Resurgence and Transseries in Quantum, Gauge and String Theories

Monday, 30 June 2014 - Friday, 4 July 2014

CERN

Scientific Programme

Talks:

Aniceto: *"Resurgence in Localizable Supersymmetric Gauge Theories"*

Argyres: *"Effective Actions of 4d Gauge Theories on* $\boldsymbol{R}^3 \times \boldsymbol{S}^1$ *with Periodic Fermions"*

Bender: *"Nonlinear Eigenvalue Problems and PT-Symmetric Quantum Mechanics"*

Beneke: "*The Operator Product Expansion as a Transseries: The O(N) Sigma-Model at Order 1/N"*

Berry: *"Divergent Series: From Thomas Bayes's Bewilderment to Today's Resurgence via the Rainbow"* (**Theory Colloquium**)

Costin: *"Resurgence, Generalized Borel Summability and Applications to*

Ionization Problems"

Couso-Santamaría: *"Resurgent Transseries in Topological String Theory"*

Delabaere: *"Resurgent Methods and the First Painlevé Equation"*

Dorey: *"The Basics of the ODE/IM Correspondence"*

Dunne: *"Resurgence in Quantum Field Theory and Quantum Mechanics"*

Écalle: *"Resurgence and the Protean Bridge Equation: Taking Stock and Looking Ahead"*

Gualtieri: *"Stokes Groupoids: A Geometric Approach to Resummation"*

Howls: *"Sum-ware over the Rainbow: Hyper, Higher and Transseries Asymptotics"*

Jentschura: *"Resurgence as a Generalization of Perturbation Theory"*

Mariño: *"Resurgence, String Theory, and the 1/N Expansion"*

Pineda: *"Phenomenology of Renormalons from Heavy Quark Physics and Lattice"*

Sauzin: *"Two Classical Examples of Resurgence"*

Scorzato: *"The Sign Problem in Lattice Field Theories and the Lefschetz Thimble"*

Shifman: *"On Renormalons in Supersymmetric Field Theories"*

Voros: *"Exact WKB Resolution of a General 1D Schrödinger Equation"*

Zinn-Justin: *"From Instanton Expansions to Exact Results"*

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Resurgent analysis and transseries are frameworks which have been extremely successful within analysis and differential equations in the past couple of decades, but which so far have not been much applied to problems arising in Theoretical Physics (although some of these ideas were already present in some form in the Physics community for a long time). They allow for the construction of nonperturbative solutions to a wide range of problems where one is dealing with asymptotic series—as is the case in most interesting problems arising in quantum mechanics, quantum field theory or string theory. It is the goal of this CERN TH-Institute / Conference to bring together researchers in Mathematics, covering topics of Resurgence, Transseries and Asymptotic Analysis, with researchers in different aspects of Theoretical Physics, from Quantum Mechanics to Gauge and String Theory, where resurgent methods may play very prominent roles. We wish to have a very explicitly interdisciplinary conference with the hope that different points-of-view and different methods and approaches will lead to new developments in this new and exciting field.

From a mathematical point-of-view, resurgence and transseries are extremely powerful methods to

understand large-order asymptotics of perturbative expansions and deal with global (non-analytic) definitions of very general classes of functions—most likely with classes of functions large enough to incorporate any physical observable one may think of. These methods are explicitly constructive and, because they start very much in parallel with usual perturbative techniques within Theoretical Physics, have the promise to allow for global nonperturbative definitions in physical frameworks where, so far, none exist. In this way, the first goal is to make the mathematical foundations clear to physicists with different backgrounds.

Moving towards quantum field theory (QFT), the goal of the resurgence program is to eventually achieve a nonperturbative continuum definition of QFT, at least in the calculable semi-classical domain. As stated above, perturbation series in QFT are generically divergent asymptotic series, typically not Borel resummable—in the sense that the resummed series is ambiguous, and where ambiguities are associated with singularities in the Borel plane. This means that, as it stands, perturbation theory does not define the full quantum theory. In quantum mechanics there are cases in which the ambiguity that arises in perturbation theory cancels against a similarly ambiguous contribution from instanton/anti-instanton events, implying a deep connection between perturbation theory and nonperturbative saddles. This mechanism of cancellation was discovered by Bogomolny and Zinn-Justin (BZJ), and is the first basic (physical) step of resurgence. This mechanism is believed to be generalizable to all nonperturbative orders via exact quantization conditions and exact WKB, implying that one can define observables in quantum mechanics in an ambiguity-free manner.

In asymptotically free gauge theories, such as Yang-Mills theory or non-linear sigma models, this mechanism does not suffice because perturbation theory develops ambiguities associated with singularities in the Borel plane which are closer to the origin by a factor of about N (the rank of the gauge group in Yang-Mills theory or the global symmetry group in non-linear sigma models) as compared to the singularities realized by instanton events. This is the IR renormalon problem, for which a well-controlled cancellation mechanism is unknown. More recently, it was understood that a large variety of gauge and field theories (either supersymmetric or not) may be continued to weakly coupled calculable regimes, by using circle (non-thermal) compactification where one has control over both perturbative and nonperturbative dynamics. This idea of continuity, combined with a generalization of the BZJ mechanism to QFT, permits a way to cancel the renormalon ambiguities at least in the semi-classical domain, and give a physical interpretation thereof. In general,

(i) resurgence provides a more refined classification of nonperturbative saddles as compared to the standard topological classification; and

(ii) it also predicts the existence of "generalized" instanton sectors and, in some cases, gives an interpretation to complex saddles that the path integration does not pass through.

Both of these are of significant importance in our understanding of QFT. The second item is related to the studies of analytic continuation of path integrals, and performing path integration over different cycles. These latter methods are also beginning to find applications in numerical Monte Carlo simulations, in problems related to sign problems in QCD, and other theories.

In this way, the second and main goal of this CERN TH-Institute is to allow for physicists with different specific interests to understand how resurgence has been used and developed in different

problems, and how particular techniques may be used cross-disciplinary and further developed towards more general results.