Two classical examples of resurgence

The aim of this talk is to provide a survey of analytical methods designed to prove resurgence, illustrated on two nonlinear situations in which resurgence appears naturally. In both examples, a nonlinear functional equation gives rise to a formal series which is generically divergent, and we show how to check the convergence of the formal Borel transform and to study its analytic continuation, so as to prove

- summability (analytic continuation in a sector with exponential bounds)

- and resurgence (analytic continuation along all the paths which avoid a certain lattice of singular points).

Summability allows one to construct analytic solutions from the fomal one; the resurgent analysis allows one to describe the geometric consequences of the divergence of the formal solution.

The results are classical, namely they are due to J. Écalle in the 80s, but there is some originality in the details of the methods which are alluded to below.

The first situation is that of a holomorphic germ at the origin of C with a simple parabolic fixed point, for which the local dynamics is classically described by means of pairs of attracting and repelling Fatou coordinates and the corresponding pairs of horn maps, of crucial importance for Écalle-Voronin's classification result and the definition of the parabolic renormalization operator.

The two Fatou coordinates are constructed by Borel-Laplace summation from one single formal series, which is proved to be Borel-summable and resurgent in
A. Dudko, D. Sauzin, "The resurgent character of the Fatou coordinates of a simple parabolic germ" (8 pages)
http://hal.archives-ouvertes.fr/hal-00849398

Then, the resurgent analysis focuses on the nature and shape of the singularities in the Borel plane, so as to establish the Bridge equation, which contains the Écalle-Voronin invariants, to be interpreted as Fourier coefficients of the horn maps. This is done, with new explicit formulas, in
A. Dudko, D. Sauzin, "On the resurgent approach to Ecalle-Voronin's invariants" (8 pages)
http://hal.archives-ouvertes.fr/hal-00849401

The second situation is that of a saddle-node, a two-dimensional vector field which is formally conjugate to an elementary normal form and for which the formal normalisation is shown to be resurgent-summable with respect to one of the variables. Here we use not only Ecalle's resurgence theory, but also Ecalle's mould calculus, a powerful combinatorial tool which yields surprisingly explicit formulas. A brief survey of the method is given in
D. Sauzin, "Initiation to mould calculus through the example of saddle-node singularities" (13 pages)
http://hal.archives-ouvertes.fr/hal-00201446
The details are in
D. Sauzin, "Mould expansions for the saddle-node and resurgence monomials" (78 pages)
http://hal.archives-ouvertes.fr/hal-00197145