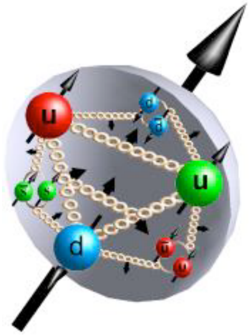
Quark Angular Momentum in a Spectator Model



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Oct. 24th @ *SPIN2014*, Beijing, China

Proton "Spin Crisis"

Experiments

$$\Delta u + \Delta d + \Delta s \approx 0.3$$

J. Ashman *et al*.(EMC), PLB(1988), NPB(1989);
E. Ageev *et al*. (COMPASS), PLB(2005),PLB(2007);
A. Airapetian *et al.* (HERMES), PRD(2007).

Where is the missing spin?

Orbital angular momentum L *?*

Gluon helicity ΔG*?*

Wigner Rotation Effect

E. P. Wigner, Annals Math. 40, 149 (1939).

Relating spinors between instant form and front form

$$\chi_T^{\uparrow} = w[(k^+ + m)\chi_F^{\uparrow} - (k^1 + ik^2)\chi_F^{\downarrow}]$$

$$\chi_T^{\downarrow} = w[(k^+ + m)\chi_F^{\downarrow} + (k^1 - ik^2)\chi_F^{\uparrow}]$$

S-wave system in the rest frame may have non-vanishing OAM in light-front formalism or IMF.

B.-Q. Ma, J. Phys. G17, L53 (1991);
B.-Q. Ma and Q.-R. Zhang, Z. Phys. C58, 479 (1993).

Angular Momentum Decomposition

Jaffe-Manohar decomposition

$$\boldsymbol{J} = \int \mathrm{d}^3x \, \psi^\dagger \tfrac{1}{2}\boldsymbol{\Sigma}\psi \;+\; \psi^\dagger(\boldsymbol{x} \times \tfrac{1}{i}\boldsymbol{\nabla})\psi$$

$$+\; \boldsymbol{E} \times \boldsymbol{A} \;+\; E^i(\boldsymbol{x} \times \boldsymbol{\nabla})A^i$$

R. Jaffe and A. Manohar, Nucl. Phys. B337, 509 (1990).

Ji decomposition

$$\boldsymbol{J} = \int \mathrm{d}^3x \, \psi^\dagger \tfrac{1}{2}\boldsymbol{\Sigma}\psi + \psi^\dagger(\boldsymbol{x} \times i\boldsymbol{D})\psi \; + \boldsymbol{x} \times (\boldsymbol{E} \times \boldsymbol{B})$$

X.-D. Ji, Phys. Rev. Lett. 78, 610 (1997).

Trans.-Longi. Decom. & GIE

Physical and pure gauge decomposition

$$\boldsymbol{A} = \boldsymbol{A}_{\text{pure}} + \boldsymbol{A}_{\text{phys}}$$

$$\boldsymbol{\nabla} \times \boldsymbol{A}_{\text{pure}} = \boldsymbol{0}, \qquad \boldsymbol{\nabla} \cdot \boldsymbol{A}_{\text{phys}} = 0$$

X.-S. Chen *et al*, Phys. Rev. Lett. 100, 232002 (2008).

Covariant version

$$A_\mu(x) = A_\mu^{\text{pure}}(x) + A_\mu^{\text{phys}}(x)$$

$$F_{\mu\nu}^{\text{pure}} = 0$$

$$D_\mu^{\text{pure}} = \partial_\mu - ig\, A_\mu^{\text{pure}}$$

Can. & Kin. Decompositions

Canonical decomposition

$$M'^{\lambda\mu\nu}_{q-spin} = \frac{1}{2}\epsilon^{\lambda\mu\nu\sigma}\,\bar{\psi}\,\gamma_\sigma\,\gamma_5\,\psi,$$

$$M'^{\lambda\mu\nu}_{q-OAM} = \bar{\psi}\,\gamma^\lambda\,(\,x^\mu\,i\,D^\nu_{pure} \;-\; x^\nu\,i\,D^\mu_{pure}\,)\,\psi,$$

$$M'^{\lambda\mu\nu}_{G-spin} = 2\,\mathrm{Tr}\,\{\,F^{\lambda\nu}\,A^\mu_{phys} \;-\; F^{\lambda\mu}\,A^\nu_{phys}\,\},$$

$$M'^{\lambda\mu\nu}_{G-OAM} = 2\,\mathrm{Tr}\,\{\,F^{\lambda\alpha}\,(\,x^\mu\,\mathcal{D}^\nu_{pure} \;-\; x^\nu\,\mathcal{D}^\mu_{pure}\,)\,A^{phys}_\alpha\,\}.$$

Kinetic (Mechanical) decomposition

$$M^{\lambda\mu\nu}_{q-spin} = M^{\lambda'\mu\nu}_{q-spin},$$

$$M^{\lambda\mu\nu}_{q-OAM} = \bar{\psi}\,\gamma^\lambda\,(\,x^\mu\,i\,D^\nu \;-\; x^\nu\,i\,D^\mu\,)\,\psi,$$

$$M^{\lambda\mu\nu}_{G-spin} = M^{\lambda'\mu\nu}_{G-spin},$$

$$M^{\lambda\mu\nu}_{g-OAM} = M^{\lambda'\mu\nu}_{G-OAM} \;+\; 2\,\mathrm{Tr}\,[\,(\,\mathcal{D}_\alpha\,F^{\alpha\lambda}\,)\,(\,x^\mu\,A^\nu_{phys} \;-\; x^\nu\,A^\mu_{phys}\,)\,]$$

E. Leader and C. Lorce, Phys. Rept. 541, 163 (2014);
M.Wakamatsu, Int. J. Mod. Phys. A29, 1430012 (2014).

Spectator Model

Proton is viewed as an active quark and a spectator.

This picture has been applied to investigate many physical observables: *structure functions, form factors, helicity, transversity, TMDs, GPDs* ... ...

Feynman, *Photon-hadron interactions* (1972);
Ma, Qing and Schmidt, PRC65, 035205(2002);PRC66, 048201(2002);
Ma, PLB375, 320(1996); Ma and Schmidt, JPG24, L71(1998); PRD58, 096008(1998);
Jakob, Mulders and Rodrigues, NPA626, 937(1997);
Bacchetta, Conti and Radici, PRD78, 074010(2008);
Hwang and Mueller, PLB660, 350(2008)
...

Light-front Wave Functions

Constrained by the quantum numbers, the spectator can be either a scalar or an axial-vector.

Scalar:

$$\frac{\bar{u}(k,s)}{\sqrt{2k^+}}\mathbf{1}\frac{u(P,\Lambda)}{\sqrt{2P^+}}\ \phi^{(s)}(x,\tilde{\boldsymbol{k}}_\perp)$$

Axial-vector:

$$\frac{\bar{u}(k,s)}{\sqrt{2k^+}}\epsilon^*_\mu(p,\lambda)\gamma^\mu\gamma_5\frac{u(P,\Lambda)}{\sqrt{2P^+}}\ \phi^{(v)}(x,\tilde{\boldsymbol{k}}_\perp)$$

$$\phi^{(s)}(x,\tilde{\boldsymbol{k}}_\perp)=\frac{g_s}{\sqrt{1-x}}x^{1-n}\left(M^2-\frac{m^2+\tilde{\boldsymbol{k}}^2_\perp}{x}-\frac{M_s^2+\tilde{\boldsymbol{k}}^2_\perp}{1-x}\right)^{-n}$$

D. Hwang and D. Mueller, Phys. Lett. B660, 350(2008); T. Liu, arXiv:1406.7709.

Electromagnetic Form Factors

Dirac and Pauli form factors

$$\left\langle P', \uparrow \left| \frac{J^+(0)}{2P^+} \right| P, \uparrow \right\rangle = F_1(Q^2)$$

$$\left\langle P', \uparrow \left| \frac{J^+(0)}{2P^+} \right| P, \downarrow \right\rangle = -\frac{\Delta^1 - i\Delta^2}{2M} F_2(Q^2)$$

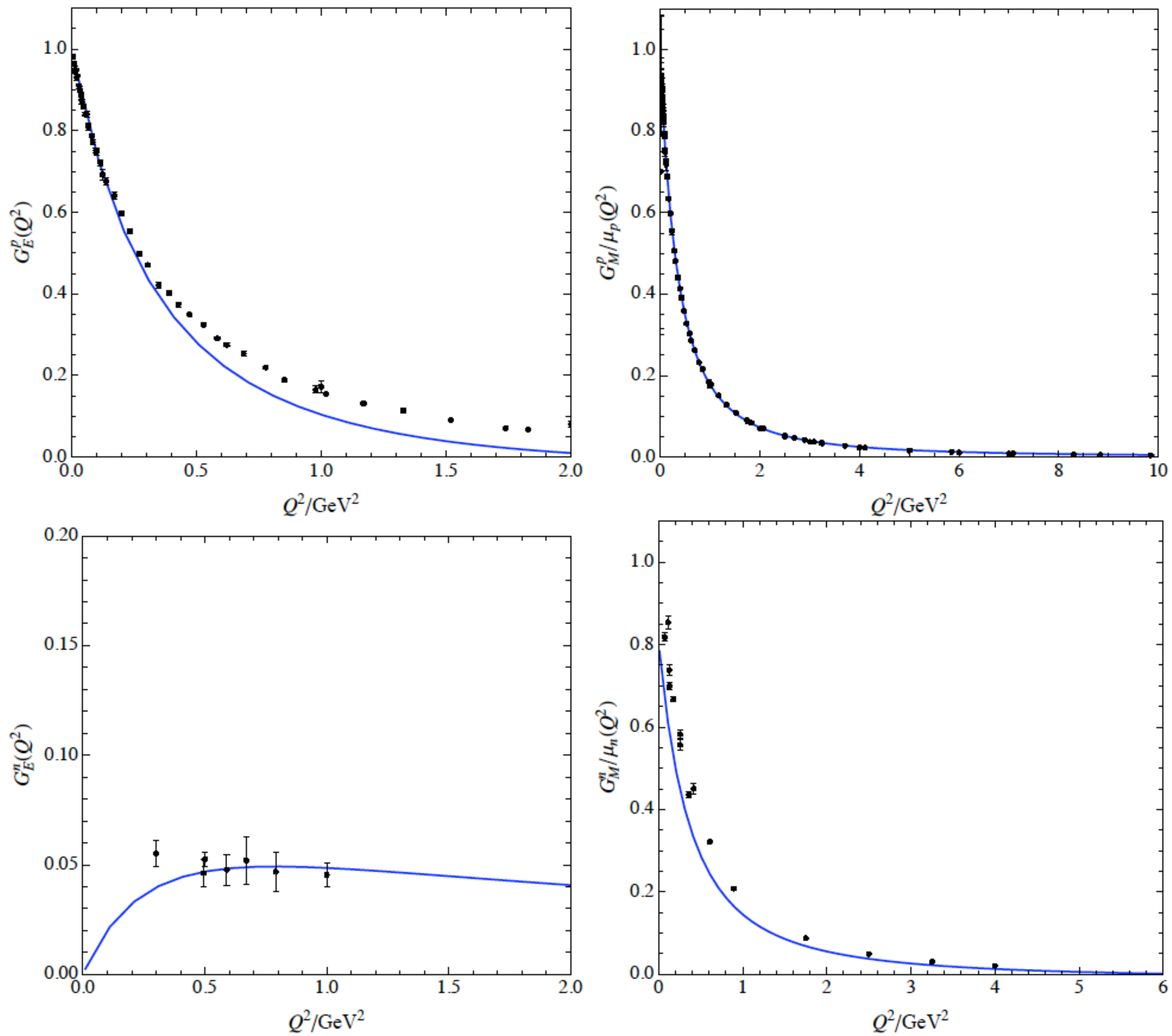
S. J. Brodsky and S. D. Drell, Phys. Rev. D22, 2236(1980).

Sachs form factors

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2),$$

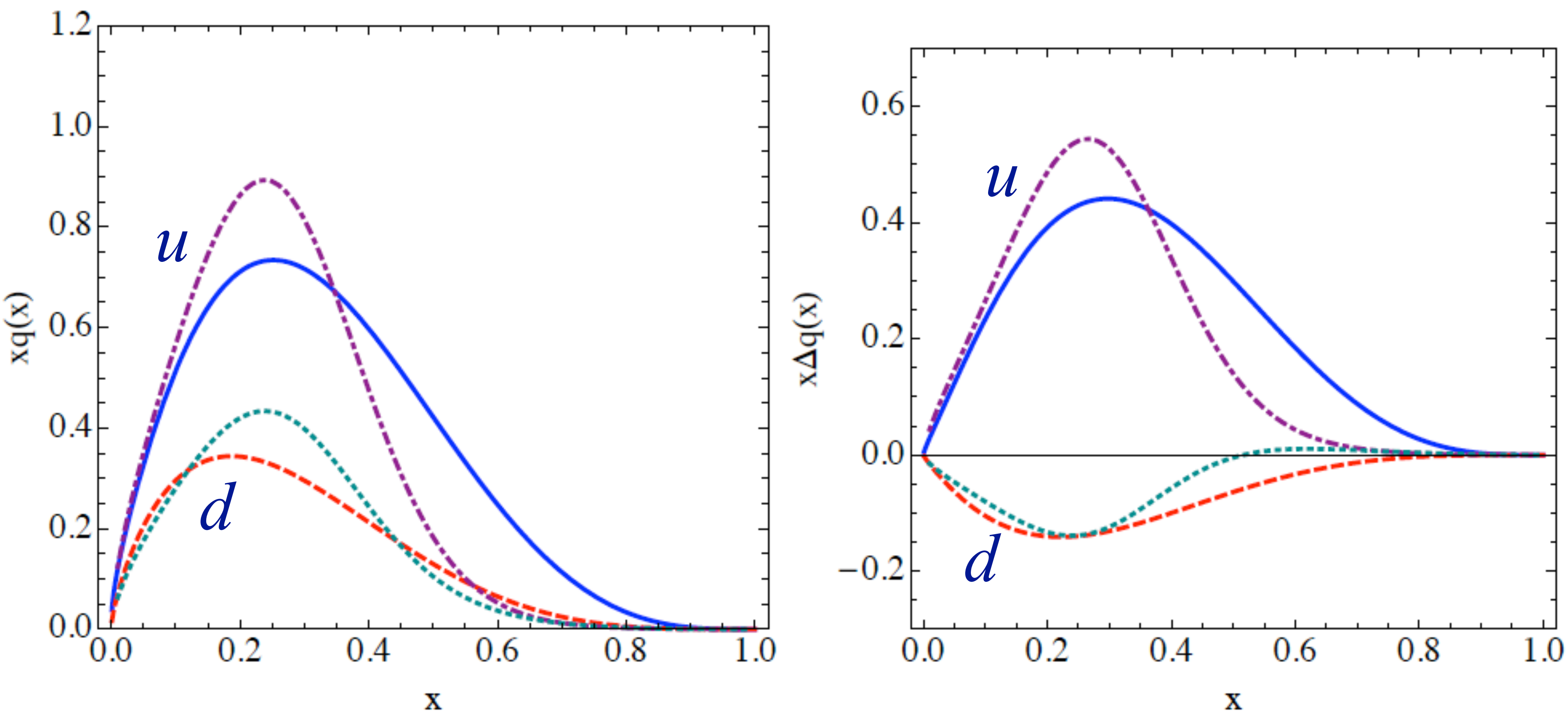
$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2).$$

EM FFs: Model Results



T. Liu, arXiv:1406.7709.

PDF & Helicity Distri.



T. Liu, arXiv:1406.7709;
A.D. Martin et al., EPJC4, 463(1998);EPJC14, 133(2000);
E. Leader et al., PRD75, 074027(2007).

Wigner Distributions

Wigner operator

E. P. Wigner, Phys. Rev. 40, 149(1932).

X. Ji, Phys. Rev. Lett. 91, 062001(2003).

$$\hat{W}^{[\Gamma]}(x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp) = \frac{1}{2}\int \frac{dz^- d^2\boldsymbol{z}_\perp}{(2\pi)^3} e^{ik\cdot z}\bar{\psi}(y-\frac{z}{2})\Gamma\boldsymbol{\mathcal{L}}[y-\frac{z}{2}, y+\frac{z}{2}]\psi(y+\frac{z}{2})|_{z^+=0}$$

C. Lorce and B. Pasquini, Phys. Rev. D84, 014015(2011).

Wilson line

$$(0, \frac{z^-}{2}, \boldsymbol{b}_\perp - \frac{\boldsymbol{z}_\perp}{2}) \to (0, \infty, \boldsymbol{b}_\perp - \frac{\boldsymbol{z}_\perp}{2}) \to (0, \infty, \boldsymbol{b}_\perp + \frac{\boldsymbol{z}_\perp}{2}) \to (0, \frac{z^-}{2}, \boldsymbol{b}_\perp + \frac{\boldsymbol{z}_\perp}{2})$$

S. Meißner, A. Metz and M. Schlegel, JHEP0908, 056(2009).

Wigner distribution

$$\rho^{[\Gamma]}(x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp, \boldsymbol{S}) = \int \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2} \langle P', \boldsymbol{S}|\, \hat{W}^{[\Gamma]}(x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp)\, |P, \boldsymbol{S}\rangle$$

Some Model Relations

T. Liu, arXiv:1406.7709.

Scalar

$$\rho_{\mathrm{UL}}^{(s)}(x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp) = -\rho_{\mathrm{LU}}^{(s)}(x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp),$$
$$\rho_{\mathrm{UT}}^{j\,(s)}(x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp) = \rho_{\mathrm{TU}}^{j\,(s)}(x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp),$$
$$\rho_{\mathrm{LT}}^{j\,(s)}(x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp) = -\rho_{\mathrm{TL}}^{j\,(s)}(x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp),$$
$$\rho_{\mathrm{TT}}^{(s)}(x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp) = \rho_{\mathrm{UU}}^{(s)}(x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp) + \rho_{\mathrm{LL}}^{(s)}(x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp).$$

Axial-vector

$$\rho_{\mathrm{UL}}^{(v)}(x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp) = \rho_{\mathrm{LU}}^{(v)}(x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp)$$

if only including transverse polarizations

OAM & Spin-Orbit Correlation

Relations to Wigner distributions

$$\ell_z(x) = \int d^2\boldsymbol{b}_\perp d^2k_\perp \boldsymbol{b}_\perp \times \boldsymbol{k}_\perp \rho_{\rm LU}(x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp),$$

$$\mathcal{C}_z(x) = \int d^2\boldsymbol{b}_\perp d^2k_\perp \boldsymbol{b}_\perp \times \boldsymbol{k}_\perp \rho_{\rm UL}(x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp).$$

C. Lorce *et al.*, Phys. Rev. D85, 114006(2012).

Model results

$$\ell_z^{(s)}(x) = \frac{g_s^2(1-x)^4}{48\pi^2[xM_s^2 + (1-x)m^2 - x(1-x)M^2]^2},$$

$$\ell_z^{(v)}(x) = -\frac{g_v^2(1-x)^2[(1-x^2) - \frac{1}{2M_v^2}(1-x)^2(m+M)^2]}{72\pi^2[xM_v^2 + (1-x)m^2 - x(1-x)M^2]^2},$$

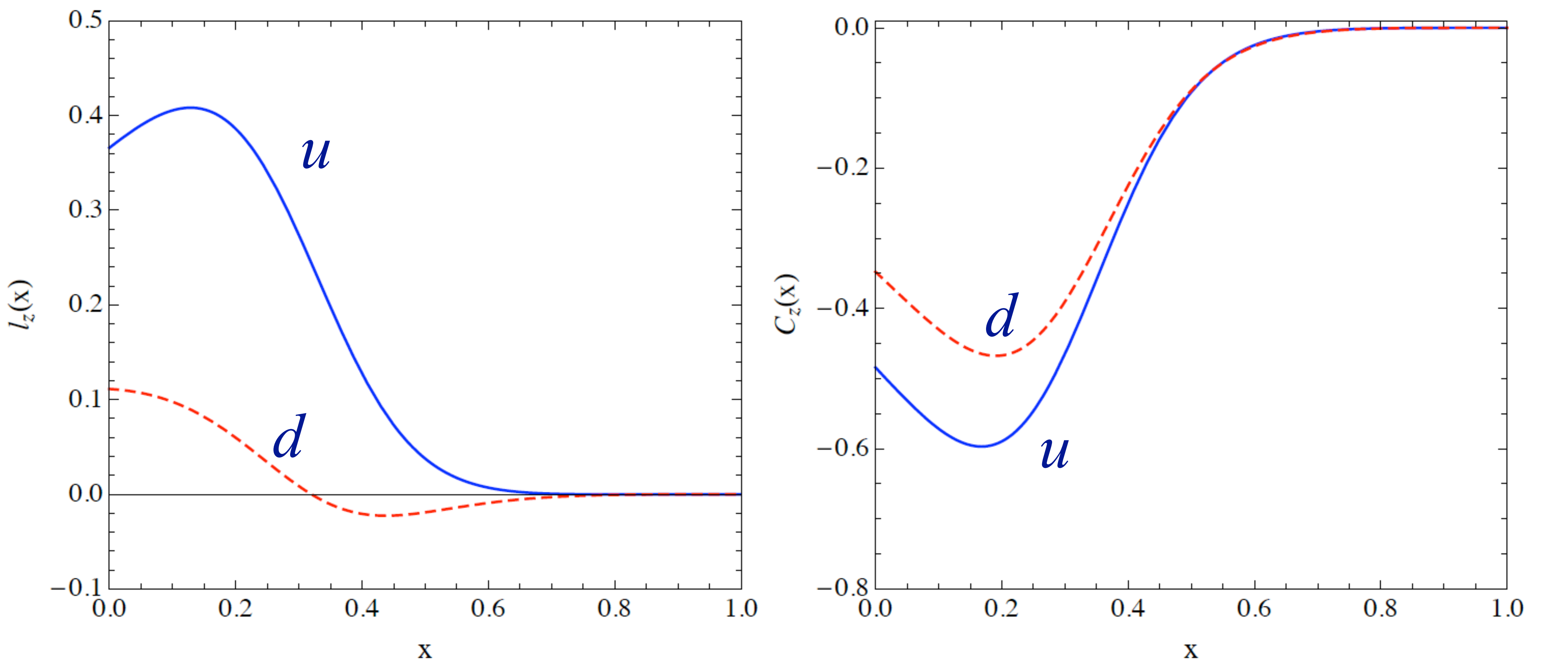
$$\mathcal{C}_z^{(s)}(x) = -\frac{g_s^2(1-x)^4}{48\pi^2[xM_s^2 + (1-x)m^2 - x(1-x)M^2]^2},$$

$$\mathcal{C}_z^{(v)}(x) = -\frac{g_v^2(1-x)^2[(1-x^2) + \frac{1}{2M_v^2}(1-x)^2(m+M)^2]}{72\pi^2[xM_v^2 + (1-x)m^2 - x(1-x)M^2]^2}.$$

T. Liu, arXiv:1406.7709.

$\ell_z^{(s)}(x) = -\mathcal{C}_z^{(s)}(x)$ *is also found in* K. Kanazawa *et al.*,PRD90, 014028(2014).

Numerical Results: OAM & Corr.



T. Liu, arXiv:1406.7709.

Mixing Distributions

Mixing distribution

$$\tilde{\rho}(x, b_x, k_y) = \int db_y dk_x \rho(x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp)$$

Probability interpretations.

Correlations between transverse momentum and transverse coordinate along two orthogonal directions reflect orbital motions.

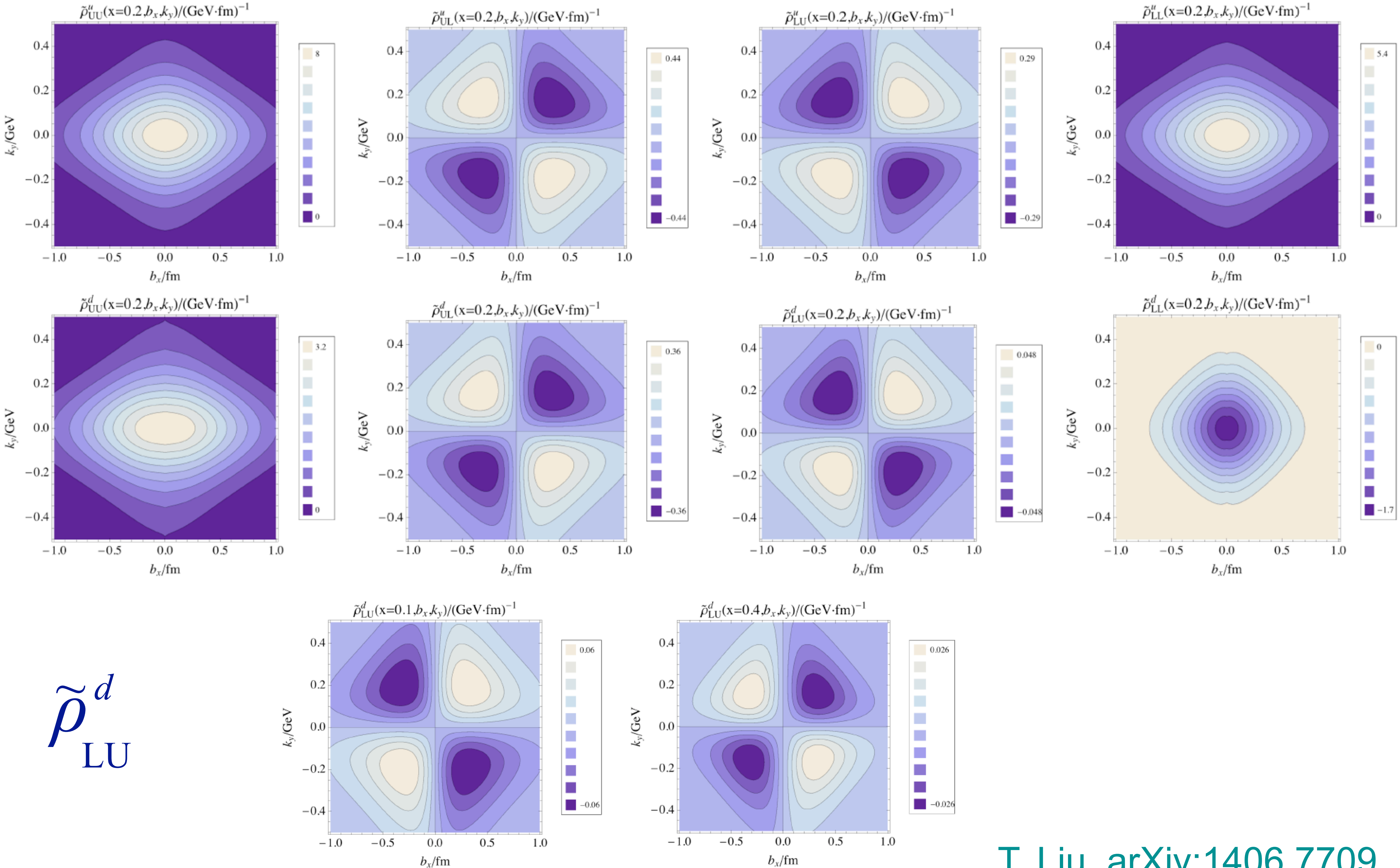
Mixing Distri.: Model Results

UU UL LU LL

u

d

$\tilde{\rho}^{d}_{\text{LU}}$



T. Liu, arXiv:1406.7709.

Gravitational Form Factors

Gravitational form factors

$$\left\langle P+q,\uparrow \left| T^{++}(0) \right| P,\uparrow \right\rangle = 2(P^+)^2 A(Q^2)$$

$$\left\langle P+q,\uparrow \left| T^{++}(0) \right| P,\downarrow \right\rangle = 2(P^+)^2 \frac{-(q^1 - iq^2)}{2m} B(Q^2)$$

Brodsky, Hwang, Ma and Schmidt, NPB593, 311(2001).

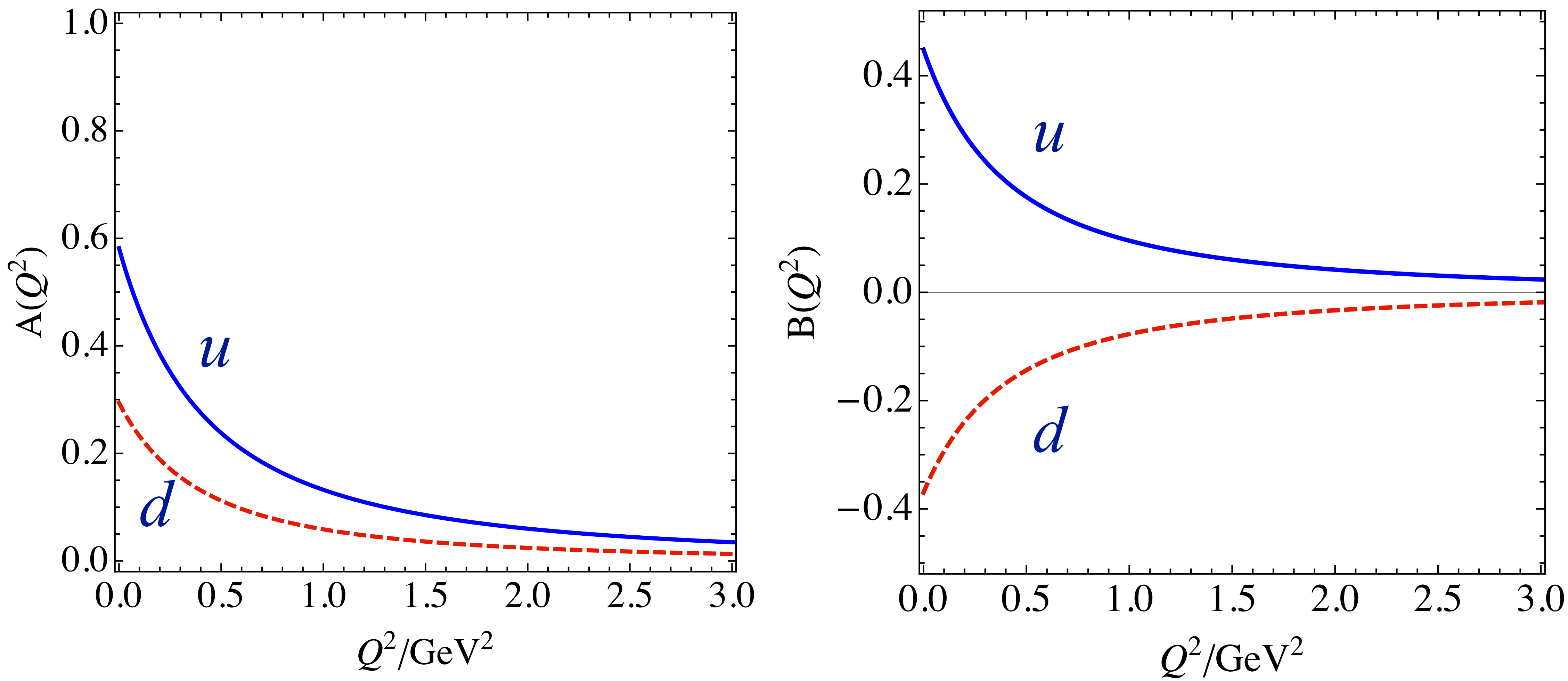
Ji's relation

$$J = \frac{1}{2}\left[A(0) + B(0)\right]$$

Kinetic angular momentum

X.-D. Ji, Phys. Rev. Lett. 78, 610 (1997).

Grav. FFs: Model Results



T. Liu and B.-Q. Ma, Phys. Rev. C89, 055202(2014).

OAM & Grav. FFs

Spin

$$S^u = 0.508 \qquad S^d = \text{-}0.135$$

OAM

$$L^u = 0.143 \qquad L^d = 0.017$$

Gravitational form factors

$$\frac{1}{2}[A^u(0) + B^u(0)] = 0.515$$

$$\frac{1}{2}[A^d(0) + B^d(0)] = \text{-}0.076$$

Summary

Due to the Wigner rotation effect, an S-wave system in rest frame may have non-vanishing orbital angular momentum in light-front formalism.

We investigated Wigner distributions in a spectator model and find some model relations.

Through Wigner distributions, we calculated quark mixing distributions where a sign change is observed for d quark orbital angular momentum at large x region.

We calculated the gravitational form factors which are usually related to the kinetic angular momentums, and found that even in no gauge field models we cannot identify the canonical angular momentums with the sum of gravitational form factors.

Thanks!