Phenomenological extraction of Transversity from COMPASS SIDIS and Belle e+e− data

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Beijing, October 20-24
COMPASS results on dihadron asymmetry

Recent results (DIS2014)

dihadron asymmetries

$\pi^+\pi^-$
COMPASS results on Collins asymmetry

Recent results (hep-ex/1408.4405)

Collins asymmetries

$\pi^+\pi^-$
Transversity from COMPASS p and d results

COMPASS results already used to extract the transversity PDFs

• Collins asymmetry \((\pi^+\pi^- \text{ COMPASS, HERMES, Belle})\)
  Torino group

• Dihadron asymmetry \((\text{COMPASS, HERMES, Belle})\)
  Pavia group
Transversity from Collins asymmetry

Anselmino et al., PRD87 2013
simultaneous fit of
HERMES p, COMPASS p & d, and Belle data
very good $\chi^2$

$$\int_0^1 dx [h_1^u(x) - \bar{h}_1^u(x)] = \delta q.$$

Bacchetta et al, 2012
model calculations
Transversity from dihadron asymmetry

\( D_{q}^{2h} \) from PYTHIA plus HERMES p, COMPASS p and d (2h), Belle data

\( \rightarrow \) linear combinations of transversity for u and d valence quark

fit with parametrisations \( \rightarrow \) transversity PDFs
Transversity from COMPASS p and d results

new:

COMPASS result on transversity measured in each x bin from pion-pair asymmetry on p and d using results of the Pavia group analysis for the FFs

C. Braun, DIS2014 (PhD Thesis)
G. Sbrizzai, this Symposium
Transversity from COMPASS p and d results

this work

published COMPASS results for the dihadron asymmetries on p and d
$A_{p}^{2h}$ and $A_{d}^{2h}$ as function of $x$

directly use the Belle data to evaluate the analyzing power
following Bacchetta et al., PRL107(2011)012001

numerical values for $4xh_{1}^{uv} - xh_{1}^{dv}$ and $xh_{1}^{uv} + xh_{1}^{dv}$ in each $x$ bin

numerical values for $xh_{1}^{uv}$ and $xh_{1}^{dv}$ in each $x$ bin

and follow a similar procedure for the Collins asymmetries,
without using parametrisations for Collins FFs and transversity PDFs

first shown @ QCD Evolution 2014
Transversity from COMPASS and Belle data

dihadron asymmetries
dihadron asymmetry – COMPASS data

\[ A^{2h} = \frac{R}{M_{2h}} \frac{\sum_{q,\bar{q}} e_q^2 x h_1^q H^L_q}{\sum_{q,\bar{q}} e_q^2 x f_1^q D_q} = \frac{\sum_{q,\bar{q}} e_q^2 x h_1^q H_q}{\sum_{q,\bar{q}} e_q^2 x f_1^q D_q} \]

measured as function of \( x, z, M \)

\[ H_q(z, M_{2h}) = \sin \theta_q \cdot \frac{R}{M_{2h}} \cdot H^L_q(z, M_{2h}) \]
dihadron asymmetry  –  COMPASS data

\[ A^{2h} = \frac{R}{M_{2h}} \sum_{q,\bar{q}} e_{q}^{2}xh_{1}^{q}H_{q}^{\perp} \left/ \sum_{q,\bar{q}} e_{q}^{2}x f_{1}^{q}D_{q} \right. \]

\[ = \frac{\sum_{q,\bar{q}} e_{q}^{2}xh_{1}^{q}H_{q}}{\sum_{q,\bar{q}} e_{q}^{2}x f_{1}^{q}D_{q}} \]

measured as function of \( x, z, M \)

\[ H_{q}(z, M_{2h}) = \sin \theta_{q} \cdot R/M_{2h} \cdot H_{q}^{\perp}(z, M_{2h}) \]

with reasonable assumptions on the FFs

\[ H_{q} = -H_{\bar{q}}, \quad H_{u} = -H_{d}, \quad H_{s} = H_{c} = 0 \]

\[ D_{u} = D_{d} = D_{\bar{u}} = D_{\bar{d}}, \quad D_{s} = D_{\bar{s}}, \quad D_{c} = D_{\bar{c}} \]

\[ D_{s} \simeq D_{c} \simeq 0.5D_{u} \]

Bacchetta et al.

PRL107(2011)012001

neglecting s and c quark contributions, and integrating over \( z, M \):

\[ A_{p}^{2h}(x) \simeq \frac{4xh_{1}^{u} - xh_{1}^{d}}{4xf_{1}^{*u} + xf_{1}^{*d}} \left/ \right. \left\langle H_{u} \right\rangle \]

\[ \left/ \right. \left\langle D_{u} \right\rangle \]

\[ A_{d}^{2h}(x) \simeq \frac{3xh_{1}^{u} + xh_{1}^{d}}{5xf_{1}^{*u} + xf_{1}^{*d}} \left/ \right. \left\langle H_{u} \right\rangle \]

from Belle data

\[ f_{1}^{*q} = f_{1}^{q} + f_{\bar{q}} \quad \text{from CTEQ} \]
Dihadron asymmetry – COMPASS data

\[ A^{2h} = \frac{R}{M_{2h}} \frac{\sum_{q, \bar{q}} e_q^2 x h_1^q H_q^F}{\sum_{q, \bar{q}} e_q^2 x f_1^q D_q} = \frac{\sum_{q, \bar{q}} e_q^2 x h_1^q H_q}{\sum_{q, \bar{q}} e_q^2 x f_1^q D_q} \]

measured as function of \( x, z, M \)

\[ H_q(z, M_{2h}) = \sin \theta_q \cdot R/M_{2h} \cdot H_q^F(z, M_{2h}) \]

with reasonable assumptions on the FFs

\[ H_q = -H_{\bar{q}}, \quad H_u = -H_d, \quad H_s = H_c = 0 \]
\[ D_u = D_d = D_{\bar{u}} = D_{\bar{d}}, \quad D_s = D_{\bar{s}}, \quad D_c = D_{\bar{c}} \]
\[ D_s \simeq D_c \simeq 0.5 D_u \]

neglecting s and c quark contributions, and integrating over \( z, M \):

\[ A^{2h}_{\rho}(x) \simeq \frac{4x h_1^{u\rho} - x h_1^{d\rho}}{4x f_1^{*u} + x f_1^{*d}} < H_u > / < D_u > \]
\[ A^{2h}_{d}(x) \simeq \frac{3x h_1^{u\rho} + x h_1^{d\rho}}{5x f_1^{*u} + x f_1^{*d}} < H_u > / < D_u > \]

only unknowns
dihadron asymmetry – Belle data

\[ a_{12} = \frac{s^2}{1 + c^2} \frac{\sum_q e_q^2 H_q H_{\bar{q}}}{\sum_q e_q^2 D_q D_{\bar{q}}} \]

measured as function of \( z_1, z_2, M_1, M_2 \)

\[ s^2 = \sin^2 \theta, \quad c^2 = \cos^2 \theta \]

\[ H_q(z, M_{2h}) = \sin \theta_q \cdot \frac{R}{M_{2h}} \cdot H_q^L(z, M_{2h}) \]

with the previous assumptions on the FFs and \( D_c \) fixed in order to reproduce the charm yield, the fully integrated \( a_{12} \) asymmetry given by Belle is

\[ a_{12}^I \approx -\frac{5}{8} \frac{s^2}{1 + c^2} \left( \frac{\langle H_u \rangle}{\langle D_u \rangle} \right)^2 \]

\[ a_{12}^I = -0.0196 \pm 0.0002 \pm 0.0022 \]

and the analysing power is

\[ \langle a_P \rangle = \frac{\langle H_u \rangle}{\langle D_u \rangle} = 0.203 \]

here we have used this value at all COMPASS \( Q^2 \), neglecting evolution effect evaluated by the Pavia group: \(~-8\%\)
dihadron asymmetry – COMPASS data

\[ A_{p,d}^{2h} \]

\[ h^+ h^- \]

\[ x \]

\[ 10^{-2} \quad 10^{-1} \]
dihadron asymmetry – transversity

present result
Transversity from COMPASS and Belle data

Collins asymmetries
Collins asymmetries

in this case, from the Belle data one has to calculate the analysing power

\[
< \alpha_{P,C} > = \frac{< H^{f_{av}} >}{< D^{f_{av}} >}
\]
Collins asymmetry – Belle data

we have used the asymmetry (corrected for charm contribution)

\[ A_{12}^{UL}(z_1, z_2) = \frac{<s^2>}{<1+c^2>} [P_U(z_1, z_2) - P_L(z_1, z_2)] \]

integrated over \( M_1, M_2 \)

where

\[
\begin{align*}
P_U(z_1, z_2) &= \frac{\sum_q e_q^2[H_{1q}^+(z_1)H_{1\bar{q}}^-(z_2) + H_{1\bar{q}}^-(z_1)H_{1q}^+(z_2)]}{\sum_q e_q^2[D_{1q}^+(z_1)D_{1\bar{q}}^-(z_2) + D_{1\bar{q}}^-(z_1)D_{1q}^+(z_2)]} \\
P_L(z_1, z_2) &= \frac{\sum_q e_q^2[H_{1q}^+(z_1)H_{1\bar{q}}^+(z_2) + H_{1\bar{q}}^-(z_1)H_{1q}^-(z_2)]}{\sum_q e_q^2[D_{1q}^+(z_1)D_{1\bar{q}}^+(z_2) + D_{1\bar{q}}^-(z_1)D_{1q}^-(z_2)]}
\end{align*}
\]

\[ H_{1q}^\pm = H_1^{\frac{1}{2}}(q \rightarrow \pi^\pm), \quad D_{1q}^\pm = D_1(q \rightarrow \pi^\pm) \]

Efremov et al., PRD73 (2006)  
Bacchetta et al., PLB659 (2008)  
Anselmino et al., PRD75 (2007)  
Seidl et al., PRD78 (2008)
Collins asymmetry – Belle data

for the FFs we have made the assumptions

\[ H_1^{fav} = H_1^+ = H_1^- = H_{1\bar{u}} = H_{1\bar{d}} \]
\[ H_1^{dis} = H_{1\bar{u}} = H_{1d}^+ = H_{1\bar{u}}^+ = H_{1\bar{d}}^- \] (same for \( D \))

ignoring the c and s quark contributions, in the case \( z_1 = z_2 = z \) it is

\[ A_{12}^{UL}(z) = \frac{< s^2 >}{< 1 + c^2 >} \left[ \frac{H_1^{fav}(z)}{D_1^{fav}(z)} \right]^2 B(z) \]

where

\[ B(z) = \frac{b(z)[1 + a^2(z)] - [1 + b^2(z)]a(z)}{b(z)[1 + b^2(z)]} \quad a(z) = \frac{H_1^{dis}(z)}{H_1^{fav}(z)} \quad b(z) = \frac{D_1^{dis}(z)}{D_1^{fav}(z)} \]

not so simple as in the 2h case \( \rightarrow \)
Collins asymmetry – Belle data

we have done 2 alternative assumptions

\[ H_1^{fav}(z) = -H_1^{dis}(z) \quad \text{i.e.} \quad a(z) = -1 \]

\[ \frac{H_1^{fav}(z)}{D_1^{fav}(z)} = -\frac{H_1^{dis}(z)}{D_1^{dis}(z)} \quad \text{i.e.} \quad a(z) = -b(z) \]

both in agreement with the considerations on the “interplay between the Collins and the dihadron FFs”
and already used / suggested / found as a result of global fits

• these assumptions allow to evaluate \( \frac{H_1^{fav}(z)}{D_1^{fav}(z)} \) in the four \( z \) bins

• the values are then fitted with a function of \( z \)
Collins asymmetry – Belle data

to obtain the analysing power
the functions are integrated over $z$
finally:

\[
\frac{\langle H_1^{fav} \rangle}{\langle D_1^{fav} \rangle} \approx 0.10 \quad \text{a1}
\]

\[
\frac{\langle H_1^{fav} \rangle}{\langle D_1^{fav} \rangle} \approx 0.18 \quad \text{a2}
\]

if the evolution of \( H_1^{fav} \) is negligible;

if the evolution of \( H_1^{fav} \) is the same as that of \( D_1^{fav} \) the analysing powers decrease by \( \sim 10\% \)
Collins asymmetry – COMPASS data

\[
A_{Coll}^{\pm}(x, z) = \frac{\sum_{q, \bar{q}} e_q^2 x h_1^q(x) \otimes H_{1q}^{\pm}(z)}{\sum_{q, \bar{q}} e_{\bar{q}}^2 x f_1^q(x) \otimes D_{1q}^{\pm}(z)}
\]

“gaussian ansatz”:

\[
A_{Coll}^{\pm}(x, z) = C_G \cdot \frac{\sum_{q, \bar{q}} e_q^2 x h_1^q(x) H_{1q}^{\pm}(z)}{\sum_{q, \bar{q}} e_{\bar{q}}^2 x f_1^q(x) D_{1q}^{\pm}(z)}
\]

\[
C_G = \frac{1}{\sqrt{1 + z^2 < p_{h_1}^2 > / < p_{H_1}^2 >}}
\]

Efremov et al., PRD73 (2006)

we have assumed

- \( C_G = 1 \)
- the previous relations among the FFs
- the s and c quark contributions to be negligible
Collins asymmetry – COMPASS data

The measured asymmetries as function of $x$ can be written as

$$
\begin{align*}
A_{Coll,p}^+ &= \frac{<H_{1}^{fav}>}{<D_{1}^{fav}>} \frac{4(xh_{1}^{u} + \alpha xh_{1}^{\bar{u}}) + (\alpha xh_{1}^{d} + xh_{1}^{\bar{d}})}{d_{p}^{+}} \\
A_{Coll,p}^- &= \frac{<H_{1}^{fav}>}{<D_{1}^{fav}>} \frac{4(\alpha xh_{1}^{u} + xh_{1}^{\bar{u}}) + (xh_{1}^{d} + \alpha xh_{1}^{\bar{d}})}{d_{p}^{-}} \\
A_{Coll,d}^+ &= \frac{<H_{1}^{fav}>}{<D_{1}^{fav}>} \frac{(xh_{1}^{u} + xh_{1}^{d})(4 + \alpha) + (xh_{1}^{\bar{u}} + xh_{1}^{\bar{d}})(1 + 4\alpha)}{d_{d}^{+}} \\
A_{Coll,d}^- &= \frac{<H_{1}^{fav}>}{<D_{1}^{fav}>} \frac{(xh_{1}^{u} + xh_{1}^{d})(4\alpha + 1) + (xh_{1}^{\bar{u}} + xh_{1}^{\bar{d}})(4 + \alpha)}{d_{d}^{-}}
\end{align*}
$$

$$
\alpha = \frac{<H_{1}^{dis}>}{<H_{1}^{fav}>} = \begin{bmatrix} a1 \\ a2 \end{bmatrix}
$$

Corresponding quantities with unpolarized PDFs and FFs from Belle

Neglecting qbar transversity, in each $x$ bin, the only unknowns are the u and d quark transversity PDFs.
Collins asymmetry – COMPASS data

\[ A_{\text{Coll},p} \]

\[ A_{\text{Coll},d} \]

proton

deuteront

un-identified hadrons
Collins asymmetry – transversity

results
Transversity from COMPASS and Belle data

results of the present work

\[ x h_1 \]

\[ x h_1^u \]

\[ x h_1^d \]

- dihadron
- Collins A1
- Collins A2
Transversity from COMPASS and Belle data

results of the present work

\[ xh_1^u \]
\[ xh_1^d \]

- dihadron
- Collins A1
- Collins A2

SPIN 2014
Transversity from COMPASS and Belle data

summary

• promising results
• the physics is simple

The work is almost over

The same method used for the Collins asymmetry could be used for the point-to-point extraction of the Boer-Mulders PDF and, even simpler, of the Sivers function
Transversity from COMPASS p and d results

method:

COMPASS results for $A_d^{2\pi}$ and $A_p^{2\pi}$ as function of $x$

use the same coefficients evaluated by A. Bacchetta et al. from Belle data [JHEP1303 (2013)119]

numerical values for $4xh_1^{uv} - xh_1^{dv}$ and $xh_1^{uv} + xh_1^{dv}$ in each $x$ bin

numerical values for $xh_1^{uv}$ and $xh_1^{dv}$ in each $x$ bin
Transversity from COMPASS p and d results

results:

\[ xh_1^{uv} \]

\[ xh_1^{dv} \]
Transversity from COMPASS and Belle data

results obtained using

- **Belle results for pion and pion-pair asymmetries**
  

- **COMPASS results on**
  - p and d dihadron asymmetries vs $x$ (integrated over $z$, $M$)
  - p and d Collins asymmetry vs $x$ (integrated over $z$, $p_T$)
    
    $h^+$ and $h^-$ assuming that all hadrons are pions

- **unpolarised PDFs and FFs parametrisations**
  - PDFs: CTEQ5D
  - FFs: DSS LO
dihadron asymmetry – transversity

present result
compared with DIS2014

\[ x h_1 \]

\[ x h_{1u} \]

\[ x h_{1d} \]

\[ h^+ h^- \]

\[ \pi^+ \pi^- \]

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