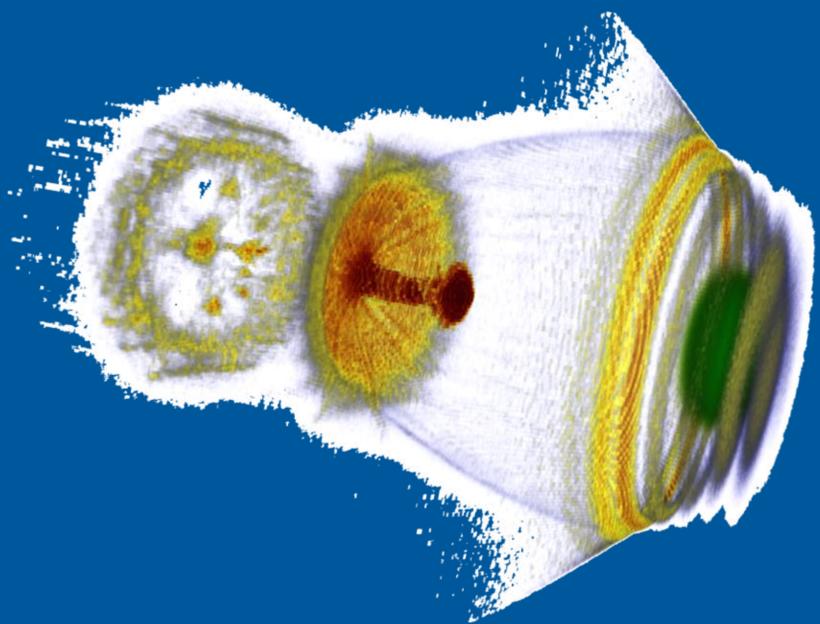
Laser-plasma based electron acceleration



Arie Irman

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LA³NET Advanced School on Laser Applications at Accelerators,
28.9 - 3.10.2014, Salamanca, Spain

Outline

- Characteristic of plasmas
- Plasma wave excitation
- Electron acceleration in plasma wave
- Different electron injection scheme
- Recent progress in LWFA

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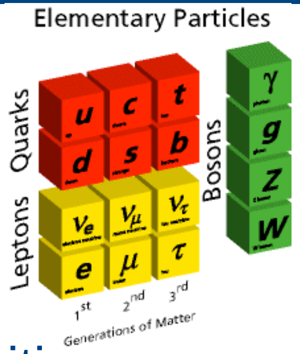
High energy charged particles

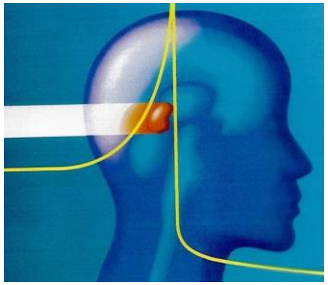
HZDR

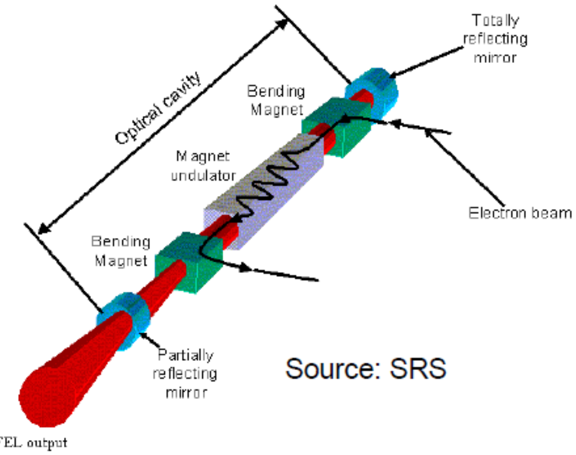
- High energy physics:
  fundamental structure of matter and energy



- Material science: semiconductor, phase transition

- Medical physics: cancer therapy

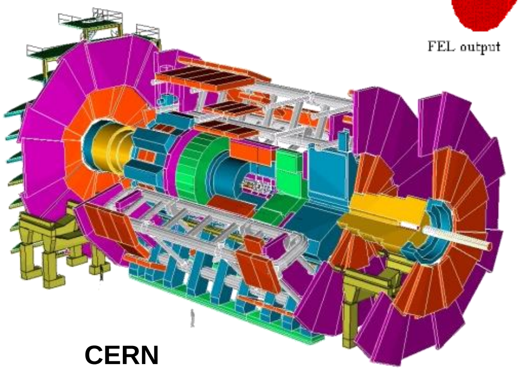




Source: SRS

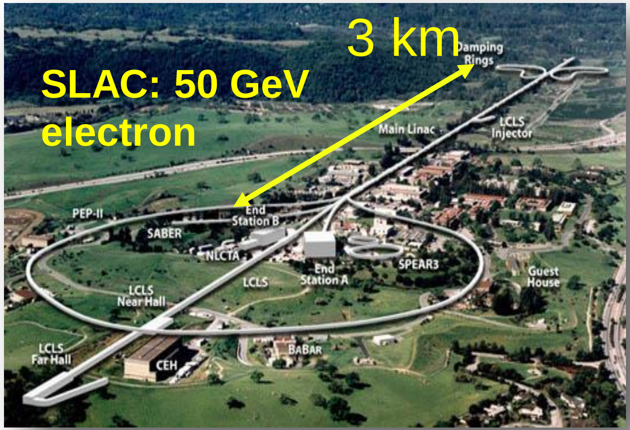
- Coherent radiation and X-ray sources:
  synchrotron and FEL

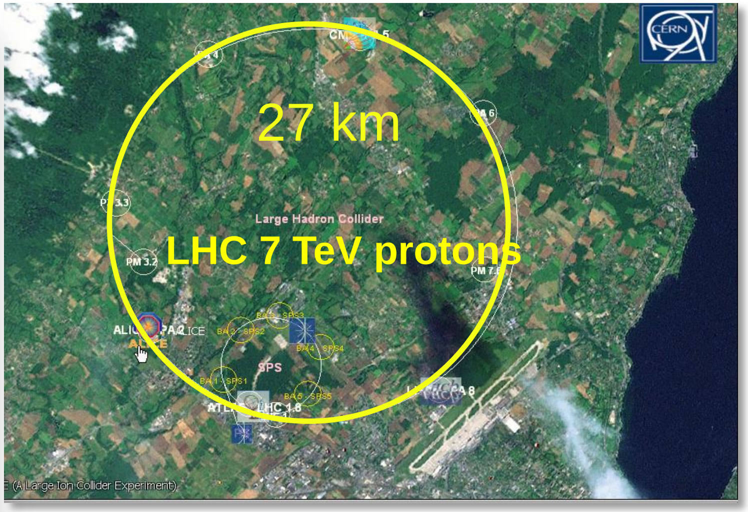
- State of the art technology:
  vacuum technology, detector



CERN

State of the art particle accelerators





Drawbacks:

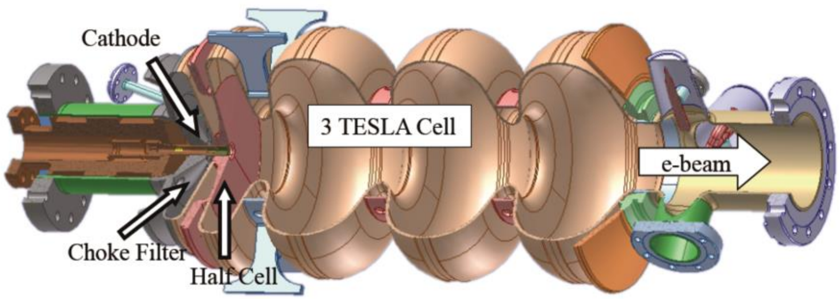
- large infrastructure: extremely expensive
- limitted access

Novel accelerator concepts need to be found !!

RF cavities versus plasmas

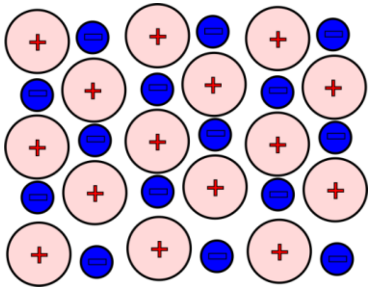
- Linac → high power RF technology (λ_{RF} ~ tens of cm)
  → accelerating field ~ a few tens of MV/m (vacuum breakdown)

3-1/2 cells Superconduting RF photoinjector ELBE-HZDR



design value:
E_{peak} = 50 MV/m
(TESLA cavities at DESY)
obtained:
$E_{peak} \approx$ 20 MV/m

- Plasmas → neutral particles, hot ions and electrons
  → space-charge electric fields >> RF electric fields in linac

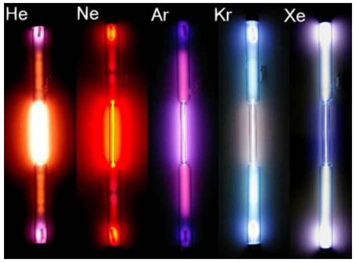


E_z ~ GV/m → > 10^3 higher than in RF linacs

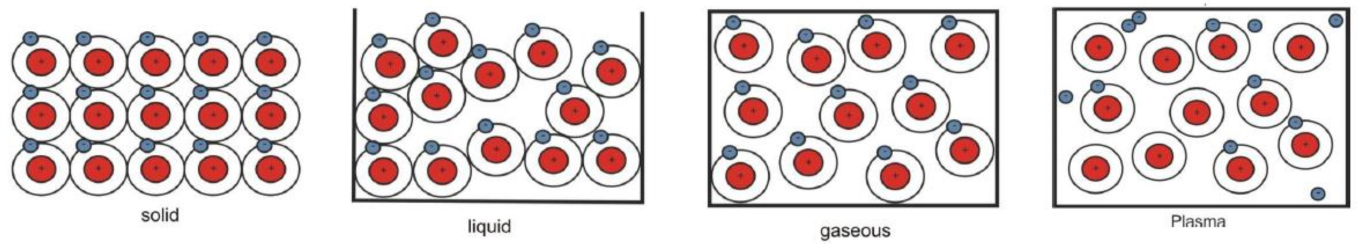
λ_{plasma} ~ tens of μm → a compact accelerator

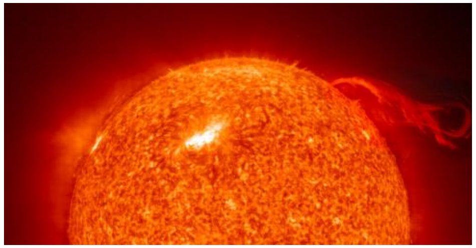
$$E_z \propto \sqrt{n_p}$$

Plasma: the fourth state of matter

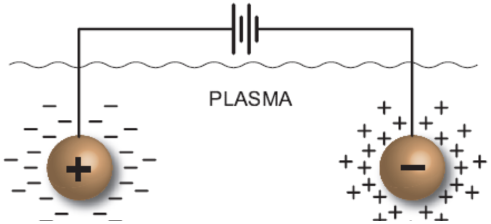


- A mixture of neutral particles, hot ions and electrons

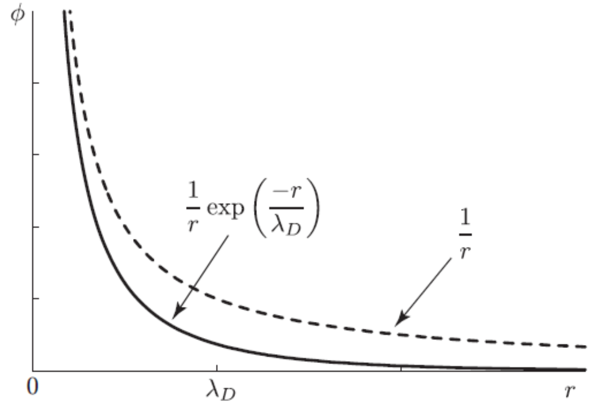




- Characteristics:
  - → Collective behavior
  - → Debye shielding, Debye length, Debye sphere



$$\lambda_D = \sqrt{\frac{k_B T_e}{4\pi n_0 e^2}} \qquad \lambda_D \ll L \qquad N_D = \frac{4\pi n_0 \lambda_D^3}{3} \qquad N_D \gg 1$$

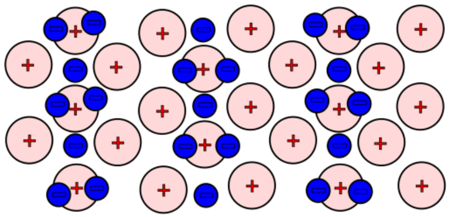


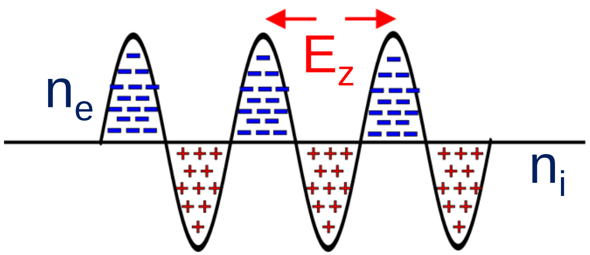
  - → Response time $\geq \frac{1}{\omega_p}$

$$n_0 = 10^{18} cm^{-3} \qquad T_e = 10\ eV (\approx 1.16 \times 10^5 K)$$
$$\lambda_D \approx 23\ nm \qquad N_D \approx 54\ electron$$

Plasmas as accelerators

plasma oscillations





If you create a charge separation in plasmas, plasma oscillations will be generated:

- it propagates with phase velocity ~ speed of the driver:
  laser-driven $v_p \sim v_g$

- frequency $\omega_p = \sqrt{\dfrac{4\pi\, n_e\, e^2}{m_e}}$ $\lambda_p = \dfrac{2\pi}{\omega_p}$

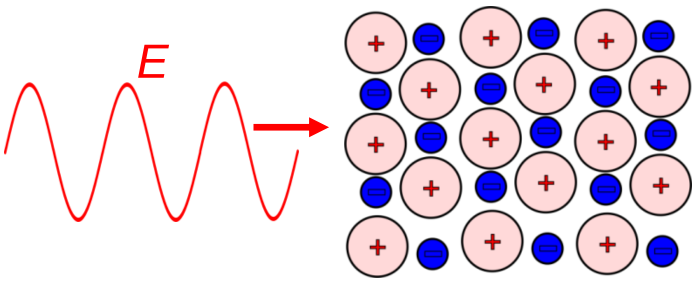
- maximum accelerating field strength

$$E_{z,\max}[V/cm] \approx 0.96 \sqrt{n_p[cm^{-3}]} \sqrt{2(\gamma_g - 1)}$$

$$\gamma_g = 1/\sqrt{1-(v_p/c)^2}$$

$$n_p = 10^{18} cm^{-3} \quad E_{z,\max} \approx 800\ GV/m \quad \lambda_p \approx 33\ \mu m$$

Propagation of light in plasmas

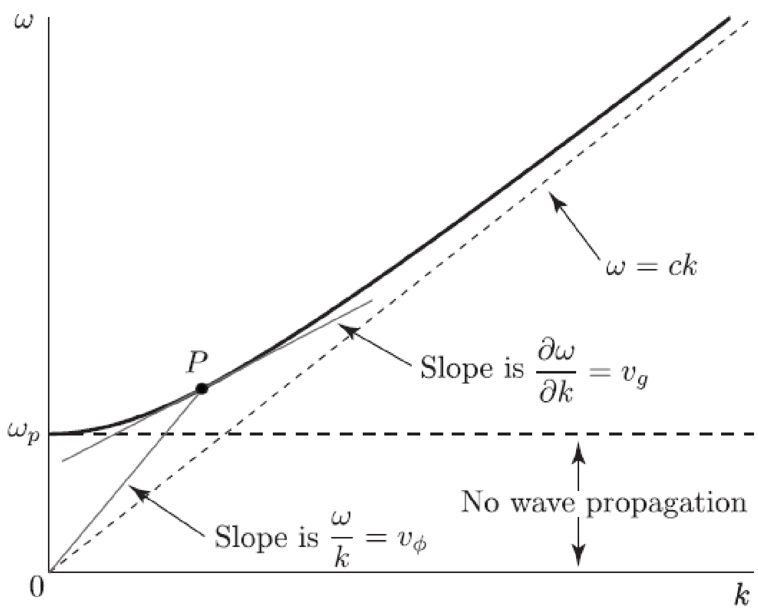


plasmas

- Electrons wiggle in the E-field (via Lorentz force)

  - ions remain immobile in their positions
  - fast time ~ laser period
  - the net charge separation is zero

$$\omega^2 = \omega_p^2 + k^2 c^2 \rightarrow \text{dispersion relation}$$



- Phase velocity

$$v_\phi = \frac{\omega}{k} = \sqrt{c^2 + \frac{\omega_p^2}{k^2}} = \frac{c}{\eta}$$

- Group velocity

$$v_g = \frac{\partial\omega}{\partial k} = c\sqrt{1 - \frac{\omega_p^2}{\omega^2}}$$

Propagation of light in plasmas

HZDR

- Plasma refractive index $\eta = \sqrt{1 - \frac{\omega_p^2}{\omega^2}}$

$\omega_p > \omega \rightarrow \eta$ Plasma refractive index becomes imaginary

$\rightarrow$ Plasma becomes not transparent

$\rightarrow$ Light will reflected by the plasma (like a mirror)

- Critical density $n_c[cm^{-3}] = \frac{m_e \omega^2}{4\pi e^2} \approx \frac{1.1 \times 10^{21}}{\lambda[\mu m]^2}$

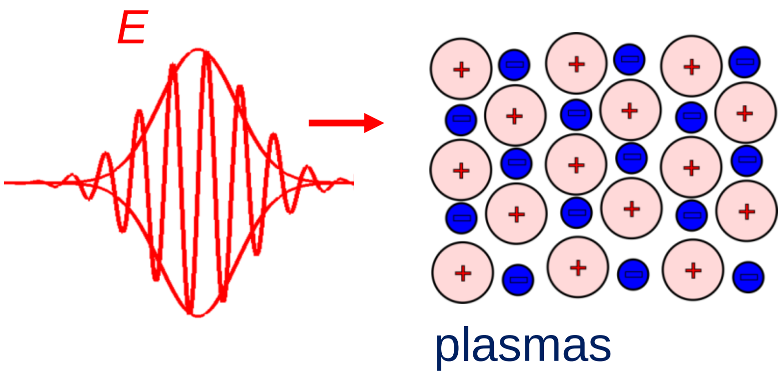
$\lambda = 0.8 \ \mu m \qquad n_c = 1.7 \cdot 10^{21} \ cm^{-3}$ $\rightarrow$ ~ 35 bar Helium gas

$\omega_p > \omega, \ n_p > n_c$ Overdense plasma

$\omega_p < \omega, \ n_p < n_c$ Underdense plasma

$\rightarrow$ Regime for laser electron acceleration

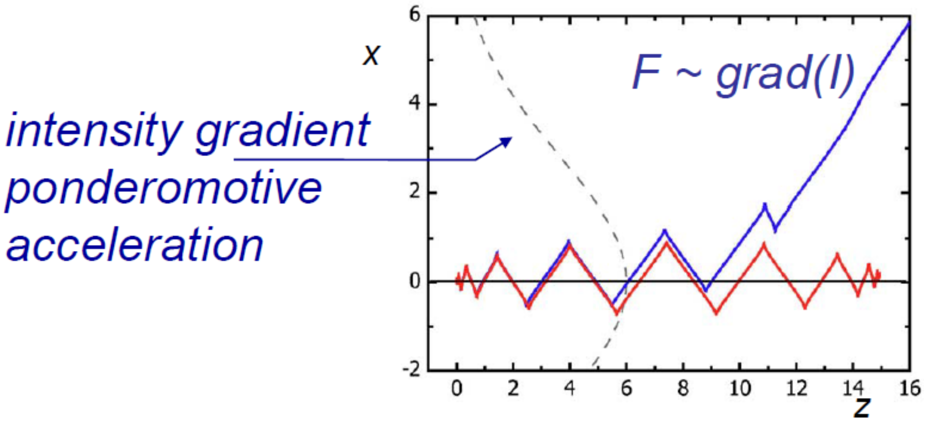
The Ponderomotive force



- Electrons wiggle in the E-field (via Lorentz force)

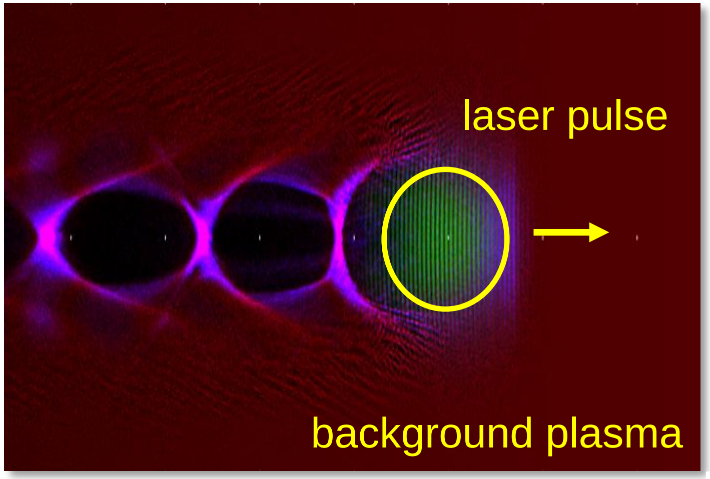
  - time-averaged force is not zero
  - fast time ~ laser period
  - slow time ~ laser envelope
  - the net charge separation is not zero

- The ponderomotive force (“light pressure”) expels electrons from high intensity region

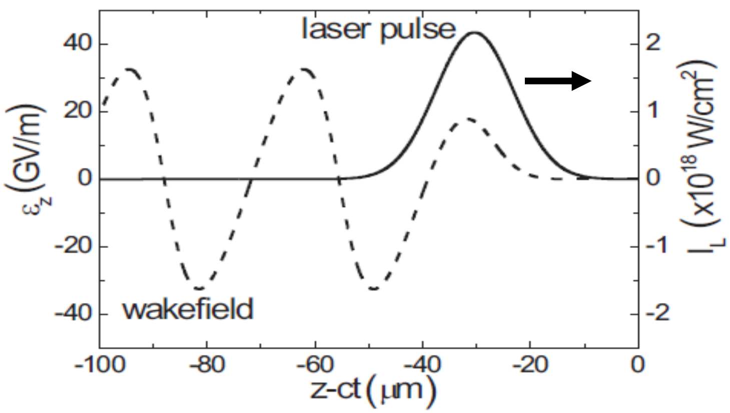


$$\overline{F}_{pond} = \left\langle m_e \frac{d\overline{v}}{dt} \right\rangle = -\frac{e^2}{4\ m_e \omega^2} \overline{\nabla} E^2(r)$$

Plasma wave excitation



T.Tajima and J.Dawson, PRL 43, 267(1979)

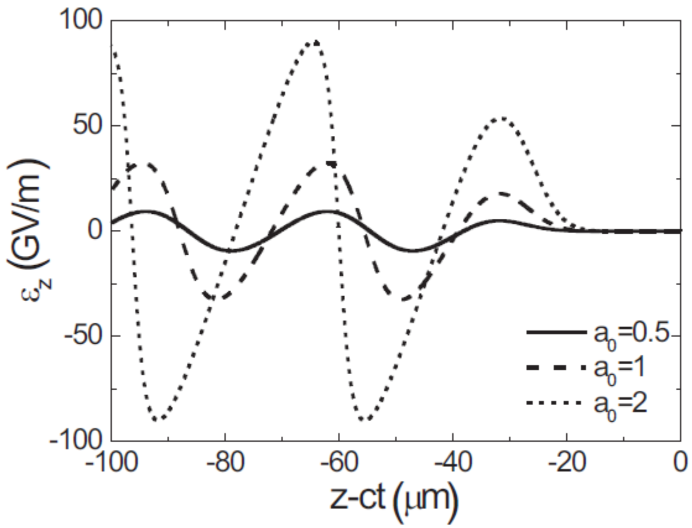


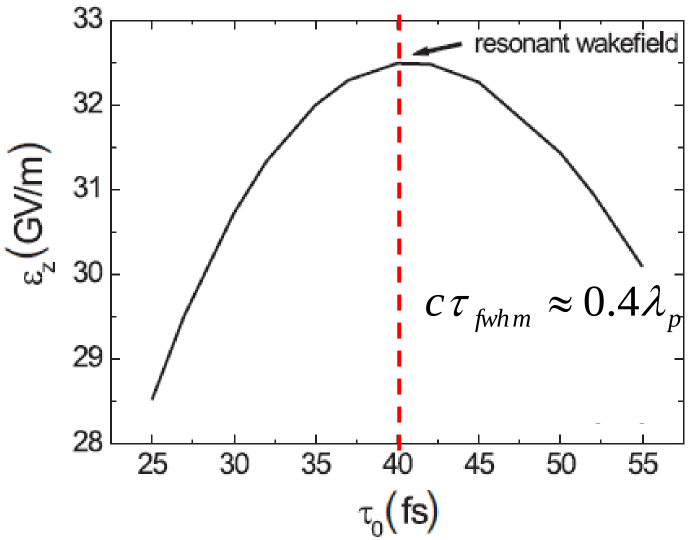
- A high intensity laser pulse can excite plasma waves > 10^{17} W/cm^2
- Driver pulse length $\ll \lambda_p$
- Ions remains immobile
- $v_\phi^{plasma} = v_g^{laser} \sim c$

1-D Laser wakefield equation

$$\frac{d^2\Phi}{d\xi^2} = \beta_g^2\gamma_g^2\left(\beta_g \frac{1}{\sqrt{1-\frac{1}{\gamma_g^2}\frac{1+a^2/2}{\Phi^2}}} - 1\right), \quad E_z = -\frac{1}{\beta_g^2}\frac{d\Phi}{d\xi}.$$

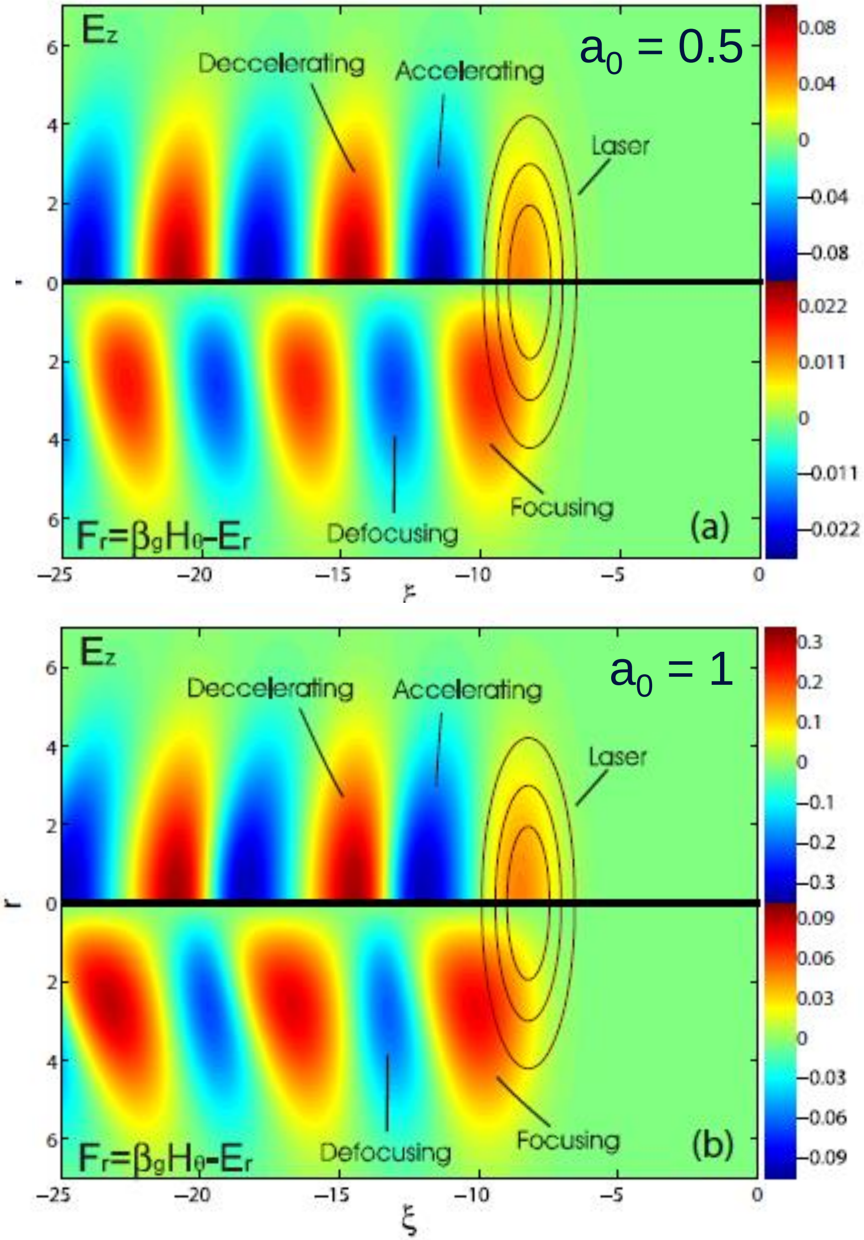
Characteristics of laser wakefield: 1D





- The amplitude of wakefield increases with increasing the laser intensity
- Linear regime: the wakefield has a sinusoidal shape
- Non-linear regime: the wakefield becomes steeper
- Plasma wavelength increases with increasing the laser intensity
- The resonance occurs when the driver laser length is ~ 0.4 λ_p

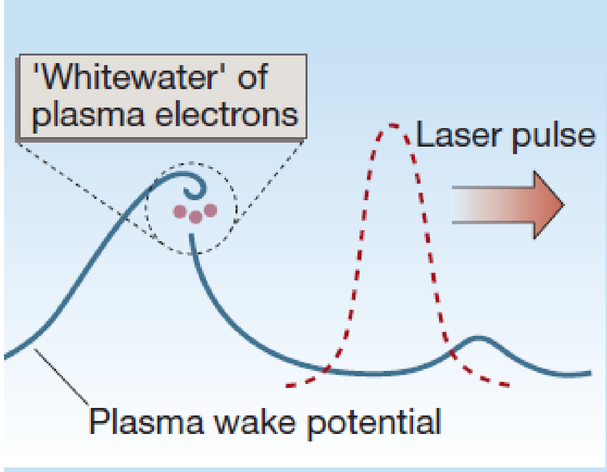
Characteristics of laser wakefield: 3D



- Existence of transverse force fields
- Focusing fields bring electrons to the axis.
- Defocusing fields scatter electrons out of the axis.
- The optimum accelerating region is the overlap region between the accelerating and the focusing region.
- The overlap region becomes larger in the non-linear wakefield.

Wave-breaking

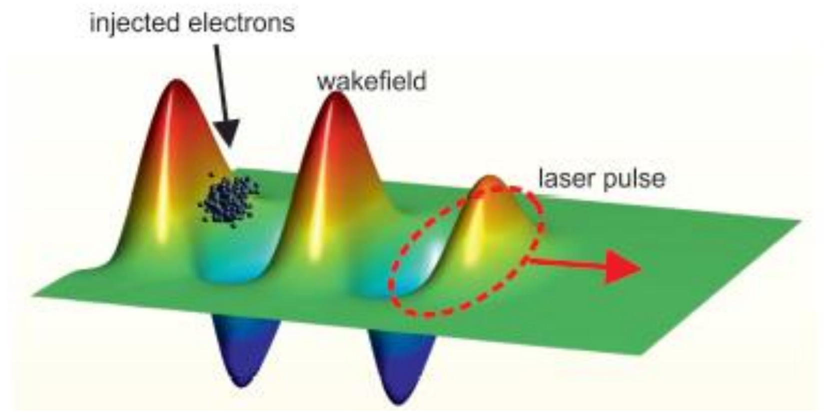
- Wave-breaking limits the maximum accelerating field strength.
- Electron longitudinal velocity > plasma wave phase velocity



$$\frac{d^2\Phi}{d\xi^2} = \beta_g^2\gamma_g^2\left(\beta_g\frac{1}{\sqrt{1-\frac{1}{\gamma_g^2}\frac{1+a^2/2}{\Phi^2}}}-1\right),$$

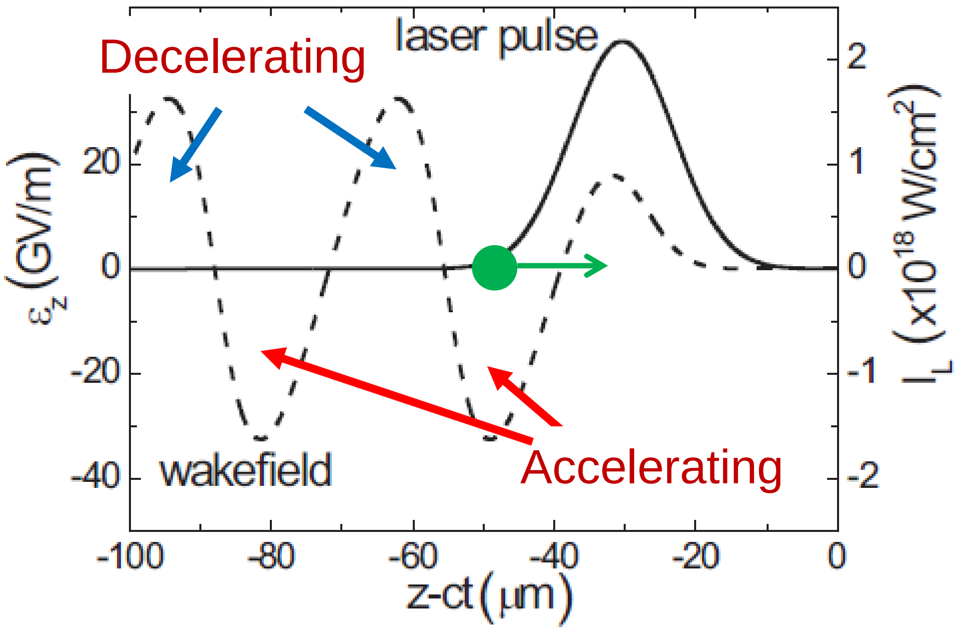
When $\frac{1}{\gamma_g^2}\frac{1+a^2/2}{\Phi^2}=1$ $\quad \frac{d^2\Phi}{d\xi^2} \rightarrow \infty$ Wave-breaking $\quad E_{z,WB} = E_0\frac{\sqrt{2(\gamma_g-1)}}{\beta_g}$

LWFA: a compact electron accelerator



Electron orbit in phase space:

- electron trapping mechanism
- acceleration
- dephasing



Equation of motion of a test electron in wakefields with amplitude E_z

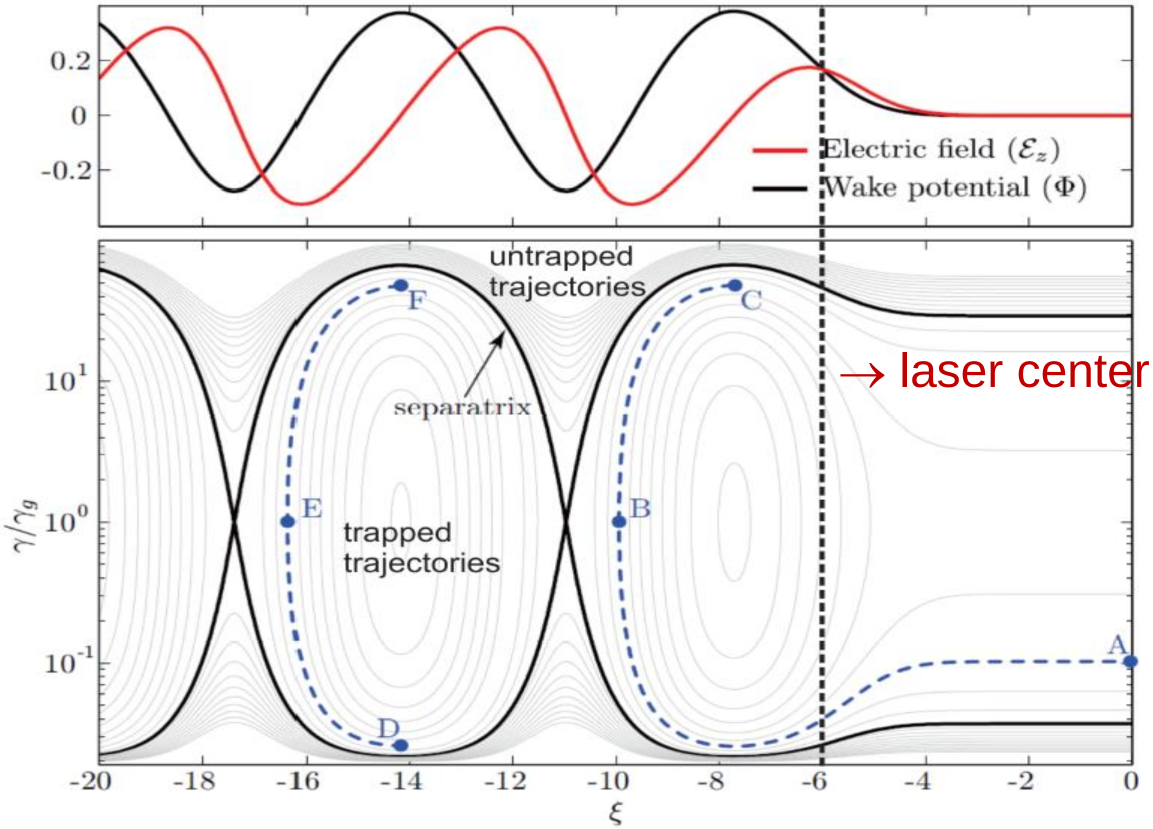
$$H(\gamma_e, \xi) = \gamma_e - \gamma_e \beta_e \beta_g - \phi(\xi)$$

$$\frac{dH(\gamma_e, \xi)}{d\tau} = 0$$

Hamiltonian constant along a given electron orbit

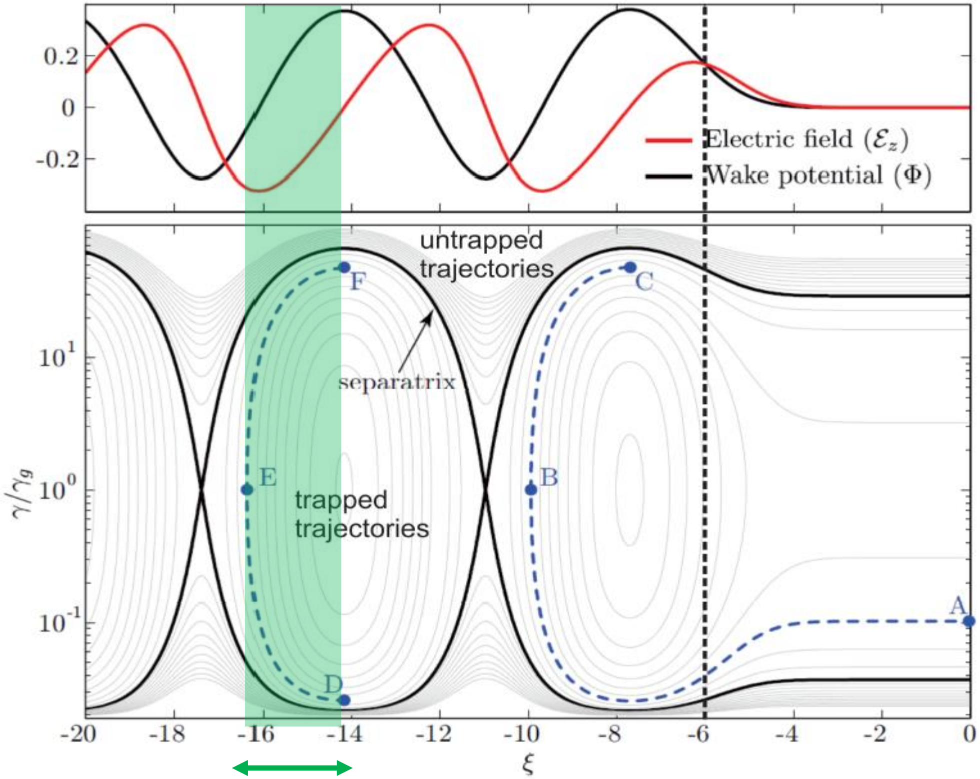
Electron dynamics in 1D plasma wave

The separatrix:
orbit that separates the region of trapped and untrapped electrons in the longitudinal phase space



Maximum energy gain

- **Dephasing length:** distance for electron to gain energy before entering the decelerating phase



L_{deph}

$$L_{deph} \approx \gamma_g^2 \lambda_p \qquad L_{deph} \sim n_p^{-3/2}$$

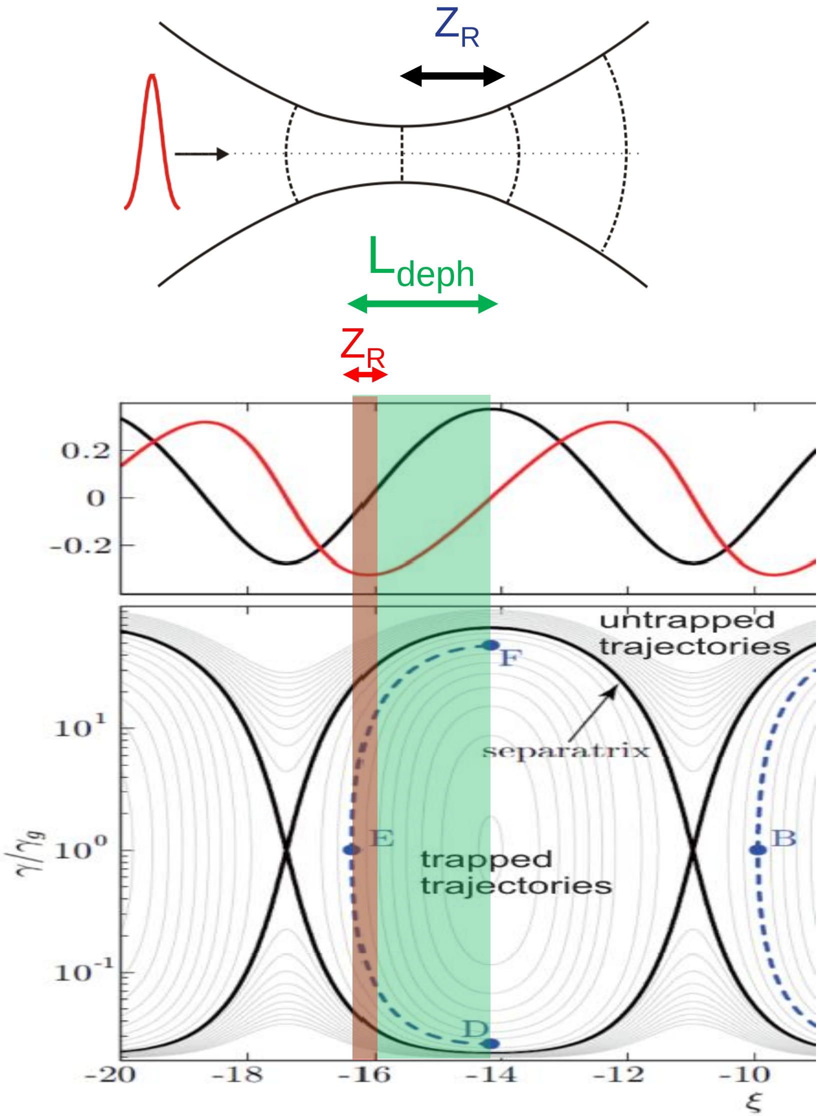
Maximum electron energy:

$$W_{\max} \sim E_z L_{deph} \qquad E_z \sim n_p^{1/2}$$

$$W_{\max} \sim n_p^{-1}$$

to gain more energy: lowering plasma density

1. Long dephasing length ≈ acceleration distance

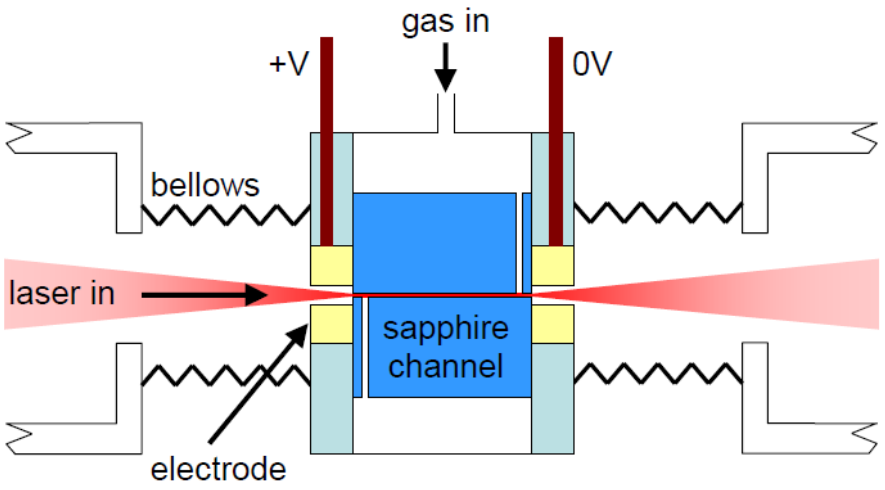


- High intensity laser over long distance
- Optical diffraction limits the acceleration length to the Rayleigh length
- Example:

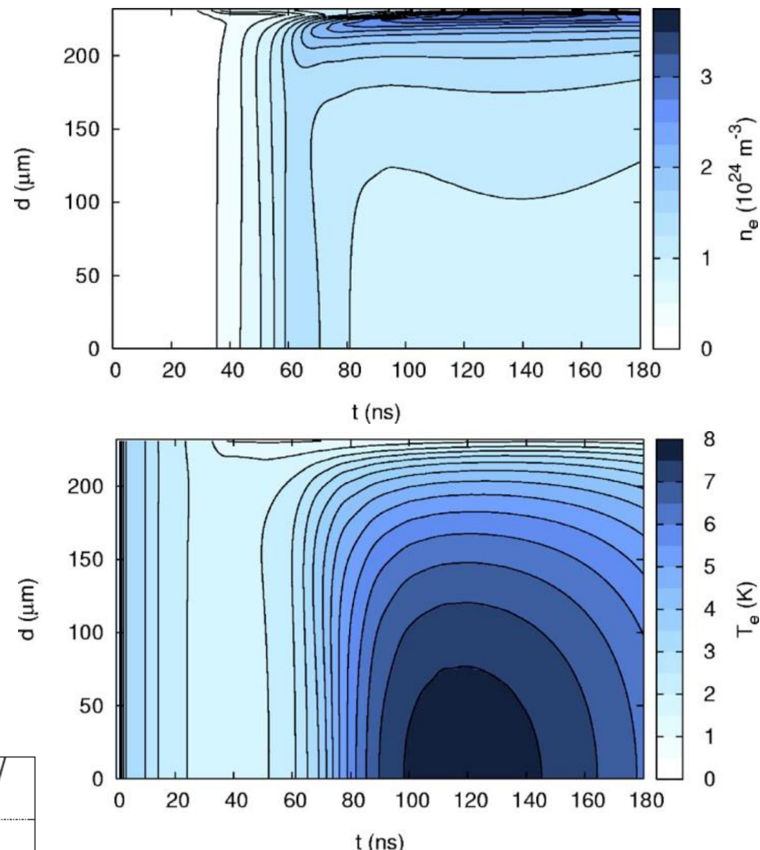
$$\lambda = 0.8\mu m \quad w_0 = 10\mu m \quad Z_R \approx 400\ \mu m$$

$$n_p \approx 10^{18}\ cm^{-3} \quad L_{deph} \approx 6\ cm$$

Preformed plasma channel



A.Gosalves, PhD Thesis 2006, Oxford University

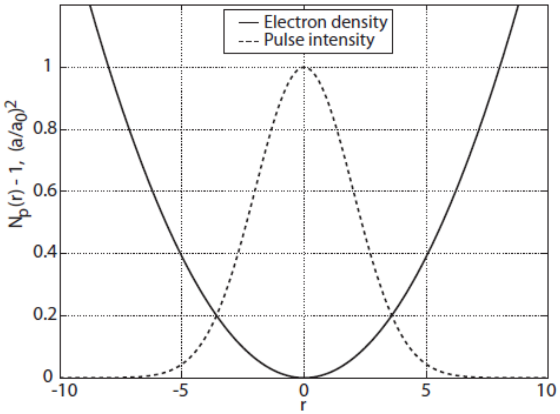


B. Broks, et.al., physplasmas 14,23501(2007)

- slow discharge: Hydrogen gas
- electron temperature higher on axis
- electron density lower on axis

$$n_p(r) = n_p(0) + \Delta(r/r_{ch})^2$$

$$w_{laser} = (r_{ch}^2 / \pi r_e \Delta)^{1/4}$$

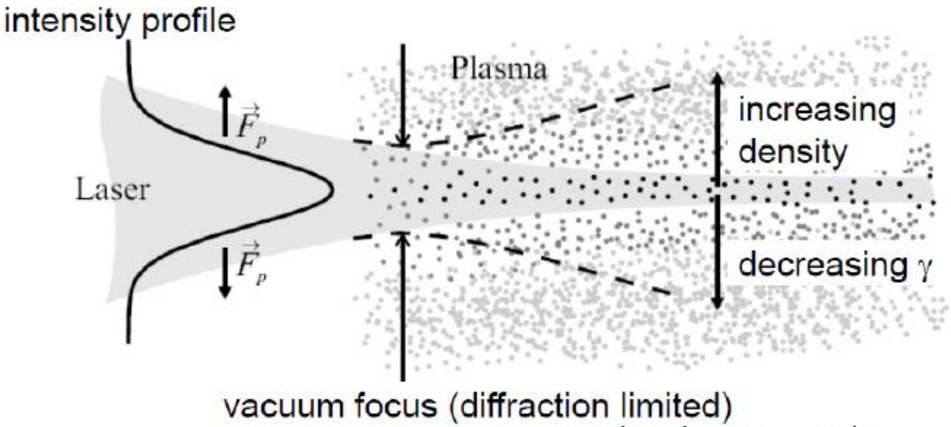


Intensity dependent plasma refractive index

$$\eta(I) = \sqrt{1-\frac{\omega_p^2}{\gamma\,\omega^2}} \cong 1-\frac{1}{2}\frac{\omega_p^2}{\omega^2}\left(1+\frac{\Delta n_e}{n_e}-2\frac{\Delta\omega}{\omega}-\frac{a_0^2}{4}\right)$$

$$\gamma = 1 + a_0^2/2$$

$$a_0^2 \sim I_L \lambda_L^2$$



M.Kaluza lect. Note,Uni Jena

Region with higher laser intensity:

- electron mass increase

relativistic quiver motion γm_e

- local plasma frequency decrease

$$\omega_p \sim (\gamma m_e)^{-1/2}$$

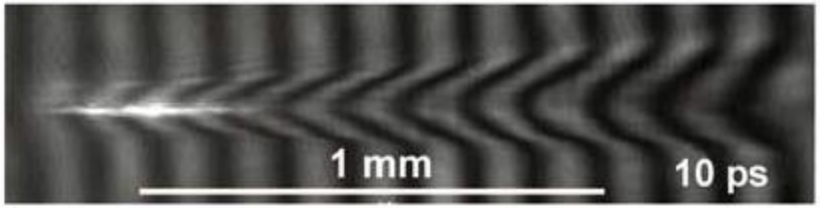
- electron density decrease

ponderomotive force + relativistic effect

Requirements:

$$P_L > P_{cr} \qquad P_{cr} = 17.4(\omega_L/\omega_p)^2 GW$$

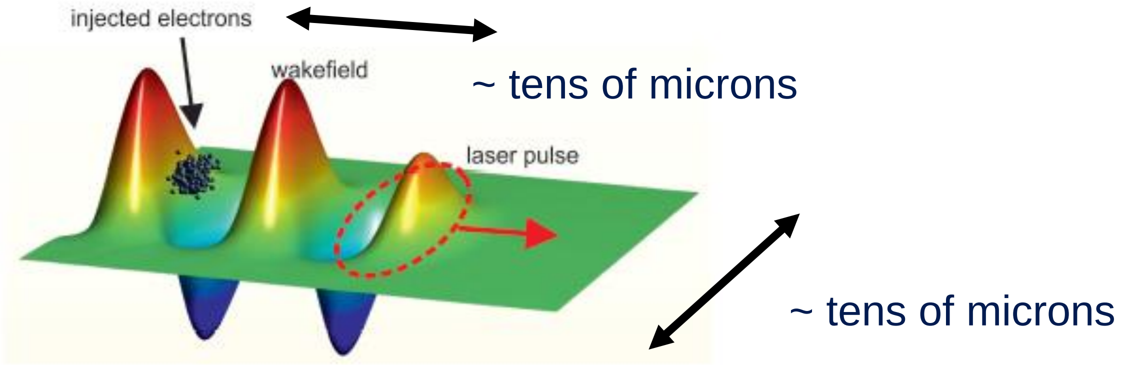
$$a_0 > \left(\frac{\omega_L}{\omega_p}\right)^{2/5}$$



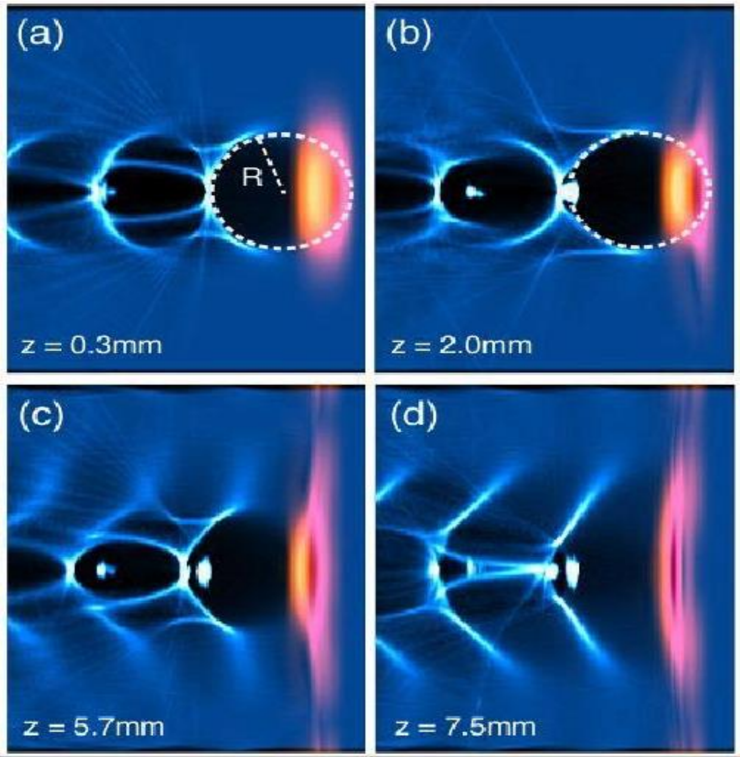
A. Maksimchuk, et.al., APB 89,201(2007)

2. fs bunch and fs time for injection → the central problem !!

- size < laser beam size
- length < plasma wavelength
- synchronization < laser pulse duration
- initial energy > trapping threshold



Self-injection in the bubble regime[1,2,3,many more]

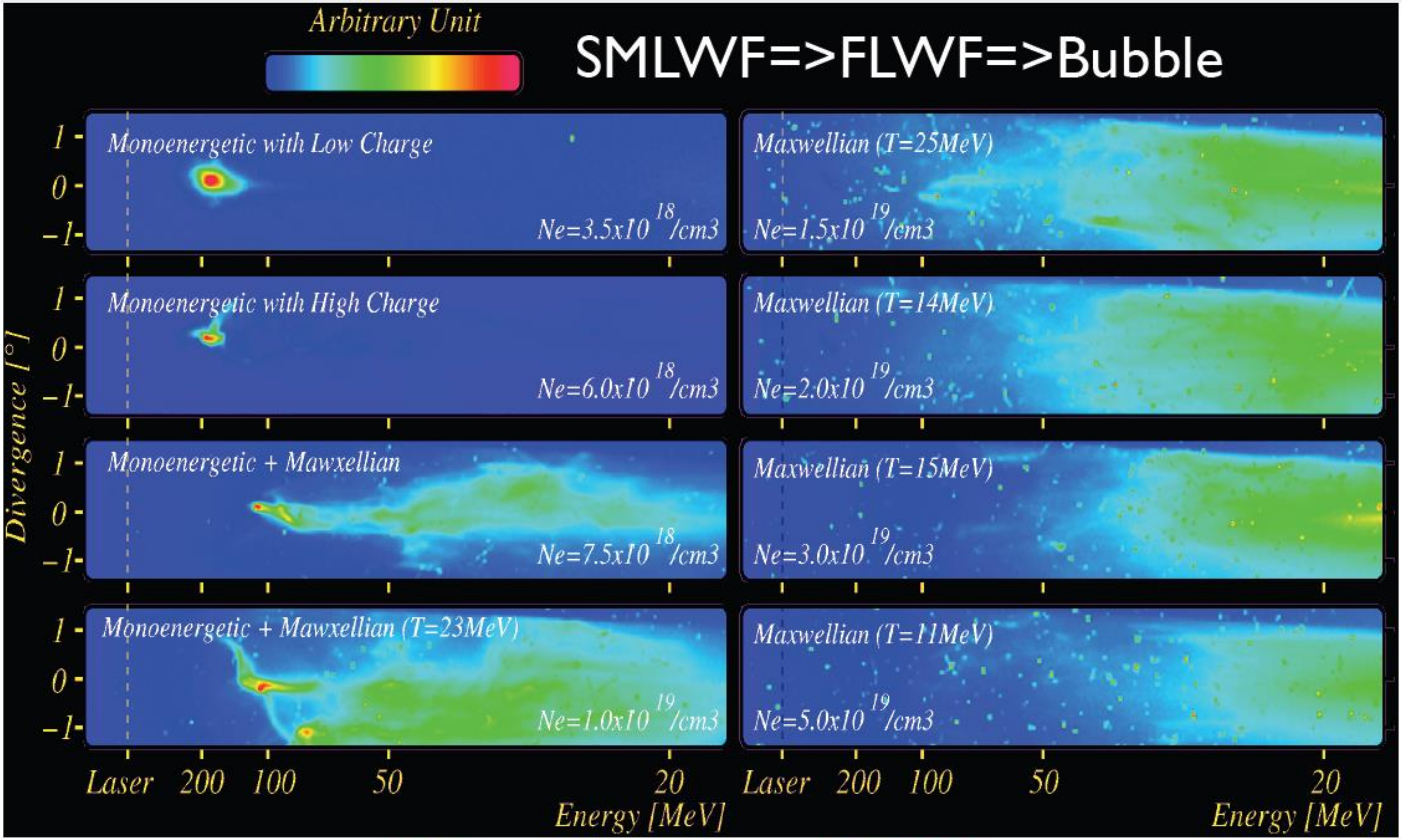


Lu, W., *et al*, PRSTAB 10,061301(2007)

- A highly nonlinear mechanism: self focusing + self steepening
- Electrons completely blown out by the F_p
- No spatial and temporal problems with injection
- Inherent problem: shot-to-shot stability is very sensitive to the fluctuation of laser's and plasma's parameters

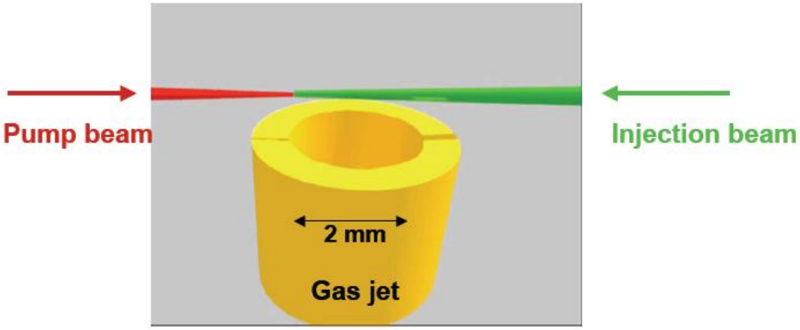
[1]Pukhov, A., *et al*, *Appl.Phys.B*.,74,355(2002), [2]Leemans, W.P., *et al*, *Nature physics2,696(2006),*[3]Mangles, S.P.D., *et al*, *Nature 431,535(2004),*[4] Geddes, C.G.R, *et al*, *Nature 431,538(2004),* [5]Faure, J., *et al*, *Nature 431,541(2004)*, many more

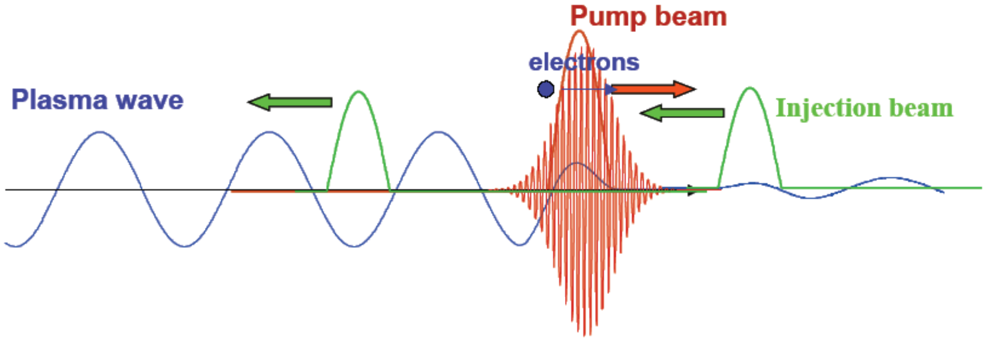
Self-injection in the bubble regime[1,2,3,many more]



V. Malka *et al.*, Phys. of Plasmas **12**, 5 (2005)

Injection by colliding pulse scheme[1,2,3]



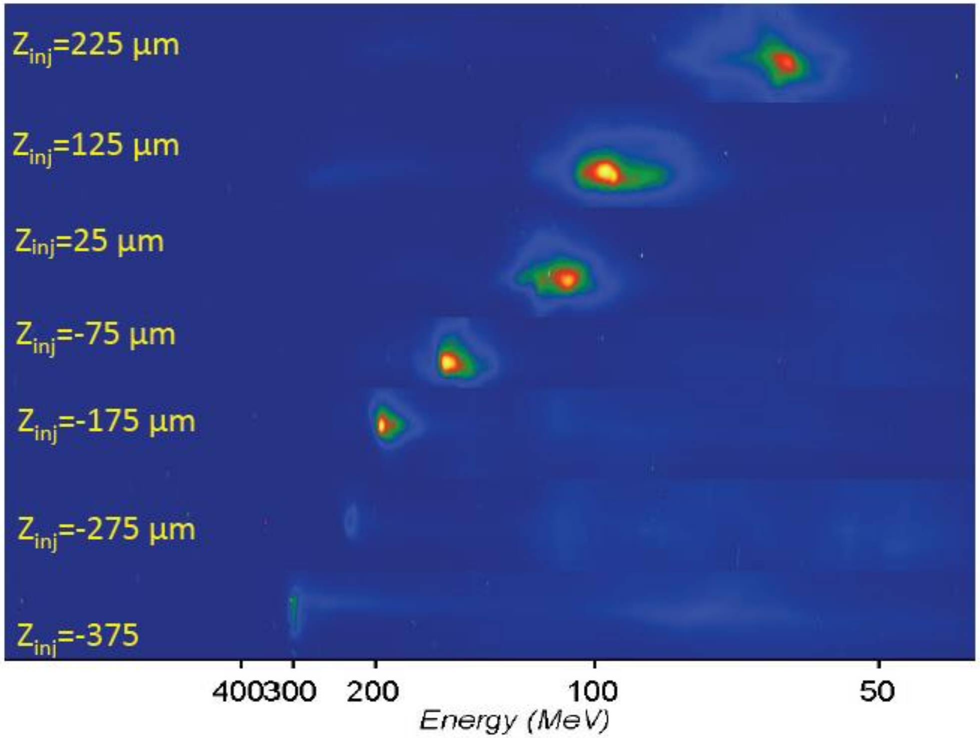


- The pump beam to drive the wakefield, the injection beam to heat up the background electrons
- Injection is local
- Better shot to shot reproducibility
- Better control over electron parameters, energy, total charge,

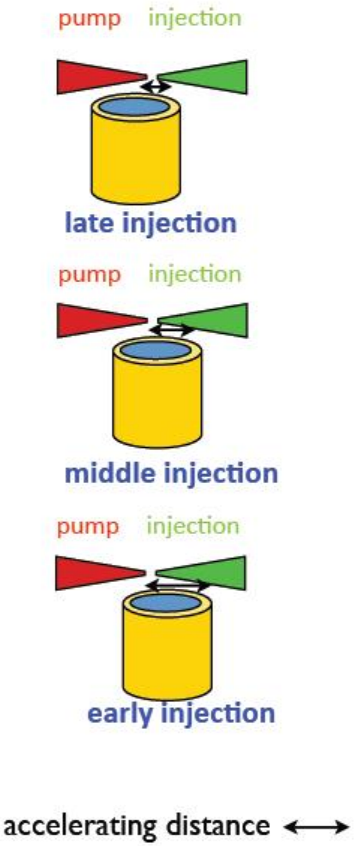
[1]Faure, J., *et al*, *Nature 444,737 (2006)*, [2]Esarey, E., *et.al.*, *PRL79,2682(1997)*, [3]Kotaki, H., *et.al.*, *PoP 11 (2004)*

Injection by colliding pulse scheme

Tuneable electron energy

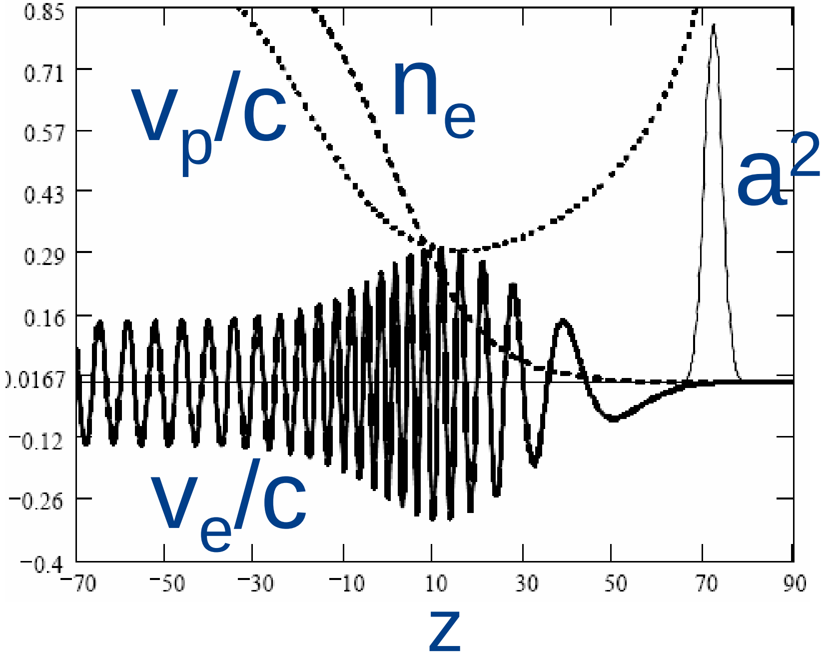


Faure, J., *et al*, *Nature 444,737 (2006)*



Density down-ramp injection

calculation by A. Khachatryan

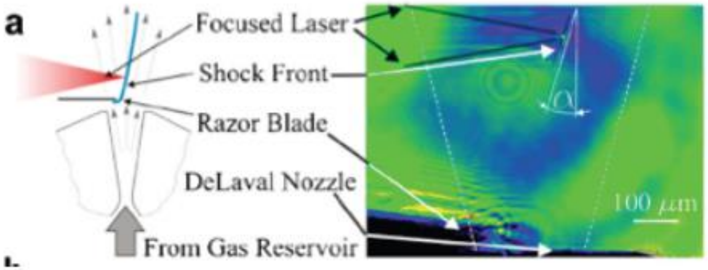


- λ_p increase as n_e decrease
- v_p decrease faster, eventhough v_g increase
- when $v_p \approx v_e$ local wavebreaking, injection and acceleration

Does not require non-linear laser pulse dynamics

➡ better stability can be expected

Density down-ramp injection



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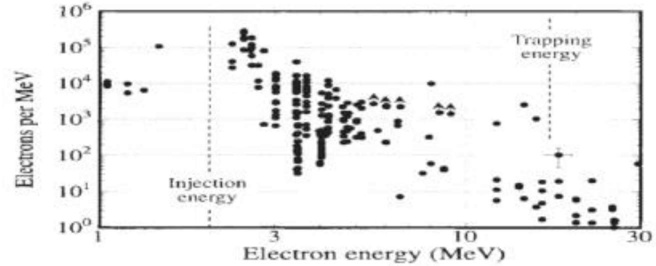
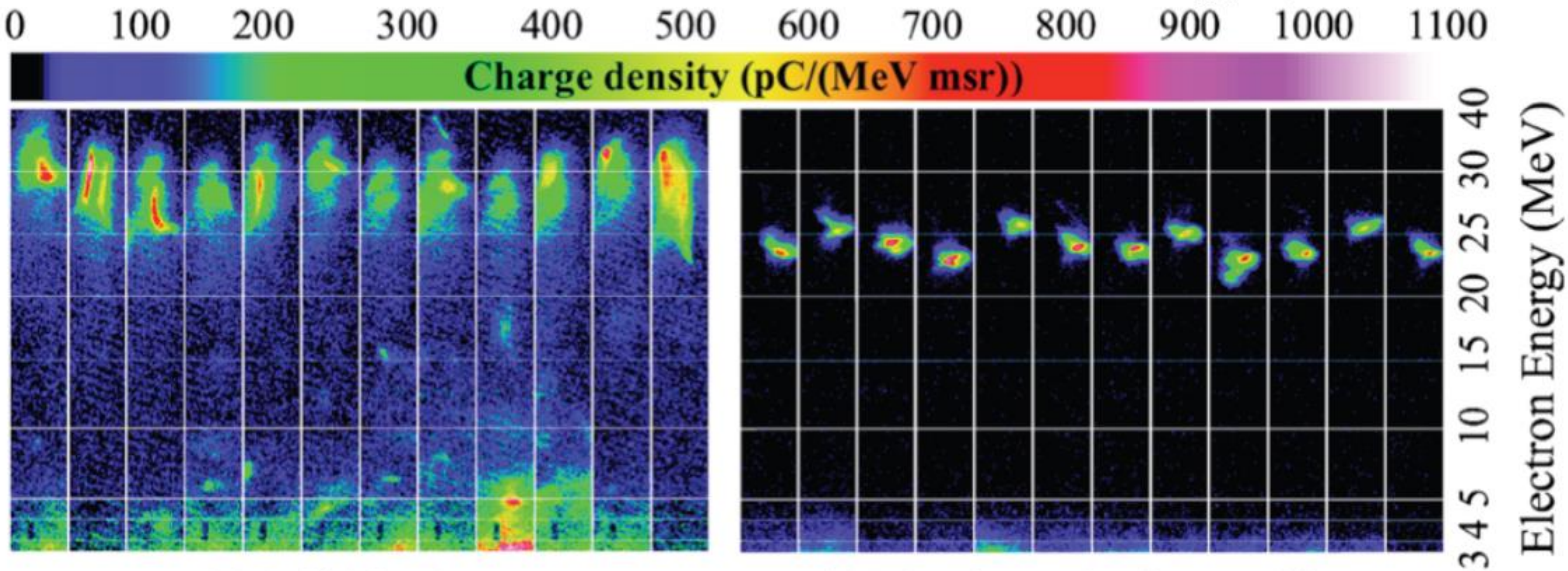




(a) Self injection

(b) Injection at density transition

K. Schmid et al., PRSTAB 13, 091301 (2010)

External electron injection

Idea:

separate the production and injection of electrons from the acceleration process ➡ similar as in conventional RF accelerators

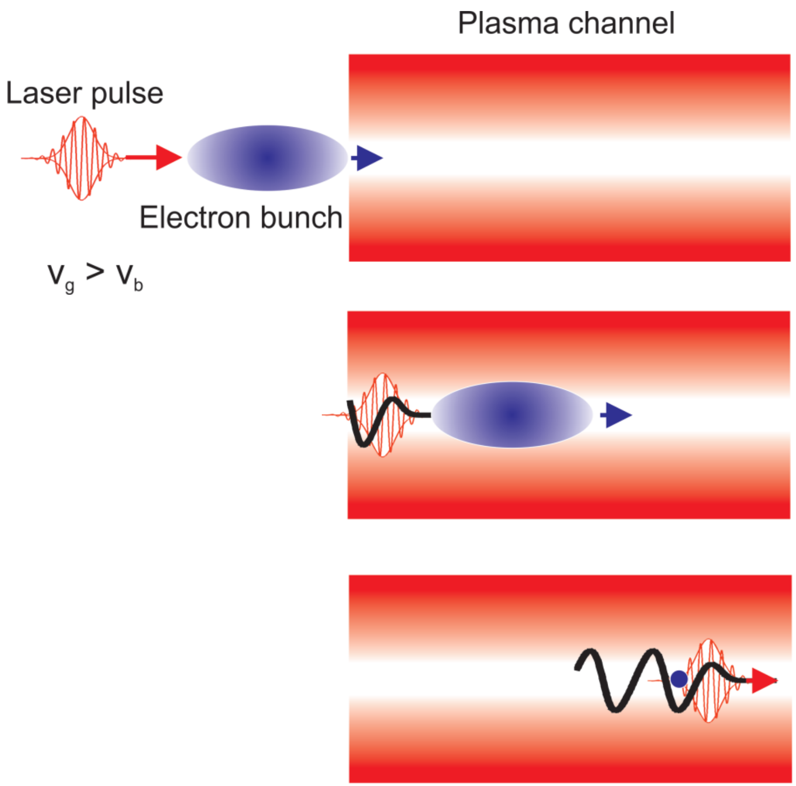
injector ⟶ booster

Injector:
- full control over electron generation
- well-characterized electron beam parameters:
  charge, emittance, energy and energy spread
- spatial and temporal control over injection
- RF photoinjector

Booster:
- linear to weakly nonlinear laser wakefield
- optical guiding for high intensity laser pulse
- operate at lower plasma density
- capillary discharge plasma channel

How to inject external electrons into the right phase of laser wakefield inside a booster stage ?

External electron injection



- Linear to weakly nonlinear wakefield
- No need ultra-short injected electron bunch
- No need fs synchronization
- No need precise transverse positioning
- Easy control over the injection time
- Scaleable to higher energies
- Promising candidate for a controlled acceleration

[1]A. G. Khachatryan *et. al Nucl.Instrum.Methods Phys. Res.A*, *566, 244 (2006),* A. G. Khachatryan *et. al., Phys. Rev. ST Accel. Beams 7, 121301 (2004),* A. G. Khachatryan, *Phys. Rev. E 65, 046504(2002)*, A. G. Khachatryan, *JETP Lett. 74, 371 (2001).*

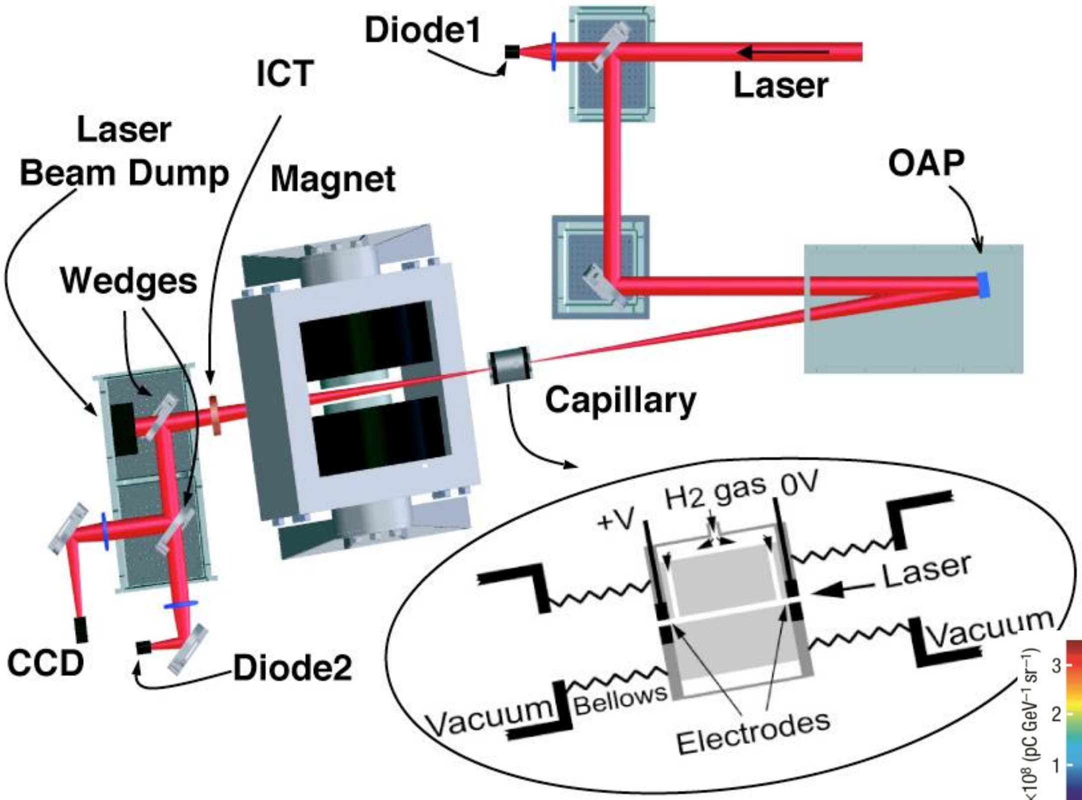
Recent progress in LWFA experiment

LETTERS

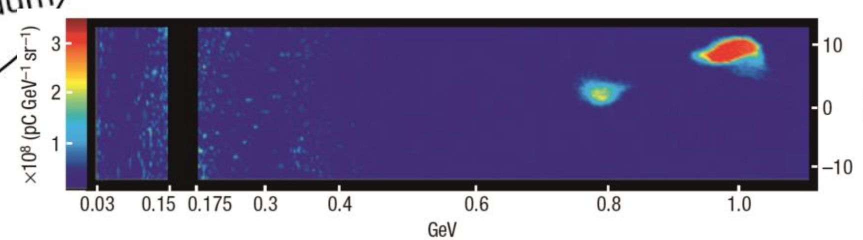
Nature Physics 2, 696(2006)

GeV electron beams from a centimetre-scale accelerator

W. P. LEEMANS[1*†], B. NAGLER[1], A. J. GONSALVES[2], Cs. TÓTH[1], K. NAKAMURA[1,3], C. G. R. GEDDES[1], E. ESAREY[1*], C. B. SCHROEDER[1] AND S. M. HOOKER[2]



- 40 TW laser pulse, plasma density $n_p = 4.3 \times 10^{18}$ cm^{-3}, 30 pC at 1 GeV with 2.5 % rms energy spread
- Acceleration distance < 3 cm, E_z > 33 GV/m



Energy scaling with laser power

ARTICLE

Received 2 Dec 2012 | Accepted 8 May 2013 | Published 11 Jun 2013

DOI: 10.1038/ncomms2988

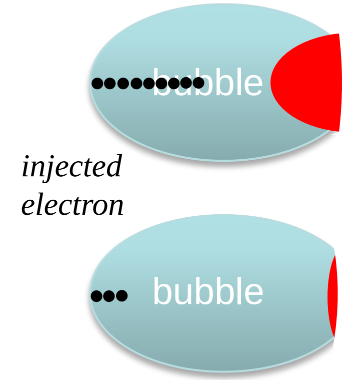
OPEN

Quasi-monoenergetic laser-plasma acceleration of electrons to 2 GeV

Xiaoming Wang[1], Rafal Zgadzaj[1], Neil Fazel[1], Zhengyan Li[1], S. A. Yi[1], Xi Zhang[1], Watson Henderson[1], Y.-Y. Chang[1], R. Korzekwa[1], H.-E. Tsai[1], C.-H. Pai[1], H. Quevedo[1], G. Dyer[1], E. Gaul[1], M. Martinez[1], A. C. Bernstein[1], T. Borger[1], M. Spinks[1], M. Donovan[1], V. Khudik[1], G. Shvets[1], T. Ditmire[1] & M. C. Downer[1]

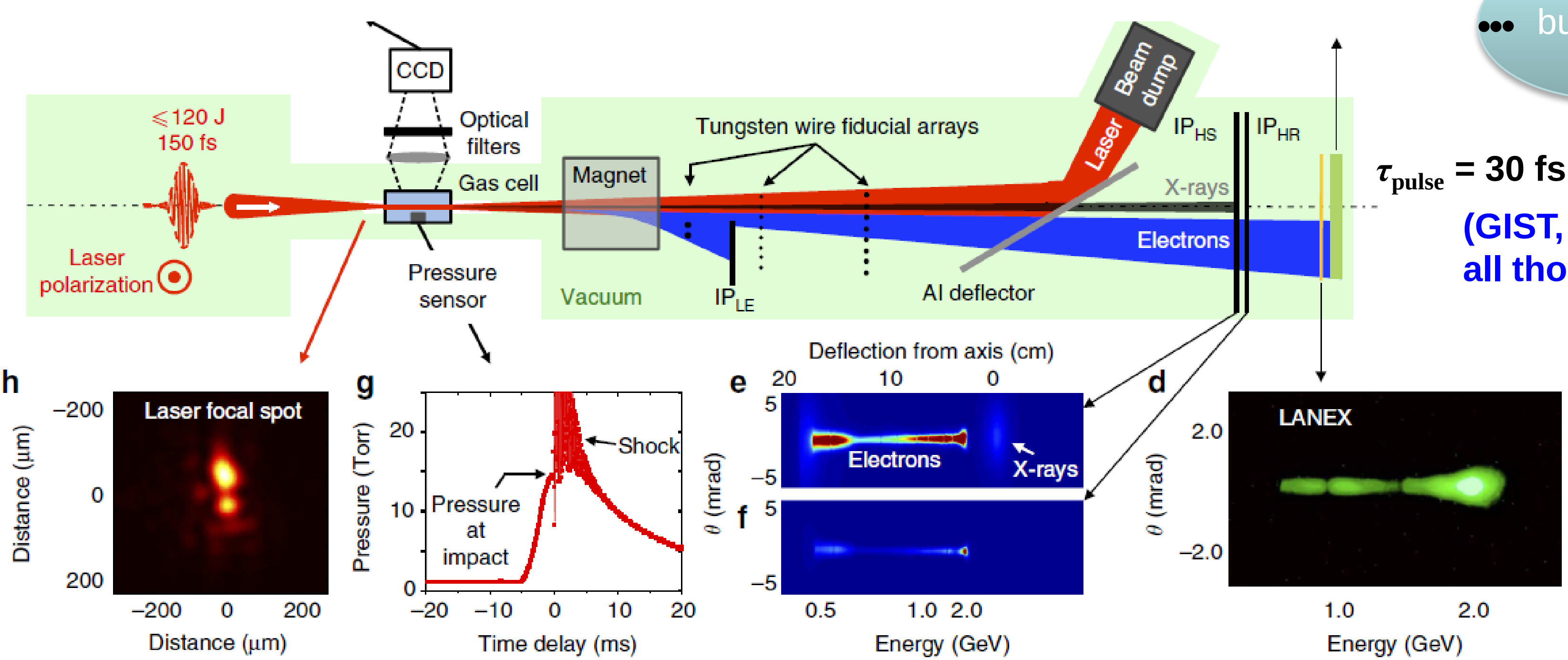
τ_{pulse} = 150 fs, 1 PW pulse

(Texas PW)



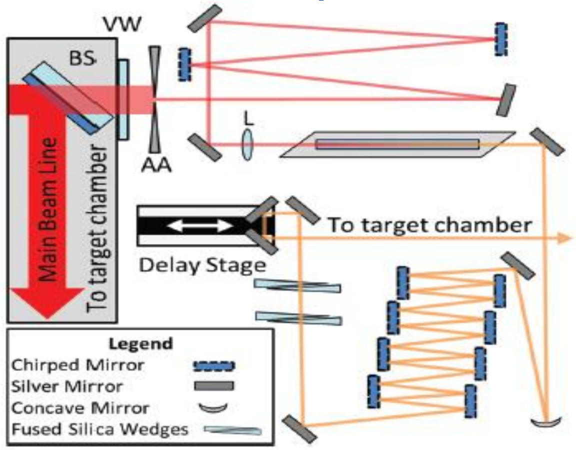
τ_{pulse} = 30 fs, 1 PW pulse

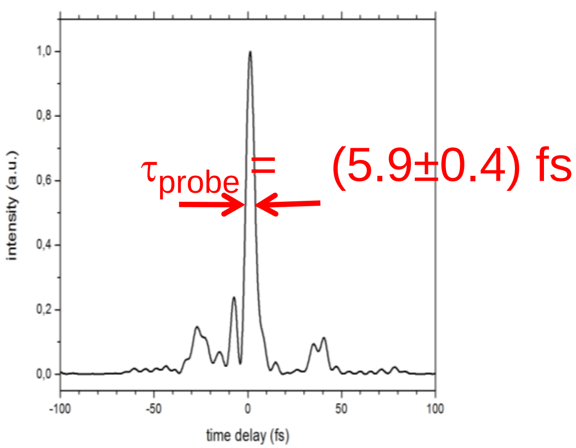
(GIST, LBNL and all those other guys)



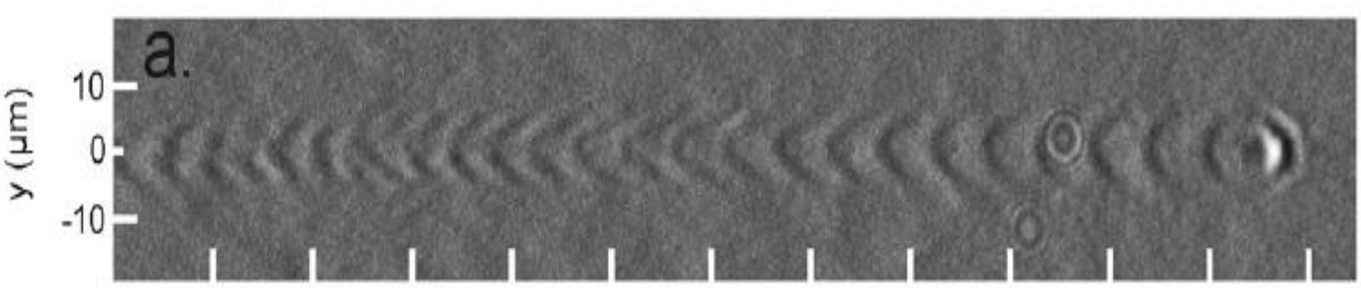
Visualization of laser wakefields

- Wakefield acceleration with 40-TW JETI-laser @ Jena, Germany: up to 0.8 J in 35 fs, f/13 focusing into H_2 gas jet
- Frequency-broadening of synchronized probe pulse in gas-filled hollow fiber + chirped-mirror compression





- 1.1 µm resolution with optimized imaging system



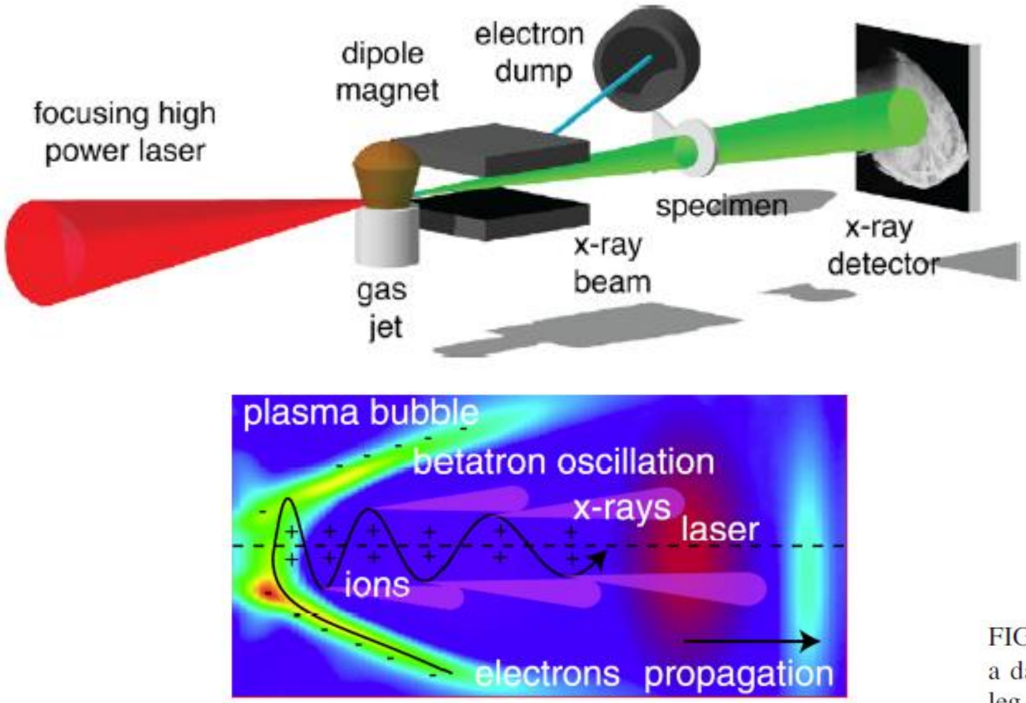
M. B. Schwab *et al.* Appl. Phys. Lett. (2013), A. Sävert *et al.* submitted (2014)

Snap-shots of non-linear, laser-driven plasma waves

APPLIED PHYSICS LETTERS **99**, 093701 (2011)

X-ray phase contrast imaging of biological specimens with femtosecond pulses of betatron radiation from a compact laser plasma wakefield accelerator

S. Kneip,[1,2,a)] C. McGuffey,[2] F. Dollar,[2] M. S. Bloom,[1] V. Chvykov,[2] G. Kalintchenko,[2] K. Krushelnick,[2] A. Maksimchuk,[2] S. P. D. Mangles,[2] T. Matsuoka,[2] Z. Najmudin,[1] C. A. J. Palmer,[1] J. Schreiber,[1] W. Schumaker,[2] A. G. R. Thomas,[2] and V. Yanovsky[2]
[1]*Blackett Laboratory, Imperial College London, London SW7 2AZ, United Kingdom*
[2]*Center for Ultrafast Optical Science, University of Michigan, Ann Arbor 48109, USA*



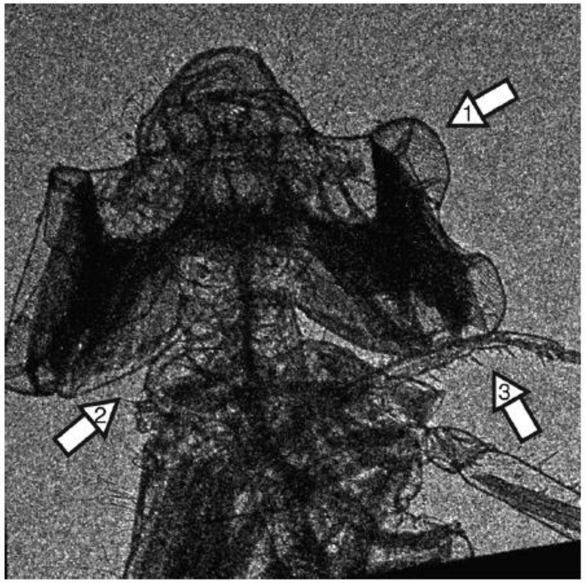
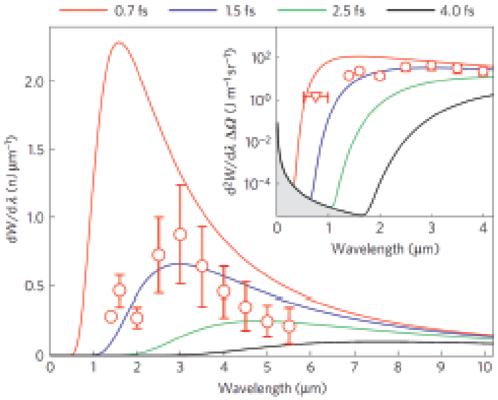


FIG. 3. Single shot 30 fs exposure x-ray phase contrast image of the head of a damselfly. Notice details of the compound eye (1), exoskeleton (2), and leg with hairs (3).

LWFA: unique bunch properties

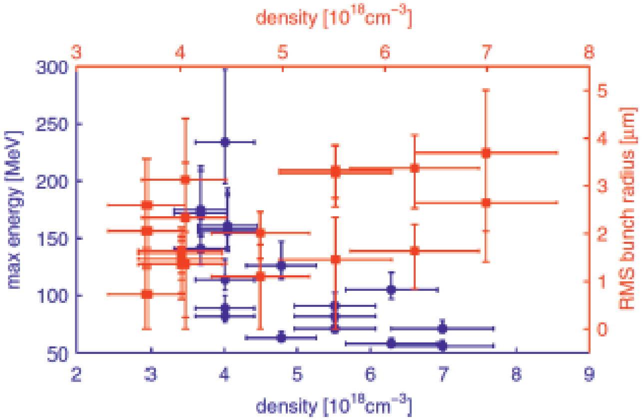
- Ultra-small size ($\sigma_{rms} << \lambda_p$) and ultra-short bunch duration ($\tau_{fwhm} << \lambda_p/c$)

CTR technique



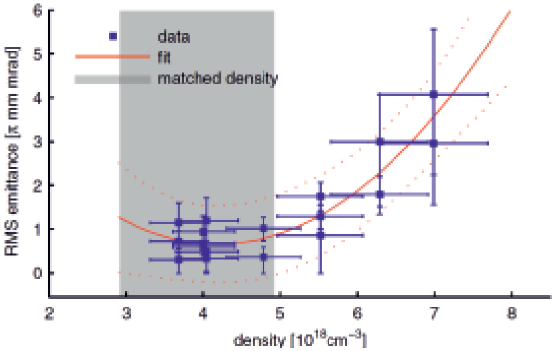
O. Lund, et.al., Nature Physics 7, 219(2011)

Betatron Xray source size



S.Kneip, et.al. Phys. Rev.ST. Accel. Beams., 15, 021302(2012)

- Normalized transverse emittance (ϵ_N<0.5 πmm-mrad) ~ linacs



S.Kneip, et.al. Phys. Rev.ST. Accel. Beams., 15, 021302(2012)

Main challenges in LWFA

- Improved control over electron injection into wakefields
  1. Coliding pulse mechanism (J.Faure, et.al.Nature 444,737(2006))
  2. Density down-ramp injection (C.G.R.Geddes,et.al.,Phys.Rev.Lettl,100,215004(2008))
- Improved beam stability: shot-to-shot reproducibility of charge, energy and energy spread, emittance
  1. Reduce the fluctuation of plasma's parameters: gas cell (J.Osterhoff ,et.al., Phys.Rev.Lett.,101,085002(2008))
  2. Full control over crucial laser's parameters:pulse's front tilt (A.Popp, et.al.,Phys.Rev.Lett.,105,215001(2010))
- Scaleability to higher energies
  1. Multi-staging technology (W.P.Leemans and E.Esarey, Phys.Today, March 2009)

Thank you for your attention!

Sachsen Zeitung

Self-consistent fluid model

The huge number of particles in plasmas

$\rightarrow$ impossible to solve Newton's equation for each particle

$\rightarrow$ hydrodynamic approach: study the motion of fluid elements

Main assumptions:

- plasma is fully ionized and initially at thermal equilibrium
- plasma is underdense: $\omega_0 >> \omega_{plasma}$
- ions are immobile
- plasma is cold: plasma electron thermal velocity << plasma wave phase velocity : $v_{th} << v_{ph}$

Towards master equations: starting point

Maxwell's equations
(EM fields)

$$\overline{\nabla}\cdot\overline{B} = 0$$ → Closed loop magnetic fields

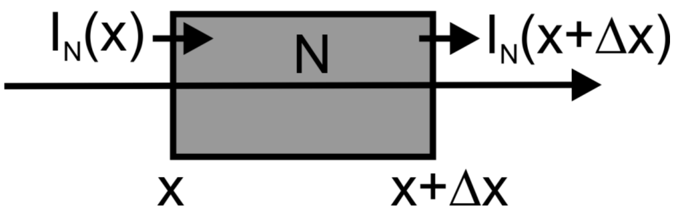
$$\overline{\nabla}\times\overline{E} = -\frac{1}{c}\frac{\partial\overline{B}}{\partial t}$$ → Faraday induction's law

$$\overline{\nabla}\cdot\overline{E} = 4\pi\rho$$ → Poisson's equation

$$\overline{\nabla}\times\overline{B} = \frac{1}{c}\left(4\pi\overline{j}+\frac{\partial\overline{E}}{\partial t}\right)$$ → Ampere's law

Continuity equation
(conservation of the number of particles)

$$\frac{\partial n_e}{\partial t}+\overline{\nabla}\cdot(n_e\overline{v})=0$$



Sources

$$\rho=-e(n_p-n_0)$$ → Charge density

$$\overline{j}=-en_p\overline{v_e}$$ → Current density

Lorentz equation
(motion of particles in an EM field)

$$\overline{F} = -e\left[\overline{E}+\left(\frac{\overline{v}}{c}\times\overline{B}\right)\right]$$

Towards master equations

HZDR

Fields can be described in term of potentials:

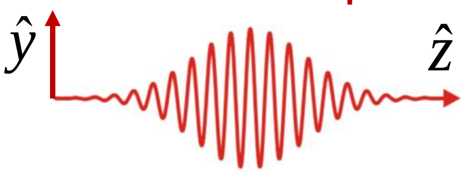
$$\mathbf{E} = -\nabla\varphi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t},$$
$$\mathbf{B} = \nabla \times \mathbf{A}.$$

where satisfies: $\nabla \cdot \mathbf{A} = 0$

In normalized parameters: $\phi = e\varphi / m_e c^2 \quad a = eA / m_e c^2$

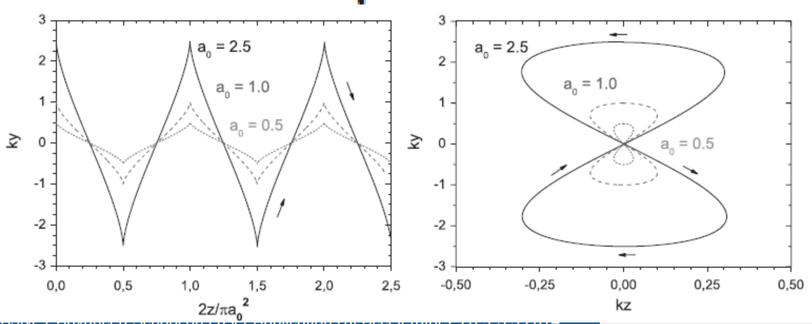
1-Dimensional case

Driver laser pulse



$$\mathbf{a}(z,t) = a_0 \exp\{-((z-ct)/\sigma_z)^2\}\,\hat{y}\exp\{-i(k_0 z - \omega_0 t)\},$$

$$a_0 = \frac{p_{y0}}{m_e c} = \sqrt{\frac{2e^2\lambda_0^2}{m_e^2 c^5 \pi} I_0},$$ $\rightarrow$ the strength of laser-plasma interaction



$a_0 \leq 1$ Non-relativistic regime

$a_0 > 1$ Relativistic regime(**v** x **B**)

λ_0=0.8 μm, a_0=1, I_0= 2.1×10^{18} W/cm²

Towards master equations

Laboratory frame → laser frame

$$z \qquad \xi = k_p\left(z - v_g t\right)$$

$$t \qquad \tau = \omega_p t$$

$$\frac{\partial}{\partial t} = \omega_p \frac{\partial}{\partial \tau} - k_p v_g \frac{\partial}{\partial \xi} \qquad \frac{\partial^2}{\partial t^2} = \left(k_p v_g \frac{\partial}{\partial \xi} - \omega_p \frac{\partial}{\partial \tau}\right)^2$$

$$\frac{\partial}{\partial z} = k_p \frac{\partial}{\partial \xi} \qquad \frac{\partial^2}{\partial z^2} = k_p^2 \frac{\partial^2}{\partial \xi^2}$$

Master equations

$$k_p \frac{\partial}{\partial \xi}\left[\gamma(1-\beta_g\beta_z)-\phi\right] = -\frac{\omega_p}{c}\frac{\partial}{\partial \tau}\gamma\beta_z,$$ momentum eq.

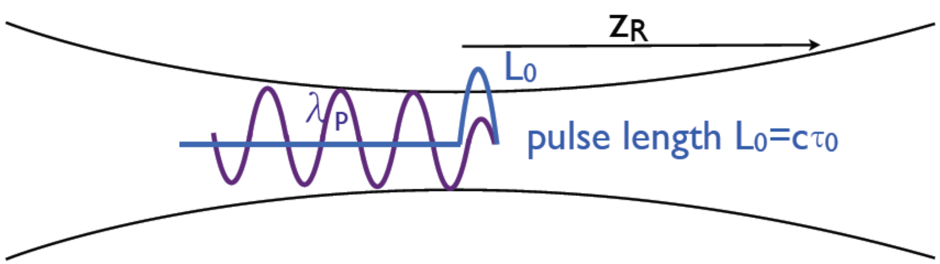
$$k_p \frac{\partial}{\partial \xi}\left[n(\beta_g-\beta_z)\right] = \frac{\omega_p}{c}\frac{\partial}{\partial \tau}n,$$ continuity eq.

$$\left[k_p^2(1-\beta_g^2)\frac{\partial^2}{\partial \xi^2}+2k_p\omega_p\frac{\beta_g}{c}\frac{\partial^2}{\partial \xi \partial \tau}-\frac{\omega_p^2}{c^2}\frac{\partial^2}{\partial \tau^2}\right]a = \beta_g^2 k_p^2 \frac{n}{n_e}\frac{a}{\gamma},$$ laser propagation in plasma eq.

$$\frac{\partial^2}{\partial \xi^2}\phi = \beta_g^2\left[\frac{n}{n_e}-1\right],$$ Poisson eq.

Quasi-static approximation:

- evolution time of the laser envelop >> the plasma response time
- this requires $\tau_{fwhm} \ll Z_R / c$



Master equations

With quasi-static approximation:

$$k_p \frac{\partial}{\partial \xi}\left[\gamma(1-\beta_g\beta_z)-\phi\right] = -\frac{\omega_p}{c}\frac{\partial}{\partial \tau}\gamma\beta_z \quad (=0),$$ momentum eq.

$$k_p \frac{\partial}{\partial \xi}\left[n(\beta_g-\beta_z)\right] = \frac{\omega_p}{c}\frac{\partial}{\partial \tau} n \quad (=0),$$ continuity eq.

$$\left[k_p^2(1-\beta_g^2)\frac{\partial^2}{\partial \xi^2} + 2k_p\omega_p\frac{\beta_g}{c}\frac{\partial^2}{\partial \xi \partial \tau} - \frac{\omega_p^2}{c^2}\frac{\partial^2}{\partial \tau^2}\right]a = \beta_g^2 k_p^2 \frac{n}{n_e}\frac{a}{\gamma},$$ laser propagation in plasma eq.

$$\frac{\partial^2}{\partial \xi^2}\phi = \beta_g^2\left[\frac{n}{n_e}-1\right],$$ Poisson eq.

1-D Laser wakefield equation

$$\frac{d^2\Phi}{d\xi^2} = \beta_g^2\gamma_g^2\left(\beta_g \frac{1}{\sqrt{1-\frac{1}{\gamma_g^2}\frac{1+a^2/2}{\Phi^2}}}-1\right), \qquad E_z = -\frac{1}{\beta_g^2}\frac{d\Phi}{d\xi}.$$

Try with Runge-Kutta

$$\gamma_g^2 = 1/(1-\beta_g^2) \qquad \Phi = 1+\phi$$

Normalized to $E_0 = m_e v_g \omega_p / e$