Thermalization of nonabelian plasmas at weak coupling

Aleksi Kurkela, CERN

Arxiv:

- 1107.5050 — Thermalization in generic setup, complete treatment of plasma instabilities
- 1108.4684 — Specialized to heavy ion collisions
- 1207.1663 — Quantitative treatment of over-occupied isotropic systems using lattice simulations
- 1209.4091 — Algorithmic development for expanding systems
- 1401.3751 — Kinetic theory description of the over-occupied system

Work in collaboration with
Guy Moore, Egang Lu, and Mark Abraao York (McGill)

Motivation

Many cases where one meets gauge theories far from equilibrium:

- Cosmology: reheating and preheating decay products, parametric resonance, etc...
- Phase transitions electro-weak etc.
- Heavy-ion collisions initial condition as far from eq. as possible

Want to know:

- How fast systems thermalize
- What are properties of matter out of eq. anomalous viscosities, etc...
- How phase transitions change out of equilibrium
- What kinds of signatures out-of-eq. systems may leave
- ...

Motivation

For generic theories, only weak coupling methods available:

- Mostly parametric estimates, not even LO results
- Even at weak coupling often non-perturbative: strong fields, secular divergences, instabilities. . .

- Weak coupling provides scale separations
- Case-by-case effective theories
  - Effective kinetic theory
  - Classical field theory
  - Hard loop effective theory/ Vlasov equations
  - . . .

Outline

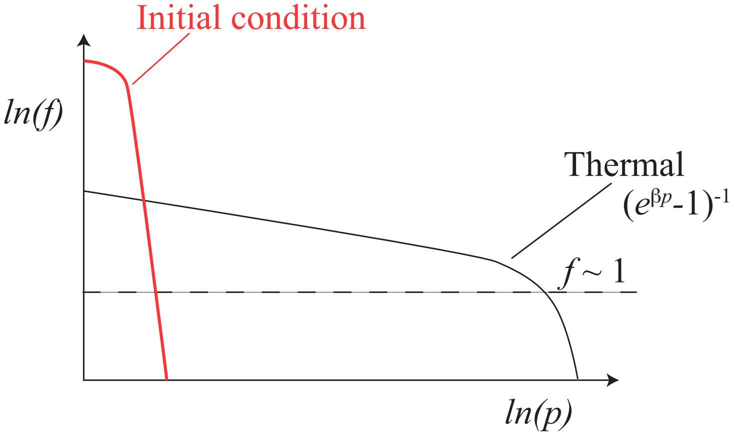
- Over-occupied, isotropic case
- Under-occupied, isotropic case
- Anisotropic systems

For another time:

- Inhomogenous systems
- Expanding systems
- Fermions
- Applications

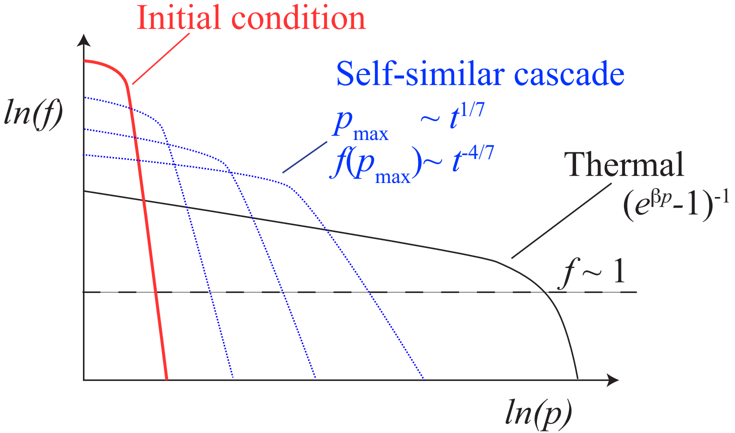
Over-occupied cascade

Simple example: what happens if you have too many soft gluons, $f \sim 1/\alpha$



Over-occupied cascade

Simple example: what happens if you have too many soft gluons, $f \sim 1/\alpha$



The system thermalises when $p_{\max} \sim T \sim \epsilon^{1/4}$

Over-occupied cascade

Energy conservation

$$\epsilon \sim \int d^3p\, p f \sim p \sim p_{\max}^4 f \Rightarrow f \propto p_{\max}^{-4}$$

Expect scattering rate Γ to be order $\Gamma t \sim 1$.

- If $\Gamma t < 1$, system has not yet relaxed to scaling solution
- If $\Gamma t > 1$, scattering fast enough to change system, reducing scattering rate

Estimate $\Gamma \sim \langle \sigma n(1+f) \rangle \sim \sigma p_{\max}^3 f^2$ solved by

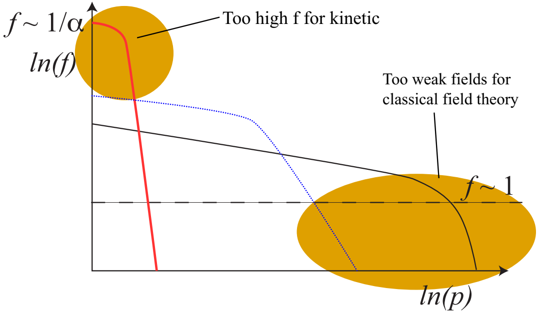
$$p_{\max} \propto t^{1/7}$$
$$f \propto t^{-4/7}$$

Expect that below $p_{\max}$, modes have had time to arrange themselves to thermal form $f(p) \sim T_*/p$

AK,Moore arXiv: 1107.5050; Blaizot, Gelis, Liao, McLerran, Venugopalan 1107.5296

Going quantitative

- Strong fields ($f \gg 1$): Classical (lat.) field theory
- But not too ($f \ll 1/\alpha$): Effective kinetic theory



Can the whole system be described with either?

- Depends on what modes are important.
- Perhaps need more than one eff. theory is the case in anisotropic systems

Classical field theory

Non-equilibrium expectation value:

$$\langle O(t)\rangle = \mathrm{Tr}\,\hat{\rho}(t_0)\hat{O}(t) = \mathrm{Tr}\,\hat{\rho}\hat{U}(t_0-t)\hat{O}\hat{U}(t-t_0)$$

$\hat{\rho}$ some non-eq. density matrix at t_0. Express in field configuration basis:

$$\int \mathcal{D}[\phi(\mathbf{x})] \int \mathcal{D}[\phi_0(\mathbf{x})]\mathrm{Tr}\;\; |\phi_0\rangle\rho(\phi_0)\langle\phi_0|\;\; \hat{U}(t_0-t)\;\; |\phi\rangle\mathcal{O}(\phi)\langle\phi|\;\; \hat{U}(t-t_0)$$

Write the two matrix elements as two path integrals ("Schwinger-Keldysh field doubling"):

$$\langle\phi_0|\hat{U}(t_0-t)|\phi\rangle \left[\langle\phi_0|\hat{U}(t_0-t)|\phi\rangle\right]^* =$$

$$\int_{\phi_+(t_0)=\phi_0}^{\phi_+(t)=\phi} \mathcal{D}[\phi_+] \int_{\phi_0}^{\phi} \mathcal{D}[\phi_-] e^{i(S[\phi_+]-S[\phi_-])}$$

Classical field theory

For classical approximation, write

$$\chi = \frac{1}{2}(\phi_- + \phi_+) \quad \pi = (\phi_- - \phi_+)$$

In a system with ϕ_+, ϕ_- large, χ large and π small:

$$S[\phi_+] - S[\phi_-] \approx \pi \frac{\delta S[\chi]}{\delta \chi}$$

Now path integral linear in π. Integral over π just gives a constraint:

$$\prod_{\mathbf{x},\tau} 2\pi\delta\left[\frac{\delta S[\chi]}{\delta\chi}\right]$$

field χ obeys classical equations of motions at all points.

$\Rightarrow$ Sample initial conditions as per density matrix, evolve classically

Occupancies on a lattice

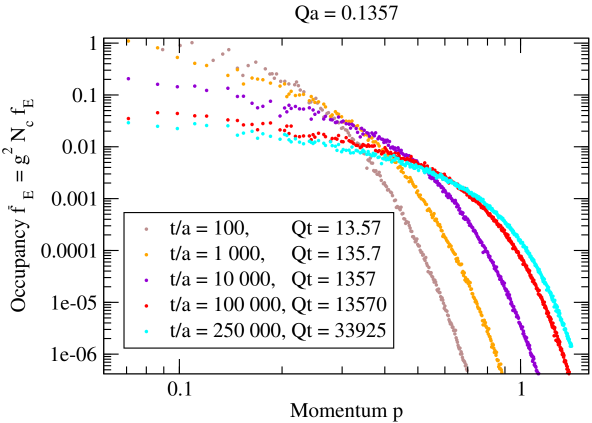
Assume free particle dispersion to define an occupancy:

$$G^{>}_{AA}(\mathbf{p}) = \int d^3x \, e^{i\mathbf{p}\cdot\mathbf{x}} \langle A^i(\mathbf{x}) A^j(0) \rangle \underset{free}{=} \frac{\mathcal{P}^{ij}(\mathbf{p})}{|\mathbf{p}|} f(p)$$

$$f(p) \equiv \frac{\delta_{ij}}{2} |\mathbf{p}| \int d^3x e^{i\mathbf{p}\cdot\mathbf{x}} \langle A^i(x) A^j(0) \rangle_{\rm coulomb}$$

- As long as modifications to disp. rel small $D_\mu \sim p_\mu + g\langle A^2 \rangle^{1/2}$, $p^2 \gg g^2 \langle A^2 \rangle$, $f(p)$ corresponds to a occupation number of gluons
- Screening scale: $m^2_{\rm screen} \sim g^2 \langle A^2 \rangle \sim g^2 \int d^3p \frac{f(p)}{p}$
  - Below $m_{\rm screen}$ physics of massive plasmons, landau damping, etc..
  - $m^2_{\rm screen} \propto g^2 p^2_{\rm max} f \propto t^{-2/7}$, particle description good for wider ranges of p at late times

Classical YM on a lattice

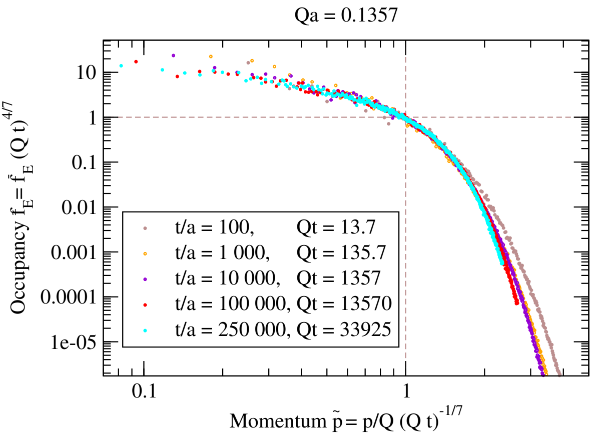


- Define scale $Q^4 = g^2\epsilon$
- In classical FT, the cascade continues forever Non-thermal fixed point

$V = 256^3$, SU(2), fixed lattice spacing, one initial condition

AK, Moore 1207.1663

Classical YM on a lattice



- Define scale $Q^4 = g^2\epsilon$
- In classical FT, the cascade continues forever Non-thermal fixed point

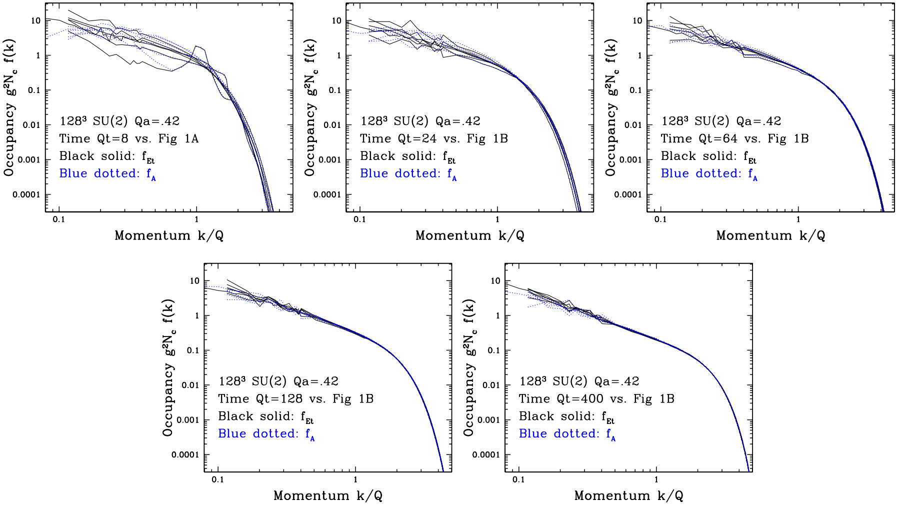
$V = 256^3$, SU(2), fixed lattice spacing, one initial condition

AK, Moore 1207.1663

Scaling reached very quickly

AK, Moore 1207.1663

6 very different initial conditions:

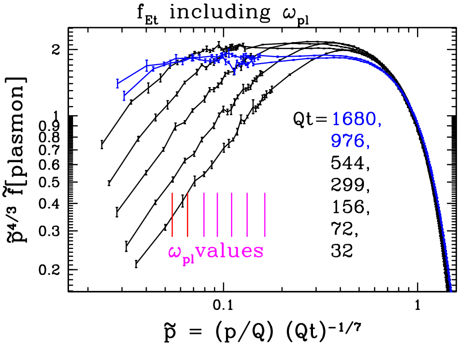


Differences vanish by $Qt \sim 64$

A mystery?

What is the form of the scaling solution?

- All the scales below $p_{\rm max}$ have had time to undergo large angle scattering and therefore have had time to reach thermal form $f(p) \propto p^{-1}$ AK, Moore 1107.5050

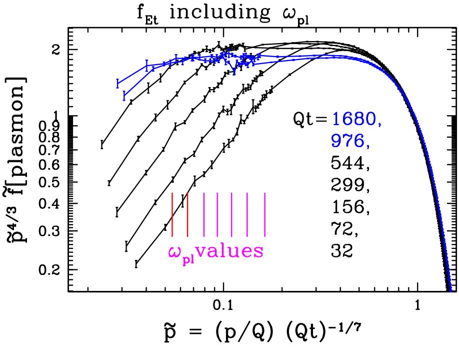


- Suggested solution by weak wave turbulence? Berges, Scheffler and Sexty 0811.4293

A mystery?

What is the form of the scaling solution?

- All the scales below $p_{\max}$ have had time to undergo large angle scattering and therefore have had time to reach thermal form $f(p) \propto p^{-1}$ AK, Moore 1107.5050



- Suggested solution by weak wave turbulence? Berges, Scheffler and Sexty 0811.4293
- Seems unfeasible to resolve by looking at ~ 1 decade of data

Kinetic theory

The forward Wightman fnct. obeys an equation of motion
$(G^{>}(x,y) = G_{+-}(x,y) = \langle \phi_+(x)\phi_-(y)\rangle)$

$$(\partial_x^2 - \partial_y^2)G^{>}(x,y) = \sum_i \int_z (G_{+i}(x,z)\Sigma_{i-}(z,y) - \Sigma_{+i}(x,z)G_{i-}(z,y))$$

Change coord to average and difference $x, y \to X + r/2, X - r/2$ and Fourier transform WRT to relative coord r

$$(\partial_x^2 - \partial_y^2) = 2\partial_X \partial_r = 2ip^\mu \partial_\mu^X$$

Assuming a free particle form of the Wightman fnct. gives function of f.
Expanding self energies gives collision terms C[f]:

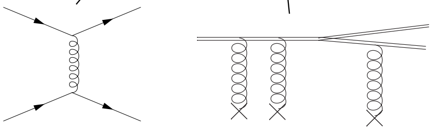
$$2p^\mu \partial_\mu f(p) = -C[f](p)$$

- Reliable if $p > m_{\rm screen}$ and $C[f]$ has expansion: $(g^2 N_c f)$

Closer look into kinetic theory

Arnold, Moore, Yaffe hep-ph/0209353

$$\frac{df}{dt} = -C_{2\leftrightarrow 2}[f] - C_{1\leftrightarrow 2}[f]$$



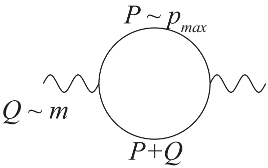
$$C_{2\leftrightarrow 2}[f] = \int_{k,p',k'} |M|^2 \left[f_p f_k (1+f_{p'})(1+f_{k'}) - f_{p'} f_{k'} (1+f_p)(1+f_k) \right]$$

- Naively $|M|^2 \sim 9 + \frac{(t-u)^2}{s^2} + \frac{(s-u)^2}{t^2} + \frac{(s-t)^2}{u^2}$
- However, t and u channels suffer from a Coulombic divergence:

$$\int |M|^2 \propto \int d^2 q_\perp \frac{1}{(q_\perp^2)^2} \longrightarrow \int d^2 q_\perp \frac{1}{(q_\perp^2 + m_{\rm screen}^2)^2}$$

  - At the screening scale $m_{\rm screen}^2 \sim g^2 \int d^3p \frac{f(p)}{p}$ medium effects important for the exchange gluon: regulate the matrix element

Soft scattering, hard loops



- Propagation of the exchange gluon ($q \sim m_{\mathrm{screen}}$) is modified dominantly by hard modes $p \sim p_{\mathrm{max}}$:

$$m_{\mathrm{screen}}^2 = g^2 \int d^3p \frac{f(p)}{p}$$

- Kinematic simplification:

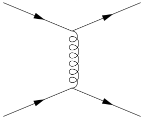
$$\Pi_T^{ij}(\mathbf{q},\omega) = -g^2 \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{f(\mathbf{p})}{p} \left[2 + \frac{(q^2 - \omega^2)(1 - (\mathbf{v_p} \cdot \mathbf{v_q})^2)}{(\mathbf{v_p} \cdot \mathbf{q} - \omega)^2} \right]$$

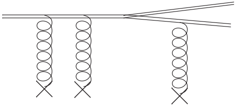
Mrówczyński, Thoma hep-ph/0001164

- Resumed propagators for soft gluons: Hard Loop theory
  - Equivalent to Vlasov equations: soft modes classical fields, hard modes classical particles. Blaizot, Iancu hep-ph/0101103

Inelastic scattering

$$\frac{df}{dt} = -C_{2\leftrightarrow 2}[f] - C_{1\leftrightarrow 2}[f]$$

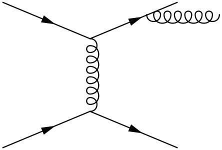




IR divergence in the elastic scattering makes soft scattering rate large

$$\Gamma_{\rm soft} \sim \sigma n(1+f) \sim p_{\rm max} f^2 \frac{p_{\rm max}^2}{m_{\rm screen}^2}, \qquad \Gamma_{\rm hard} \sim p_{\rm max} f^2$$

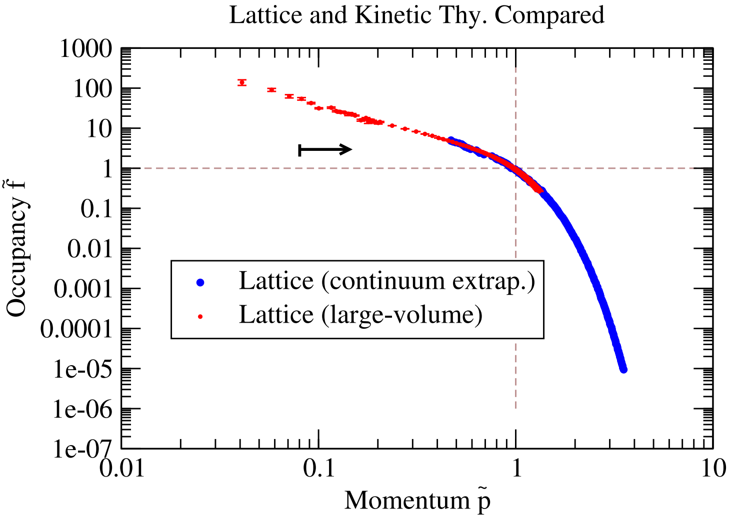
Each time a particle undergoes a soft scattering, has g^2 chance to split



$\Gamma_{\rm split} \sim g^2 \Gamma_{\rm soft}(1+f) \sim \Gamma_{\rm hard}$

As important as elastic scattering

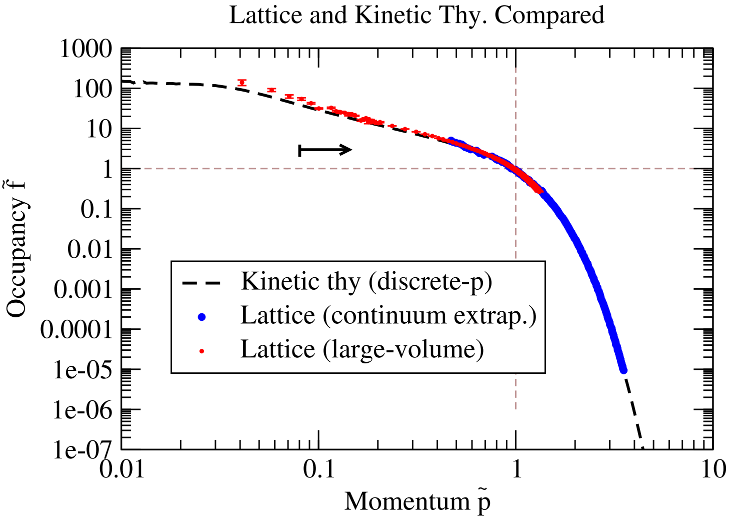
Apples to apples comparison:



Abraao York, AK, Lu, Moore 1401.3751.

Large-volume: (Qa)=0.2, (QL)=51.2, Cont. extr.: down to (Qa)=0.1, (QL)=25.6 Both at Qt=2000, $\tilde{m} = 0.08$

Apples to apples comparison:



Explicit demonstration of field-particle duality!

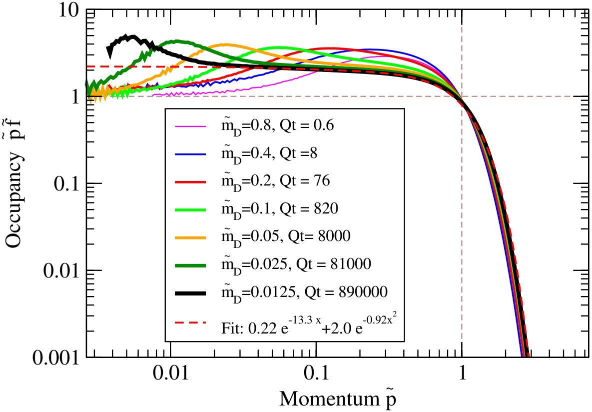
Abraao York, AK, Lu, Moore 1401.3751.

Large-volume: (Qa)=0.2, (QL)=51.2, Cont. extr.: down to (Qa)=0.1, (QL)=25.6 Both at Qt=2000, $\tilde{m} = 0.08$

Coming back to the mystery:

Kinetic theory simulations $\sim$ 1000 times faster

What is the power law form of the scaling solution?

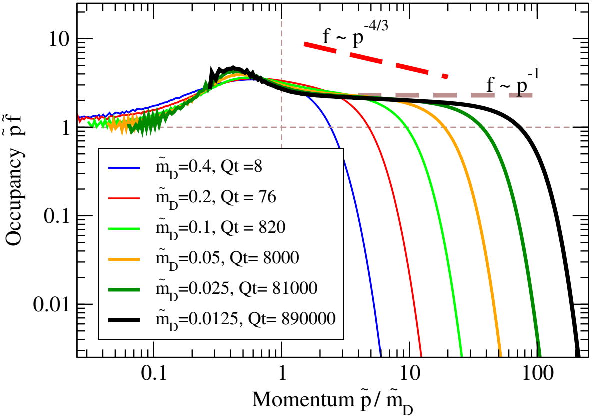


Solution to the mystery: No $p^{-4/3}$ scaling, rather large regions of special physics at m_{screen} and p_{max}

Coming back to the mystery:

Kinetic theory simulations ~ 1000 times faster

What is the power law form of the scaling solution?

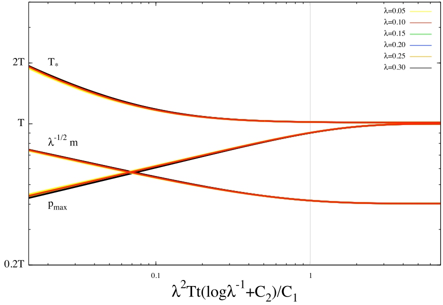


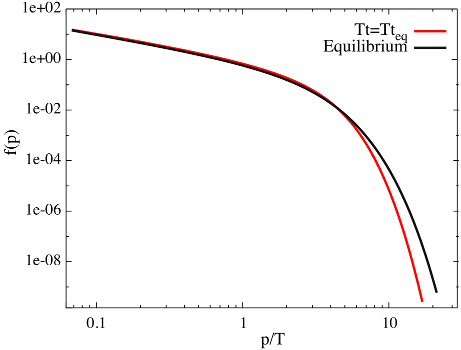
Solution to the mystery: No $p^{-4/3}$ scaling, rather large regions of special physics at m_{screen} and p_{max}

End of the cascade

with Egang Lu, 1403.xxxx

Kinetic theory can address the relaxation from the scaling form to equilibrium

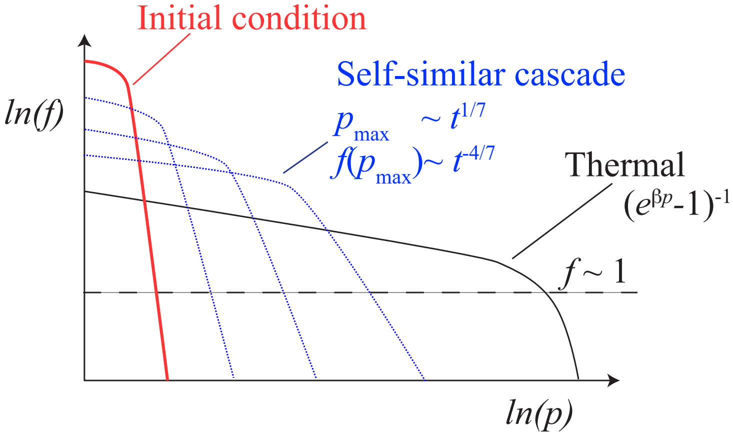




- T_* apparent temperature of the IR tail
- At $t_{\rm eq} \sim T/g^4$ of course!, $T_* \sim p_{\rm max} \sim T$

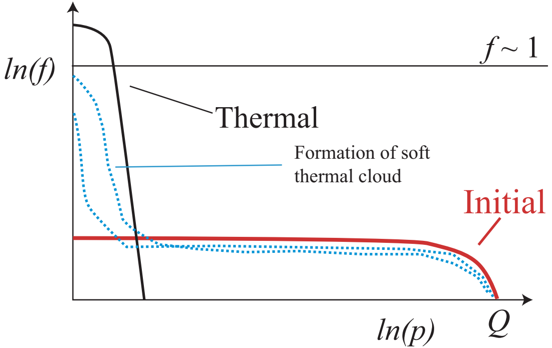
Overoccupied cascade: summary

- The thermalization under control at all time scales
  - Fast relaxation to scaling form (classical)
  - Parametrically long time in scaling form (classical and kinetic)
  - Relaxation to equilibrium form (kinetic)
- (semi-)classical physics due to scale separations



Small initial occupancy

Just the same thing backwards?? NO!



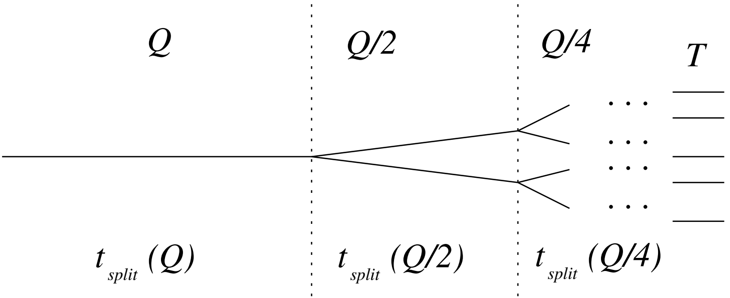
Soft inelastic scattering fast: build soft particle distribution

Radiation cascade

Hard particles move in soft thermal bath, radiate

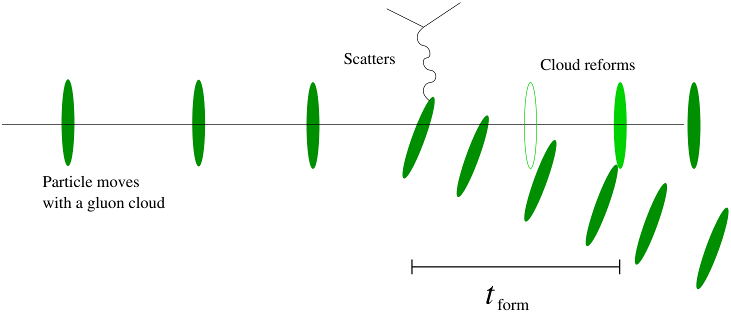
Hard splitting slow

Once the hard particles have had time to split democratically, they cascade quickly to IR



$t_{\rm split}$ governed by physics of Landau-Pomeranchuck-Migdal suppression

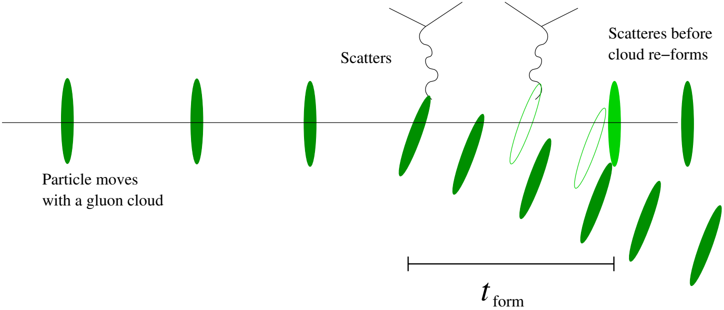
Landau-Pomeranchuck-Migdal suppression



- Scattering kicks a gluon from the virtual cloud
- Cloud reforms when the wave-packets separate

$$t_{\text{form}} \sim \frac{\text{trans. size}}{\text{trans. vel.}} \sim \frac{1/p_\perp}{p_\perp/p} \sim \frac{p}{p_\perp^2}$$

Landau-Pomeranchuck-Migdal suppression



- Frequent or soft scattering: cloud hasn't re-formed Coulomb div.!
- At most one emission per t_{form} $\Rightarrow$Reduced rate:

$$p_\perp^2 \equiv \hat{q}\, t_{\mathrm{form}} \Rightarrow \Gamma_{\mathrm{emit}}(p) \sim g^2 t_{\mathrm{form}}^{-1} \sim g^2 \sqrt{\frac{\hat{q}}{p}}$$

QED: Landau, Pomeranchuk, Migdal 1953. QCD: Baier Dokshitzer Mueller Peigne Schiff hep-ph/9607355

Effective $C_{1\leftrightarrow 2}$ matrix element revisited Arnold, Moore, Yaffe hep-ph/0209353

$$C_{1\leftrightarrow 2} \sim \int dp\, \gamma^{p}_{k,p-k} \left[f_p(1+f_k)(1+f_{p-k}) - f_k f_{p-k}(1+f_p) \right]$$

$$\gamma^{p'}_{p,k} \sim \underbrace{\frac{p'^4 + p^4 + k^4}{p'^3 p^3 k^3}}_{\text{DGLAP split-kernel}} \int \frac{d^2 h}{(2\pi)^2} \mathbf{h} \cdot \mathrm{Re}\mathbf{F}(\mathbf{h}; p', p, k)$$

$$2\mathbf{h} = i\delta E(\mathbf{h})\mathbf{F}(\mathbf{h}) + \frac{g^2 N_c}{2} \int \frac{d^2\mathbf{q}}{(2\pi)^2} \left[T_* \left(\frac{1}{\mathbf{q}^2} - \frac{1}{\mathbf{q}^2 + m^2_{\text{screen}}} \right) \right] \times \left(3\mathbf{F}(\mathbf{h}) - \mathbf{F}(\mathbf{h} - p\mathbf{q}) - \mathbf{F}(\mathbf{h} - k\mathbf{q}) - \mathbf{F}(\mathbf{h} + p\mathbf{q}) \right) \tag{1}$$

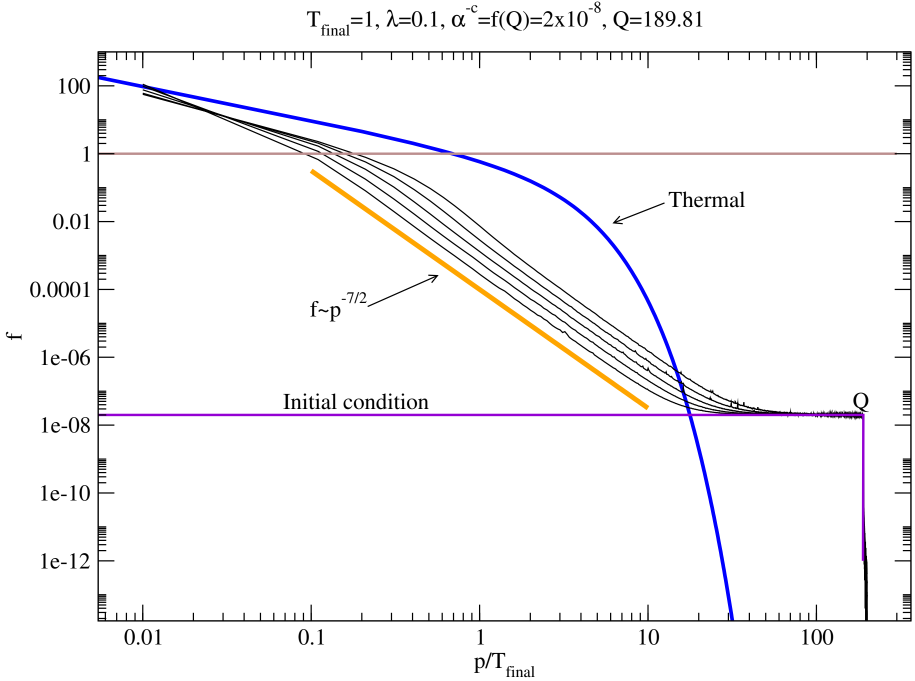
Where sensitivity to the medium comes from

- δE is the difference of energies of one gluon with momentum p' compared the two with k, p': depends on effective masses
- Apparent termperature $T_* \sim \int_{\mathbf{p}} f(1+f)/(\int_{\mathbf{p}} f/p) \sim \hat{q}/m^2_{\text{screen}}$

Bottom-up thermalization Baier, Mueller, Schiff, Son hep-ph/0009237, AK, Moore 1107.5050

- Hard particles collide with each other and emit LPM suppressed radiation $n_{\rm daughter}(p) \sim n_{\rm hard}\Gamma_{\rm emit}(p)t \propto p^{-1/2} \Rightarrow f \propto tp^{-7/2}$
-
-
-

with Egang Lu, 1403.xxxx



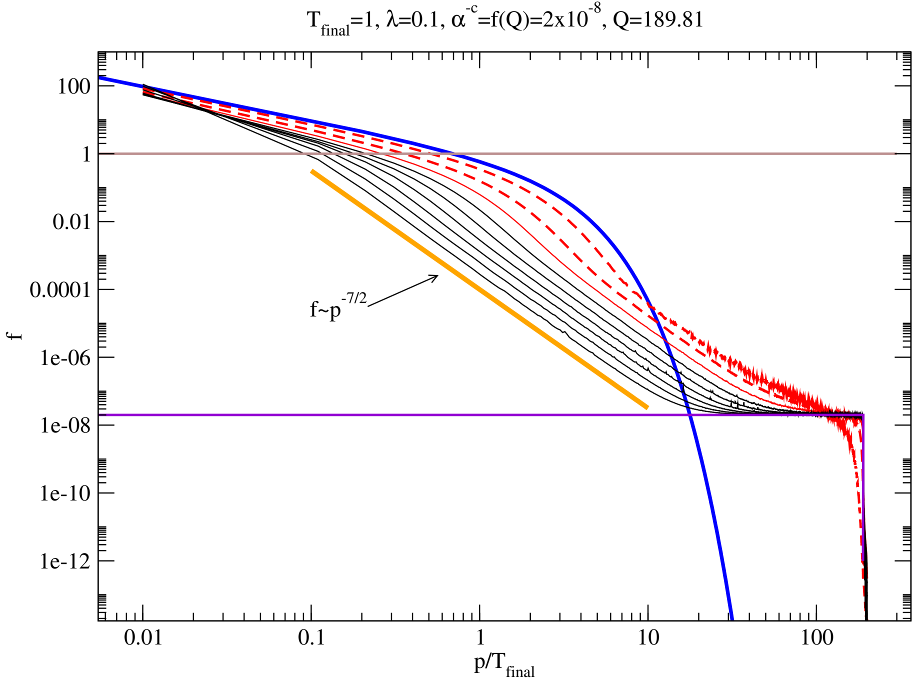
Bottom-up thermalization
Baier, Mueller, Schiff, Son hep-ph/0009237, AK, Moore 1107.5050

- Hard particles collide with each other and emit LPM suppressed radiation $n_{\rm daughter}(p) \sim n_{\rm hard}\Gamma_{\rm emit}(p)t \propto p^{-1/2} \Rightarrow f \propto tp^{-7/2}$
- Soft particles become numerous enough to start dominating screening and scattering. Thermalize among themselves
- Radiation from the hard particles heats up the soft thermal bath

$$T^4 \sim \epsilon_{\rm soft} \sim n_{\rm hard} k_{\rm split} \quad \text{with} \quad \Gamma_{\rm emit}(k_{\rm split})t \sim 1$$

-

with Egang Lu, 1403.xxxx



Bottom-up thermalization Baier, Mueller, Schiff, Son hep-ph/0009237, AK, Moore 1107.5050

- Hard particles collide with each other and emit LPM suppressed radiation $n_{\rm daughter}(p) \sim n_{\rm hard}\Gamma_{\rm emit}(p)t \propto p^{-1/2} \Rightarrow f \propto tp^{-7/2}$

- Soft particles become numerous enough to start dominating screening and scattering. Thermalize among themselves

- Radiation from the hard particles heats up the soft thermal bat

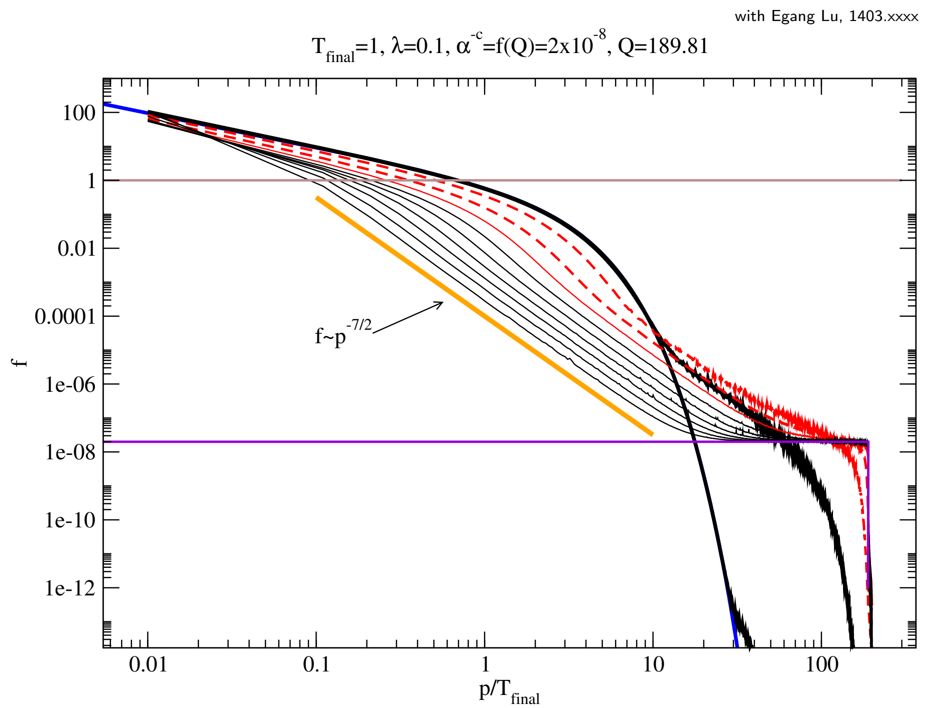
$$T^4 \sim \epsilon_{\rm soft} \sim n_{\rm hard} k_{\rm split} \quad \text{with} \quad \Gamma_{\rm emit}(k_{\rm split})t \sim 1$$

- Thermalization when all the hard particles have had time to undergo hard splitting

$$k_{\rm split} \sim Q \sim g^4 \hat{q}_T t_{\rm therm}^2, \qquad \hat{q}_T \sim g^4 T^3, \qquad T^4 \sim n_{\rm hard} Q$$

$$\text{so that } t_{\rm therm} \sim 1/(g^4 T)\sqrt{Q/T}$$

Naively $\hat{q}_T t \sim Q^2 \to t_{\rm naive} \sim 1/(g^4 T)(Q/T)^2$



with Egang Lu, 1403.xxxx
T_final=1, λ=0.1, α^-c=f(Q)=2x10^-8, Q=189.81
f~p^-7/2
f
p/T_final
100
1
0.01
0.0001
1e-06
1e-08
1e-10
1e-12
0.01
0.1
1
10
100

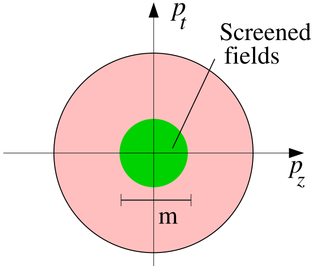
Anisotropic systems, anisotropic screening:

Mrowczynski

Isotropic distributions:

- Screening stabilizes soft E-fields

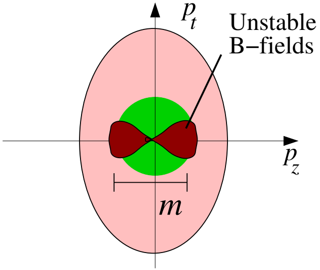
$$\omega_{pl}^2 \sim m_{\rm screen}^2 \sim g^2 \int d^3p \frac{f(p)}{p}$$



- B-fields induces a rotation on $f(p) \to$ No screening for static B-fields

Anisotropic distributions:

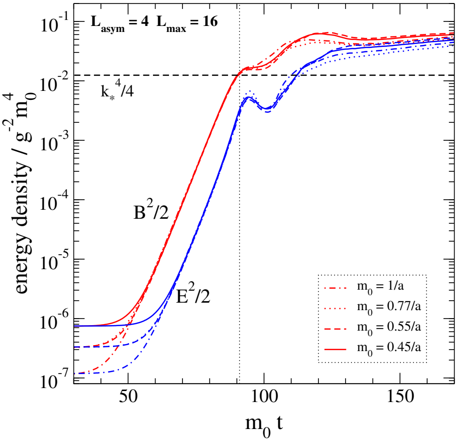
- B-field induces non-trivial rotation:
  - Some B-fields stabilized
  - ...others destabilized: Plasma-unstable modes



- Unstable modes grow exponentially. . .

Saturation of the instabilities.

Instabilities and their saturation can be simulated using Hard loop/lattice Vlasov simulations



Bödeker and Rummukainen (0705.0180) and many others. . .

Anisotropic systems

- Are plasma instabilities important?

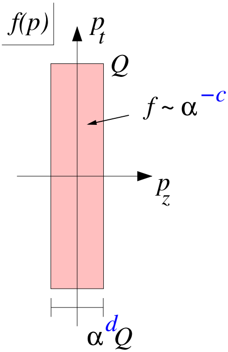
  (elastic scattering, inelastic scattering, other instabilities,...)

Anisotropic systems

- Are plasma instabilities important?

  (elastic scattering, inelastic scattering, other instabilities,...)

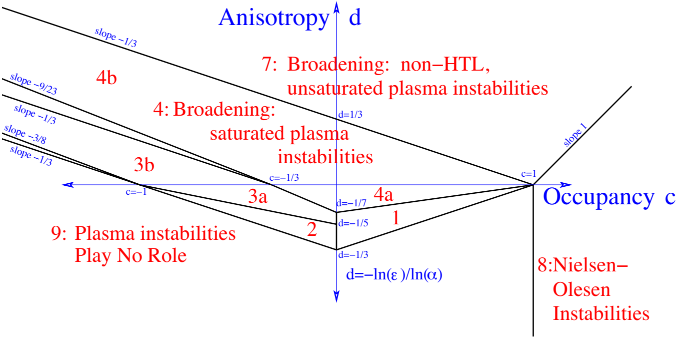
- Depends on the system:
  - The more anisotropic the system is, the stronger instabilities
  - Depends also on occupancies

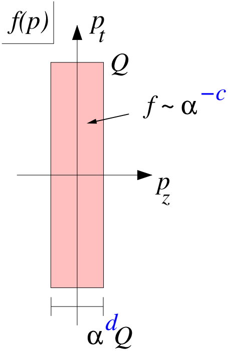


Anisotropic systems

- Are plasma instabilities important?

  (elastic scattering, inelastic scattering, other instabilities,...)
- Depends on the system:
  - The more anisotropic the system is, the stronger instabilities
  - Depends also on occupancies





($\alpha_s \ll 1$, AK, Moore 1107.5050)

Summary

- At weak coupling many non-eq. systems can be mapped to classical eff. theories and studied numerically
  - Classical field theory
  - kinetic theory
  - Vlasov equations
  - ...similarly at strong coupling: classical gravity

- The physics is very different from scalar theories
  - Complicated screening
  - Small angle scattering
  - LPM
  - Instabilities

- Applications:
  - Heavy-ion collisions AK, Moore 1108.4684
  - Reheating Harigaya, Mukaida 1312.3097
  - ...