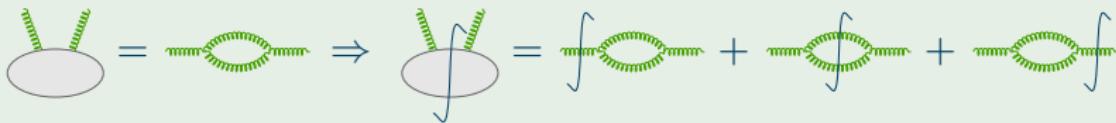
$$\begin{aligned} \text{Diagram 1} &= \sum_{\sigma} f^{a_1 a_2 x} f^{x a_3 a_4} \tilde{F}_{\mu_1 \dots \mu_4}(k_1, \dots, k_4) \\ \text{Diagram 2} &= \text{Diagram 1} \\ \text{Diagram 3} &= C_1 \text{Diagram 4} + C_2 \text{Diagram 5} + C_3 \text{Diagram 6} \\ \text{Diagram 4} &= C_3 \text{Diagram 7} + C_2 \text{Diagram 8} + C_1 \text{Diagram 9} \end{aligned}$$

The diagrams consist of green wavy lines and blue double-headed arrows. The first diagram has two wavy lines meeting at a point. The second diagram has a wavy line and a double-headed arrow meeting at a point. The third diagram has two double-headed arrows meeting at a point. The fourth through ninth diagrams involve a yellow wavy line connecting a green wavy line to a blue double-headed arrow, which then connects to a light gray oval labeled \tilde{F} . The coefficients C_1, C_2, C_3 are placed next to the first three diagrams.

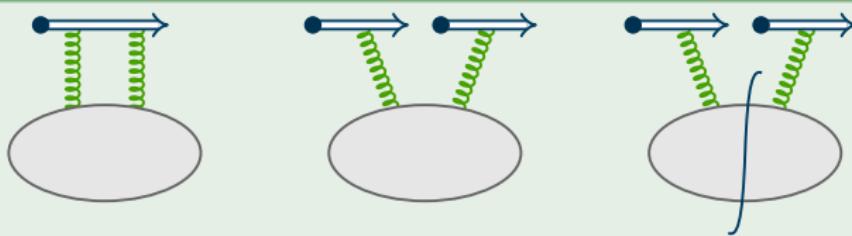
Final-State Cut

Cutting

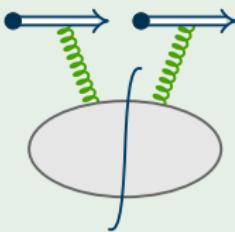
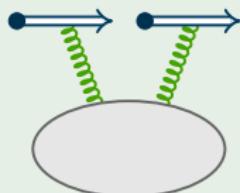
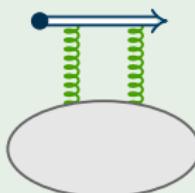
- cutting happens at infinity \Rightarrow no momentum cut in Wilson line
- cut line doesn't cross segment, only *between* segments
 \Rightarrow only blob is cut
- define cut blob as sum of possible cuttings, e.g.:



- denote number of segments before the cut by M_c

$m = 2$ 

$m = 2$



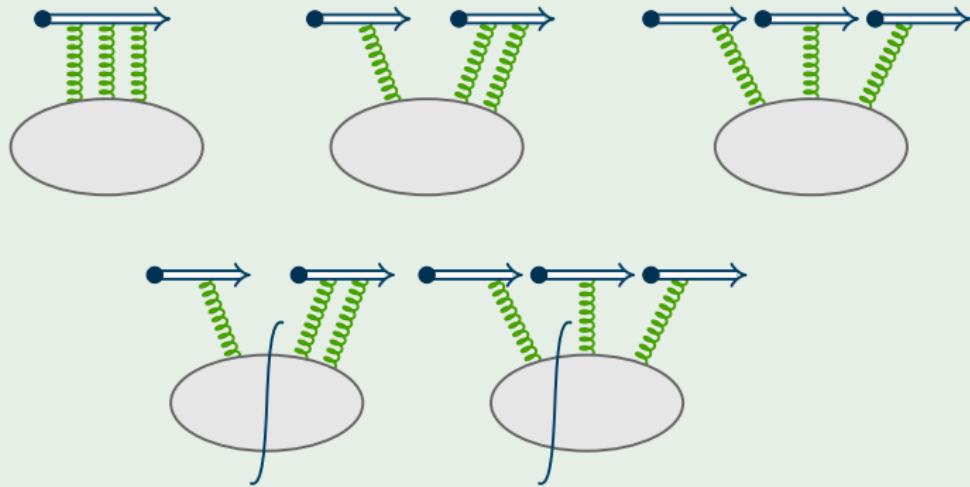
$$\sum_{J=1}^M \text{Diagram } J + \left(\sum_{K=2}^{M_c} \sum_{J=1}^{K-1} + \sum_{K=M_c+2}^M \sum_{J=M_c+1}^{K-1} \right) \text{Diagram } K \text{ and } J + \sum_{K=M_c+1}^M \sum_{J=1}^{M_c} \text{Diagram } K \text{ and } J$$

Diagrammatic expansion equation:

The equation shows the expansion of a sum of diagrams. The first term is a sum from $J=1$ to M of a diagram with a single horizontal line and a green spring. The second term is a sum from $K=2$ to M_c and from $J=1$ to $K-1$ of a diagram with two horizontal lines and two green springs. The third term is a sum from $K=M_c+2$ to M and from $J=M_c+1$ to $K-1$ of a diagram with two horizontal lines and two green springs. The fourth term is a sum from $K=M_c+1$ to M and from $J=1$ to M_c of a diagram with two horizontal lines and two green springs.

Expanding the Set of Basic Diagrams

$m = 3$





Outline

- 1 Wilson Line Exponentials
- 2 Linear Wilson Lines
- 3 Piecewise Linear Wilson Lines: Methodology
- 4 Example Calculations

Reusability

LO 2-Gluon Blob Connecting 2 Segments

$$\begin{aligned} \text{Diagram: Two gluons (black arrows) connecting via a gluon loop (green wavy line).} \\ &\stackrel{\text{LC}}{=} \frac{\alpha_s}{2\pi} \left(\frac{1}{\epsilon^2} + \frac{\pi}{3} \right) \left(\frac{n_K \cdot n_J}{2} \frac{\mu^2}{\eta^2} \right)^\epsilon \\ &\stackrel{\text{off-LC}}{=} \frac{\alpha_s}{2\pi} \chi \coth \chi \left(\frac{1}{\epsilon} + \Upsilon \right) \left[\frac{1}{4} \left(n_K^2 n_J^2 - (n_K \cdot n_J)^2 \right) \frac{\mu^2}{\eta^2} \right]^\epsilon \\ &\quad \left(\Upsilon = 2 \ln 2 + \ln n_K^2 + 2 \ln(1 + e^\chi) + \chi - \frac{1}{\chi} (\text{Li}_2 e^\chi - \text{Li}_2 e^{-\chi}) \right) \end{aligned}$$

Reusability

⇒ difference only in path parameters r_{J_i}, \hat{n}_{J_i} and path constants $\Phi(J_i)$!

TMD Example

TMD Wilson Line Self



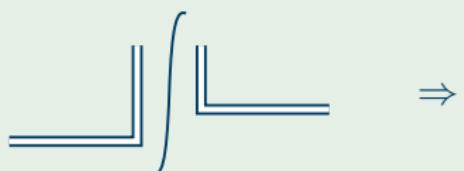
SIDIS

$$\begin{array}{l} \bullet \rightarrow n^-, 0^-, \mathbf{0}_\perp \\ \bullet \rightarrow \mathbf{n}_\perp, +\infty^-, \mathbf{0}_\perp \end{array}$$

$$\begin{array}{l} \leftarrow \bullet -\mathbf{n}_\perp, +\infty^-, \mathbf{r}_\perp \\ \leftarrow \bullet -n^-, r^-, \mathbf{r}_\perp \end{array}$$

TMD Example

TMD Wilson Line Self



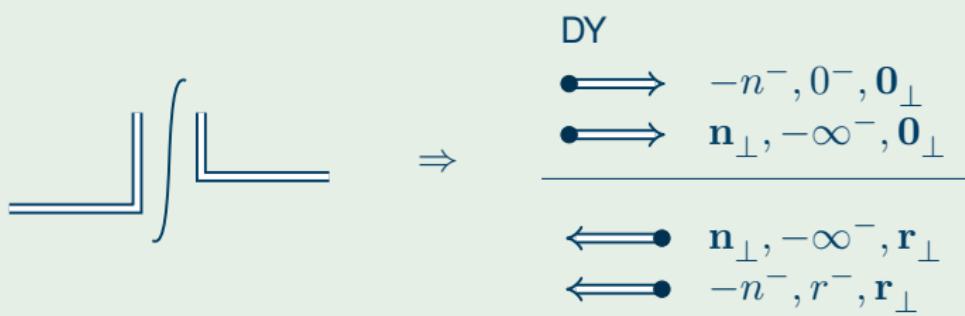
DY

$$\begin{array}{l} \xrightarrow{\hspace{1cm}} -n^-, 0^-, \mathbf{0}_\perp \\ \xrightarrow{\hspace{1cm}} \mathbf{n}_\perp, -\infty^-, \mathbf{0}_\perp \end{array}$$

$$\begin{array}{l} \xleftarrow{\hspace{1cm}} \mathbf{n}_\perp, -\infty^-, \mathbf{r}_\perp \\ \xleftarrow{\hspace{1cm}} -n^-, r^-, \mathbf{r}_\perp \end{array}$$

TMD Example

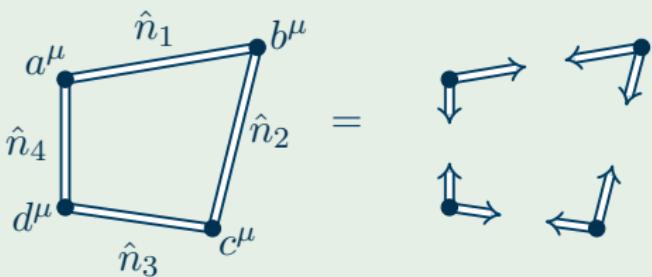
TMD Wilson Line Self



- Difference *only* in sign of n^- and $\pm\infty^-$. No difference in path constants!
- At this level still a gauge invariant, all-order statement.

Quadrilateral Wilson Loop

Quadrilateral Wilson Loop



Quadrilateral Wilson Loop

First Order

$$\text{Diagram of a quadrilateral loop} = \sum_{J=1}^M \text{Diagram of a horizontal line with a green wavy loop labeled } J + \sum_{K=1}^M \sum_{J=1}^{K-1} (-)^{\phi_K + \phi_J} \text{Diagram of two horizontal lines connected by a green wavy loop labeled } K \text{ and } J$$

Result for Light-Like Loop

$$\begin{aligned} \mathcal{U}_2 &= \frac{\alpha_s C_F}{\pi} (-2\pi\mu^2)^\epsilon \Gamma(1-\epsilon) \times \dots \\ &\quad \dots \times \left[\frac{1}{\epsilon^2} ((b-d)^2 - i\eta)^\epsilon + \frac{1}{\epsilon^2} ((c-a)^2 - i\eta)^\epsilon \right] \end{aligned}$$

Conjecture

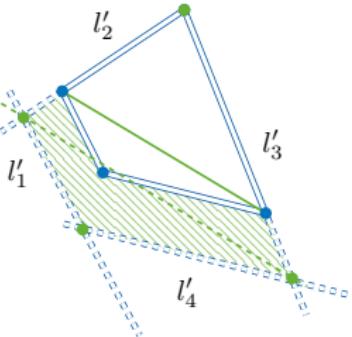
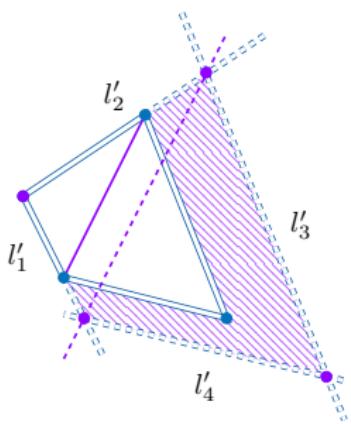
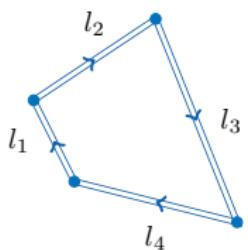
Geometric Evolution of Light-Like Quadrilateral

$$\left(\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} \right) \left\langle \frac{\delta}{\delta \ln \Sigma} \right\rangle \ln \mathcal{W}_\gamma = - \sum_{\text{cusps}} \Gamma_{\text{cusp}}$$

Gamma cusp at NLO:

$$\Gamma_{\text{cusp}}(g) = \frac{\alpha_s}{\pi} C_F + \left(\frac{\alpha_s}{\pi} \right)^2 C_F \left(C_A \left(\frac{67}{36} - \frac{\pi^2}{12} \right) - N_f \frac{5}{18} \right)$$

Conjecture



Conclusions

Conclusions & Outlook

- framework to minimize number of diagrams for piecewise linear Wilson lines
 - lesser diagrams in exchange for more general (and thus more complicated) integrals
 - interesting for $M > 2$
-
- clean up result & calculate higher orders
 - try framework for TMD Wilson line structure

Conclusions

Thank You!