Fermionic Symmetry Protected Topological Phase Induced by Interaction

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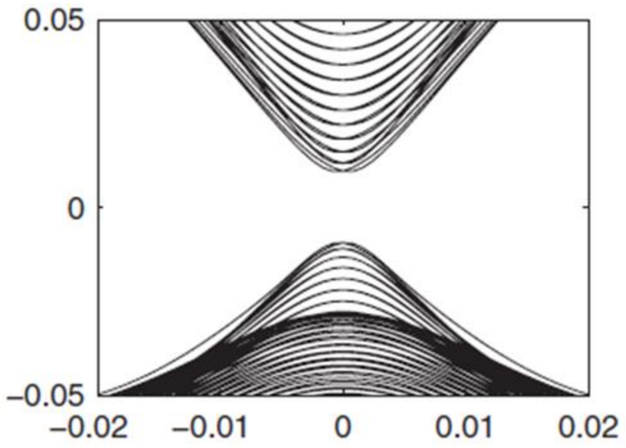
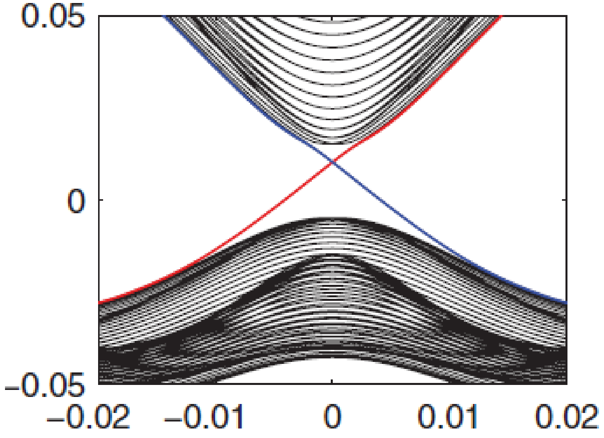
Outline

1. Background and motivation
2. Study of a specific model
3. Conclusion

Background and Motivation



Band insulator



2D topological insulator

Key properties of TI:

- ■Bulk gap
- ■Gapless edge states protected by time reversal symmetry and U(1)
- ■No symmetry breaking

Mass terms

$B(\psi_\uparrow^+\psi_\downarrow + \psi_\downarrow^+\psi_\uparrow)$ breaks ***T***

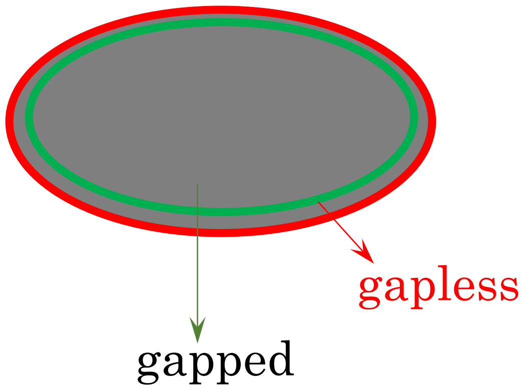
$\Delta(\psi_\uparrow^+\psi_\downarrow^+ + \psi_\downarrow\psi_\uparrow)$ breaks ***U(1)***

X. Qi, S.C. Zhang RevModPhys.83.1057

Symmetry Protected Topological phase (SPT phase)

Key points of SPT

- No symmetry breaking
- bulk is gapped
- boundary excitation is gapless
- protected by symmetry



Classification of SPT

- Free fermion systems are classified by K-theory

q	$\pi_0(R_q)$	$d=1$	$d=2$	$d=3$
0	$\mathbb{Z}$		no symmetry (p_x+ip_y, e.g., SrRu)	T only (^{3}He-B)
1	$\mathbb{Z}_2$	no symmetry (Majorana chain)	T only ($(p_x+ip_y)\uparrow+(p_x-ip_y)\downarrow$)	T and Q (BiSb)
2	$\mathbb{Z}_2$	T only ($(TMTSF)_2X$)	T and Q (HgTe)	
3	0	T and Q		
4	$\mathbb{Z}$			
5	0			
6	0			
7	0			no symmetry

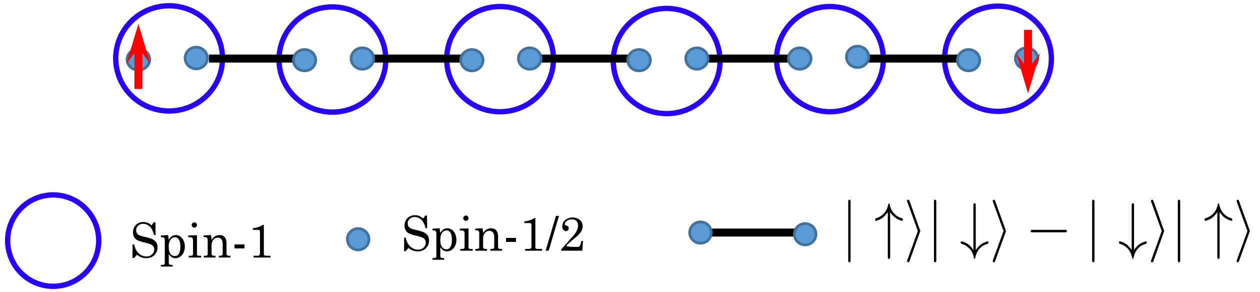
A. Kitaev, arXiv:0901.2686

For more symmetry group in Xiao-Gang Wen PhysRevB.85.085103

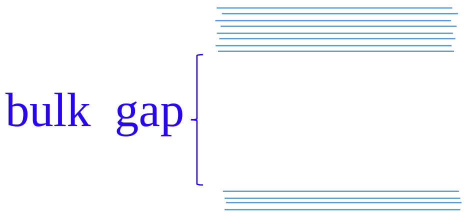
■ SPT phases also exist in Bosonic systems

- *S=1* Haldane phase (Bosonic) *SO(3)* symmetry

$$H = \sum_i J_1 \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1}$$



  - Finite excitation gap
  - No symmetry breaking
  - *SO(3)* symmetry protected spin-1/2 edge states
  - Degeneracy of entanglement spectrum



Haldane 83; Affleck, Kennedy, Lieb, Tasaki 87
Gu, Wen 09; Pollmann et.al 09; Chen,Gu,Wen 11

■ entanglement spectrum for 1D

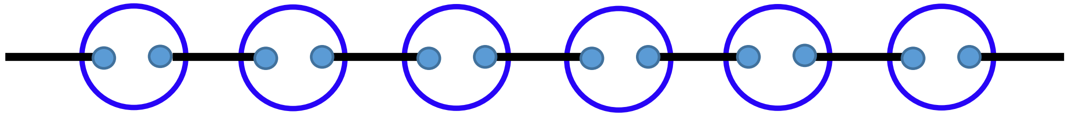
$\rho_A = \mathrm{Tr}_\mathrm{B}\,(\rho)$, ρ density matrix of the ground state

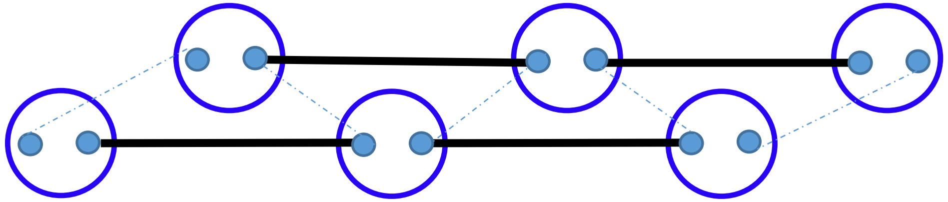
Entanglement spectrum: eigenvalues of ρ_A

Entanglement spectrum

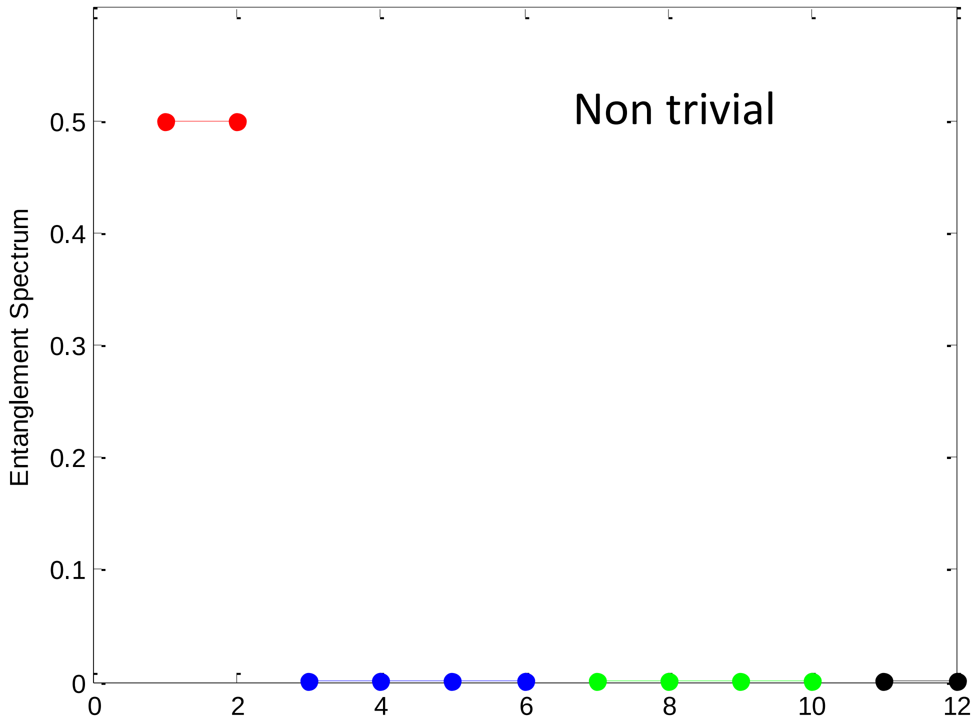
$$H = \sum_i J_1 S_i \cdot S_{i+1} + J_2 S_i \cdot S_{i+2}$$

SO(3) symmetry



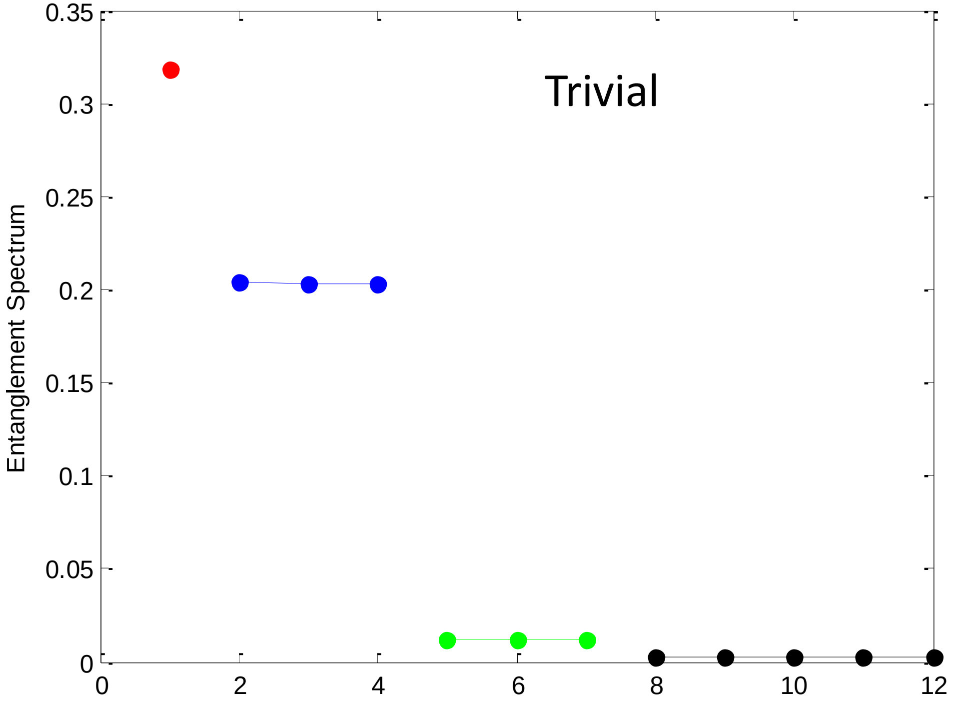


J_2/J_1=0.3



Entanglement spectrum

J_2/J_1=0.9



Entanglement spectrum

Classification of Bosonic SPT

- 1D by the projective representation

SO(3) : 2 projective representation

$$H = \sum_i J_1 \mathbf{S}_i \cdot \mathbf{S}_{i+1} + J_2 \mathbf{S}_i \cdot \mathbf{S}_{i+2}$$

: 2 phases

SO(3) symmetry

Xie Chen, Zheng-Cheng Gu, and Xiao-Gang Wen,2011

- 2D or higher by group cohomology

Chen, Gu, **ZXL**, Wen, Science, 338, 1604 (2012);
Chen, Gu, **ZXL**, Wen, PRB, 87, 155114 (2013)

Classification of Interacting Fermionic SPT

- 1D by projective representation
- 2D or higher partially by super-cohomology
- effect of interaction

1D superconductor

Symmetry	Free classification	With interactions
$U(1) \times Z_2^T$	$\mathbb{Z}$	$\mathbb{Z}_4$
$Z_n \times Z_2^T$ (n even)	$\mathbb{Z}$	$\mathbb{Z}_4$
$Z_n \times Z_2^T$ (n odd and $n > 1$)	$\mathbb{Z}$	$\mathbb{Z}_2$

2D superconductor

Time reversal and mirror reflect	Z	Z_8

It seems that interactions reduce the classification. However, it maybe not true!

Evelyn Tang and Xiao-Gang Wen arXiv:1204.0520v2 Hong Yao and Shinsei Ryu, arXiv :1202.5805

Model

$$H = H_0 + H_U + H_J$$

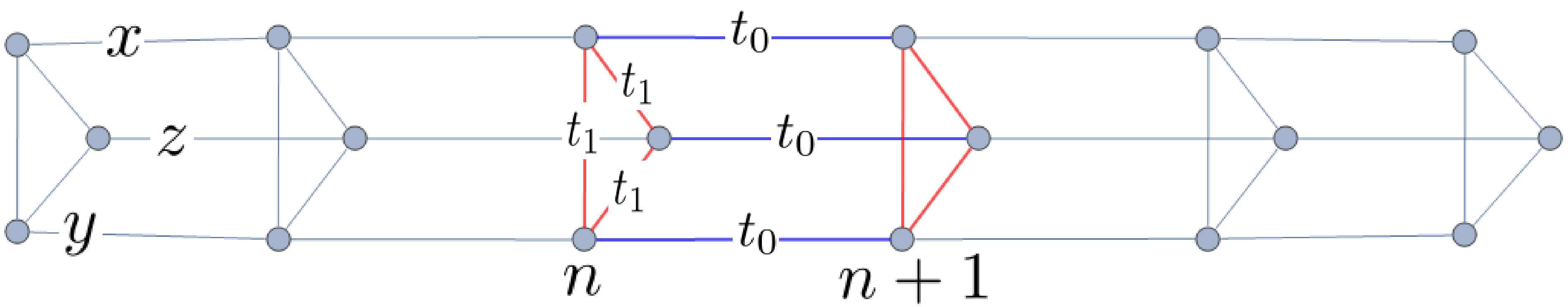
$$H_0 = -\{\sum_{i,\alpha} t_0(c^\dagger_{i,\alpha} c_{i+1,\alpha}) + \sum_{i,\{\alpha,\hat{\alpha}\}} t_1 c^\dagger_{i,\alpha} c_{i,\hat{\alpha}} + h.c.\} + \lambda N_i$$

$$N_i = n_{i,x} + n_{i,y} + n_{i,z}$$

$$S^\alpha = \sum_{\beta,\gamma} i\varepsilon^{\alpha\beta\gamma} c^\dagger_\beta c_\gamma$$

$$H_U = U \sum_i (N_i - 1)^2 \qquad H_J = J \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+1}$$

$$\alpha = x, y, z, \{\alpha, \hat{\alpha}\} = \{x, y\}, \{y, z\}, \{z, x\}$$



Symmetry and classification

■The model : U(1) and time reversal symmetry T

$$U_\theta c_\alpha U_\theta^{-1} = c_\alpha e^{i\theta}$$
$$T c_\alpha T^{-1} = -c_\alpha . \quad T^2 = 1$$

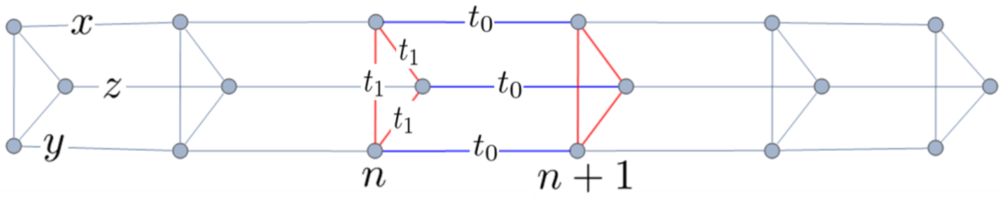
q	$\pi_0(R_q)$	$d = 1$
0	$\mathbb{Z}$	
1	$\mathbb{Z}_2$	no symmetry (Majorana chain)
2	$\mathbb{Z}_2$	T only ($(TMTSF)_2X$)
3	0	T and Q
4	$\mathbb{Z}$	
5	0	
6	0	
7	0	

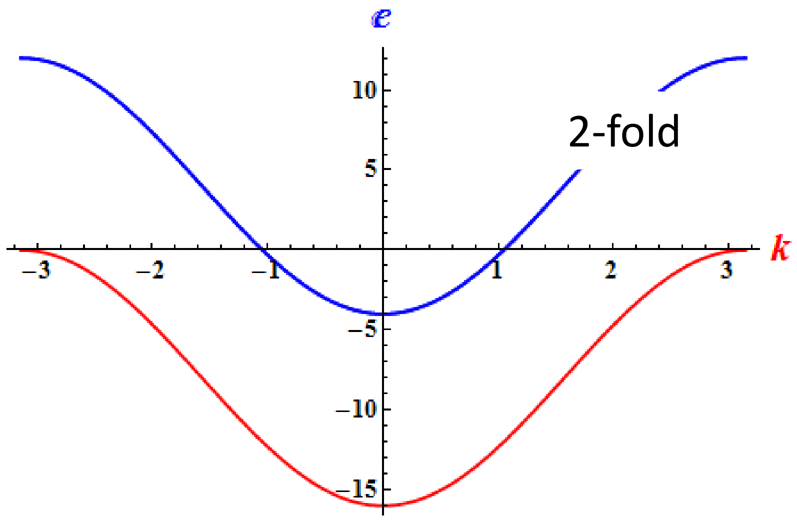
■Classification

free case, only the trivial phase.

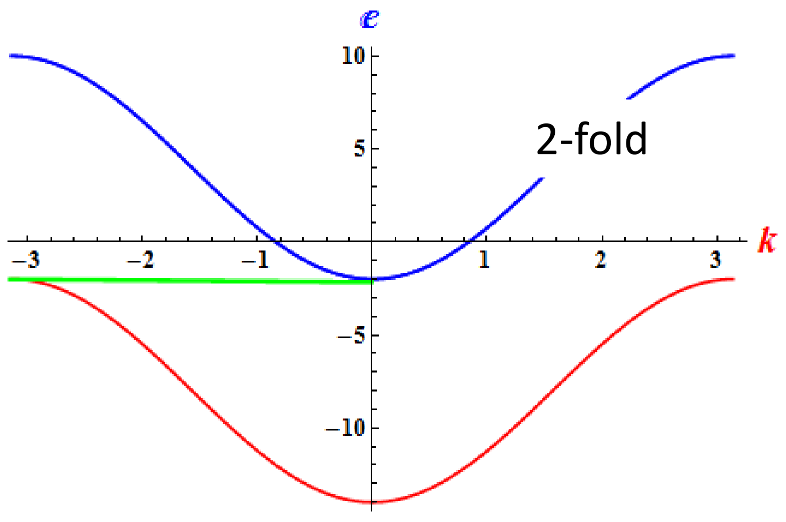
2 projective representation → two SPT phases.

Non-interacting limit U=0, J=0

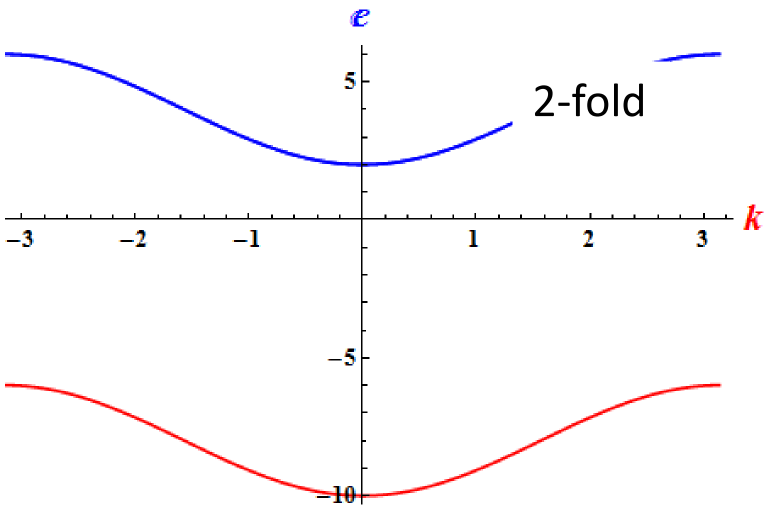




$t_1 < \frac{8}{3} t_0$

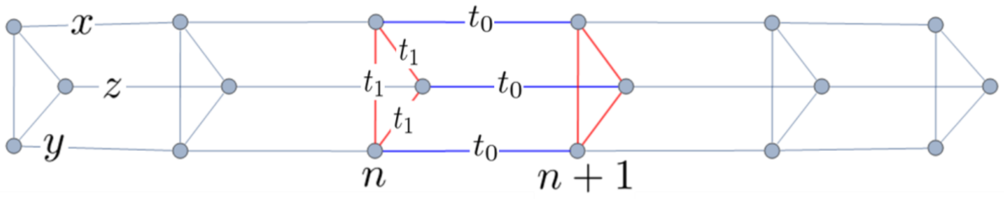


$t_1 = \frac{8}{3} t_0$

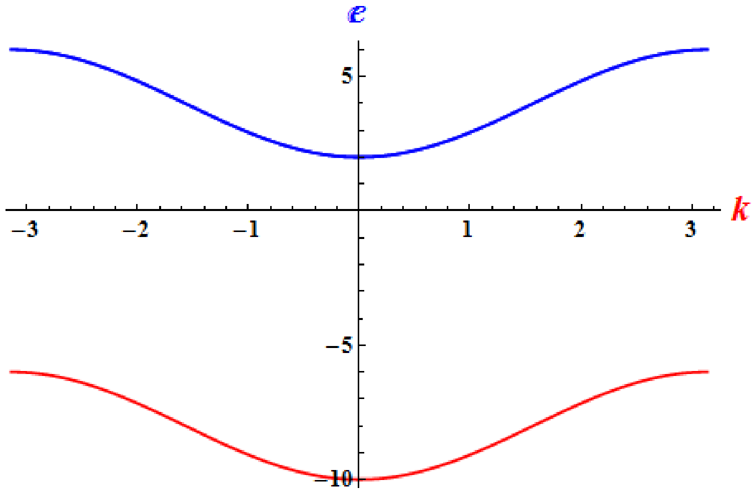


$t_1 > \frac{8}{3} t_0$

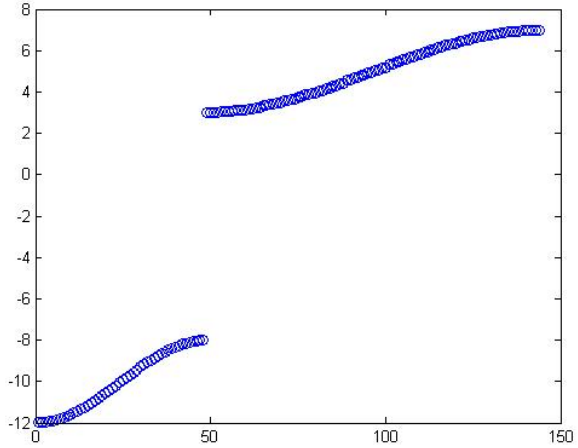
Non-interacting limit U=0, J=0



SPT only focus on gapped phases. $In\ \frac{1}{3}\,filling\ case$



$$t_1 > \frac{8}{3}t_0$$

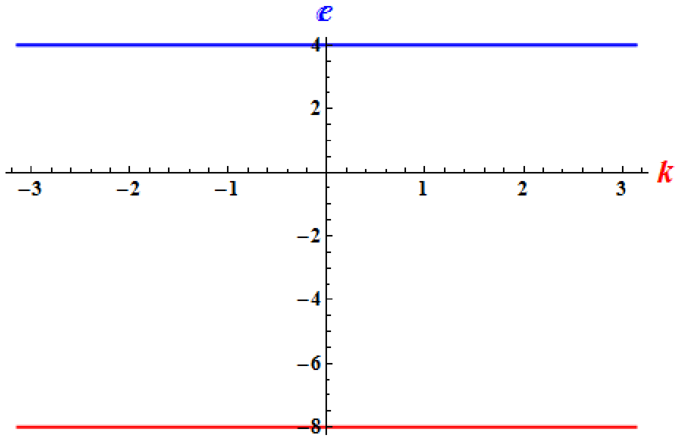


Open boundary, no zero mode, indicating the trivialness of phase

$$In\ region\ of\ t_1 > \frac{8}{3}t_0,\ only\ trivial\ phase,\ i.e\ band\ insulator$$

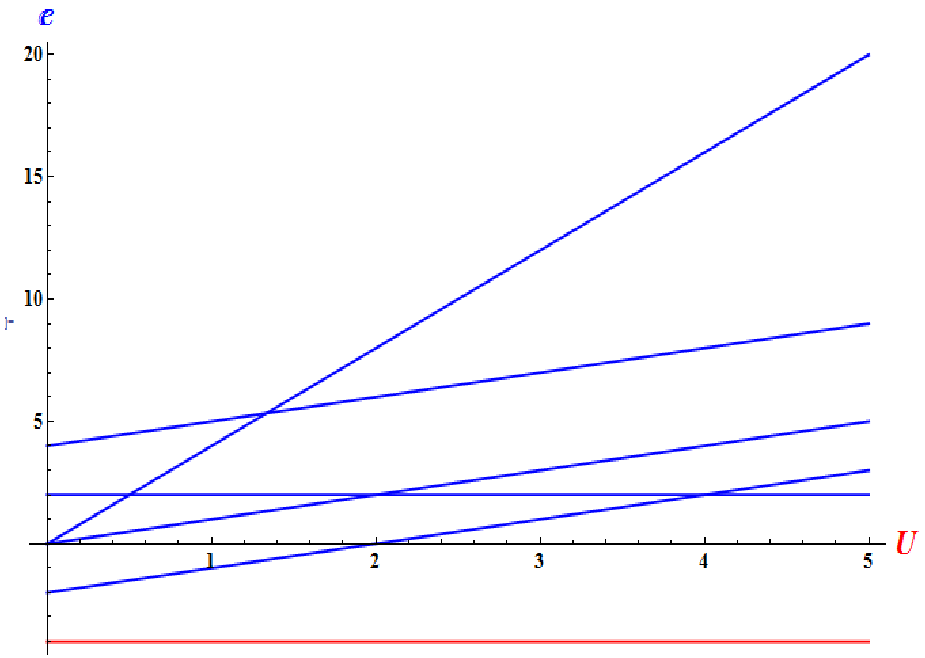
Interacting case $t_1 \gg t_0$

Add Hubbard interaction $H_u = U \sum_i (N_i - 1)^2$



$\frac{t_1}{t_0} \to \infty, U = 0$

the charge are *localized* *at each rung in ground state.*



the band insulator and the Mott insulator are *adiabatically connected*

Interacting case

■ Add Heisenberg-like interaction $H_J = J\sum_i \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1}$,

■ This term competes to exhibit the Haldane phase，
so in the limit $J \to \infty$, the system would fall into Haldane phase.

Interacting case

t_0 *terms* →metal phase,

t_1 *and* U →trivial insulator phase,

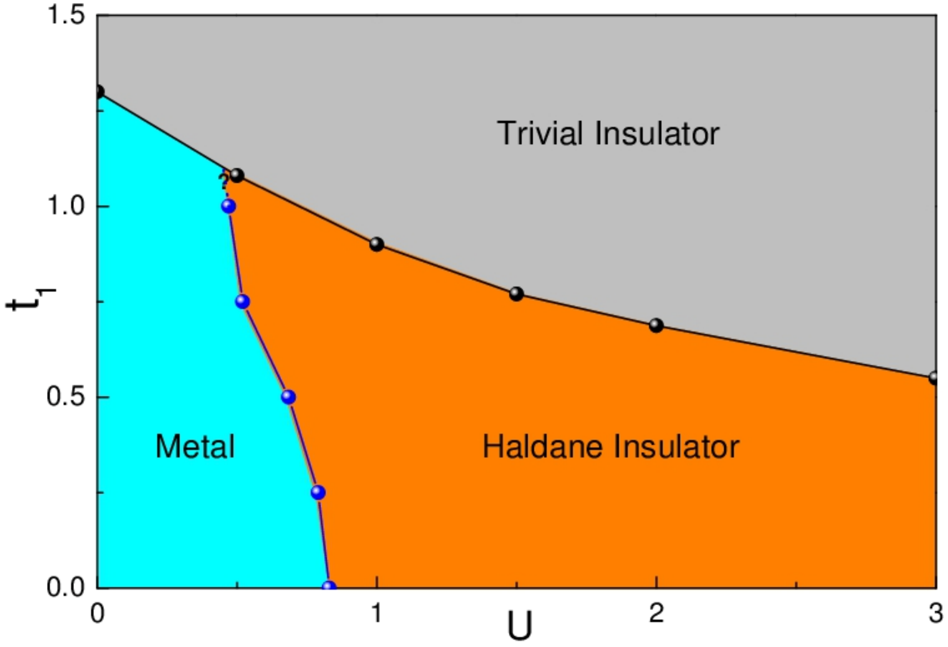
J *terms* →nontrivial Haldane phase.

Competition

For finite-parameters region, the system maybe exist the trivial and nontrivial gapped phases.

Interacting case

$Set\ t_0 = 1, J = 0.5\ in\ DMRG\ simulation$



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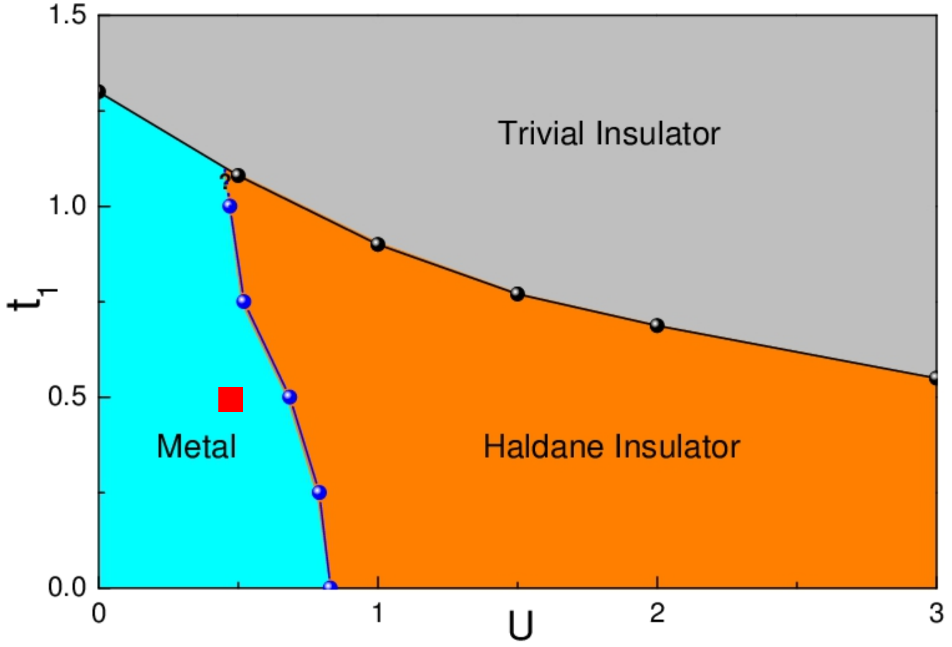

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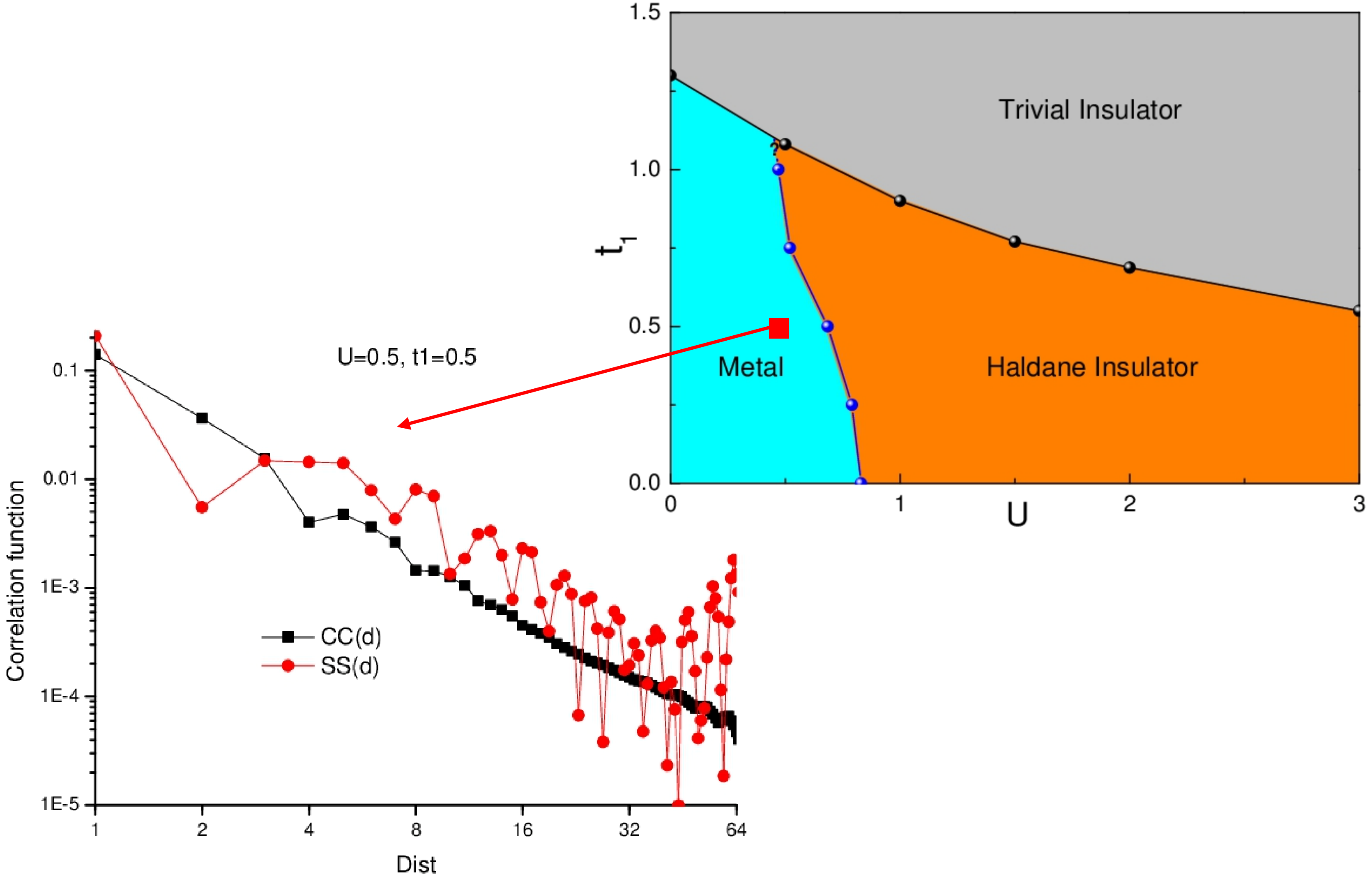

Interacting case

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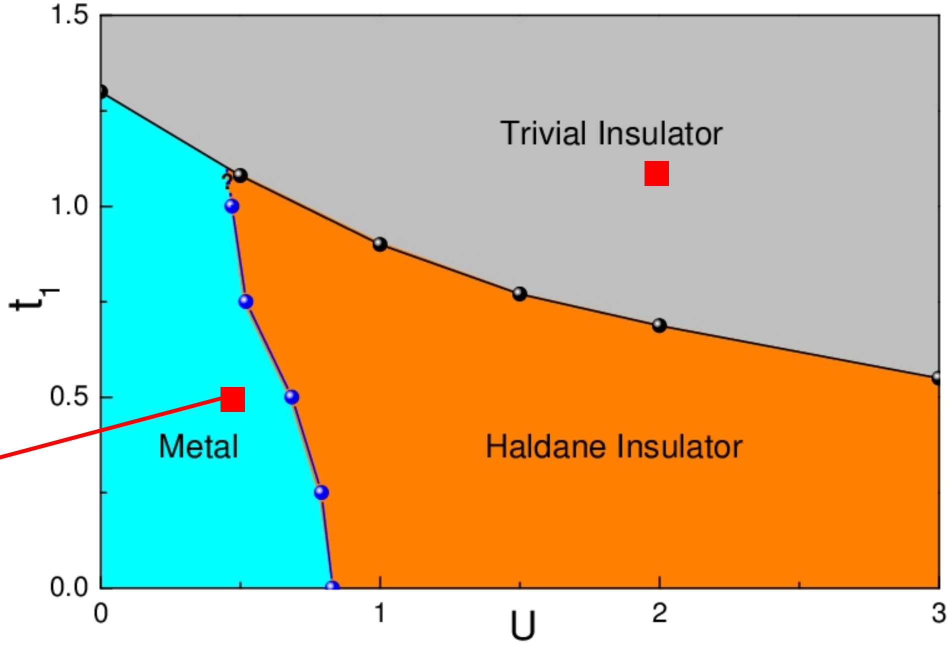

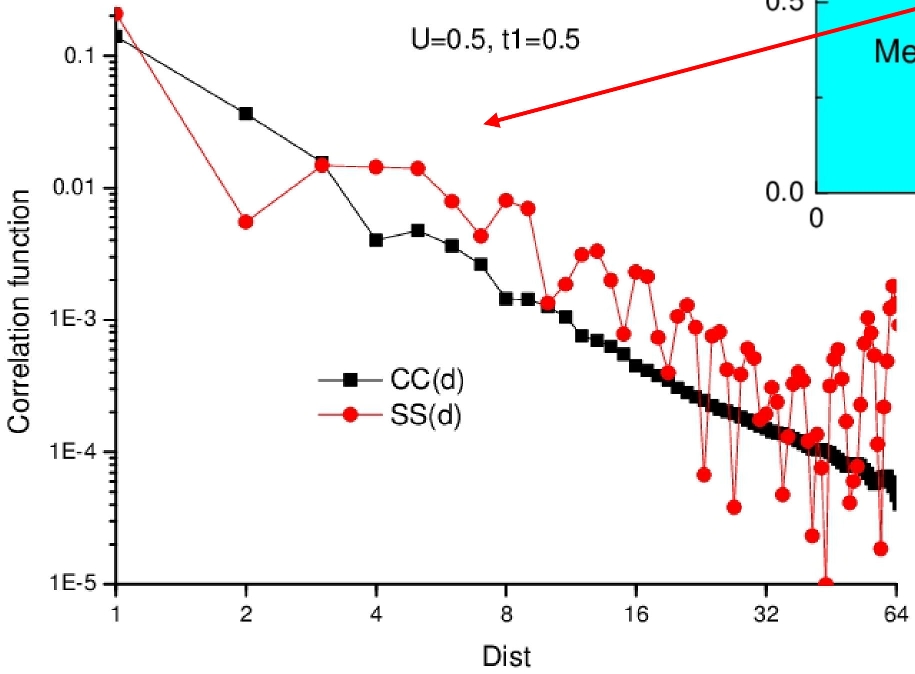

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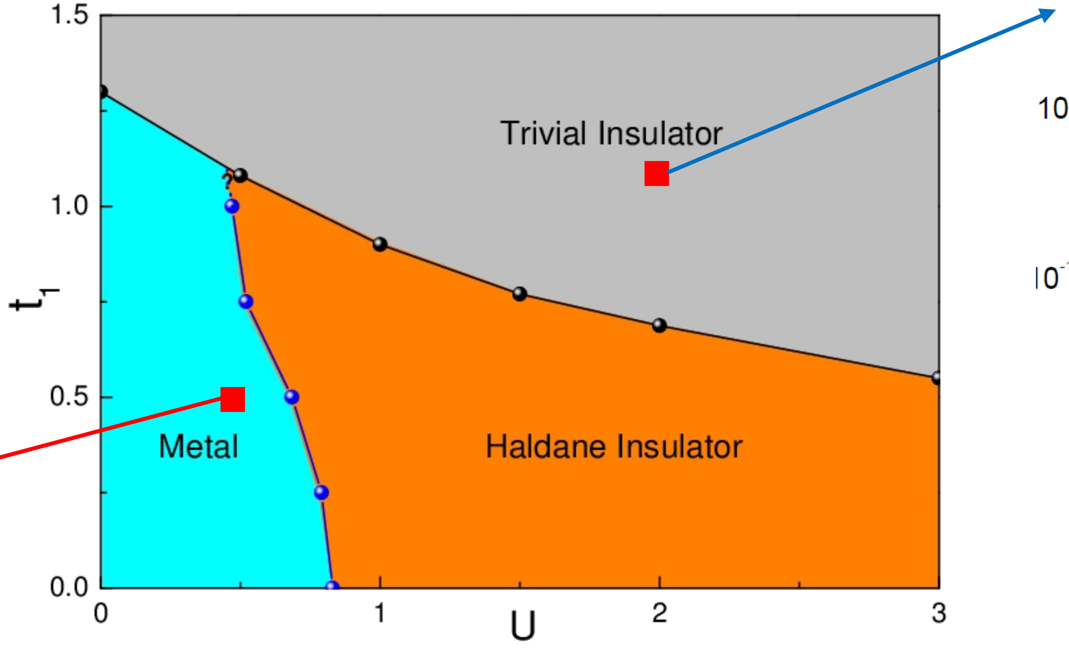
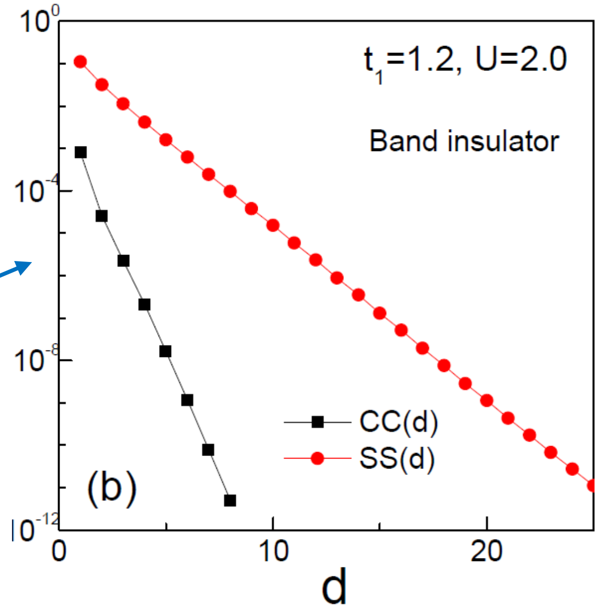
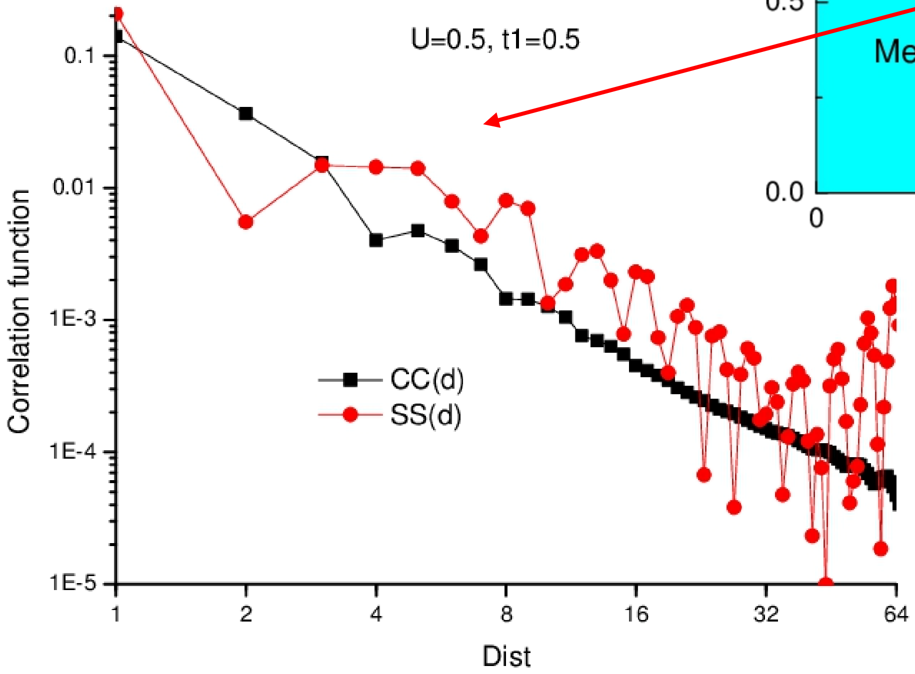
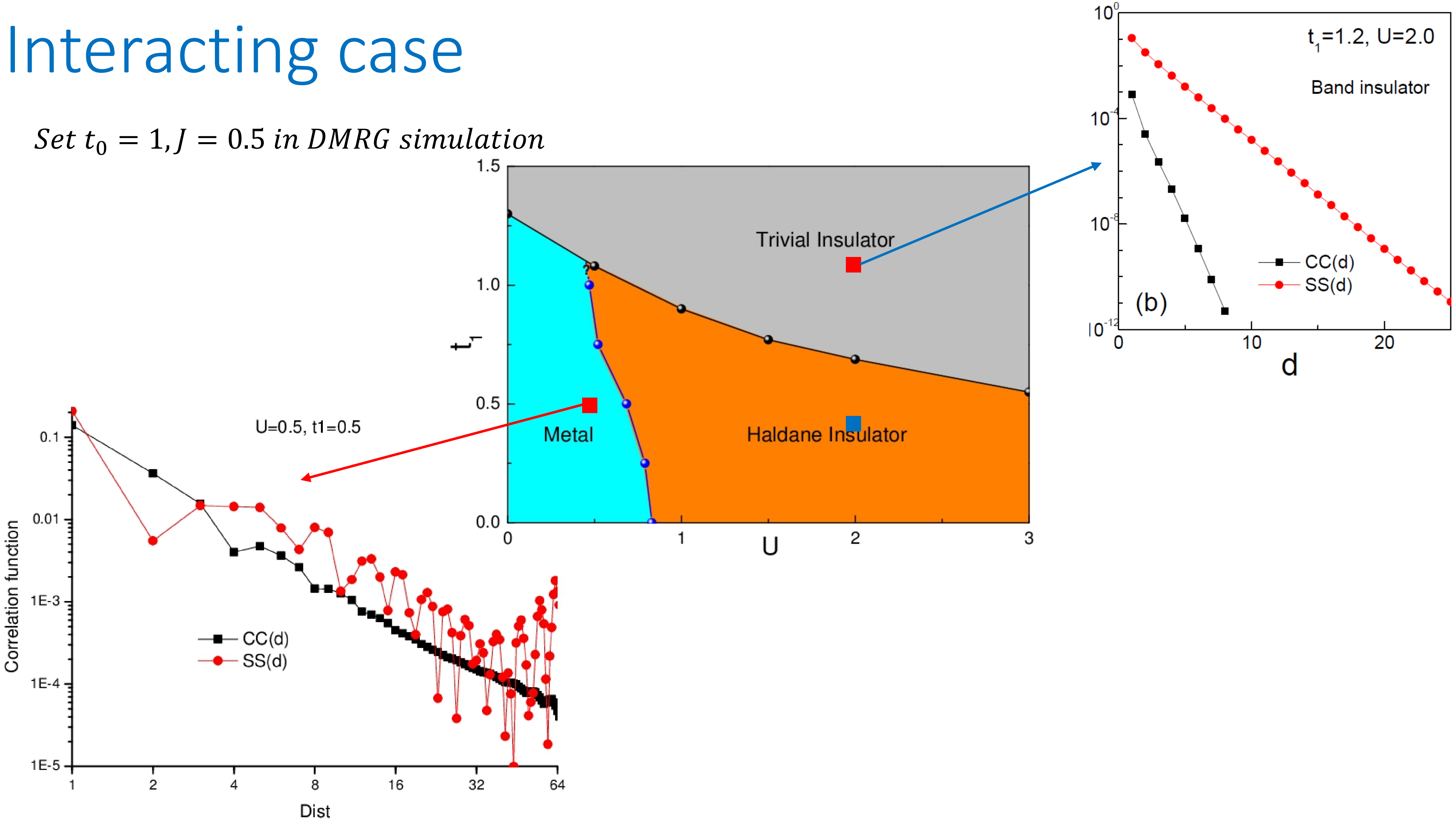






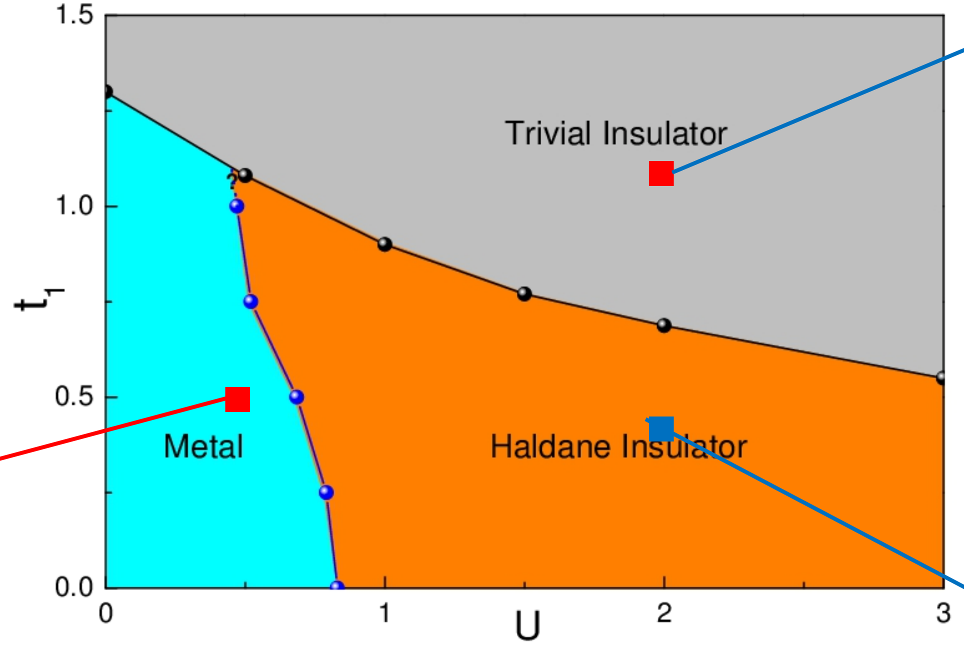
Interacting case

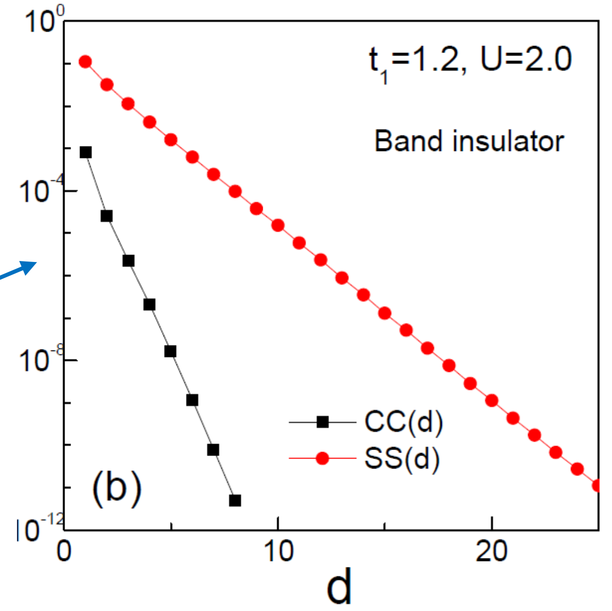
Set $t_0 = 1, J = 0.5$ *in DMRG simulation*

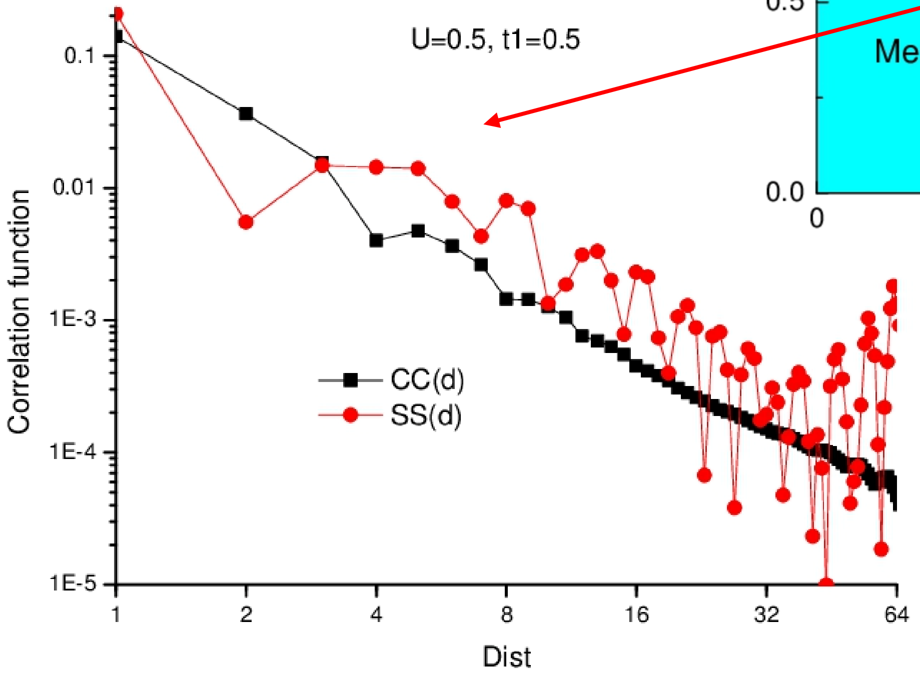


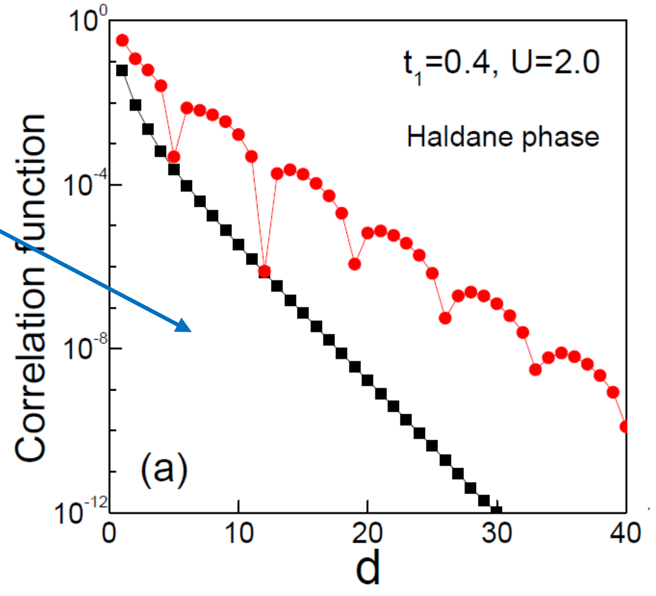
Interacting case

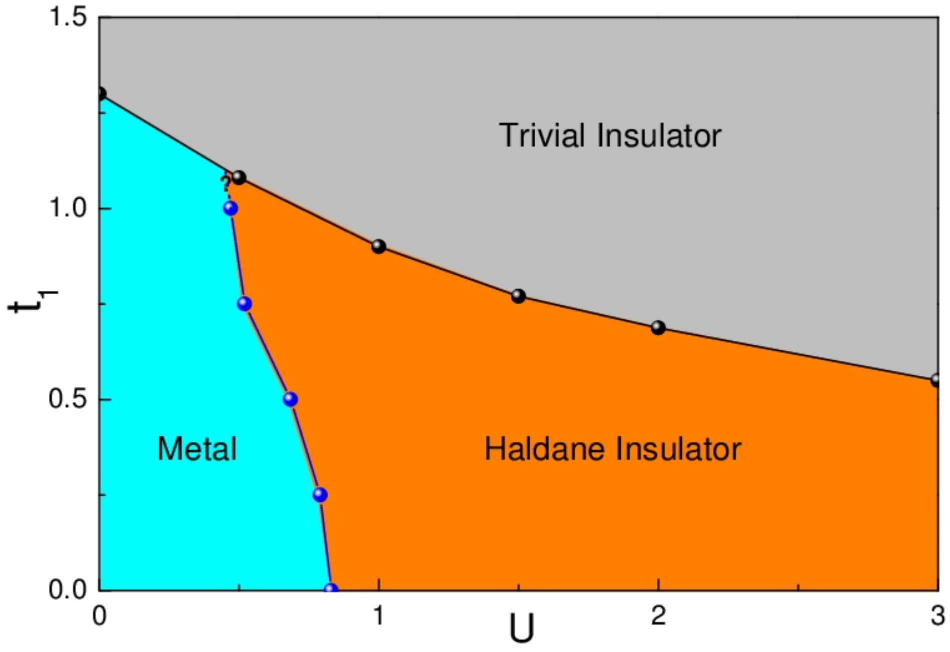
Set $t_0 = 1, J = 0.5$ *in DMRG simulation*



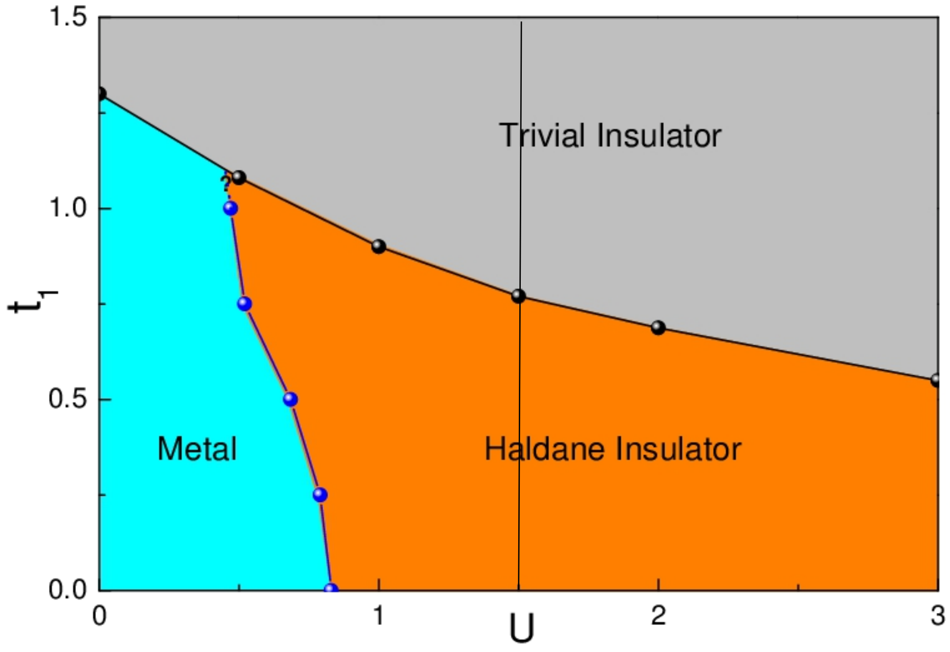








1.5
1.0
0.5
0.0
t_1
Trivial Insulator
Metal
Haldane Insulator
?
0
1
2
3
U



1.5
1.0
0.5
0.0
t_1
0
1
2
3
U
Trivial Insulator
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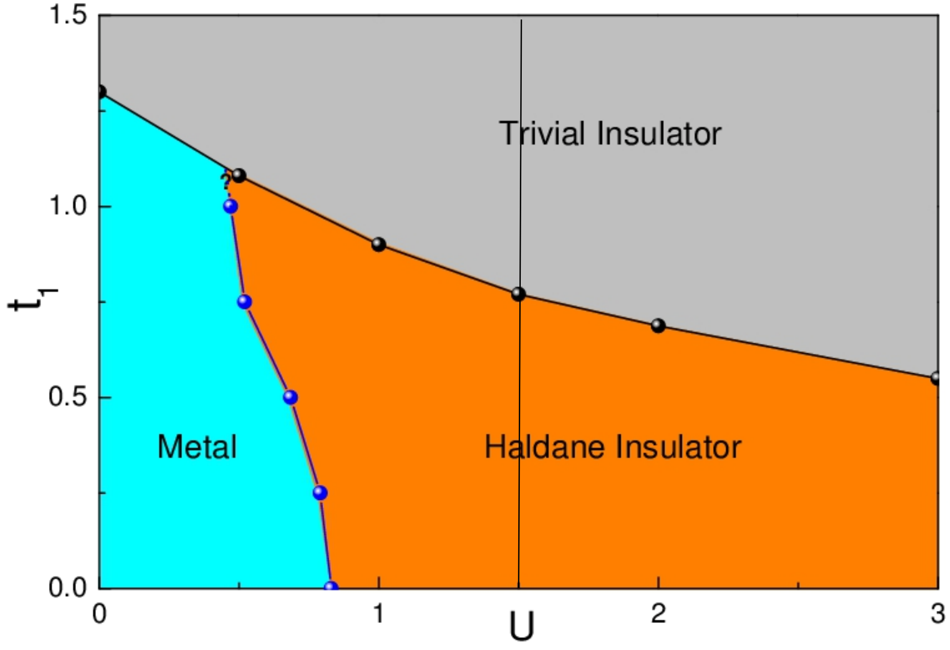
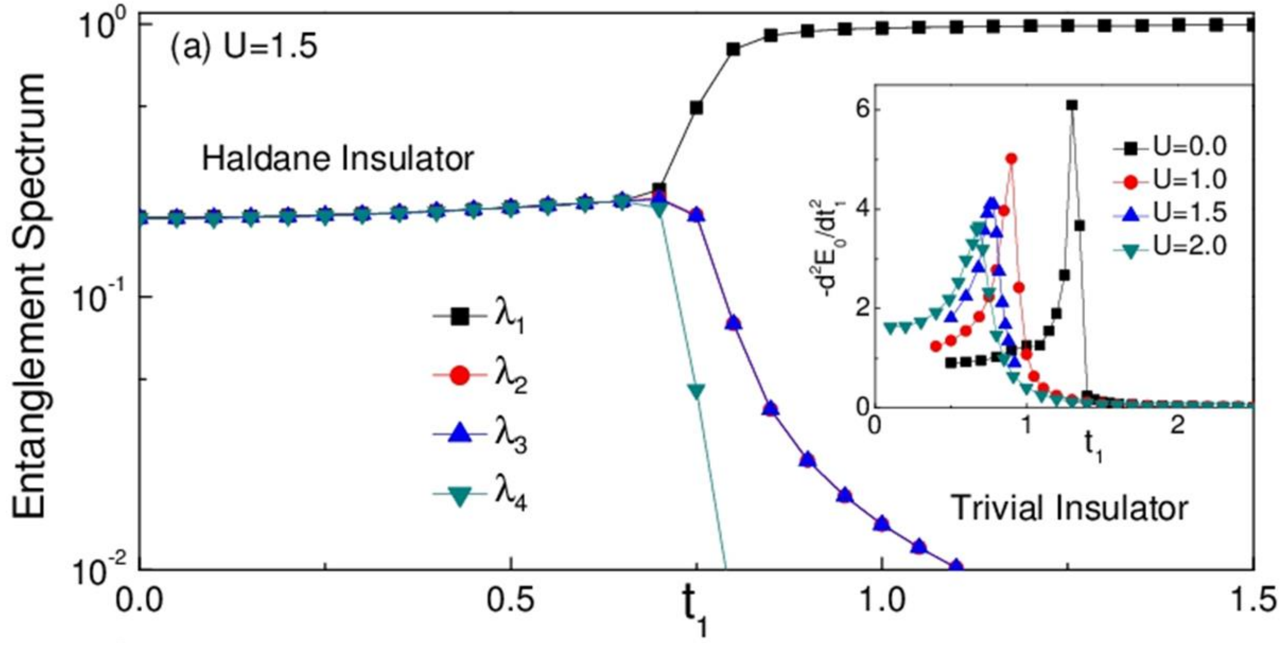
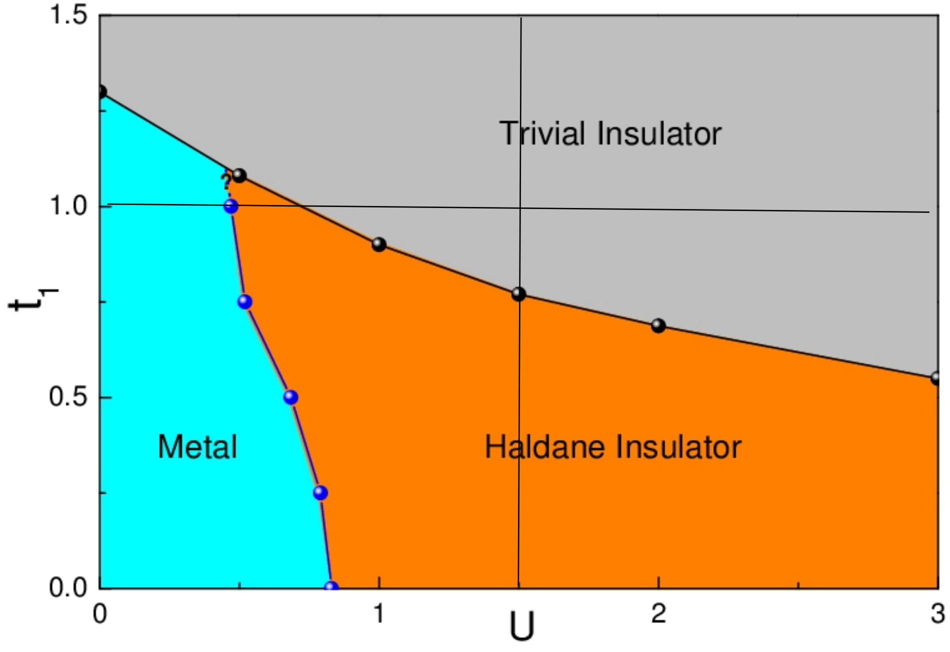
?



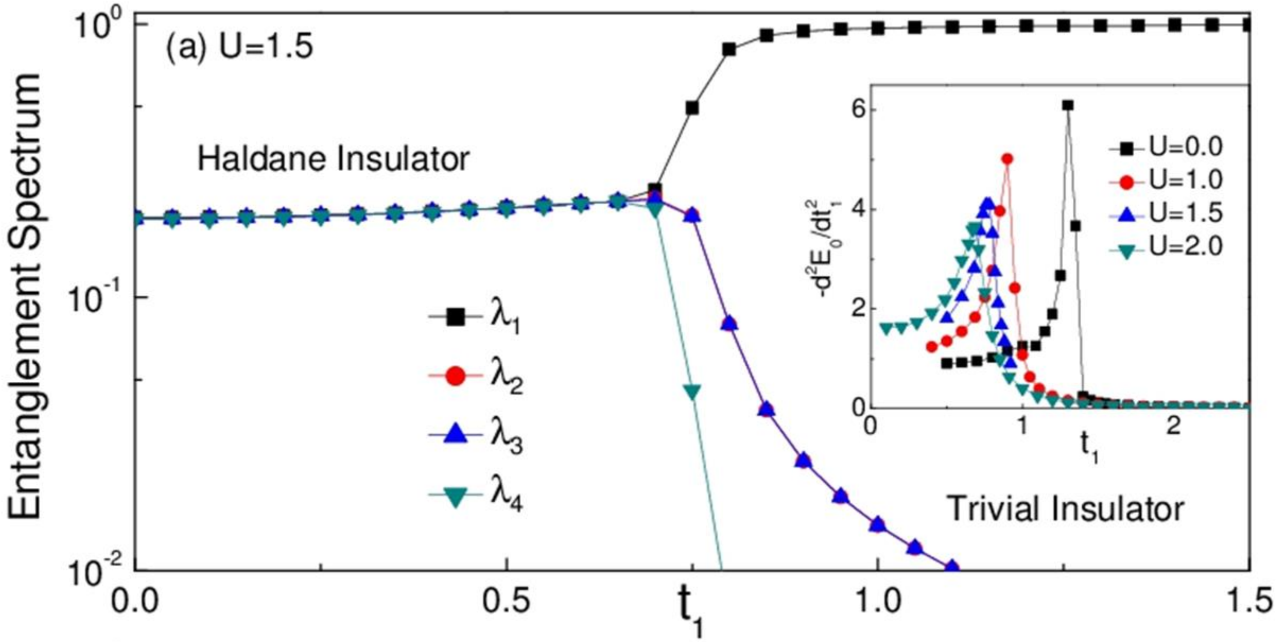
Trivial Insulator
Metal
Haldane Insulator
?
t_1
U



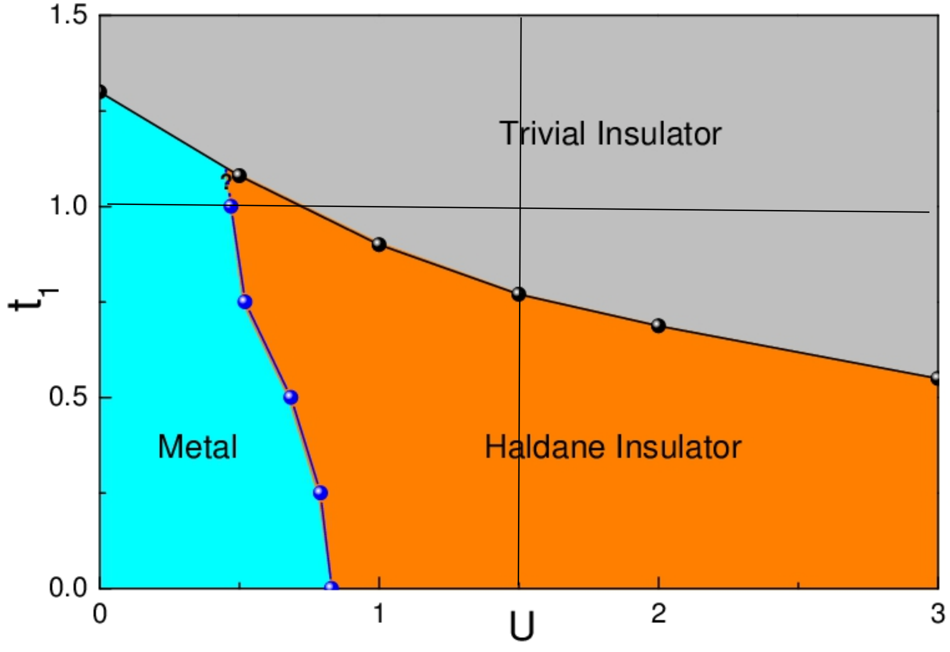
(a) U=1.5
Haldane Insulator
Trivial Insulator
Entanglement Spectrum
λ_1
λ_2
λ_3
λ_4
t_1
$-d^2E_0/dt_1^2$
U=0.0
U=1.0
U=1.5
U=2.0



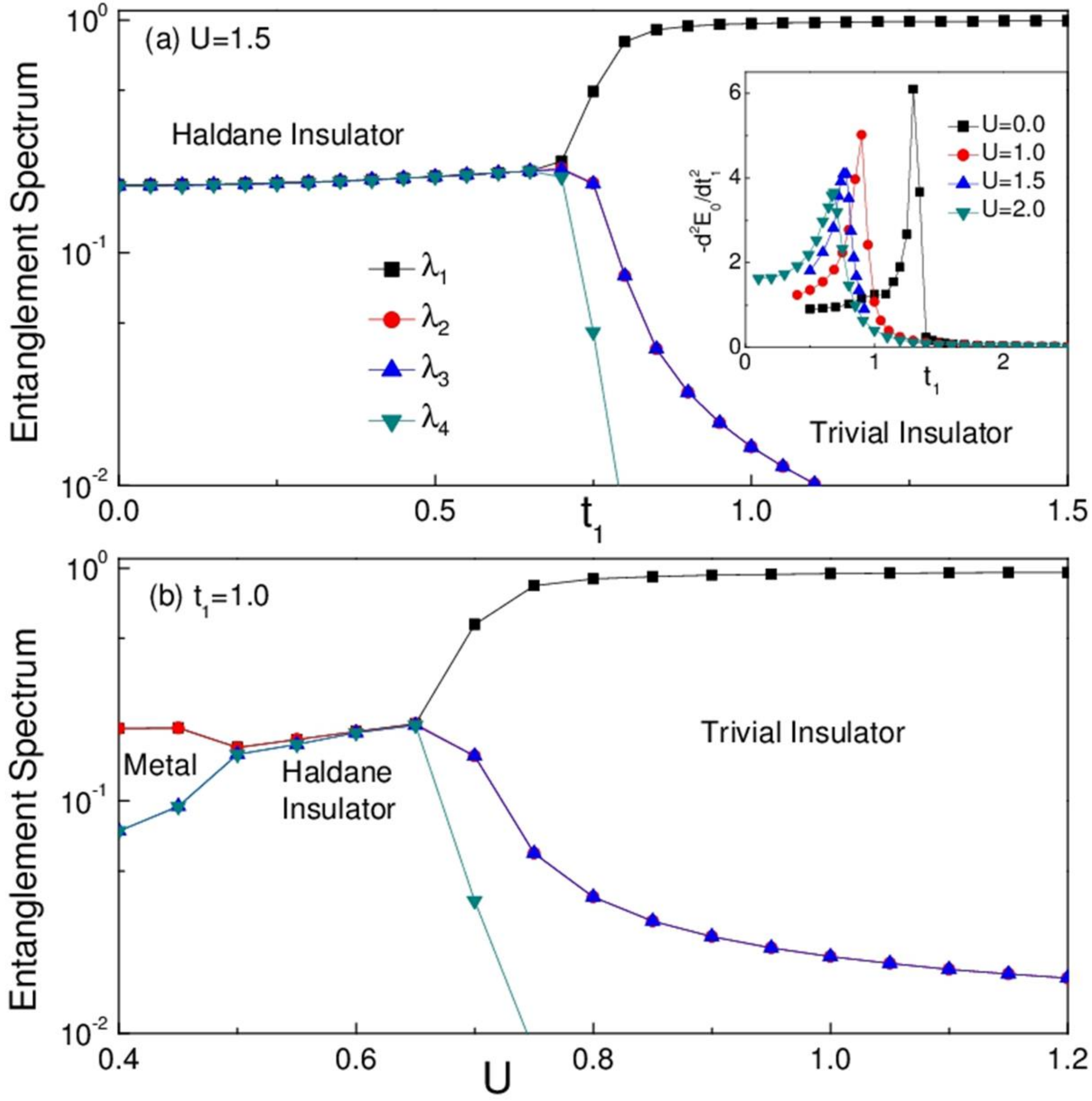
Trivial Insulator
Metal
Haldane Insulator
t1
U
1.5
1.0
0.5
0.0
0
1
2
3
?



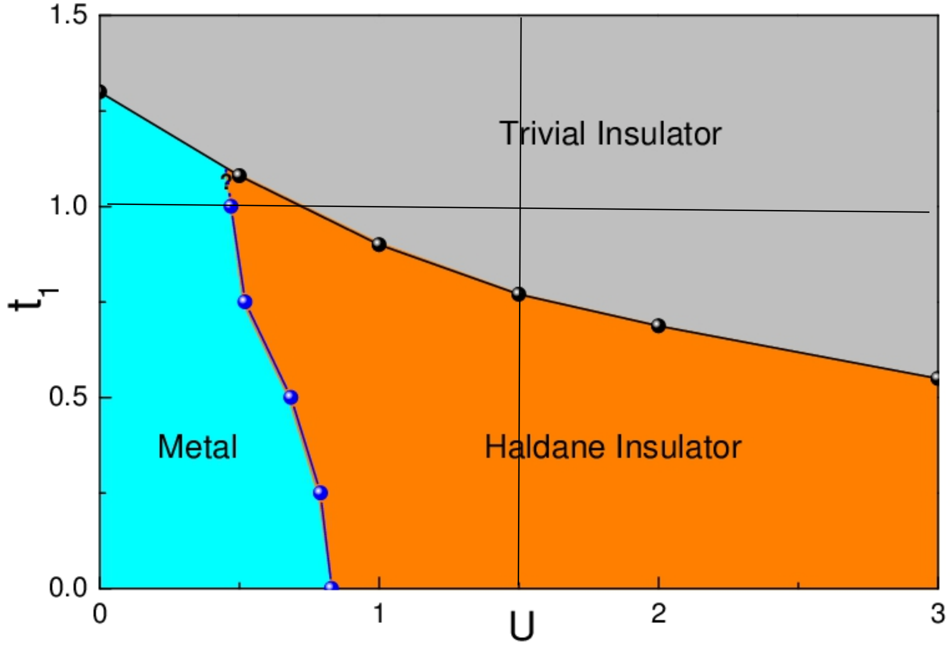
(a) U=1.5
Haldane Insulator
Trivial Insulator
Entanglement Spectrum
λ_1
λ_2
λ_3
λ_4
t_1
10^0
10^{-1}
10^{-2}
0.0
0.5
1.0
1.5
$-d^2E_0/dt_1^2$
U=0.0
U=1.0
U=1.5
U=2.0



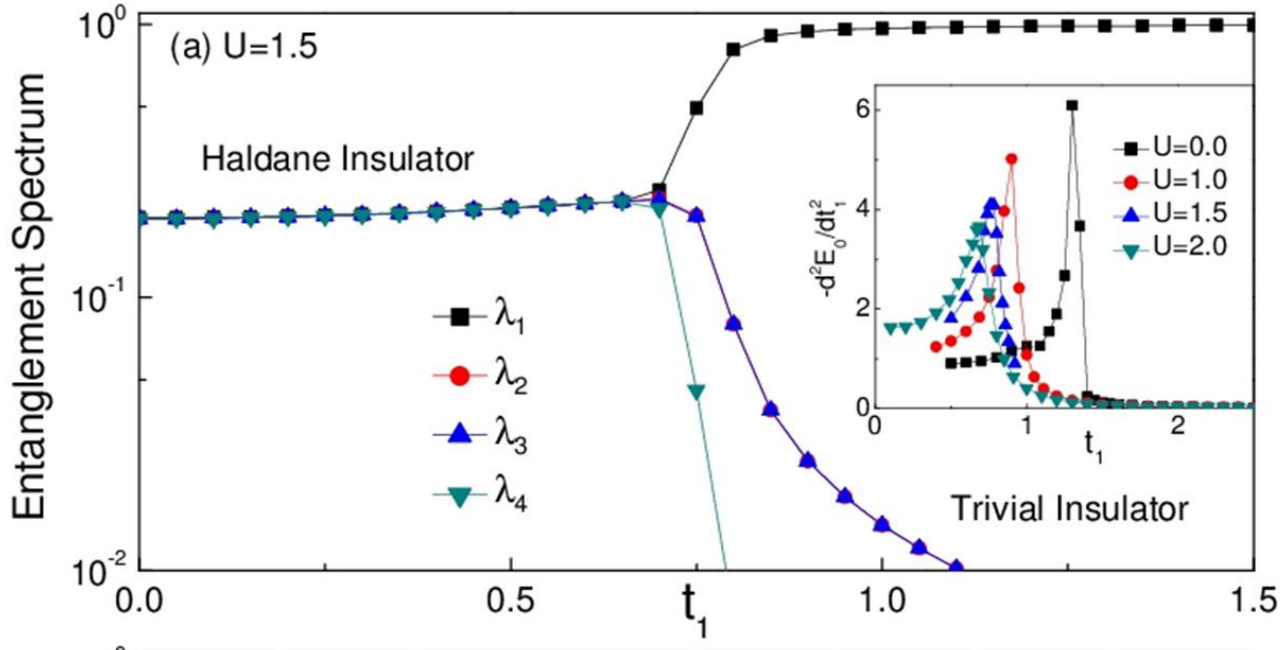
Trivial Insulator
Metal
Haldane Insulator
?
t1
U
1.5
1.0
0.5
0.0
0
1
2
3

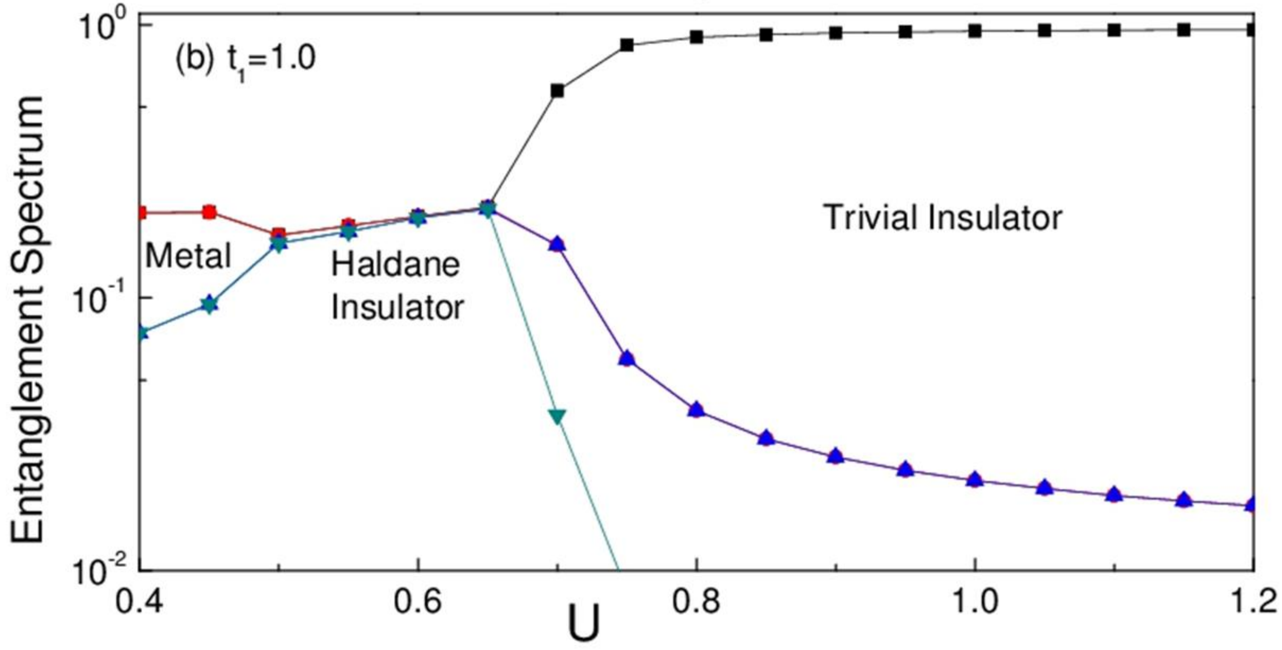


(a) U=1.5
Haldane Insulator
Trivial Insulator
Entanglement Spectrum
λ1
λ2
λ3
λ4
-d²E0/dt1²
U=0.0
U=1.0
U=1.5
U=2.0
t1
0.0
0.5
1.0
1.5
(b) t1=1.0
Metal
Haldane
Insulator
Trivial Insulator
Entanglement Spectrum
U
0.4
0.6
0.8
1.0
1.2



We find two phases in the interacting case：
trivial insulator and Haldane insulator





Conclusion

- introduction of SPT

- Interaction can induce new SPT phases for fermionic systems.

Thanks for your attention!