



TMD evolution for Collins asymmetries
in e^+e^- annihilation and SIDIS

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LBL

in collaboration with Kang, Prokudin and Yuan

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Outlines

- Energy evolution in TMD factorization
- Collins asymmetry in e^+e^- annihilation and SIDIS
- Summary

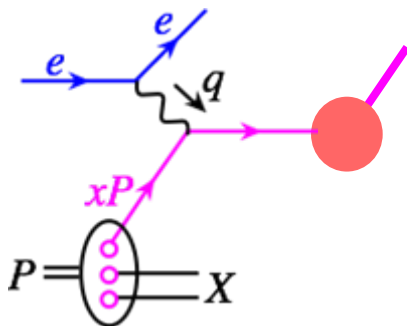
Why do we need QCD evolution?

TMD factorization is applicable in case two different scales are observed in processes such as SIDIS, Drell-Yan, W/Z production in hadron-hadron collisions. Kinematical regime: $Q_T \ll Q$

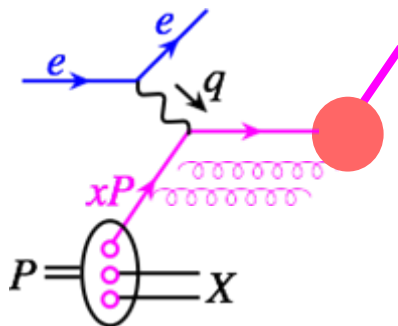
For SIDIS Q_T is transverse momentum of final parton

And Q is the invariant mass of virtual photon

Double and Single logarithms will appear order by order in perturbative calculations



$$f(x)$$



$$f(x, k_{\perp}; Q)$$

$$\left(\alpha_s \ln^2 \frac{Q^2}{Q_T^2} \right)^n$$

$$\left(\alpha_s \ln \frac{Q^2}{\mu^2} \right)^n$$

TMD evolution in TMD parton distributions

With CSS evolution equation, evolution starts from $\mu_b = c/b$, $c = 2e^{-\gamma_E}$

$$\tilde{f}(x, b; Q) = \tilde{f}(x, b; \mu_b) e^{-S_{pert}(b)}$$

where

$$\tilde{f}(x, b; Q) = \int d^2 k_{\perp} e^{-ik_{\perp} b} f(x, k_{\perp}; Q)$$

$$S^{PT}(b) = \int_{\mu_b}^Q \frac{d\mu}{\mu} \left(A \ln \frac{Q^2}{\mu^2} + B \right) \quad \text{Perturbative Sudakov factor}$$

$$A = \sum_{n=1} \left(\frac{\alpha_s}{\pi} \right)^n A^{(n)}$$

$$B = \sum_{n=1} \left(\frac{\alpha_s}{\pi} \right)^n B^{(n)}$$

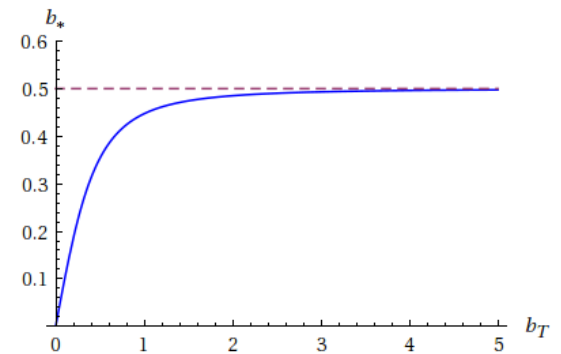
Calculation is perturbative, valid only in region $b \ll 1/\Lambda_{QCD}$

Fourier transform in momentum space involves non-perturbative region

$$f(x, k_{\perp}; Q) = \int_0^{\infty} \frac{bdb}{2\pi} J_0(k_{\perp} b) \tilde{f}(x, b; Q)$$

Non perturbative region needs to be treated.
Common method b^* prescription

$$b_* = \frac{b}{\sqrt{1 + b^2/b_{max}^2}}$$



$$\tilde{f}(x, b; Q) = \tilde{f}(x, b_*; c/b_*) e^{-S_{pert}(b_*)} e^{-S_{NP}(b)}$$

Non perturbative Sudakov factor

$$S_{\text{NP}}^{\text{SIDIS}}(Q, b) = g_2 \ln \left(\frac{b}{b_*} \right) \ln \left(\frac{Q}{Q_0} \right) + \left(\frac{g_1}{2} + g_3 \left(\frac{x_0}{x_B} \right)^\lambda + \frac{g_h}{z_h^2} \right) b^2$$

A new non-perturbative Sudakov factor is used.

Where $x_0=0.01$, $Q_0^2=2.4\text{GeV}^2$, and $\lambda=0.2$

g_1 , g_2 and g_3 are free parameters, from the fitting of Drell-Yan processes, and g_h is from SIDIS

In our fit, we choose $b_{\text{max}}=1.5\text{GeV}^{-1}$

$$\tilde{f}^j(x, b_*; c/b_*) = \sum_{j'=q,g} \int_x^1 \frac{d\hat{x}}{\hat{x}} C_{j/j'} \left(\frac{x}{\hat{x}}, b_*, c/b_* \right) f^{j'}(x; c/b_*)$$

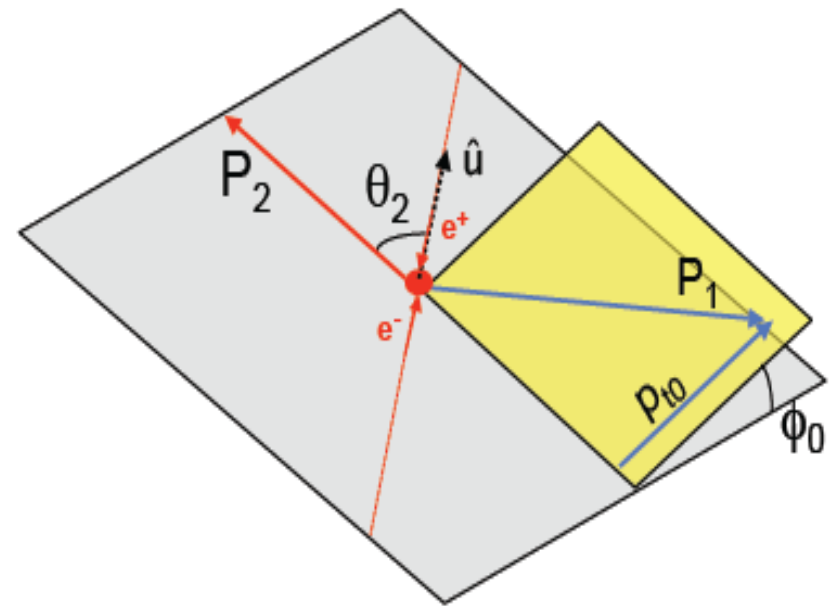
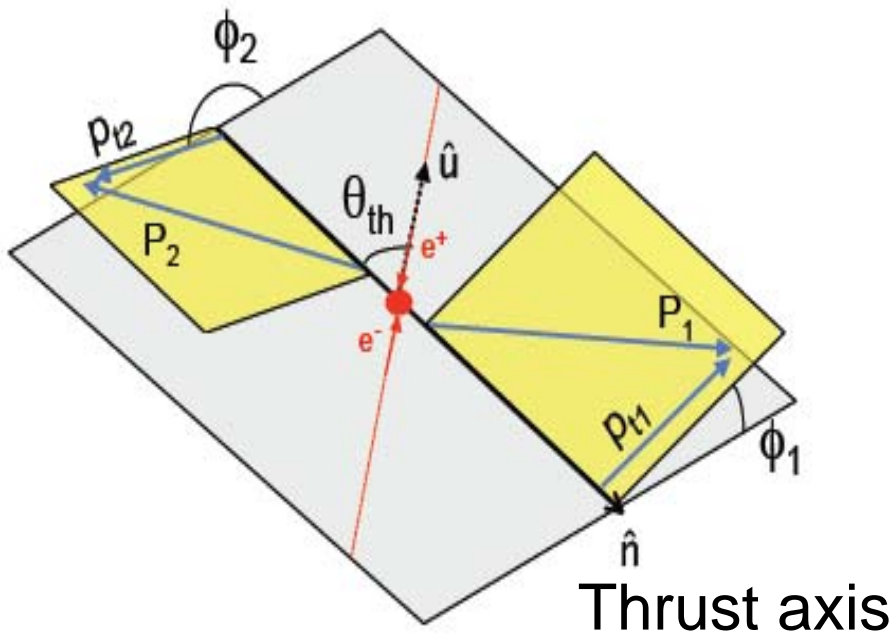
$$C = \sum_{n=1} \left(\frac{\alpha_s}{\pi} \right)^n C^{(n)} \quad \text{Wilson coefficient}$$

Collinear PDF

transversity is related to a twist-2 collinear PDF

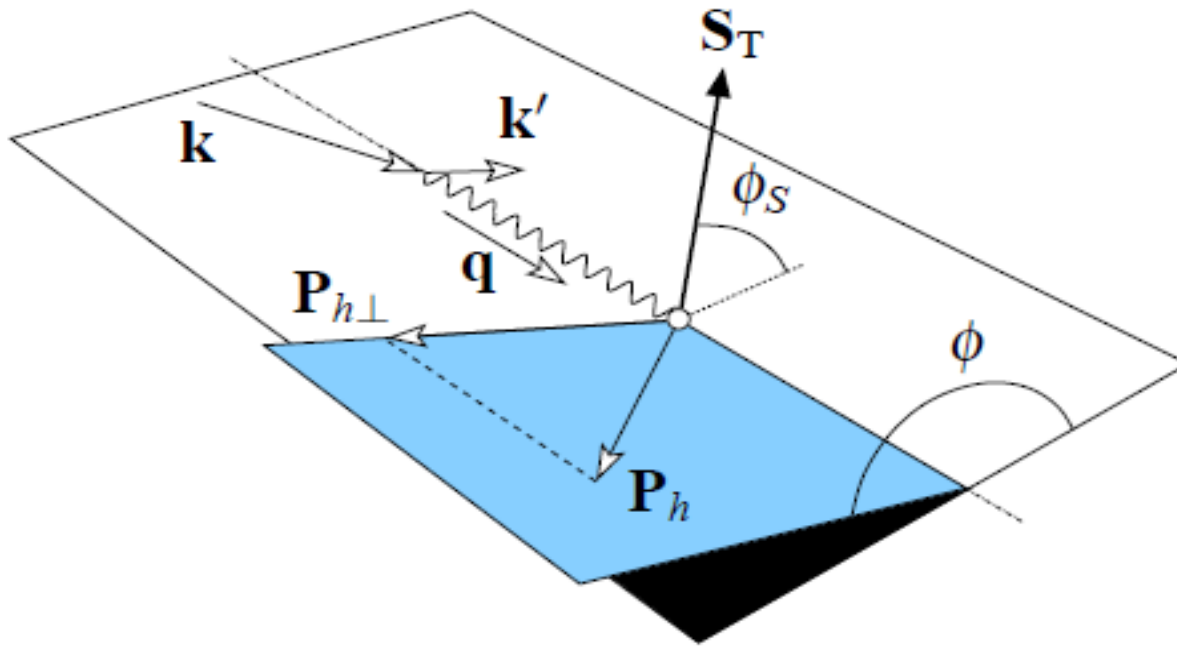
Collins function is related to twist-3 collinear PDF

Collins asymmetries in $e^+e^- \rightarrow hh+X$



The Collins asymmetries are proportional to $\cos(\phi_1 + \phi_2)$ or $\cos(2\phi_0)$

Collins asymmetries in SIDIS



The Collins asymmetries is proportional to $\sin(\phi+\phi_S)$

Transversity and Collins FF Kang-Prokudin-Sun-Yuan 2014

Parametrizations: $h_1^q(x, Q_0) \propto N_q^h x^{a_q} (1-x)^{b_q} \frac{1}{2} (f_1(x, Q_0) + g_1(x, Q_0))$

Transversity

Favoured and unfavoured Collins FF

$$\hat{H}_{fav}^{(3)}(z, Q_0) = N_u^c z^{\alpha_u} (1-z)^{\beta_u} D_{\pi^+/u}(z, Q_0)$$

$$\hat{H}_{unf}^{(3)}(z, Q_0) = N_d^c z^{\alpha_d} (1-z)^{\beta_d} D_{\pi^+/d}(z, Q_0)$$

Total 13 parameters: $N_u^h, N_d^h, a_u, a_d, b_u, b_d, N_u^c, N_d^c, \alpha_u, \alpha_d, \beta_d, \beta_u, g_c$

SIDIS data used: HERMES, COMPASS, JLAB – 140 points

e+e- data used: BELLE, BABAR including P_T dependence – 122 points

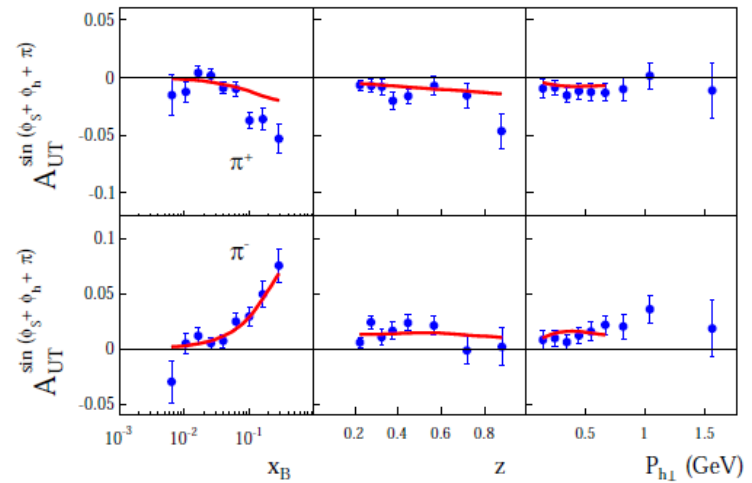
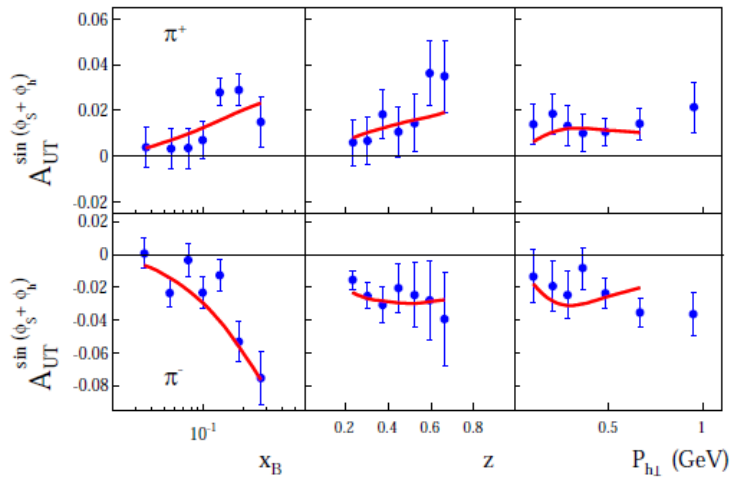
$$\chi_{min}^2 / n_{d.o.f} = 0.89$$

Transversity and Collins FF Kang-Prokudin-Sun-Yuan 2014

$$\ell P \rightarrow \pi^\pm X$$

HERMES

COMPASS



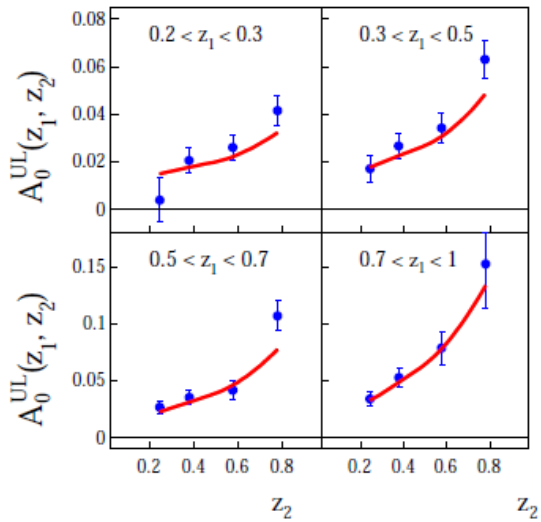
$$1 \lesssim \langle Q^2 \rangle \lesssim 6 \text{ GeV}^2$$

$$1 \lesssim \langle Q^2 \rangle \lesssim 21 \text{ GeV}^2$$

Transversity and Collins FF Kang-Prokudin-Sun-Yuan 2014

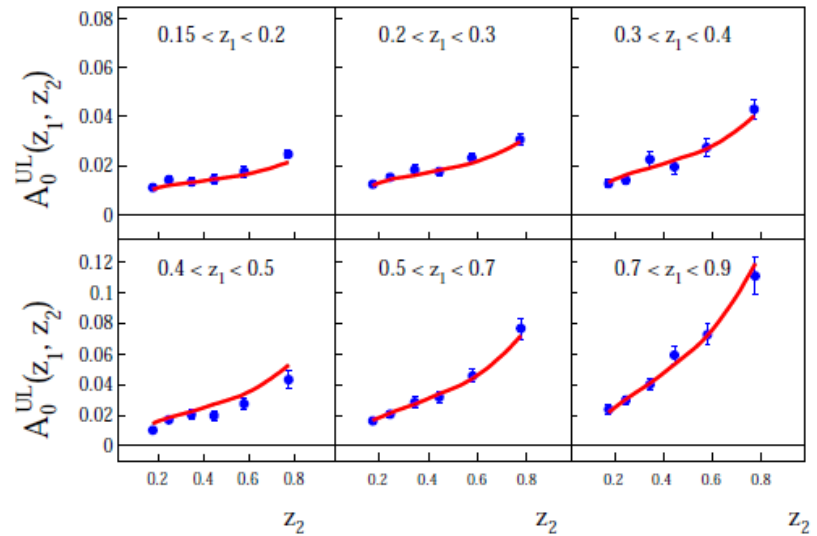
$$e^+e^- \rightarrow \pi\pi X$$

BELLE



$$Q^2 = 110 \text{ GeV}^2$$

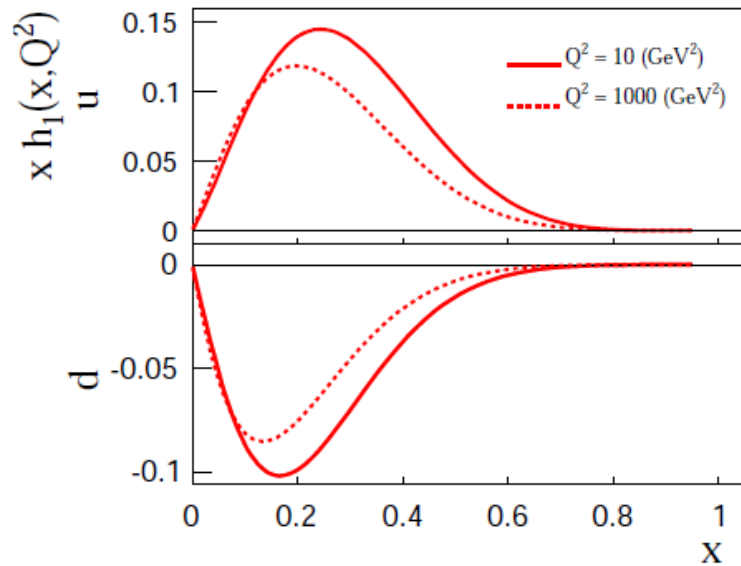
BABAR



$$Q^2 = 110 \text{ GeV}^2$$

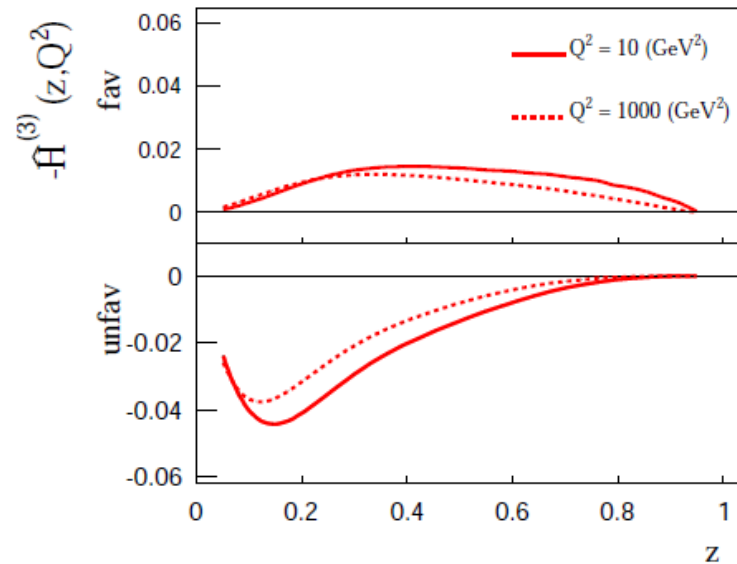
Transversity and Collins FF Kang-Prokudin-Sun-Yuan 2014

Transversity



Positive u and negative d transversity

Collins



Positive favoured and negative unfavoured Collins FF

Compatible with LO extraction [Anselmino et al 2009](#)

Transversity and Collins FF

Kang-Prokudin-Sun-Yuan 2014

What are evolution effects?

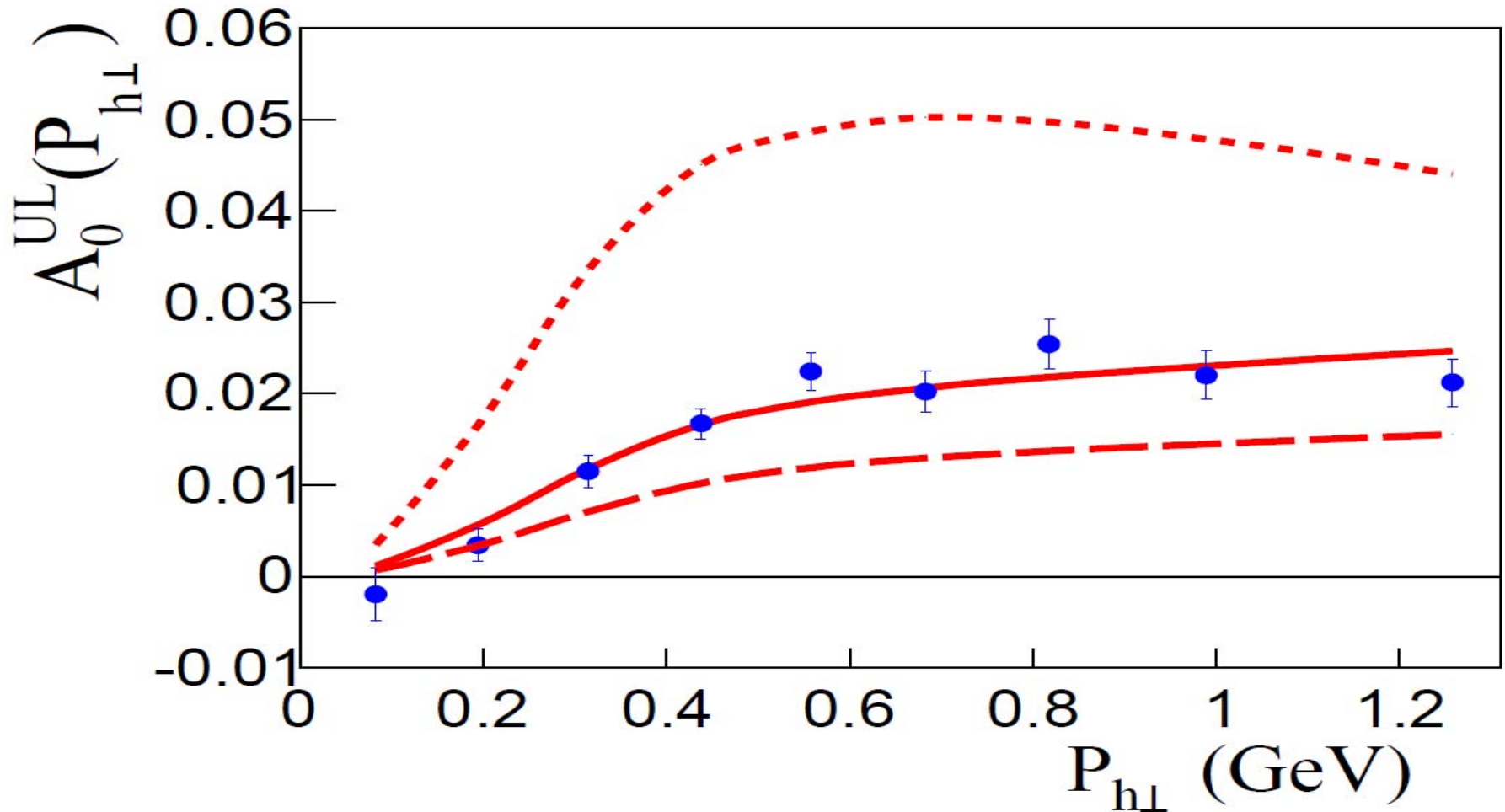
$$e^+e^- \rightarrow \pi\pi X$$

No evolution:

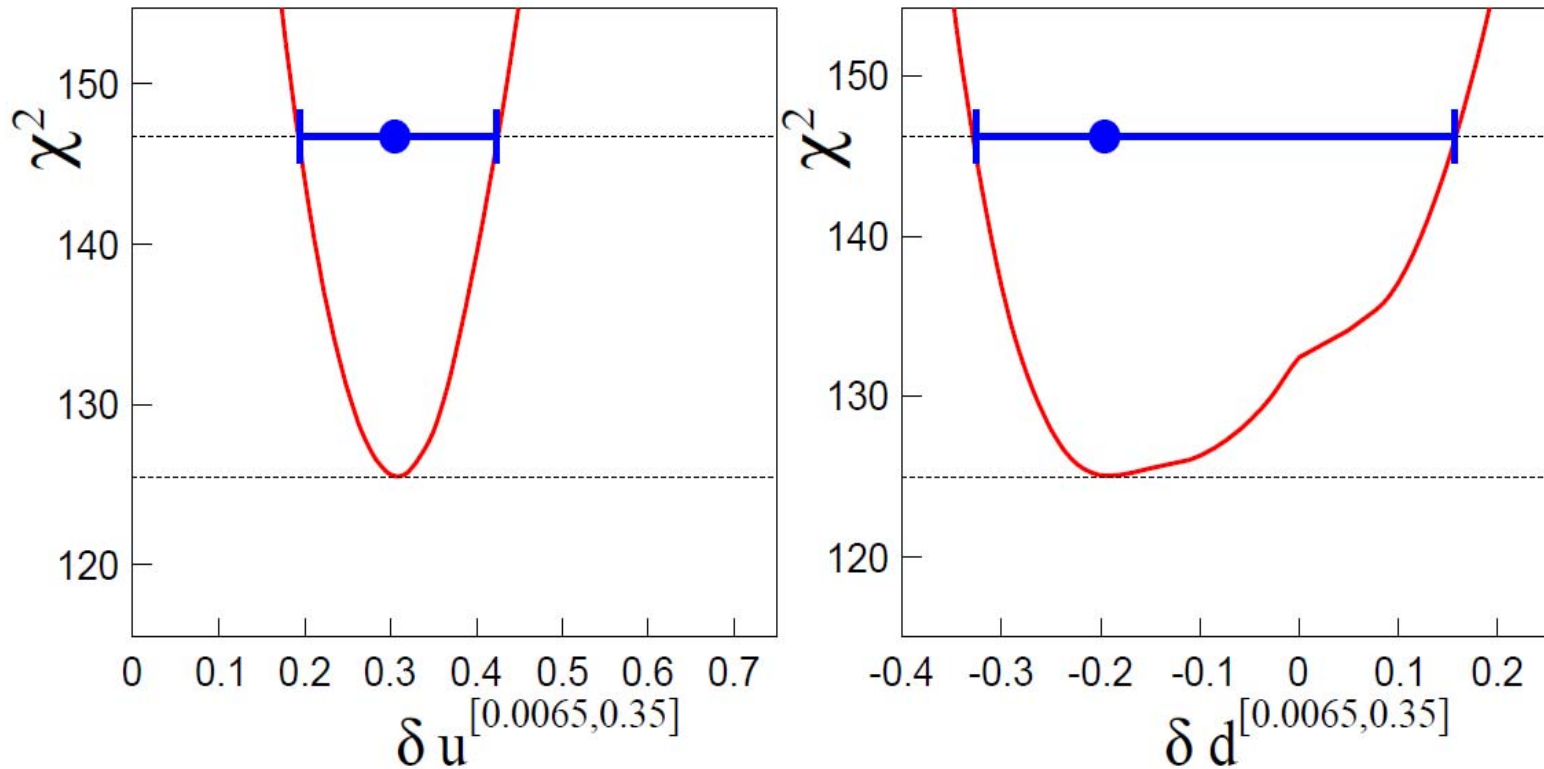
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$$Q^2 = 2.4 \text{ GeV}^2$$

No A_2



Scan δq for transversity



$$\delta u^{[0.0065, 0.35]} = +0.30_{-0.11}^{+0.12}$$

$$\delta d^{[0.0065, 0.35]} = -0.20_{-0.13}^{+0.35}$$

$$\delta q^{[x_{\min}, x_{\max}]}(Q^2) \equiv \int_{x_{\min}}^{x_{\max}} dx h_1^q(x, Q^2) \quad \text{at } Q^2 = 10 \text{ GeV}^2$$

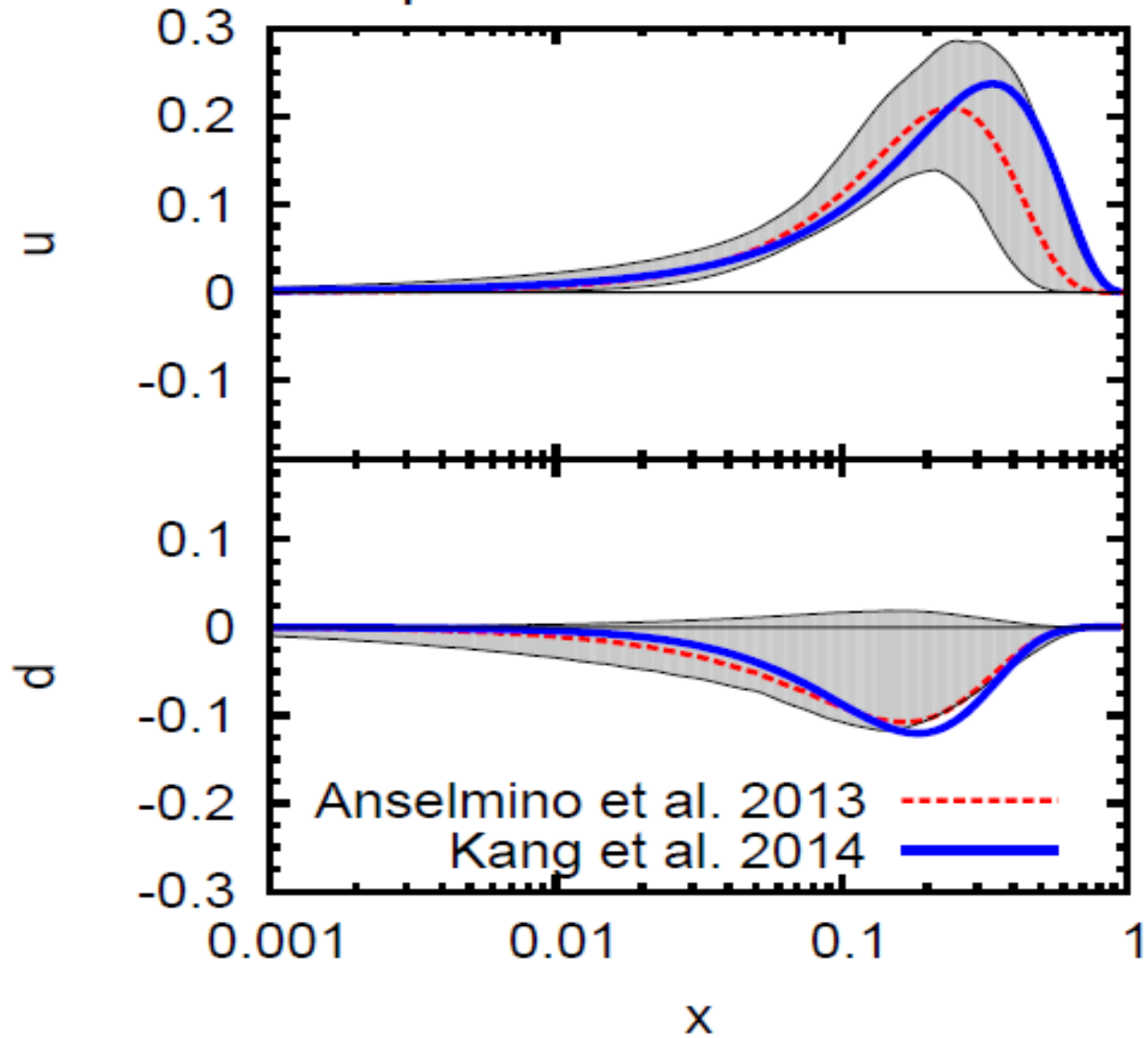
Summary

- TMD evolution is studied for the Collins effects in e^+e^- annihilation and SIDIS
- Collins functions and transversity are fitted from the existing data at BELLE , BABAR, JLAB, COMPASS and HERMES with CSS resummation scheme



Thank you very much!

$x h_1(x) \quad Q^2=2.4 \text{ GeV}^2$



Approaches to TMD evolution

Collins-Soper-Sterman (CSS) resummation framework

Collins-Soper-Sterman 1985
ResBos: C.P. Yuan, P. Nadolsky
Qiu-Zhang 1999, Vogelsang etc...
Kang-Xiao-Yuan 2011, Sun-Yuan 2013
Prokudin-Kang-Sun-Yuan 2014

“New” Collins approach

Collins 2011
Aybat-Rogers 2011,
Aybat-Collins-Rogers-Qiu, 2012
Aybat-Prokudin-Rogers 2012
Anselmino-Boglione-Melis 2012
Prokudin-Bacchetta 2013
Echevarria-Idilbi-Kang-Vitev 2014

Soft Collinear Effective Theory (SCET)

Echevarria-Idilbi-Schafer-Scimemi 2012
D'Alesio-Echevarria-Melis-Scimemi 2014