Three dimensional imaging of the nucleon: TMD (Theory/Phenomenology)

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Introduction

After the talks:

Parallel-II: S1

16:30	Overview of TMD Evolution	BOER, Daniel

Parallel-V: S3

08:45	Transverse Momentum and Spin Dependent Distribution Functions	MULDERS, Piet J.

and many others ………

Where should I start?

Introduction

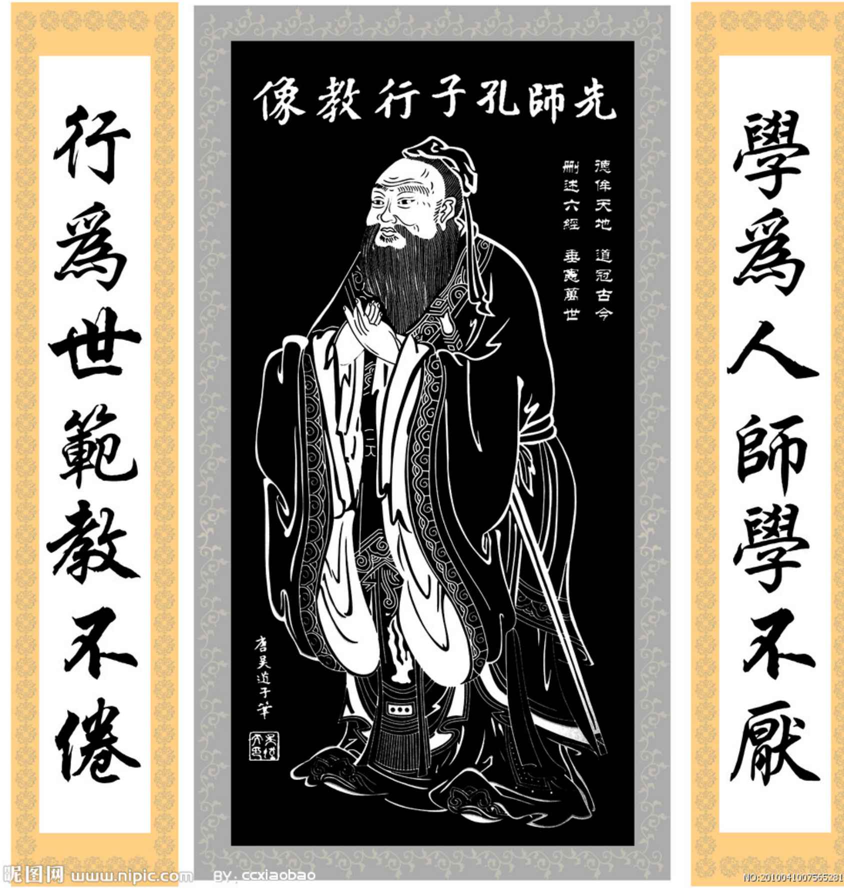
《论语》

子曰：“温故而知新，
可以为师矣”

“The Analects of Confucius”

Confucius said:

“One can be a master if he gets to know new things by reviewing the old knowledge”.



I therefore start with inclusive DIS and one dimensional imaging of the nucleon.

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Contents

Inclusive deep inelastic scattering (DIS) $e^- + N \to e^- + X$

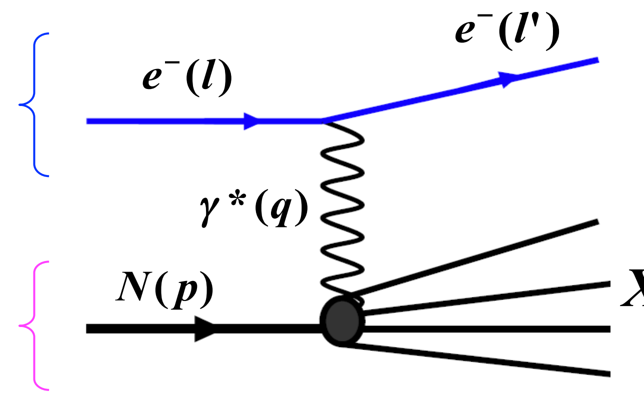
"温故而知新，
可以为师矣"

The differential cross section

$$d\sigma = \frac{\alpha_{em}^2}{sQ^4} L^{\mu\nu}(l,\lambda_l,l',\lambda_l')W_{\mu\nu}(q,p,S)\frac{d^3l'}{2E'}$$

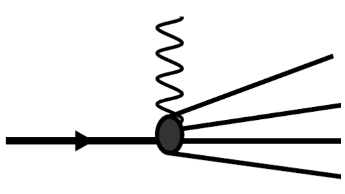
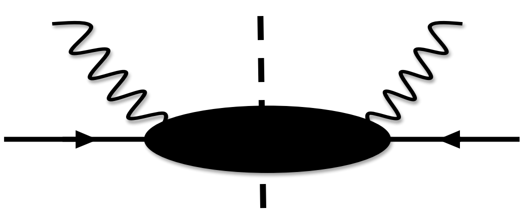
leptonic tensor

hadronic tensor



$$-q^2 = Q^2 \qquad x_B = \frac{Q^2}{2q\cdot p}, \qquad y = \frac{q\cdot p}{l\cdot p}$$

The hadronic tensor: $W_{\mu\nu}(q,p,S) = \sum_X \langle p,S \,|\, J_\mu(0) \,|\, X\rangle\langle X \,|\, J_\nu(0) \,|\, p,S\rangle (2\pi)^4 \delta^4(p+q-p_X)$

$W_{\mu\nu}(q,p,S) =$ |  $|^2$ = 

$|\mathcal{M}|^2$ $\qquad\qquad$ $\mathcal{M}\times\mathcal{M}^*$

Inclusive deep inelastic scattering (DIS) $e^- + N \to e^- + X$

"温故而知新，可以为师矣"

Kinematics (Lorentz invariance, symmetries, conservation laws….)：

Gauge invariance: $q^\mu W_{\mu\nu}(q,p,S) = 0$

Parity invariance: $W_{\mu\nu}(\tilde{q},\tilde{p},-\tilde{S}) = W^{\mu\nu}(q,p,S)$

Hermiticity: $W^*_{\mu\nu}(q,p,S) = W_{\nu\mu}(q,p,S)$

general form

$$\Rightarrow W_{\mu\nu}(q,p,S) = W^{(S)}_{\mu\nu}(q,p) + iW^{(A)}_{\mu\nu}(q,p,S)$$

$$W^{(S)}_{\mu\nu}(q,p) = 2(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2})F_1(x,Q^2) + \frac{1}{xQ^2}(q_\mu + 2xp_\mu)(q_\nu + 2xp_\nu)F_2(x,Q^2)$$

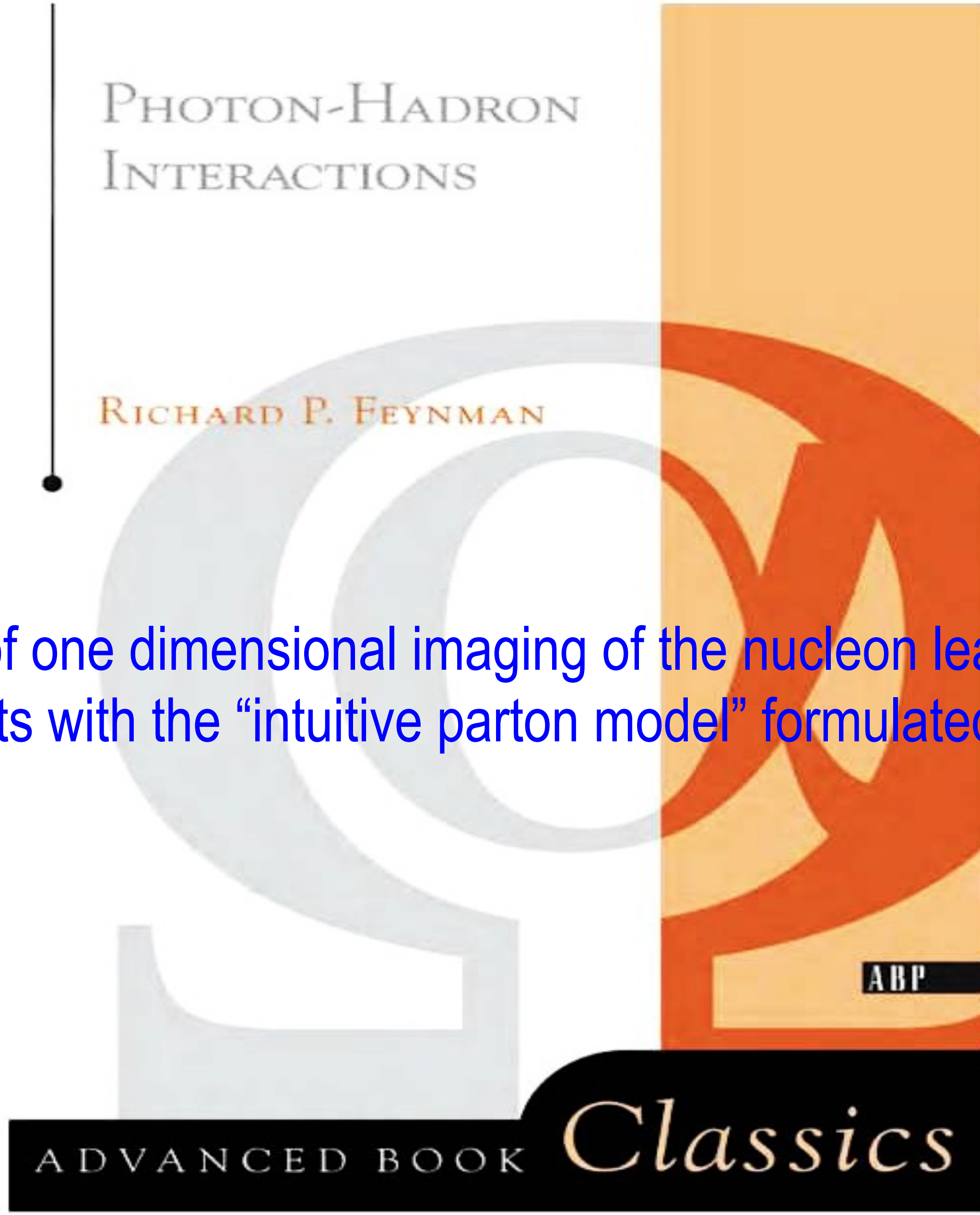
$$W^{(A)}_{\mu\nu}(q,p,S) = \frac{2M\varepsilon_{\mu\nu\rho\sigma}q^\rho}{p\cdot q}\left\{S^\sigma g_1(x,Q^2) + (S^\sigma - \frac{S\cdot q}{p\cdot q}p^\sigma)g_2(x,Q^2)\right\}$$

$$\frac{d\sigma^{unp}}{dxdy} = \frac{4\pi\alpha^2 s}{Q^4}\left\{xy^2F_1(x,Q^2) + (1 - y - \frac{xyM^2}{s})F_2(x,Q^2)\right\}$$

$$\frac{d\Delta\sigma}{dxdy} = \frac{4\pi\alpha^2 s}{Q^4}\left\{xy(2 - y - \frac{2xyM^2}{s})g_1(x,Q^2) + 8\frac{x^2yM^2}{s}g_2(x,Q^2)\right\}$$

“Original / Intuitive” Parton Model

“温故而知新，
可以为师矣”



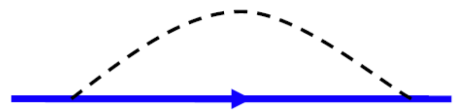
Our knowledge of one dimensional imaging of the nucleon learned from DIS experiments starts with the “intuitive parton model” formulated e.g. in this book.

"Original / Intuitive" Parton Model

"温故而知新，可以为师矣"

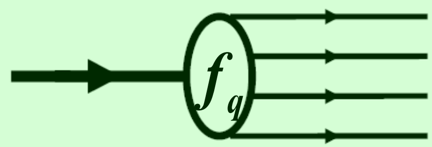
Feynman (1969);
Bjorken & Paschos (1969)

The model:

Virtual processes such as 

Because of time dilatation, in the infinite momentum frame, they exist forever.

⇨ A fast moving proton = A beam of free partons



⇨ $$|\mathcal{M}(eN \to eX)|^2 = \sum_q \int dx\, f_q(x)\, |\hat{\mathcal{M}}(eq \to eq)|^2$$

scattering amplitude squared

$f_q(x)$: parton number density, known as PDF

$x = k/p$: momentum fraction carried by the parton

⇨ $$F_1(x) = \sum_q e_q^2 f_q(x) \qquad g_1(x) = \sum_q e_q^2 \Delta f_q(x) \qquad g_1(x) + g_2(x) = \sum_q e_q^2 \delta f_q(x)$$

$$F_2(x) = 2x F_1(x)$$

"Original / Intuitive" Parton Model

"温故而知新，可以为师矣"

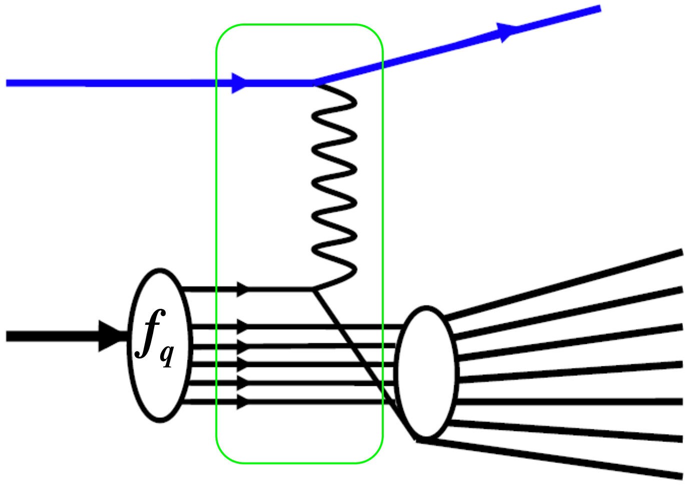
It is just the impulse approximation!

Impulse Approximation:

(1) during the interaction of lepton with parton,
interaction between partons is neglected;

(2) lepton interacts only with one single parton;

(3) interaction with different partons adds incoherently.



Approximation: What is neglected? Controllable?

Parton distribution functions: A proper (quantum field theoretical) definition?

⟹ A quantum field theoretical formulation ?

Quantum field theoretical formulation of parton model

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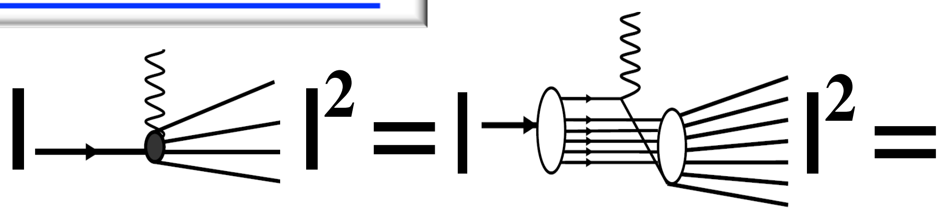
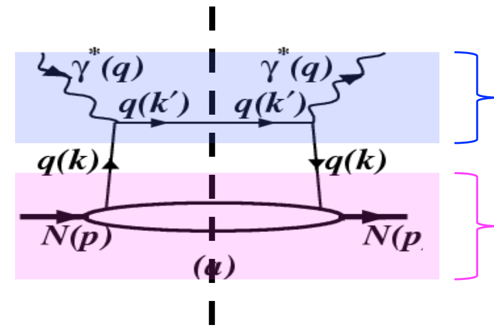
Parton model without QCD:

$$W_{\mu\nu}(q,p,S) =$$





$$W_{\mu\nu}(q,p,S) = \int \frac{d^4k}{(2\pi)^4} Tr\left[\hat{H}_{\mu\nu}(k,q)\hat{\phi}(k,p,S)\right]$$

the calculable hard part $\hat{H}_{\mu\nu}(k,q) = \gamma_\mu(\not{k}+\not{q})\gamma_\nu(2\pi)\delta_+\left((k+q)^2\right)$

the quark-quark correlator $\hat{\phi}(k,p,S) = \int d^4z e^{ikz}\langle p,S|\bar{\psi}(0)\psi(z)|p,S\rangle$

Collinear approximation: $p \approx p^+\bar{n}, \quad k \approx xp$

$$\Rightarrow \quad W_{\mu\nu}(q,p) \approx \left[(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}) + \frac{1}{2xq\cdot p}(q+2xp)_\mu(q+2xp)_\nu\right] f_q(x)$$

operator expression of the number density : $f_q(x) = \int \frac{dz^-}{2\pi} e^{ixp^+z^-}\langle p|\bar{\psi}(0)\frac{\gamma^+}{2}\psi(z)|p\rangle$

no local (color) gauge invariance!

Inclusive DIS with "multiple gluon scattering"

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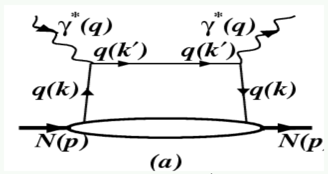
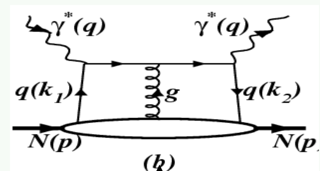
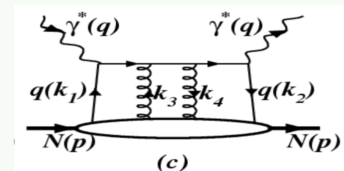
To get the gauge invariance, we need to take the "multiple gluon scattering" into account

$W_{\mu\nu}(q,p,S) =$ 

$+$ 

$+$ 

$+ \cdots$

$$W_{\mu\nu}(q,p,S) = W^{(0)}_{\mu\nu}(q,p,S) + W^{(1)}_{\mu\nu}(q,p,S) + W^{(2)}_{\mu\nu}(q,p,S) + \ldots$$

$$W^{(0)}_{\mu\nu}(q,p,S) = \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\hat{H}^{(0)}_{\mu\nu}(k,q)\hat{\phi}^{(0)}(k,p,S)\right]$$

$$W^{(1)}_{\mu\nu}(q,p,S) = \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4} \mathrm{Tr}\left[\hat{H}^{(1)\rho}_{\mu\nu}(k_1,k_2,q)\hat{\phi}^{(1)}_{\rho}(k_1,k_2,p,S)\right]$$

no gauge invariance!

Collinear approximation:

- ✪ Approximating the hard part at $k = xp$: $\hat{H}^{(0)}_{\mu\nu}(k,q) \approx \hat{H}^{(0)}_{\mu\nu}(x)$ $\quad \hat{H}^{(1)\rho}_{\mu\nu}(k_1,k_2,q) \approx \hat{H}^{(1)\rho}_{\mu\nu}(x_1,x_2)$
- ✪ Keeping only the longitudinal component of the gluon field: $A_\rho(y) \approx n \cdot A(y) \frac{p_\rho}{n \cdot p}$
- ✪ Using Ward identities, e.g., $p_\rho \hat{H}^{(1,L)\rho}_{\mu\nu}(x_1,x_2) = \frac{\hat{H}^{(0)}_{\mu\nu}(x_1)}{x_2 - x_1 - i\varepsilon}$ all $\hat{H}^{(j)}_{\mu\nu}(x_i)$'s reduce to $\hat{H}^{(0)}_{\mu\nu}(x)$
- ✪ Adding all the terms together ⇒

Inclusive DIS: LO pQCD, leading twist

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$$W_{\mu\nu}(q,p,S) \approx \tilde{W}^{(0)}_{\mu\nu}(q,p,S) = \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\hat{\Phi}^{(0)}(k;p,S)\hat{H}^{(0)}_{\mu\nu}(x)\right]$$

LO & leading twist

$$\hat{\Phi}^{(0)}(k;p,S) = \int d^4z e^{ikz} \langle p,S|\bar{\psi}(0)\mathcal{L}(0,z)\psi(z)|p,S\rangle$$

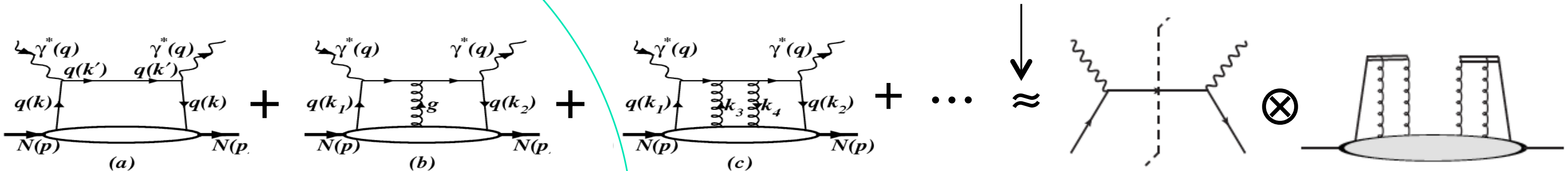
Un-integrated quark-quark correlator: contain QCD interactions, gauge invariant !

$$\mathcal{L}(0,z) = \mathcal{L}^\dagger(-\infty,0)\mathcal{L}(-\infty,z) \qquad \mathcal{L}(-\infty,z) = Pe^{ig\int_{-\infty}^{z^-} dy^- A^+(0,y^-,\vec{z}_\perp)}$$

gauge link

Graphically:

collinear approximation



Since

$$\hat{H}^{(0)}_{\mu\nu}(x) = \hat{h}^{(0)}_{\mu\nu}\delta(x-x_B)$$
$$\hat{h}^{(0)}_{\mu\nu} = \gamma_\mu \not{n} \gamma_\nu$$

$$\Rightarrow \quad \tilde{W}^{(0)}_{\mu\nu}(q,p,S) = \int dx \mathrm{Tr}\left[\hat{\Phi}^{(0)}(x;p,S)\, h^{(0)}_{\mu\nu}\right]\delta(x-x_B)$$

$$\hat{\Phi}^{(0)}(x;p,S) \equiv \int \frac{d^4k}{(2\pi)^4}\delta(x-k^+/p^+)\hat{\Phi}^{(0)}(k;p,S) = \int dz^- e^{ixp^+z^-}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,z^-)\psi(0,z^-,\vec{0}_\perp)|p,S\rangle$$

(only) one dimensional imaging of the nucleon via inclusive DIS.

Inclusive DIS: LO pQCD, leading & higher twists

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Collinear expansion:

Ellis, Furmanski, Petronzio, (1982,1983)
Qiu, Sterman (1990,1991)

- Expanding the hard part at $k = xp$:

$$\hat{H}^{(0)}_{\mu\nu}(k,q) = \hat{H}^{(0)}_{\mu\nu}(x) + \frac{\partial \hat{H}^{(0)}_{\mu\nu}(x)}{\partial k^\rho} \omega_\rho{}^{\rho'} k_{\rho'} + \ldots\ldots$$

$$\hat{H}^{(1)\rho}_{\mu\nu}(k_1,k_2,q) = \hat{H}^{(1)\rho}_{\mu\nu}(x_1,x_2) + \frac{\partial \hat{H}^{(1)\rho}_{\mu\nu}(x_1,x_2)}{\partial k_1^\sigma} \omega_\sigma{}^{\sigma'} k_{1\sigma'} + \ldots\ldots$$

$$\hat{H}^{(0)}_{\mu\nu}(x) \equiv \hat{H}^{(0)}_{\mu\nu}(k = xp, q)$$

$$\frac{\partial \hat{H}^{(0)}_{\mu\nu}(x)}{\partial k^\rho} \equiv \left.\frac{\partial \hat{H}^{(0)}_{\mu\nu}(k,q)}{\partial k^\rho}\right|_{k=xp}$$

- Decomposition of the gluon field:

$$A_\rho(y) = n \cdot A(y) \frac{p_\rho}{n \cdot p} + \omega_\rho{}^{\rho'} A_{\rho'}(y)$$

- Using the Ward identities such as,

$$\frac{\partial \hat{H}^{(0)}_{\mu\nu}(x)}{\partial k^\rho} = -\hat{H}^{(1)\rho}_{\mu\nu}(x,x), \qquad p_\rho \hat{H}^{(1,L)\rho}_{\mu\nu}(x_1,x_2) = \frac{\hat{H}^{(0)}_{\mu\nu}(x_1)}{x_2 - x_1 - i\varepsilon}$$

to replace the derivatives etc.

- Adding all the terms with the same hard part together ⟹

$$x = k^+ / p^+$$

$$\omega_\rho{}^{\rho'} \equiv g_\rho{}^{\rho'} - \bar{n}_\rho n^{\rho'}$$

$$\omega_\rho{}^{\rho'} k_{\rho'} = (k - xp)_\rho$$

$$k^\pm = \frac{1}{\sqrt{2}}(k_0 \pm k_3)$$

$$n = (0,1,\vec{0}_\perp)$$

$$\bar{n} = (1,0,\vec{0}_\perp)$$

Inclusive DIS: LO pQCD, leading & higher twists

"温故而知新，可以为师矣"

$$W_{\mu\nu}(q,p,S)=\tilde{W}^{(0)}_{\mu\nu}(q,p,S)+\tilde{W}^{(1)}_{\mu\nu}(q,p,S)+\tilde{W}^{(2)}_{\mu\nu}(q,p,S)+\ldots$$

twist-2, twist-3 and twist-4 contributions

$$\tilde{W}^{(0)}_{\mu\nu}(q,p,S)=\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\hat{\Phi}^{(0)}(k,p,S)\hat{H}^{(0)}_{\mu\nu}(x)\right]$$

$$\hat{\Phi}^{(0)}(k,p,S)=\int d^4z e^{ikz}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,z)\psi(z)|p,S\rangle$$

gauge invariant quark-quark correlator

twist-3, twist-4 and even higher twist contributions

$$\tilde{W}^{(1)}_{\mu\nu}(q,p,S)=\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\mathrm{Tr}\left[\hat{\Phi}^{(1)}_{\rho'}(k_1,k_2,p,S)\hat{H}^{(1)\rho}_{\mu\nu}(x_1,x_2)\omega_\rho^{\ \rho'}\right]$$

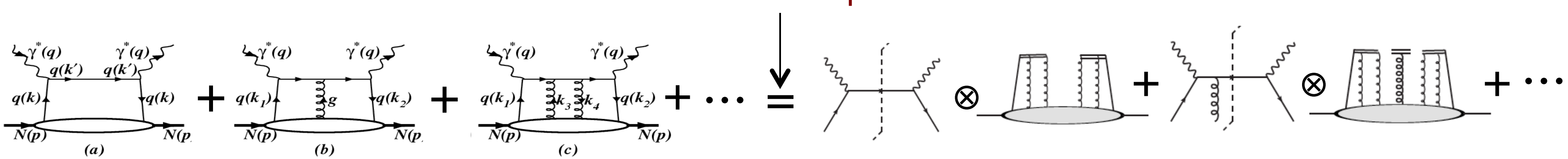
$$\hat{\Phi}^{(1)}_{\rho}(k_1,k_2,p,S)=\int d^4z d^4y e^{ik_1y+ik_2(z-y)}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,y)D_\rho(y)\mathcal{L}(y,z)\psi(z)|p,S\rangle$$

$$D_\rho(y)=-i\partial_\rho+gA_\rho(y)$$

gauge invariant quark-gluon-quark correlator

⇨ A consistent framework for inclusive DIS $e^-N\rightarrow e^-X$ including leading & higher twists

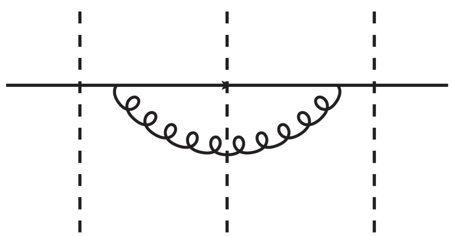
collinear expansion

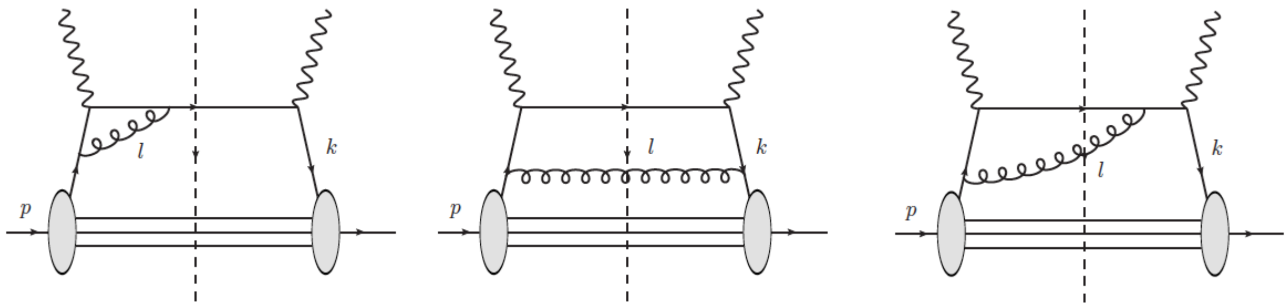


Inclusive DIS: Higher order pQCD

Factorization theorem and QCD evolution of PDFs

“Loop diagram contributions”





(a) (b) (c)

factorization & resummation →

Higher order pQCD contributions;
Evolution of PDFs.

Inclusive DIS and parton model: brief summary

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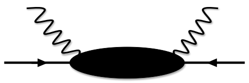
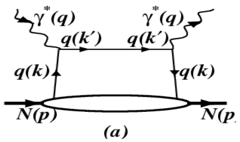
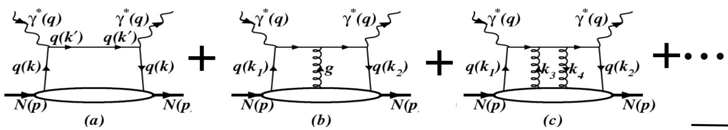
List of to do’s --- the recipe



kinematics (symmetries, … …) → general form of the cross section



parton model without QCD interaction → (collinear approximation) → leading order pQCD, leading twist, no evolution, no gauge invariance



parton model + “multiple gluon scattering” → (collinear expansion) → leading order pQCD, leading & higher twist, no evolution, but gauge invariance

parton model + “multiple gluon scattering” + “loop diagram contributions” → (collinear expansion + factorization & resummation) → leading & higher order pQCD, leading & higher twist, evolution & gauge invariance

experiments → parameterizations

Global QCD analysis and PDFLIB

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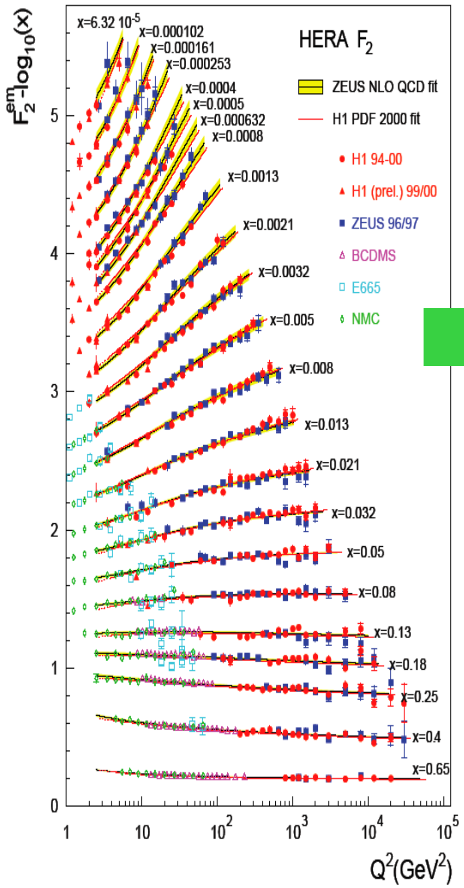
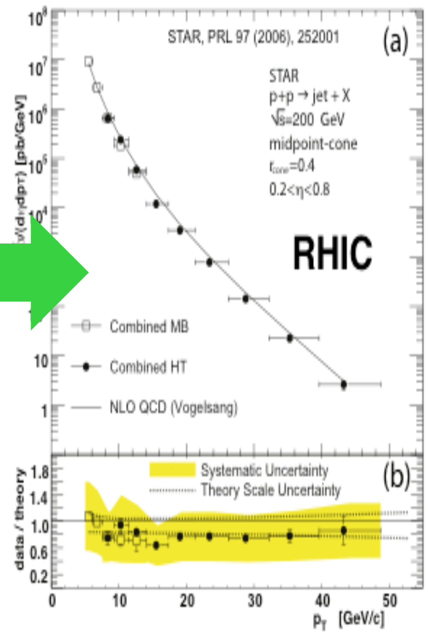
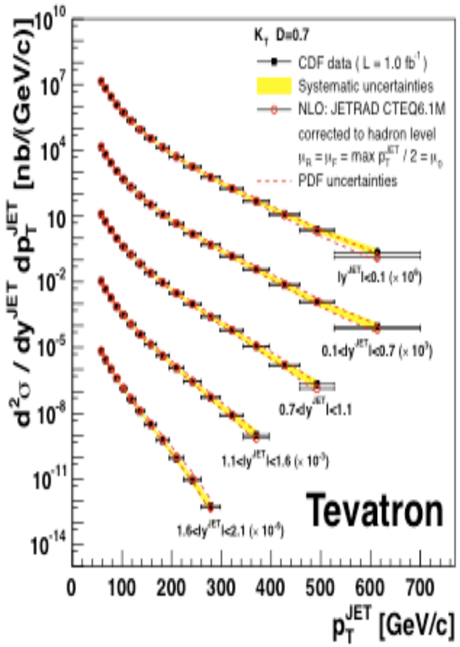
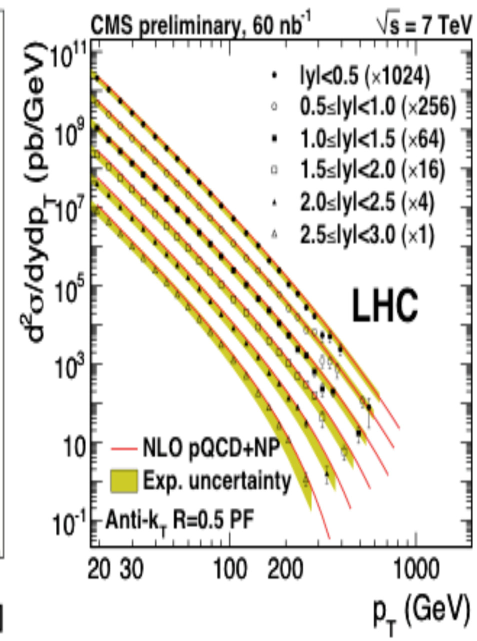
Very successful!









parameterize at 0.3 TeV e-p (HERA), predict p-p and p-p-bar at 0.2, 1.96, and 7 TeV.

The need for three dimensional imaging of the nucleon

“温故而知新，可以为师矣”

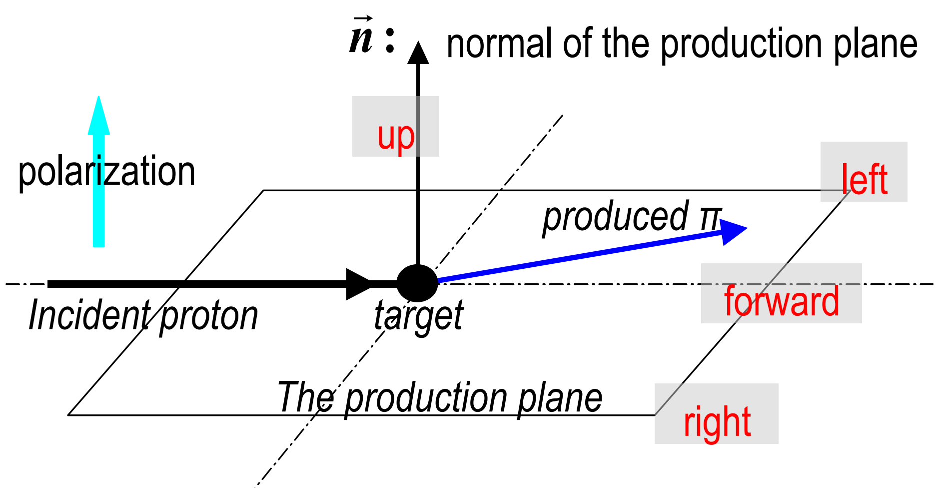
Single-spin left-right asymmetry

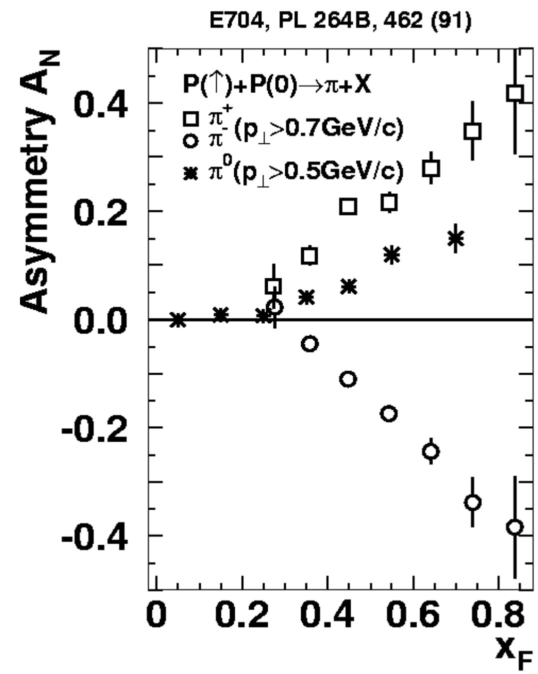
where study of TMD PDFs started.

$$p(\uparrow) + p \to \pi + X$$

$$A_N = \frac{N_L - N_R}{N_L + N_R}$$

azimuthal asymmetry $A_N = A^{\cos\phi}$





<u>Theory:</u> Kane, Pumplin, Repko (1978), pQCD leads to $a_N[q(\uparrow)q \to q\,q] = 0$

The need for three dimensional imaging of the nucleon

"温故而知新，可以为师矣"

Single-spin left-right asymmetry studies

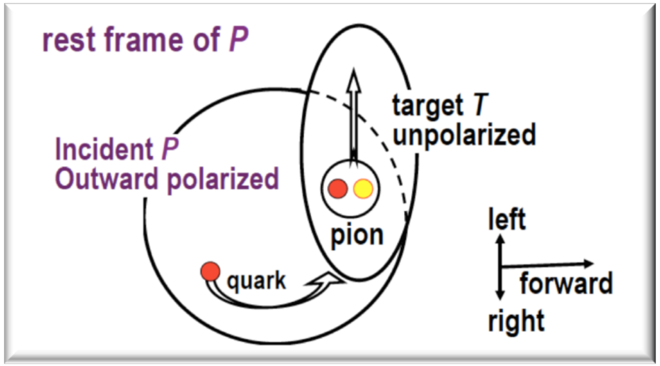
a brief history for Sivers function

1991, Sivers: asymmetric quark distribution in polarized nucleon (Sivers function)

$$f_q(x,k_\perp;S_\perp) = f_q(x,k_\perp) + \frac{1}{M}(\vec{k}_\perp \times \hat{\vec{p}})\cdot \vec{S}_\perp \; f_{1T}^\perp(x,k_\perp)$$

1993, Boros, Liang & Meng:

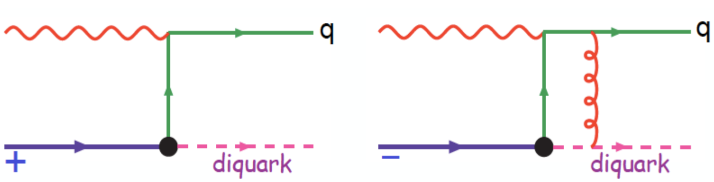
an intuitive picture: quark orbital angular momentum + "surface effect"



1993, Collins: P&T invariance $\Longrightarrow$ $f_{1T}^\perp(x,k_\perp) = 0$ (proof of non-existence of Sivers effect).

2002, Brodsky, Hwang, Schmidt: take "final state interaction" into account,

consider



$\Longrightarrow$ $f_{1T}^\perp(x,k_\perp) \neq 0$

2002, Ji, Yuan: "final state interaction" = "gauge link".

Lesson: do not forget the gauge link!

The need for three dimensional imaging of the nucleon

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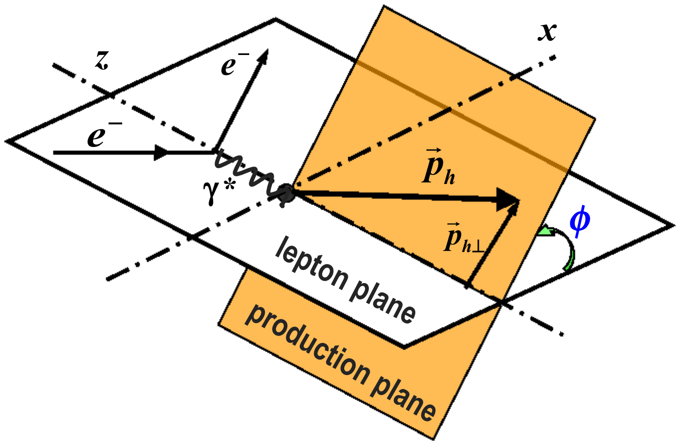
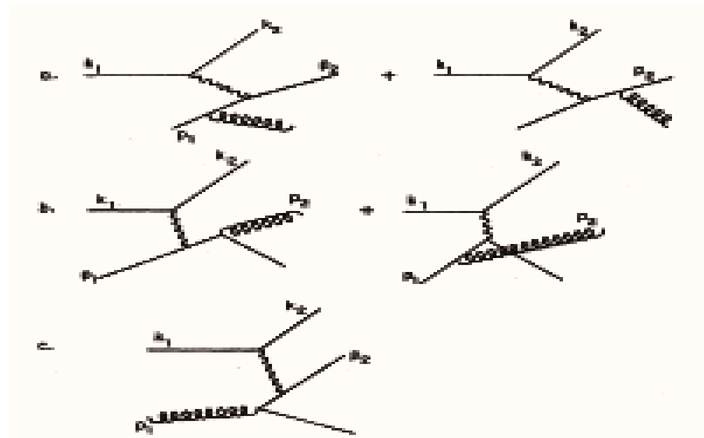
Azimuthal asymmetry studies: $e^- + N \to e^- + q(jet) + X$

1977, Georgi & Politzer: gluon radiation ⇨ azimuthal asymmetry ⇨ “Clean test to pQCD”





1978, Cahn: generalize parton model to include an intrinsic $\vec{k}_\perp$:

“Cahn effect”

$$\langle \cos\varphi \rangle = -\frac{|\vec{k}_\perp|}{Q} \frac{2(2-y)\sqrt{1-y}}{1+(1-y)^2} \quad \text{(twist 3)} \qquad \langle \cos 2\phi \rangle = \frac{|\vec{k}_\perp|^2}{Q^2} \frac{2(1-y)}{1+(1-y)^2} \quad \text{(twist 4)}$$

higher twist, nevertheless significant ! $|\vec{k}_\perp| \sim 0.3 - 0.7 GeV$ $\quad |\vec{k}_\perp|/Q \sim 0.1$

⇨ **Lesson: do not forget higher twists!**

The need for three dimensional imaging of the nucleon

"温故而知新，可以为师矣"

A short summary:

Study of single-spin asymmetry ⟹ gauge link

Study of azimuthal asymmetry ⟹ higher twists

⟹ collinear expansion!

⟹ We need to use the field theoretical formulation rather than the intuitive parton model

Contents

TMD PDFs defined via quark-quark correlator

The quark-quark correlator $\hat{\Phi}^{(0)}(k;p,S)=\int d^4z e^{ikz}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,z)\psi(z)|p,S\rangle$

integrate over k^- : $\hat{\Phi}^{(0)}(x,k_\perp;p,S)=\int dz^- d^2z_\perp e^{i(xp^+z^- - \vec{k}_\perp\cdot\vec{z}_\perp)}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,z)\psi(z)|p,S\rangle$

$$
\begin{aligned}
\hat{\Phi}^{(0)}(x,k_\perp;p,S)=\frac{1}{2}\Big[\ & \Phi_S^{(0)}(x,k_\perp;p,S) && \text{——— scalar}\\
&+\gamma_5\ \Phi_{PS}^{(0)}(x,k_\perp;p,S) && \text{——— pseudo-scalar}\\
&+\lambda^\alpha\ \Phi_\alpha^{(0)}(x,k_\perp;p,S) && \text{——— vector}\\
&+\gamma_5\lambda^\alpha\ \tilde{\Phi}_\alpha^{(0)}(x,k_\perp;p,S) && \text{——— axial vector}\\
&+i\gamma_5\sigma^{\alpha\beta}\ \Phi_{T\alpha\beta}^{(0)}(x,k_\perp;p,S)\Big] && \text{——— tensor}
\end{aligned}
$$

e.g.: $\Phi_\alpha^{(0)}(x,k_\perp;p,S)=\frac{1}{2}\mathrm{Tr}\left[\gamma_\alpha\hat{\Phi}^{(0)}(x,k_\perp;p,S)\right]$

$$=\int d^4z e^{ikz}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,z)\frac{\gamma_\alpha}{2}\psi(z)|p,S\rangle$$

TMD PDFs defined via quark-quark correlator

The Lorentz decomposition

totally 8(twist 2)+16(twist 3)+8(twist 4) components

$$\Phi_S^{(0)}(x,k_\perp;p,S) = M\left[e(x,k_\perp) + \frac{\varepsilon_{\perp\rho\sigma}k_\perp^\rho S_T^\sigma}{M} e_T^\perp(x,k_\perp)\right]$$

twist-3

$$\Phi_\alpha^{(0)}(x,k_\perp;p,S) = p_\alpha\left[f_1(x,k_\perp) + \frac{\varepsilon_{\perp\rho\sigma}k_\perp^\rho S_T^\sigma}{M} f_{1T}^\perp(x,k_\perp)\right]$$

twist-2

$$+\ k_{\perp\alpha}\left[f^\perp(x,k_\perp) + \frac{\varepsilon_{\perp\rho\sigma}k_\perp^\rho S_T^\sigma}{M} f_T^{\perp 1}(x,k_\perp)\right] + \varepsilon_{\perp\alpha\rho}k_\perp^\rho\left[\lambda f_L^\perp(x,k_\perp) + \frac{k_\perp\cdot S_T}{M} f_T^{\perp 2}(x,k_\perp)\right]$$

$$+\frac{M^2}{p^+}n_\alpha\left[f_3(x,k_\perp) + \frac{\varepsilon_{\perp\rho\sigma}k_\perp^\rho S_T^\sigma}{M} f_{3T}^\perp(x,k_\perp)\right]$$

twist-4

$$p = p^+\bar{n} + \frac{M^2}{2p^+}n \qquad S = \lambda\frac{p^+}{M}\bar{n} + S_T - \lambda\frac{M^2}{2p^+}n \qquad \varepsilon_{\perp\rho\sigma} = \varepsilon_{\alpha\beta\rho\sigma}\bar{n}^\alpha n^\beta$$

See e.g., Goeke, Metz, Schlegel, PLB 618, 90 (2005);
Mulders, talk yesterday, and lectures in 17th Taiwan nuclear physics summer school, Aug. 25-28, 2014; and …………..

TMD PDFs defined via quark-quark correlator

The Lorentz decomposition

totally 8(twist 2)+16(twist 3)+8(twist 4) components

$$\Phi_{PS}^{(0)}(x,k_\perp;p,S) = M\left[\lambda e_L(x,k_\perp) + \frac{k_\perp\cdot S_T}{M} e_T(x,k_\perp)\right]$$

$$\tilde{\Phi}_{\alpha}^{(0)}(x,k_\perp;p,S) = p_\alpha\left[\lambda g_{1L}(x,k_\perp) + \frac{k_\perp\cdot S_T}{M} g_{1T}(x,k_\perp)\right]$$

$$+ S_{T\alpha} g_T^{\perp 1}(x,k_\perp) + \frac{k_{\perp\alpha}}{M}\left[\lambda g_L^\perp(x,k_\perp) + \frac{k_\perp\cdot S_T}{M} g_T^{\perp 2}(x,k_\perp)\right] - \frac{\varepsilon_{\perp\alpha\beta}k_\perp^\beta}{M} g^\perp(x,k_\perp)$$

$$+\frac{M^2}{p^+} n_\alpha\left[\lambda g_{3L}(x,k_\perp) + \frac{k_\perp\cdot S_T}{M} g_{3T}^\perp(x,k_\perp)\right]$$

$$\Phi_{\rho\alpha}^{(0)}(x,k_\perp;p,S) = p_{[\rho}S_{T\alpha]}h_{1T}(x,k_\perp) + \frac{p_{[\rho}k_{\perp\alpha]}}{M}\left[\lambda h_{1L}^\perp(x,k_\perp) - \frac{k_\perp\cdot S_T}{M} h_{1T}^\perp(x,k_\perp)\right] - \frac{p_{[\rho}\varepsilon_{\perp\alpha]\beta}k_\perp^\beta}{M} h_1^\perp(x,k_\perp)$$

$$+ \frac{S_{T[\rho}k_{\perp\alpha]}}{M} h_T^\perp(x,k_\perp) - \varepsilon_{\perp\rho\alpha} h(x,k_\perp) + \bar{n}_{[\rho}n_{\alpha]}\left[\lambda h_L(x,k_\perp) - \frac{k_\perp\cdot S_T}{M} h_T(x,k_\perp)\right]$$

$$+\frac{M^2}{p^+}\left\{n_{[\rho}S_{T\alpha]}h_{3T}(x,k_\perp) + \frac{n_{[\rho}k_{\perp\alpha]}}{M}\left[\lambda h_{3L}^\perp(x,k_\perp) + \frac{k_\perp\cdot S_T}{M} h_{3T}^\perp(x,k_\perp)\right] - \frac{n_{[\rho}\varepsilon_{\perp\alpha]\beta}k_\perp^\beta}{M} h_3^\perp(x,k_\perp)\right\}$$

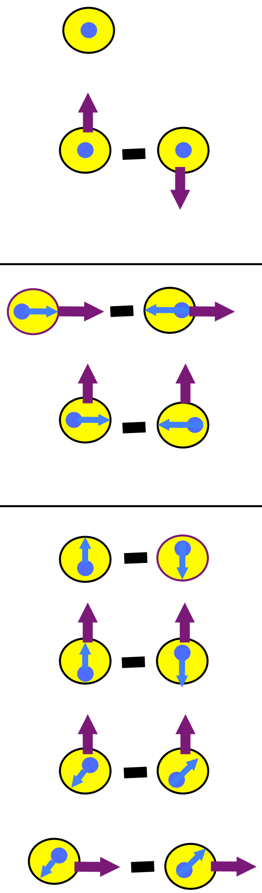
twist-3

twist-2

twist-4

TMD PDFs defined via quark-quark correlator

At leading twist (twist 2) f, g, h: quark un-, longitudinally, transversely polarized

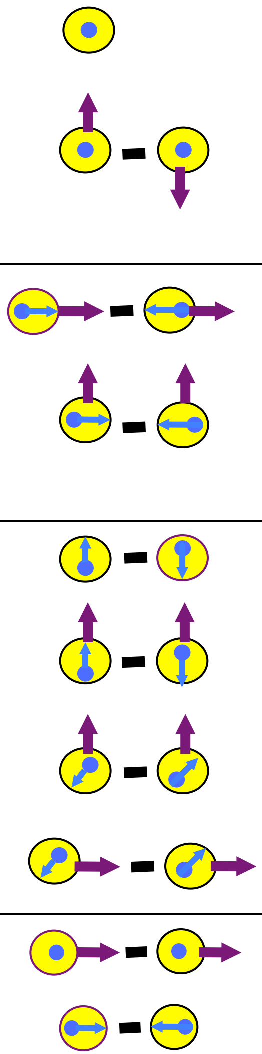


	if no gauge link	integrate over $k_\perp$	
$f_1(x,k_\perp)$		$q(x)$	number density
$f_{1T}^{\perp}(x,k_\perp)$	0	×	Sivers function
$g_{1L}(x,k_\perp)$		$\Delta q(x)$	helicity distribution
$g_{1T}^{\perp}(x,k_\perp)$		×	Worm gear: trans-helicity
$h_1^{\perp}(x,k_\perp)$	0	×	Boer-Mulders function
$h_{1T}(x,k_\perp)$		$\delta q(x)$	transversity distribution
$h_{1T}^{\perp}(x,k_\perp)$			pretzelocity
$h_{1L}^{\perp}(x,k_\perp)$		×	Worm gear: longi-transversity

TMD PDFs defined via quark-quark correlator

At twist 3: (16)

they are NOT the probability distributions but are related to different polarization cases as indicated in the table



$e(x,k_\perp)$, $f^\perp(x,k_\perp)$	0	$\frac{f_1(x,k_\perp)}{x}$	$e(x)$, $\times$	number density
$e_T^\perp(x,k_\perp)$,	0		$\times$	Sivers function
$f_T^{\perp 1}(x,k_\perp)$, $f_T^{\perp 2}(x,k_\perp)$	0	0	$\times$, $\times$	
$e_L(x,k_\perp)$, $g_L^\perp(x,k_\perp)$	0	$\frac{g_{1L}(x,k_\perp)}{x}$	$\times$, $\times$	helicity distribution
$e_T(x,k_\perp)$,	0		$\times$	
$g_T(x,k_\perp)$, $g_T^\perp(x,k_\perp)$	0	$\frac{g_{1T}(x,k_\perp)}{x}$	$g'_T(x)$	Worm gear: trans-helicity
$h(x,k_\perp)$	0		$\times$	Boer-Mulders function
$h_T^\perp(x,k_\perp)$	$\frac{h_{1T}^\perp(x,k_\perp)}{x}$		$\times$	transversity distribution
$h_T(x,k_\perp)$	$\frac{k_\perp^2 h_{1T}^\perp(x,k_\perp)}{M^2 x}$		$\times$	pretzelocity
$h_L(x,k_\perp)$	$\frac{k_\perp^2 h_{1L}^\perp(x,k_\perp)}{M^2 x}$		$h_L(x)$	Worm gear: longi-transversity
$f_L^\perp(x,k_\perp)$	0		$\times$	
$g^\perp(x,k_\perp)$	0		$\times$	

TMD PDFs defined via quark-quark correlator

Twist-3 TMD PDFs defined via quark-gluon-quark correlator

$$\hat{\Phi}^{(1)}(x,k_\perp;p,S) = \int dz^- d^2 z_\perp e^{i(xp^+z^- - \vec{k}_\perp \cdot \vec{z}_\perp)} \langle p,S | \bar{\psi}(0) \mathcal{L}(0,z) D(z) \psi(z) | p,S \rangle$$

However, they are NOT independent!

$$\gamma \cdot D(z)\psi(z) = 0 \Rightarrow \begin{cases} x\Phi^{(0)}_{\perp\rho}(x,k_\perp;p,S) = -\frac{n^\alpha}{p^+}\left[\mathrm{Re}\,\Phi^{(1)}_{\alpha\rho}(x,k_\perp;p,S) - \varepsilon_{\perp\rho}{}^{\sigma}\,\mathrm{Im}\,\tilde{\Phi}^{(1)}_{\alpha\sigma}(x,k_\perp;p,S)\right] \\ x\tilde{\Phi}^{(0)}_{\perp\rho}(x,k_\perp;p,S) = -\frac{n^\alpha}{p^+}\left[\mathrm{Re}\,\tilde{\Phi}^{(1)}_{\alpha\rho}(x,k_\perp;p,S) + \varepsilon_{\perp\rho}{}^{\sigma}\,\mathrm{Im}\,\Phi^{(1)}_{\alpha\sigma}(x,k_\perp;p,S)\right] \end{cases}$$

all the twist-3 components involved in SIDIS defined via quark-gluon-quark correlator → replaced by → the corresponding twist-3 components defined via quark-quark correlator

↑ the relationships obtained from equation of motion

See e.g., Y.K. Song, J.H. Gao, ZTL & X.N. Wang, PRD (2014); …….

TMD PDFs & FFs defined via quark-quark correlators

Twist-2 TMD PDFs

nucleon polarization ↓ / quark polarization →	U	L	T
U	$f_1(x,k_\perp)$ number density		$h_1^\perp(x,k_\perp)$ Boer-Mulders function
L		$g_{1L}(x,k_\perp)$ helicity distribution	$h_{1L}^\perp(x,k_\perp)$ Worm-gear/longi-transversit
T	$f_{1T}^\perp(x,k_\perp)$ Sivers function	$g_{1T}^\perp(x,k_\perp)$ Worm-gear/trans-helicity	$h_{1T}(x,k_\perp)$ transversity distribution $h_{1T}^\perp(x,k_\perp)$ pretzelosity

Twist-2 TMD FFs

nucleon polarization ↓ / quark polarization →	U	L	T
U	$D_1(z,k_{F\perp})$ number density		$H_1^\perp(z,k_{F\perp})$ Collins function
L		$G_{1L}(z,k_{F\perp})$ spin transfer	$H_{1L}^\perp(z,k_{F\perp})$
T	$D_{1T}^\perp(z,k_{F\perp})$	$G_{1T}^\perp(z,k_{F\perp})$	$H_{1T}(z,k_{F\perp})$ spin transfer $H_{1T}^\perp(z,k_{F\perp})$

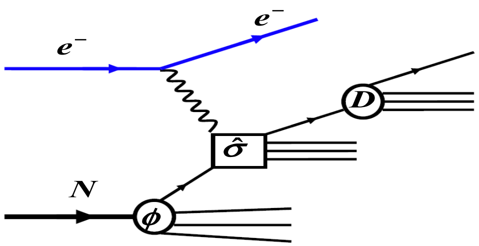
Twist-3 TMD PDFs

nucleon polarization	U	L	T
U	$e(x,k_\perp), f^\perp(x,k_\perp)$ number density	$g^\perp(x,k_\perp)$	$h(x,k_\perp)$ Boer-Mulders function
L	$f_L^\perp(x,k_\perp)$	$e_L(x,k_\perp), g_L^\perp(x,k_\perp)$ helicity distribution	$h_L(x,k_\perp)$ Worm gear/ longi-transversity
T	$e_T^\perp(x,k_\perp)$, $f_T^{\perp 1}(x,k_\perp), f_T^{\perp 2}(x,k_\perp)$ Sivers function	$e_T(x,k_\perp)$, $g_T(x,k_\perp), g_T^\perp(x,k_\perp)$ Worm gear/ trans-helicity	$h_T^\perp(x,k_\perp)$ transversity distribution $h_T(x,k_\perp)$ pretzelosity

Contents

Access TMDs via semi-inclusive high energy reactions

Semi-inclusive reactions



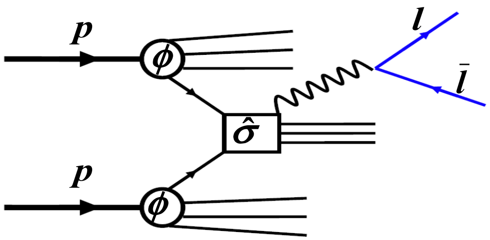
DIS: $e+N \to e+h+X$

DESY, COMPASS, Jefferson Lab, EIC

TMD PDFs:

$f_1, f_{1T}^{\perp}, g_{1L}, g_{1T}, h_1, h_1^{\perp}, h_{1L}^{\perp}, h_{1T}^{\perp} \ldots$

TMD FFs: $D_1, H_1^{\perp}, \ldots$

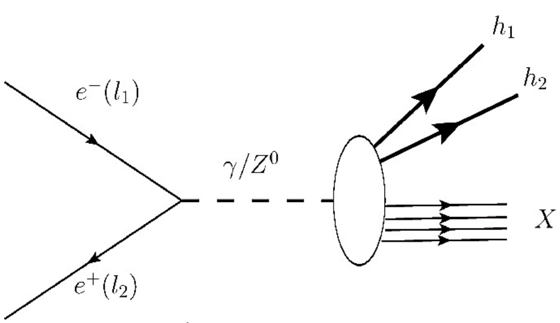


Drell-Yan: $p+p \to l+\bar{l}+X$

FNAL, BNL, JPARC, GSI

TMD PDFs:

$f_1, f_{1T}^{\perp}, g_{1L}, g_{1T}, h_1, h_1^{\perp}, h_{1L}^{\perp}, h_{1T}^{\perp} \ldots$



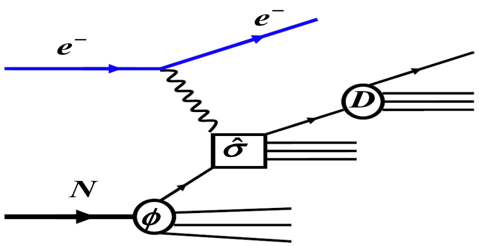
$e^- + e^+ \to h_1 + h_2 + X$

BESIII, BELLE, CERN, BABAR

TMD FFs: $D_1, H_1^{\perp}, \ldots$

Semi-inclusive high energy reactions: Kinematics

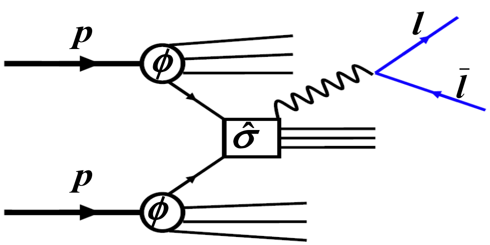
Semi-inclusive reactions: general form of the hadronic tensors and cross sections



DIS: $e+N\rightarrow e+h+X$

Gourdin, NPB 49, 501 (1972);
Kotzinian, NPB 441, 234 (1995);
Diehl, Sapeta, EPJ C41, 515 (2005);
Bacchetta, Diehl, Goeke, Metz, Mulders, Schlegel, JHEP 02, 093 (2007);
……

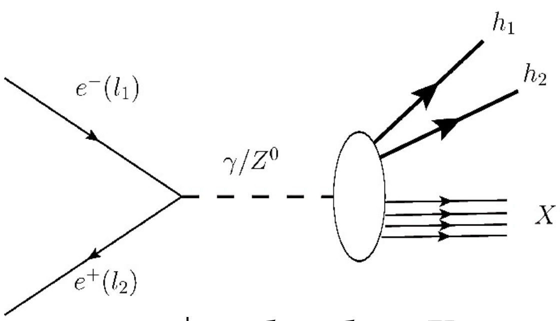
→ 18 independent "structure functions" for spinless hadron h



Drell-Yan: $p+p\rightarrow l+\bar{l}+X$

Arnold, Metz, Schlegel, Phys. Rev. D79 ,034005 (2009)

→ 48 independent "structure functions"



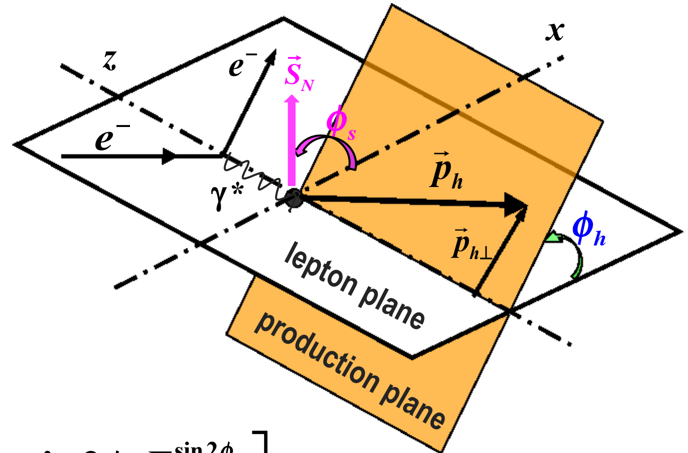
$e^- + e^+ \rightarrow h_1 + h_2 + X$

Pitonyak, Schlegel, Metz, Phys.Rev. D89, 054032(2014)

→ 72 independent "structure functions" for spin-1/2 hadrons

Semi-inclusive high energy reactions: Kinematics

Semi-inclusive reaction $e + N \to e + h + X$



$$\frac{d\sigma}{dx\,dy\,dz\,d^2 p_{h\perp}} = \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\times$$

$$\Big\{F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)}\cos\phi_h F_{UU}^{\cos\phi_h} + \varepsilon\cos 2\phi_h F_{UU}^{\cos 2\phi_h}$$

$$+\lambda_l\sqrt{2\varepsilon(1-\varepsilon)}\sin\phi_h F_{LU}^{\sin\phi_h} + \lambda\left[\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_h F_{UL}^{\sin\phi_h} + \varepsilon\sin 2\phi_h F_{UL}^{\sin 2\phi_h}\right]$$

$$+\lambda_l\lambda\left[\sqrt{1-\varepsilon^2}F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_h F_{LL}^{\cos\phi_h}\right]$$

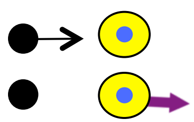
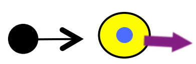
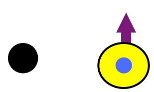
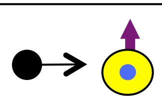
$$+|\vec{S}_T|\Big[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) + \varepsilon\sin(\phi_h+\phi_S)F_{UT,T}^{\sin(\phi_h+\phi_S)} + \varepsilon\sin(3\phi_h-\phi_S)F_{UT,T}^{\sin(3\phi_h-\phi_S)}$$

$$+\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_S F_{UT}^{\sin\phi_S} + \sqrt{2\varepsilon(1+\varepsilon)}\sin(2\phi_h-\phi_S)F_{UT}^{\sin(2\phi_h-\phi_S)}\Big]$$

$$+\lambda_l|\vec{S}_T|\left[\sqrt{1-\varepsilon^2}\cos(\phi_h-\phi_S)F_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\varepsilon(1-\varepsilon)}\left(\cos\phi_S F_{LT}^{\cos\phi_S} + \cos(2\phi_h-\phi_S)F_{LT}^{\cos(2\phi_h-\phi_S)}\right)\right]\Big\}$$

$$\varepsilon = \left(1-y-\frac{1}{4}\gamma^2y^2\right)\Big/\left(1-y+\frac{1}{2}y^2+\frac{1}{4}\gamma^2y^2\right) \qquad \gamma = 2Mx/Q$$

LO pQCD & leading twist parton model results for the structure functions

	$F_{UU,T} = \mathcal{C}\left[f_1 D_1\right]$ $\quad F_{UU,L} = 0$	$F_{UU}^{\cos\phi_h} = 0$	$F_{UU}^{\cos 2\phi_h} = \mathcal{C}\left[w_1 h_1^{\perp} H_1^{\perp}\right]$
	$F_{LU}^{\sin\phi_h} = 0$	$F_{UL}^{\sin\phi_h} = 0$	$F_{UL}^{\sin 2\phi_h} = \mathcal{C}\left[w_1 h_{1L}^{\perp} H_1^{\perp}\right]$
	$F_{LL} = \mathcal{C}\left[g_{1L} D_1\right]$	$F_{LL}^{\cos\phi_h} = 0$	
	$F_{UT,T}^{\sin(\phi_h-\phi_S)} = -2\mathcal{C}\left[w_2 f_{1T}^{\perp} D_1\right]$ $F_{UT,L}^{\sin(\phi_h-\phi_S)} = 0$	$F_{UT}^{\sin\phi_S} = 0$ $F_{UT}^{\sin(2\phi_h+\phi_S)} = 0$	$F_{UT}^{\sin(\phi_h+\phi_S)} = -2\mathcal{C}\left[w_3 h_{1T} H_1^{\perp}\right]$ $F_{UT}^{\sin(3\phi_h-\phi_S)} = \mathcal{C}\left[w_4 h_{1T}^{\perp} H_1^{\perp}\right]$
	$F_{LT}^{\cos(\phi_h-\phi_S)} = \mathcal{C}\left[w_2 g_{1T} D_1\right]$	$F_{LT}^{\cos\phi_S} = 0$	$F_{LT}^{\cos(2\phi_h-\phi_S)} = 0$

$$\mathcal{C}\left[w_i f D\right] \equiv x \sum_q e_q^2 \int d^2k_\perp d^2k_{F\perp} \delta^{(2)}(\vec{k}_\perp - \vec{k}_{F\perp} - \vec{p}_{h\perp}/z)\, w_i\, f_q(x,k_\perp) D_q(z,k_{F\perp})$$

$$w_1 = -\left[2(\hat{\vec{p}}_{h\perp}\cdot\vec{k}_{F\perp})(\hat{\vec{p}}_{h\perp}\cdot\vec{k}_\perp) - \vec{k}_\perp\cdot\vec{k}_{F\perp}\right]/MM_h, \qquad w_2 = \hat{\vec{p}}_{h\perp}\cdot\vec{k}_\perp/M, \qquad w_3 = \hat{\vec{p}}_{h\perp}\cdot\vec{k}_{F\perp}/M_h$$

$$w_4 = \left[2(\hat{\vec{p}}_{h\perp}\cdot\vec{k}_\perp)(\vec{k}_\perp\cdot\vec{k}_{F\perp}) + \vec{k}_\perp^2(\hat{\vec{p}}_{h\perp}\cdot\vec{k}_{F\perp}) - 4(\hat{\vec{p}}_{h\perp}\cdot\vec{k}_\perp)(\hat{\vec{p}}_{h\perp}\cdot\vec{k}_{F\perp})^2\right]/2M^2M_h$$

Semi-inclusive high energy reactions: LO & Leading twist results

LO pQCD & leading twist parton model results for the cross section

$$\frac{d\sigma}{dxdydzd^2p_{h\perp}} = \frac{\alpha^2}{xyQ^2} \times$$

$$\Big\{ (1-y+\frac{1}{2}y^2)\mathcal{C}\left[f_1 D_1\right] + (1-y)\cos 2\phi_h \mathcal{C}\left[w_1 h_1^\perp H_1^\perp\right]$$

Boer-Mulders ⊗ Collins

$$+\lambda_l \lambda y(1-\frac{1}{2}y)\mathcal{C}\left[g_{1L} D_1\right]$$

Worm-gear ⊗ Collins

$$+\lambda(1-y)\sin 2\phi_h \mathcal{C}\left[w_1 h_{1L}^\perp H_1^\perp\right]$$

Sivers ⊗ unpolarized FF

$$+|\vec{S}_T|\Big(-2(1-y+\frac{1}{2}y^2)\sin(\phi_h-\phi_S)\mathcal{C}\left[w_2 f_{1T}^\perp D_1\right]$$

$$-2(1-y)\sin(\phi_h+\phi_S)\mathcal{C}\left[w_3 h_{1T} H_1^\perp\right] + (1-y)\sin(3\phi_h-\phi_S)\mathcal{C}\left[w_4 h_{1T}^\perp H_1^\perp\right]\Big)$$

transversity ⊗ Collins

pretzelosity ⊗ Collins

$$+\lambda_l |\vec{S}_T| y(1-\frac{1}{2}y)\cos(\phi_h-\phi_S)\mathcal{C}\left[w_2 g_{1T} D_1\right]\Big\}$$

Worm-gear ⊗ unpolarized FF

Semi-inclusive high energy reactions

Semi-inclusive reactions

Similar for $p+p \to l+\bar{l}+X$ and $e^- + e^+ \to h_1 + h_2 + X$

We have: (1) General form of
the hadronic tensor and cross sections
in terms of "structure functions";

(2) Leading order pQCD & leading twist parton model results
in terms of leading twist TMD PDFs and FFs.

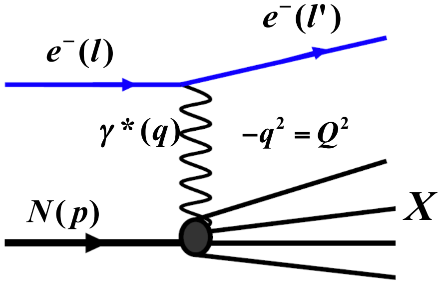
Going beyond
LO pQCD and/or leading twist

⇨

collinear expansion
factorization … …

Collinear expansion in high energy reactions

Inclusive DIS $e^- N \to e^- X$

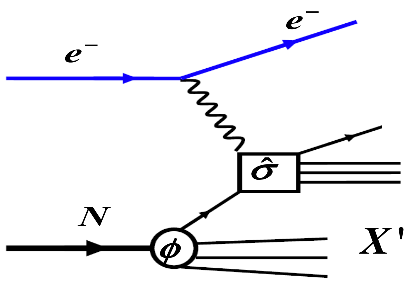


Yes!

where collinear expansion was first formulated.

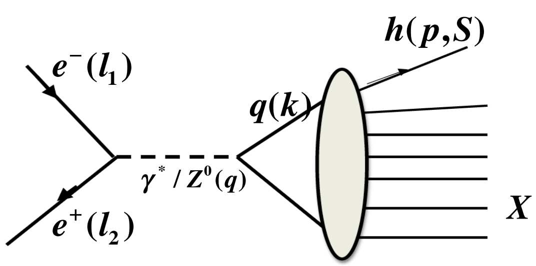
R. K. Ellis, W. Furmanski and R. Petronzio,
Nucl. Phys. B207,1 (1982); B212, 29 (1983).

Semi-Inclusive DIS
$e + N \to e + q(jet) + X$



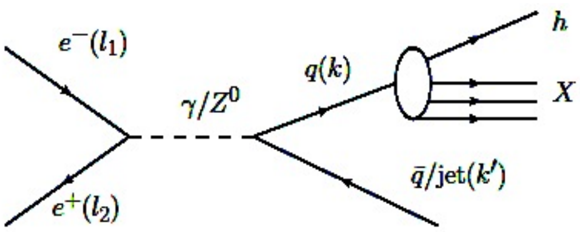
Yes!
ZTL & X.N. Wang,
PRD (2007);

Inclusive
$e^- + e^+ \to h + X$



Yes!
S.Y. Wei, Y.K Song, ZTL,
PRD (2014);

Semi-Inclusive
$e^- + e^+ \to h + \bar{q}(jet) + X$



Yes!
S.Y. Wei, K.B. Chen, Y.K Song,
ZTL, 1410.4314 [hep-ph] (2014).

Successfully to all processes where only ONE hadron is explicitly involved.

Semi-Inclusive DIS: $e^- + N \to e^- + q(jet) + X$

$$W_{\mu\nu}^{(si)}(q,p,S,k') = \tilde{W}_{\mu\nu}^{(0,si)}(q,p,S,k') + \tilde{W}_{\mu\nu}^{(1,si)}(q,p,S,k') + \tilde{W}_{\mu\nu}^{(2,si)}(q,p,S,k') + \ldots$$

twist-2, twist-3 and twist-4 contributions

$$\tilde{W}_{\mu\nu}^{(0,si)}(q,p,S,k') = \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}[\hat{\Phi}^{(0)}(k,p,S)\hat{H}_{\mu\nu}^{(0)}(x)]\,(2E_{k'})(2\pi)^3\delta^3(\vec{k}'-\vec{k}-\vec{q})$$

$$\hat{\Phi}^{(0)}(k,p,S) = \int d^4z e^{ikz}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,z)\psi(z)|p,S\rangle$$

twist-3, twist-4 and even higher twist contributions

$$\tilde{W}_{\mu\nu}^{(1,si)}(q,p,S,k') = \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\sum_{c=L,R}\mathrm{Tr}[\hat{\Phi}_{\rho'}^{(1)}(k_1,k_2,p,S)\,\hat{H}_{\mu\nu}^{(1,c)\rho}(x_1,x_2)\,\omega_\rho^{\ \rho'}]\,(2E_{k'})(2\pi)^3\delta^3(\vec{k}'-\vec{k}_c-\vec{q})$$

$$\hat{\Phi}_{\rho}^{(1)}(k_1,k_2,p,S) = \int d^4z d^4y e^{ik_1y+ik_2(z-y)}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,y)D_\rho(y)\mathcal{L}(y,z)\psi(z)|p,S\rangle$$

⇨ A consistent framework for $e^- N \to e^- + q(jet) + X$ at LO pQCD including higher twists

ZTL & X.N. Wang, PRD (2007);
Y.K. Song, J.H. Gao, ZTL & X.N. Wang, PRD (2011) & PRD (2014).

Semi-Inclusive DIS: $e^- + N \rightarrow e^- + q(jet) + X$

A complete twist-3 result for polarized $e(\lambda_l) + N(\lambda, S_\perp) \rightarrow e + q + X$

$$A(y) = 1 + (1-y)^2$$
$$B(y) = 2(2-y)\sqrt{1-y}$$
$$C(y) = y(2-y)$$
$$D(y) = 2y\sqrt{1-y}$$

$$\frac{d\sigma}{dxdyd^2k_\perp} = \frac{2\pi\alpha_{em}^2}{Q^2 y}(W_{UU} + \lambda_l W_{LU} + S_\perp W_{UT} + \lambda_l W_{LU} + \lambda_l \lambda W_{LL} + \lambda_l S_\perp W_{LT})$$

$$W_{UU}(x,k_\perp,\phi) = A(y)f_q(x,k_\perp) - \frac{2x|\vec{k}_\perp|}{Q}B(y)f_q^\perp(x,k_\perp)\cos\phi$$

$$W_{UT}(x,k_\perp,\phi,\phi_s) = \frac{|\vec{k}_\perp|}{M}A(y)f_{1T}^\perp(x,k_\perp)\sin(\phi-\phi_s) + \frac{2xM}{Q}B(y)\left\{\frac{k_\perp^2}{2M^2}f_T^\perp(x,k_\perp)\sin(2\phi-\phi_s) + f_T(x,k_\perp)\sin\phi_s\right\}$$

$$W_{LU}(x,k_\perp,\phi) = -\frac{2x|\vec{k}_\perp|}{Q}D(y)g^\perp(x,k_\perp)\sin\phi$$

$$W_{UL}(x,k_\perp,\phi) = -\frac{2x|\vec{k}_\perp|}{Q}B(y)f_L^\perp(x,k_\perp)\sin\phi$$

$$W_{LL}(x,k_\perp,\phi) = C(y)g_{1L}(x,k_\perp) - \frac{2x|\vec{k}_\perp|}{Q}D(y)g_L^\perp(x,k_\perp)\cos\phi$$

$$W_{LT}(x,k_\perp,\phi,\phi_s) = \frac{|\vec{k}_\perp|}{M}C(y)g_{1T}^\perp(x,k_\perp)\cos(\phi-\phi_s) - \frac{2xM}{Q}D(y)\left[g_T(x,k_\perp)\cos\phi_s - \frac{k_\perp^2}{2M^2}g_T^\perp(x,k_\perp)\cos(2\phi-\phi_s)\right]$$

Y.K. Song, J.H. Gao, ZTL, X.N. Wang, PRD (2014).

Semi-Inclusive DIS: $e^- + N \to e^- + q(jet) + X$

A complete twist-4 result for un-polarized $e + N \to e + q + X$

Cahn effects

$$\frac{d\sigma}{dxdyd^2k_\perp} = \frac{2\pi\alpha_{em}^2}{Q^2 y}\Big\{[1+(1-y)^2]f_1(x,k_\perp) +$$

$$-4(2-y)\sqrt{1-y}\frac{|\vec{k}_\perp|}{Q}xf^\perp(x,k_\perp)\cos\phi +$$

$$-4(1-y)\frac{|\vec{k}_\perp|^2}{Q^2}x[\varphi_3^{(1)\perp}(x,k_\perp) - \tilde{\varphi}_3^{(1)\perp}(x,k_\perp)]\cos 2\phi$$

$$+8(1-y)\frac{2x^2M^2}{Q^2}f_3(x,k_\perp)$$

$$-2[1+(1-y)^2]\frac{|\vec{k}_\perp|^2}{Q^2}x[\varphi_3^{(2,L)\perp}(x,k_\perp) - \tilde{\varphi}_3^{(2,L)\perp}(x,k_\perp)]\Big\}$$

twist 2

twist 3

twist 4

Y.K. Song, J.H. Gao, ZTL, X.N. Wang, PRD (2011).

$$\hat{\varphi}_\rho^{(1)}(x,k_\perp) = \ldots + \frac{1}{2}\left(k_{\perp\rho}k_{\perp\sigma} - \frac{1}{2}k_\perp^2 d_{\rho\sigma}\right)\gamma^\sigma\varphi_3^{(1)\perp}(x,k_\perp) + \frac{i}{2}\gamma_5\gamma^\sigma\left(k_{\perp[\rho}\varepsilon_{\perp\sigma]\gamma}k_\perp^\gamma\right)\tilde{\varphi}_3^{(1)\perp}(x,k_\perp) + \ldots$$

$$\hat{\varphi}_{\rho\sigma}^{(2,L)}(x,k_\perp) = -\frac{1}{4}p\left(k_\perp^2 d_{\rho\sigma}\right)\varphi_3^{(2,L)\perp}(x,k_\perp) + \frac{1}{4}\gamma_5 p\left(k_{\perp[\rho}\varepsilon_{\perp\sigma]\gamma}k_\perp^\gamma\right)\tilde{\varphi}_3^{(2,L)\perp}(x,k_\perp) + \ldots$$

talk by Y.K. Song, Parallel-VII: S3 (Friday).

Semi-Inclusive e⁺e⁻ annihilation: $e^+ + e^- \to h + \bar{q}(jet) + X$

$$W_{\mu\nu}^{(si)}(q,p,S,k') = \tilde{W}_{\mu\nu}^{(0,si)}(q,p,S,k') + \tilde{W}_{\mu\nu}^{(1,si)}(q,p,S,k') + \tilde{W}_{\mu\nu}^{(2,si)}(q,p,S,k') + \ldots$$

twist-2, twist-3 and twist-4 contributions

$$\tilde{W}_{\mu\nu}^{(0,si)}(q,p,S,k') = \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\hat{\Xi}^{(0)}(k,p,S)\hat{H}_{\mu\nu}^{(0)}(z)\right](2E_{k'})(2\pi)^3\delta^3(\vec{k}' - \vec{k} - \vec{q})$$

$$\hat{\Xi}^{(0)}(k,p,S) = \frac{1}{2\pi}\sum_X \int d^4\xi e^{-ik\xi}\langle 0|\mathcal{L}^\dagger(0,\infty)\psi(0)|hX\rangle\langle hX|\bar{\psi}(\xi)\mathcal{L}(\xi,0)|0\rangle$$

twist-3, twist-4 and even higher twist contributions

$$\tilde{W}_{\mu\nu}^{(1,L,si)}(q,p,S,k') = \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4} \mathrm{Tr}\left[\hat{\Xi}^{(1,L)}(k_1,k_2;p,S)\hat{H}_{\mu\nu}^{(1,L)\rho}(z_1,z_2)\omega_\rho^{\ \rho'}\right](2E_{k'})(2\pi)^3\delta^3(\vec{k}' - \vec{k}_c - \vec{q})$$

$$\hat{\Xi}_\rho^{(1,L)}(k_1,k_2,p,S) = \frac{1}{2\pi}\sum_X \int d^4\xi d^4\eta e^{-ik_1\xi - i(k_2-k_1)\eta}\langle 0|\mathcal{L}(0,y)D_\rho(\eta)\mathcal{L}^\dagger(y,z)\psi(0)|hX\rangle\langle hX|\bar{\psi}(\xi)\mathcal{L}(\xi,\infty)|0\rangle$$

$$D_\rho(y) = -i\partial_\rho + gA_\rho(y)$$

⇨ A consistent framework for $e^-e^+ \to h + \bar{q}(jet) + X$ at LO pQCD including higher twists.

S.Y. Wei, K.B. Chen, Y.K Song, ZTL, 1410.4314 [hep-ph] (2014).

Semi-Inclusive e+e- annihilation: $e^+ + e^- \to h + \bar{q}(jet) + X$

A complete twist-3 result for spin-0, ½ and 1 hadron in $e^+ + e^- \to h(\lambda, S_\perp) + \bar{q}(jet) + X$

$$\frac{d\sigma^{(si,unp)}}{dydzd^2k'_\perp} = \frac{\alpha^2\chi}{2\pi Q^2}\Big\{T_0^q(y)\hat{D}_1(z,k'_\perp) + \frac{4}{zQ^2}[T_2^q(y)l_\perp\cdot k'_\perp\hat{D}^\perp(z,k'_\perp) + T_3^q(y)\epsilon_\perp^{l_\perp k'_\perp}\Delta\hat{D}^\perp(z,k'_\perp)]\Big\}.$$

$$\frac{d\sigma^{(si,V\,pol)}}{dydzd^2k'_\perp} = \frac{\alpha^2\chi}{2\pi Q^2}\Big\{T_0^q(y)\frac{\epsilon_\perp^{k'_\perp S_\perp}}{M}\hat{D}_{1T}^\perp(z,k'_\perp) + T_1^q(y)[\lambda_h\Delta\hat{D}_{1L}(z,k'_\perp) + \frac{k'_\perp\cdot S_\perp}{M}\Delta\hat{D}_{1T}^\perp(z,k'_\perp)]$$
$$+ \frac{4\lambda_h}{zQ^2}[T_2^q(y)\epsilon_\perp^{l_\perp k'_\perp}\hat{D}_L^\perp(z,k'_\perp) + T_3^q(y)l_\perp\cdot k'_\perp\Delta\hat{D}_L^\perp(z,k'_\perp)] + \frac{4\epsilon_\perp^{k'_\perp S_\perp}}{zMQ^2}[T_2^q(y)l_\perp\cdot k'_\perp\hat{D}_T^\perp(z,k'_\perp) + T_3^q(y)\epsilon_\perp^{l_\perp k'_\perp}\Delta\hat{D}_T^\perp(z,k'_\perp)]$$
$$+ \frac{4M}{zQ^2}[T_2^q(y)\epsilon_\perp^{l_\perp S_\perp}\hat{D}_T(z,k'_\perp) + T_3^q(y)l_\perp\cdot S_\perp\Delta\hat{D}_T(z,k'_\perp)]\Big\}.$$

$$\frac{d\sigma^{(si,LL)}}{dydzd^2k'_\perp} = \frac{\alpha^2\chi}{2\pi Q^2}S_{LL}\Big\{T_0^q(y)\hat{D}_{1LL}(z,k'_\perp) + \frac{4}{zQ^2}[T_2^q(y)(l_\perp\cdot k'_\perp)\hat{D}_{LL}^\perp(z,k'_\perp) + T_3^q(y)\epsilon_\perp^{l_\perp k'_\perp}\Delta\hat{D}_{LL}^\perp(z,k'_\perp)]\Big\},$$

....

S.Y. Wei, K.B. Chen, Y.K Song, ZTL, 1410.4314 [hep-ph] (2014).

talk by S.Y. Wei, in Parallel-II: S5 (Monday)

Semi-Inclusive e$^+$e$^-$ annihilation: $e^+ + e^- \to h + \bar{q}(jet) + X$

A complete twist-3 result for spin-0, ½ and 1 hadron in $e^+ + e^- \to h(\lambda, S_\perp) + \bar{q}(jet) + X$

$$A^{\cos\phi}_{unp,em}(y,z,p_T) = -\frac{2|\vec{p}_T|}{z^2 Q}\frac{\tilde{B}(y)}{A(y)}\frac{\sum_q e_q^2 D^{\perp q\to h}(z,p_T)}{\sum_q e_q^2 D_1^{q\to h}(z,p_T)}$$

twist-3 azimuthal asymmetry for spin 0 hadrons

$$P^{(0,em)}_{hn}(y,z,p_T) = -\frac{|\vec{p}_T|}{zM}\frac{\sum_q e_q^2 D_{1T}^{\perp q\to h}(z,p_T)}{\sum_q e_q^2 D_1^{q\to h}(z,p_T)}$$

twist-2 polarization transverse to the production plane.

$$S^{(0,em)}_{LL}(y,z,p_T) = \frac{\sum_q e_q^2 D_{1LL}^{q\to h}(z,p_T)}{2\sum_q e_q^2 D_1^{q\to h}(z,p_T)}$$

(spin alignment)

twist-2 tensor polarizations for spin 1 hadrons

$$S^{(t,0,em)}_{LL}(y,z,p_T) = -\frac{2|\vec{p}_T|}{3zM}\frac{\sum_q e_q^2 D_{1LT}^{q\to h}(z,p_T)}{2\sum_q e_q^2 D_1^{q\to h}(z,p_T)}$$

$$S^{(nn,0,em)}_{TT}(y,z,p_T) = -\frac{2|\vec{p}_T|^2}{3z^2M^2}\frac{\sum_q e_q^2 D_{1TT}^{\perp q\to h}(z,p_T)}{2\sum_q e_q^2 D_1^{q\to h}(z,p_T)}$$

…………………

much less explored!

S.Y. Wei, K.B. Chen, Y.K Song, ZTL, 1410.4314 [hep-ph] (2014).

LO pQCD & twist-3 calculations have been done for processes where two hadrons are involved.

$$e^- + N \to e^- + h + X$$
$$e^+ + e^- \to h_1 + h_2 + X$$
$$h_1 + h_2 \to l + \bar{l} + X$$

Mulders, Tangerman, NPB461, 197 (1996);
Boer, Jakob, Mulders, NPB 504, 345, (1997);
Bacchetta, Diehl, Goeke, Metz, Mulders, Schlegel, JHEP 02, 093 (2007);
Lu, Schmidt, PRD 82, 114022 (2010);
……………

before collinear expansion

(1) Draw Feynman diagrams with “multiple gluon scattering” to one gluon exchange;

(2) Insert gauge link in the correlators wherever needed to keep them gauge invariant;

(3) Do calculations to the order (1/Q).

$$e + N \to e + h + X$$
$$e^+ + e^- \to h_1 + h_2 + X$$
no collinear expansion

→

$$e + N \to e + q + X$$
$$e^+ + e^- \to h + \bar{q} + X$$
with collinear expansion

Semi-inclusive high energy reactions: TMD factorization

Semi-inclusive DIS: $e + N \to e + h + X$

Leading twist

factorization theorem: verified to one loop order, argued to be valid to all orders

Collins, Soper, NPB (1981,1982); Collins, Sterman, Soper, NPB (1985);
Ibildi, Ji, Ma, Yuan, PRD (2004); Ji, Ma, Yuan, PLB (2004), PRD (2005);
Collins, Oxford Press 2011; ....

talk by D. Boer, Parallel-II: S1

$$F(x_B, z, p_{h\perp}, Q^2) = \sum_q e_q^2 \int d^2k_\perp d^2k_{F\perp} d^2l_\perp \delta^2(z\vec{k}_\perp + \vec{k}_{F\perp} + \vec{l}_\perp - \vec{p}_{h\perp})$$
$$\times q(x_B, k_\perp, \mu^2, x_B\varsigma, \rho)\hat{q}(z, k_{F\perp}, \mu^2, \hat{\varsigma}/z, \rho) S(l_\perp, \mu^2, \rho) H(Q^2, \mu^2, \rho)$$

$$W^{\mu\nu}(x_B, z, p_{h\perp}, Q) = \sum_q |\mathcal{H}_q(Q,\mu)^2|^{\mu\nu} \int d^2k_\perp d^2k_{F\perp} \delta^2(\vec{k}_\perp + \vec{k}_{F\perp} - \vec{p}_{h\perp})$$
$$\times q(x_B, k_\perp, S; \mu, \varsigma_F) D_{h/q}(z, zk_{F\perp}, \mu^2; \varsigma_D) + Y(Q, p_{h\perp}) + \mathcal{O}((\Lambda/Q)^a)$$

TMD parameterizations: QCD evolution

TMD evolution:

partial list of the dedicated publications recently

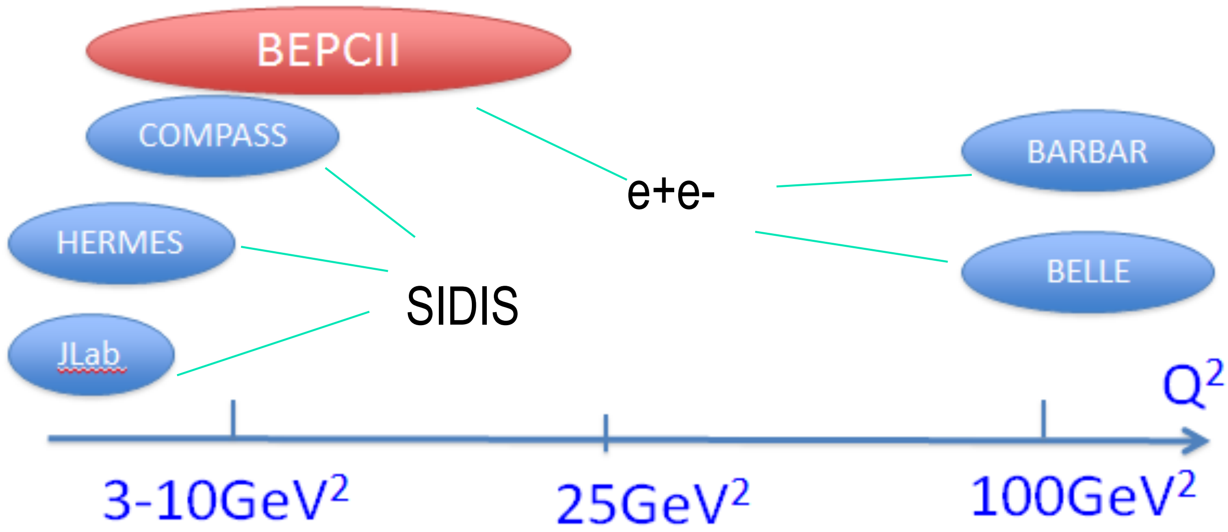
Developing very fast!

talk by D. Boer, Parallel-II: S1

- …………
- Evolution of TMD distribution and fragmentation functions
  Henneman, Boer, Mulders, NPB 620, 331 (2002);
- QCD evolution of the TMD correlations
  Zhou, Yuan, Liang, PRD 79, 114022 (2009);
- TMD parton distribution and fragmentation functions with QCD evolution
  Aybat, Rogers, PRD 83, 114042 (2011);
- QCD evolution of the Sivers function
  Aybat, Collins, Qiu, Rogers, PRD 85, 034043 (2012);
- Strategy towards the extraction of the Sivers function with TMD evolution
  Anselmino, Boglione, Melis, PRD 86, 014028 (2012);
- TMD evolution: Matching SIDIS to Drell-Yan and W/Z boson production
  Sun, Yuan, PRD 88, 114012 (2013);
- QCD evolution of the Sivers asymmetry
  Echevarria, Idilbi, Kang, Vitev, PRD 89, 074013 (2014);
- Limits on TMD evolution from semi-inclusive deep inelastic scattering at moderate Q
  Aidala, Field, Gamberg, Rogers, PRD 89, 094002 (2014);
- Unified treatment of the QCD evolution of all (un-)polarized TMDs: Collins function as a study case
  Echevarria, Idilbi, Scimemi, PRD 90, 014003 (2014);
- ……………

Parameterizations & evolution

Measurements



talks by: STOLARSKI Marcin, Plenary Session III (Tuesday)
FATEMI Renee, Plenary Session III (Tuesday)
ROSTOMYAN Armine, Plenary Session III (Tuesday)
MARTIN Anna, PARSAMYAN Bakur, ZHAO, Yuxiang, MAKKE Nour, MAO Weijuan, GIORDANO Francesca, GUAN Yinghui, and many many in the parallel sessions.

(1) transverse momentum dependence

Gaussian anzatz:

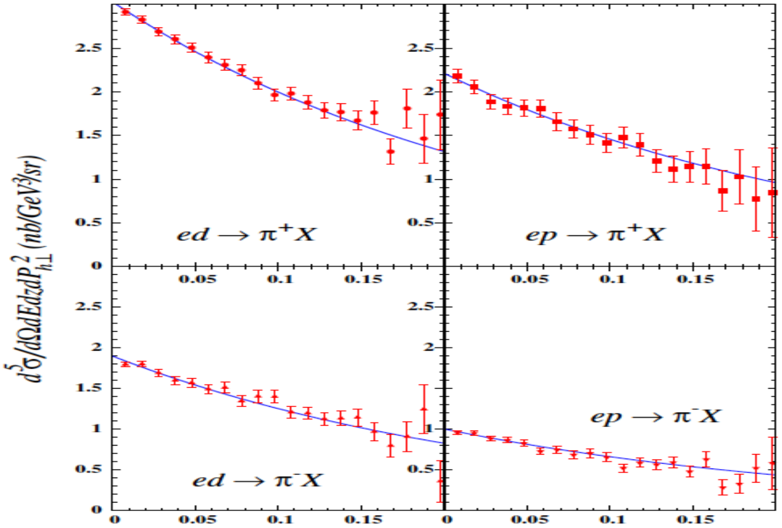
$$f_1(x,k_\perp) = f_1(x)\frac{1}{\pi\langle \vec{k}_\perp^2\rangle} e^{-\vec{k}_\perp^2/\langle \vec{k}_\perp^2\rangle}$$

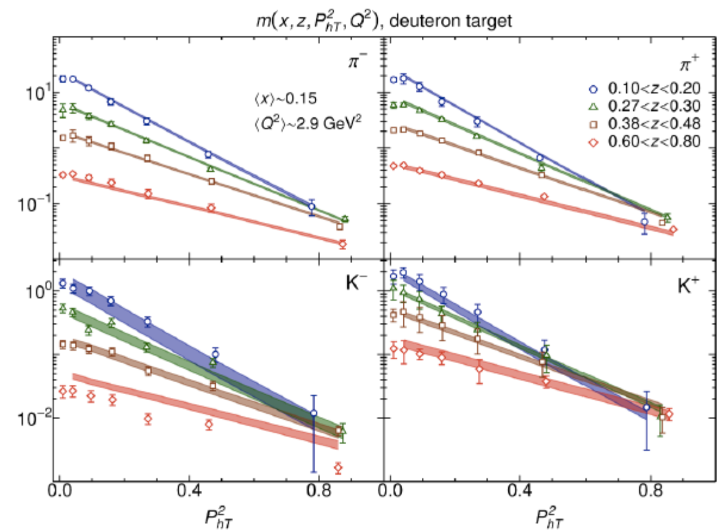
$$D_1(z,k_{F\perp}) = D_1(z)\frac{1}{\pi\langle \vec{k}_{F\perp}^2\rangle} e^{-\vec{k}_{F\perp}^2/\langle \vec{k}_{F\perp}^2\rangle}$$

- the width is fitted
- the form is tested
- flavor dependence

See, e.g.,
Anselmino, Boglione, D'Alesio, Kotzinian, Murgia, Prokudin, PRD 71, 074006 (2005);
Anselmino, Boglione, D'Alesio, Kotzinian, Murgia, Prokudin, Tuerk, PRD 75, 054032 (2007);
Schweitzer,Teckentrup, Metz, PRD 81, 094019 (2010);
Signori, Bacchetta, Radicic, Schnelle, JHEP 11,194 (2013);
Anselmino, Boglione, Gonzalez, Melis, Prokudin, JHEP 04, 005 (2014);
………………

first phase: without evolution





TMD parameterizations: first phase

(2) Sivers function

first phase: without evolution

Efremov, Goeke, Menzel, Metz, Schweitzer, PLB 612, 233 (2005);
Collins, Efremov, Goeke, Menzel, Metz, Schweitzer, PRD 73, 014021 (2006);
Arnold, Efremov, Goeke, Schlegel, Schweitzer, 0805.2137[hep-ph] (2008);

Bochum fits

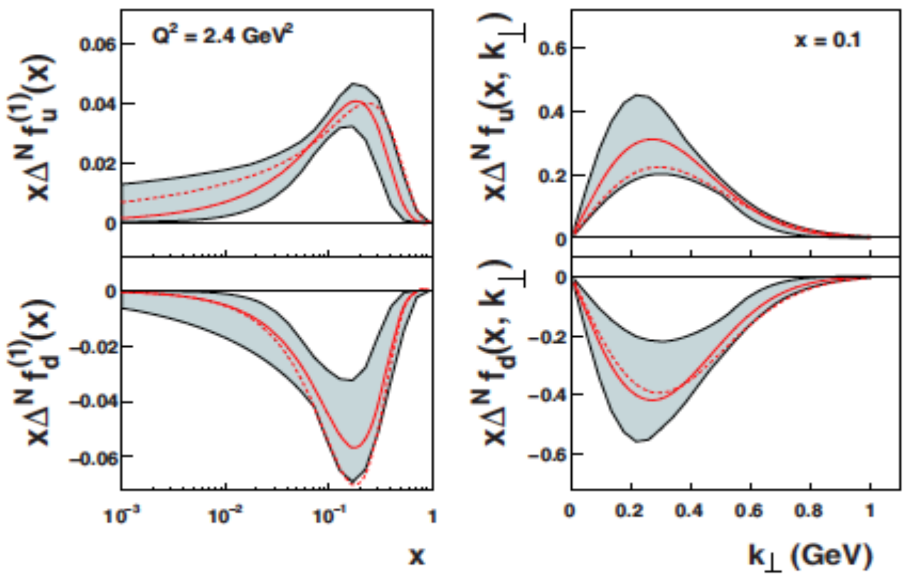
Anselmino, Boglione, D'Alesio, Kotzinian, Murgia, Prokudin, PRD 71, 074006 (2005);
Anselmino, Boglione, D'Alesio, Kotzinian, Melis, Murgia, Prokudin, Tuerk, EPJA 39, 89 (2009);

Torino fits

Vogelsang, Yuan, PRD 72, 054028 (2005);

Vogelsang-Yuan fits

Bacchetta, Radici, PRL 107, 212001 (2011);



$$\Delta^N f_{q/p^\uparrow}(x,k_\perp) = 2\mathcal{N}_q(x)h(k_\perp)f_{q/p}(x,k_\perp)$$

$$\mathcal{N}_q(x) = \mathcal{N}_q x^{\alpha_q}(1-x)^{\beta_q}\frac{(\alpha_q+\beta_q)^{\alpha_q+\beta_q}}{\alpha_q^{\alpha_q}\beta_q^{\beta_q}}$$

$$h(k_\perp) = \sqrt{2e}\frac{|\vec{k}_\perp|}{M_1}e^{-\vec{k}_\perp^2/M_1^2}$$

Already different sets of parameterizations, though not very much different from each other.

(3) Transversity & Collins function

first phase: without evolution

Simultaneous extraction of transversity and Collins function

Anselmino, Boglione, D'Alesio, Kotzinian, Murgia, Prokudin, PRD 75, 054032 (2007);
Anselmino, Boglione, D'Alesio, Melis, Murgia, Prokudin, PRD 87, 094019 (2013);
……………..

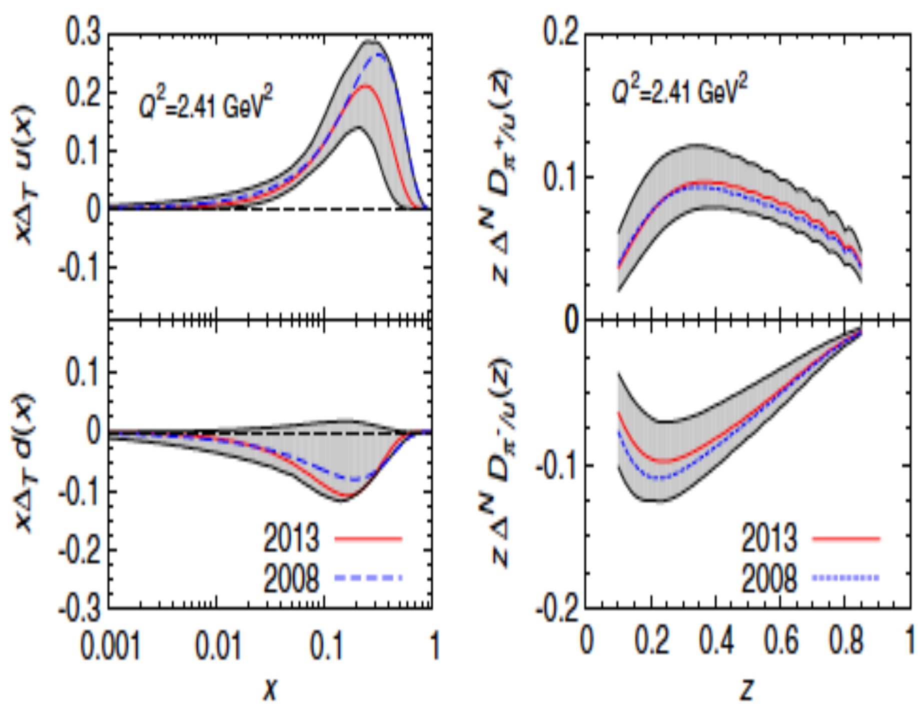
$$\Delta_T q(x,k_\perp) = \frac{1}{2}\mathcal{N}_q^T(x)\left[f_{q/p}(x)+\Delta q(x)\right]\frac{e^{-\vec{k}_\perp^2/\langle \vec{k}_\perp^2\rangle_T}}{\pi\langle \vec{k}_\perp^2\rangle_T}$$

$$\Delta^N D_{h/q^\uparrow}(z,k_{F\perp}) = 2\mathcal{N}_q^C(z)\,D_{h/q}(z)\,h(k_{F\perp})\frac{e^{-\vec{k}_{F\perp}^2/\langle \vec{k}_{F\perp}^2\rangle}}{\pi\langle \vec{k}_{F\perp}^2\rangle}$$

$$\mathcal{N}_q^T(x) = \mathcal{N}_q^T x^\alpha (1-x)^\beta \frac{(\alpha+\beta)^{\alpha+\beta}}{\alpha^\alpha \beta^\beta}$$

$$\mathcal{N}_q^C(z) = \mathcal{N}_q^C z^\gamma (1-z)^\delta \frac{(\gamma+\delta)^{\gamma+\delta}}{\gamma^\gamma \delta^\delta}$$

$$h(k_{F\perp}) = \sqrt{2e}\,\frac{|\vec{k}_{F\perp}|}{M_F}\,e^{-\vec{k}_{F\perp}^2/M_F^2}$$



(4) Boer-Mulders, pretzelosity, ……

first phase: without evolution

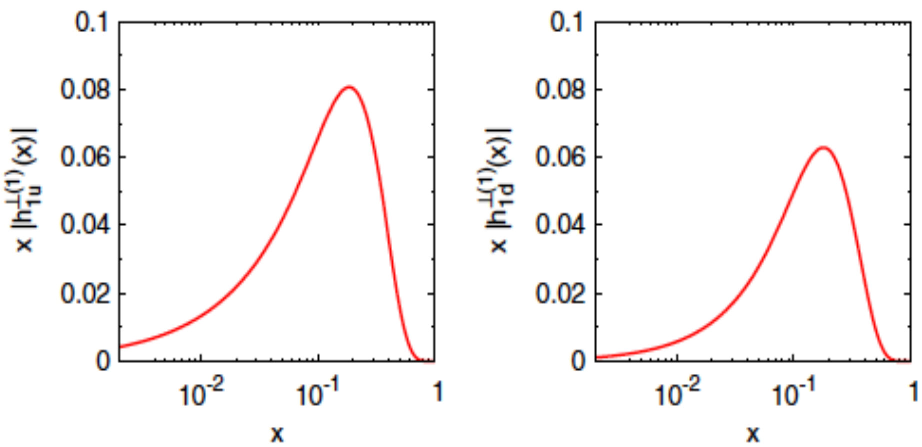
Zhang, Lu, Ma, Schmidt, PRD 77, 054011 (2008); D78, 034035 (2008);
Barone, Prokudin, Ma, PRD 78, 045022 (2008);
Barone, Melis, Prokudin, PRD 81, 114026 (2010);
Lu, Schmidt, PRD 81, 034023 (2010);
……………..

$$h_1^{\perp q}(x,k_\perp) = \lambda_q f_{1T}^{\perp q}(x,k_\perp)$$

$$f_{1T}^{\perp}(x,k_\perp) = 2\mathcal{N}_q(x) h(k_\perp) f_{1q}(x,k_\perp)$$

$$\mathcal{N}_q(x) = \mathcal{N}_q x^{\alpha_q}(1-x)^{\beta_q}\frac{(\alpha_q+\beta_q)^{\alpha_q+\beta_q}}{\alpha_q^{\alpha_q}\beta_q^{\beta_q}}$$

$$h(k_\perp) = \sqrt{2e}\,\frac{M_p}{M_1}e^{-\vec{k}_\perp^2/M_1^2}$$



nonzero Boer-Mulders function from SIDIS data on $\langle \cos 2\phi \rangle$

Attn.: twist-4 contribution of Cahn effect?

TMD parameterizations: QCD evolution

TMD evolution:

partial list of the dedicated publications recently

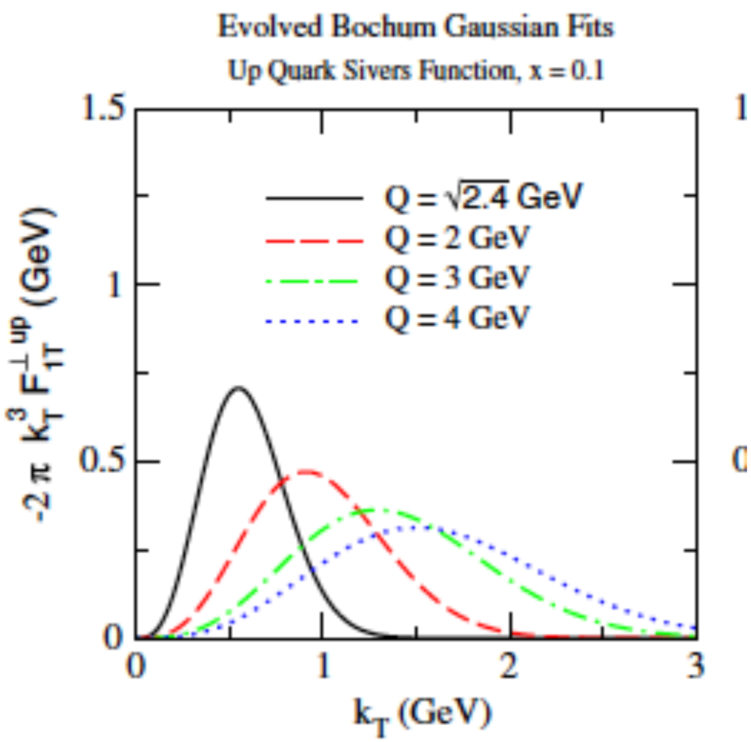
Developing very fast!

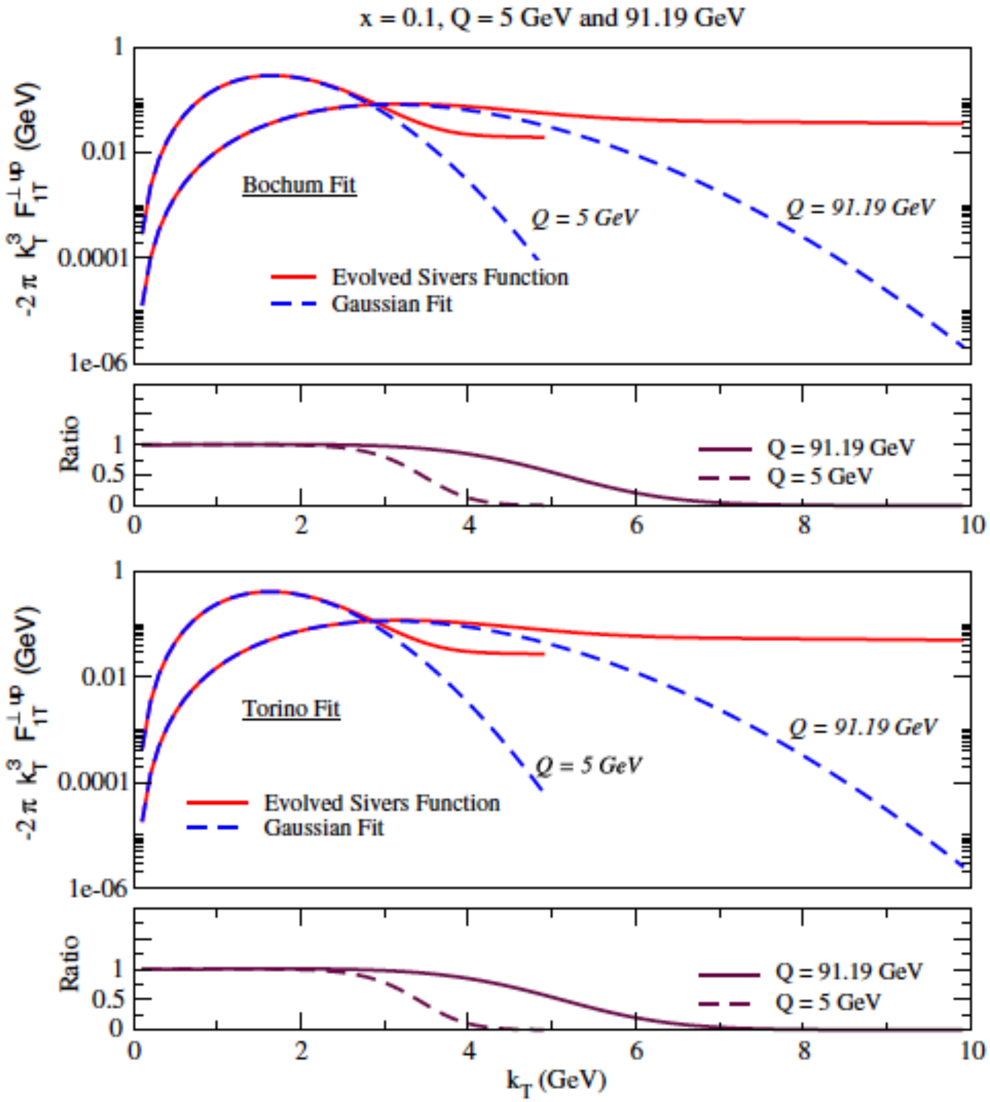
talk by D. Boer, Parallel-II: S1

- ………….
- Evolution of TMD distribution and fragmentation functions
  Henneman, Boer, Mulders, NPB 620, 331 (2002);
- QCD evolution of the TMD correlations
  Zhou, Yuan, Liang, PRD 79, 114022 (2009);
- TMD parton distribution and fragmentation functions with QCD evolution
  Aybat, Rogers, PRD 83, 114042 (2011);
- QCD evolution of the Sivers function
  Aybat, Collins, Qiu, Rogers, PRD 85, 034043 (2012);
- Strategy towards the extraction of the Sivers function with TMD evolution
  Anselmino, Boglione, Melis. PRD 86, 014028 (2012);
- TMD evolution: Matching SIDIS to Drell-Yan and W/Z boson production
  Sun, Yuan, PRD 88, 114012 (2013);
- QCD evolution of the Sivers asymmetry
  Echevarria, Idilbi, Kang, Vitev, PRD 89, 074013 (2014);
- Limits on TMD evolution from semi-inclusive deep inelastic scattering at moderate Q
  Aidala, Field, Gamberg, Rogers, PRD 89, 094002 (2014);
- Unified treatment of the QCD evolution of all (un-)polarized TMDs: Collins function as a study case
  Echevarria, Idilbi, Scimemi, PRD 90, 014003 (2014);
- ……………

TMD parameterizations: QCD evolution

TMD evolution:

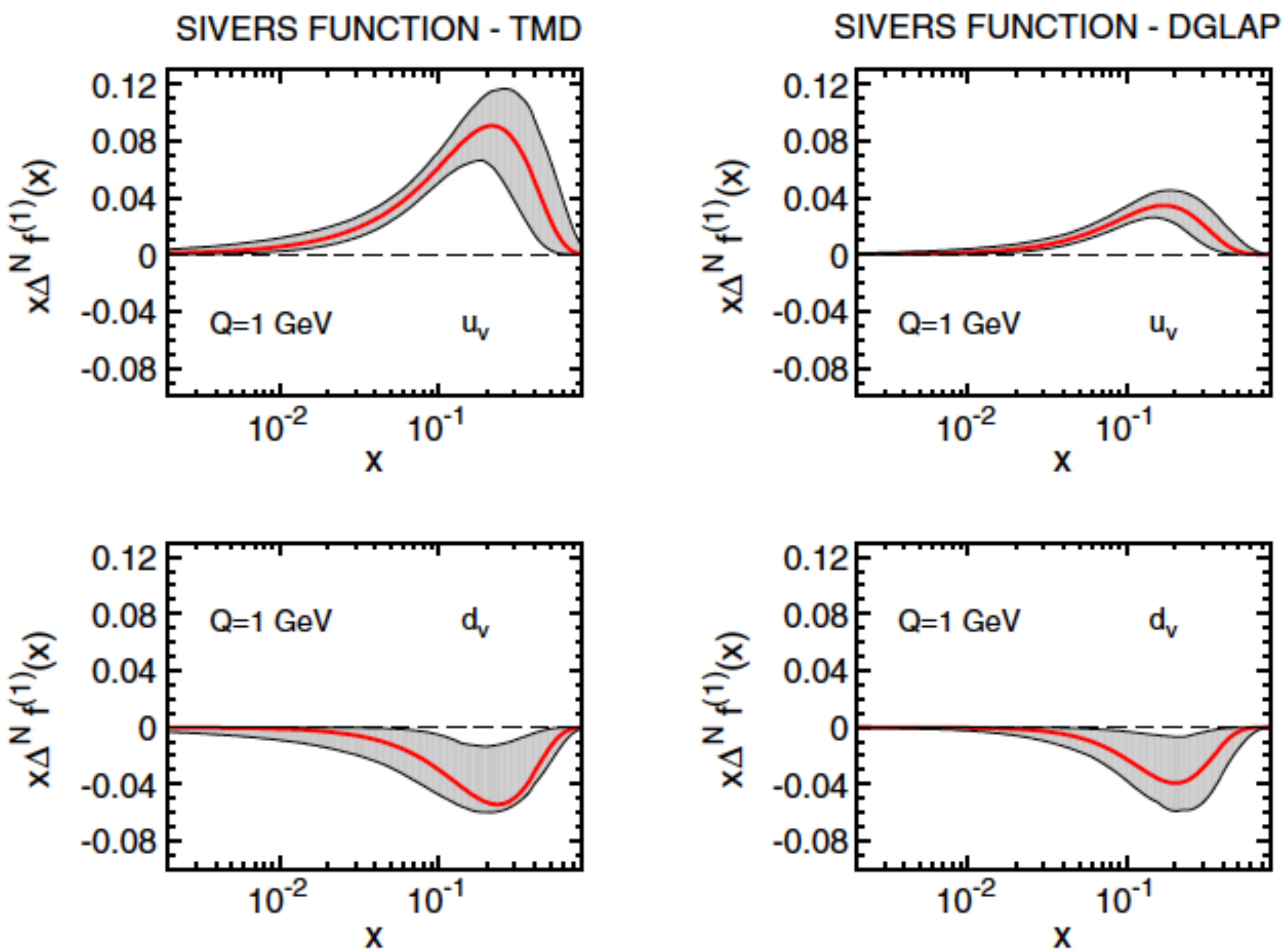




Aybat, Collins,Qiu, Rogers,
PRD 85, 034043 (2012)

TMD parameterizations: QCD evolution

TMD evolution:



Anselmino, Boglione, Melis, PRD 86, 014028 (2012).

TMD parameterizations: TMDlib & TMDplotter

First version is there!

DESY 14-059
NIKHEF 2014-024
YITP-SB-14-24
August 2014

TMDlib and TMDplotter:
library and plotting tools for
transverse-momentum-dependent parton distributions
Version 1.0.0

F. Hautmann[1,2], H. Jung[3,4], M. Krämer[3],
P. J. Mulders[5,6], E. R. Nocera[7], T. C. Rogers[8], A. Signori[5,6]

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[2] Dept. of Theoretical Physics, University of Oxford, UK
[3] DESY, Hamburg, FRG
[4] University of Antwerp, Belgium
[5] Department of Physics and Astronomy, VU University Amsterdam, the Netherlands
[6] Nikhef, the Netherlands
[7] Università degli Studi di Milano and INFN Milano, Italy
[8] C.N. Yang Institute for Theoretical Physics, Stony Brook University, USA

Abstract

Transverse-momentum-dependent distributions (TMDs) are central in high-energy physics from both theoretical and phenomenological points of view. In this manual we introduce the library, TMDlib, of fits and parameterisations for transverse-momentum-dependent parton distribution functions (TMD PDFs) and fragmentation functions (TMD FFs) together with an online plotting tool, TMDplotter. We provide a description of the program components and of the different physical frameworks the user can access via the available parameterisations.

Summary & Outlook

- Rapid developing
- Much progress
- Bright future

Apologize for many aspects not covered

- TMD and Wigner function;
- model studies;
- nuclear dependence;
- hyperon polarization;
- Drell-Yan;
- generalized PDFs
- …………

Thank you for your attention!