Fermionic Symmetry Protected Topological Phase Induced by Interaction

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Outline

1. Background and motivation
2. Study of a specific model
3. Conclusion
Background and Motivation

Key properties of TI:
- Bulk gap
- Gapless edge states protected by time reversal symmetry and U(1)
- No symmetry breaking

Mass terms

\[ B(\psi_\uparrow^+\psi_\downarrow + \psi_\downarrow^+\psi_\uparrow) \quad \Delta(\psi_\uparrow^+\psi_\downarrow^+ + \psi_\downarrow\psi_\uparrow) \]

breaks \( T \)  \quad \text{breaks} \quad \text{U}(1)

X. Qi, S.C. Zhang RevModPhys.83.1057
Symmetry Protected Topological phase (SPT phase)

Key points of SPT

- No symmetry breaking
- bulk is gapped
- boundary excitation is gapless
- protected by symmetry
Classification of SPT

- Free fermion systems are classified by K-theory

<table>
<thead>
<tr>
<th>q</th>
<th>$\pi_0(R_3)$</th>
<th>$d = 1$</th>
<th>$d = 2$</th>
<th>$d = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\mathbb{Z}$</td>
<td>no symmetry ((p_x + ip_y, \text{e.g., SrRu}))</td>
<td>(T) only (^{3}\text{He-B})</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>$\mathbb{Z}_2$</td>
<td>no symmetry ((\text{Majorana chain}))</td>
<td>(T) only ((p_x + ip_y)^\uparrow + (p_x - ip_y)^\downarrow)</td>
<td>(T) and (Q) ((\text{BiSb}))</td>
</tr>
<tr>
<td>2</td>
<td>$\mathbb{Z}_2$</td>
<td>(T) only (((\text{TMTSF})_2X))</td>
<td>(T) and (Q) ((\text{HgTe}))</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>(T) and (Q)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>$\mathbb{Z}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>no symmetry</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
SPT phases also exist in Bosonic systems

- $S=1$ Haldane phase (Bosonic) $SO(3)$ symmetry

$$H = \sum_i J_1 S_i \cdot S_{i+1}$$

- Finite excitation gap
- No symmetry breaking
- $SO(3)$ symmetry protected spin-1/2 edge states
- Degeneracy of entanglement spectrum

[Diagram]: Spin-1 and Spin-1/2 states with bulk gap.
entanglement spectrum for 1D

\[ \rho_A = \text{Tr}_B (\rho) , \quad \rho \text{ density matrix of the ground state} \]

Entanglement spectrum: eigenvalues of \( \rho_A \)
Entanglement spectrum

\[ H = \sum_j J_1 s_i \cdot s_{i+1} + J_2 s_i \cdot s_{i+2} \]

SO(3) symmetry

\[ J_2/J_1 = 0.3 \]

Non trivial

\[ J_2/J_1 = 0.9 \]

Trivial
Classification of Bosonic SPT

- 1D by the projective representation

\[ H = \sum J_1 S_i \cdot S_{i+1} + J_2 S_i \cdot S_{i+2} \quad \text{SO}(3) \text{ symmetry} \]

\[ \text{SO}(3) : 2 \text{ projective representation} \]

- 2D or higher by group cohomology


Chen, Gu, ZXL, Wen, Science, 338, 1604 (2012);
Chen, Gu, ZXL, Wen, PRB, 87, 155114 (2013)
Classification of Interacting Fermionic SPT

- 1D by projective representation
- 2D or higher partially by super-cohomology
- effect of interaction

1D superconductor

<table>
<thead>
<tr>
<th>Symmetry</th>
<th>Free classification</th>
<th>With interactions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U(1) \times Z^T_2$</td>
<td>$\mathbb{Z}$</td>
<td>$\mathbb{Z}_4$</td>
</tr>
<tr>
<td>$Z_n \times Z^T_2$ ($n$ even)</td>
<td>$\mathbb{Z}$</td>
<td>$\mathbb{Z}_4$</td>
</tr>
<tr>
<td>$Z_n \times Z^T_2$ ($n$ odd and $n &gt; 1$)</td>
<td>$\mathbb{Z}$</td>
<td>$\mathbb{Z}_2$</td>
</tr>
</tbody>
</table>

2D superconductor

| Time reversal and mirror reflect | $\mathbb{Z}$ | $\mathbb{Z}_8$ |

It seems that interactions reduce the classification. However, it maybe not true!

Hong Yao and Shinsei Ryu, arXiv:1202.5805
Model

\[ H = H_0 + H_U + H_J \]

\[ H_0 = -\left\{ \sum_{i,\alpha} t_0 (c_{i,\alpha}^\dagger c_{i+1,\alpha}) + \sum_{i,\{\alpha,\hat{\alpha}\}} t_1 c_{i,\alpha}^\dagger c_{i,\hat{\alpha}} + h.c. \right\} + \lambda N_i \]

\[ H_U = U \sum_i (N_i - 1)^2 \quad H_J = J \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+1} \]

\[ N_i = n_{i,x} + n_{i,y} + n_{i,z} \]

\[ S^\alpha = \sum_{\beta,\gamma} i\varepsilon_{\alpha\beta\gamma} c_\beta^\dagger c_\gamma \]

\[ \alpha = x, y, z, \{\alpha, \hat{\alpha}\} = \{x, y\}, \{y, z\}, \{z, x\} \]
Symmetry and classification

The model: \( U(1) \) and time reversal symmetry \( T \)

\[
U_\theta c_\alpha U_\theta^{-1} = c_\alpha e^{i\theta} \\
T c_\alpha T^{-1} = -c_\alpha. \quad T^2 = 1
\]

Classification

Free case, only the trivial phase.

2 projective representation \( \rightarrow \) two SPT phases.
Non-interacting limit \( U=0, J=0 \)

\[ t_1 < \frac{8}{3} t_0 \]
\[ t_1 = \frac{8}{3} t_0 \]
\[ t_1 > \frac{8}{3} t_0 \]
Non-interacting limit \( U=0, J=0 \)

SPT only focus on gapped phases. \( in \ \frac{1}{3} \) filling case

\( t_1 > \frac{8}{3} t_0 \)

Open boundary, no zero mode, indicating the trivialness of phase

In region of \( t_1 > \frac{8}{3} t_0 \), only trivial phase, i.e. band insulator
Interacting case $t_1 \gg t_0$

Add Hubbard interaction $H_u = U \sum_i (N_i - 1)^2$

When $\frac{t_1}{t_0} \to \infty, U = 0$
the charge are localized at each rung in ground state.

the band insulator and the Mott insulator are adiabatically connected
Interacting case

- Add Heisenberg-like interaction $H_J = J \sum_i S_i \cdot S_{i+1}$,

- This term competes to exhibit the Haldane phase, so in the limit $J \to \infty$, the system would fall into Haldane phase.
Interacting case

$t_0$ terms $\rightarrow$ metal phase,
$t_1$ and $U$ $\rightarrow$ trivial insulator phase,
$J$ terms $\rightarrow$ nontrivial Haldane phase.

For finite-parameters region, the system maybe exist the trivial and nontrivial gapped phases.
Interacting case

Set $t_0 = 1, J = 0.5$ in DMRG simulation
Interacting case

Set $t_0 = 1, J = 0.5$ in DMRG simulation.
Interacting case

Set $t_0 = 1, J = 0.5$ in DMRG simulation
Interacting case

Set $t_0 = 1, J = 0.5$ in DMRG simulation

![Graph showing the relationship between $U$, $t_1$, and correlation function with different phases (Trivial Insulator, Metal, Haldane Insulator).](image)
Interacting case

Set $t_0 = 1, J = 0.5$ in DMRG simulation
Interacting case

Set $t_0 = 1, J = 0.5$ in DMRG simulation.
Interacting case

Set $t_0 = 1, J = 0.5$ in DMRG simulation
We find two phases in the interacting case: trivial insulator and Haldane insulator.
Conclusion

- introduction of SPT

- Interaction can induce new SPT phases for fermionic systems.
Thanks for your attention!