Laser Based Accelerator Diagnostics

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Talk Outline

- Recap description of EM pulses
  - Ultra-short pulses
  - Fourier relationship time-frequency
  - Effect of phase on pulse shape
- Longitudinal profile measurements
  - Overview of direct and radiative techniques
  - EO processes as nonlinear frequency mixing
  - Implementations of EO measurements
    - Spectral decoding
    - Temporal decoding
    - Spectral up-conversion
  - New scheme being developed in Daresbury
  - Alignment induced distortions in EO techniques
  - Methods to overcoming material bandwidth limitations
- Summary

Basic Description of an Ultra-short Pulse

Assuming linear polarisation we can construct a simple pulse:

$$E(t) = \underbrace{A(t)}_{\text{envelope}}\ \underbrace{\cos(\varphi(t))}_{\text{carrier}}$$

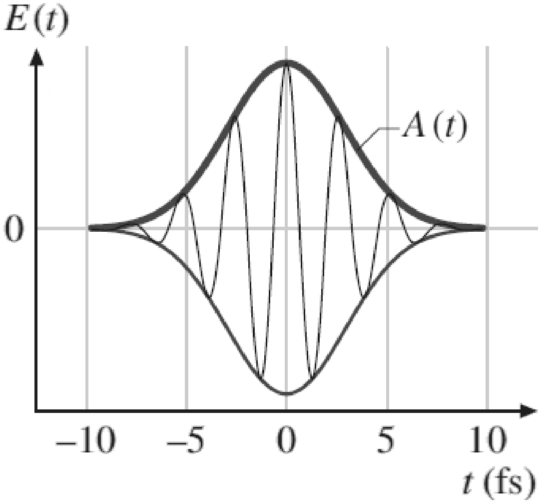
where

$$\begin{aligned}\varphi(t) = {} & \varphi_0 + \frac{d\varphi}{dt}t + \frac{1}{2}\frac{d^2\varphi}{dt^2}t^2 + \cdots \\ & \varphi_0 + \omega_o t + \frac{1}{2}\beta t^2 + \cdots\end{aligned}$$

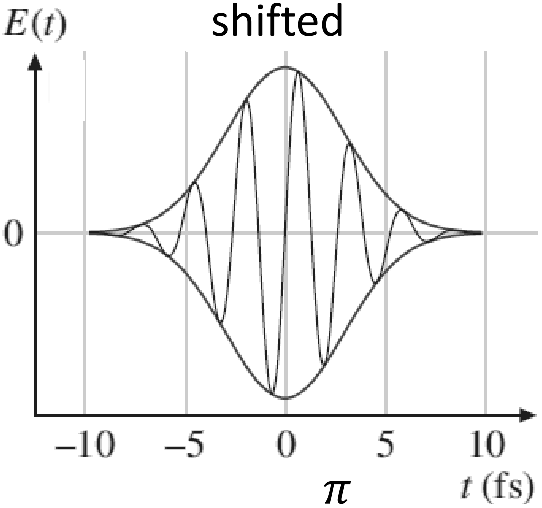
t, time. φ_0, absolute phase. ω_0, ‘carrier’ frequency.

Can define an instantaneous frequency $\dfrac{d\varphi(t)}{dt} = \omega_{\,inst} = \omega_o + \dfrac{1}{2}\beta t + \cdots$

Assuming a 800nm carrier wave with Gaussian envelope:

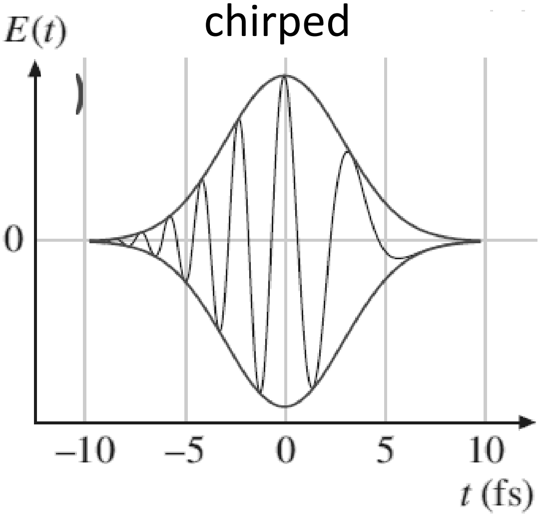


$\varphi_0 = 0$



shifted

$\varphi_0 = \dfrac{\pi}{2}$



chirped

$\beta \neq 0$

Basic Description of an Ultra-short Pulse

Often it is easier to deal with a complex field (e.g. for Fourier analysis)

$$\boldsymbol{E(t) = A(t)\,cos(\varphi(t))}$$

$$\boldsymbol{= \frac{1}{2}A(t)e^{i\varphi(t)} + c.c.}$$

Allows us to modify phase via multiplication by a complex number…

Usually use the complex amplitude, $\tilde{E}(t)$ to describe the pulse

$$\boxed{\tilde{E}(t) \propto A(t)e^{-i\varphi(t)}}$$

N.B. Carrier frequency removed

Amplitude (Real) **Phase (Complex)**

Fourier Relationship

Useful to swap between frequency and time descriptions:

- Generally, time domain is what we require knowledge of. Also, nonlinear processes easier to calculate.
  - *Convolution Theorem*: multiplication of time domain equivalent to a convolution in the frequency domain. FFT and multiplication can be more computationally efficient.
- Dispersion and propagation more easily analysed in frequency domain.

$$\boxed{\begin{aligned} \tilde{E}(t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{E}(\omega)\, e^{i\omega t} d\omega \\ \tilde{E}(\omega) &= \int_{-\infty}^{\infty} \tilde{E}(t)\, e^{-i\omega t} dt \end{aligned}}$$

Frequency Domain

Complex spectral amplitude, $\tilde{E}(\omega)$:

$$\boxed{\tilde{E}(\omega) = S(\omega)e^{i\varphi(\omega)}}$$

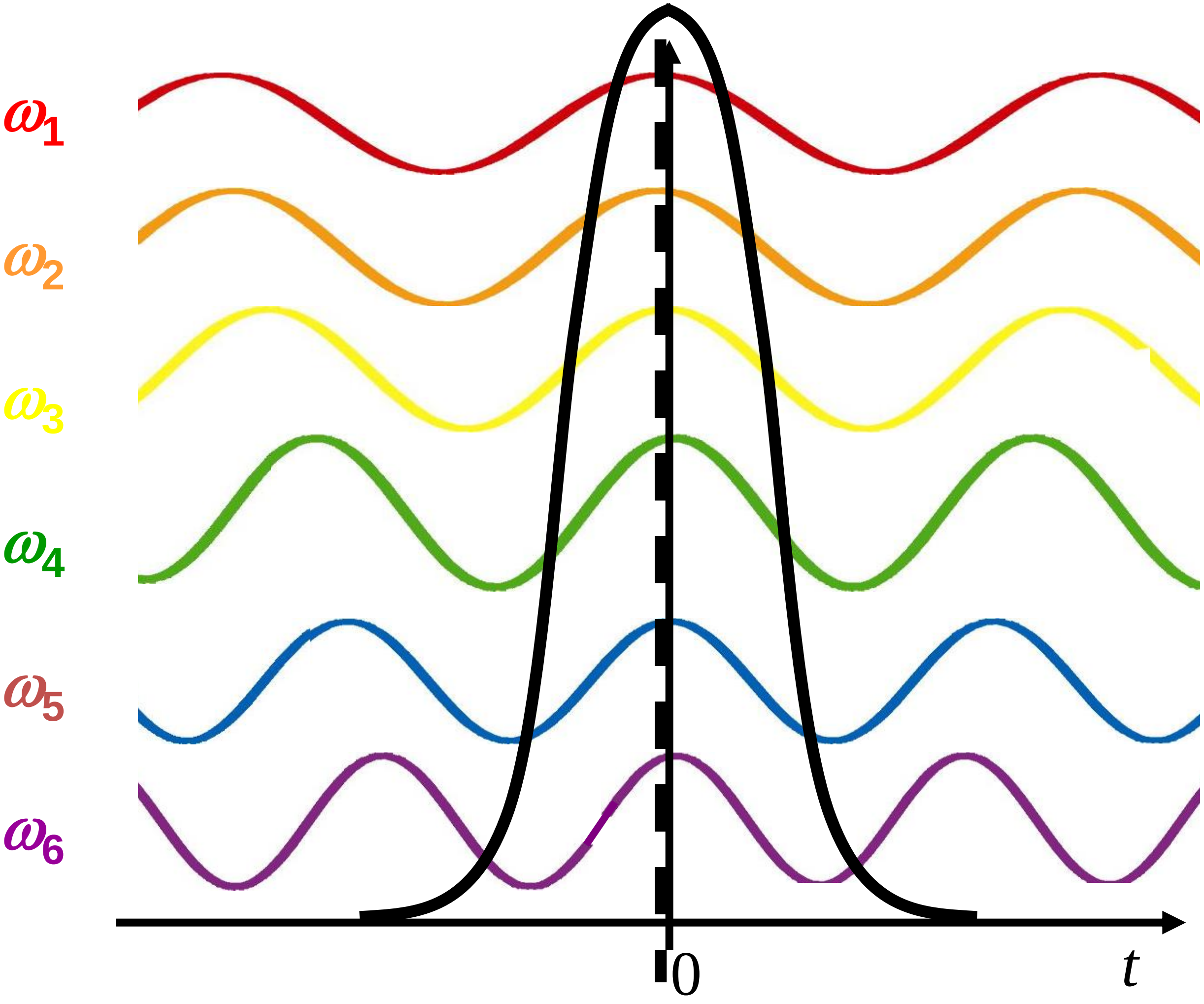
Similarly to the phase in time, It is helpful to consider the phase as a Taylor series

$$\varphi(\omega) = \underbrace{\varphi(\omega_0)}_{\text{“absolute”}} + \underbrace{\frac{d\varphi(\omega_0)}{d\omega}(\omega - \omega_0)}_{\text{“linear”}} + \underbrace{\frac{1}{2}\frac{d^2\varphi(\omega_0)}{d\omega^2}(\omega - \omega_0)^2}_{\text{“quadratic”}} + \underbrace{\frac{1}{6}\frac{d^3\varphi(\omega_0)}{d\omega^3}(\omega - \omega_0)^3}_{\text{“cubic”}} + \cdots$$

It is important to consider the effect of the spectral phase on the temporal profile

Effect of the Spectral Phase

The spectral phase is the phase of each frequency in the wave-form.



All of these frequencies have zero phase. So this pulse has:

$$\boxed{\varphi(\omega) = 0}$$

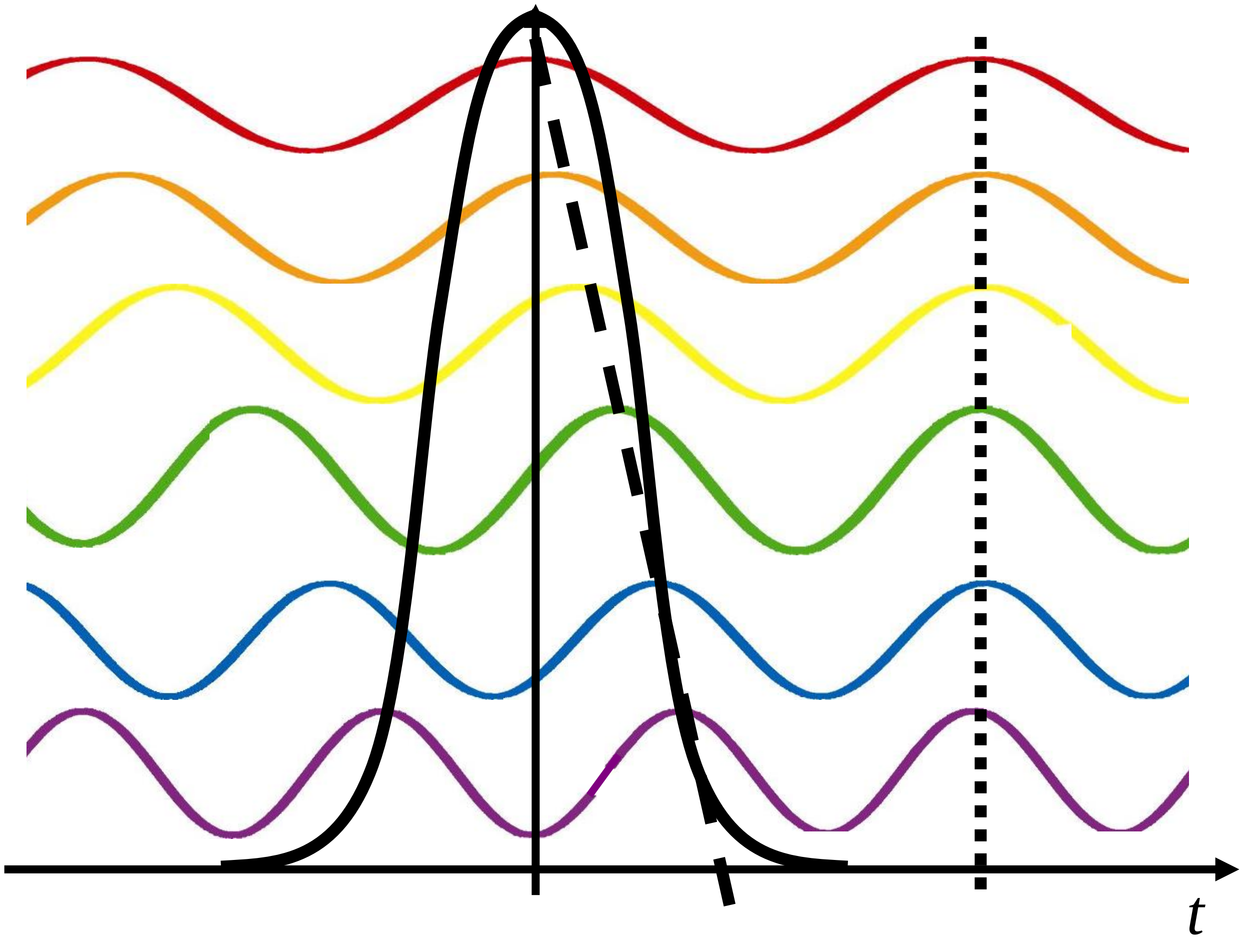
Note that this has constructive interference @ $t = 0$.

And it has cancellation everywhere else.

"Transform limited pulse" – cannot get any shorter for the given spectral content

Effect of the Spectral Phase

Now set a phase that varies with frequency : $\boxed{\varphi(\omega) = a\omega}$



$\varphi(\omega_1) = 0$

$\varphi(\omega_2) = 0.2\,\pi$

$\varphi(\omega_3) = 0.4\,\pi$

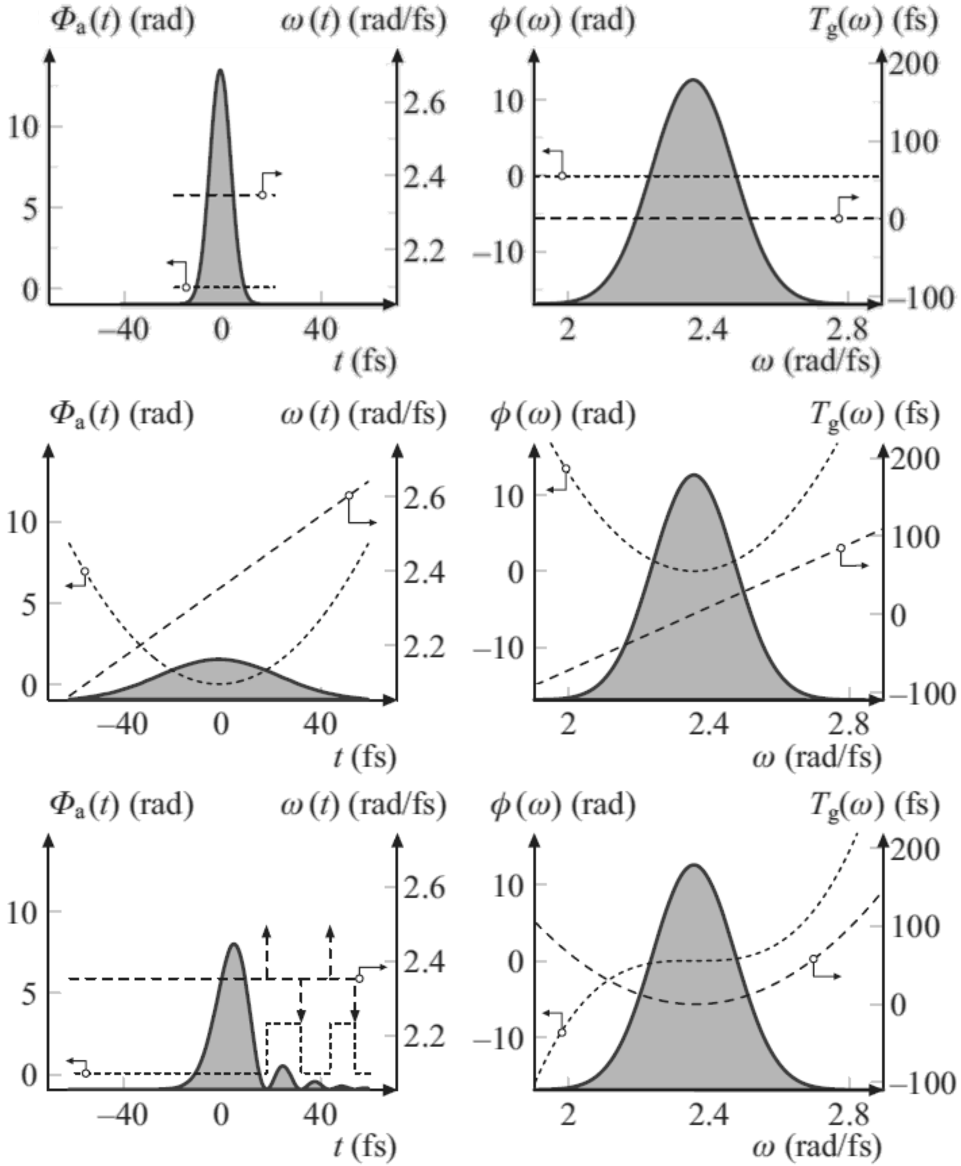
$\varphi(\omega_4) = 0.6\,\pi$

$\varphi(\omega_5) = 0.8\,\pi$

$\varphi(\omega_6) = \pi$

Effect of the Spectral Phase

Resulting Profile ← Spectrum



Transform Limited

Quadratic Phase

Creates a linear chirp, as seen earlier

"Group delay" $\dfrac{d\varphi(\omega)}{d\omega} = \tau_g$

Cubic Phase

The need for (femtosecond) longitudinal diagnostics

1. **Advanced Light Sources: 4th - 5th generation**

Free-Electron Lasers kA peak currents required for collective gain

τ = 200fs FWHM, 200pC (<2008, standard) ⇒ 10fs FWHM,10pC (>2008, increasing interest)

2. **Particle Physics:** Linear Colliders (ILC, CLIC) e^+-e^- and others
   short bunches, high charge, high quality - for *luminosity*
   - *~300fs rms, ~1nC stable, known (smooth?) longitudinal profiles*

3. **LPWAs:** Laser-plasma accelerators produce ultra-short electron bunches!
   - *1-5 fs FWHM (and perhaps even shorter in future), ~20pC + future FELs*

Diagnostics needed for…

- Verification of electron beam optics
- Machine tune-up & optimisation
- Machine longitudinal feedback (non-invasive)

Significant influence on bunch profile from

wakefields, space charge, CSR, collective instabilities… machine stability & drift

⇒ ***must have a single-shot diagnostic***

Two distinct classes of diagnostics

Grouped by similar physics and capabilities / limitations

Direct Particle Techniques

$\rho(t) \rightarrow \rho(x)$

longitudinal → transverse imaging

- Transverse Deflecting Cavities

$\rho(t) \rightarrow \rho(x') \rightarrow \rho(x)$

- RF zero-phasing

$\rho(t) \rightarrow \rho(\gamma) \rightarrow \rho(x)$

"Radiative" Techniques

$\rho(t) \rightarrow E(t)$

propagating & non-propagating

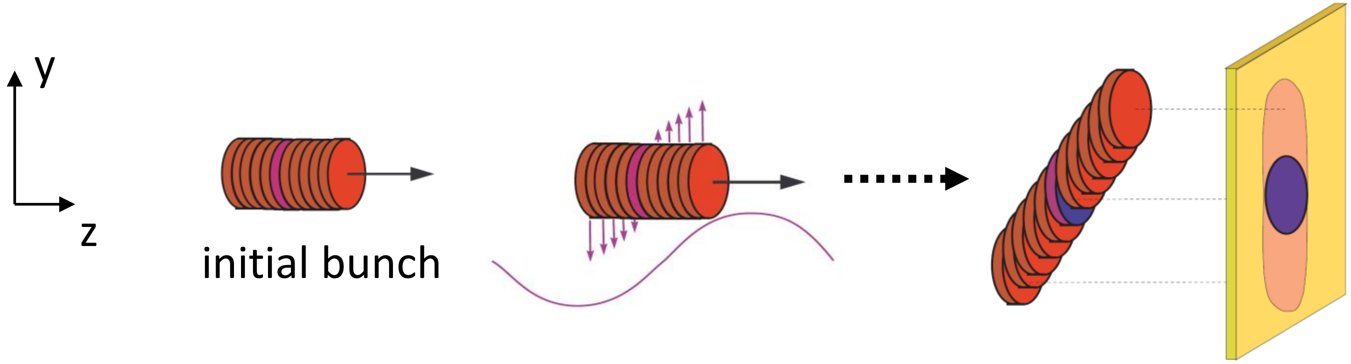
Spectral domain:

- CTR, CDR, CSR
  (spectral characterisation)
- Smith-Purcell
- Electro-Optic

Time domain:

- Electro-Optic
- Optical Replica/Transposition
- CTR, CDR (autocorrelation)

Transverse Deflecting Cavities (TDC)



cavity: transverse kick

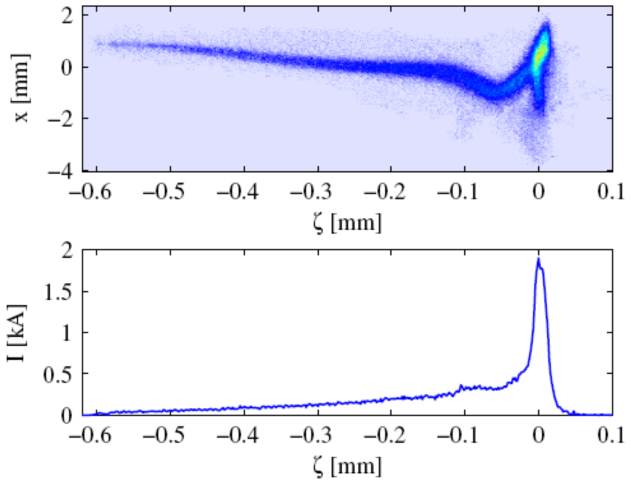
beam optics : transverse streak

$$\Delta y'_{\mathrm{cav}}(z) = \frac{eV}{pc}\sin(\frac{2\pi z}{\lambda_{\mathrm{cav}}} + \phi)$$

$$\Delta y_{\mathrm{screen}}(z) = \left\{\sqrt{\beta_{\mathrm{c}}\beta_{\mathrm{s}}}\,\sin(\Delta\psi)\right\}\,\Delta y'_{\mathrm{cav}}(z)$$

Time resolution scaling

$$\alpha \left\{ \begin{array}{l} \text{deflection gradient} \\ \gamma^{-1/2} \end{array} \right.$$



FLASH :

27 fs resolution

-improvements discussed tomorrow

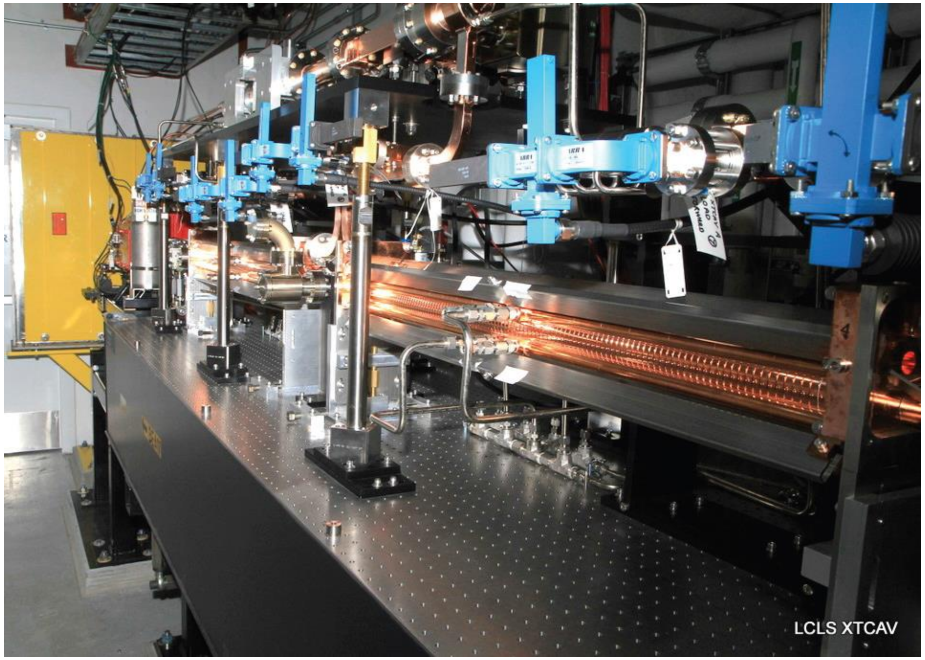
Diagnostic capabilities linked to beam optics

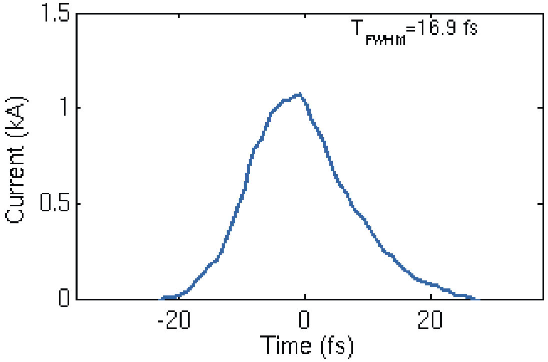
Disadvantage - destructive to beam

Rohrs et al. Phys Rev ST (2009)

LCLS XTCAV X-band transverse deflecting cavity

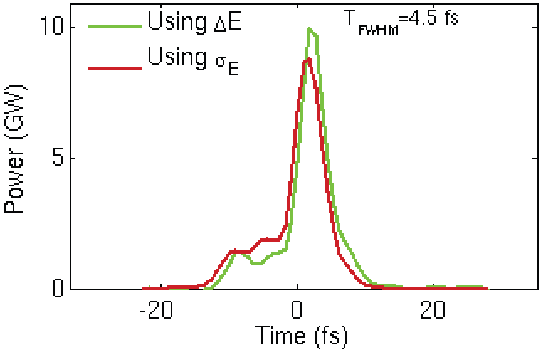
(Y. Ding et al, FEL 2013, NYC)





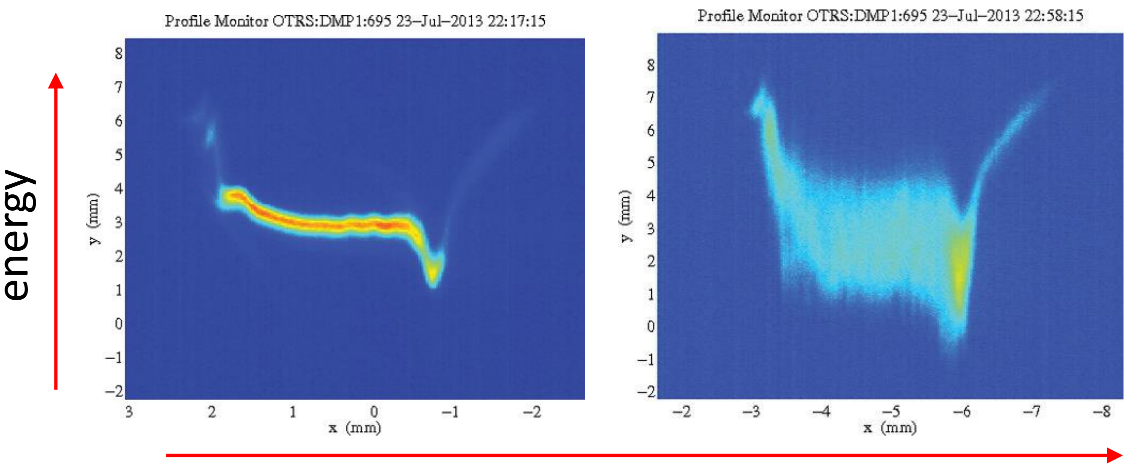
Electrons

20pC, 1keV photon energy examples



X-rays

Bunch head on the left



energy

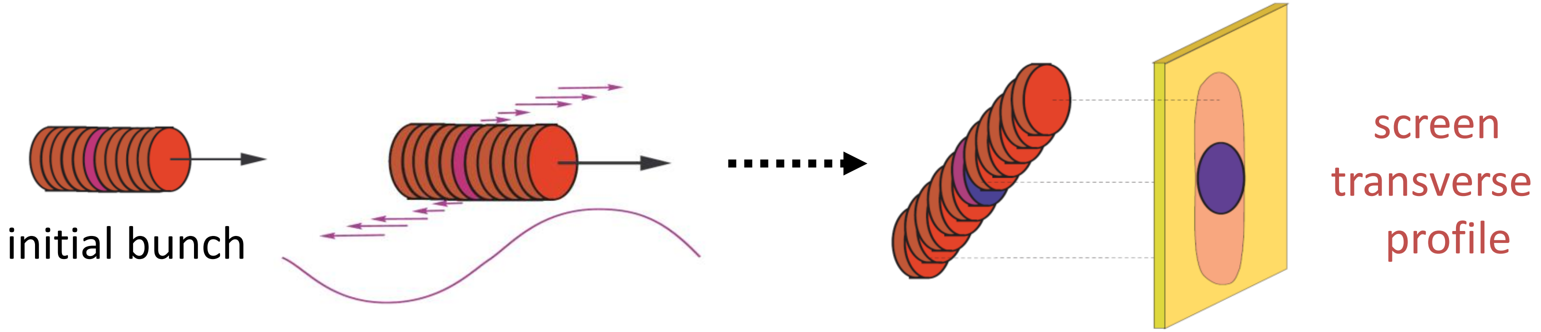
time

FEL-OFF

FEL-ON

(~1mJ pulse energy)

RF zero phasing



cavity:
z-dependent accel/deceleration

beam optics:
energy dispersion

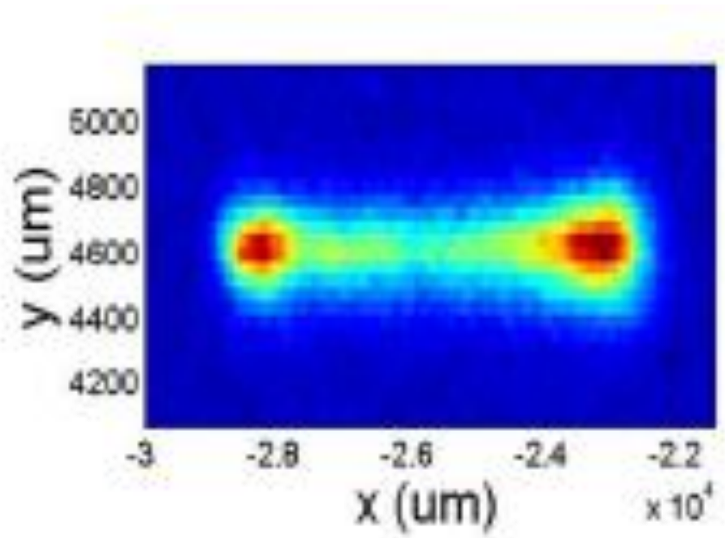
- Introduce energy chirp to beam via “linear” near-zero crossover of RF
- Measure energy spread with downstream spectrometer ⇒ infer initial bunch profile

time resolution dependent on:

- gradient of energy gain
- dispersion of spectrometer
- initial energy spread

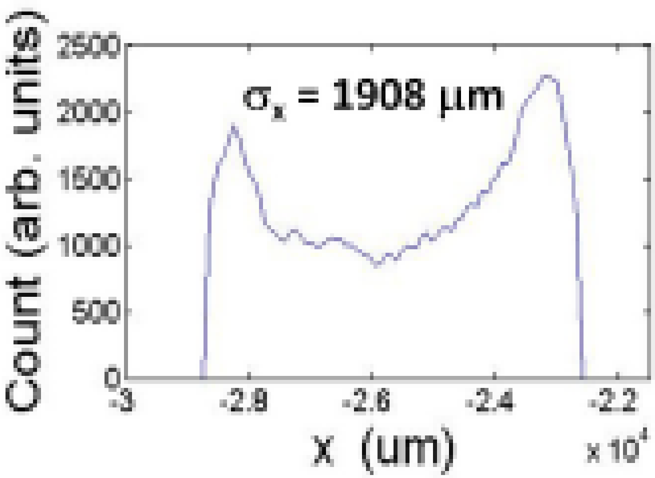
Disadvantage - destructive to beam

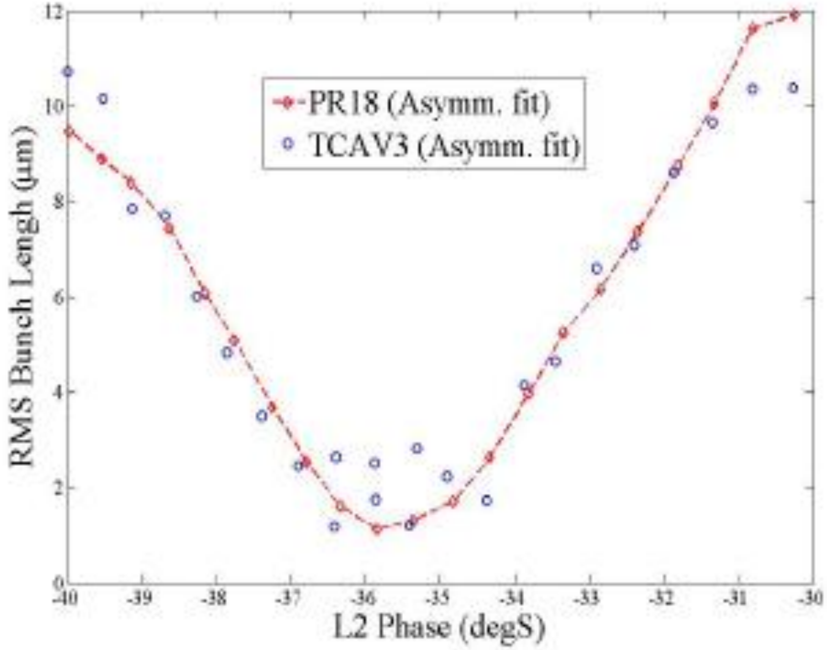
RF zero-phasing examples



SLAC LCLS: at 4.3 &14 GeV

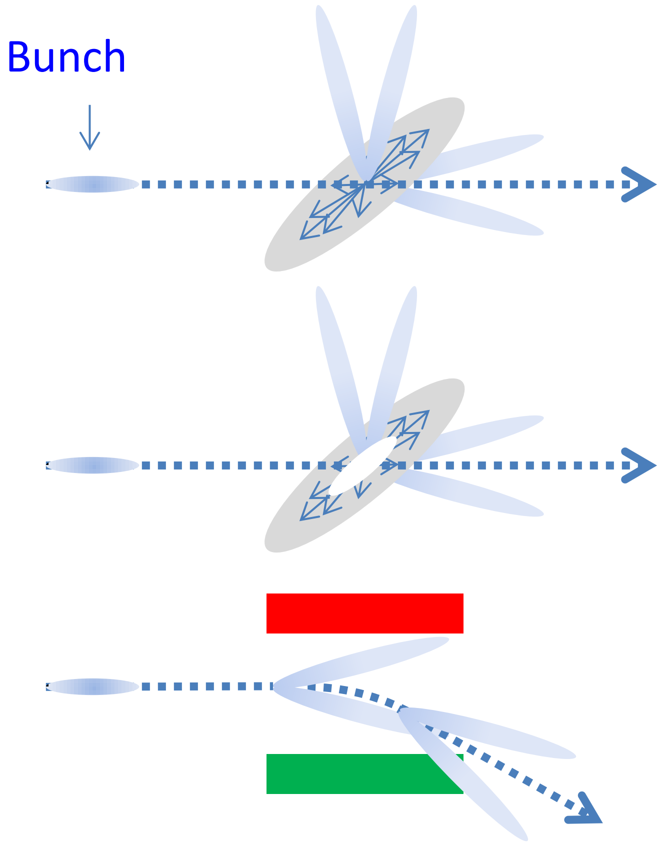
- 550m of linac at RF zero crossing!
- 6m dispersion on A-line spectrometer





Spectral domain radiative techniques

Radiation, emitted in cone (not TEM00!)



Coherent transition radiation (CTR)

Bunch field sets up currents which re-radiate
Can think of as a reflection of the Coulomb field
"destructive"

Coherent diffraction radiation (CDR)

Similar to CTR but with a hole in the screen
Can lose shorter wavelengths
Also Smith-Purcell radiation (SP) similar but extra complication due to interference

Coherent synchrotron radiation (CSR)

Or "edge" version, CER
Need to divert the beam!

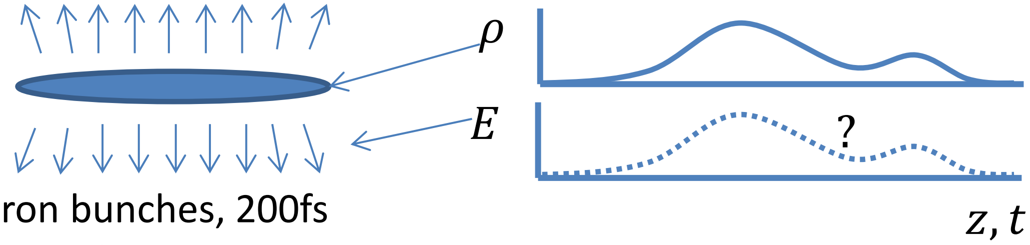
Bunch form factor $\Longrightarrow$ $F(\lambda) \equiv \left|\int_{-\infty}^{\infty} f(z) e^{-i\frac{2\pi z}{\lambda}} dz\right|^2$ $\Longrightarrow$ far-IR / mid-IR spectrum

Usually only spectrum measured, but temporal measurements possible (EO)…

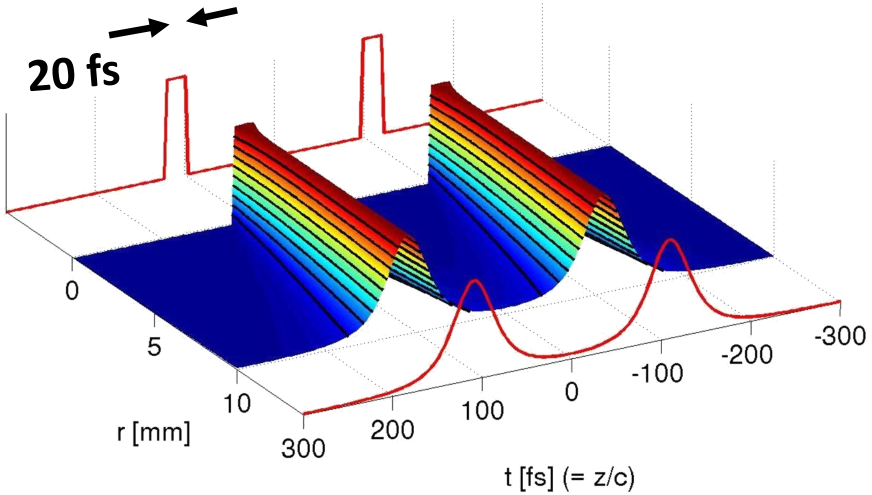
no direct detectors are fast enough!

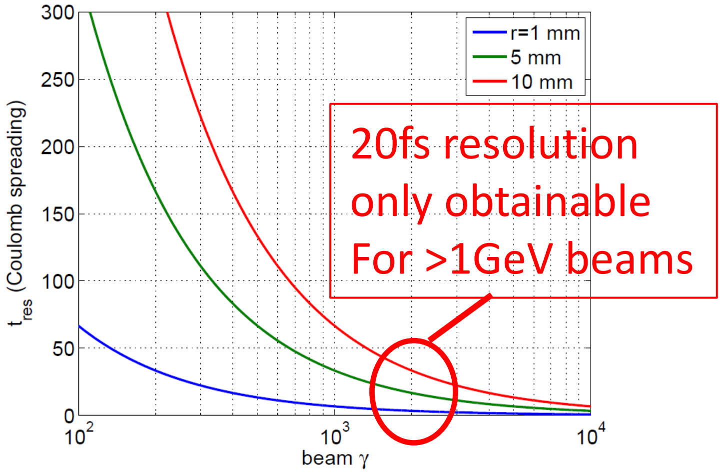
Common Problem - Field at Source

Field radiated or probed is related to Coulomb field near electron bunch



20fs electron bunches, 200fs separation, $\gamma = 1000$





20fs resolution only obtainable For >1GeV beams

High γ is an advantage!

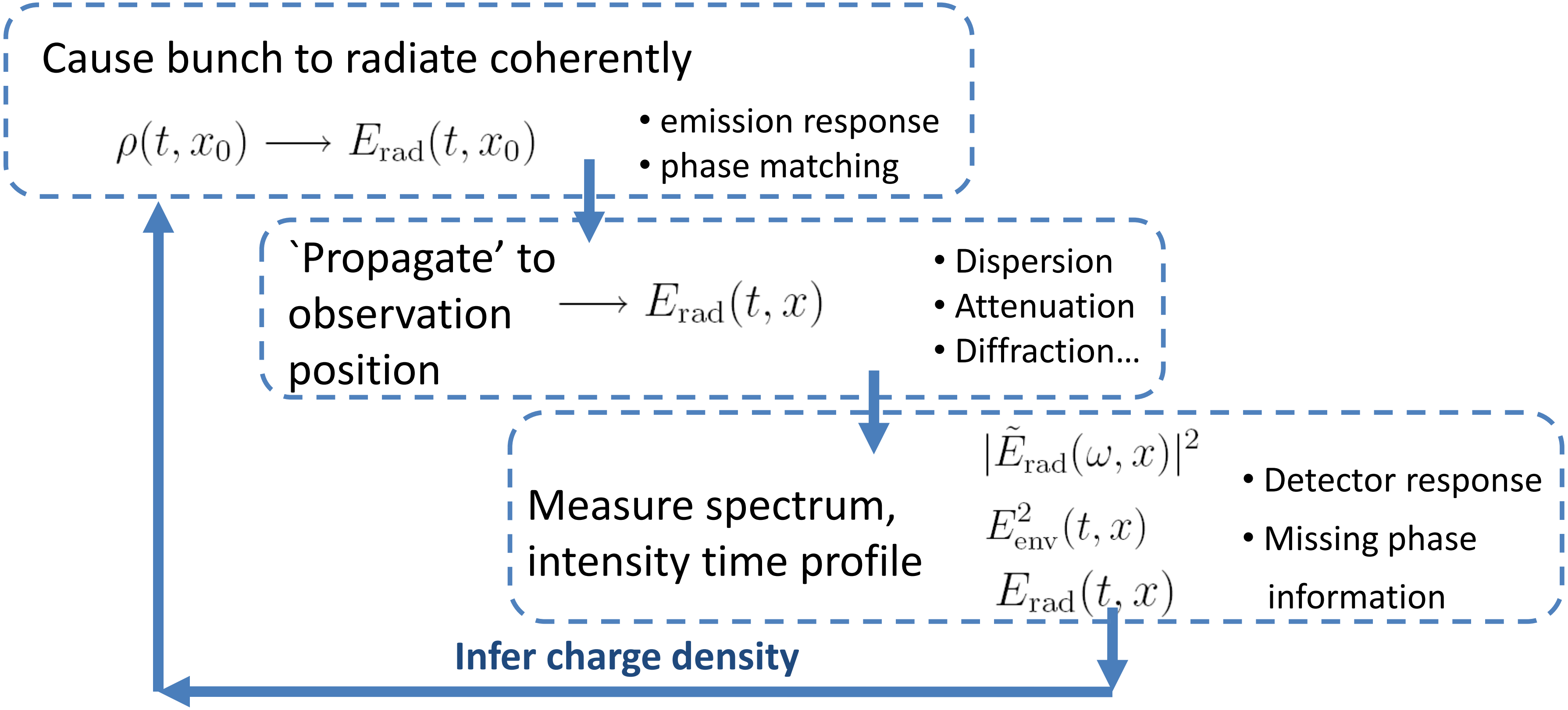
Time response & spectrum of field dependent on spatial position, R:

$$\delta t \sim 2R/c\gamma$$

$\Rightarrow$ ultrafast time resolution needs close proximity to bunch

(N.B. equally true of CTR, CDR, Smith-Purcell, Electro-Optic, etc.)

General Methodology for "Radiative" Techniques



Cause bunch to radiate coherently

$\rho(t, x_0) \longrightarrow E_{\mathrm{rad}}(t, x_0)$

- emission response
- phase matching

\`Propagate' to observation position $\longrightarrow E_{\mathrm{rad}}(t, x)$

- Dispersion
- Attenuation
- Diffraction…

Measure spectrum, intensity time profile

$|\tilde{E}_{\mathrm{rad}}(\omega, x)|^2$

$E^2_{\mathrm{env}}(t, x)$

$E_{\mathrm{rad}}(t, x)$

- Detector response
- Missing phase information

Infer charge density

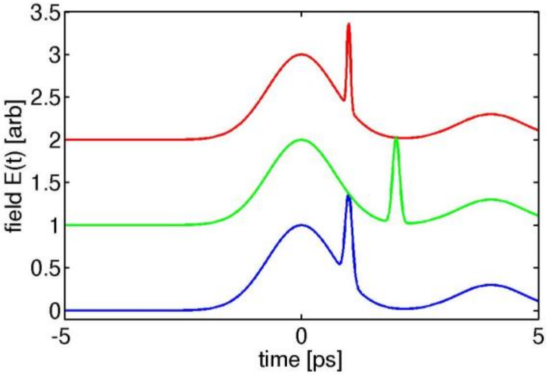
Techniques & limitations:

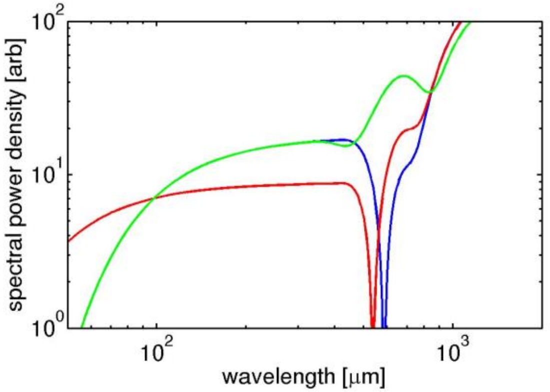
CSR/CTR : propagation effects; detector response; missing phase

CDR : as for CSR/CTR; plus emission response

Electro-Optic: detector response

Spectral domain radiative techniques





- More than an octave spanning in frequency
- Short wavelengths describe the fast structure
- Long wavelengths <u>required</u> for bunch reconstruction

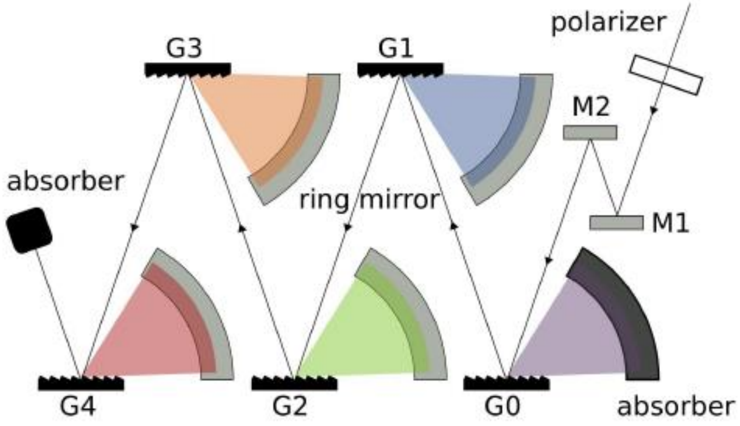
For: Simplicity (not always!)
Empirical machine information, real time
Information on fast and slow structure

Against: No explicit time profile
(but reconstruction *may* be possible)
Significant calibration issues

Need to consider diffraction effects and Gouy phase shifts

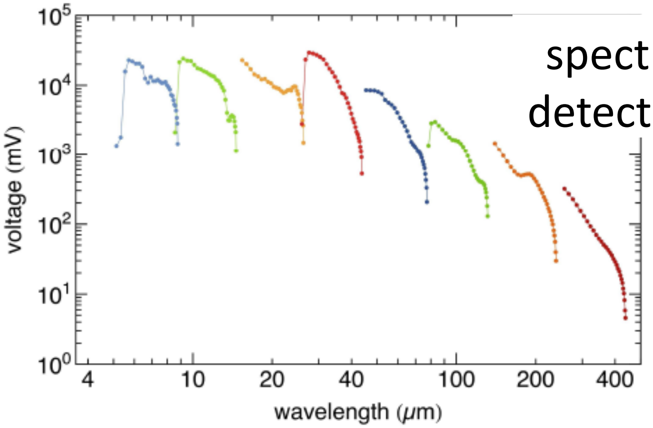
Good example: single shot CTR spectrometer at FLASH

cascaded dispersive grating elements, and pyro-electric detector arrays



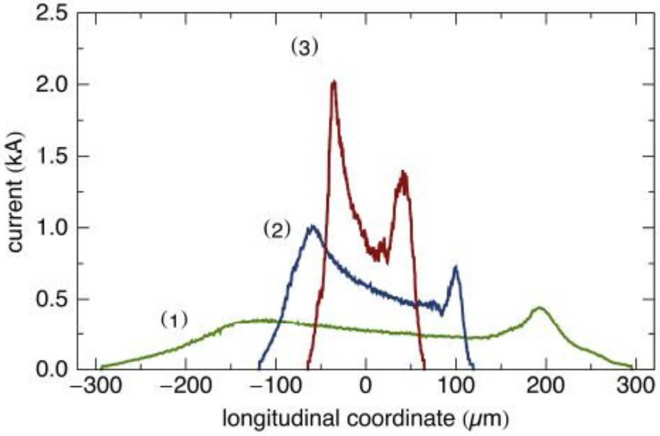
Cannot just use a single grating!

E. Hass et al., Proc. SPIE 8778, May 2013

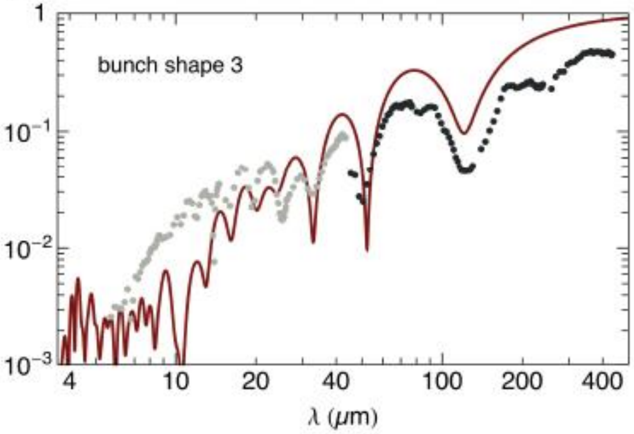


spectrometer & detector response

Deflecting cavity bunch profiles



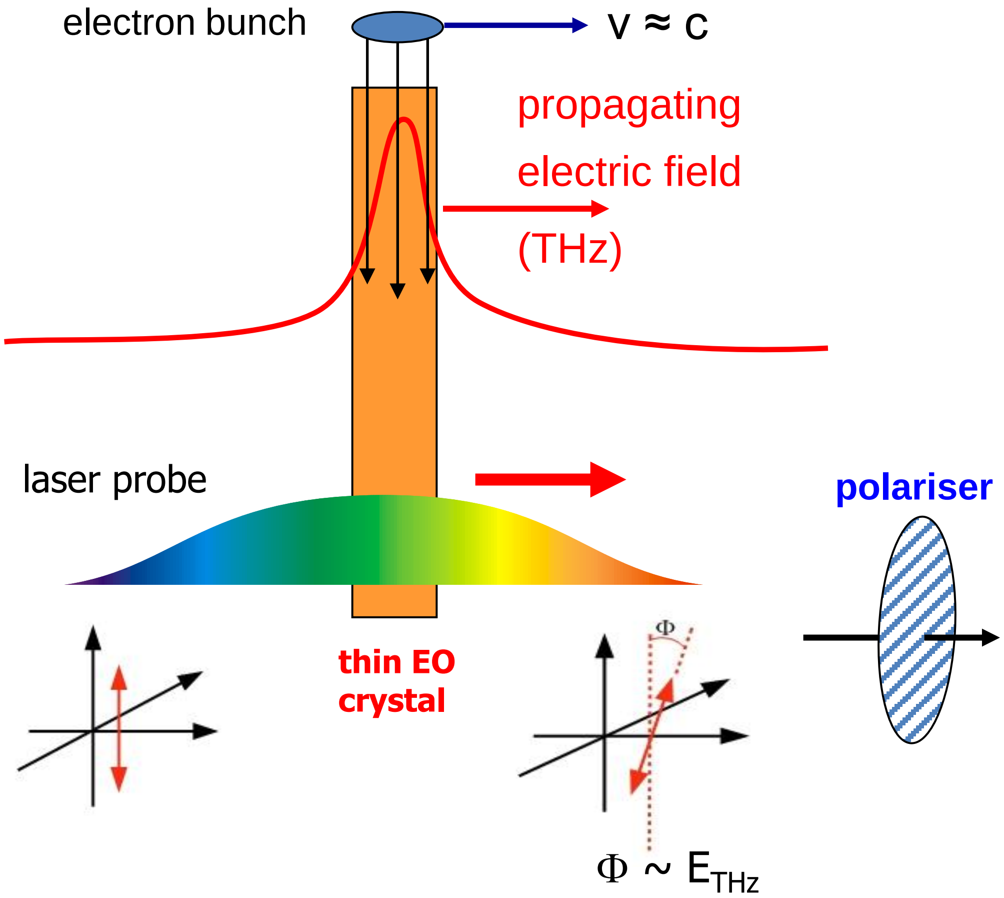
Measured & calculated spectra



Similar concepts applied at HZDR ELBE facility (O. Zarini et al, LA^{3}NET workshop, Dresden, April 2014) and at SLAC LCLS (T. J. Maxwell et al, PRL 111, 184801, 2013)

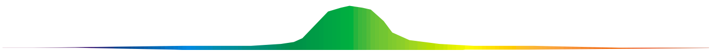
Q) How can we measure the time profile unambiguously?
A) Electro-Optic Measurements

Encode Coulomb field on to an optical probe pulse - from Ti:Sa or fibre laser



Decoding: several options available now that we are dealing with an optical pulses! depends on resolution required

Can obtain the temporal variations in a single laser pulse

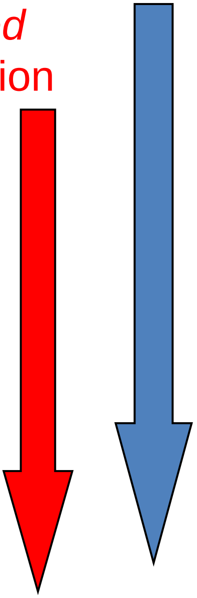


Detect polarisation rotation proportional to E or E^2, depending on set-up

*(allows **all-optical (intra-beamline)** pickup of relativistic bunch Coulomb field)*

Range of Electro-Optic Techniques

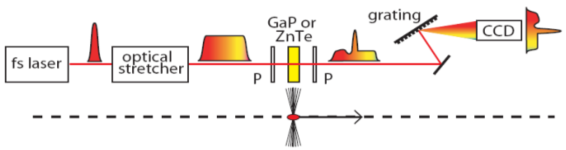
Variations in read-out of optical temporal signal



complexity

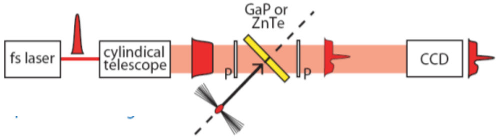
demonstrated
time resolution

Spectral Decoding



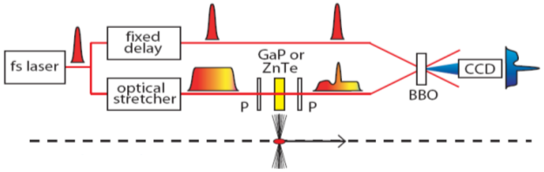
- Chirped optical input
- Spectral readout
- Use time-wavelength relationship

Spatial Encoding



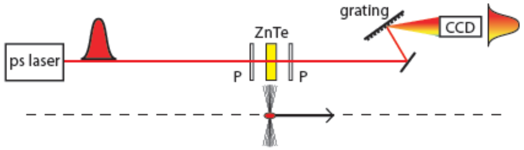
- Ultrashort optical input
- Spatial readout (EO crystal)
- Use time-space relationship

Temporal Decoding



- Long pulse + ultrashort pulse gate
- Spatial readout (cross-correlator crystal)
- Use time-space relationship

Spectral Upconversion/ EO Transposition



- quasi-monochomatic optical input (long pulse)
- Spectral readout
- Use FROG-related techniques to recover bunch info

The Physics of EO Encoding

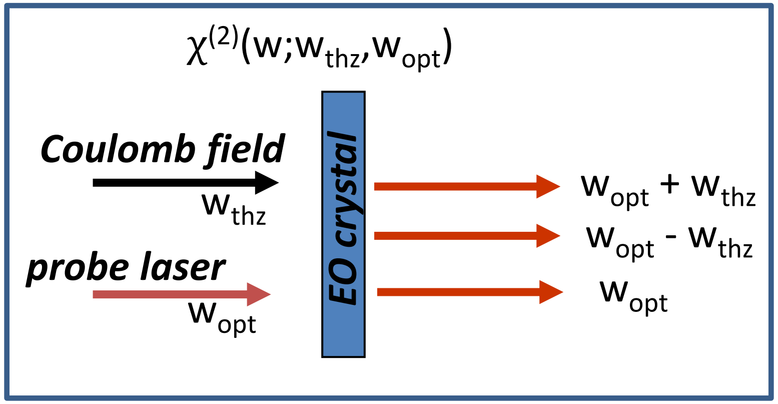
Standard Description

Pockels effect induces a phase change which is detected via polarization measurements. Assumes THz pulse has small bandwidth w.r.t. probe.

This is not true for short bunches!
A common misconception.

More Rigorous Description – nonlinear frequency mixing

- No assumptions made on bunch profile or on laser probe
- Dispersion straightforward in frequency domain



Non-linear response in EO crystal

$$P(\omega_1+\omega_2)=\chi^{(2)}E_1(\omega_1)E_2(\omega_2)$$

Convolve over all combinations of optical and Coulomb frequencies

$$P(\omega)=\int\chi^{(2)}E_1(\omega_1)E_2(\omega-\omega_1)\mathrm{d}\omega_1$$

The Physics of EO Encoding

Wave equation for $\chi^{(2)}$ frequency mixing

non-linear properties

$$\left[\frac{\partial}{\partial z}+\beta^{\mathrm{opt}}(\omega)\right]\widetilde{A}(\omega, z) = \frac{\mathrm{i}\omega}{2c\eta}\times\int_{-\infty}^{\infty}\mathrm{d}\Omega\,\chi^{(2)}(\omega;\Omega,\omega-\Omega)$$
$$\times\underbrace{\exp[\mathrm{i}\Delta k(\Omega,\omega)z-\beta^{\mathrm{THz}}(\Omega)z]}_{\text{linear material properties}}\widetilde{A}_{\mathrm{THz}}(\Omega)\widetilde{A}(\omega-\Omega, z)\,.$$

sum & difference mixing included

Coulomb / THz field

input optical field

Simple solution within small signal approximation...

$$\boxed{\widetilde{A}(\omega, z)=\widetilde{A}_0(\omega)\mathrm{e}^{-z\beta_{\mathrm{opt}}}+\frac{\mathrm{i}}{2c\eta}\mathrm{e}^{-z\beta_{\mathrm{opt}}}\omega\int\mathrm{d}\omega'\,\widetilde{A}^{\mathrm{THz}}_{\mathrm{eff}}(\omega-\omega')\widetilde{A}(\omega')\,,}$$

where material properties define an “effective” THz field....

$$\widetilde{A}^{\mathrm{THz}}_{\mathrm{eff}}(\omega)\equiv\widetilde{A}^{\mathrm{THz}}(\omega)\chi^{(2)}(\omega)\left[\frac{\exp(\mathrm{i}\Delta\widetilde{k}(\omega,\omega^{\mathrm{opt}})z)-1}{i\Delta\widetilde{k}(\omega,\omega^{\mathrm{opt}})}\right]$$

Very general... describes CW, ultrafast transform limited and arbitrarily chirped pulses

Jamison et al.
Opt. Lett 31 1753 (2006)

The Physics of EO Encoding

Simplified forms:

Frequency domain

$$\tilde{E}^{\mathrm{opt}}_{\mathrm{out}}(\omega) = \tilde{E}^{\mathrm{opt}}_{\mathrm{in}}(\omega) + i\omega a\tilde{E}^{\mathrm{opt}}_{\mathrm{in}}(\omega) * \left[\tilde{E}^{\mathrm{Coul}}(\omega)\tilde{R}(\omega)\right]$$

geometry dependent (repeat for each principle axis)

optical probe spectrum (complex)

convolution over all combinations of optical and Coulomb frequencies

THz spectrum (complex)

propagation & nonlinear efficiency

Time domain

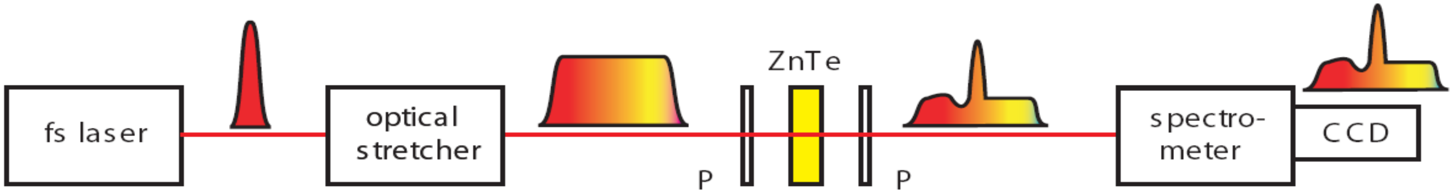
$$\begin{array}{ccc} E_{\mathrm{x}}(t) & & \tilde{E}_{\mathrm{x}}(\omega) \\ & \rightarrow & \\ E_{\mathrm{y}}(t) & & \tilde{E}_{\mathrm{y}}(\omega) \end{array}$$

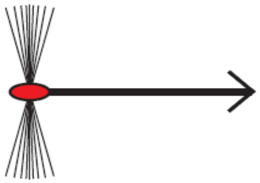
$$E^{\mathrm{opt}}_{\mathrm{out}}(t) = E^{\mathrm{opt}}_{\mathrm{in}}(t) + a\left[E^{\mathrm{Coul}}(t) * R(t)\right]\frac{\mathrm{d}}{\mathrm{d}t}E^{\mathrm{opt}}_{\mathrm{in}}(t)$$

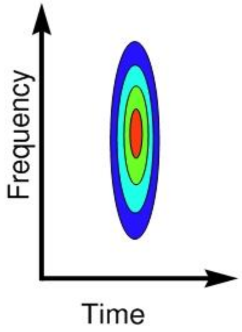
S.P. Jamison Opt. Lett. v**31** no.11 p1753

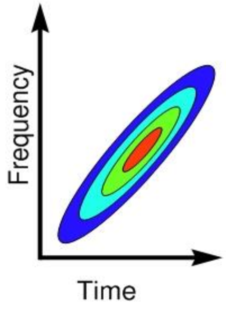
Spectral Decoding

Apply instantaneous-frequency chirp to probe to produce a ω↔t mapping

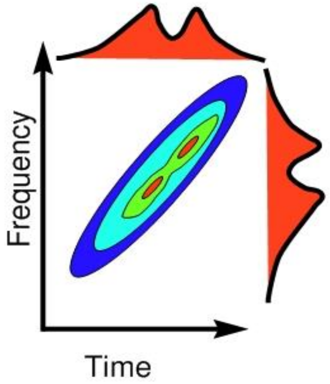








$$\tau \equiv \beta(\omega - \omega_0)$$



works well for "slow" modulations

fast modulation $\Longrightarrow$ broad bandwidth

- Measure probe intensity I(λ)
- known (initial) λ(t)

$\Rightarrow$ infer I(t)

very fast modulations destroy initial frequency-time map

Spectral Decoding Resolution

Under restrictions, the convolution in the EO effect has the mathematical form of a Fourier transform

Consider (positive) optical frequencies from mixing

$$\tilde{M}(\omega) = \int_{-\infty}^{\infty} \mathrm{d}\Omega\, \tilde{E}_{\mathrm{opt}}(\omega - \Omega)\tilde{E}_{THz}(\Omega)$$

Positive and negative Coulomb (THz) frequencies allowed - sum and diff mixing

Linear chirped pulse:

$$\tilde{E}_{\mathrm{opt}}(\omega) = A(\omega)\exp(-i\beta(\omega-\omega_0)^2).$$

Assume A(w) varying slowly over bunch frequency span

$$\tilde{M}(\omega) = \exp(-i\beta(\omega-\omega_0)^2)A(\omega)\int \exp(-i\beta\Omega^2)\tilde{E}_{THz}(\Omega)\mathrm{e}^{i\Omega\tau}$$

$$\boxed{\tau \equiv \beta(\omega - \omega_0)}$$

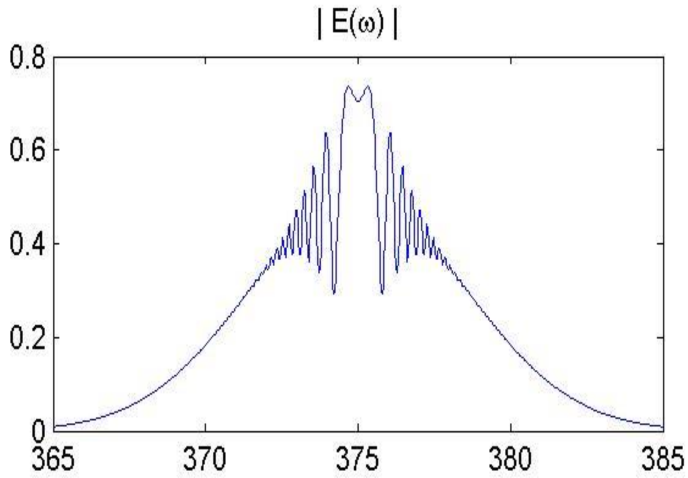
delay to frequency map from chirp

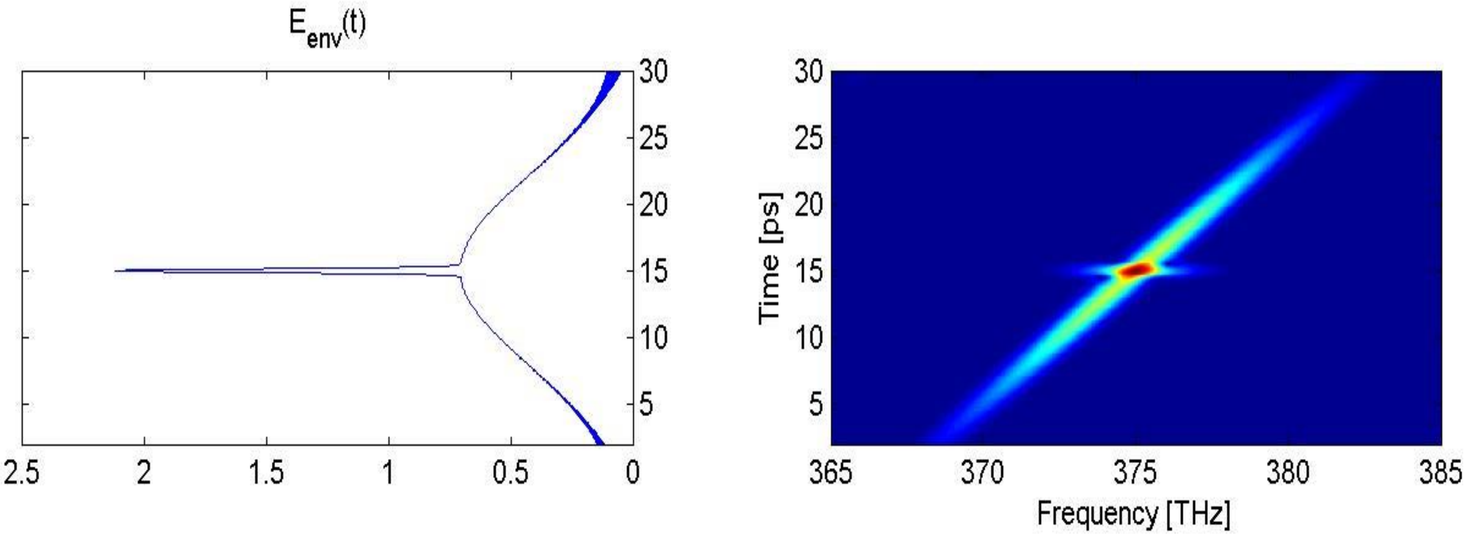
Fourier transform of product is:

$$FT^{-1}\left[\exp(-i\beta\Omega^2)\right] = \frac{1}{2\pi}\sqrt{\frac{\pi}{\beta}}\exp\left(\frac{i\tau^2}{4\beta} - \frac{i\pi}{4}\right) \quad * \quad E_{THz}(\tau)$$

Examples

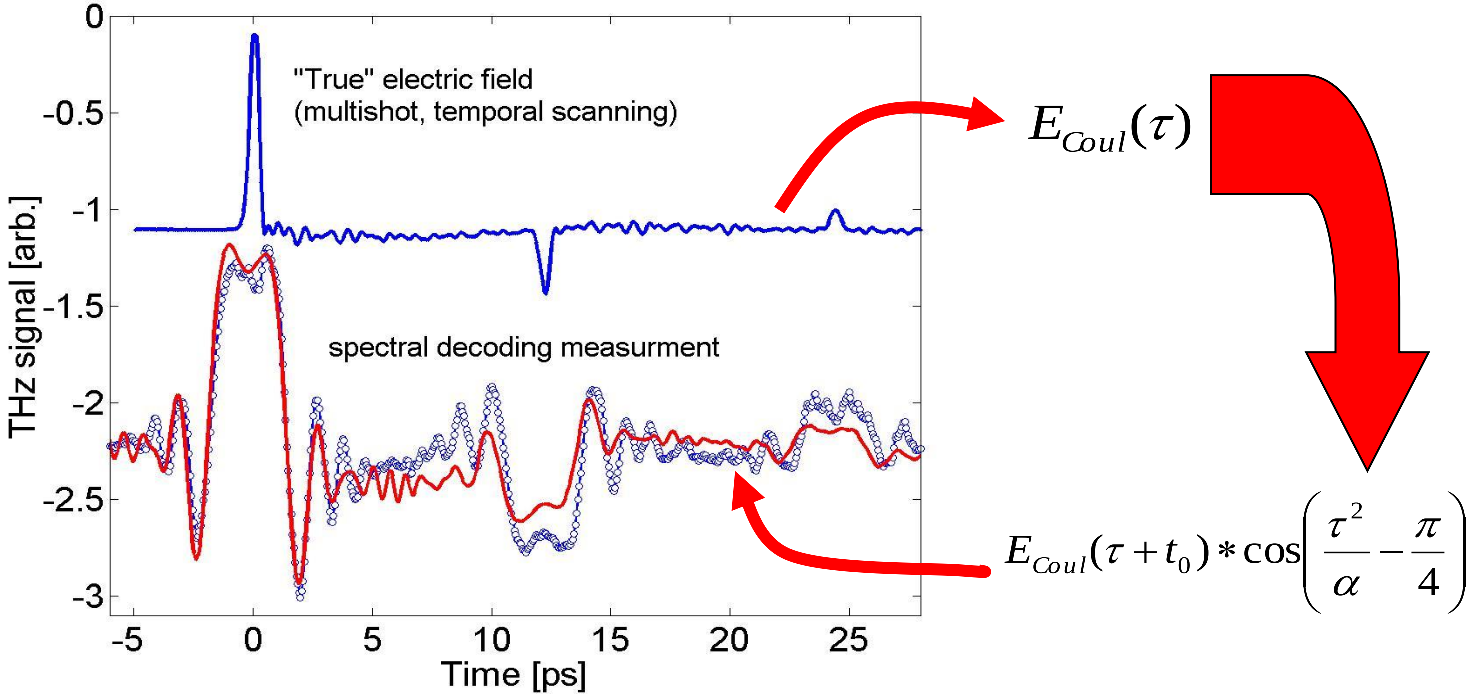
Short bunch modulation :
Spectral interpretation fails





Experimental Confirmation of Resolution

Extreme case confirming the cosine “time resolution function”



$E_{Coul}(\tau)$

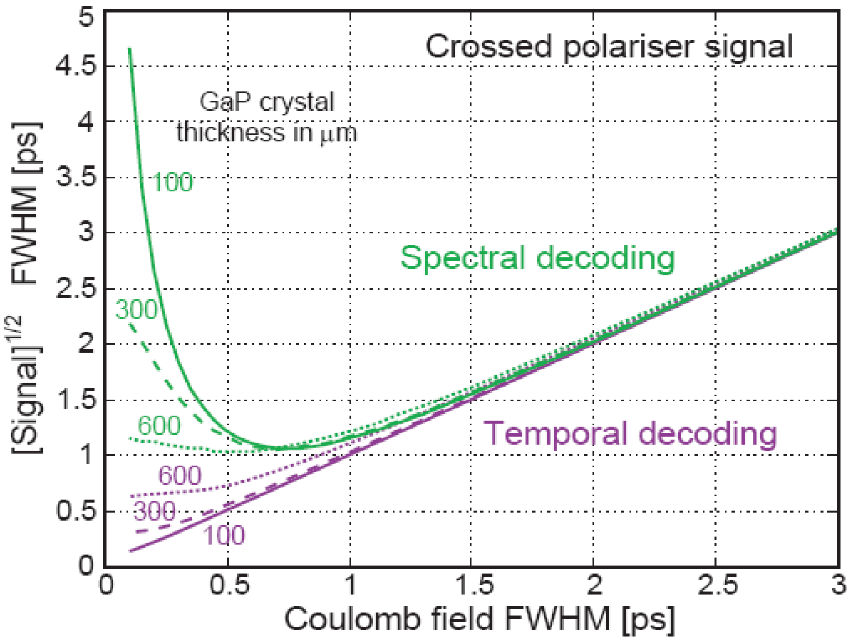
$E_{Coul}(\tau + t_0) * \cos\left(\frac{\tau^2}{\alpha} - \frac{\pi}{4}\right)$

$$\begin{aligned} S^{BD}(\omega) &\equiv I_{\text{opt}}^{\text{in}}(\omega) - I_{\text{opt}}^{\text{in}}(\omega) \\ &\propto I_{\text{opt}}^{\text{in}}(\omega) \left\{ E_{\text{Coul}}(\tau + t_0) * \cos\left(\frac{\tau^2}{4\beta} - \frac{\pi}{4}\right) \right\}. \end{aligned}$$

Spectral Decoding Resolution

Rely on t-λ relationship of input pulse for interpreting output optical spectrum.

Resolution limits come from the fact that the EO-generated optical field doesn't have the same t-λ relationship



For optical pulse of 45fs FWHM chirped to 6.2ps FWHM

temporal resolution limits:

EOSD limited by chirp

Can relate to FWHM durations…

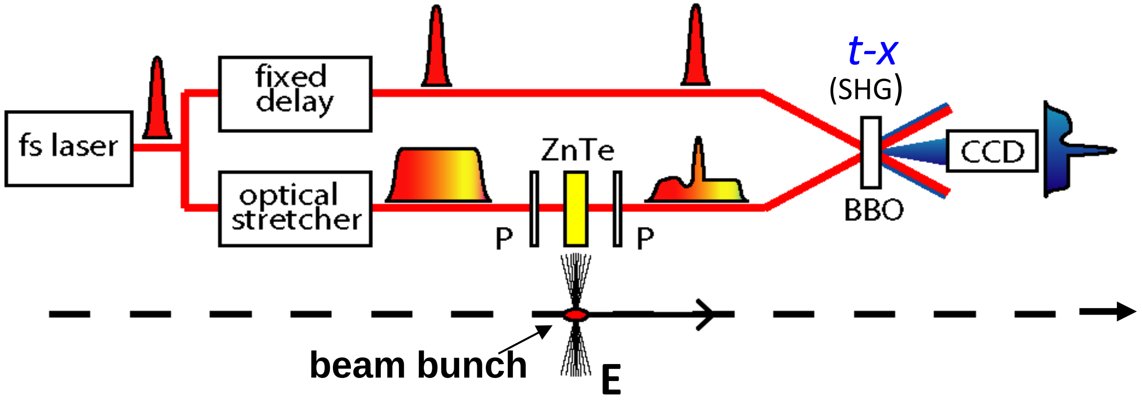
$$\tau_{\lim} = \sqrt{12\pi\beta}$$

$$\tau_{\lim} = 2.61\sqrt{T_0 T_c} \qquad ; \text{for a Gaussian pulse}$$

Conclusion:
Unlikely to get better than **~1.0ps FWHM** But…
Attractive **simplicity** for low time resolution measurements
e.g. injector diagnostics

Temporal Decoding (EOTD)

(currently best demonstrated time resolution)



Temporal profile of probe pulse

→ **Spatial image of SHG pulse**

<u>Thin</u> EO crystal (ZnTe or GaP) produces an *optical temporal replica* of Coulomb field

Measure optical replica with *t-x* mapping in 2nd Harmonic Generation (SHG)

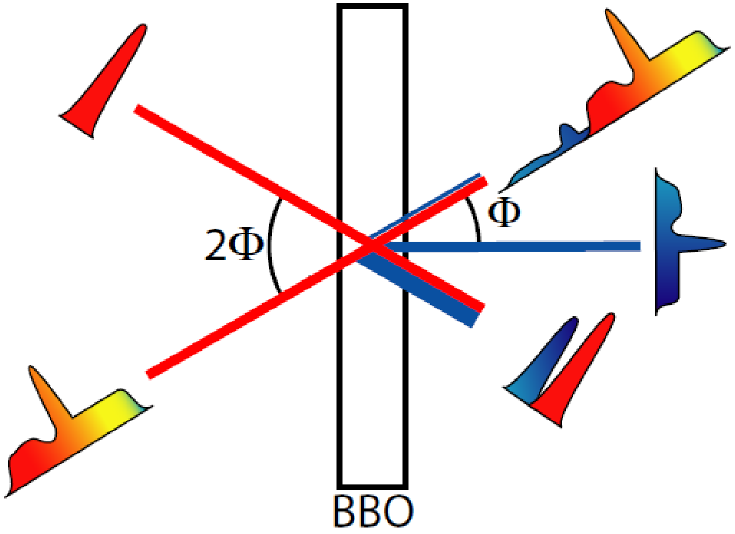
- *stretched & chirped* laser pulse leaving EO crystal assembly measured by short laser pulse via single-shot cross correlation in BBO crystal
- large (~1mJ) laser pulse energy required (via Ti:Sa amplifier)

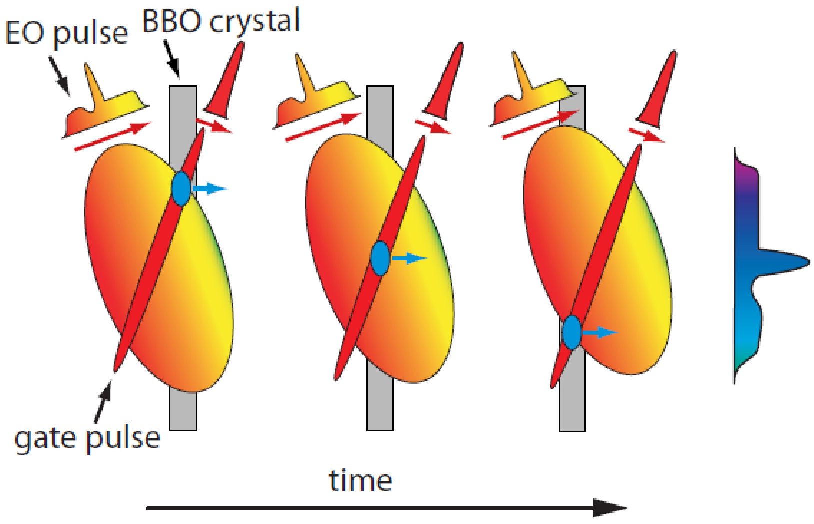
Technique limited by

- gate pulse duration (~50 fs, although FROG, etc. could improve)
- EO encoding efficiency, phase matching

Practical limitations: *complexity of laser systems involved*
transporting short-pulse laser (gate pulse only)

Temporal Decoding





$$I_{SHG}(x \leftarrow t) \propto \int I_{probe}(\tau) I_{gate}(t-\tau)\mathrm{d}\tau$$

- **Resolution is limited by gate duration (+phase matching)**

Practical implementation limits gate to >40fs fwhm
(laser transport, cross-correlator phase matching/signal levels)

- **Weak probe due to EO material damage limits…**
- **Compensated by intense gate**

Signal/noise issues from this mismatch in intensities

Images : Bernd Steffen
PhD thesis

EOTD Electro-optic diagnostics at FLASH

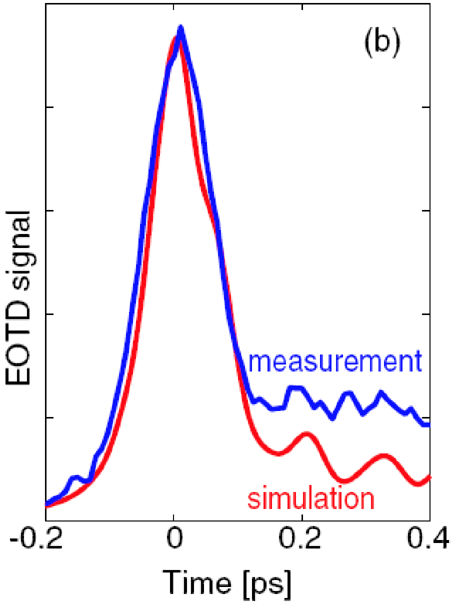
- temporal decoding
- spectral decoding
- benchmarking against TDC

60 – 200μm thick GaP detector

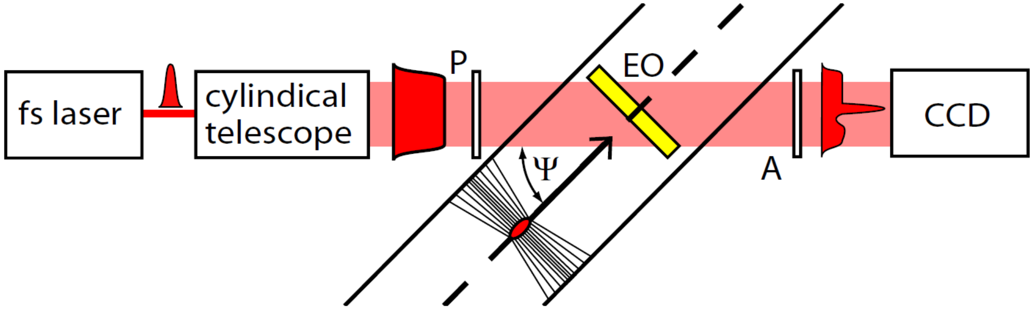
- 450 MeV, γ ~1000
- bunches with peak + pedestal structure
- 20% charge in ~100 fs spike

Time resolution σ_z ~ 90fs (rms)

Temporal Decoding Diagnostic



EO Spatial Encoding



Similar concept to temporal encoding

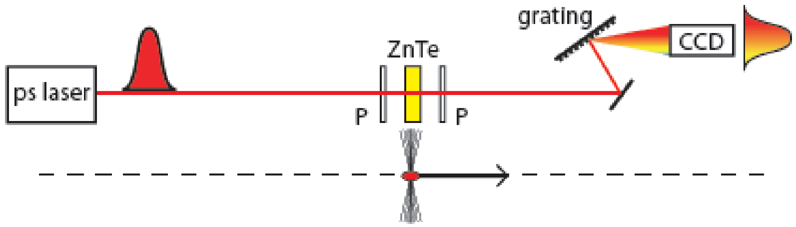
- Crossing angle creates a time to space mapping of Coulomb field in probe
- Lower pulse energy requirements than EOTD – no SHG
- Resolution limit is ultimately the same as in EOTD – duration of probe pulse

But…

- Phasematching efficiency and material response not matched.
- Geometric smearing can reduce resolution.

(simple)
Spectral upconversion diagnostic

measure the bunch Fourier spectrum...



... accepting loss of phase information
& explicit temporal information

... gaining potential for determining
information on even shorter structure

... gaining measurement simplicity

Long pulse, narrow bandwidth, probe laser

$$\tilde{E}^{\rm opt}_{\rm out}(\omega) = \underbrace{\tilde{E}^{\rm opt}_{\rm in}(\omega)} + i\omega a \underbrace{\tilde{E}^{\rm opt}_{\rm in}(\omega) *} \left[\tilde{E}^{\rm Coul}(\omega)\tilde{R}(\omega)\right]$$

→ ***δ-function***

same physics
as "standard" EO

$$\tilde{E}(\omega_0+\Omega) = \tilde{E}(\omega_0) + i\omega a\tilde{E}(\omega_0)\left[\tilde{E}^{\rm Coul}(\Omega)\tilde{R}(\Omega)\right]$$

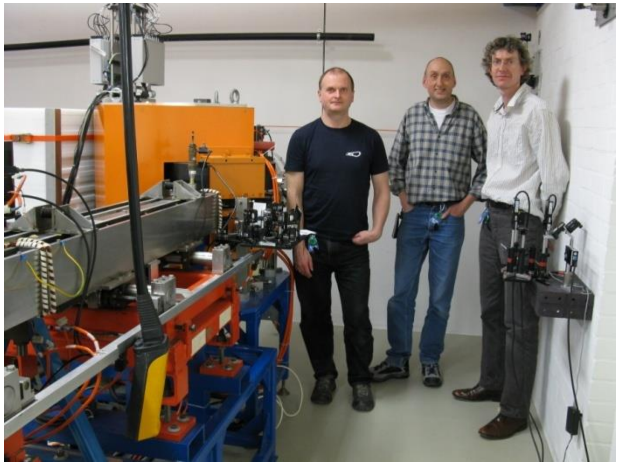
$(\Omega \text{ can be } < 0)$

different observational
outcome

NOTE: the long probe is still converted to optical replica

Spectral upconversion diagnostic

First demonstration experiments at FELIX

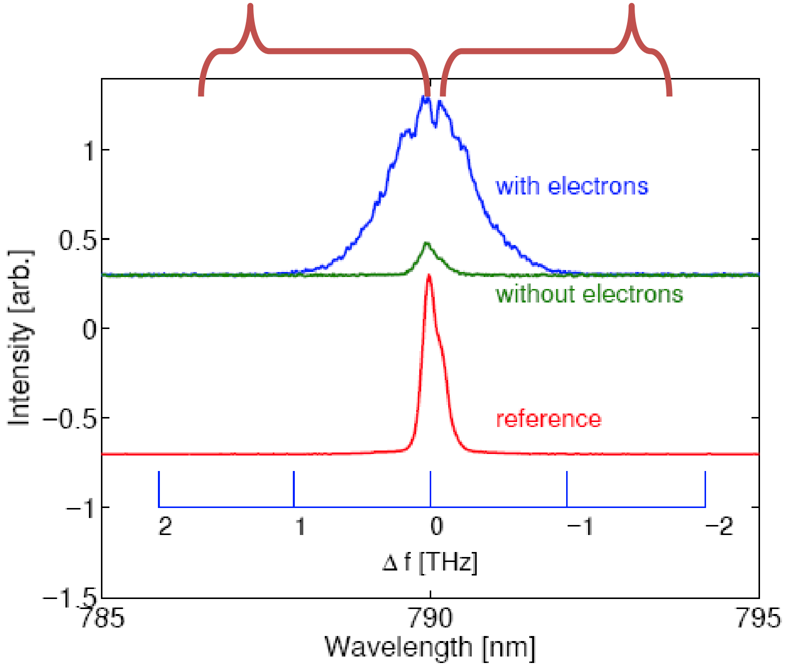


sum
frequency mixing

$$\tilde{E}(\omega_0+\Omega) = i\omega a\tilde{E}(\omega_0)\,\tilde{E}^{\mathrm{Coul}}(\Omega)\tilde{R}(\Omega)$$

difference
frequency mixing

$$\tilde{E}(\omega_0-\Omega) = i\omega a\tilde{E}(\omega_0)[\{\tilde{E}^{\mathrm{Coul}}(\Omega)\}^*\tilde{R}^*(\Omega)]$$

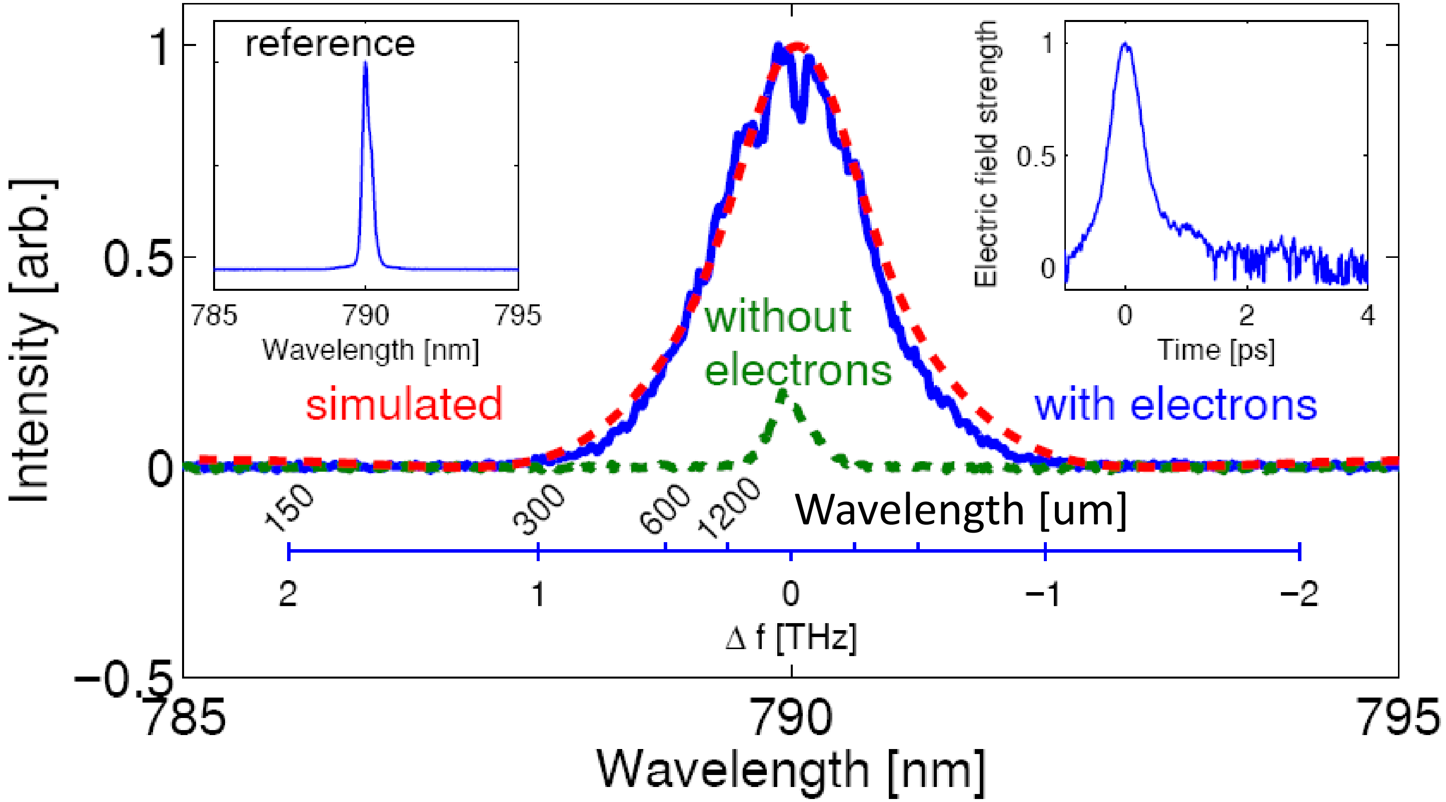


Applied Physics Letters, **96** 231114 (2010)

Measures long wavelength components

non-propagating *spectral components which are not accessible to radiative techniques (CSR/CTR/SP)*

Right down to DC!



~650fs FWHM Coulomb field

These experiments had less than ideal laser: ~5ps, not very narrow spectrum

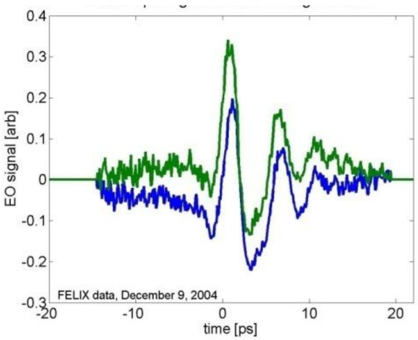
General status of electro-optic...

Many *demonstrations*...

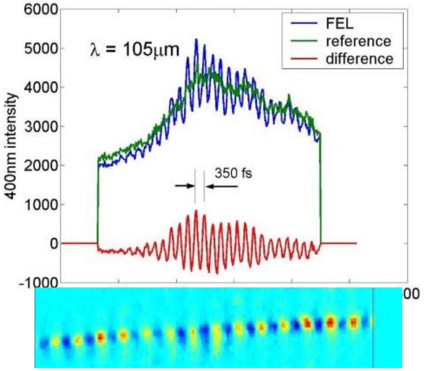
Accelerator Bunch profile -	FLASH, FELIX, SLAC, SLS, ALICE, FERMI ....
Laser Wakefield experiments -	CLF, MPQ, Jena, Berkley, ...
Emitted EM (CSR, CTR, FEL) -	FLASH, FELIX, SLS, ...

Temporal Decoding @FLASH

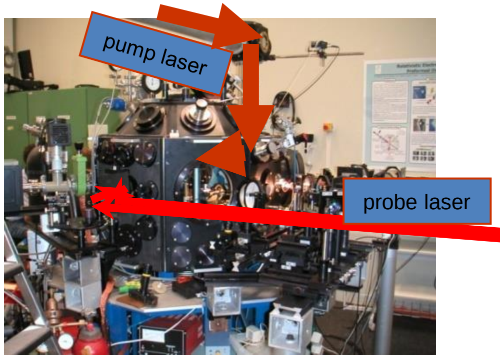
CSR @FELIX



Mid-IRFEL lasing @FELIX



Laser Wakefield
@ Max Planck Garching



Few facility implementations: remaining as experimental / demonstration systems

- Complex & temperamental laser systems
- Time resolution “stalled” at ~100fs

Phys Rev Lett **99** 164801 (2007)
Phys. Rev. ST, **12** 032802 (2009)

EO Transposition

Project at ASTeC, Daresbury and Univ. of Dundee funded by CLIC UK

From earlier: nonlinear frequency mixing

$$\tilde{E}^{\mathrm{opt}}_{\mathrm{out}}(\omega) = \tilde{E}^{\mathrm{opt}}_{\mathrm{in}}(\omega) + i\omega a \tilde{E}^{\mathrm{opt}}_{\mathrm{in}}(\omega) * \left[\tilde{E}^{\mathrm{Coul}}(\omega)\tilde{R}(\omega)\right]$$

$$E^{\mathrm{opt}}_{\mathrm{out}}(t) = E^{\mathrm{opt}}_{\mathrm{in}}(t) + a \underbrace{\left[E^{\mathrm{Coul}}(t) * R(t)\right]}_{\text{envelope}} \underbrace{\frac{\mathrm{d}}{\mathrm{d}t} E^{\mathrm{opt}}_{\mathrm{in}}(t)}_{\text{optical field}}$$

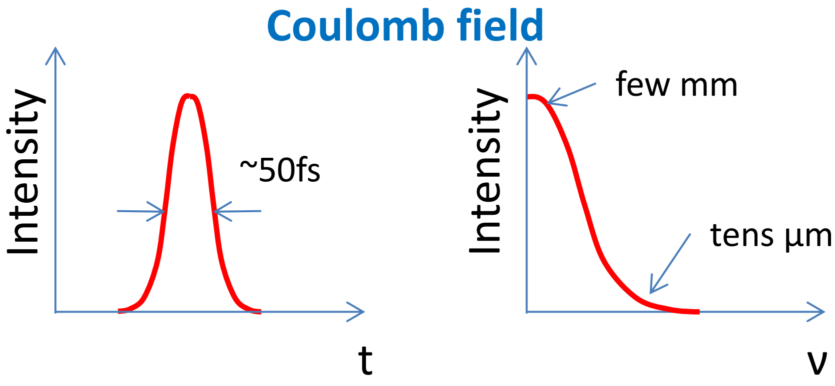
$$\begin{matrix} E_{\mathrm{x}}(t) \\ E_{\mathrm{y}}(t) \end{matrix} \quad \longrightarrow \quad \begin{matrix} \tilde{E}_{\mathrm{x}}(\omega) \\ \tilde{E}_{\mathrm{y}}(\omega) \end{matrix}$$

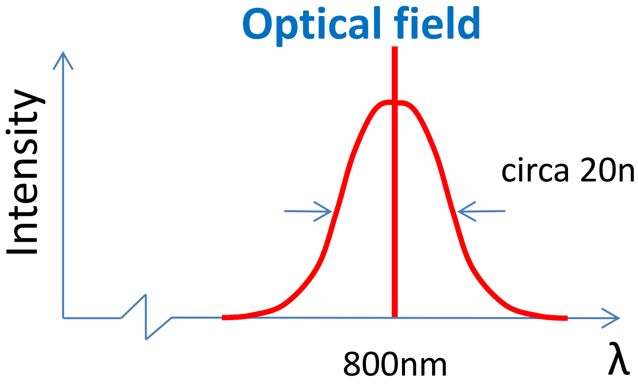
Coulomb spectrum shifted to optical region

Coulomb pulse temporally replicated in optical pulse

S.P. Jamison Opt. Lett. v**31** no.11 p1753

Consider a single frequency probe and short coulomb field "pulse"

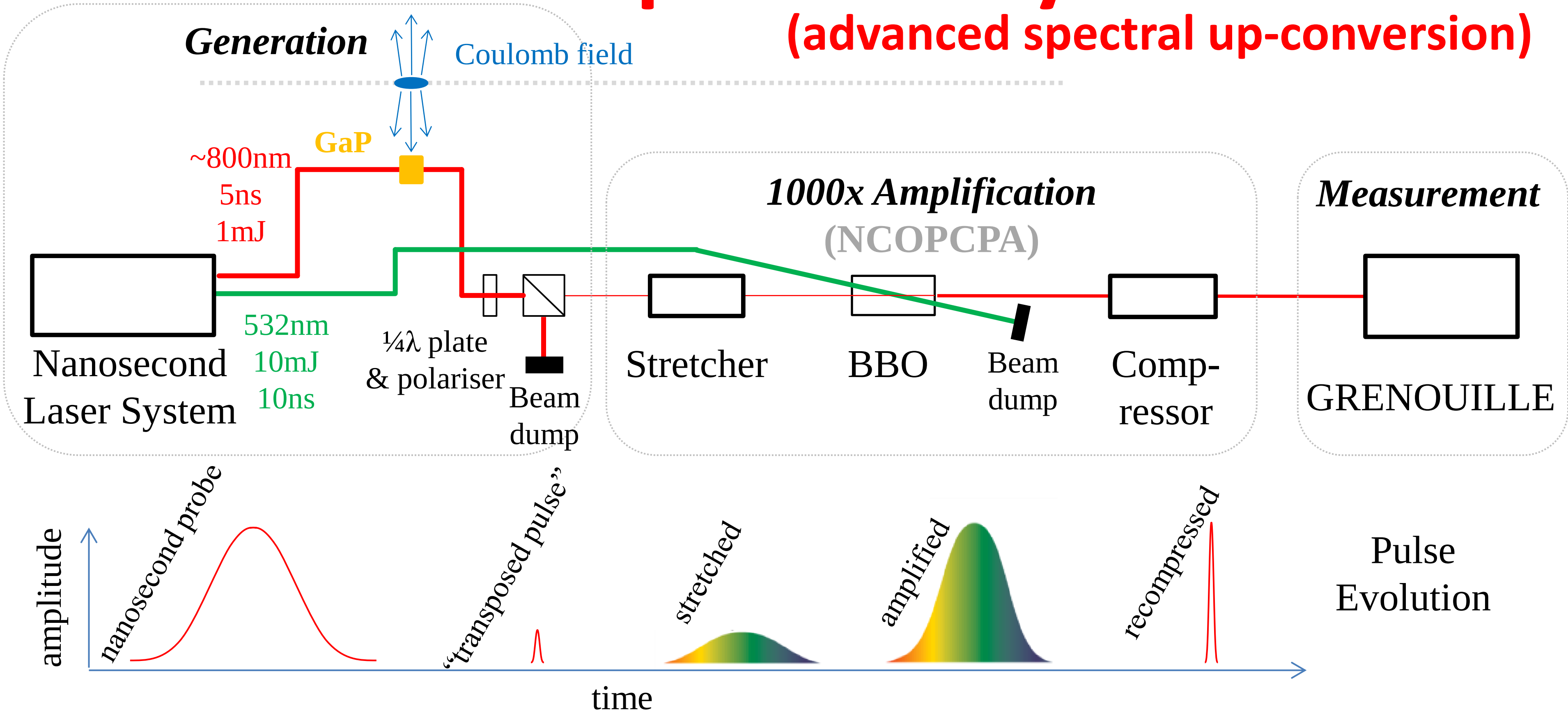

Coulomb field


Optical field

This process preserves the spectral phase information!

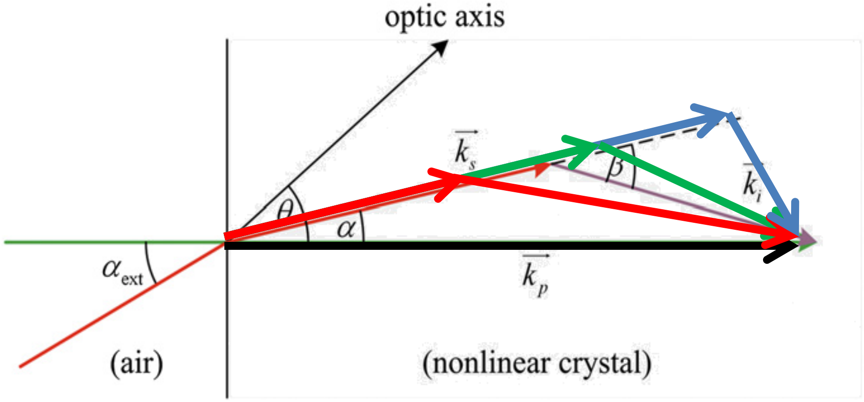
EO Transposition System

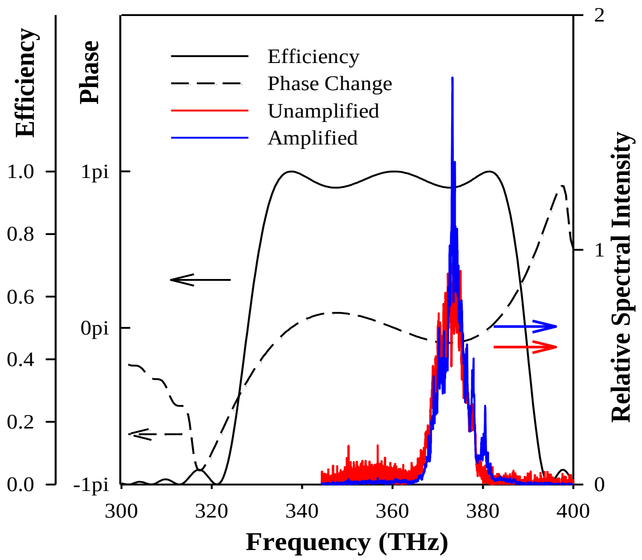
(advanced spectral up-conversion)



1. Nanosecond laser derived single frequency probe brings reliability
2. “Electro-Optic Transposition” of probe encodes temporal profile
3. Non-collinear optical parametric chirped pulse amplification (NCOPCPA) amplifies signal
4. Full spectral amplitude and phase measured via FROG
5. Coulomb field, and hence bunch profile, calculated via time-reversed propagation of pulse

Parametric Amplification





$$k_p \sin\alpha = k_i \sin\beta, \quad k_p \cos\alpha = k_s + k_i \cos\beta \quad v_{gs} = v_{gi}\cos\beta$$

$$\sin\alpha = \left[\frac{1-\left({}^{v_{gs}}/_{v_{gi}}\right)^2}{1+2{}^{v_{gs}k_s}/_{v_{gi}k_i}+\left({}^{k_s}/_{k_i}\right)^2}\right]^{1/2} \qquad \sin\theta = \frac{n_{pe}}{k_p}\left[\frac{\left({}^{2\pi}/_{\lambda_p}\right)^2 n_{po}^2 - k_p^2}{n_{po}^2 - n_{pe}^2}\right]^{1/2}$$

In BBO it is possible to arrange the phasematching condition such that a very large range of frequencies are phasematched.
Of interest for us is that for a pump of 532nm and θ ~ 23.8° and α ~ 2.4°

Δk ~0 over >100nm centred circa 825nm!

Pumping with 350MW/cm² should give ~1000x gain over 2cm

Why Grenouille?

Problem: Unknown phase

What we want to know

$$E(t) = Re\left(\sqrt{I(t)}e^{i(\omega_o t-\phi(t))}\right)$$

<-Fourier->

$$\tilde{E}(\omega)=\sqrt{S(\omega)}e^{-i\varphi(\omega)}$$

"Carrier" frequency

Can't measure

Spectrum

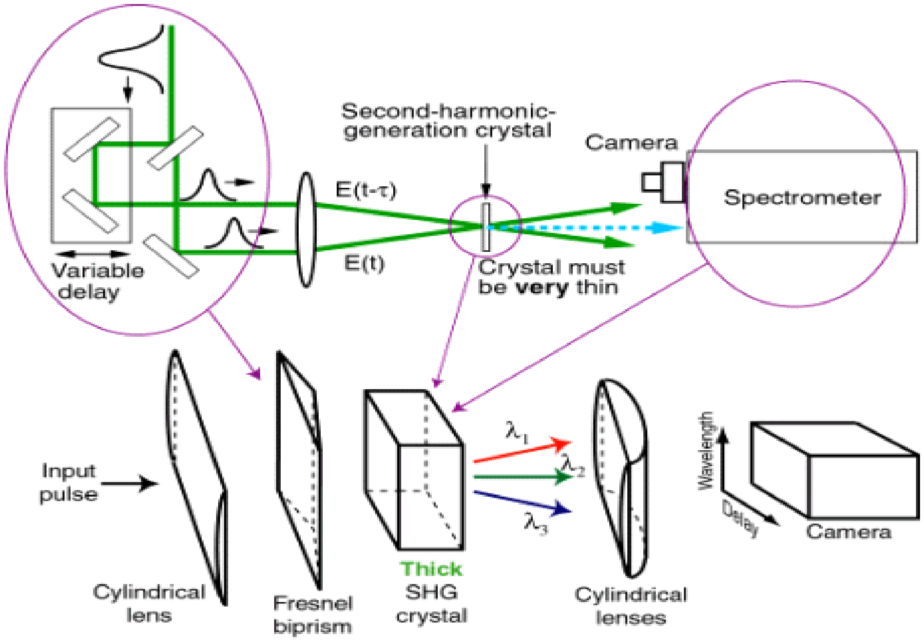
Spectral Phase

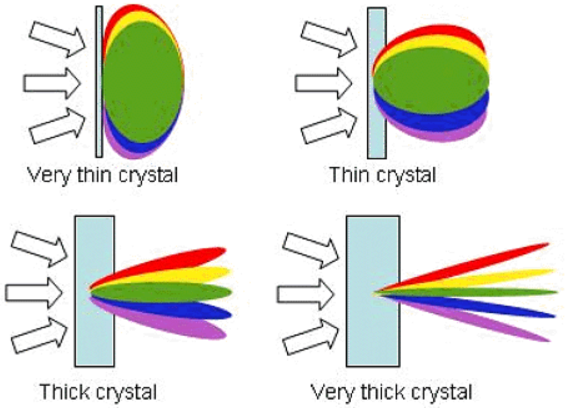
This will be important for improving bandwidth…

Can be retrieved via…

Solution: Frequency Resolved Optical Gating (FROG), a **standard and robust** optical diagnostic.

Retrieves spectral intensity and phase from spectrally resolved autocorrelation.





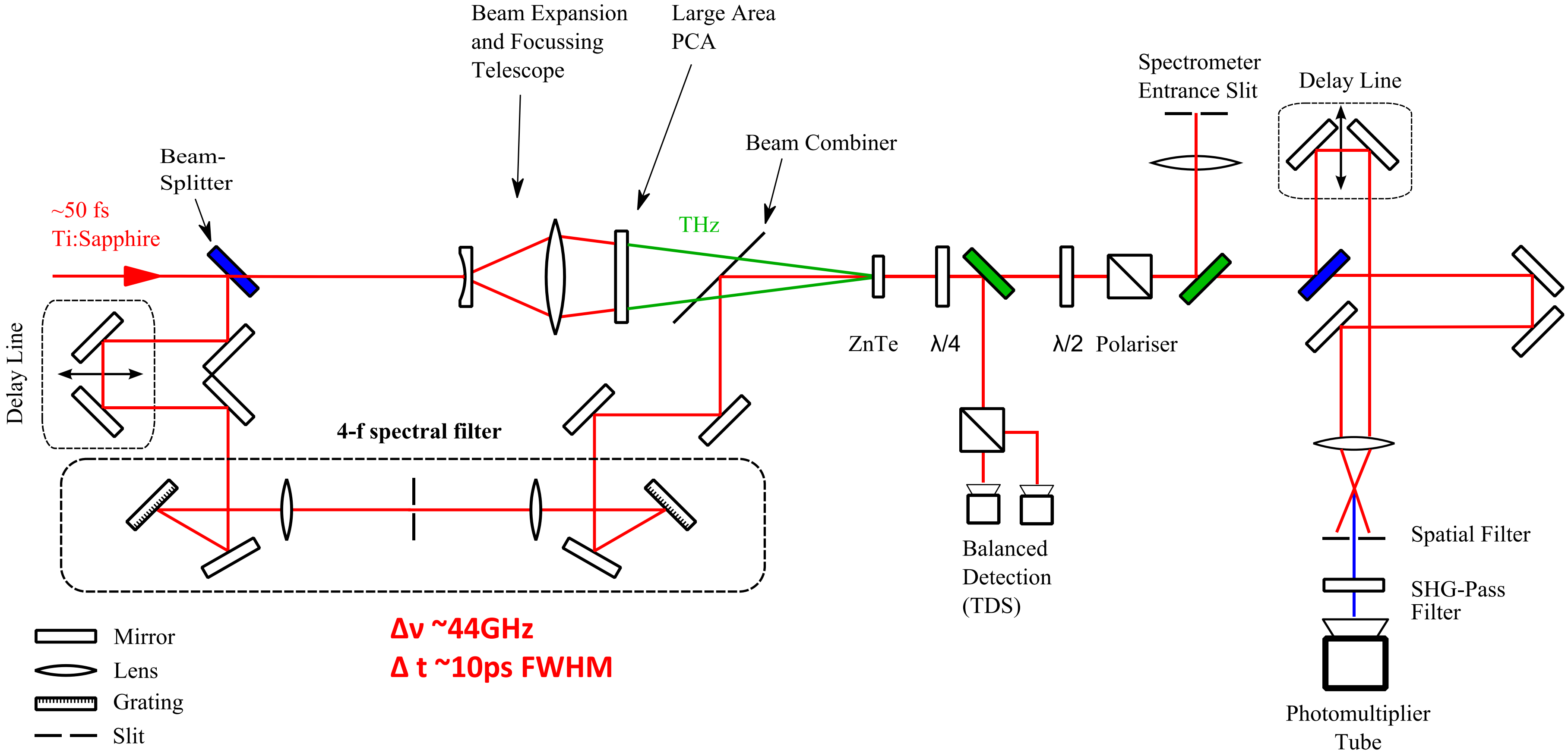
- **The most sensitive "auto gating" measurement**
- **Self-gating avoids timing issues (no need for a fs laser)**
- **Single shot measurement possible**
- **Requires minimum pulse energy of > 10 nJ**
- **Commercial systems offer > 1 µJ**

Characterisation of EO Transposition

Femtosecond laser-based test bed

Auston switch THz source mimics Coulomb field.

Well-characterised spectral and temporal profile.



Δν ~44GHz

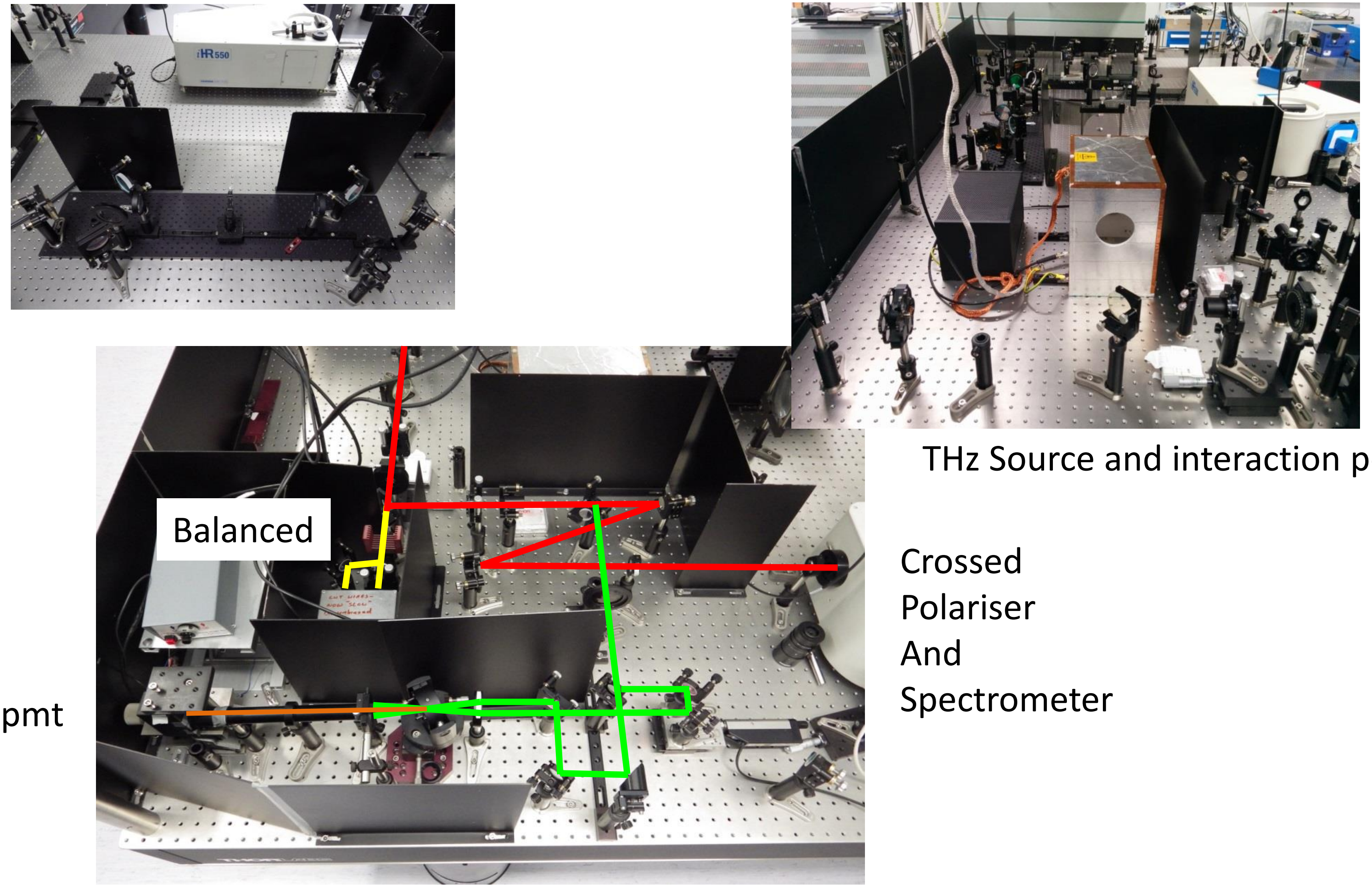
Δ t ~10ps FWHM

Femtosecond laser pulse spectrally filtered to produce narrow bandwidth probe

Switchable diagnostics – Balanced sampling, Crossed Sampling, and **Autocorrelation**

Experimental System

4-f filter



THz Source and interaction point

Crossed
Polariser
And
Spectrometer

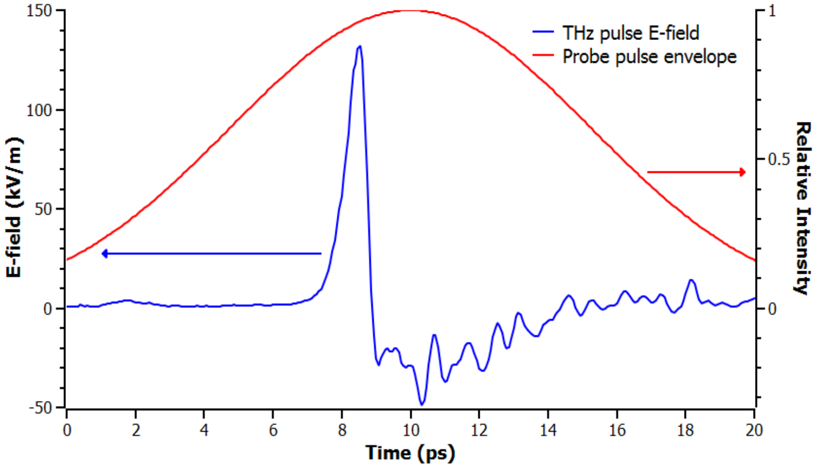
pmt

Autocorrelator

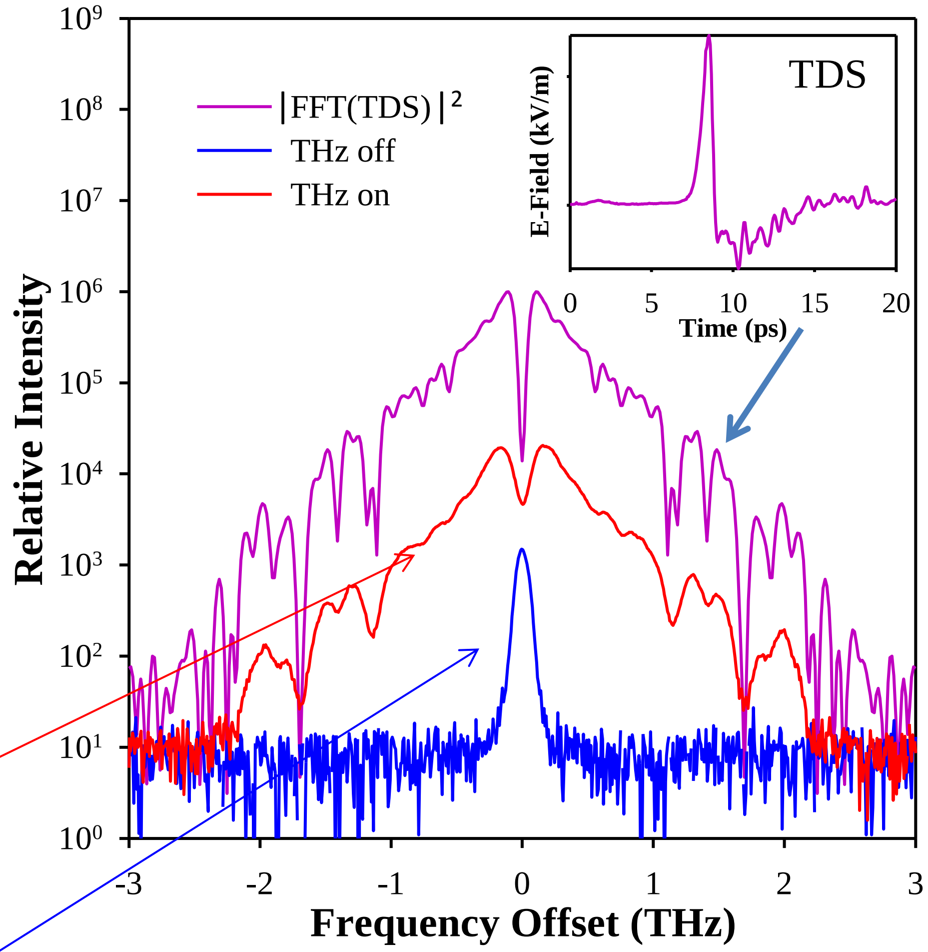
Tests at Daresbury Lab

Input pulse characteristics

Optical probe length	Δt ~ 10ps
Optical probe energy	S ~ 28nJ
THz field strength max	E ~ 132kV/m



Output characteristics (4mm ZnTe)

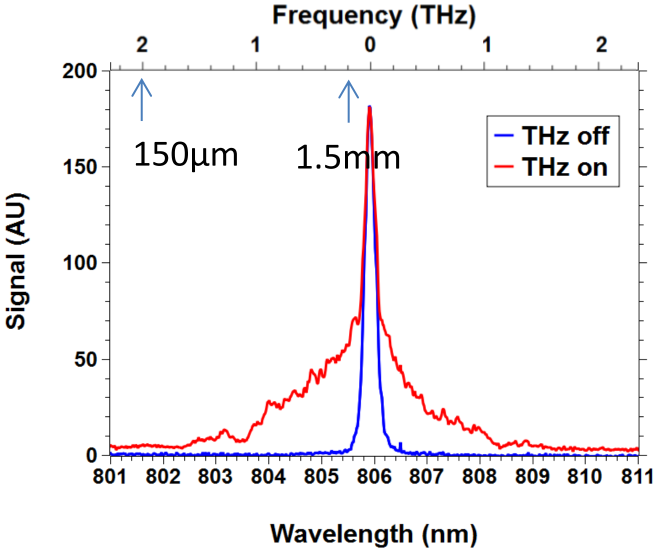


Total energy ~470pJ

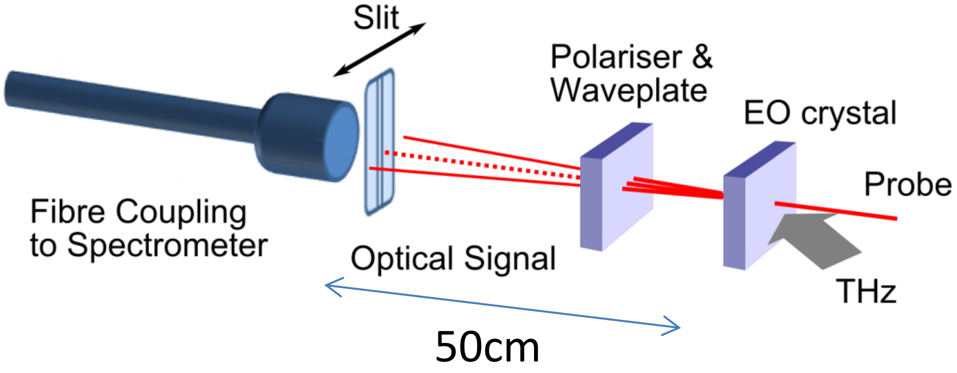
Leaking probe

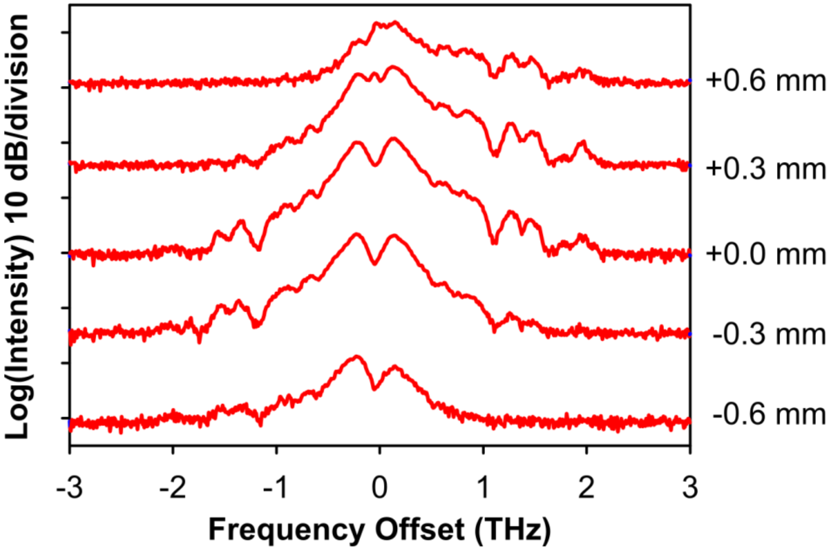
Alignment Issues in EO Systems

Early measurements of spectra often asymmetric and weak/unobservable



Adjustment of the THz alignment could modify the observed spectral sidebands!

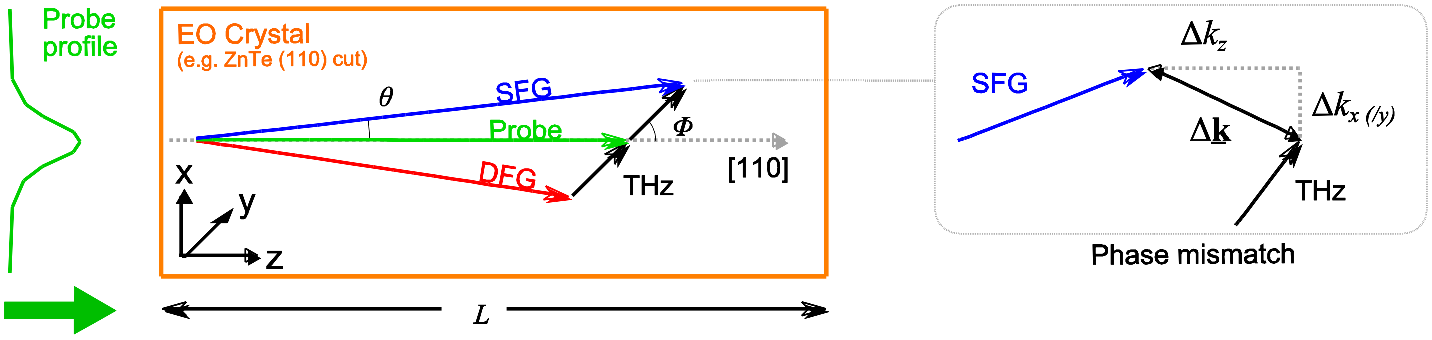




Understanding this effect is crucial to correctly performing any EO measurement!

Non-collinear Phase Matching

A natural consequence of considering nonlinear processes
is that phase matching must be considered!



Phase mismatch

Polarisation field set up by probe and THz (Coulomb) field:

$$\tilde{P}(\omega_3,\underline{\mathbf{r}}) = \chi^{(2)}\tilde{E}^{\mathrm{opt}}(\omega_1,\underline{\mathbf{r}})\tilde{E}^{\mathrm{THz}}(\omega_2,\underline{\mathbf{r}})$$

Expand fields into envelope and carrier:

$$\tilde{P}(\omega_3,\underline{\mathbf{r}}) = \chi^{(2)}\tilde{A}_1(\omega_1,\underline{\mathbf{r}})\tilde{A}_2(\omega_2,\underline{\mathbf{r}})\exp\left(i(\underline{\mathbf{k}}_1+\underline{\mathbf{k}}_2)\cdot\underline{\mathbf{r}})\right)$$

Then solve paraxial wave equation using Gaussian transverse profiles:

$$Eff(\omega_3,\theta,\varphi)= \exp\left(-\frac{1}{2}\left(\sigma_x^2\Delta k_x^2+\sigma_y^2\Delta k_y^2\right)\right)\frac{\exp(i\Delta k_z L)-1}{\Delta k_z L}$$

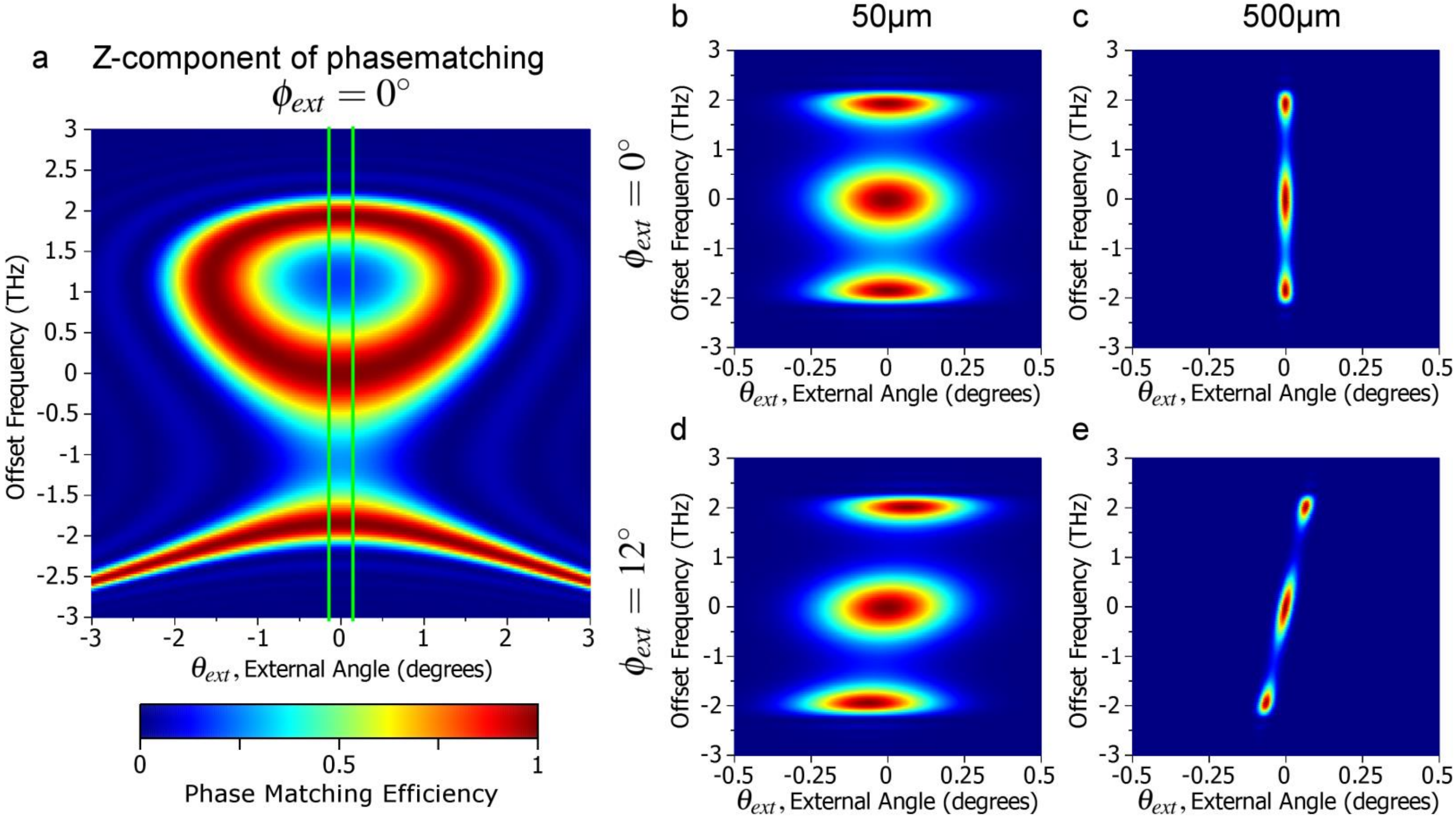
Same form as derived in NLO literature

Predictions and Validation

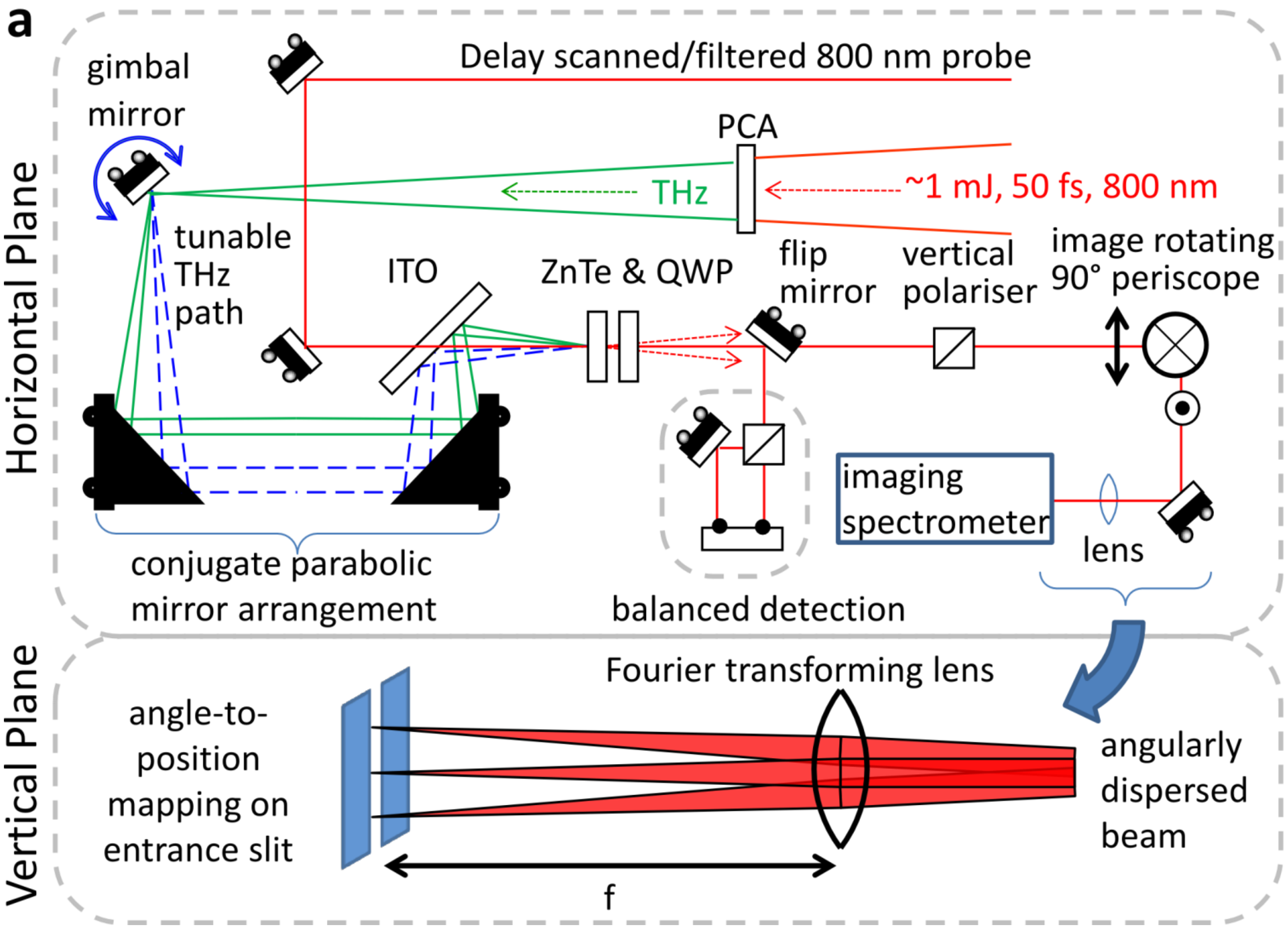
Phase matching efficiencies calculated in Matlab

Code iterates through THz frequencies and calculates efficiency for a range of upconversion directions

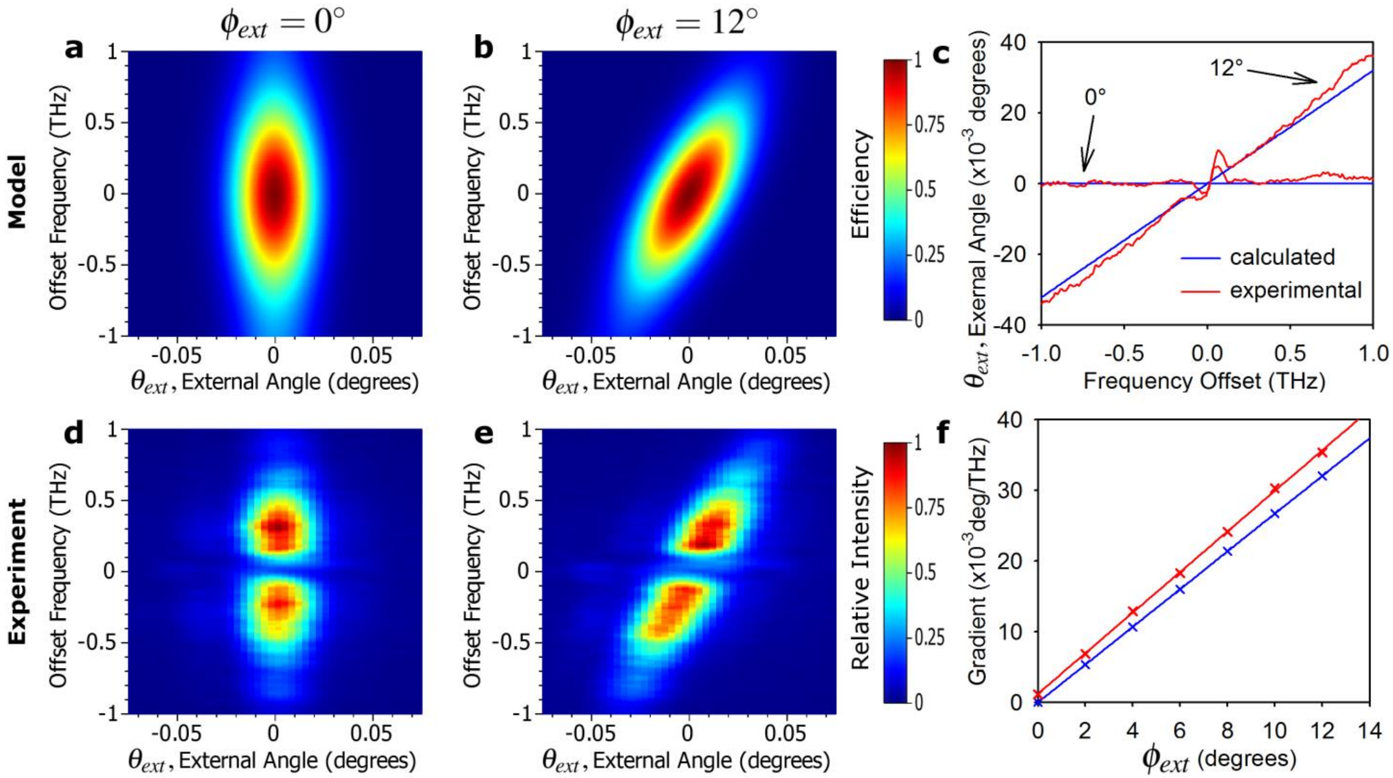
$$\exp\left(-\frac{1}{2}\left(\sigma_x^2\Delta k_x^2+\sigma_y^2\Delta k_y^2\right)\right)\frac{\exp(i\Delta k_z L)-1}{\Delta k_z L}$$



Experimental Validation



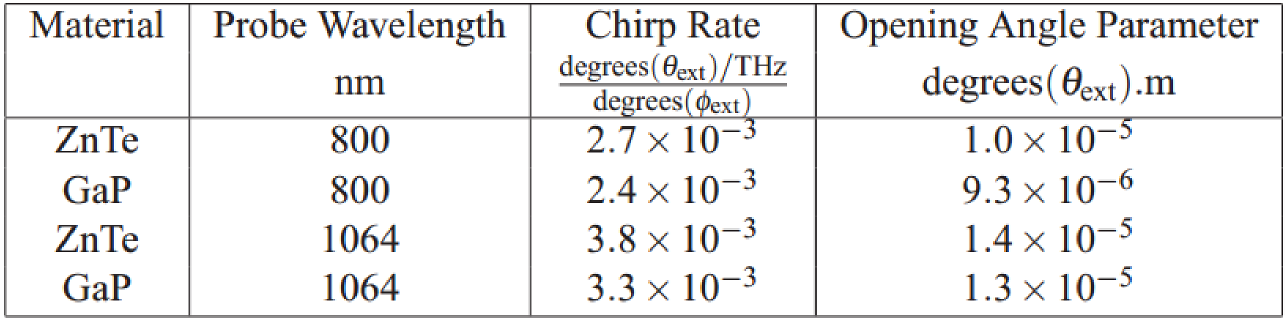
Results



Confirmed predictions of model.

Phasematching Summary

- We now have a proper understanding of the issues
- Have shown that correct management of the optical beam is essential for any EO system
- Could well have been the cause of difficulties with EO systems in the past!
- Enabled us to produce rule of thumb guides for common systems:



Material	Probe Wavelength nm	Chirp Rate $\frac{\text{degrees}(\theta_{\text{ext}})/\text{THz}}{\text{degrees}(\phi_{\text{ext}})}$	Opening Angle Parameter $\text{degrees}(\theta_{\text{ext}}).\text{m}$
ZnTe	800	2.7×10^{-3}	1.0×10^{-5}
GaP	800	2.4×10^{-3}	9.3×10^{-6}
ZnTe	1064	3.8×10^{-3}	1.4×10^{-5}
GaP	1064	3.3×10^{-3}	1.3×10^{-5}

Full paper with guidelines on system design:
D. Walsh, Opt. Express 22, 12028-12037 (2014)

Temporal Resolution

EO transposition scheme is now limited by materials:

- Phase matching and absorption bands in ZnTe/GaP.
- Other materials are of interest, such as DAST or poled polymers, but there are questions over the lifetime in accelerator environments.

Collaborative effort with MAPS group at the University of Dundee on development of novel EO materials

- Potential to produce an enhancement of nonlinear processes through metallic nanoparticles.
- THz field induced second harmonic TFISH enhancement being investigated.
- Surface nonlinear effects…

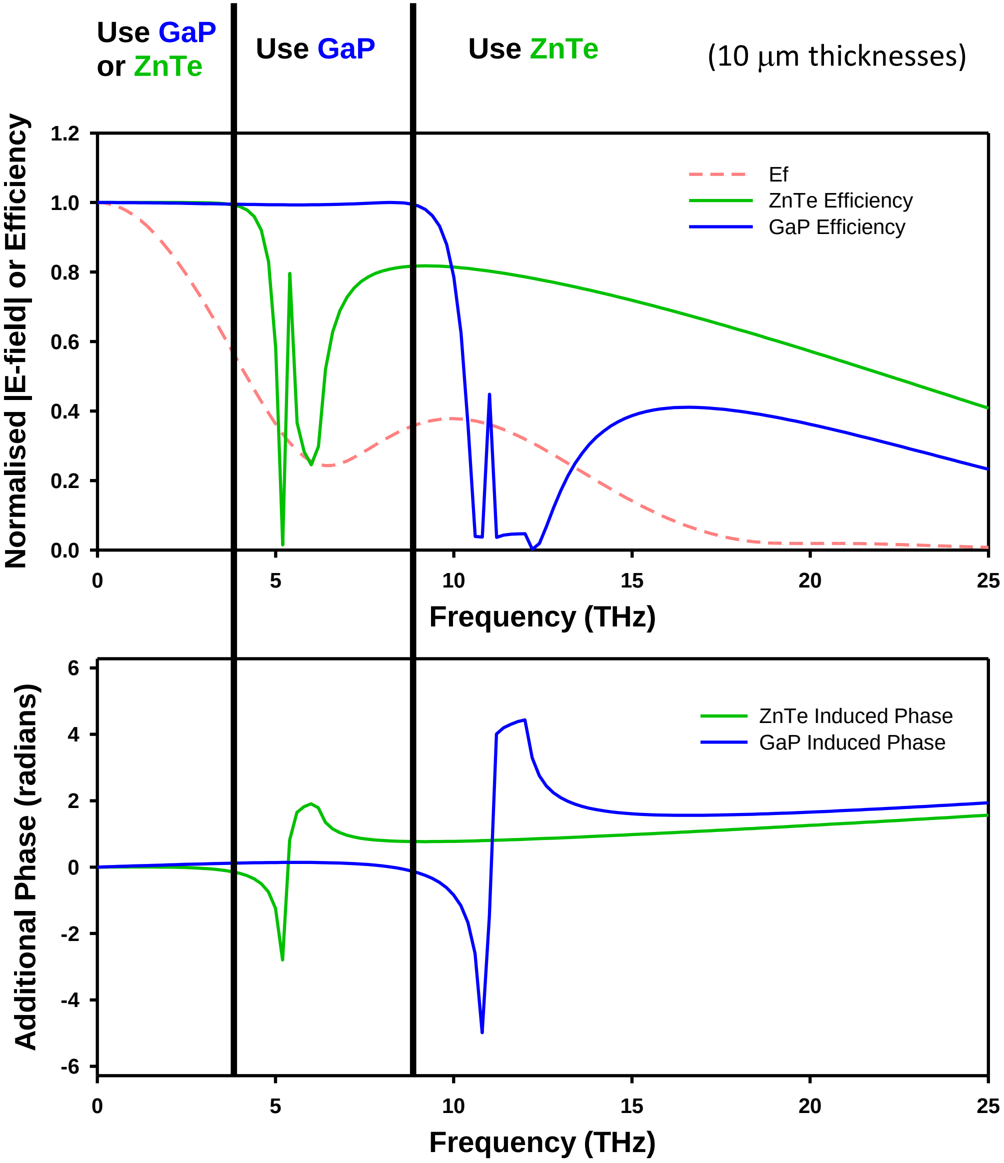
A key property of the EO Transposition scheme may be exploited

- FROG (Grenouille) retrieves the spectral amplitude and phase
- At frequencies away from absorptions etc. the spectrum should still be faithfully retrieved
- Potential to run two, “tried and tested”, crystals with complementary response functions side by side to record FULL spectral information!

Spectral Compositing of Multiple Crystals

- Phasematching not the whole story
  - Dips caused by absorption near phonons
  - Phase distortion near absorptions become very large
  - Distortions in $\chi^{(2)}$ near absorptions
- Discard data around the absorption lines
- Fill in the blanks with different crystals

In theory seems sound.
Not yet demonstrated.



Summary

- Discussed importance of the spectral phase
- Compared EO to other methods
- Summarised EO methods and limits
- New diagnostic – EOT
- Effect of misalignments
- Potential to (soon?) increase resolution limit of EOT