Accelerator Physics and Limitations

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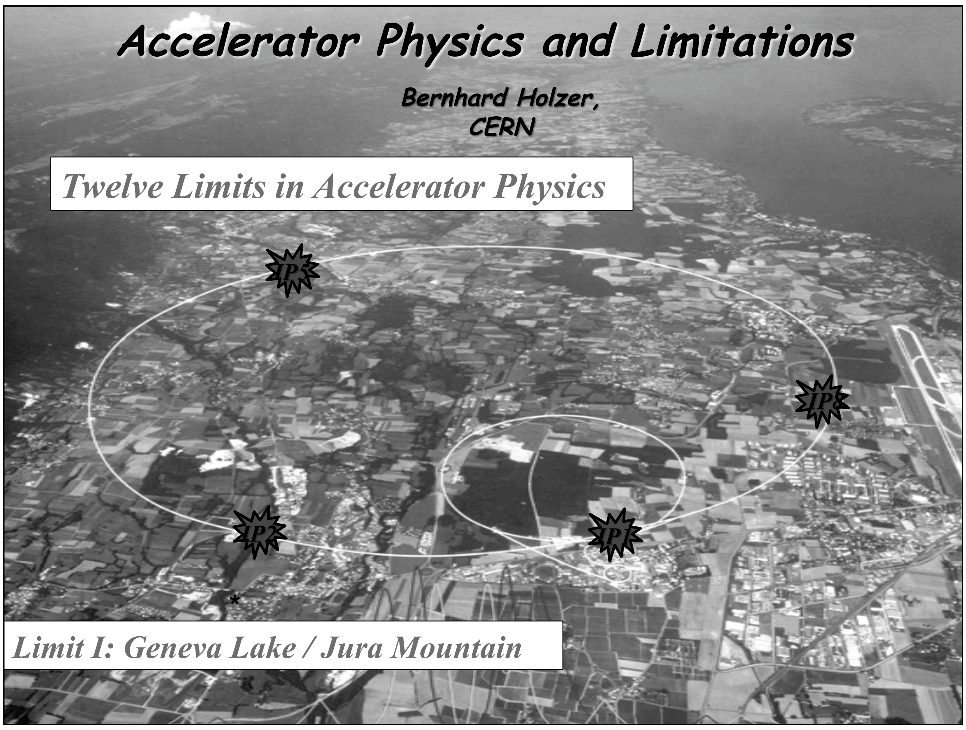
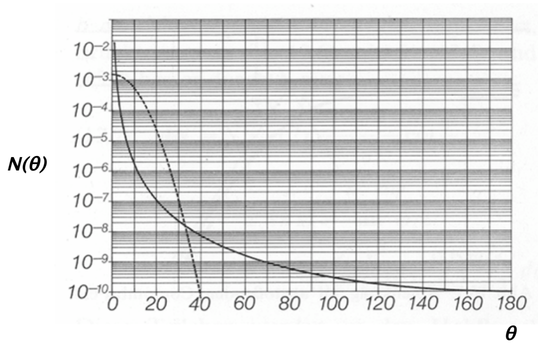


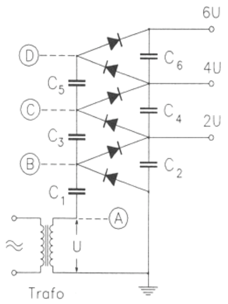
A Bit of History

$$N(\theta) = \frac{N_i n t Z^2 e^4}{(8\pi\varepsilon_0)^2 r^2 K^2} * \frac{1}{\sin^4(\theta/2)}$$



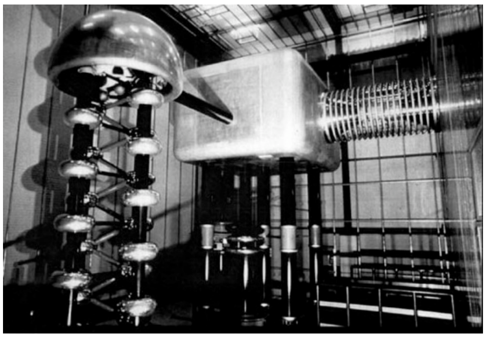
Rutherford Scattering, 1911
Using radioactive particle sources:
α-particles of some MeV energy

Electrostatic Machines:
The Cockcroft-Walton Generator



1928: Encouraged by Rutherford Cockcroft and Walton start the design & construction of a high voltage generator to accelerate a proton beam

1932: First particle beam (protons) produced for nuclear reactions: splitting of Li-nuclei with a proton beam of 400 keV



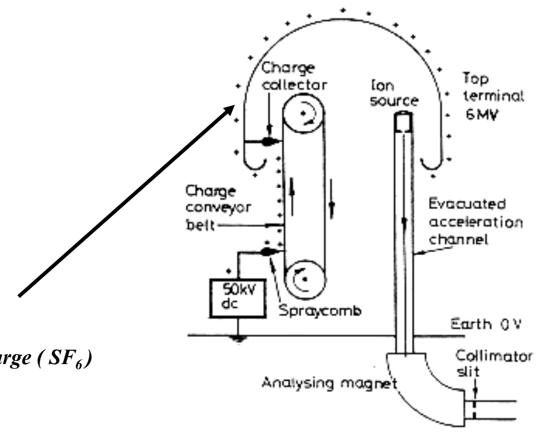
Particle source: Hydrogen discharge tube on 400 kV level
Accelerator: evacuated glas tube
Target: Li-Foil on earth potential

Technically: rectifier circuit, built of capacitors and diodes (Greinacher)

Problem:
DC Voltage can only be used once

Electrostatic Machines:
(Tandem –) van de Graaff Accelerator (1930 ...)

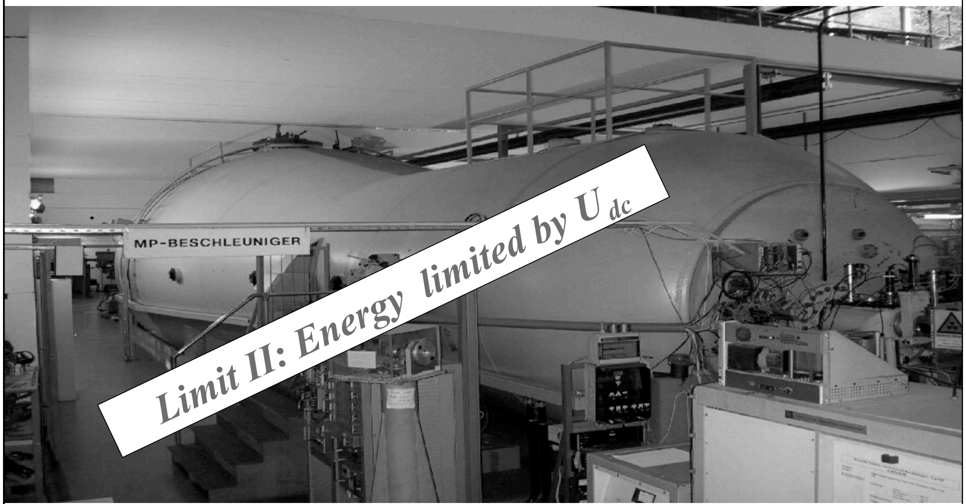
creating high voltages by mechanical transport of charges



**** Terminal Potential: U ≈ 12 ...28 MV***
using high pressure gas to suppress discharge (SF_6)

Problems: * Particle energy limited by high voltage discharges
**** high voltage can only be applied once per particle ...***
... or twice ?

The „Tandem principle": Apply the accelerating voltage twice ...
... by working with negative ions (e.g. H^-) and
stripping the electrons in the centre of the structure

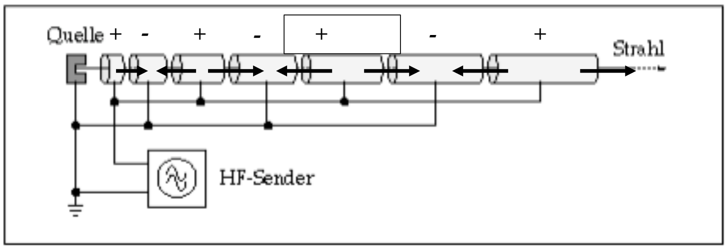


Example for such a „steam engine": 12 MV-Tandem van de Graaff Accelerator at MPI Heidelberg

The first RF-Accelerator: „Linac"

1928, Wideroe: how can the acceleration voltage be applied several times to the particle beam

schematic Layout:



Energy gained after n acceleration gaps

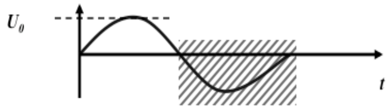
$$E_n = n * q * U_0 * \sin\psi_s$$

n *number of gaps between the drift tubes*
q *charge of the particle*
U_0 *Peak voltage of the RF System*
Ψ_S *synchronous phase of the particle*

** acceleration **of the proton in the** first gap*
** voltage has to be „flipped" **to get the right sign in the second gap →** RF voltage*
*→ shield the particle in drift tubes **during the negative half wave of the RF voltage***

Wideroe-Structure: the drift tubes

shielding of the particles during the negative half wave of the RF



Time span of the negative half wave: $\tau_{RF}/2$

Length of the Drift Tube: $l_i = v_i * \frac{\tau_{rf}}{2}$

Kinetic Energy of the Particles $E_i = \frac{1}{2} m v^2$

$$\rightarrow v_i = \sqrt{2E_i/m}$$

$$l_i = \frac{1}{\nu_{rf}} * \sqrt{\frac{i * q * U_{0*\sin\psi_s}}{2m}}$$

valid for non relativistic particles ...

Alvarez-Structure: 1946, surround the whole structure by a rf vessel

Energy: ≈ 20 MeV per Nucleon $\beta \approx 0.04 \dots 0.6$, Particles: Protons/Ions

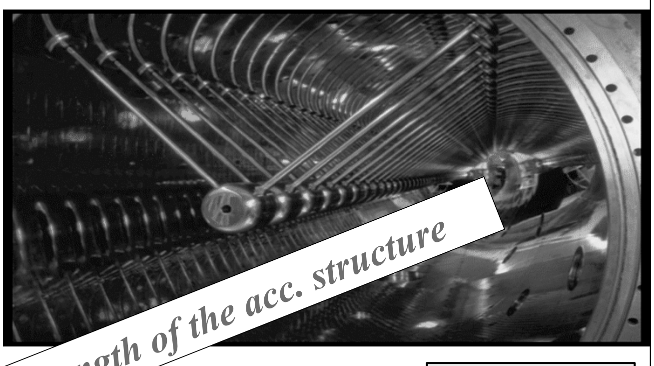
Accelerating structure of a Proton Linac (DESY Linac III)

E_{total} = ***988*** MeV

m_0c^2 = ***938*** MeV

p = ***310*** MeV / c

E_{kin} = ***50*** MeV



Limit III: length of the acc. structure

Beam energies

Energy Gain per „Gap“: $\boxed{W = q\, U_0 \sin \omega_{RF} t}$

reminder of some re

rest energy m_0c^2

total energy $E = \gamma * E_0 = \gamma * m_0 c^2$ ***momentum*** $E = \sqrt{p^2c^2 + m^2c^4}$

kinetic energy $E_{kin} = E_{total} - m_0c^2$

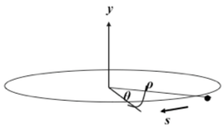
1.) Introduction and Basic Ideas

„ ... in the end and after all it should be a kind of circular machine“
→ need transverse deflecting force

Lorentz force $\vec{F} = q * (\cancel{\vec{E}} + \vec{v} \times \vec{B})$

typical velocity in high energy machines: $v \approx c \approx 3*10^8 \, m/s$

The ideal circular orbit



circular coordinate system

condition for circular orbit:

Lorentz force $F_L = e\, v\, B$

centrifugal force $F_{centr} = \dfrac{\gamma\, m_0\, v^2}{\rho}$

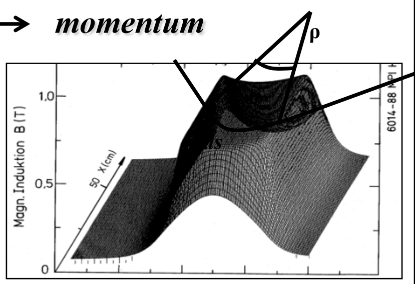
$$\frac{\gamma\, m_0\, v^{\cancel{2}}}{\rho} = e\, \cancel{v}\, B$$

$$\boxed{\frac{p}{e} = B\, \rho}$$

$B\,\rho =$ **"beam rigidity"**

Limit IV: The Magnetic Guide Field ⟷ momentum

Circular Orbit: dipole magnets to define the geometry



field map of a storage ring dipole magnet

$$\alpha = \frac{ds}{\rho} \approx \frac{dl}{\rho} = \frac{Bdl}{B\rho}$$

The angle run out in one revolution must be 2π so … for a full circle

$$\alpha = \frac{\int Bdl}{B\,\rho} = 2\pi \quad \rightarrow \quad \int Bdl = 2\pi\, \frac{p}{q}$$

… defines the integrated dipole field

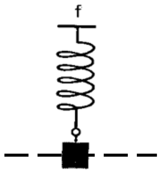
LHC: 7000 GeV Proton storage ring
dipole magnets N = 1232
l = 15 m
q = +1 e

$$\int B\, dl \approx N\, l\, B = 2\pi\; p/e$$

$$B \approx \frac{2\pi\;\; 7000\;\; 10^9\, eV}{1232\;\; 15\, m\;\; 3\; 10^8\, \frac{m}{s}\;\; e} = \underline{\underline{8.3\, Tesla}}$$

Focusing Properties – Short Excursion to Classical Mechanics

classical mechanics:
pendulum



there is a restoring force, proportional to the elongation x:

$$m*\frac{d^2x}{dt^2} = -c*x$$

general solution: free harmonic oszillation

$$x(t) = A*\cos(\omega t + \varphi)$$

Storage Ring: ***we need a Lorentz force that rises as a function of the distance to .... ?***

.... the design orbit

$$F(x) = q*v*B(x)$$

Quadrupole Magnets:

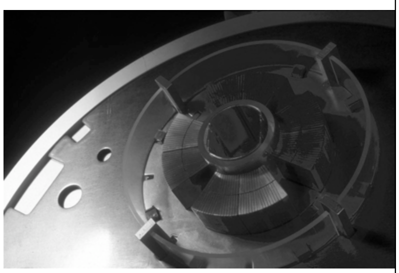
required: ***focusing forces to keep trajectories in vicinity of the ideal orbit***

linear increasing Lorentz force

linear increasing magnetic field $\quad B_y = g\,x \qquad B_x = g\,y$

normalised quadrupole field:

$$\longrightarrow \qquad k = \frac{g}{p/e}$$



LHC main quadrupole magnet

$$g \approx 25\,...\,220\ \ T/m$$

simple rule:

$$k = 0.3\,\frac{g(T/m)}{p(GeV/c)}$$

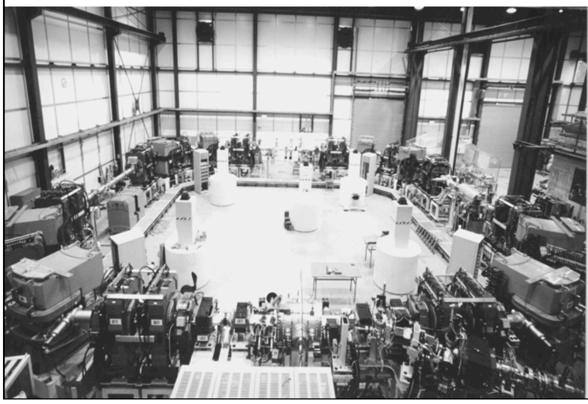
what about the vertical plane:
... Maxwell

$$\vec{\nabla}\times\vec{B} = \cancel{\vec{j}} + \cancel{\frac{\partial\vec{E}}{\partial t}} = 0 \qquad \Rightarrow \qquad \frac{\partial B_y}{\partial x} = \frac{\partial B_x}{\partial y} = g$$

The Equation of Motion:

$$\frac{B(x)}{p/e} = \frac{1}{\rho} + k\,x + \cancel{\frac{1}{2!} m\, x^2} + \cancel{\frac{1}{3!} n\, x^3} + \ldots$$

only terms linear in x, y taken into account dipole fields
quadrupole fields



Separate Function Machines:

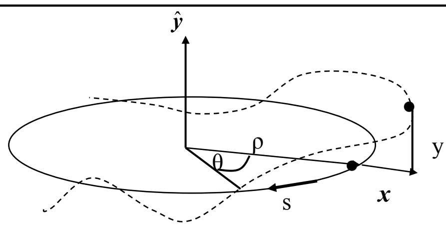
Split the magnets and optimise them according to their job:

bending, focusing etc

Example:
heavy ion storage ring TSR

* *man sieht nur dipole und quads → linear*

The Equation of Motion:



* ***Equation for the horizontal motion:***

$$x'' + x\left(\frac{1}{\rho^2} + k\right) = 0$$

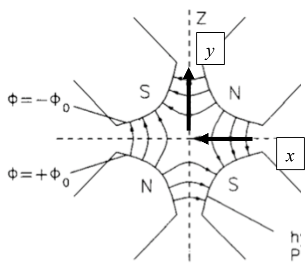
x = particle amplitude
x' = angle of particle trajectory (wrt ideal path line)

* ***Equation for the vertical motion:***

$\frac{1}{\rho^2} = 0$ ***no dipoles … in general …***

$k \leftrightarrow -k$ ***quadrupole field changes sign***

$$y'' - k\,y = 0$$



Solution of Trajectory Equations

Define … hor. plane: $K = 1/\rho^2 - k$

… vert. Plane: $K = k$

$$\left.\begin{array}{l} K = 1/\rho^2 - k \\ K = k \end{array}\right\} \quad \boldsymbol{x'' + K\, x} = 0$$

Differential Equation of harmonic oscillator … with spring constant K

Ansatz: $x(s) = a_1 \cdot \cos(\omega s) + a_2 \cdot \sin(\omega s)$

general solution: linear combination of two independent solutions

$$x'(s) = -a_1\omega \sin(\omega s) + a_2\omega \cos(\omega s)$$

$$x''(s) = -a_1\omega^2 \cos(\omega s) - a_2\omega^2 \sin(\omega s) = -\omega^2 x(s) \longrightarrow \omega = \sqrt{K}$$

general solution:

$$x(s) = a_1 \cos(\sqrt{K} s) + a_2 \sin(\sqrt{K} s)$$

determine a_1 , a_2 by boundary conditions:

$$s = 0 \longrightarrow \begin{cases} x(0) = x_0 \quad , \quad a_1 = x_0 \\ x'(0) = x'_0 \quad , \quad a_2 = \dfrac{x'_0}{\sqrt{K}} \end{cases}$$

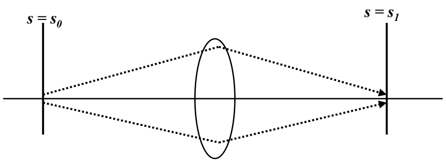
Hor. Focusing Quadrupole K > 0:

$$x(s) = x_0 \cdot \cos(\sqrt{|K|} s) + x'_0 \cdot \frac{1}{\sqrt{|K|}} \sin(\sqrt{|K|} s)$$

$$x'(s) = -x_0 \cdot \sqrt{|K|} \cdot \sin(\sqrt{|K|} s) + x'_0 \cdot \cos(\sqrt{|K|} s)$$

For convenience expressed in matrix formalism:

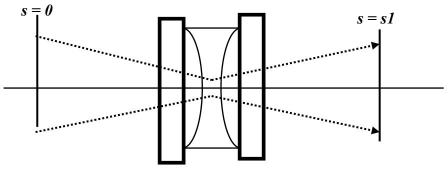
$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M_{foc} * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$



$$M_{foc} = \begin{pmatrix} \cos(\sqrt{|K|} s) & \dfrac{1}{\sqrt{|K|}} \sin(\sqrt{|K|} s \\ -\sqrt{|K|} \sin(\sqrt{|K|} s) & \cos(\sqrt{|K|} s) \end{pmatrix}_0$$

hor. defocusing quadrupole:

$$x'' - K\,x = 0$$



Remember from school:

$$f(s) = \cosh(s) \quad , \quad f'(s) = \sinh(s)$$

Ansatz: $x(s) = a_1 \cdot \cosh(\omega s) + a_2 \cdot \sinh(\omega s)$

$$M_{defoc} = \begin{pmatrix} \cosh\sqrt{|K|}\,l & \frac{1}{\sqrt{|K|}}\sinh\sqrt{|K|}\,l \\ \sqrt{|K|}\sinh\sqrt{|K|}\,l & \cosh\sqrt{|K|}\,l \end{pmatrix}$$

drift space:

K = 0

$$M_{drift} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$

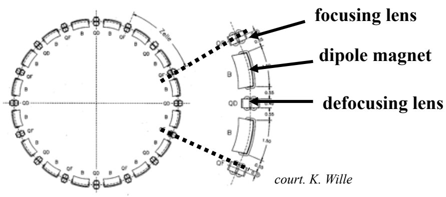
! with the assumptions made, the motion in the horizontal and vertical planes are independent „ ... the particle motion in x & y is uncoupled“

Transformation through a system of lattice elements

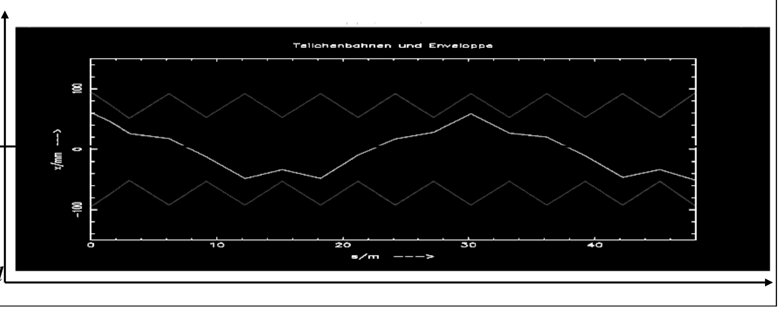
combine the single element solutions by multiplication of the matrices

$$M_{total} = M_{QF} * M_D * M_{QD} * M_{Bend} * M_{D^*} .....$$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s2} = M(s_2, s_1) * \begin{pmatrix} x \\ x' \end{pmatrix}_{s1}$$

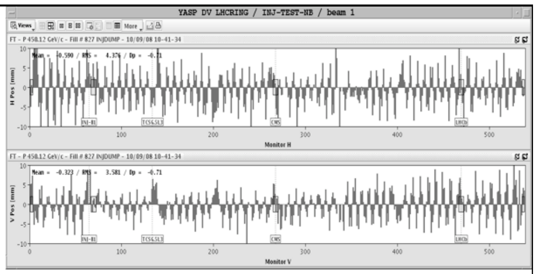


in each accelerator element the particle trajectory corresponds to the movement of a harmonic oscillator „



typical values in a strong foc. machine: $x \approx$ mm, $x' \leq$ mrad

5.) Orbit & Tune:



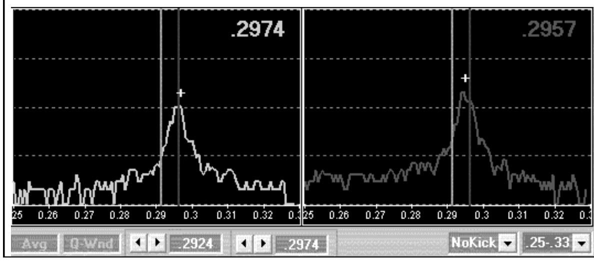
Tune: number of oscillations per turn

64.31
59.32

Relevant for beam stability:
non integer part

LHC revolution frequency: 11.3 kHz $0.31*11.3 = 3.5\,kHz$

We treat the transverse movement of the particles along the accelerator as harmonic oscillations with a well defined amplitude and (Eigen-) frequency.



To avoid resonance problems -> keep the tune away from resonance conditions

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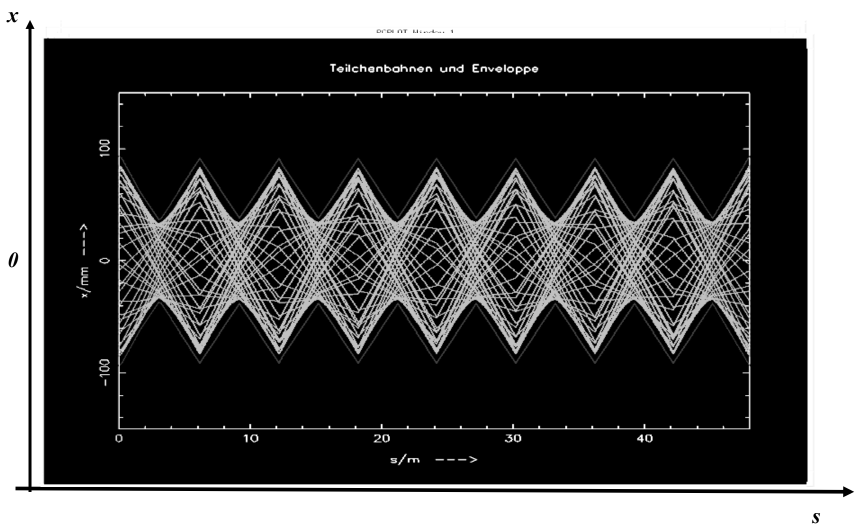
Question: what will happen, if the particle performs a second turn ?

... or a third one or ... 10^{10} turns



Astronomer Hill:

differential equation for motions with periodic focusing properties „Hill's equation"

Example: particle motion with periodic coefficient

equation of motion: $x''(s) - k(s)x(s) = 0$

restoring force $\neq$ const,
k(s) = depending on the position s
k(s+L) = k(s), periodic function

we expect a kind of quasi harmonic oscillation: amplitude & phase will depend on the position s in the ring.

6.) The Beta Function

General solution of Hill´s equation:

(i) $$x(s) = \sqrt{\varepsilon}\,\sqrt{\beta(s)} \cdot \cos(\psi(s) + \phi)$$

ε, Φ = integration constants determined by initial conditions

β(s) periodic function given by focusing properties of the lattice ↔ quadrupoles

$$\beta(s+L) = \beta(s)$$

Inserting (i) into the equation of motion …

$$\psi(s) = \int_0^s \frac{ds}{\beta(s)}$$

Ψ(s) = „phase advance" of the oscillation between point „0" and „s" in the lattice. For one complete revolution: number of oscillations per turn „Tune"

$$\boldsymbol{Q_y = \frac{1}{2\pi}\oint \frac{ds}{\beta(s)}}$$

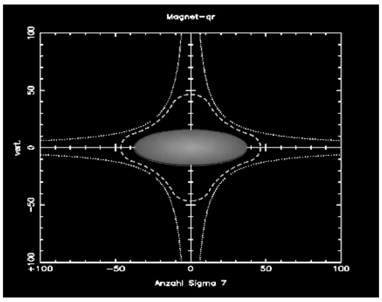
The Beta Function

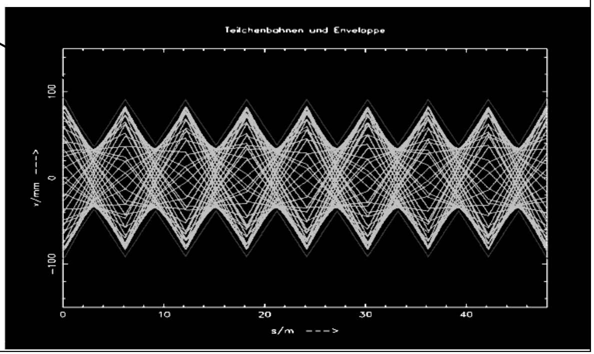
Amplitude of a particle trajectory:

$$x(s) = \sqrt{\varepsilon} * \sqrt{\beta(s)} * \cos(\psi(s) + \varphi)$$

Maximum size of a particle amplitude

$$\boxed{\hat{x}(s) = \sqrt{\varepsilon}\sqrt{\beta(s)}}$$





β determines the beam size (... the envelope of all particle trajectories at a given position "s" in the storage ring.

It reflects the periodicity of the magnet structure.

7.) Beam Emittance and Phase Space Ellipse

general solution of Hill equation

$$\begin{cases} (1) \quad x(s) = \sqrt{\varepsilon}\,\sqrt{\beta(s)}\,\cos(\psi(s)+\phi) \\ (2) \quad x'(s) = -\dfrac{\sqrt{\varepsilon}}{\sqrt{\beta(s)}}\,\{\alpha(s)\cos(\psi(s)+\phi) + \sin(\psi(s)+\phi)\} \end{cases}$$

from (1) we get

$$\cos(\psi(s)+\phi) = \frac{x(s)}{\sqrt{\varepsilon}\,\sqrt{\beta(s)}}$$

$$\alpha(s) = \frac{-1}{2}\beta'(s)$$

$$\gamma(s) = \frac{1+\alpha(s)^2}{\beta(s)}$$

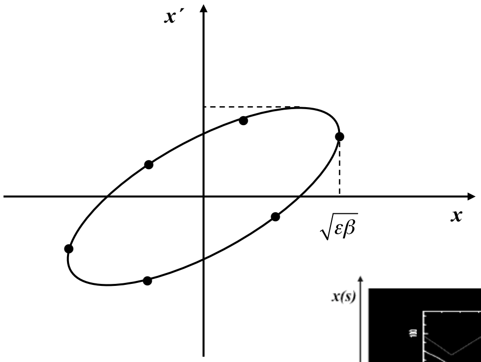
Insert into (2) and solve for ε

$$\varepsilon = \gamma(s)\,x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s)\,x'^2(s)$$

**** ε is a constant of the motion … it is independent of „s"***
**** parametric representation of an ellipse in the x x' space***
**** shape and orientation of ellipse are given by α, β, γ***

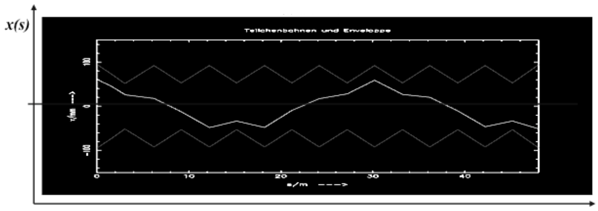
Beam Emittance and Phase Space Ellipse

$$\varepsilon = \gamma(s)\, x^2(s) + 2\alpha(s) x(s) x'(s) + \beta(s)\, x'^2(s)$$



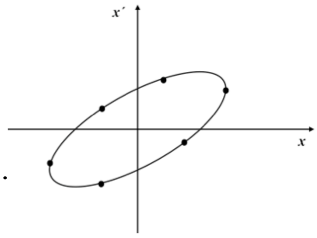
Liouville: in reasonable storage rings area in phase space is constant.

A = π*ε=const



ε beam emittance = woozilycity of the particle ensemble, intrinsic beam parameter, cannot be changed by the foc. properties.
Scientifiquely speaking: area covered in transverse x, x´ phase space … and it is constant !!!

Phase Space Ellipse



particel trajectory: $x(s) = \sqrt{\varepsilon}\sqrt{\beta(s)}\cos\{\psi(s) + \phi\}$

max. Amplitude: $\hat{x}(s) = \sqrt{\varepsilon\beta}$ $\longrightarrow$ ***x´ at that position …***

… put $\hat{x}(s)$ ***into*** $\varepsilon = \gamma(s)\, x^2(s) + 2\alpha(s) x(s) x'(s) + \beta(s)\, x'^2(s)$ ***and solve for x´***

$$\varepsilon = \gamma \cdot \varepsilon\beta + 2\alpha\sqrt{\varepsilon\beta} \cdot x' + \beta x'^2$$

$$\longrightarrow \quad x' = -\alpha \cdot \sqrt{\varepsilon / \beta}$$

* ***A high β-function means a large beam size and a small beam divergence. … et vice versa !!!*** **!**

* ***In the middle of a quadrupole β = maximum, α = zero*** $\Big\}$ $x' = 0$

… and the ellipse is flat

Phase Space Ellipse

$$\varepsilon = \gamma(s)\, x^2(s) + 2\alpha(s) x(s) x'(s) + \beta(s)\, x'^2(s)$$

$$\alpha(s) = \frac{-1}{2}\beta'(s)$$

$$\gamma(s) = \frac{1+\alpha(s)^2}{\beta(s)}$$

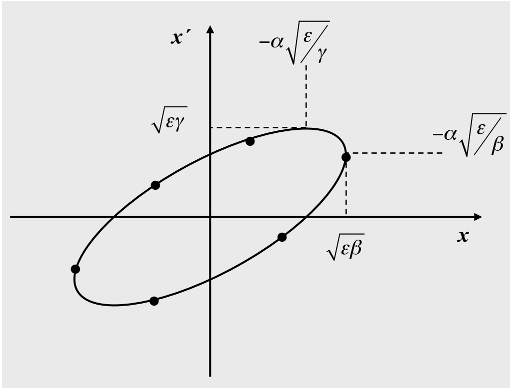
$$\longrightarrow \quad \varepsilon = \frac{x^2}{\beta} + \frac{\alpha^2 x^2}{\beta} + 2\alpha \cdot x x' + \beta \cdot x'^2$$

… solve for x′ $\quad x'_{1,2} = \dfrac{-\alpha \cdot x \pm \sqrt{\varepsilon\beta - x^2}}{\beta}$

… and determine $\hat{x}'$ ***via:*** $\quad \dfrac{dx'}{dx} = 0$

$$\longrightarrow \quad \hat{x}' = \sqrt{\varepsilon\gamma}$$

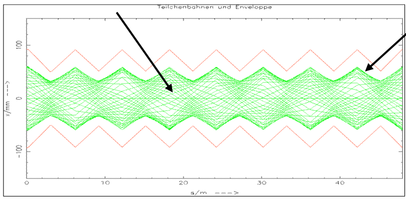
$$\longrightarrow \quad \hat{x} = \pm\alpha\sqrt{\varepsilon/\gamma}$$



shape and orientation of the phase space ellipse depend on the Twiss parameters β α γ

Emittance of the Particle Ensemble:

$$x(s) = \sqrt{\varepsilon}\sqrt{\beta(s)} \cdot \cos(\Psi(s) + \phi) \qquad \boxed{\hat{x}(s) = \sqrt{\varepsilon}\sqrt{\beta(s)}}$$



single particle trajectories, N ≈ 10 [11] per bunch

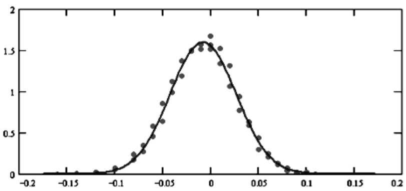
Gauß Particle Distribution: $\quad \rho(x) = \dfrac{N \cdot e}{\sqrt{2\pi}\sigma_x} \cdot e^{-\frac{1}{2}\frac{x^2}{\sigma_x^2}}$

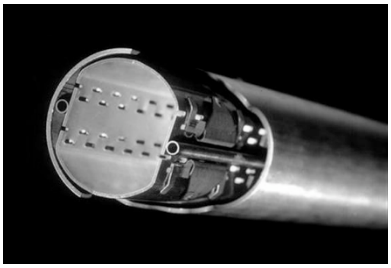
particle at distance 1 σ from centre ↔ 68.3 % of all beam particles

LHC: $\beta = 180\, m$

$\varepsilon = 5 * 10^{-10}\, m\, rad$

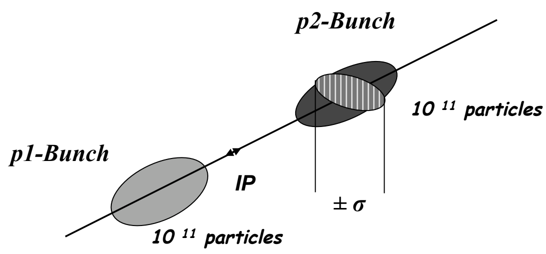
$$\sigma = \sqrt{\varepsilon * \beta} = \sqrt{5*10^{-10}\, m * 180\, m} = 0.3\, mm$$





aperture requirements: $r_0 = 12 * \sigma$

21.) Luminosity



Example: Luminosity run at LHC

$\beta_{x,y} = 0.55\ m$ $\qquad f_0 = 11.245\ kHz$

$\varepsilon_{x,y} = 5*10^{-10}\ rad\ m$ $\qquad n_b = 2808$

$\sigma_{x,y} = 17\ \mu m$

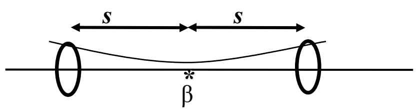
$I_p = 584\ mA$

$$L = \frac{1}{4\pi e^2 f_0 n_b} * \frac{I_{p1} I_{p2}}{\sigma_x \sigma_y}$$

$$L = 1.0*10^{34}\ \frac{1}{cm^2 s}$$

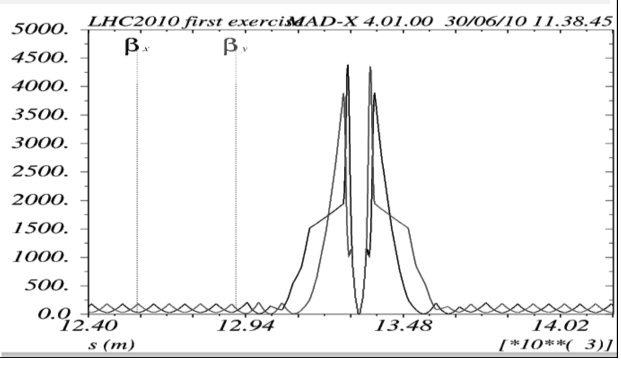
β-Function in a Drift:

let's assume we are at a symmetry point in the center of a drift.



$$\beta(s) = \beta_0 + \frac{s^2}{\beta_0} \quad !!!$$

At the end of a long symmetric drift space the beta function reaches its maximum value in the complete lattice.
-> here we get the largest beam dimension.
-> keep l as small as possible



8 individually powered quadrupole magnets are needed to match the insertion (... at least)

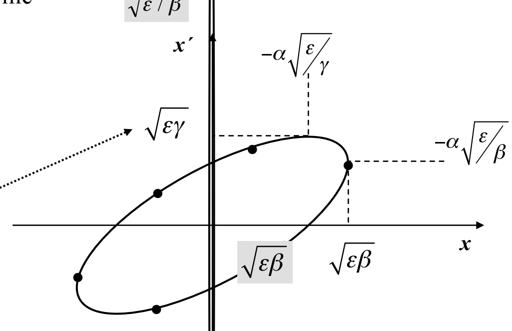
Mini-β Insertions: Betafunctions

A mini-β insertion is always a kind of special symmetric drift space.
→greetings from Liouville

$$\alpha^* = 0$$

$$\gamma^* = \frac{1+\alpha^2}{\beta} = \frac{1}{\beta^*}$$

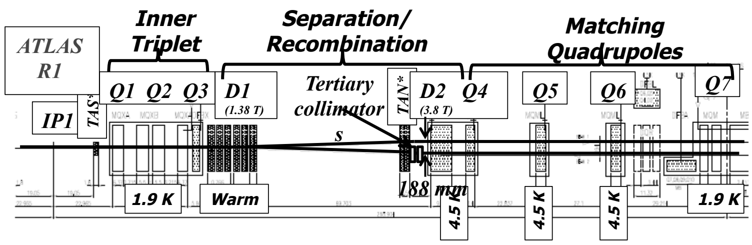
$$\sigma'^* = \sqrt{\frac{\varepsilon}{\beta^*}}$$

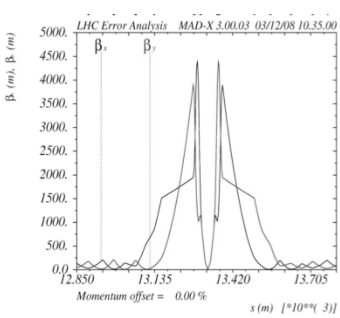


$$\beta^* = \frac{\sigma^*}{\sigma'^*}$$

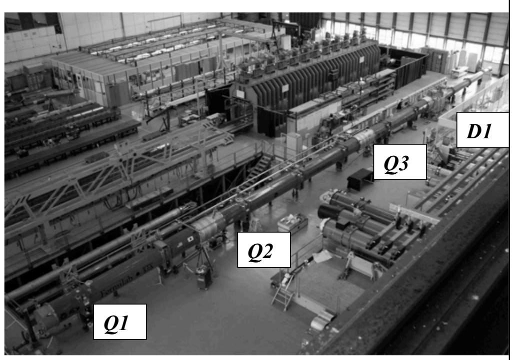
at a symmetry point β is just the ratio of beam dimension and beam divergence.

The LHC Insertions

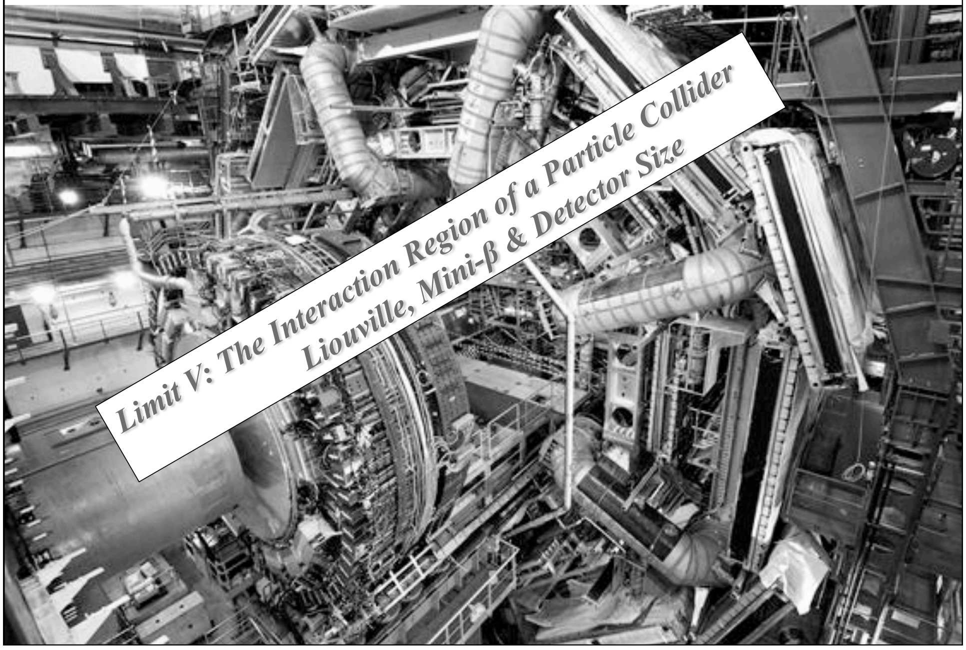




mini β optics



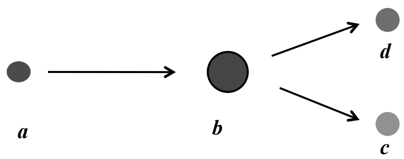
ATLAS detector in LHC for 7x7 TeV interactions



Limit VI: Fixed Target Machines
The (Problem of the) Centre of Mass Energy

Fixed Target experiments

accelerated particle beam hits a target at rest $a + b \rightarrow c + d$



Lab system: $p_b^{lab} = 0,\ E_b^{lab} = m_b c^2$

Centre of mass system: $p_b^{cm} + p_b^{cm} = 0$

relativistic total energy $E^2 = p^2c^2 + (mc^2)^2$

and for a single particle as well as for system of particles the overall rest energy is constant

... invariance of the 4momentum scalar product

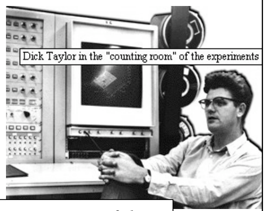
$$\sum_i E_i^2 - (\sum_i p_i^2)c^2 = M^2c^4 = const$$

$$\left(E_a^{cm} + E_b^{cm}\right)^2 - \left(p_a^{cm} + p_b^{cm}\right)^2 c^2 = \left(E_a^{lab} + E_b^{lab}\right)^2 - \left(p_a^{lab} + p_b^{lab}\right)^2 c^2$$

The (Problem of the) Centre of Mass Energy

Fixed Target experiments:

$$\left(E_a^{cm}+E_b^{cm}\right)^2-\underbrace{\left(p_a^{cm}+p_b^{cm}\right)}_{=0}{}^2c^2=\left(E_a^{lab}+E_b^{lab}\right)^2-\underbrace{\left(p_a^{lab}+p_b^{lab}\right)}_{=p_a^{lab}}{}^2c^2$$

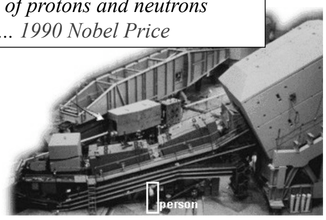


$$W^2=\left(E_a^{cm}+E_b^{cm}\right)^2=\left(E_a^{lab}+m_bc^2\right)^2-\left(p_a^{lab}c\right)^2$$
$$=2E_a^{lab}m_bc^2+\left(m_a{}^2+m_b{}^2\right)c^4$$

for $E_a^{lab} >> m_ac^2,\ m_bc^2$

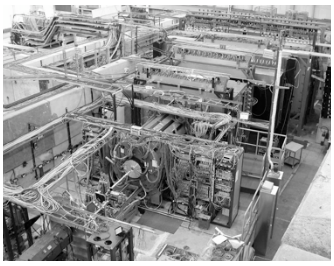
$$=> \quad W\approx\sqrt{2E_a^{lab}m_bc^2}$$

Taylor/Kendall/Friedman: Discovery of the quark structure of protons and neutrons
1966-1978 ..... 1990 Nobel Price



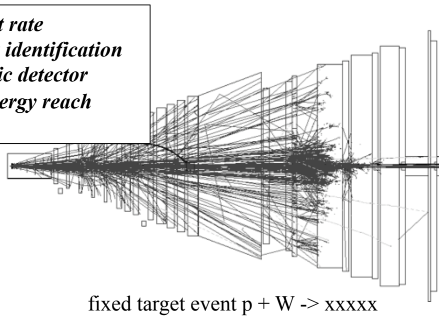
For high energies in the centre of mass system, fixed target machines are not effective.
... → need for colliding beams

Fixed target experiments:



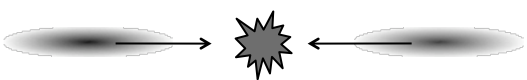
HARP Detector, CERN

high event rate
easy track identification
asymmetric detector
limited energy reach

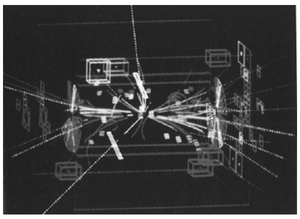


fixed target event p + W -> xxxxx

Collider experiments:



low event rate (luminosity)
challenging track identification
symmetric detector
$\boldsymbol{E_{lab} = E_{cm}}$



Z0 boson discovery at the UA2 experiment (CERN).
The Z0 boson decays
into a e+e− pair, shown as white dashed lines.

Limit VI: Fixed Target Machines
→ go for particle colliders

The (Problem of the) Centre of Mass Energy

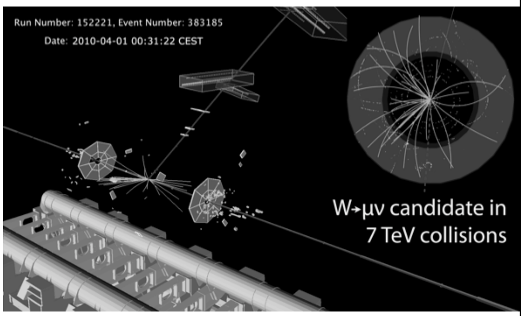
Colliding Beams experiments:

$$\left(E_a^{cm} + E_b^{cm}\right)^2 - \underbrace{\left(p_a^{cm} + p_b^{cm}\right)^2}_{=0} c^2 = \left(E_a^{lab} + E_b^{lab}\right)^2 - \underbrace{\left(p_a^{lab} + p_b^{lab}\right)^2}_{p_a^{lab} = -p_b^{lab}} c^2$$

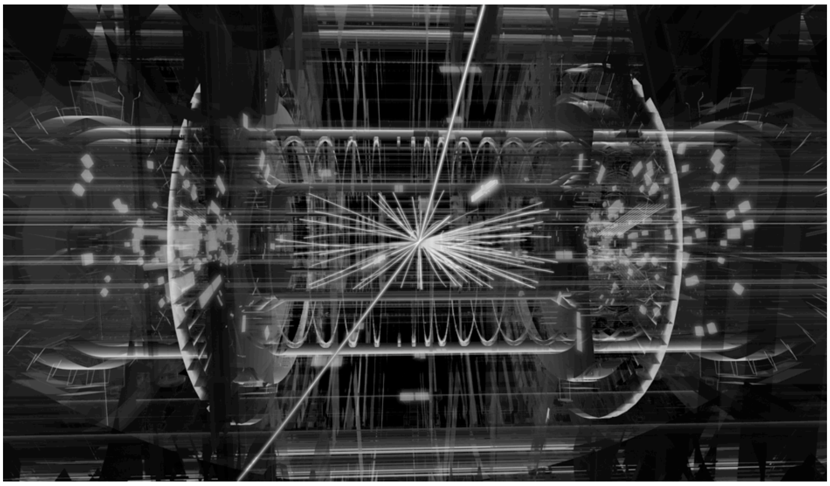
$$W^2 = \left(E_a^{cm} + E_b^{cm}\right)^2$$

$$\Rightarrow \quad W = 2E_a^{lab}$$

The full lab energy is available in the center of mass system.
Prize to pay: we have to build colliders ... beam sizes = µm



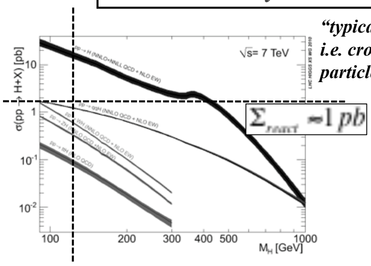
Limit VII: Nature ... or the cross sections of HEP



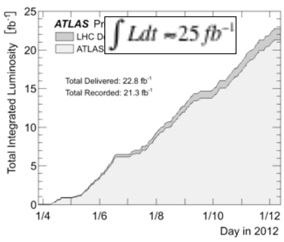
ATLAS event display: Higgs => two electrons & two muons

The High light of the year

production rate of events is determined by the cross section Σ_{react} and a parameter L that is given by the design of the accelerator:
… the luminosity



"typical particle size"
i.e. cross section for particle production



accumulated collision rate in LHC run 1

$$1b = 10^{-24}\,cm^2 \;=\; 1/mio * 1/mio * 1/mio * \frac{1}{100}\,mm^2$$

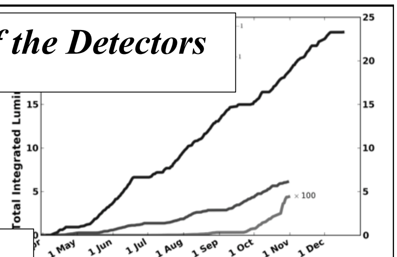
The particles are "very small"

$$R = L * \Sigma_{react} \approx 10^{-12} b \cdot 25 \frac{1}{10^{-15} b} = some 1000\, H$$

The luminosity is a storage ring quality parameter and depends on beam size (β !) and stored current

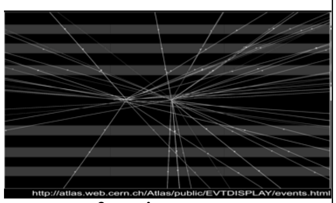
$$L = \frac{1}{4\pi e^2 f_0\, b} * \frac{I_1 * I_2}{\sigma^*_x * \sigma^*_y}$$

Limit VIII: Data Taking Efficiency of the Detectors "event pile up"

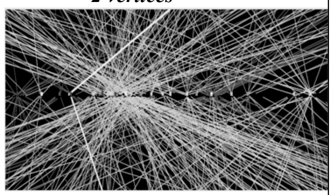


The LHC Performance in Run 1

	Design	2012
Momentum	*7 TeV /c*	*4 TeV/c*
Luminosity (cm^{-2}s^{-1})	*10^{34}*	*$7.7*10^{33}$*
Protons per bunch 10^{11}	*1.15*	*1.50*
Number of bunches/beam	*2808*	*1380*
Nominal bunch spacing	*25 ns*	*50ns*
rms beam size (arc)	*300 µm*	*350 µm*
rms beam size IP	*17 µm*	*20 µm*



2 vertices

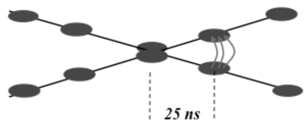


20 vertices

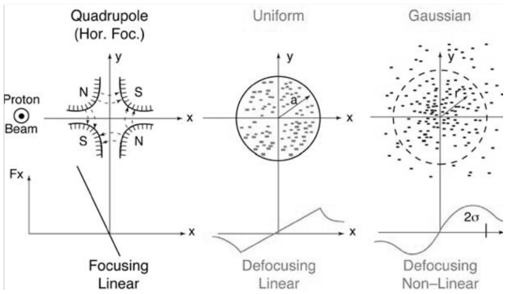
Storage ring colliders are very efficient machines:
Bunch collision Frequency: 40 Mhz = 1/25ns

Limit IX: Luminosity Limit due to Beam-Beam Effect

Beam-Beam-Effect



the colliding bunches influence each other
=> change the focusing properties of the ring !!
for LHC a strong non-linear defoc. effect



court. K. Schindl

most simple case:
linear beam beam tune shift

$$\Delta Q_x = \frac{\beta_x^* * r_p * N_p}{2\pi\,\gamma_p(\sigma_x + \sigma_y) * \sigma_x}$$

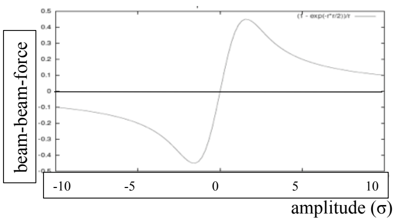
⇒puts a limit to N_p

Eigenfrequency of the paticles is changed due to the beam beam interaction
Particles are pushed onto resonances and are lost.

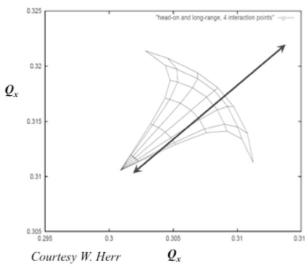
Luminosity Limits

Beam-Beam-Effect

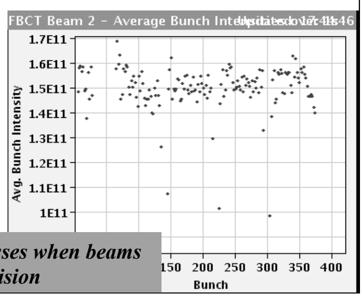
the space charge of the colliding bunche lead to a strong non-linear defoc. effect and possibly to particle loss.



$$L = \frac{1}{4\pi}\left(f_{rev} N_{p1} n_b\right)\left(\frac{\gamma N_{p2}}{\varepsilon_n \beta^*}\right) \cdot F \cdot W$$



effect of beam-beam force in LHC run1



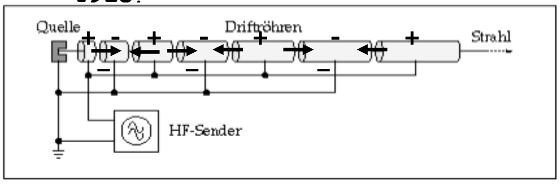
observed particle losses when beams are brought into collision

Limit X: RF Acceleration & Momentum Spread

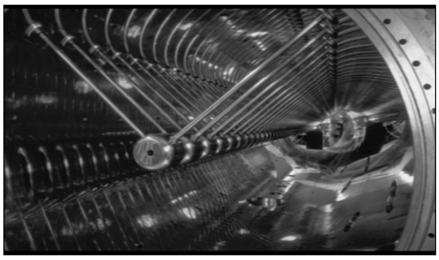
Energy Gain per „Gap“:

$$\boxed{W = q\, U_0 \sin \omega_{RF} t}$$

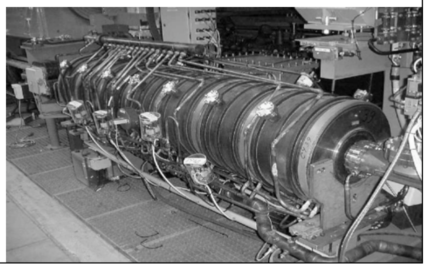
1928.



drift tube structure at a proton linac (GSI Unilac)



500 MHz cavities in an electron storage ring

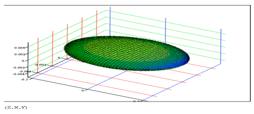


** RF Acceleration: multiple application of the same acceleration voltage; brillant idea to gain higher energies*

Problem: panta rhei !!!

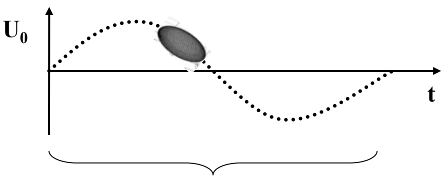
(Heraklit: 540-480 v. Chr.)

How do we accelerate ???



Bunch length of Electrons ≈ 1cm

Example: HERA RF:



$\lambda = 60\ cm$

$$\left.\begin{array}{l} \nu = 500\,MHz \\ c = \lambda\,\nu \end{array}\right\} \lambda = 60\ cm$$

$$\sin(90^o) = 1$$
$$\sin(84^o) = 0.994$$

$$\frac{\Delta U}{U} = 6.0\ 10^{-3}$$

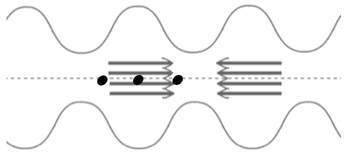
typical momentum spread of an electron bunch: $\frac{\Delta p}{p} \approx 1.0\ 10^{-3}$

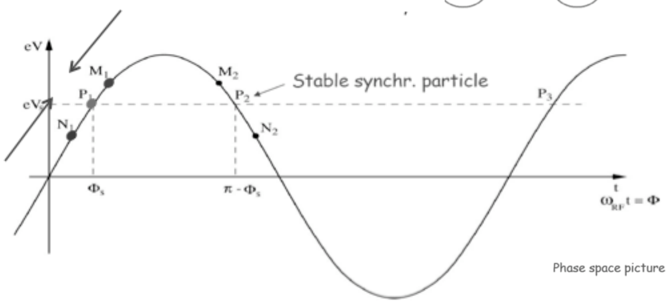
The Acceleration for Δp/p≠0
"Phase Focusing" *below transition*

ideal particle •

particle with Δp/p > 0 • ***faster***

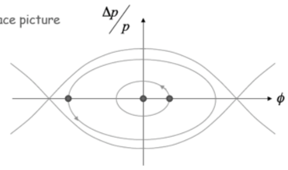
particle with Δp/p < 0 • ***slower***





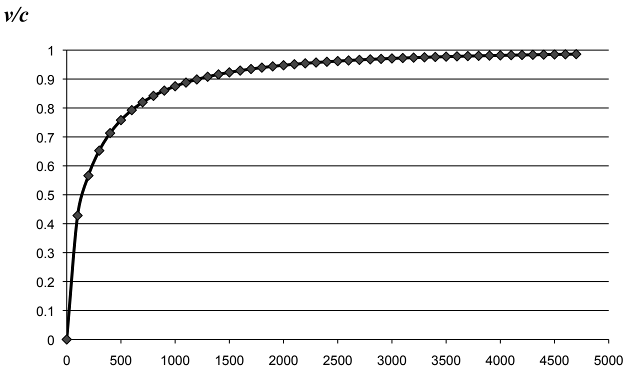
Focussing effect in the longitudinal direction
keeping the particles close together
... forming a "bunch"

Phase space picture

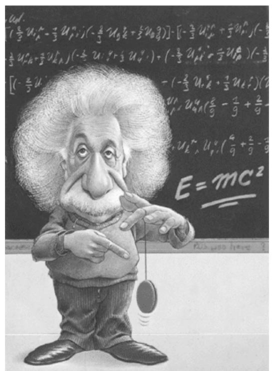


... so sorry, here we need help from Albert:

$$\gamma = \frac{E_{total}}{mc^2} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \longrightarrow \frac{v}{c} = \sqrt{1 - \frac{mc^2}{E^2}}$$



kinetic energy of a proton



... some when the particles
do not get faster anymore

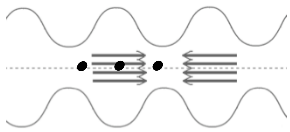
.... but heavier !

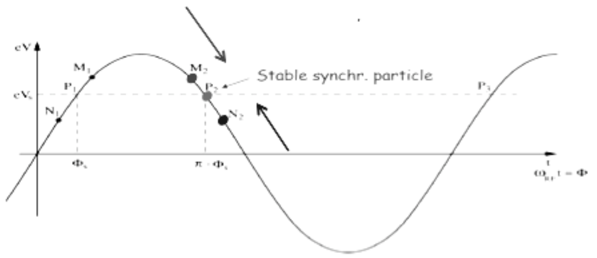
The Acceleration for Δp/p≠0
"Phase Focusing" *above transition*

ideal particle

particle with Δp/p > 0 ***heavier***

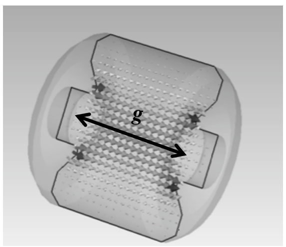
particle with Δp/p < 0 ***lighter***





oscillation frequency: $f_s = f_{rev}\sqrt{-\frac{h\alpha_s}{2\pi}\cdot\frac{qU_0\cos\phi_s}{E_s}}$ ***≈ some Hz***

Energy Gain in RF structures:
Transit Time Factor to optimise the cavities



Oscillating field at frequency **ω** *(amplitude is assumed to be constant all along the gap)*

$$E_z = E_0\cos\omega t = \frac{V}{g}\cos\omega t$$

Consider a particle passing through the middle of the gap at time t=0 : $z = vt$

The total energy gain is: $\Delta W = \frac{eV}{g}\int_{-g/2}^{g/2}\cos\omega\frac{z}{v}dz$

$$\Delta W = eV\frac{\sin\theta/2}{\theta/2} = eVT$$

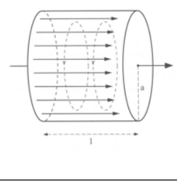
$T = \frac{\sin\theta/2}{\theta/2}$ *transit time factor (0 < T < 1)*

$\theta = \frac{\omega g}{v}$ *transit angle*

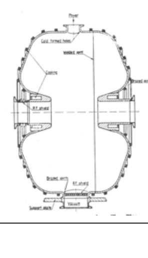
ideal case: $T = \frac{\sin\theta/2}{\theta/2} \rightarrow 1 \quad \Leftrightarrow \quad \theta/2 \rightarrow 0$

el. static accelertors $\omega \rightarrow 0$

minimise acc. gap $g \rightarrow 0$

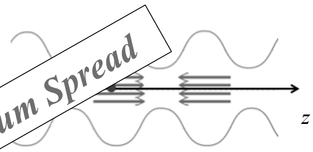


!



RF Cavities, Acceleration and Energy Gain

$$dW = dE = eE_z ds \quad \Rightarrow \quad W = e\int E_z ds = eV$$



Limit X: RF Acceleration & Momentum Spread

Energy Gain per turn / per passage … an acc. structure is limited by electric discharge… at the electrical field and so the achievable acceleration … MV/m”

typical values: …es LEP $\Delta E/\Delta s = 5\ MV/m$
…e of the art (ILC) $\Delta E/\Delta s = 30\ MV/m$

... which defines the number of resonators installed in the ring.

Synchrotron Radiation

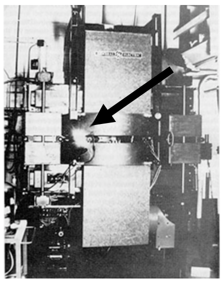
In a circular accelerator charged particles lose energy via emission of intense light.

$$P_s = \frac{2}{3}\alpha\hbar c^2 \frac{\gamma^4}{\rho^2}$$ *radiation power*

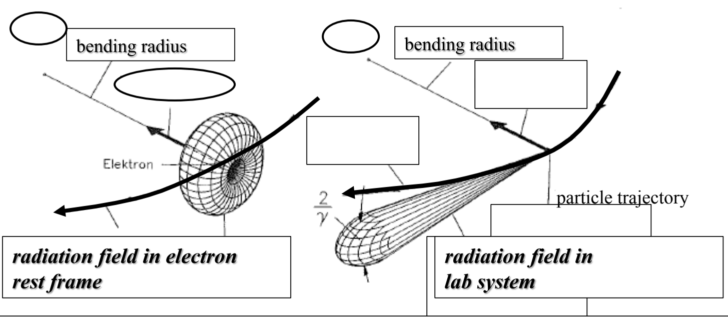
$$\Delta E = \frac{4}{3}\pi\alpha\hbar c \frac{\gamma^4}{\rho}$$ *energy loss*

$$\omega_c = \frac{3}{2}\frac{c\gamma^3}{\rho}$$ *critical frequency*

$$\alpha \approx \frac{1}{137}$$
$$\hbar c \approx 197\ MeV\ fm$$



1946 observed for the first time in the General Electric Synchrotron

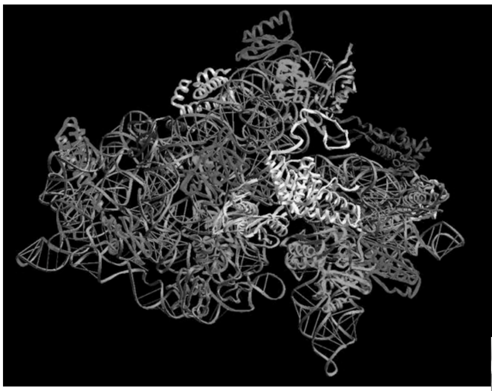


radiation field in electron rest frame

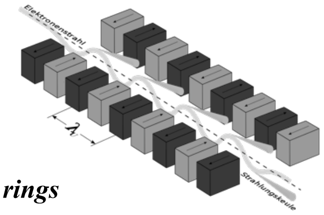
radiation field in lab system

court. K. Wille

Synchrotron Radiation as useful tool

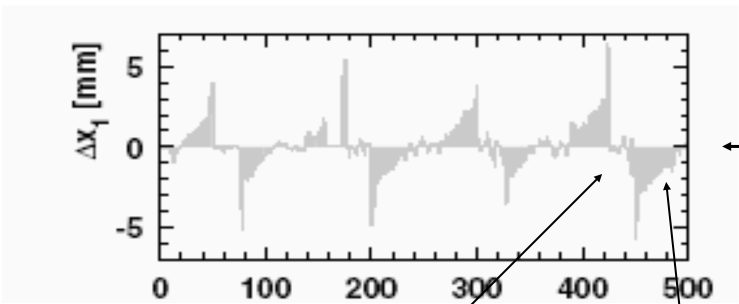


structure analysis with highest resolution Ribosome molecule



Undulator to enhance the synchrotron radiation in e+/e- storage rings

Synchrotron Radiation as aggravating effect in High Energy Rings
„Sawtooth Effect“ at LEP (CERN)

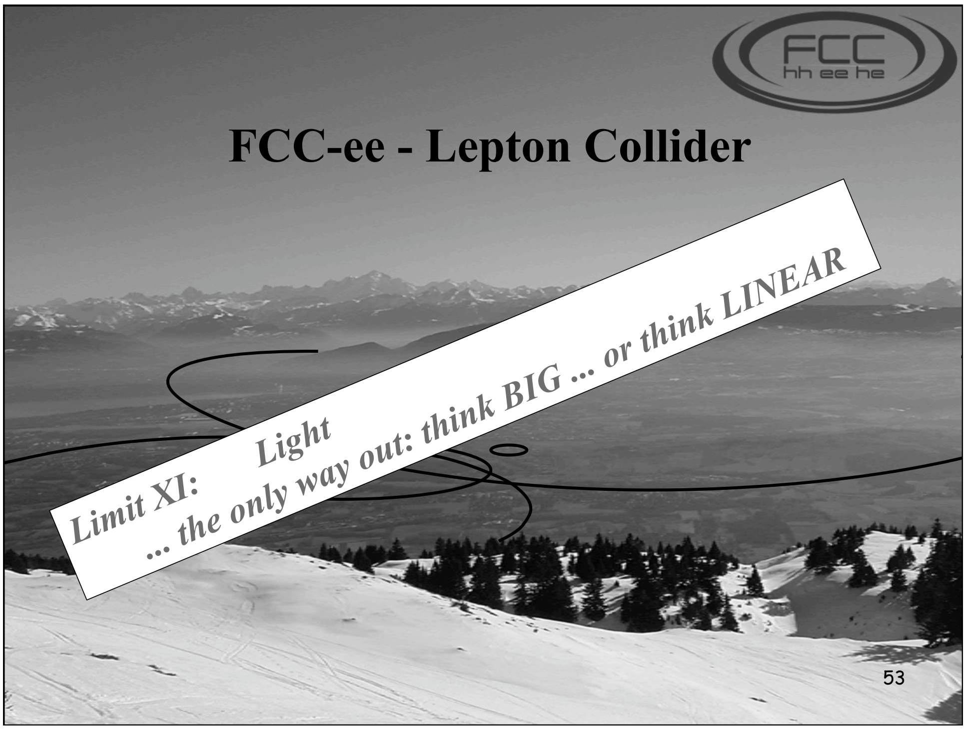


Electron course

BPM Number

In the straight sections they are accelerated by the rf cavities so much that they „overshoot“ and reach nearly the outer side of the vacuum chamber.

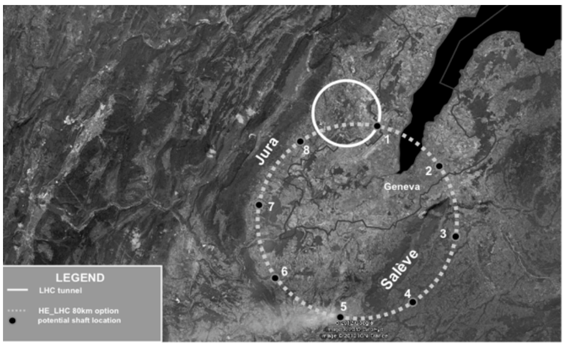
In the arc the electron beam loses so much energy in ea octant that the particle are running more and more on a dispersion trajectory.



Planning the next generation e+ / e- Ring Colliders

Design Parameters FCC-ee

E = 175 GeV / beam
L = 100 km



$$\Delta U_0(keV) \approx \frac{89 * E^4(GeV)}{\rho}$$

$$\Delta U_0 \approx 8.62\ GeV$$

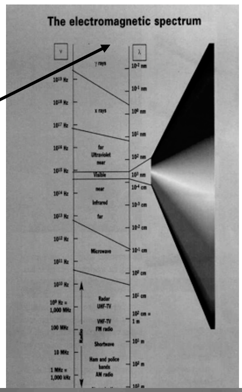
$$\Delta P_{sy} \approx \frac{\Delta U_0}{T_0} * N_p = \frac{10.4 * 10^6 eV * 1.6 * 10^{-19} Cb}{263 * 10^{-6} s} * 9 * 10^{12}$$

$$\Delta P_{sy} \approx 47\ MW$$

Circular e+ / e- colliders are severely limited by synchrotron radiation losses and have to be replaced for higher energies by linear accelerators

Example: FCC

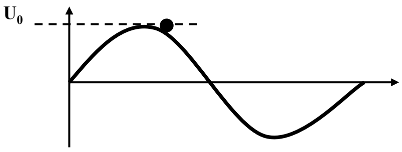
Typical Energy of the Photons $E_{crit} = 1.2\ MeV$



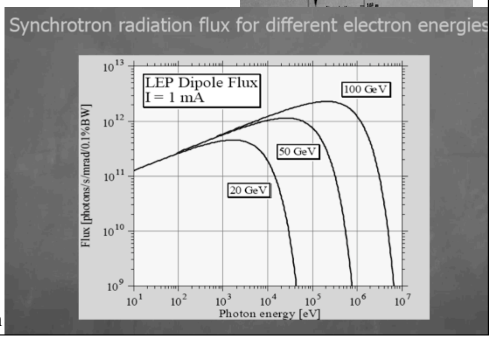
reminder: visible light ≈ some eV

Energy Loss per Turn $\Delta E_{turn} = 8.2 GeV$

Cavity Voltage to compensate losses



$U_{\Sigma cav} = 11 GV$

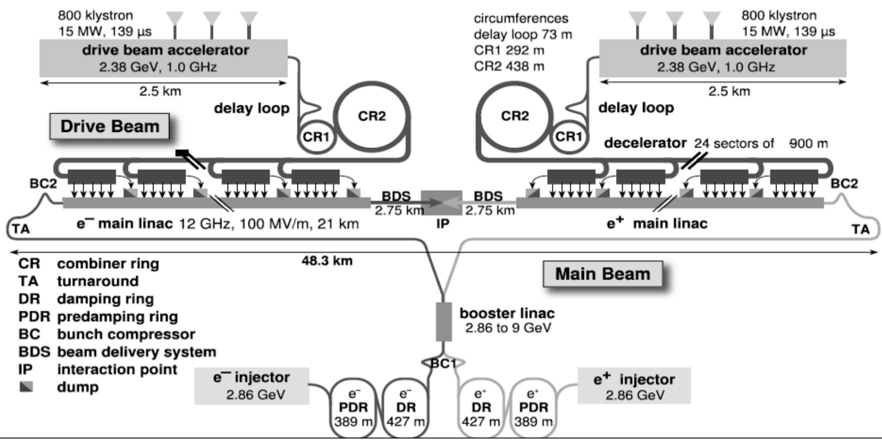


court. L. Rivkin

Limit XII: Once more: the Accelating Gradient

CLIC ... a future Linear e+/ e- Accelerator

Avoid bending magnets => no synchrotron radiation losses
=> energy gain has to be obtained in ONE GO

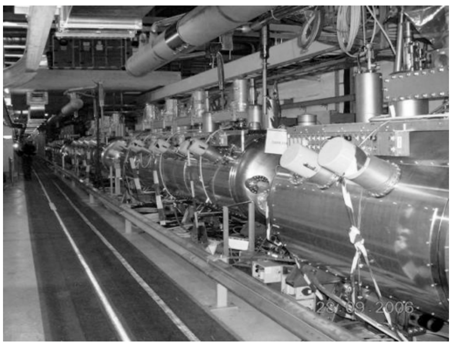


Description [units]	500 GeV	3 TeV
Total (peak 1%) luminosity	$2.3\ (1.4)\times 10^{34}$	$5.9\ (2.0)\times 10^{34}$
Total site length [km]	13.0	48.4
Loaded accel. gradient [MV/m]	80	100
Main Linac RF frequency [GHz]	12	12
Beam power/beam [MW]	4.9	14
Bunch charge [10^9 e^+/e^-]	6.8	3.72
Bunch separation [ns]	0.5	0.5
Bunch length [μm]	72	44
Beam pulse duration [ns]	177	156
Repetition rate [Hz]	50	50
Hor./vert. norm. emitt. [$10^{-6}/10^{-9}$m]	2.4/25	0.66/20
Hor./vert. IP beam size [nm]	202/2.3	40/1

CLIC Parameter List

The LHC RF system

LHC ... as a low gradient example 16 MV / 27000m



Bunch length (4σ)	***ns***	***1.06***
Energy spread (2σ)	***10^{-3}***	***0.22***
Synchr. rad. loss/turn	***keV***	***7***
RF frequency	***MHz***	***400***
RF voltage/beam	***MV***	***16***
Energy gain/turn	*keV*	*485*

4xFour-cavity cryo module 400 MHz, 16 MV/beam

For the fun of it ...
energy gain per turn = 485 keV
takes 14.4 Mio turns to get to 7 TeV
summs up to 387 Mio km

going linear we have to be much more efficient

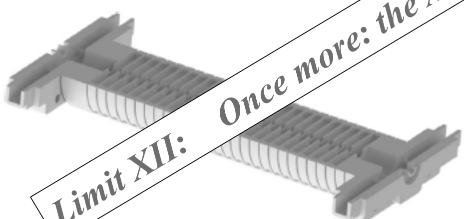
Linear Colliders need the highest feasible Accelerating Gradient

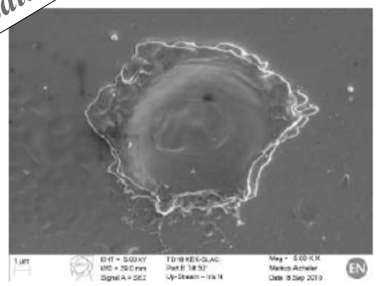
RF break downs have to be studied and understood in detail and pushed to the limit.

as they have impact on

=> the accelerator performance (luminosity)
=> beam quality
=> and the accelerating structure itself

Limit XII: Once more: the Accelating Gradient





" how far can we go and how much can we optimise such a future accelerator before we reach technical limits and how can we push these limits ? "

Resume:

In order to reach higher energies and keep the machines still "compact" we need acceleration techniques that are much more efficient than the status quo.

We urgently need new and better ideas.

And we need them NOW.

court. Z. Najmudin