



High
Luminosity
LHC

Fringe Field modeling status

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Contents

- Non linear fringe field effects
 - 4 different tracking models
 - Pro & Con
- Test of quadrupoles
 - Triplet
 - Q4 MQYY
- Proposed strategy for SixTrack

4 tracking methods

- I. Lie Tracking: calculation of transfer map using 3D magnetic field data
- II. RK4: 4th order symplectic integrator (by A. Chao)
- III. Lee-Withing fringe field model
- IV. Forest & Milutinovic hard edge fringe field model

I. Symplectic integrator of z-dependent Hamiltonian

$$K(x, p_x, y, p_y, \delta, l, z, p_z; \sigma) \approx -\delta + \frac{(p_x - a_x)^2}{2(1 + \delta)} + \frac{(p_y - a_y)^2}{2(1 + \delta)} - a_z + p_z$$

- $a_x \equiv a_x(x, y, \mathbf{z}) = \frac{qA_x(x, y, \mathbf{z})}{P_0 c}$; $a_y = a_y(x, y, \mathbf{z}) = \frac{qA_y(x, y, \mathbf{z})}{P_0 c}$; $a_z = a_z(x, y, \mathbf{z}) = \frac{qA_z(x, y, \mathbf{z})}{P_0 c}$;
- σ is the independent variable with $d\sigma = dz$
- (\mathbf{z}, p_z) is the fourth canonical pairs, needed to have the explicit dependence on \mathbf{z}

The solution of the equation of motion (Transfer Map) for this Hamiltonian using Lie algebra formalism is:

$$M(\sigma) = \exp(-: \sigma K :)$$

The transfer map $M(\sigma)$ can be replaced by a product of symplectic maps which approximates it (symplectic integration).

Reference:

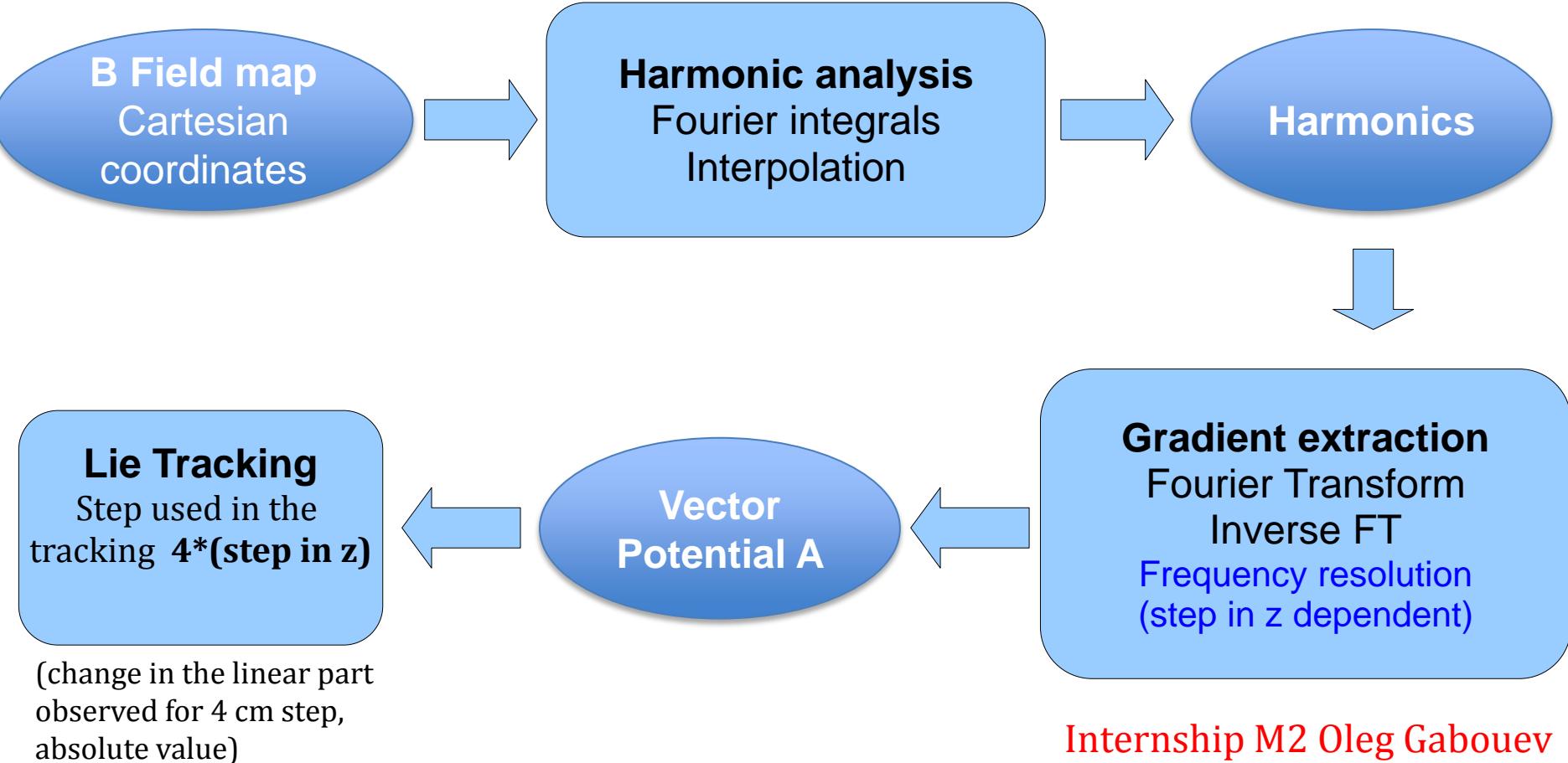
Y. Wu, E. Forest and D. S. Robin, Phys. Rev. E 68, 046502, 2003

I. Transfer map

The second half of iterations for K_1 , K_2 and K_3 is missing in the table.

	K_1	K_2	K_3		K_4		
	$-\frac{\Delta\sigma}{2}(p_z - \delta)$	$\frac{\Delta\sigma}{2}a_z$	$-\int a_x dx$	$-\frac{\Delta\sigma}{2} \frac{(\mathbf{p}_x)^2}{2(1 + \delta)}$	$\int a_x dx$	$-\int a_y dy$	$-\Delta\sigma \frac{(\mathbf{p}_y)^2}{2(1 + \delta)}$
x				$+ \frac{p_x \Delta\sigma}{2(1 + \delta)}$			
p_x		$+ \frac{\partial a_z \Delta\sigma}{\partial x} \frac{1}{2}$	$-a_x$		$+a_x$	$- \int \frac{\partial a_y}{\partial x} dy$	$+ \int \frac{\partial a_y}{\partial x} dy$
y							$+ \frac{p_y \Delta\sigma}{(1 + \delta)}$
p_y		$+ \frac{\partial a_z \Delta\sigma}{\partial y} \frac{1}{2}$	$- \int \frac{\partial a_x}{\partial y} dx$		$+ \int \frac{\partial a_x}{\partial y} dx$	$-a_y$	$+a_y$
l	$-\frac{\Delta\sigma}{2}$			$- \frac{(p_x)^2 \Delta\sigma}{4(1 + \delta)^2}$			$- \frac{(p_y)^2 \Delta\sigma}{2(1 + \delta)^2}$
δ							
z	$+ \frac{\Delta\sigma}{2}$						
p_z		$+ \frac{\partial a_z \Delta\sigma}{\partial z} \frac{1}{2}$	$- \int \frac{\partial a_x}{\partial z} dx$		$+ \int \frac{\partial a_x}{\partial z} dx$	$- \int \frac{\partial a_y}{\partial z} dy$	$+ \int \frac{\partial a_y}{\partial z} dy$

I. From 3D magnetic field data to tracking

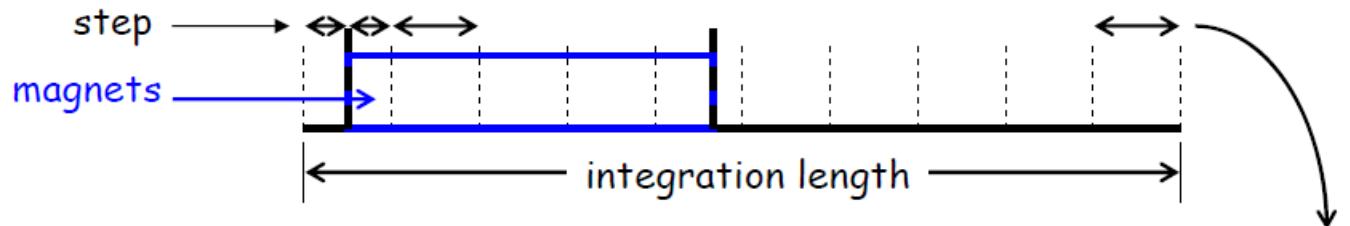


Internship M2 Oleg Gabouev

I. Lie Tracking

- Pro
 - realistic description of magnet design
 - can separate/study in the tracking the contribution of the different multipoles
- Con
 - harmonics reconstruction limited to the radius of analysis
 - numerical integration slow than kicks (step size 3 cm)
 - not trivial to perform a statistical DA study (i.e. integrate multipoles errors)

II. RK4



$$\frac{d^2\vec{r}}{ds^2} = \frac{1}{B\rho} \frac{d\vec{r}}{ds} \times \vec{B}$$

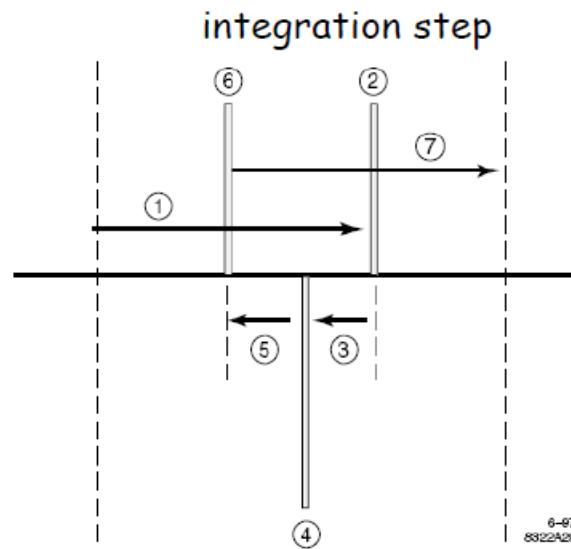


Figure 7.3: Seven steps in the 4-th order symplectic integration.

A. Chao Lectures

June 24 2009

ILC-CLIC LET Beam Dynamics
Workshop

20

II. RK4

- Pro
 - realistic description of magnet design
 - no further elaboration of the 3D magnetic field data needed
- Con
 - numerical integration slow than kicks (step size order of mm)
 - not trivial to perform a statistical DA study (i.e. integrate multipoles errors)
 - not possible to separate/study the contribution of the different multipoles

but it is the reference for tracking on one single element

III. Lee-Withing

First order:

$$\Delta x = \frac{k_0}{12} [(x^3 + 3xy^2)]$$

$$\Delta p_x = \frac{-k_0}{12} [3px(x^2 + y^2)]$$

Extension to 3rd order:

"Intrinsic Third Order Aberrations in Electrostatic and Magnetic Quadrupoles",
R.Baartman, TRIUMF

$$\Delta x \approx \frac{k_0}{12} [(x^3 + 3xy^2)] - \frac{k(s_0)I_1}{6} (5x^3 + 9xy^2) + \frac{I_1}{2} x(P_x^2 - P_y^2) + I_1 P_x (xP_x + yP_y)$$

$$\begin{aligned} \Delta P_x \approx & \frac{-k_0}{12} [3P_x(x^2 + y^2) - 6xyP_y] + \frac{1}{72} [6k'(s_0)I_1 + k'''(s_0)I_3] (4x^3 + 12xy^2) \\ & + \frac{k(s_0)I_1}{6} (15x^2P_x + 9P_xy^2 + 18yP_yx) - \frac{I_1}{2} P_x(P_x^2 - P_y^2) \end{aligned}$$

III. Lee-Withing

- Pro
 - faster than numerical integration
 - it needs just two kicks: one at the entrance and another at the exit of the hard edge quadrupole with opposite signs
- Con
 - it is not symplectic according to SixTrack
 - higher order multipoles not included
 - no dependence on the magnet aperture included in the model
 - no dependence on the fringe field shape included in the model

IV. Forest hard edge fringe field model

Skew Hard edge kicks:

Rotation of -45°

$$\Delta x = \frac{-k_0}{6} \frac{y^3}{1 + \delta}$$

$$\Delta p_x = \frac{k_0}{6} \left[\frac{3p_y x^2}{1 + \delta} \right]$$

Rotation of 45°

"LEADING ORDER HARD EDGE FRINGE FIELDS EFFECTS EXACT IN $(1 + \delta)$ AND CONSISTENT WITH MAXWELL'S EQUATIONS FOR RECTILINEAR MAGNETS", É. FOREST and J. MILUTINOVIC, Nuclear Instruments and Methods in Physics Research A269 (1988) 474-482

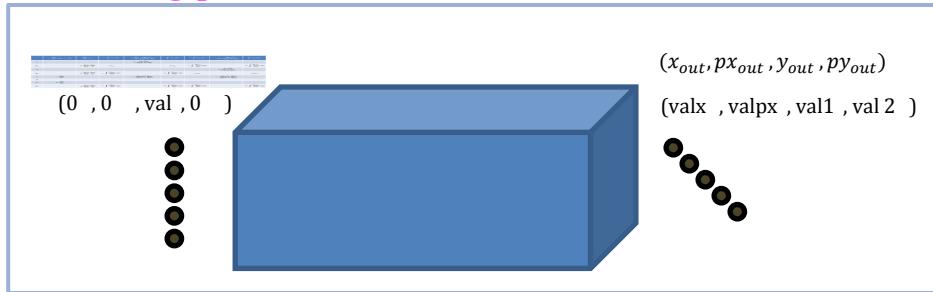
N.B. These kicks are symplectic

IV. Forest hard edge fringe field model

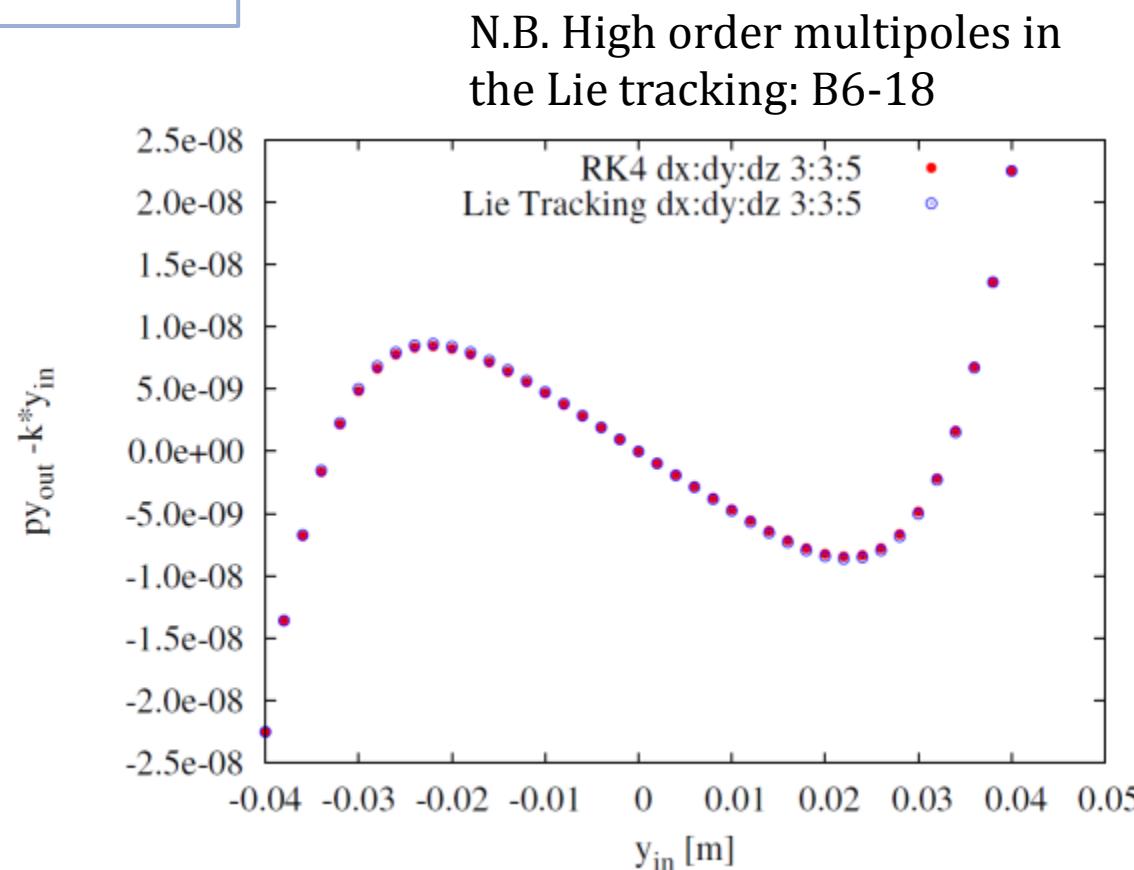
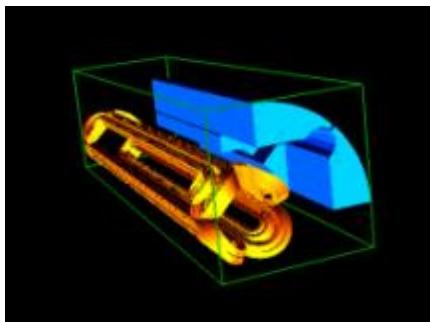
- Pro
 - faster than numerical integration
 - it needs just two kicks: one at the entrance and another at the exit of the hard edge quadrupole with opposite signs
- Con
 - higher order multipoles not included
 - no dependence on the magnet aperture included in the model
 - no dependence on the fringe field shape included in the model

RK4 vs Lie Tracking

Tracking procedure:

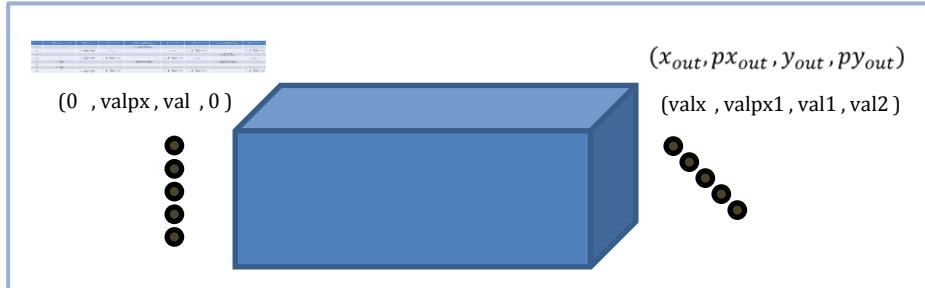


Quadrupoles analyzed:



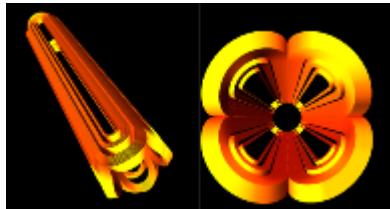
Lie tracking vs Forest

Tracking procedure:

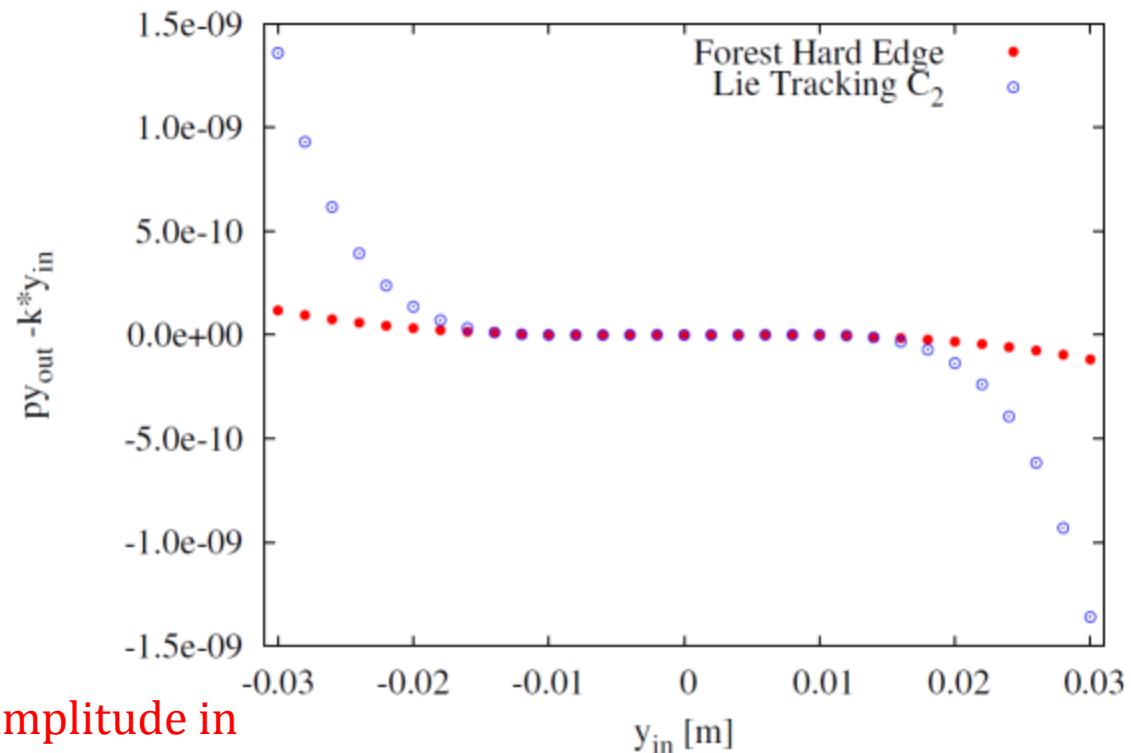


Quadrupole analyzed:

Q4 MQYY sym



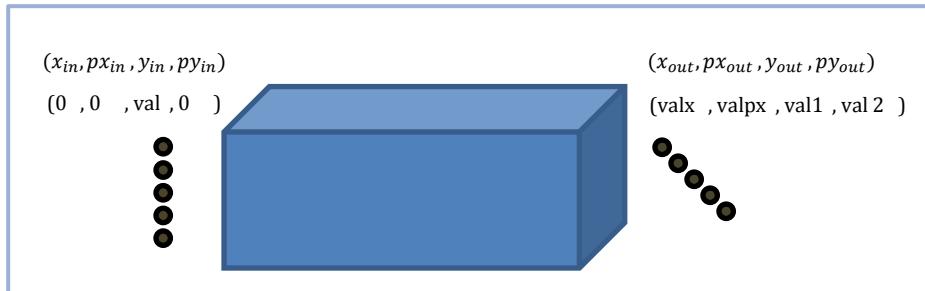
Quadrupole field in the Lie tracking only



Agreement for amplitude in
the range $\sim [-1.5:1.5]$ cm

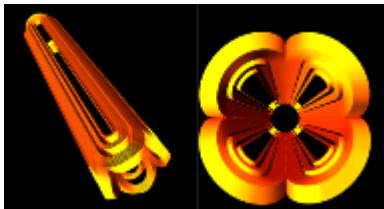
4 tracking comparison: Q4 asym vs sym and high order multipoles

Tracking procedure:

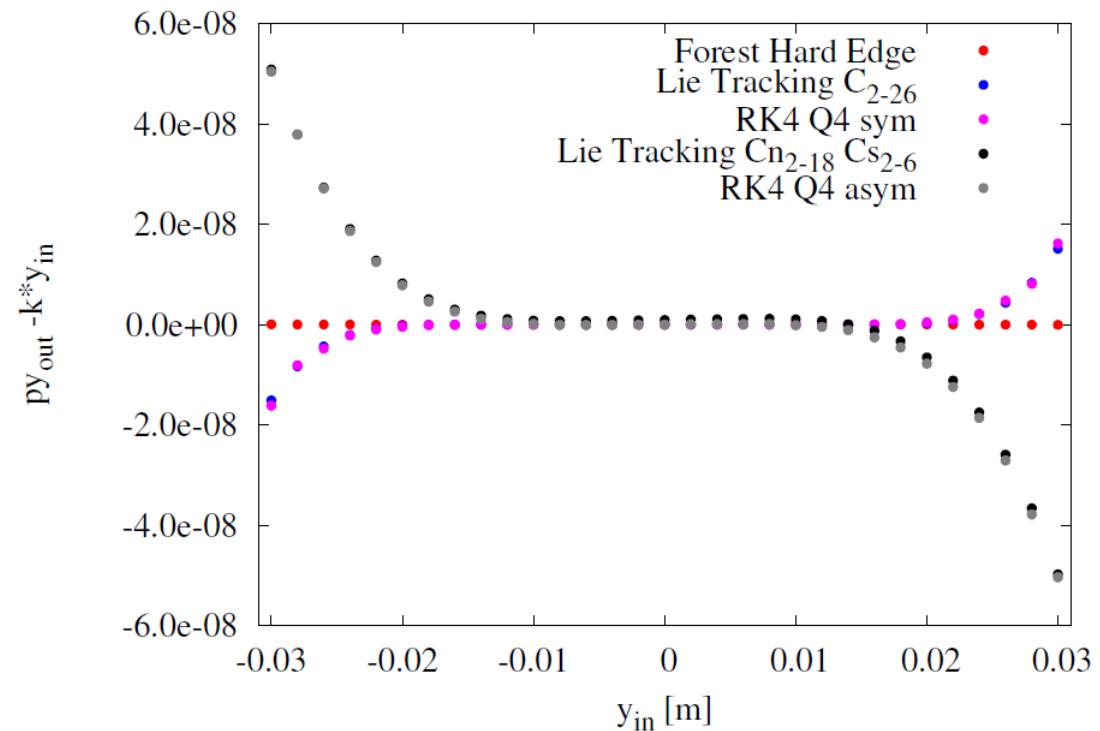
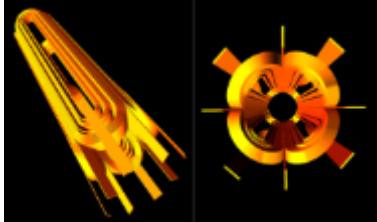


Quadrupoles analyzed:

Q4 MQYY sym

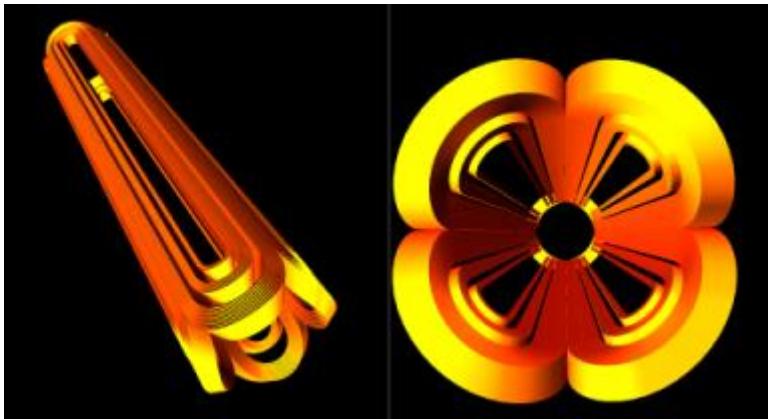


Q4 MQYY asym



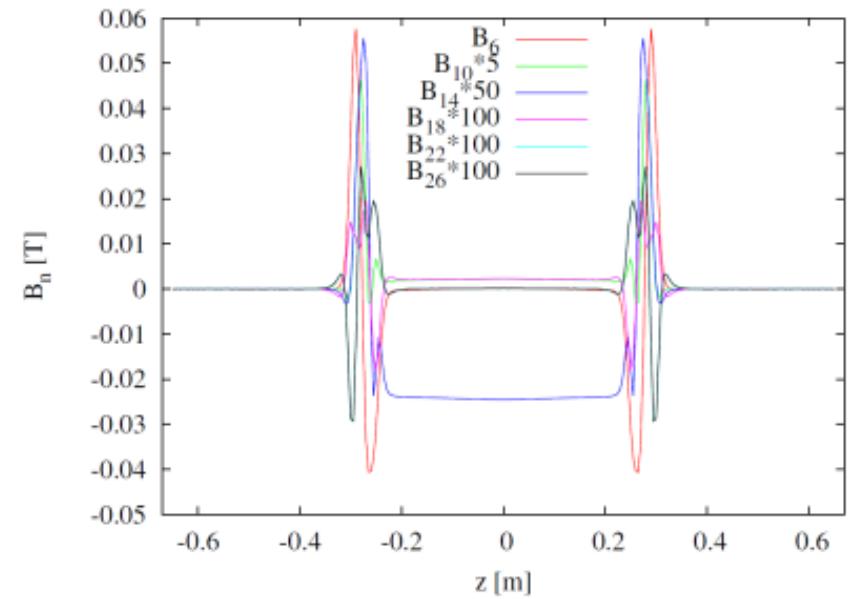
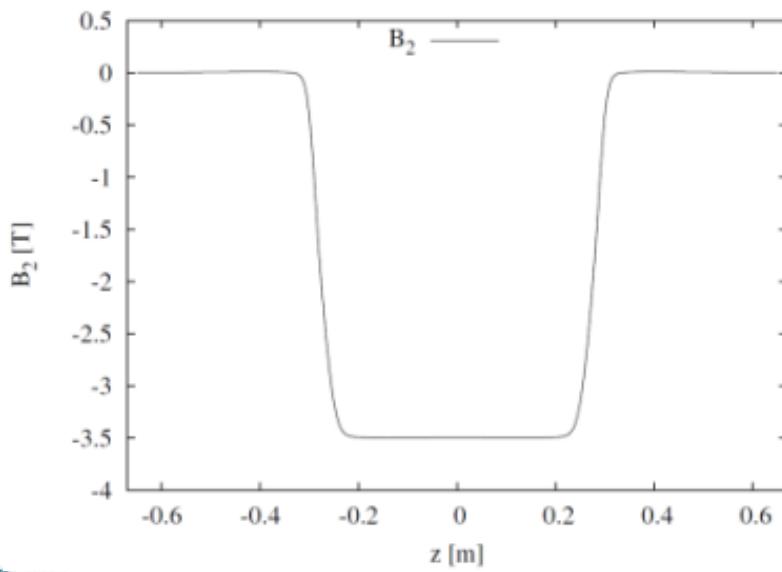
Q4 design (symmetric)

M. Segreto (CEA SACM/LEAS)



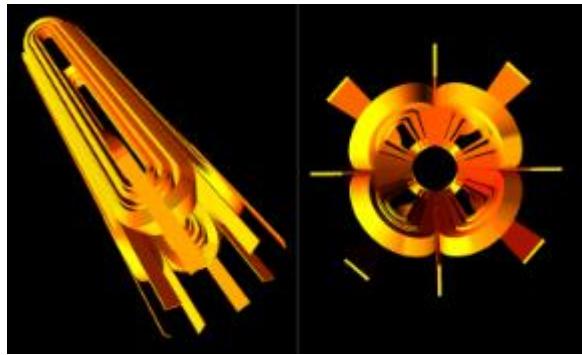
Q4 MQYY , G=120 T/m @1.9 K, $\emptyset = 90$ mm, L=3.5m

⇒ Prototype scaled version



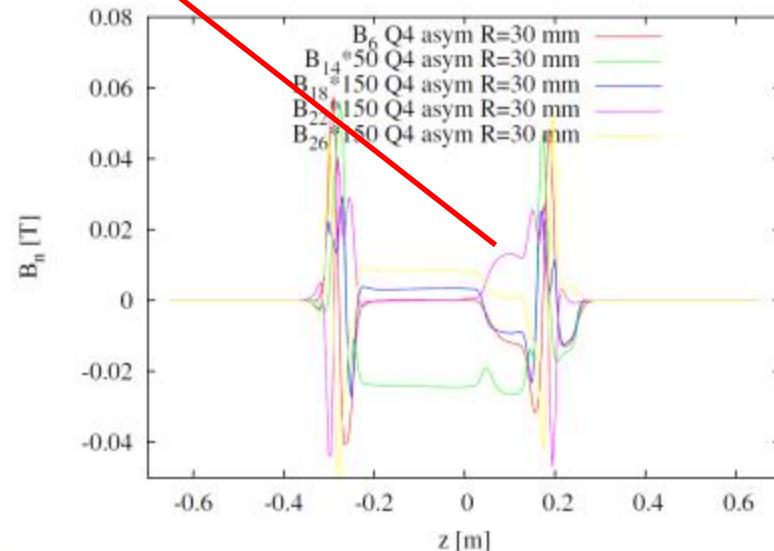
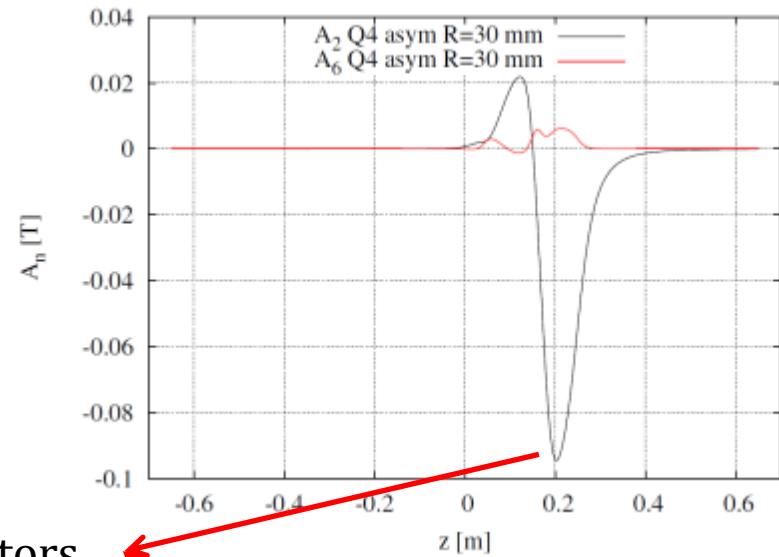
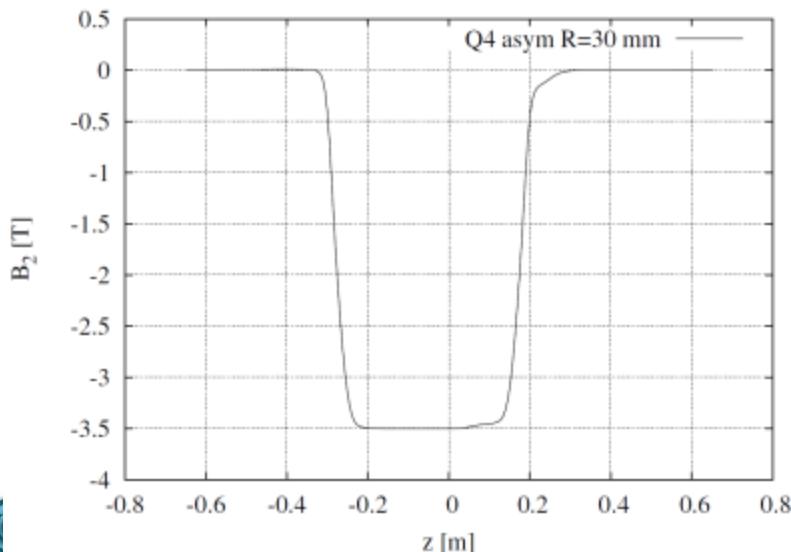
Q4 design (asymmetric)

M. Segreti (CEA SACM/LEAS)



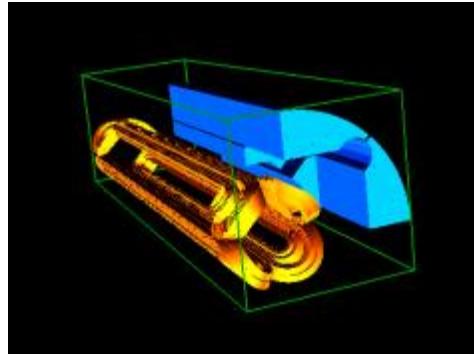
Q4 MQYY , G=120 T/m @1.9 K, $\emptyset = 90$ mm, L = 3.5 m

Side of connectors



Triplet design

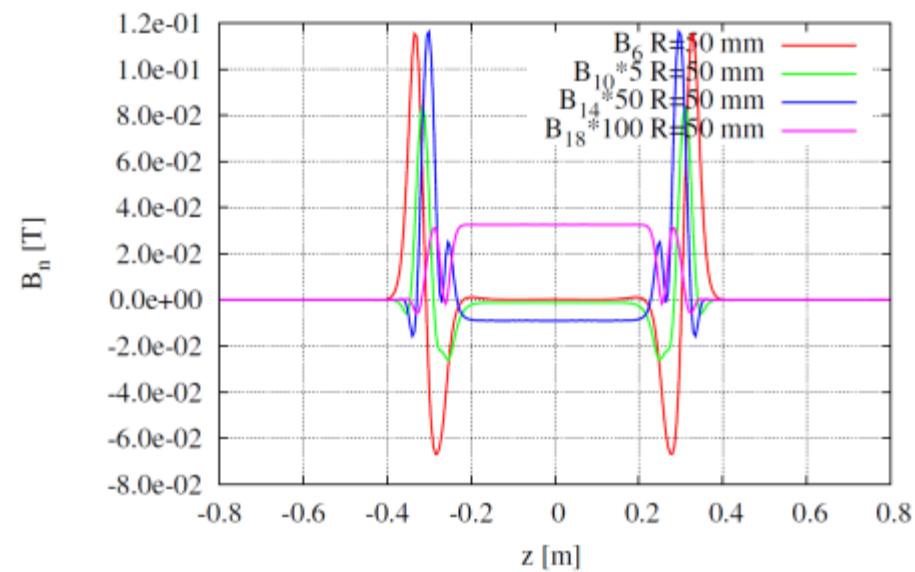
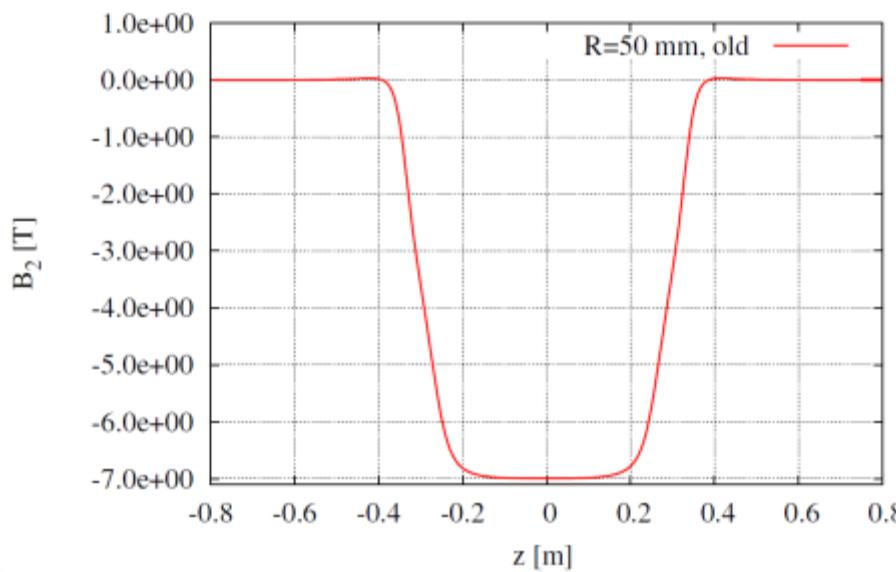
Susana Izquierdo Bermudez (CERN)



old
 $dx:dy:dz = 3:3:5$
[mm]

$G=140 \text{ T/m}$, $\emptyset = 150 \text{ mm}$

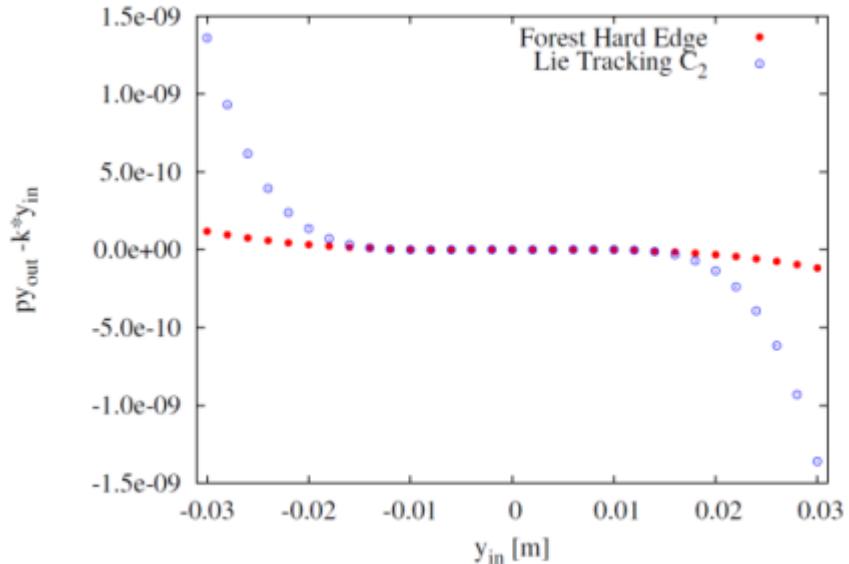
\Rightarrow Prototype scaled version



Proposed strategy for SixTrack

Quadrupole field

- Find an equivalent kick and drift to match the Lie Tracking with C_2 only



High order multipole

- Use multipole kicks as the multipole errors table used in field quality study

Higher order multipoles (triplet)

skew	mean	uncertainty	random	normal	mean	uncertainty	random
a3	0	0.800	0.800	b3	0	0.820	0.820
a4	0	0.650	0.650	b4	0	0.570	0.570
a5	0	0.430	0.430	b5	0	0.420	0.420
a6	0	0.310	0.310	b6	0.800	0.550	0.550
a7	0	0.152	0.095	b7	0	0.095	0.095
a8	0	0.088	0.055	b8	0	0.065	0.065
a9	0	0.064	0.040	b9	0	0.035	0.035
a10	0	0.040	0.032	b10	0.075	0.100	0.100
a11	0	0.026	0.0208	b11	0	0.0208	0.0208
a12	0	0.014	0.014	b12	0	0.0144	0.0144
a13	0	0.010	0.010	b13	0	0.0072	0.0072
a14	0	0.005	0.005	b14	-0.020	0.0115	0.0115

$$B_y + iB_x = 10^{-4} B_{ref} \times \sum_{n=1}^{\infty} (b_n + ia_n) \left(\frac{x + iy}{r_0} \right)^{n-1}$$

IT field quality specifications
at $r_0 = 50$ mm
(« IT_errortable_v66 »)

Yuri Nosochkov,
3rd Joint HiLumi LHC-LARP Meeting

harmonics	skew	normal
6	0,003	-4,034
10	0,003	-0,559
14	0,003	-0,114
18	0,003	-0,251
22		0,026
26		0,002

Integral (1.6 m)/10⁻⁴ Bref at $r_0 = 50$ mm for a **symmetric** design

- The higher order multipoles in the fringe field region seem not to be taken into account in the errors table
- A full table with the errors U and R maybe not possible but we can add the mean values

Higher order multipoles (Q4)

skew	mean	uncertainty	random	normal	mean	uncertainty	random
a3	0	0.682	1.227	b3	0	1.282	1.500
a4	0	0.428	0.893	b4	0	0.483	0.465
a5	0	0.177	0.406	b5	0	0.203	0.431
a6	0	0.484	0.277	b6	0	5.187	1.487
a7	0	0.094	0.189	b7	0	0.094	0.189
a8	0	0.193	0.257	b8	0	0.193	0.257
a9	0	0.088	0.088	b9	0	0.088	0.088
a10	0	0.120	0.120	b10	0	3.587	0.956
a11	0	0.326	0.489	b11	0	0.326	0.489
a12	0	0.445	0.222	b12	0	0.445	0.222
a13	0	0.606	0.303	b13	0	0.606	0.303
a14	0	0.827	0.413	b14	0	2.067	0.413
a15	0	1.127	0.564	b15	0	1.127	0.564

Q4 field errors at $r_0 = 30$ mm
 (« Q4_errortable_v1 »)

Yuri Nosochkov,
 3rd Joint HiLumi LHC-LARP Meeting

harmonics	skew	norm
2	20,549	4550,286
6	-1,854	4,406
10	0,380	-1,403
14	-0,134	0,480
18	-0,003	-0,003
22		-0,039
26		-0,029

Integral (1.3 m) / $10^{-4} B_{ref}$ at $r_0 = 30$ mm for an **asymmetric** design

- Important a2 component
- Uncertainty of b6, b10 and b14 in Q4 errors table are bigger than mean value in the fringe field (...what they do include ?)

Conclusion

- Lie Tracking and RK4 agree for different magnets design. Lie tracking limited at the radius used in the harmonic analysis.
- Hard Edge model of fringe field and Lie tracking (considering C_2 only) agree for small particle amplitude (ex: $\sim \pm 1.5$ cm for the Q4 magnets design)
- Important effect of fringe field high order multipoles and asymmetry of the two magnet ends
- Numerical integration slow and not trivial to do a statistical DA study with it.

Proposed strategy for SixTrack

- Find an equivalent kick and drift to match the Lie Tracking with C_2 only
- Use multipole kicks as the multipole errors table used in field quality study



cern.ch

I. Application

If A_x , A_y and A_z are non zero and K split as:

- $K_1 = p_z - \delta$
- $K_2 = -a_z$
- $K_3 = \left(\frac{(p_x - a_x)^2}{2(1+\delta)} \right)$
- $K_4 = \left(\frac{(p_y - a_y)^2}{2(1+\delta)} \right)$

The second order integrator writes

$$\begin{aligned} \mathcal{M}_2(\Delta\sigma) &= \exp\left(-\frac{\Delta\sigma}{2}(p_z - \delta)\right) \exp\left(\frac{\Delta\sigma}{2}a_z\right) \exp\left(-\int a_x dx\right) \exp\left(-\frac{\Delta\sigma}{2}\frac{(p_x)^2}{2(1+\delta)}\right) \\ &\quad \exp\left(\int a_x dx\right) \exp\left(-\int a_y dy\right) \exp\left(-\Delta\sigma\frac{(p_y)^2}{2(1+\delta)}\right) \exp\left(\int a_y dy\right) \exp\left(-\int a_x dx\right) \\ &\quad \exp\left(-\frac{\Delta\sigma}{2}\left(\frac{(p_x)^2}{2(1+\delta)}\right)\right) \exp\left(\int a_x dx\right) \exp\left(\frac{\Delta\sigma}{2}a_z\right) \exp\left(-\frac{\Delta\sigma}{2}(p_z - \delta)\right) \end{aligned}$$

using

$$\begin{aligned} \exp(-\Delta\sigma K_4) &= \exp\left(-\Delta\sigma\left(\frac{(p_y - a_y)^2}{2(1+\delta)}\right)\right) \\ &= \exp\left(-\int a_y dy\right) \exp\left(-\Delta\sigma\frac{(p_y)^2}{2(1+\delta)}\right) \exp\left(\int a_y dy\right) \end{aligned}$$

Explicit dependence on z

⇒ The number of iterations needed can be reduced choosing a Gauge transformation, so that $A_x=0$ or $A_y=0$

I. Computation of the vector potential in cartesian coordinates

$$A_x = \sum_m \sum_l \sum_{p=0:2:m} \sum_{q=0}^l -\frac{1}{m} \frac{(-1)^l m!}{2^{2l} l!(l+m)!} \binom{m}{p} \binom{l}{q} C_{m,\alpha}^{[2l+1]}(z) i^p x^{m-p+2l-2q+1} y^{p+2q}$$

$$A_y = \sum_m \sum_l \sum_{p=0:2:m} \sum_{q=0}^l -\frac{1}{m} \frac{(-1)^l m!}{2^{2l} l!(l+m)!} \binom{m}{p} \binom{l}{q} C_{m,\alpha}^{[2l+1]}(z) i^p x^{m-p+2l-2q} y^{p+2q+1}$$

$$A_z = \sum_m \sum_l \sum_{p=0:2:m} \sum_{q=0}^l \frac{1}{m} \frac{(-1)^l m!(2l+m)}{2^{2l} l!(l+m)!} \binom{m}{p} \binom{l}{q} C_{m,\alpha}^{[2l]}(z) i^p x^{m-p+2l-2q} y^{p+2q}$$

with

$$[(x + iy)^m] = \sum_{p=0}^m \binom{m}{p} x^{m-p} (iy)^p = \sum_{p=0:2:m} \binom{m}{p} x^{m-p} (iy)^p + \sum_{p=1:2:m} \binom{m}{p} x^{m-p} (iy)^p$$

$$(x^2 + y^2)^l = \sum_{q=0}^l \binom{l}{q} x^{2l-2q} y^{2q}$$

References:

A. J. Dragt, www.physics.umd.edu/dsat

I. The generalized gradients

The z-dependent coefficients can be calculated using the multipole expansion of the magnetic field:

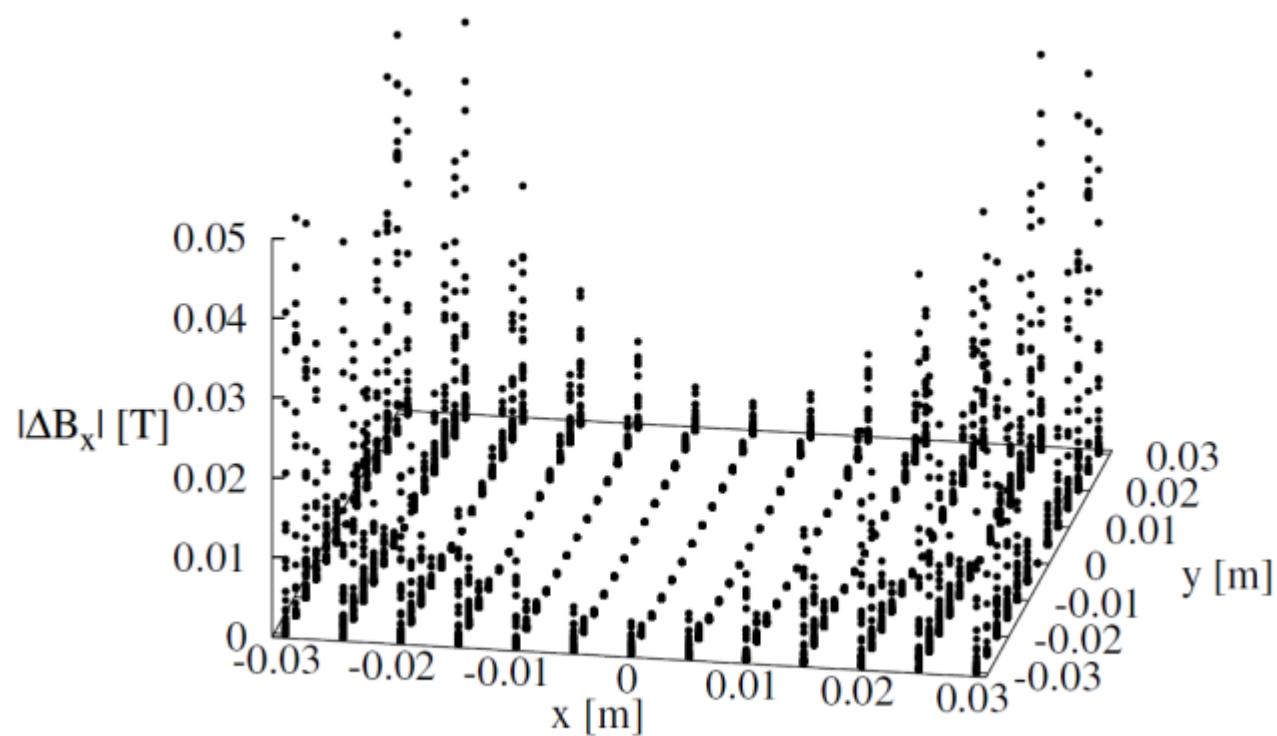
$$C_{m,\alpha=s}^{[n]}(z) = \frac{i^n}{2^m m! \sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{e^{ikz} k^{m+n-1}}{I'_m(kR_{analysis})} \tilde{B}_m(R_{analysis}, k) dk$$

where: $I'_m(kR)$ is the derivative of the modified Bessel function

$$\tilde{B}_m(R, k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-ikz} B_m(R, z) dz$$

$$B_r(R, \phi, z) = \sum_{m=1}^{\infty} B_m(R, z) \sin(m\phi) + A_m(R, z) \cos(m\phi)$$

Field reconstruction



Q4 design

(asymmetric: dx:dy:dz 5:5:5)

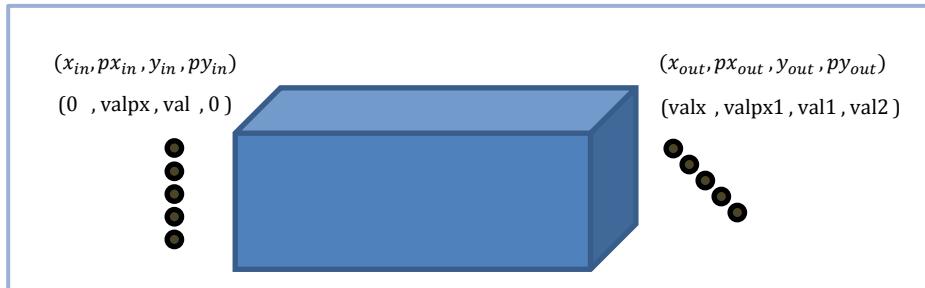
with errors

harmonics	Integral/ 10^{-4} at R=30 mm	
	Norm	Skew
2	-15926.	-71.92
6	-15.42	6.49
10	4.91	-1.33
14	-1.68	$4.7e^{-1}$
18	$1.16e^{-2}$	$-8.8e^{-2}$
22	$1.37e^{-1}$	
26	$1.02e^{-1}$	

harmonics	Integral/ 10^{-4} at R=30 mm	
	Norm	Skew
2	-15915.	-72.86
6	-7.41	6.29
10	6.55	-1.33
14	-1.49	$4.7e^{-1}$
18	$3.87e^{-2}$	$-8.8e^{-2}$
22	$1.46e^{-1}$	
26	$0.66e^{-1}$	

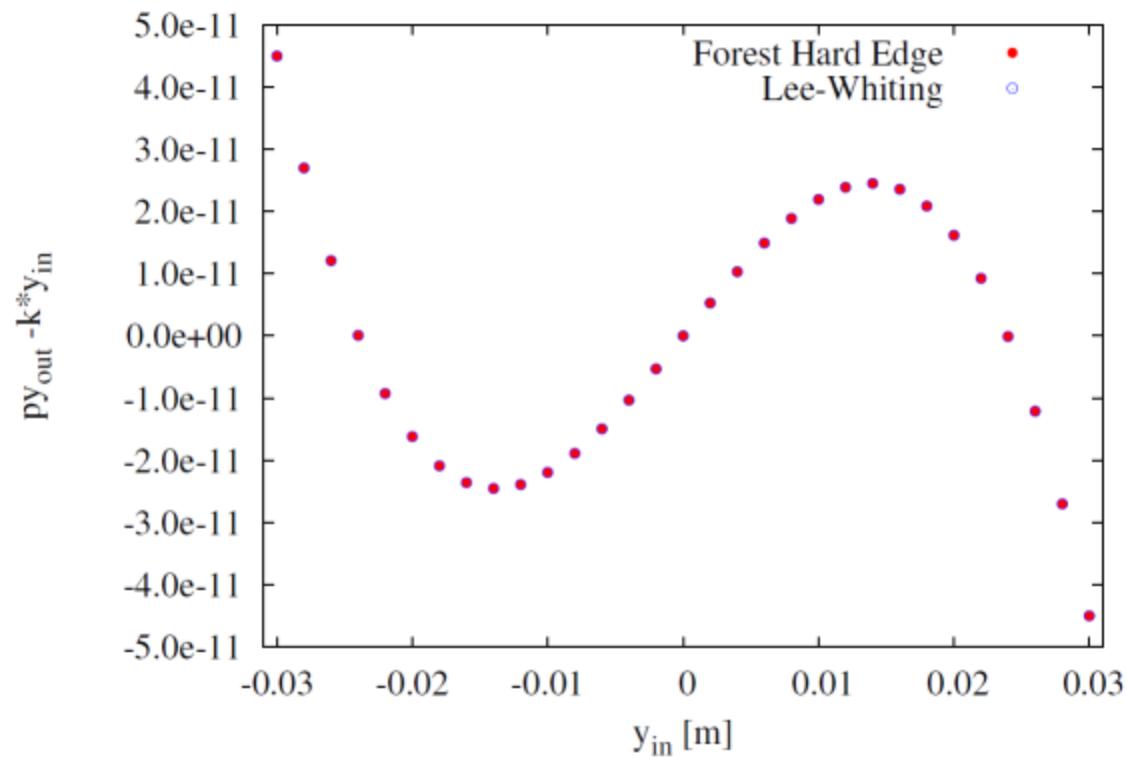
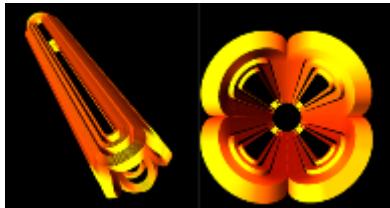
Results: hard edge Fringe Field models

Tracking procedure:



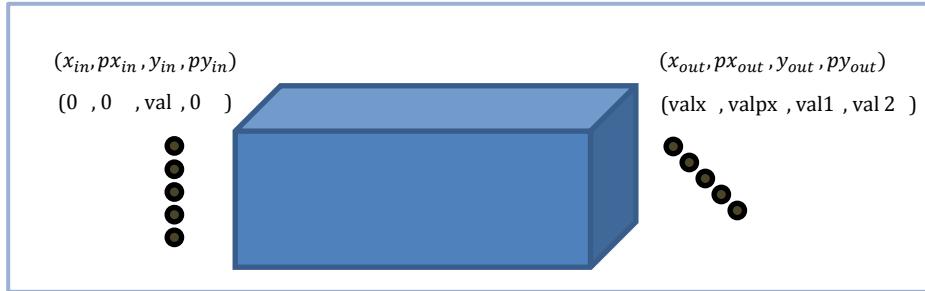
Quadrupole analyzed:

Q4 MQYY sym



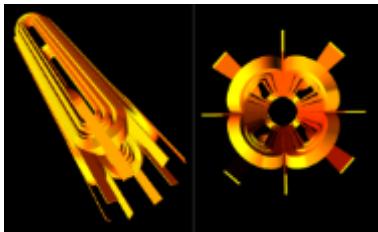
Lie Tracking and magnetic field grid steps

Tracking procedure:



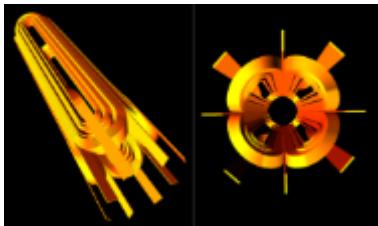
Quadrupoles analyzed:

Q4 MQYY asym



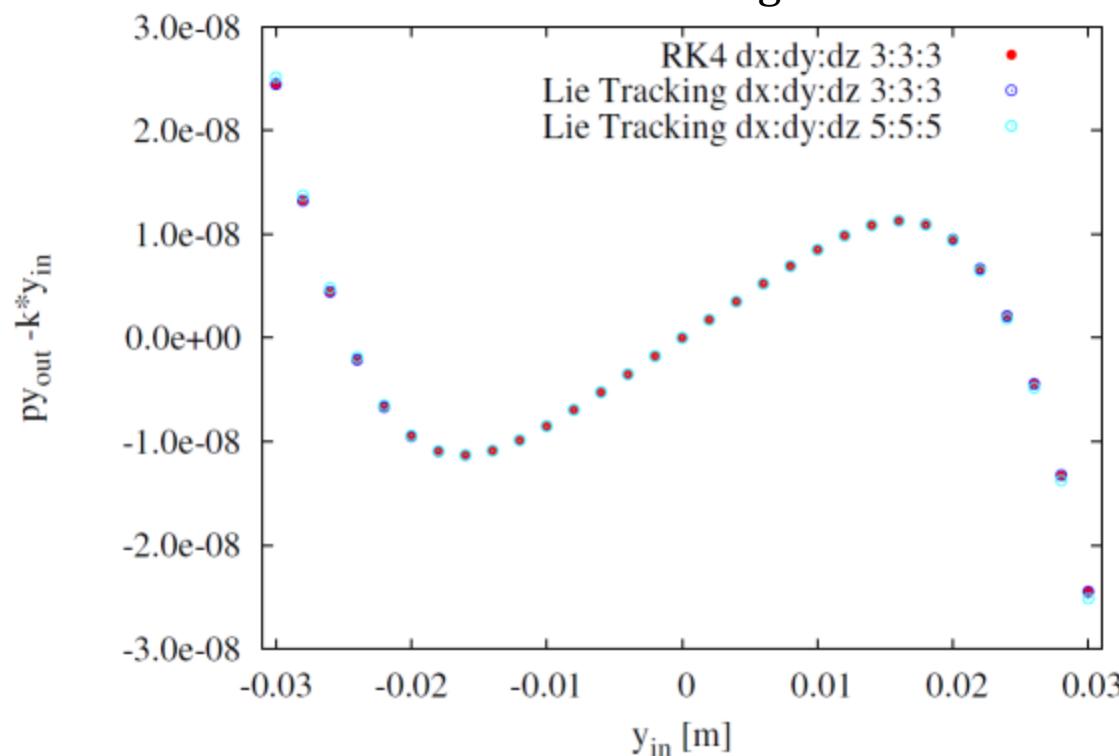
dx:dy:dz
5 : 5 : 5

Q4 MQYY asym



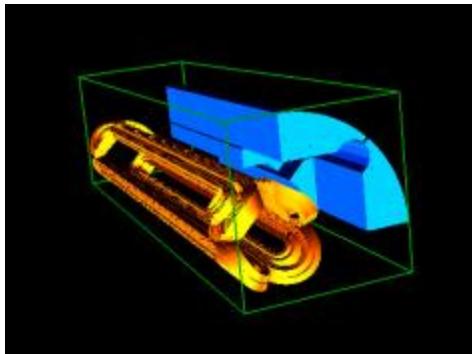
dx:dy:dz
3 : 3 : 3

N.B. High order multipoles in the Lie tracking: B6-18 C2-14

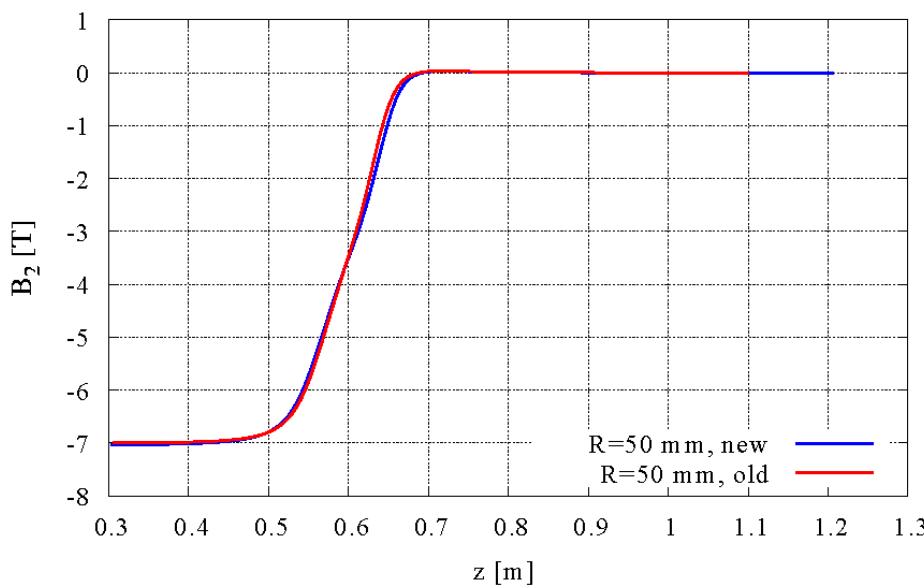


Triplet design

Susana Izquierdo Bermudez (CERN)



old $dx:dy:dz$
 3 : 3 : 5
 [mm]
new $dx:dy:dz$
 3 : 3 : 3



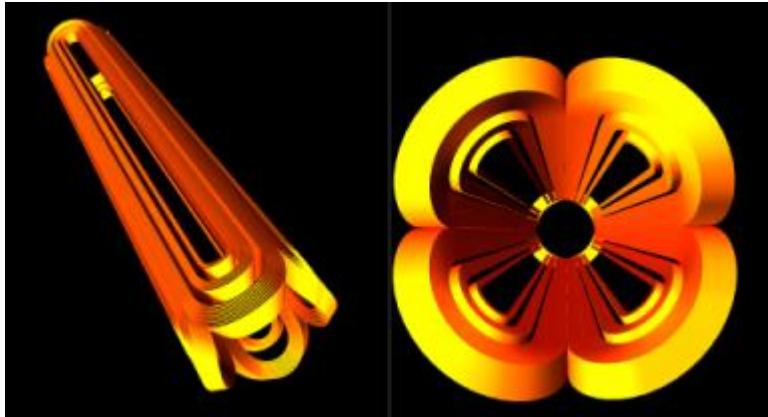
$G=140 \text{ T/m}$, $\emptyset = 150 \text{ mm}$

Integrated $B_n / 10^{-4}$ at $R = 50 \text{ mm}$ for two symmetric side of the magnet end

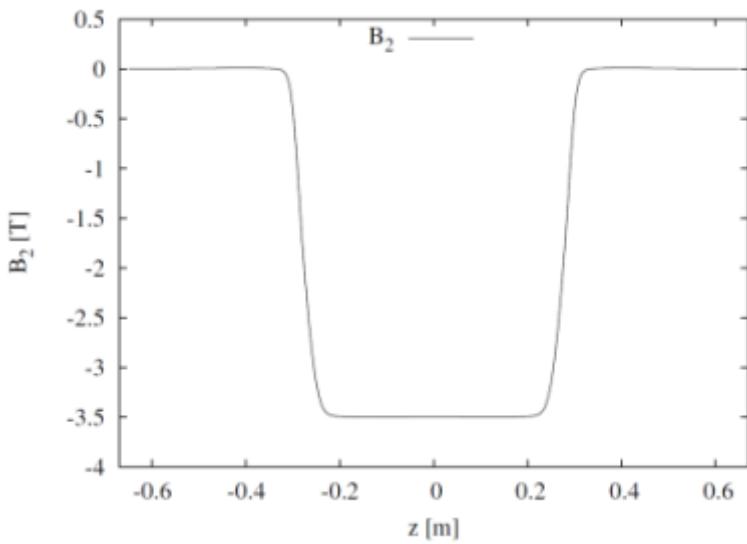
Harmonics	Old map February 2013	New map August 2013
B6	28.24	70.77
B10	3.91	7.23
B14	0.80	4.23
B18	1.76	1.41
B22	-0.18	0.03
B26	-0.015	0.2
A1	-0.02	0.18
A3	-0.02	0.18
A5	-0.02	0.18
A7	-0.02	0.18
A9	-0.02	0.18

(?)

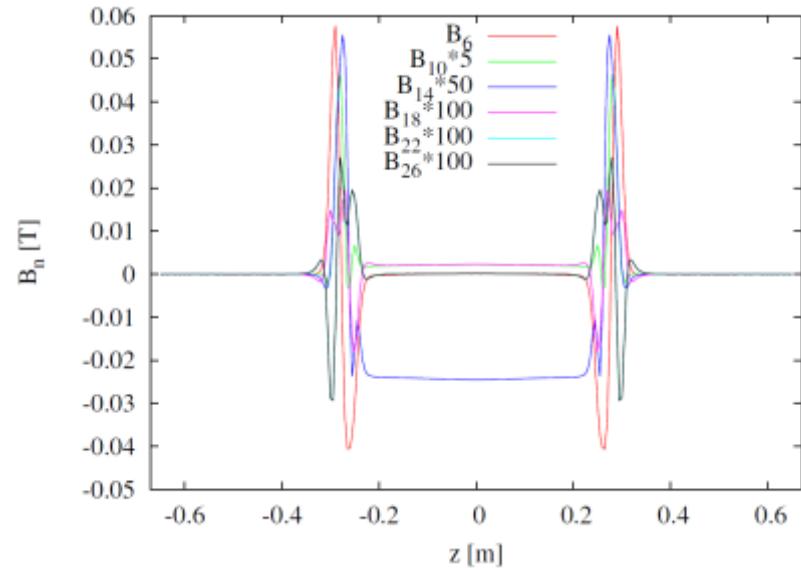
Q4 design (symmetric dx:dy:dz 5:5:5 [mm])



M. Segreti (CEA SACM/LEAS)



harmonics	Integral at R=30 mm
2	-1,94058
6	-3,71327e-5
10	5,5493e-4
14	-1,93321e-4
18	1,60133e-5
22	8,26182e-6
26	1,63571e-5

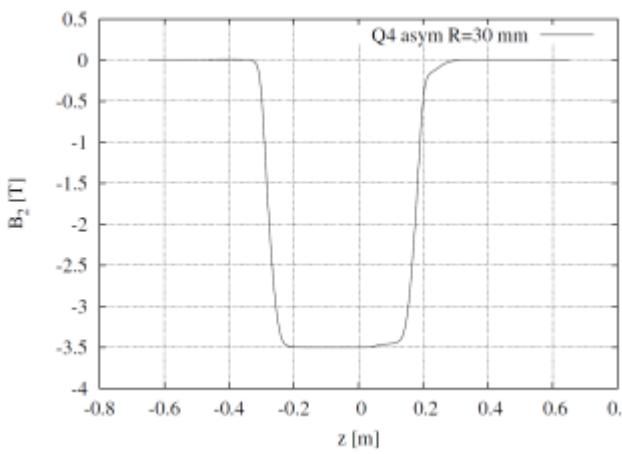
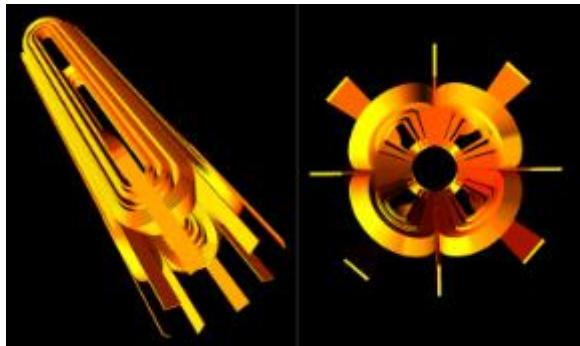


Example: Q4 MQYY , G=120 T/m @1.9 K, $\emptyset = 90$ mm, L=3.5m

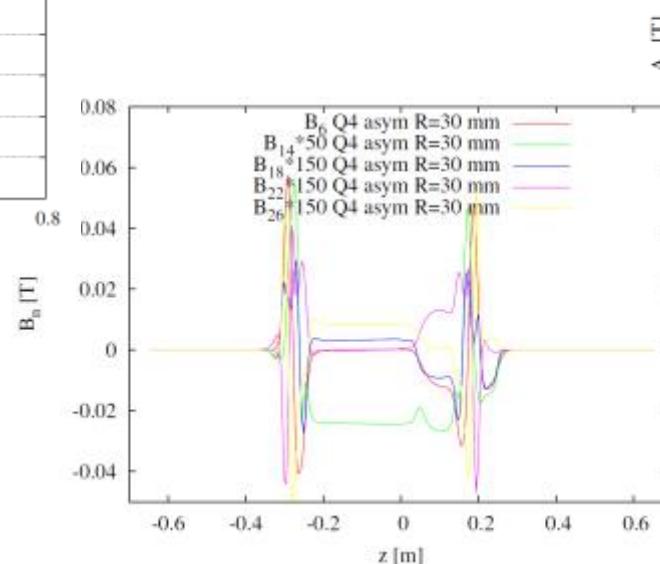
Q4 design

(asymmetric: dx:dy:dz 5:5:5)

M. Segreti (CEA SACM/LEAS)

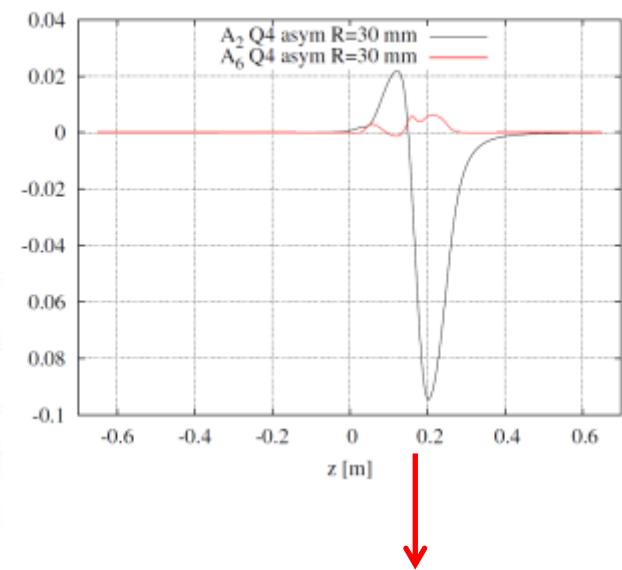


harmonics	Integral at R=30 mm Norm	Skew
2	-1.5926	-0.007192
6	-0.001542	6.49e ⁻⁴
10	4.91e ⁻⁴	-1.33e ⁻⁴
14	-1.68e ⁻⁴	4.7e ⁻⁵
18	1.164e ⁻⁶	-8.8e ⁻⁶
22	1.37e ⁻⁵	
26	1.02e ⁻⁵	



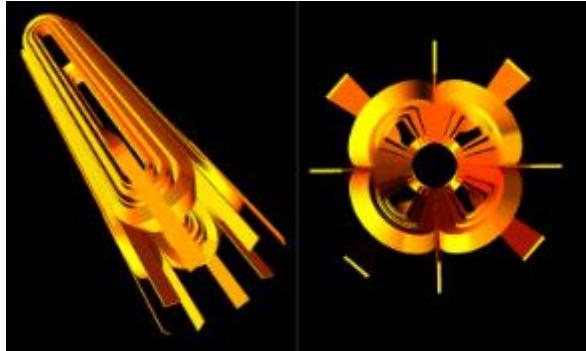
Example: Q4 MQYY ,
G=120 T/m @1.9 K,
 $\emptyset = 90$ mm, L = 3.5 m

Scaled version



Side of connectors

Q4 design (asymmetric: dx:dy:dz 3:3:3)



M. Segreti (CEA SACM/LEAS)

harmonics	Integral at R=30 mm	
	Norm	Skew
2	-1.5926	-0.007192
6	-0.001516	6.45e ⁻⁴
10	4.52e ⁻⁴	-1.22e ⁻⁴
14	-1.61e ⁻⁴	3.8e ⁻⁵
18	3.37e ⁻⁵	-1.37e ⁻⁶
22	3.48e ⁻⁶	
26	-1.73e ⁻⁶	