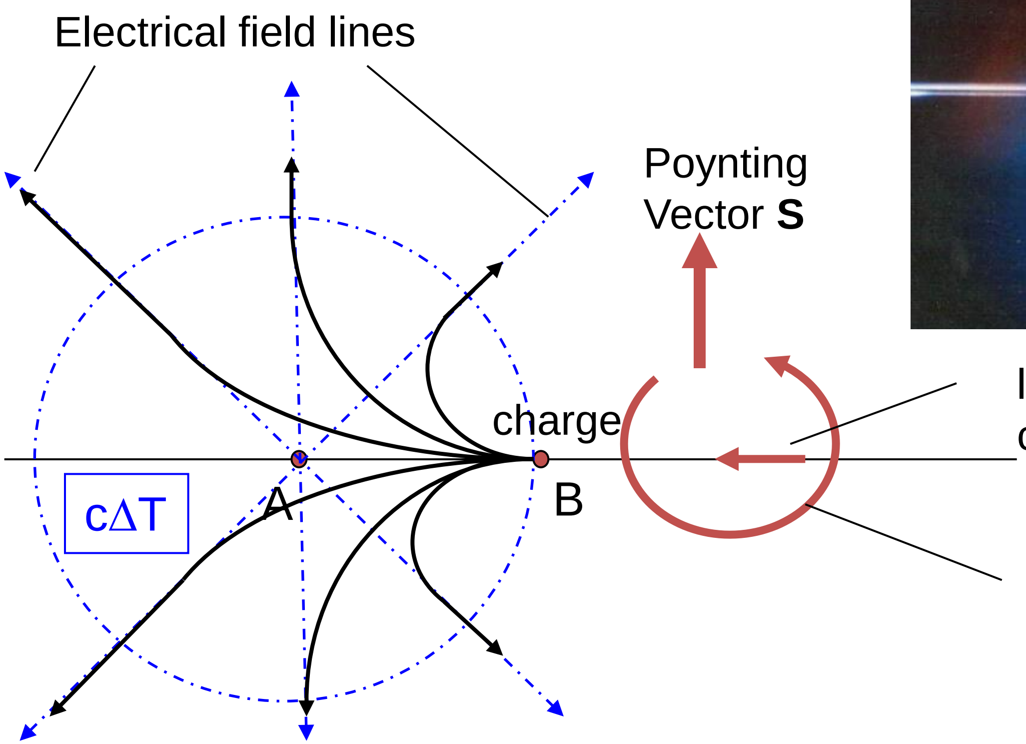
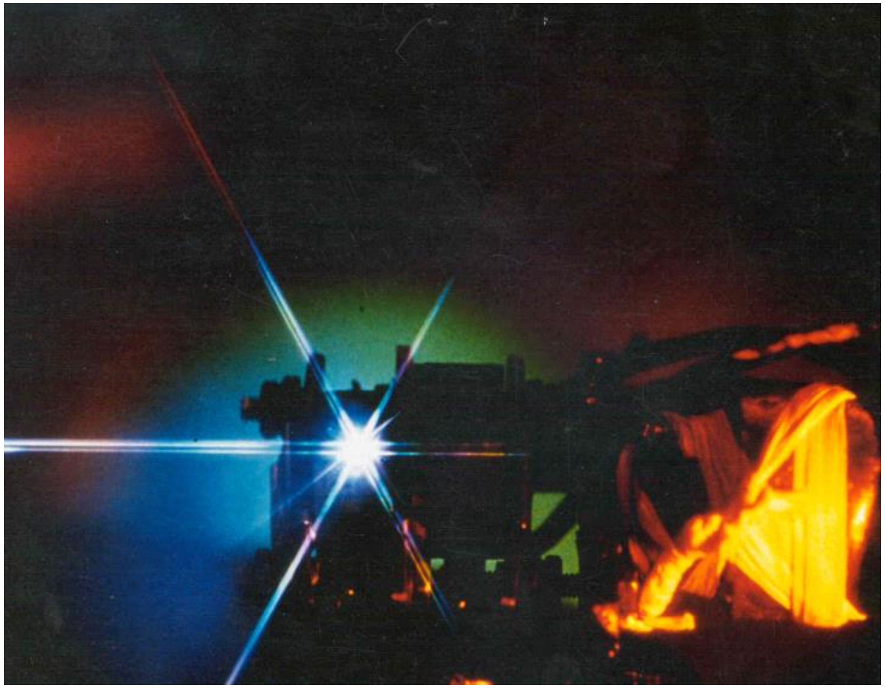
FEL I : Introduction to FELs

Atoosa Meseck

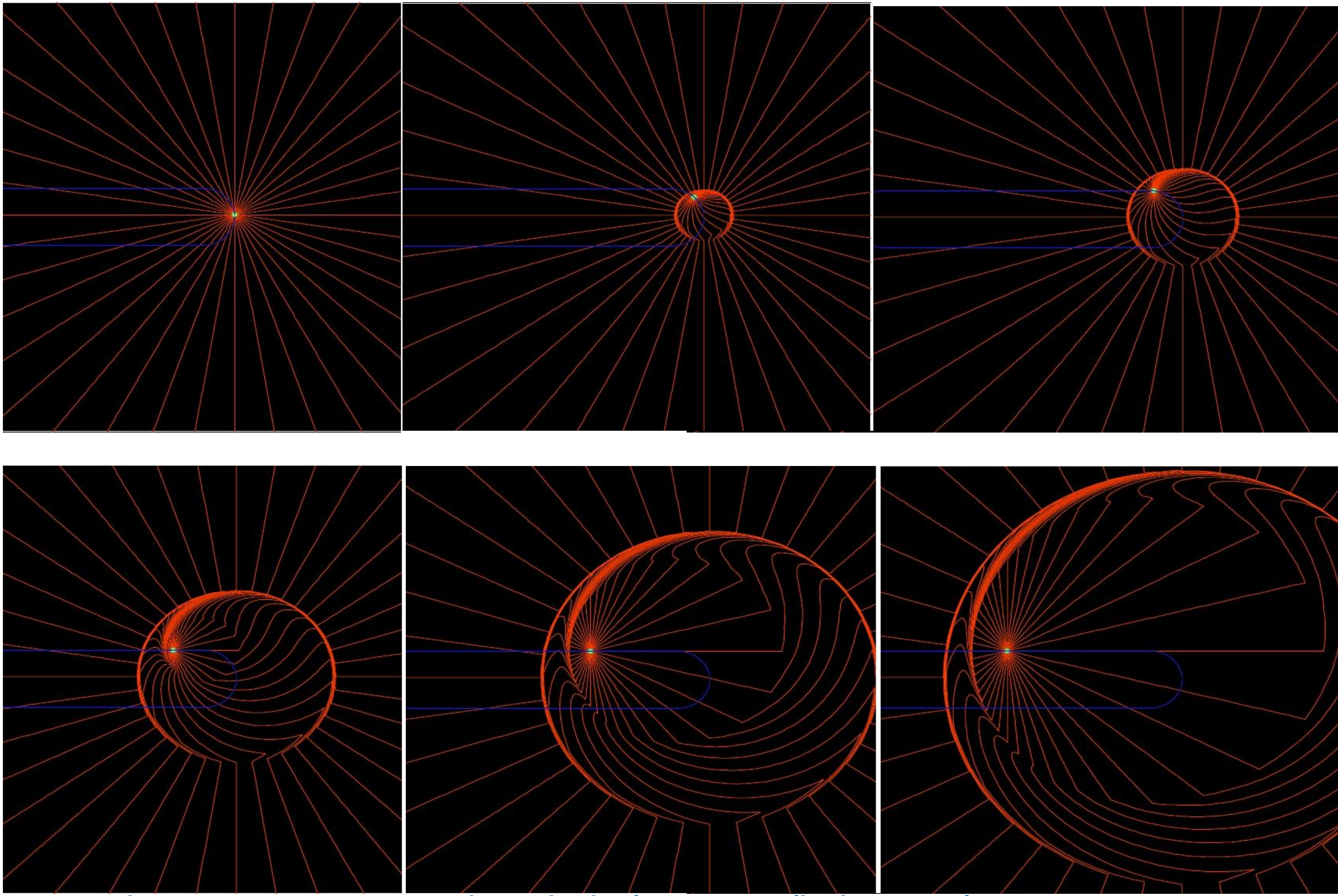
Synchrotron Radiation

Phenomenologically:
A consequence of the finite value of the velocity of light.





Synchrotron Radiation: Electrical Field Component



Retarded Potentials!

1873 $\Rightarrow$ J.C. Maxwell formulated his unifying electromagnetic theory.

1887 $\Rightarrow$ H. Hertz succeeded to generate, emit and receive again electromagnetic waves.

1898 Lienard
1900 Wiechert } *Independently, derived the expressions for "retarded" potentials of point charges.*

Lienard-Wiechert Potentials relate the scalar and vector potential of electromagnetic fields at the observation point to the location of the emitting charges and currents at the time of emission.

Retarded Potentials

Lienard-Wiechert Potentials

$$\varphi(t) = \frac{1}{4\pi\varepsilon_0} \frac{q}{\left\lfloor r\left(1 - \vec{n}\cdot\vec{\beta}\right)\right\rfloor_{ret}} \qquad \vec{A}(t) = \frac{q}{4\pi c^2\varepsilon_0} \frac{\vec{v}}{\left\lfloor r\left(1 - \vec{n}\cdot\vec{\beta}\right)\right\rfloor_{ret}}$$

And the electromagnetic fields:

$$\nabla\cdot\vec{A} + \frac{1}{C}\frac{\partial\varphi}{\partial t} = 0$$

Lorentz gauge

$$\vec{B} = \nabla\times\vec{A} \qquad \vec{E} = -\nabla\varphi - \frac{\partial\vec{A}}{\partial t}$$

Moving Charge

updates the direction of the field toward the instantaneous position of the charge

Coulomb regime

$$\vec{E}(t)=\frac{q}{4\pi\varepsilon_0}\left[\frac{\vec{n}-\vec{\beta}}{\left(1-\vec{n}\cdot\vec{\beta}\right)^3\gamma^2}\frac{1}{r^2}\right]_{ret}+$$

radiation regime

$$\frac{q}{4\pi c\varepsilon_0}\left[\frac{\vec{n}\times\left(\vec{n}-\vec{\beta}\right)\times\dot{\vec{\beta}}}{\left(1-\vec{n}\cdot\vec{\beta}\right)^3}\frac{1}{r}\right]_{ret}$$

magnetic field

$$\vec{B}(t)=\frac{1}{c}\left[\vec{n}\times\vec{E}\right]$$

Poynting Vector

Observer time

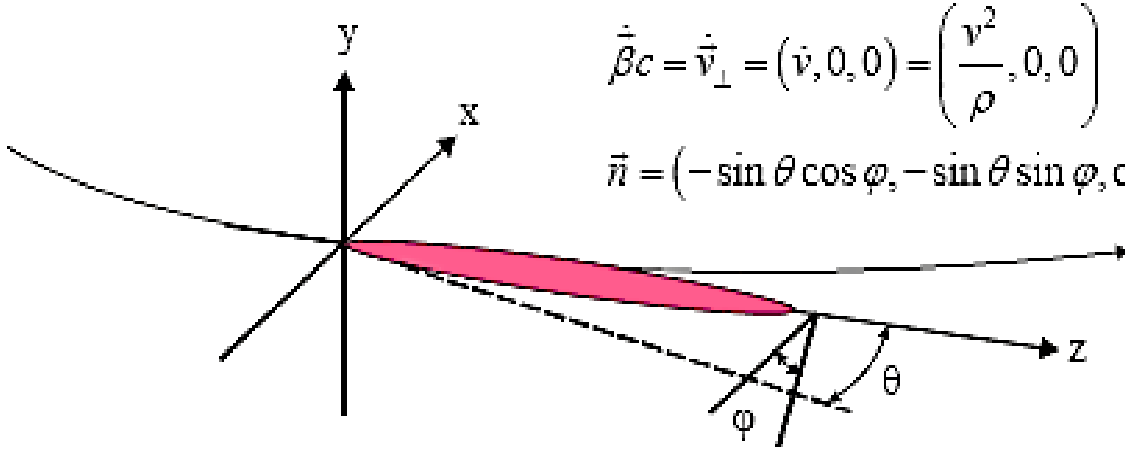
$$\vec{S} = c^2 \varepsilon_0 \left[\vec{E} \times \vec{B} \right]_r = c\varepsilon_0 \left[\vec{E} \times \left(\vec{E} \times \vec{n} \right) \right]_r = -c\varepsilon_0 \vec{E}^2 \, \vec{n} \Big|_r$$

retarded time

$$\vec{S}_r = \vec{S} \frac{dt}{dt_r} = -c\varepsilon_0 \vec{E}^2 \left[(1 + \vec{\beta}\vec{n})\vec{n} \right]_r$$

Radiation in a Solid Angle

$$\frac{dP}{d\Omega} = -\vec{n}\vec{S}R^2\Big|_r = c\varepsilon_0\vec{E}^2\left(1+\vec{\beta}\vec{n}\right)R^2\Big|_r \quad = \frac{c}{4\pi} r_e mc^2 \frac{R^5}{c^3 r^5}\left\{\vec{n}\times\left[\left(\vec{n}+\vec{\beta}\right)\times\dot{\vec{\beta}}\right]\right\}^2$$



$$\dot{\vec{\beta}}c = \dot{\vec{v}}_\perp = \left(\dot{v},0,0\right) = \left(\frac{v^2}{\rho},0,0\right) \qquad \vec{\beta}c = \vec{v} = \left(0,0,v\right)$$

$$\vec{n} = \left(-\sin\theta\cos\varphi, -\sin\theta\sin\varphi, \cos\theta\right)$$

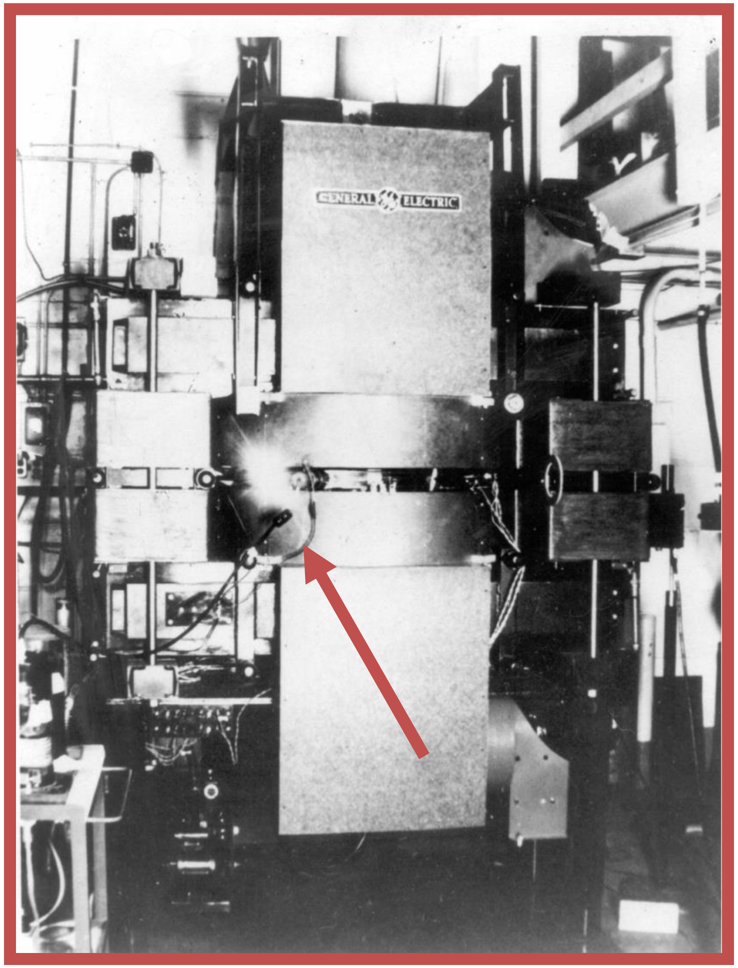
$$\vec{n}\times\left[\left(\vec{n}+\vec{\beta}\right)\times\beta\right] = \left(\vec{n}+\vec{\beta}\right)\left(\vec{n}\vec{\beta}\right) - \vec{\beta}\left(1+\vec{n}\vec{\beta}\right)$$

$$r^5 = R^5\left(1+\vec{n}\vec{\beta}\right)^5 = R^5\left(1-\beta\cos\theta\right)^5$$

$$\frac{dP}{d\Omega} = \frac{r_e mc^3}{4\pi}\frac{\beta^4}{\rho^2}\frac{\left(1-\beta\cos\theta\right)^2 - \left(1-\beta^2\right)\sin^2\theta\cos^2\varphi}{\left(1-\beta\cos\theta\right)^5} \qquad \theta_\gamma \approx \pm\frac{1}{\gamma}$$

Observed for the First Time

April 24, 1947
visible radiation was Observed for the first time at the 70 MeV Synchrotron built at General Electric.
Since then, this radiation is called Synchrotron Radiation.



The theory was developed by Ivanenko, Pomeranchuk 1944, and Schwinger 1946.

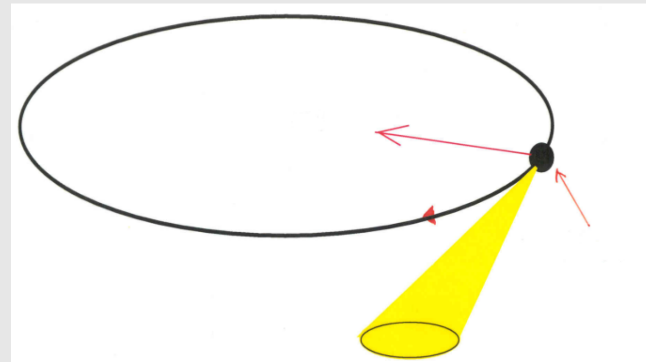
Radiation Power

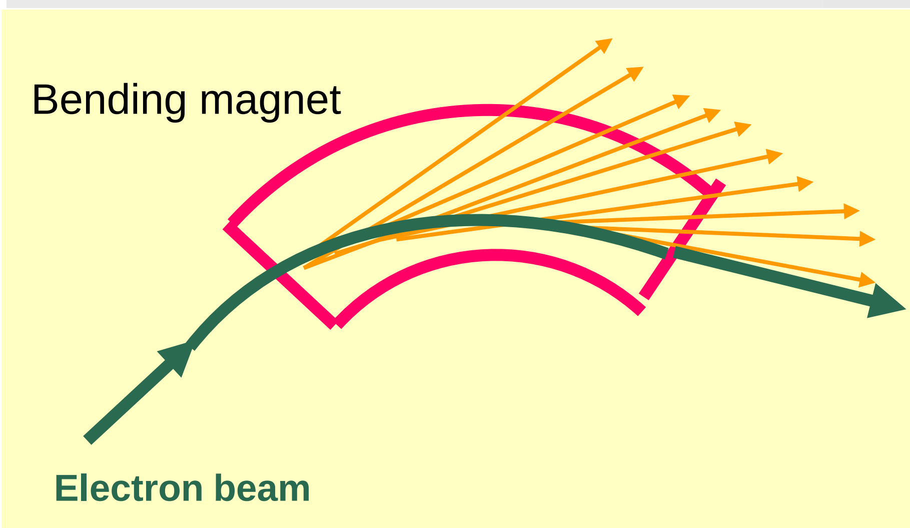
Synchrotron radiation power

Power emitted is proportional to:

$$P_\gamma = \frac{cC_\gamma}{2\pi} \cdot \frac{E^4}{\rho^2}$$

$$C_\gamma = \frac{4\pi}{3}\frac{r_e}{(m_e c^2)^3} = 8.858 \cdot 10^{-5}\left[\frac{\text{m}}{\text{GeV}^3}\right]$$





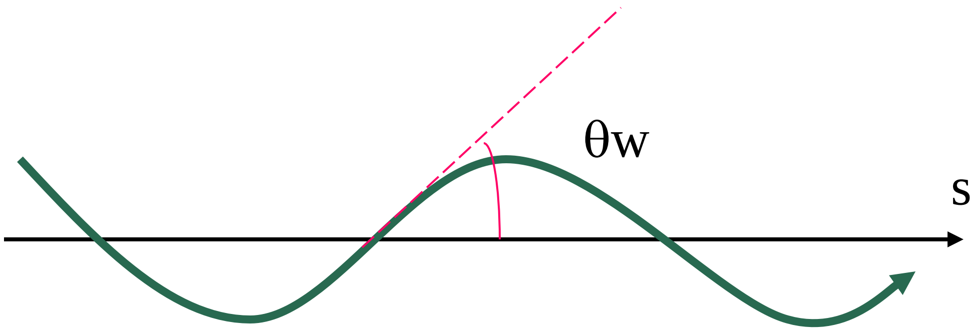
Energy loss per turn:

$$U_0 = C_\gamma \cdot \frac{E^4}{\rho}$$

ρ = Bending radius
E = Electron beam energy
γ = Lorentz factor

Wiggler and Undulator

Oscillation frequency: $\Omega_w = k_u \beta \cdot c$



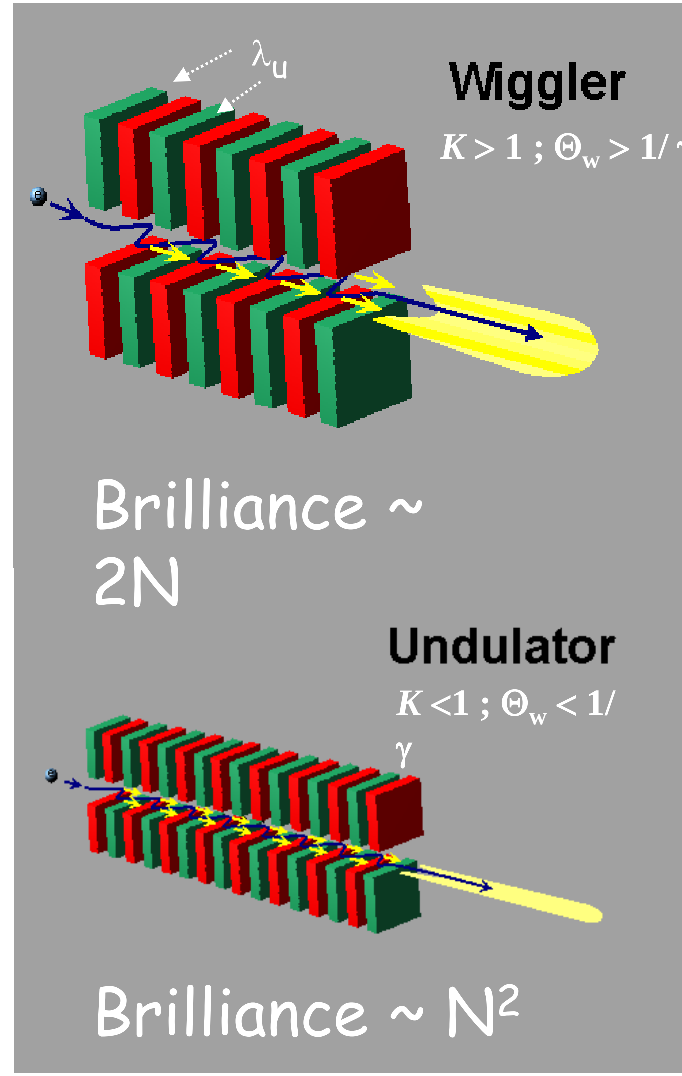
Trajectory

$$\theta_w = \frac{1}{\gamma} \cdot \frac{\lambda_u \cdot e \cdot B}{2\pi \cdot m_e \cdot c} = \frac{K}{\gamma}$$

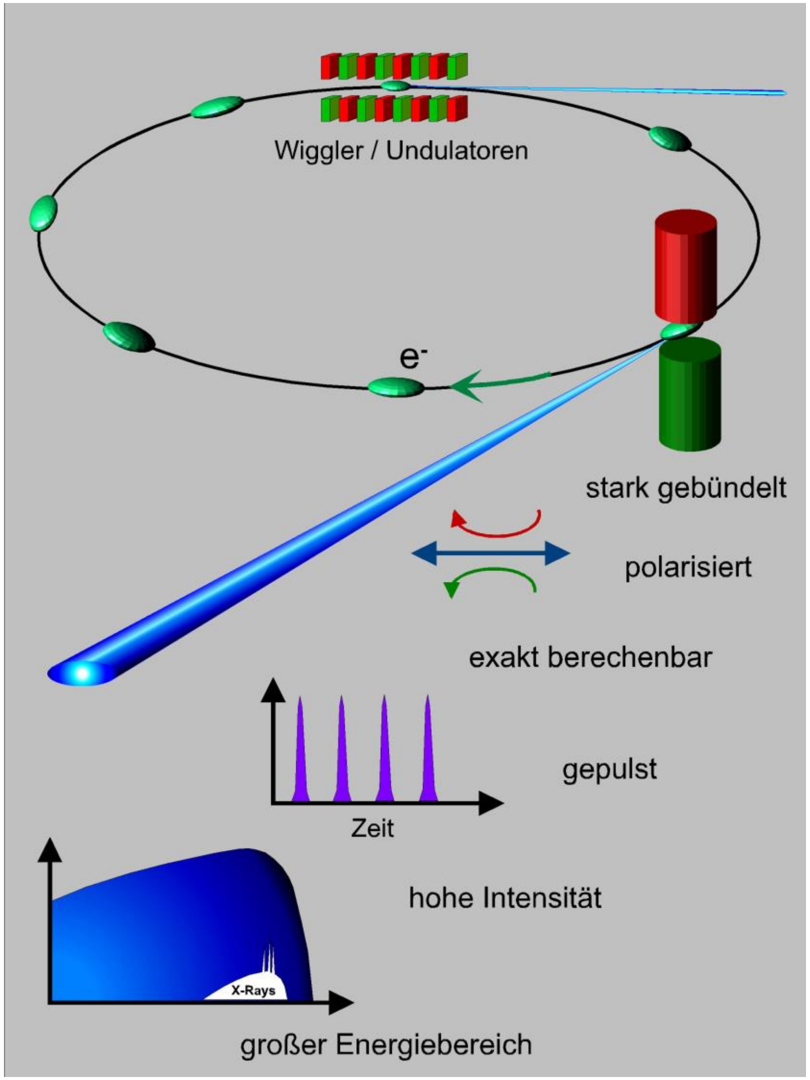
Undulator parameter

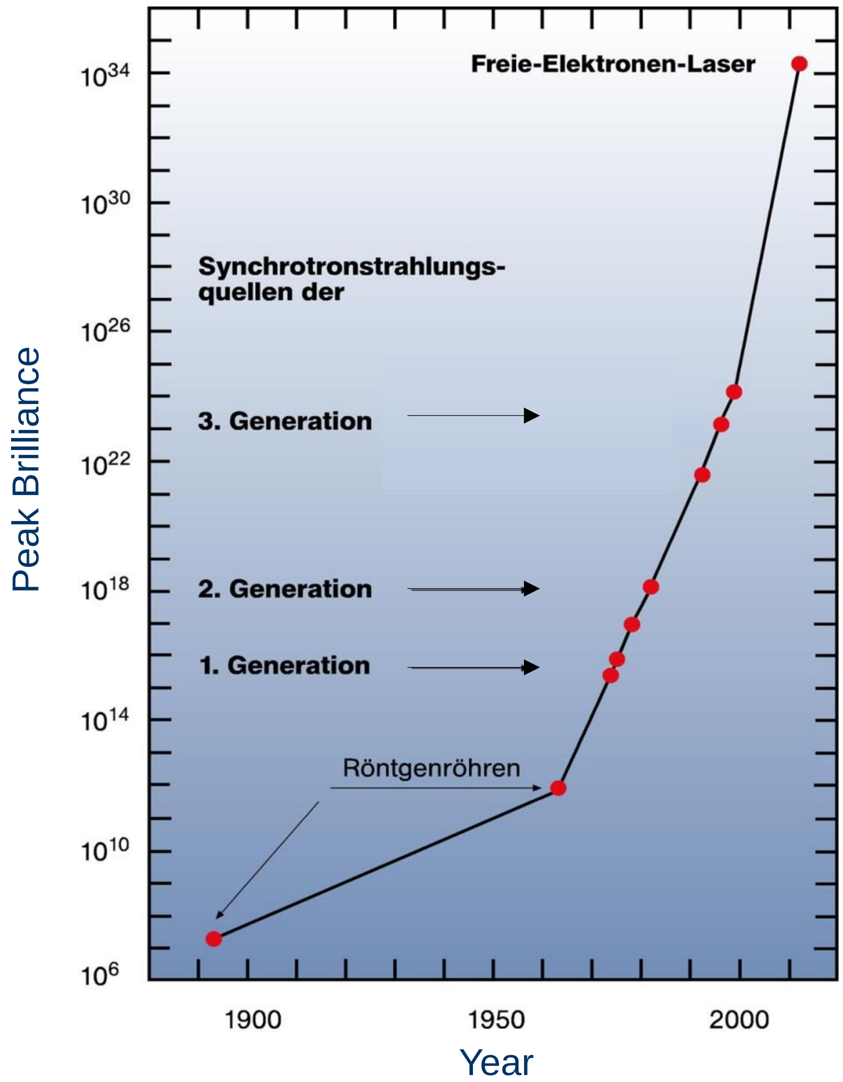
Res. Wavelength:

$$\lambda_w = \frac{\lambda_u}{2 \cdot \gamma^2} \cdot \left[1 + \frac{K^2}{2} + \gamma^2 \cdot (\theta_0)^2 \right]$$



Synchrotron Radiation Sources





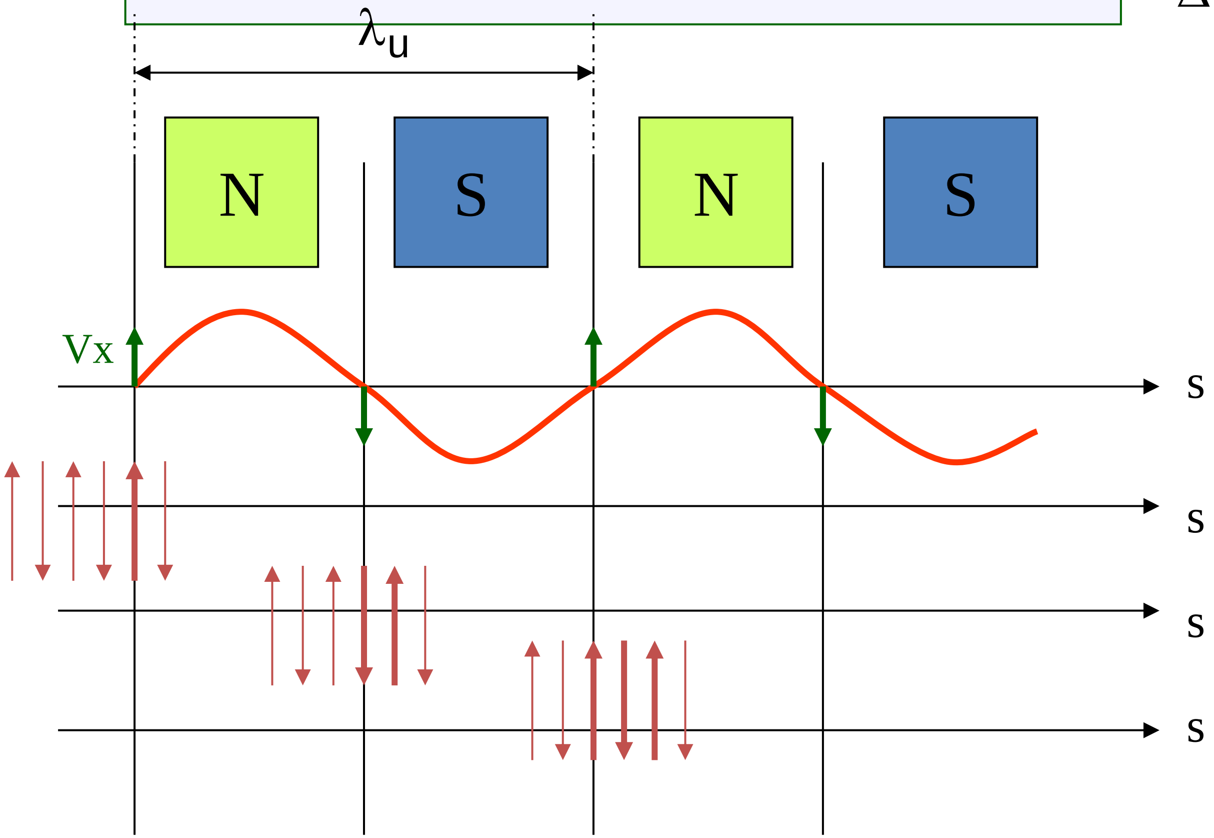
FEL- Interaction

Interchange between electron beam and radiation field:

$$\Delta W = -e \cdot \int \vec{E}_L \, d\vec{s} = -e \cdot \int \vec{v} \cdot \vec{E}_L \, dt$$

$\Delta W = 0 \qquad \vec{v} \perp \vec{E}_L$

$\Delta W \neq 0 \qquad \vec{v} \parallel \vec{E}_L$



Resonance condition for FEL:

$$\lambda_L = \frac{\lambda_u}{2 \cdot \gamma^2} \cdot \left(1 + \frac{K^2}{2}\right)$$

FEL: Equations of Motion

$$\frac{d}{ds}\psi(s) = 2 \cdot \frac{N \cdot k_u}{\gamma_r} \cdot \Delta\gamma\ (s)$$

$$\gamma_r = \gamma\left(\frac{d}{ds}\psi(s) = 0\right)$$

$$\Delta\gamma = \gamma - \gamma_r$$

$$\frac{d}{ds}\Delta\gamma(s) = \frac{-k_u \cdot K_L \cdot K}{2 \cdot \gamma_r} \cdot \sqrt{F(N \cdot \eta)} \cdot \sin(\psi(s))$$

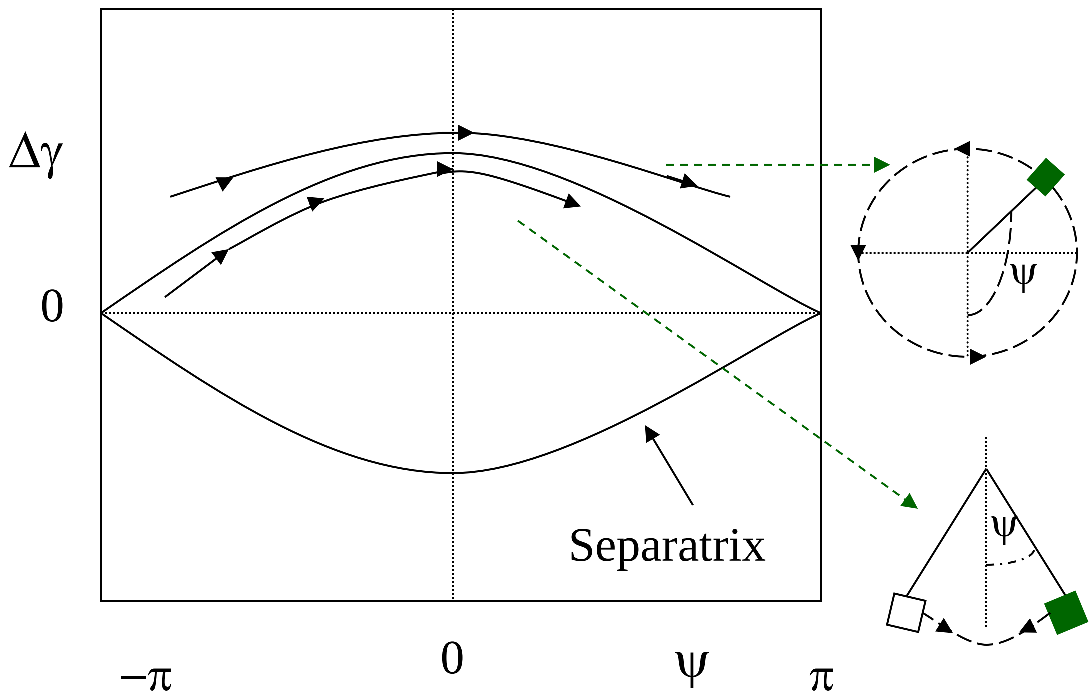
$$K_L = \frac{e \cdot E_{L,0}}{k_u \cdot m_e \cdot c^2}$$

Pendulum:

$$\frac{d^2}{ds^2}\psi(s) + (\Omega_L)^2 \cdot \sin(\psi(s)) = 0$$

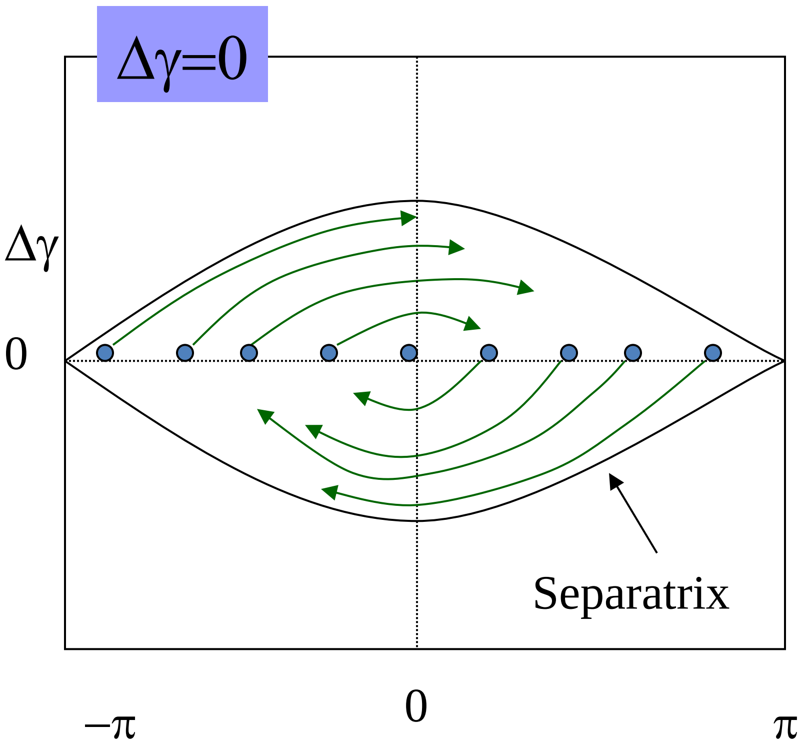
Frequency:

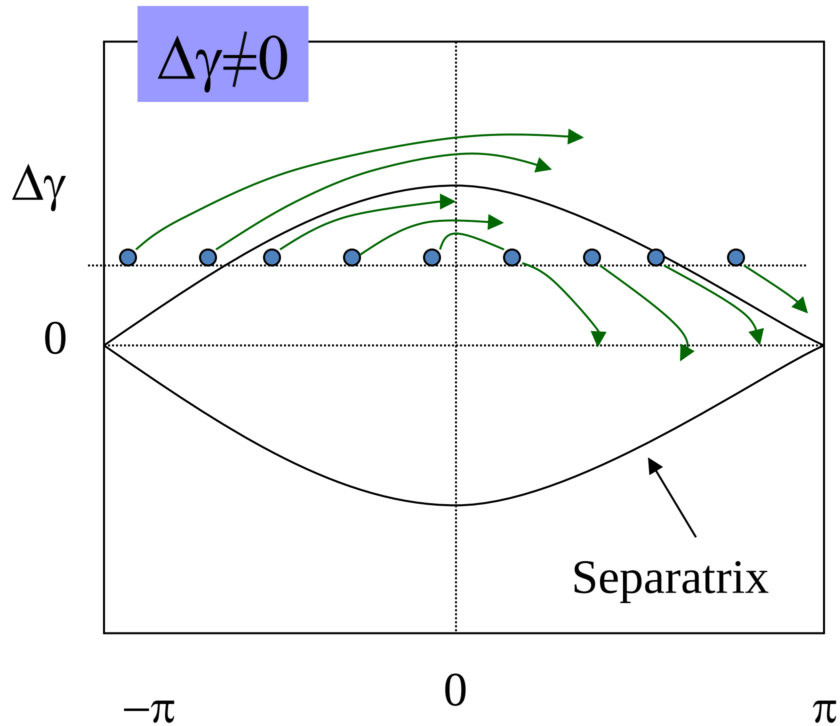
$$\Omega_L = \frac{N \cdot k_u \cdot K_L \cdot K}{(\gamma_r)^2} \cdot \sqrt{F(N \cdot \eta)}$$



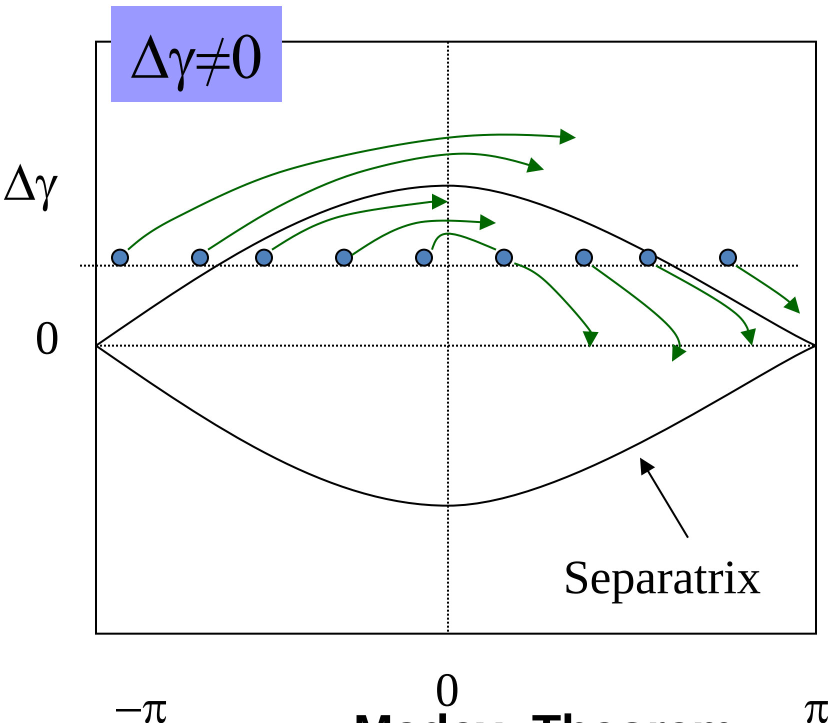
Amplification (Low Gain)

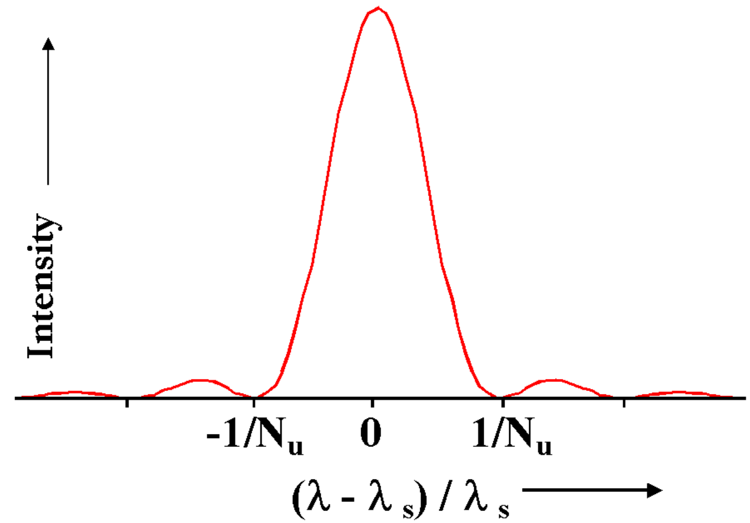
- The amplification of radiation intensity depends on the electron density.
- For small electron densities the amplification per turn is small by the Undulator.
- For $\Delta\gamma > 0$ a net intensity amplification is expected.

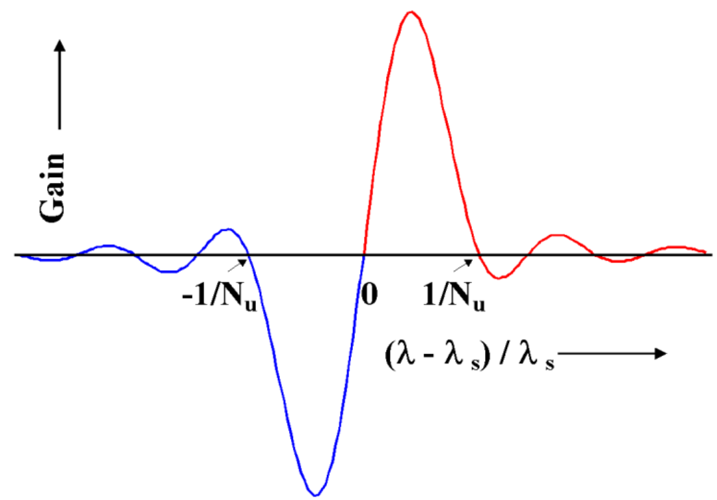




Low Gain



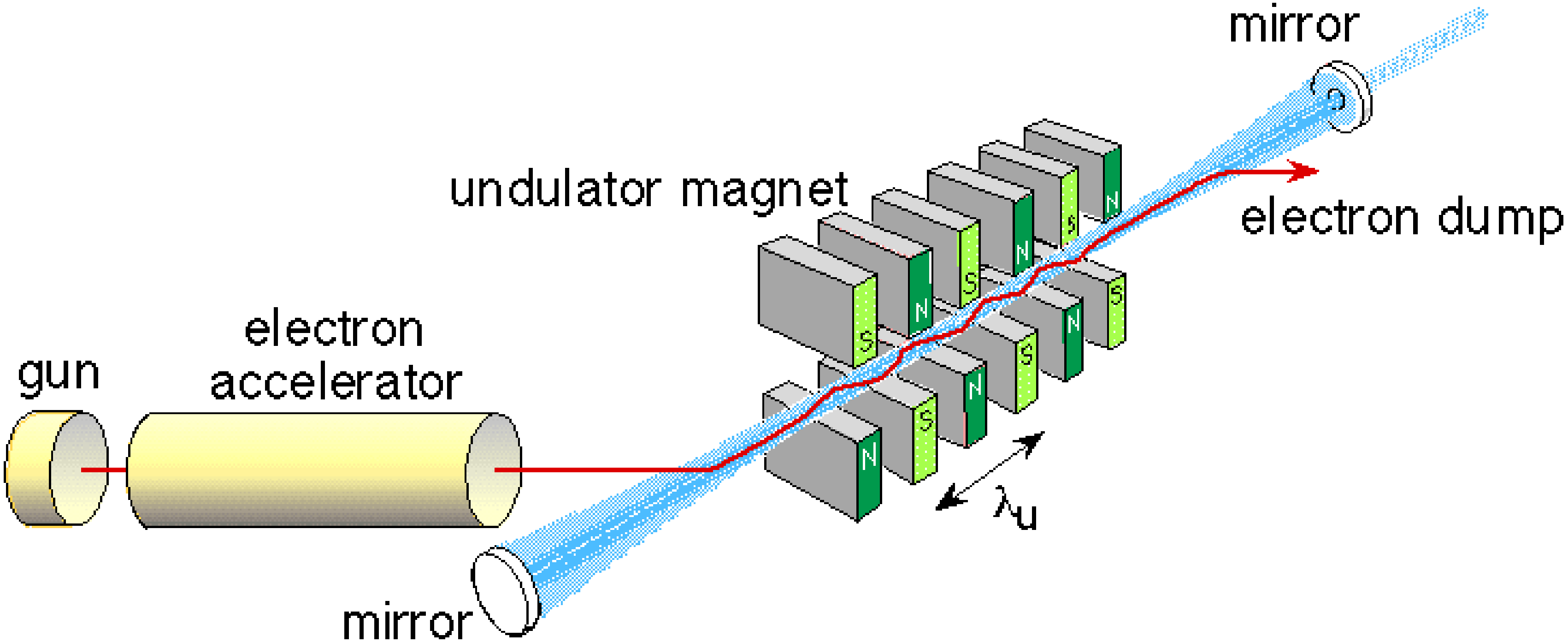




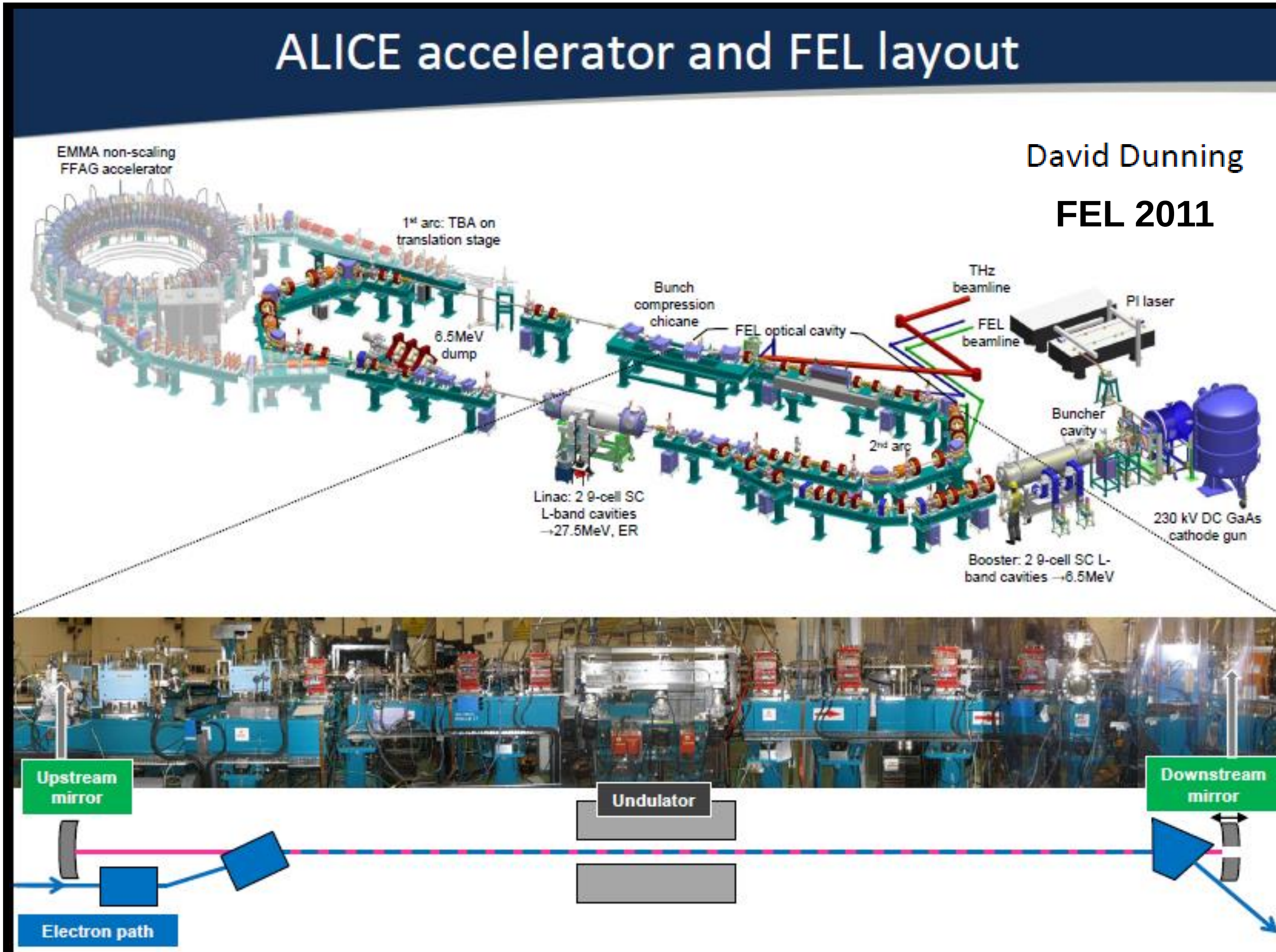
Madey -Theorem:

The amplification (Gain) is proportional to the negative derivative of the “resonance-curve” of the spontaneous undulator spectrum.

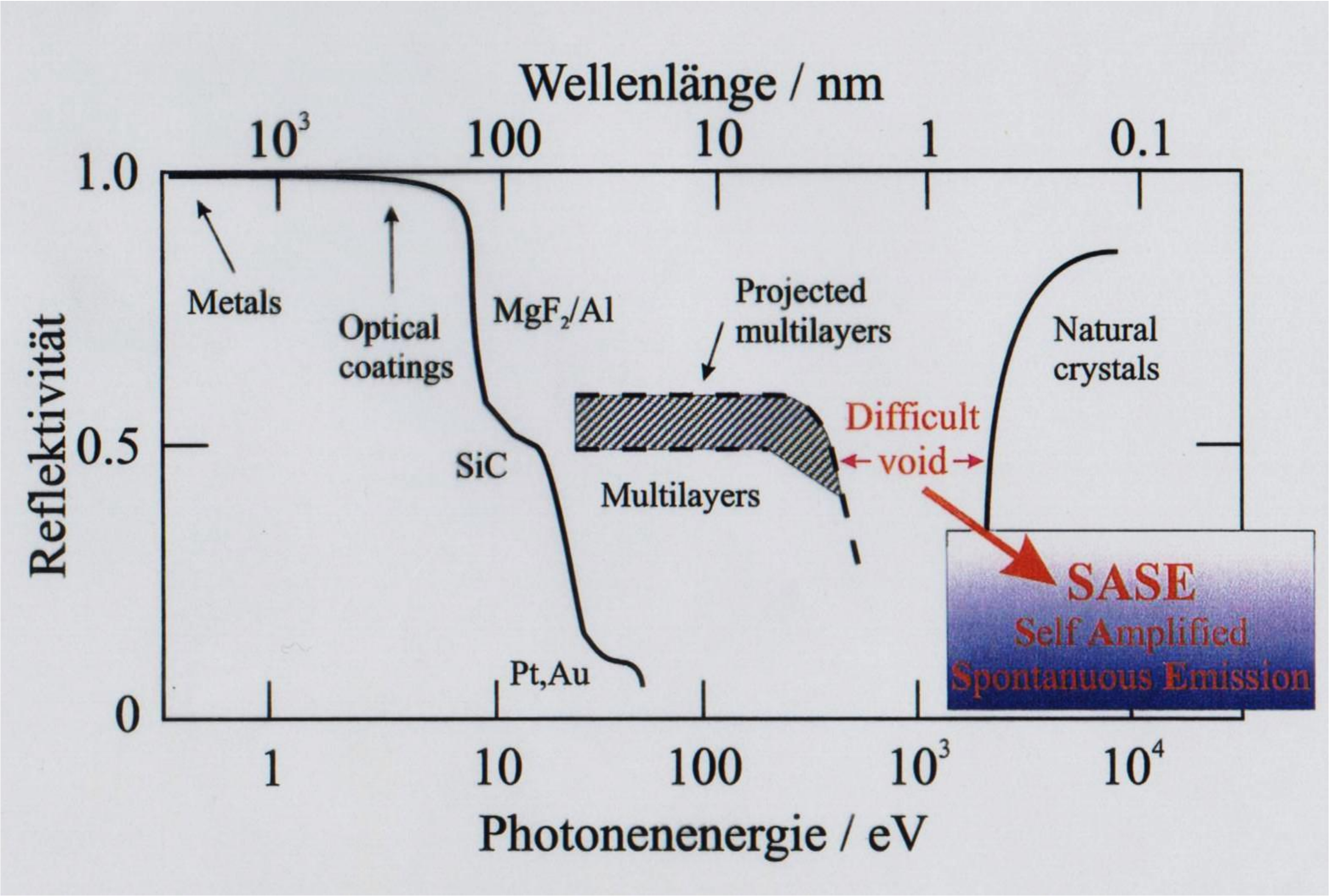
Low Gain FEL


mirror
undulator magnet
electron dump
electron
accelerator
gun
N
S
λu
mirror

Low-Gain FEL Example : ALICE

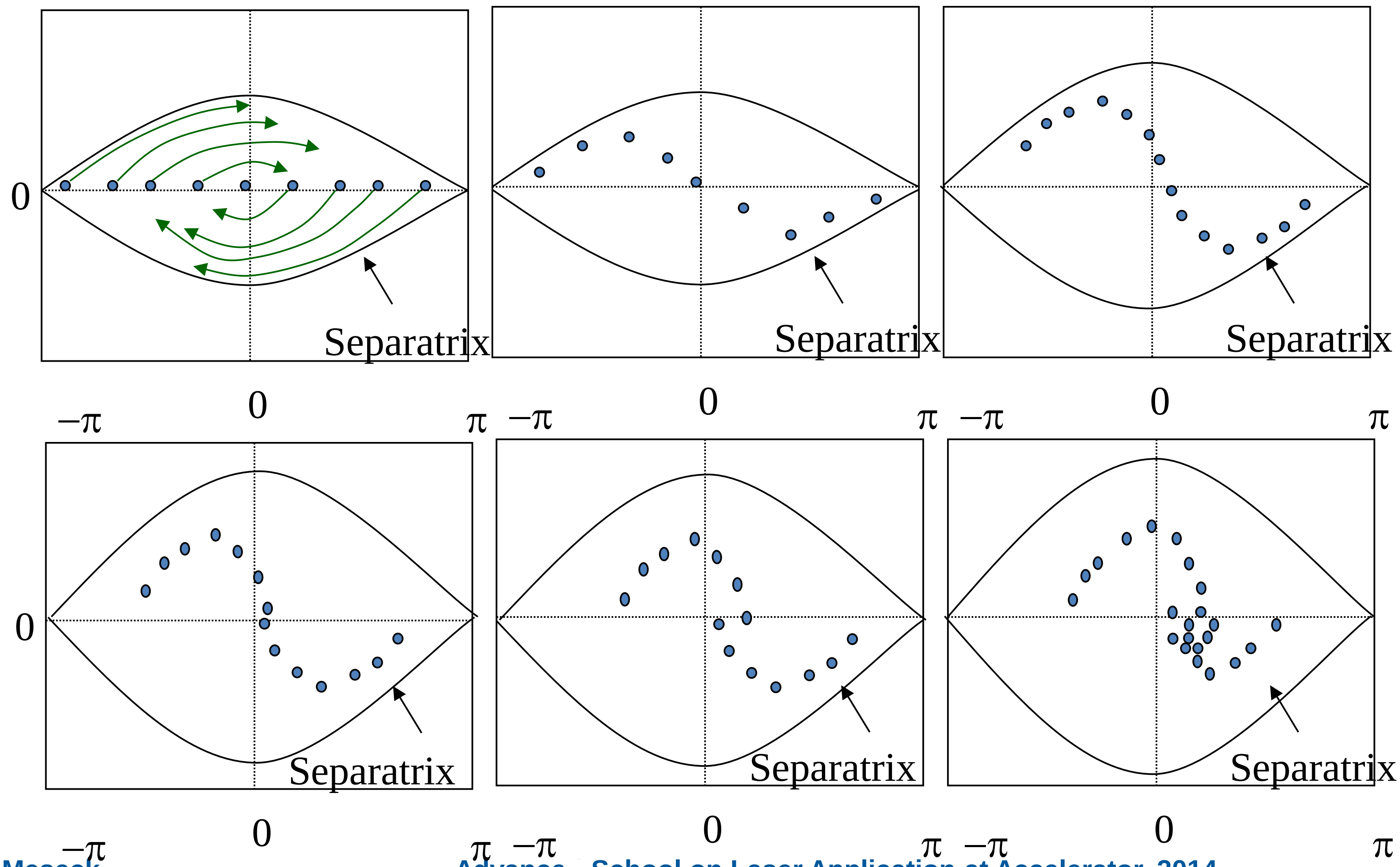


Mirrors for FELs



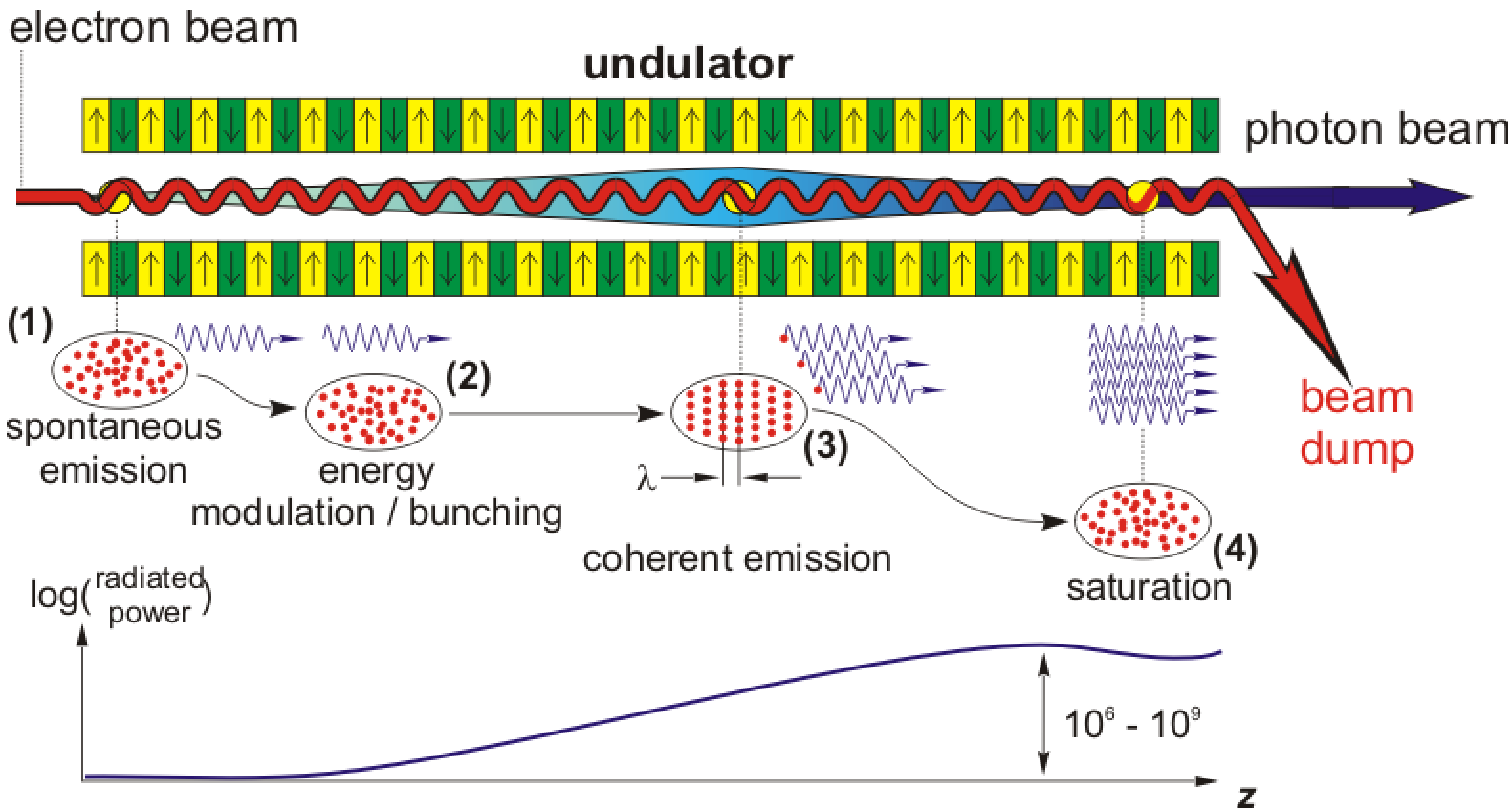
High Gain - SASE FEL

- Extremely high electron densities lead to a permanent amplification of the radiation intensity.
- The electrons are bundled into packages: micro-bunching
- The electrons radiate coherently.



High Gain - SASE FEL

- Extremely high electron densities lead to a permanent amplification of the radiation intensity.
- The electrons are bundled into packages: Micro-bunching
- The electrons radiate coherently.



SASE FEL: Efficiency Parameter

A fundamental scaling parameter for a SASE FEL is the dimensionless Pierce parameter:

$$\rho = \left[\frac{K^2 f_b^2 \lambda_u^2}{32 \lambda_p^2} \right]^{1/3}$$

Radiation wavelength:

$$\lambda = \frac{\lambda_u}{2\gamma^2} \left(1 + K^2\right)$$

Radiation Power:

$$P_{out} \approx \rho \cdot P_{beam} = \rho \cdot (I \cdot E)$$

Once the FEL interaction has started, the radiation intensity starts to grow exponentially along the undulator. The e-folding length of the radiation power called the gain length is given by

$$l_g = \frac{\lambda_u}{4\pi \sqrt{3}\, \rho}$$

The fact that the repulsive space charge force inside the bunch counteracts the formation of microbunches needs to be taken into account::

$$\lambda_p = \frac{2\pi}{\sqrt{2I \big/ \left(\gamma^3 I_A \sigma^2\right)}}$$

SASE FEL: Slippage , bandwidth, cooperation

The radiation propagates faster than the electrons. It “slips” by λ per undulator period; thus electrons communicate with the ones in front, only if their separation is less than the total slippage:

$$S = N_u \lambda$$

The high gain FEL cuts and amplifies only a narrow frequency band of the initial spectrum. The typical bandwidth of the amplified spectrum is of the order of :

$$\frac{\Delta\lambda}{\lambda} \approx 2\rho$$

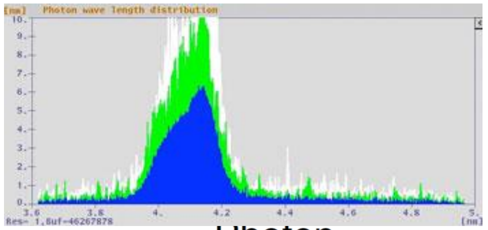
A single shot spectrum of a radiation pulse having the duration T contains spikes with a typical width of 1/T. The number of spikes in the spectrum and thus in the pulse profile is about:

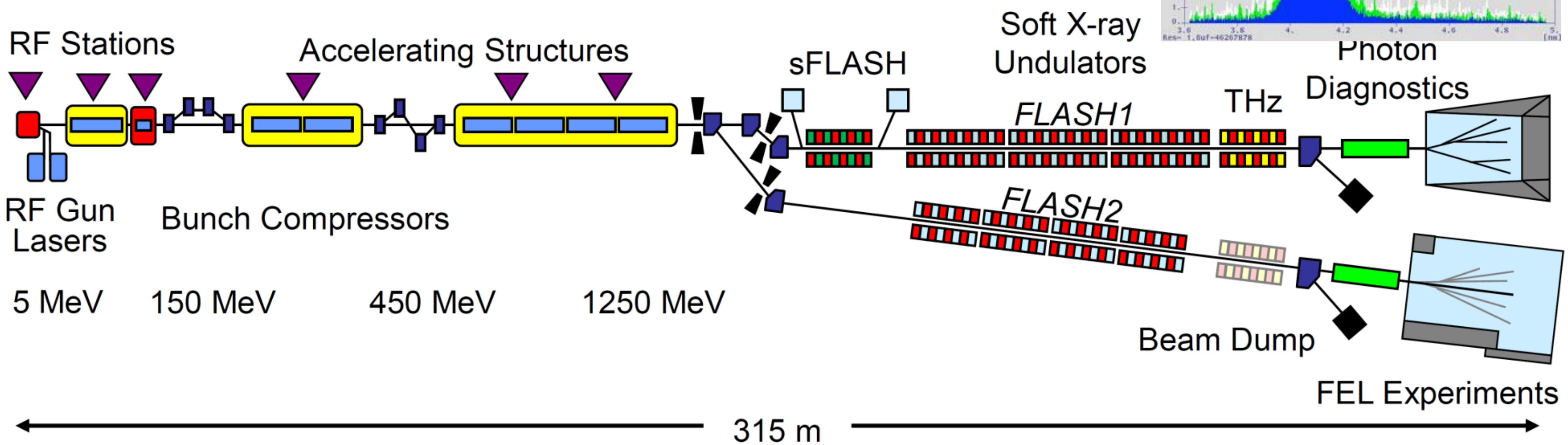
$$\frac{2\pi c T 2\rho}{\lambda}$$

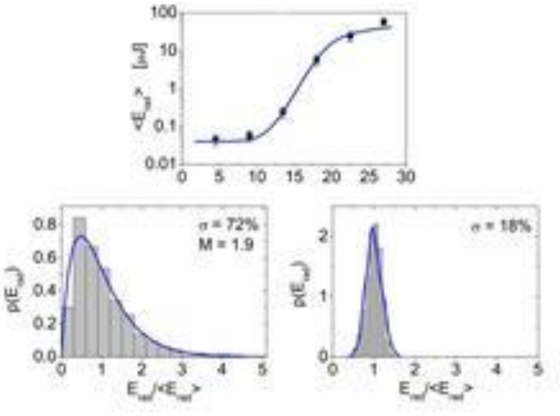
Each spike (wavepacket) has a length of λ / 4πρ. Thus, the cooperation length (slippage in one gain length) is defined as:

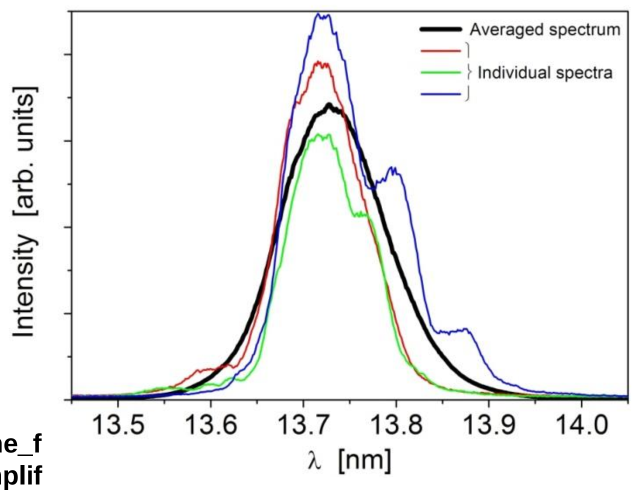
$$l_c = \frac{l_g}{\lambda_u}\lambda = \frac{\lambda}{4\pi\sqrt{3}\rho}$$

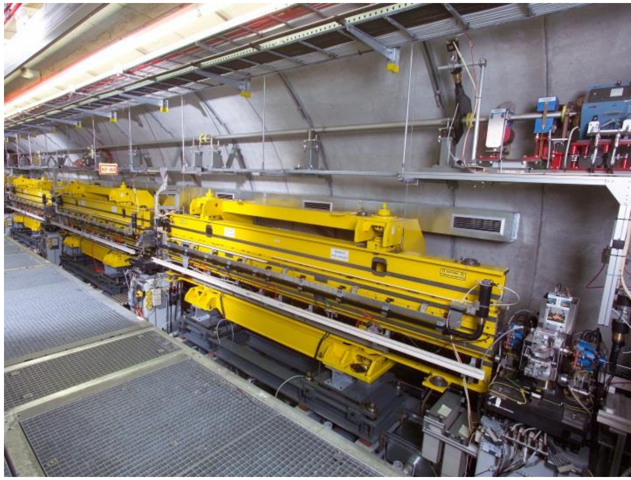
SASE-FEL Example : FLASH





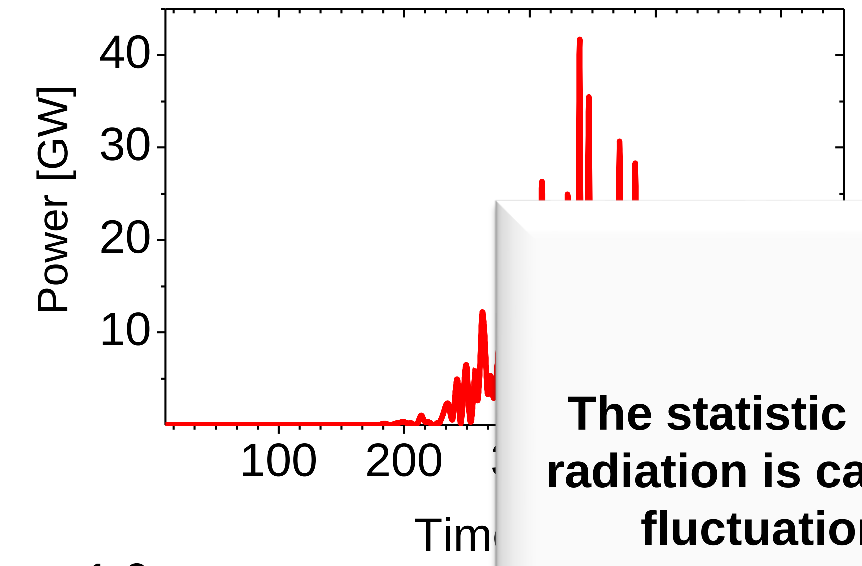


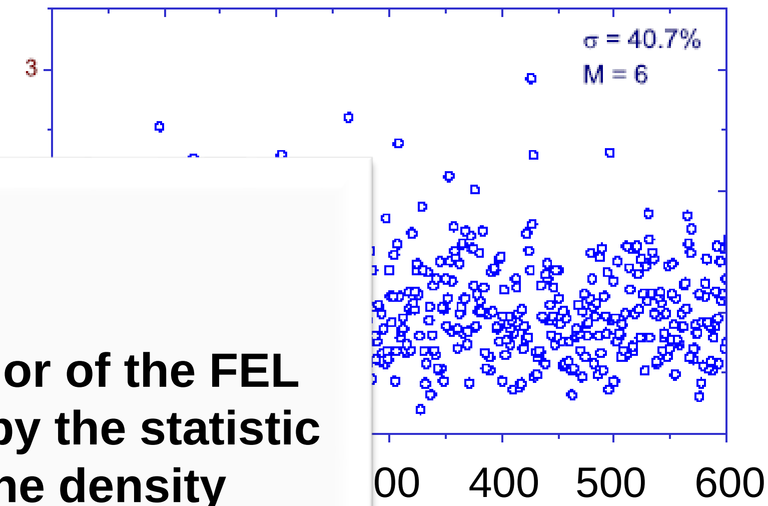




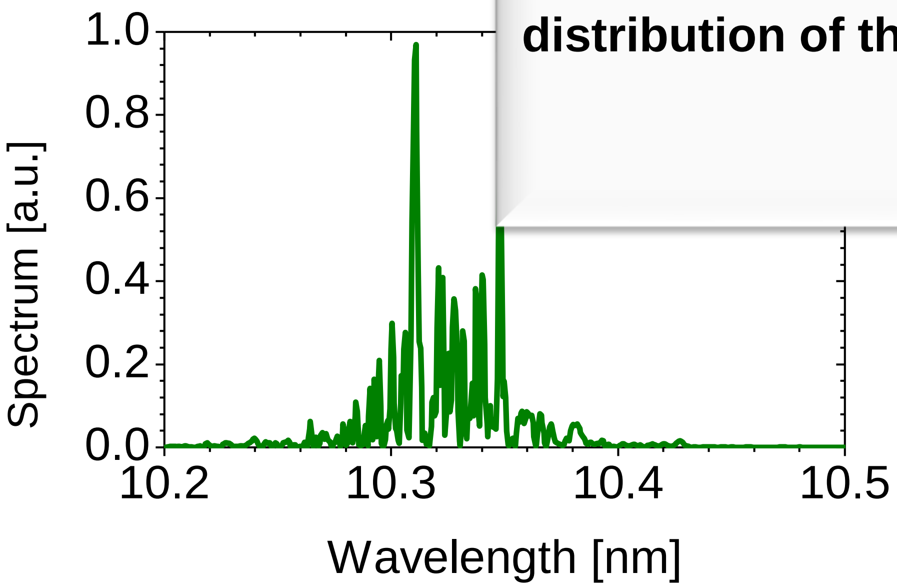
http://photon_science.desy.de/facilities/flash/the_free_electron_laser/how_it_works/sase_self_amplified_spontaneous_emission/index_eng.html

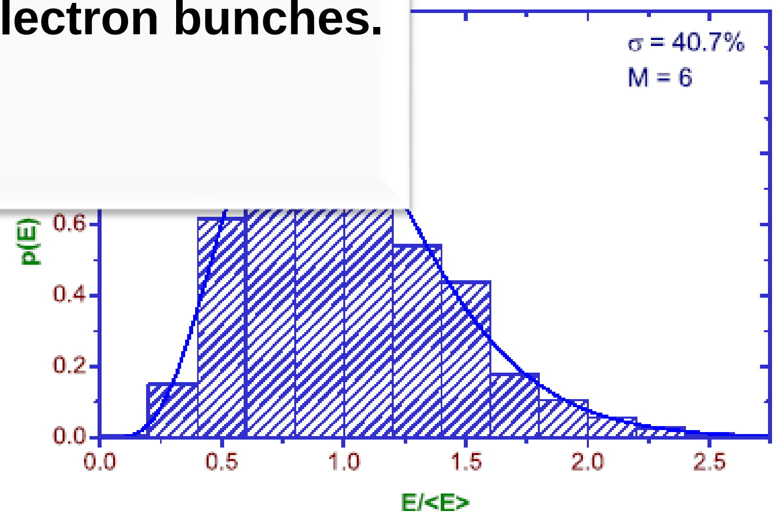
Spectral Properties of the SASE -FELs





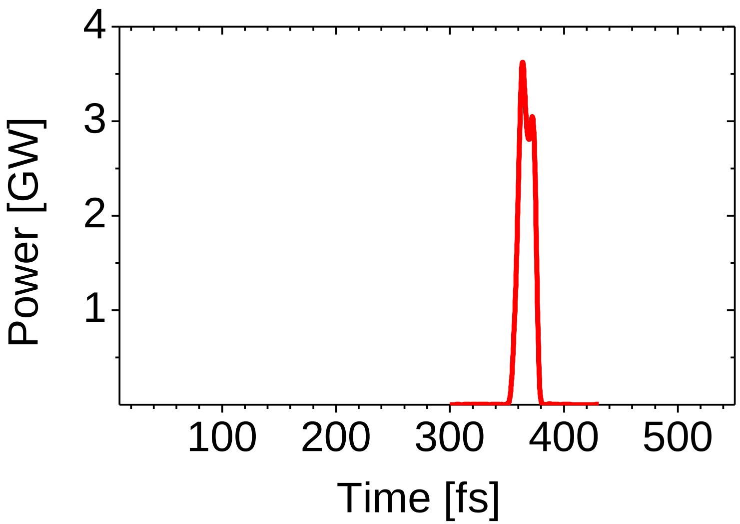
The statistic behavior of the FEL radiation is caused by the statistic fluctuations in the density distribution of the electron bunches.

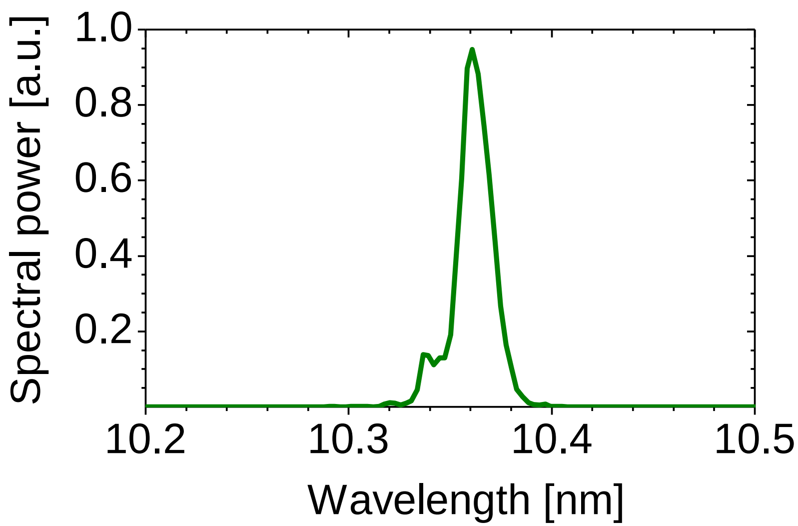




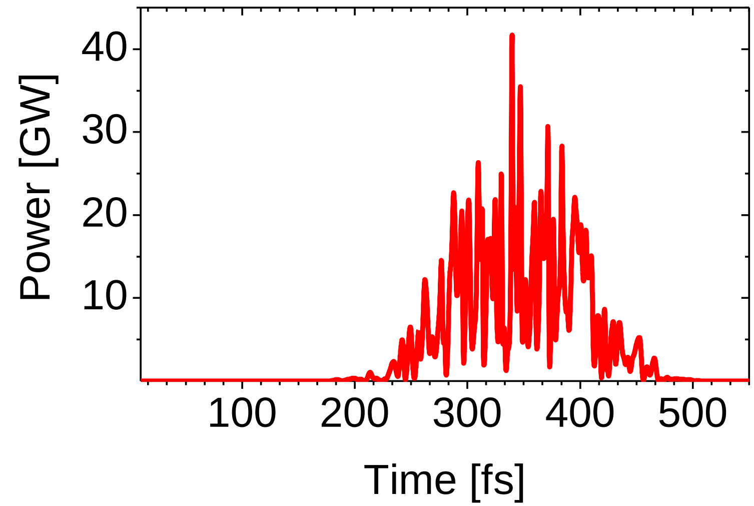
Benefits of Seeding

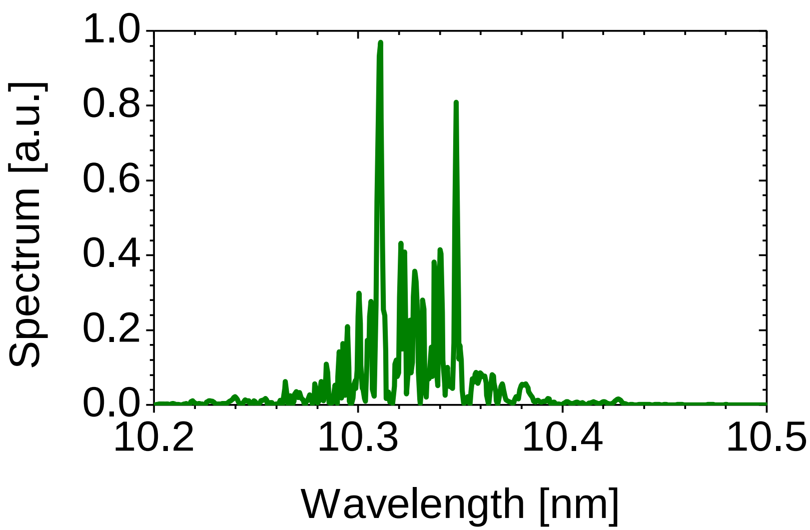
HGHG



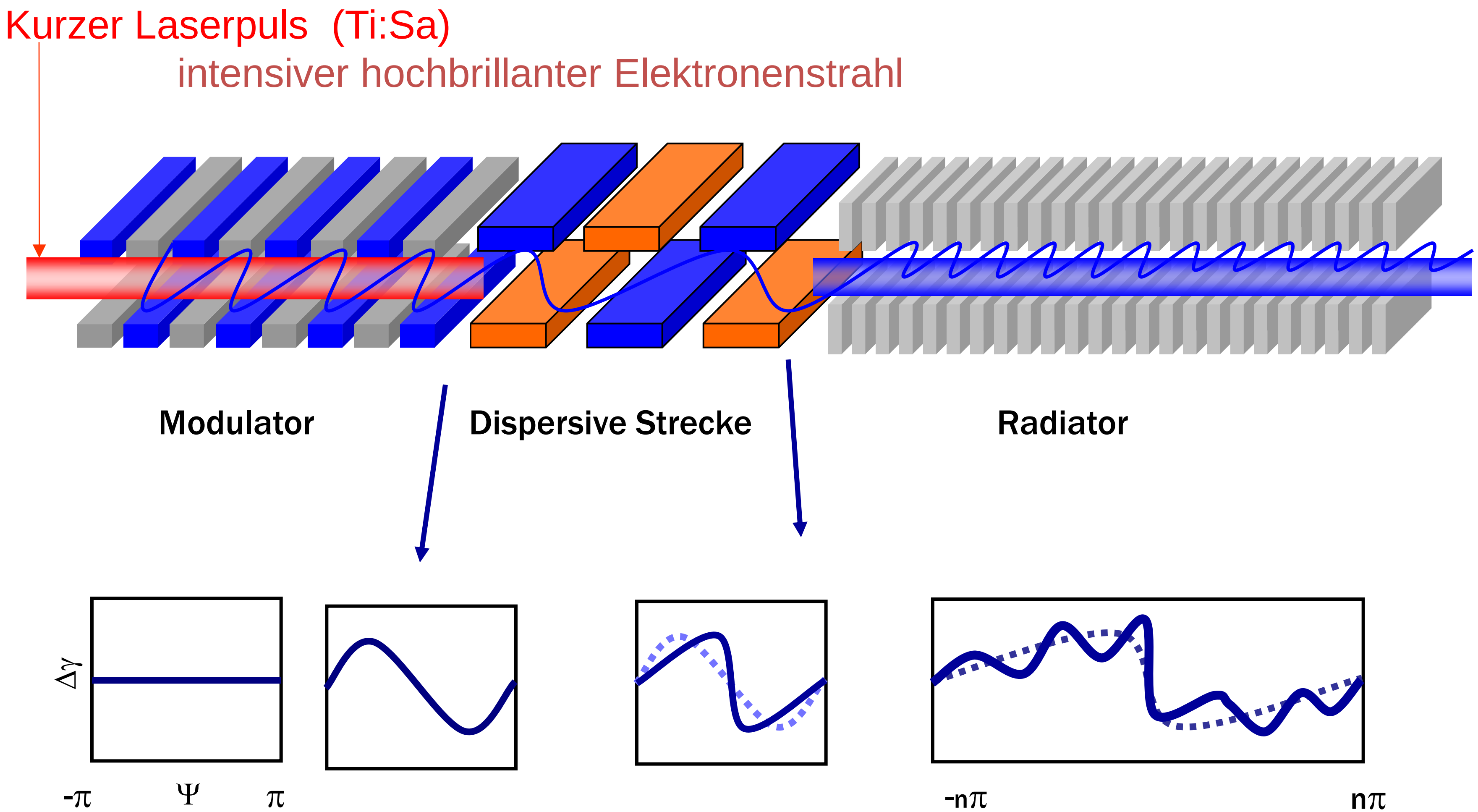


SASE



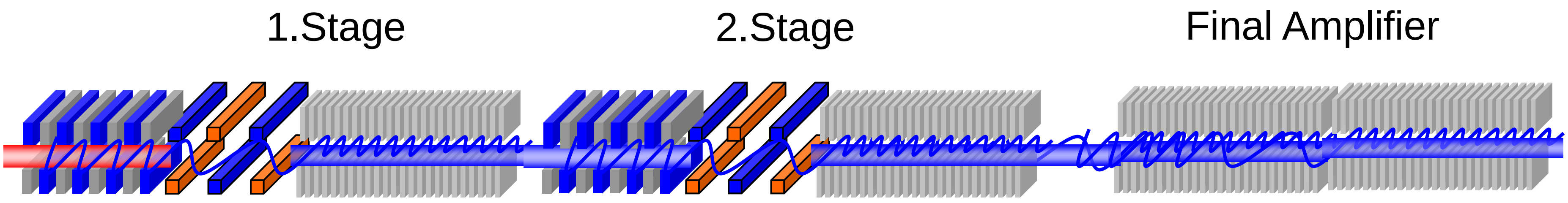


High Gain Harmonic Generation (HGHG)*



*Developed by L.-H. Yu et al.,BNL
Phys. Rev. A44/8 (1991) 5178

Cascaded HGHG-FEL

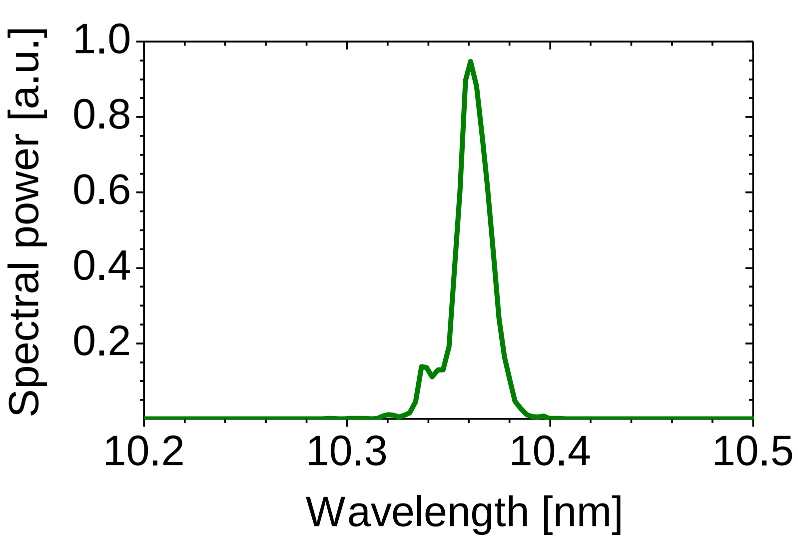


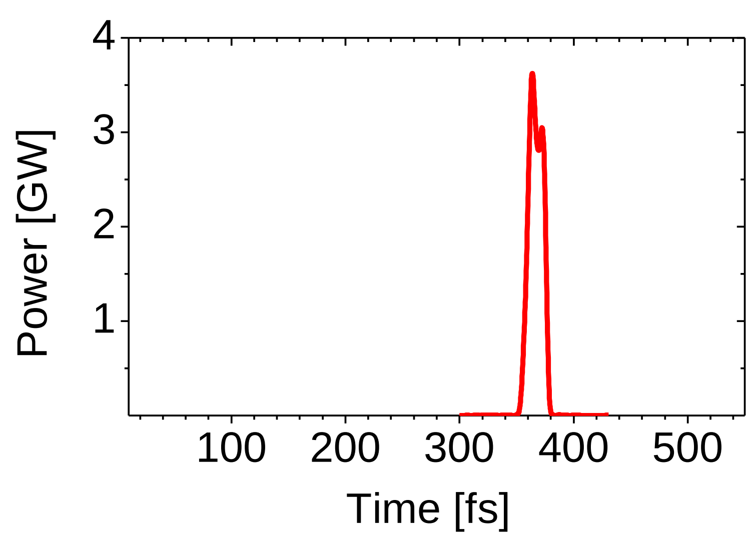
Laser
λ_s

$\lambda_{r1} = \lambda_s / n_1$

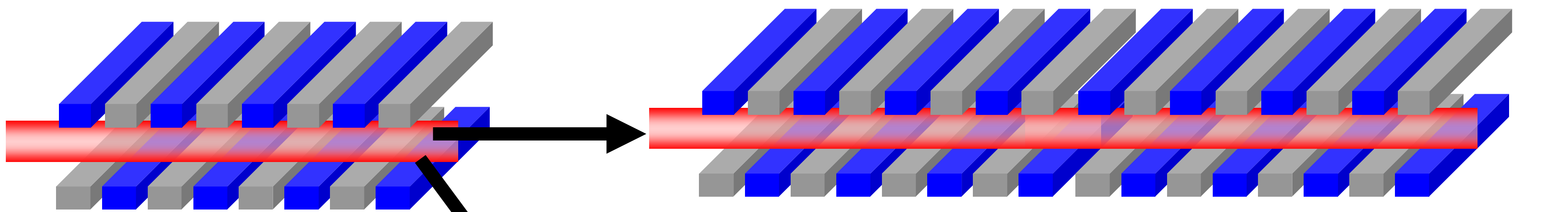
$\lambda_{r2} = \lambda_s / n_1 / n_2$

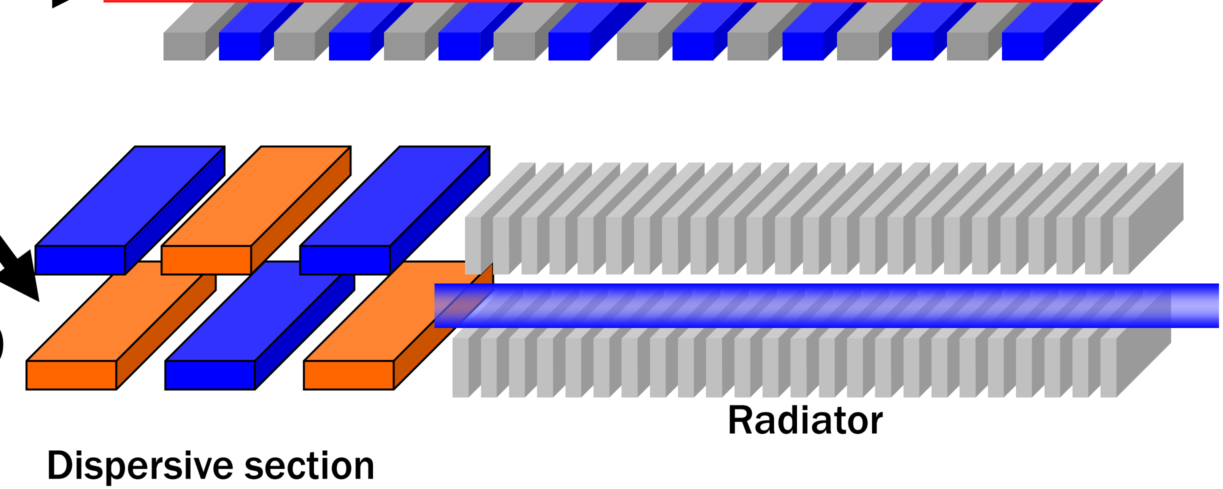
$\lambda_f = \lambda_{r2}$





Seeding : Radiation





1. Energy modulation
2. Coherent emission (fundamental or harmonics)
3. Amplification

Expected Power:

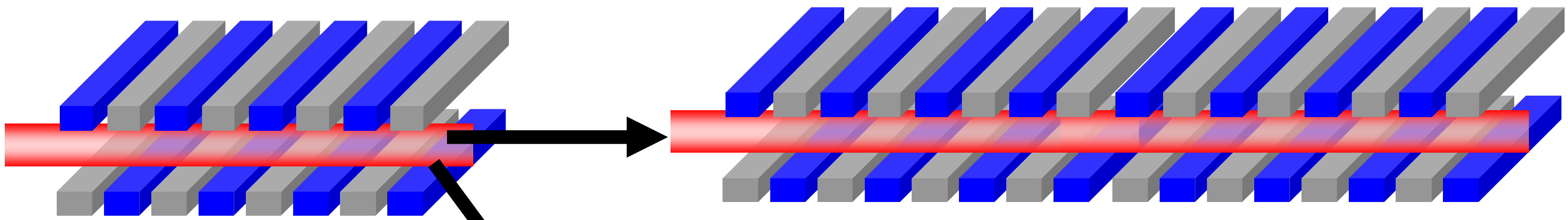
$$P_{coh.} \sim b_n^2 \frac{I^2}{\varepsilon^2} \frac{K^2}{\gamma^2}$$

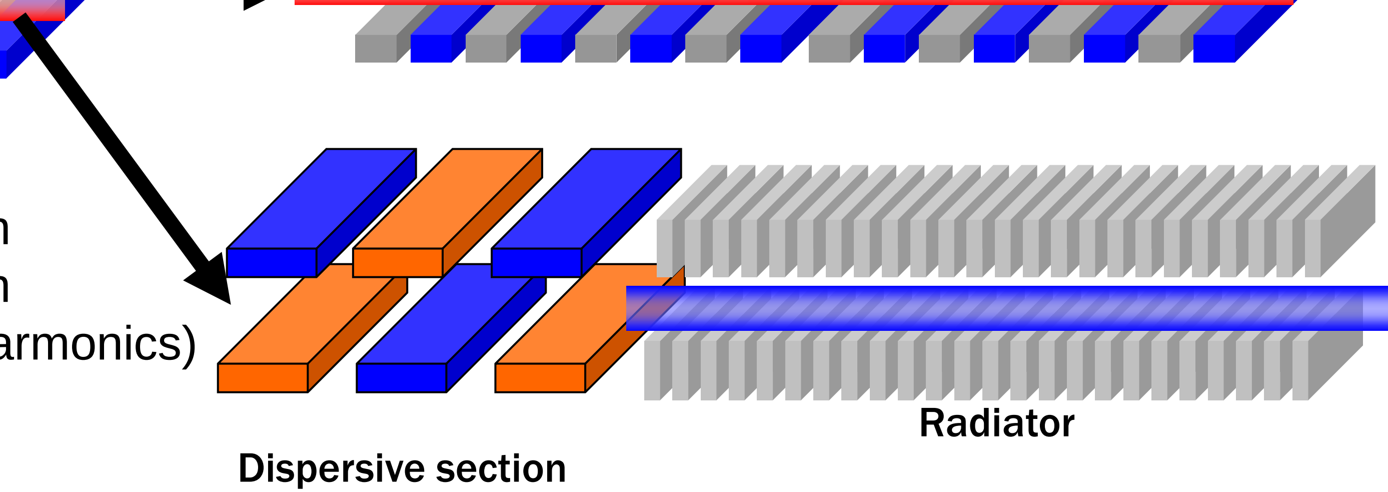
For given period length, the desired Wavelength determines this ratio via resonance condition.

Electron beam brightness is determined by the injector and accelerator.

Microbunching depends on the energy modulation.

Seeding: Electron Beam





1. Energy modulation
2. Coherent emission (fundamental or harmonics)
3. Amplification

necessary Modulation:

$$\Delta\gamma \geq n\,\sigma_\gamma$$

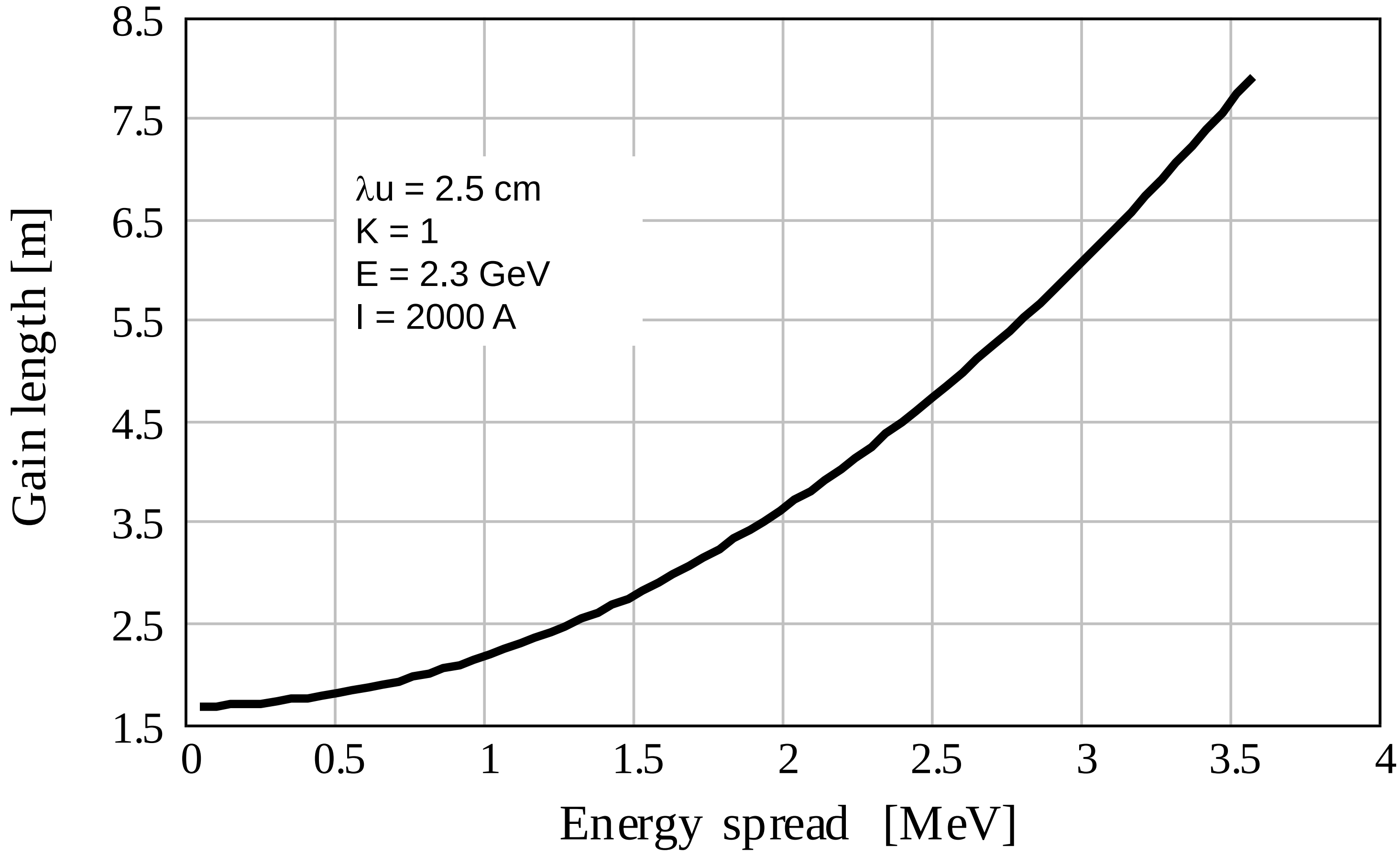
Energy modulation by the seed:

$$\Delta\gamma \sim \frac{K}{\gamma} L_{\mathrm{mod}} \sqrt{\frac{P_{seed}}{\lambda_s}}$$

Total energy deviation at the radiator entrance:

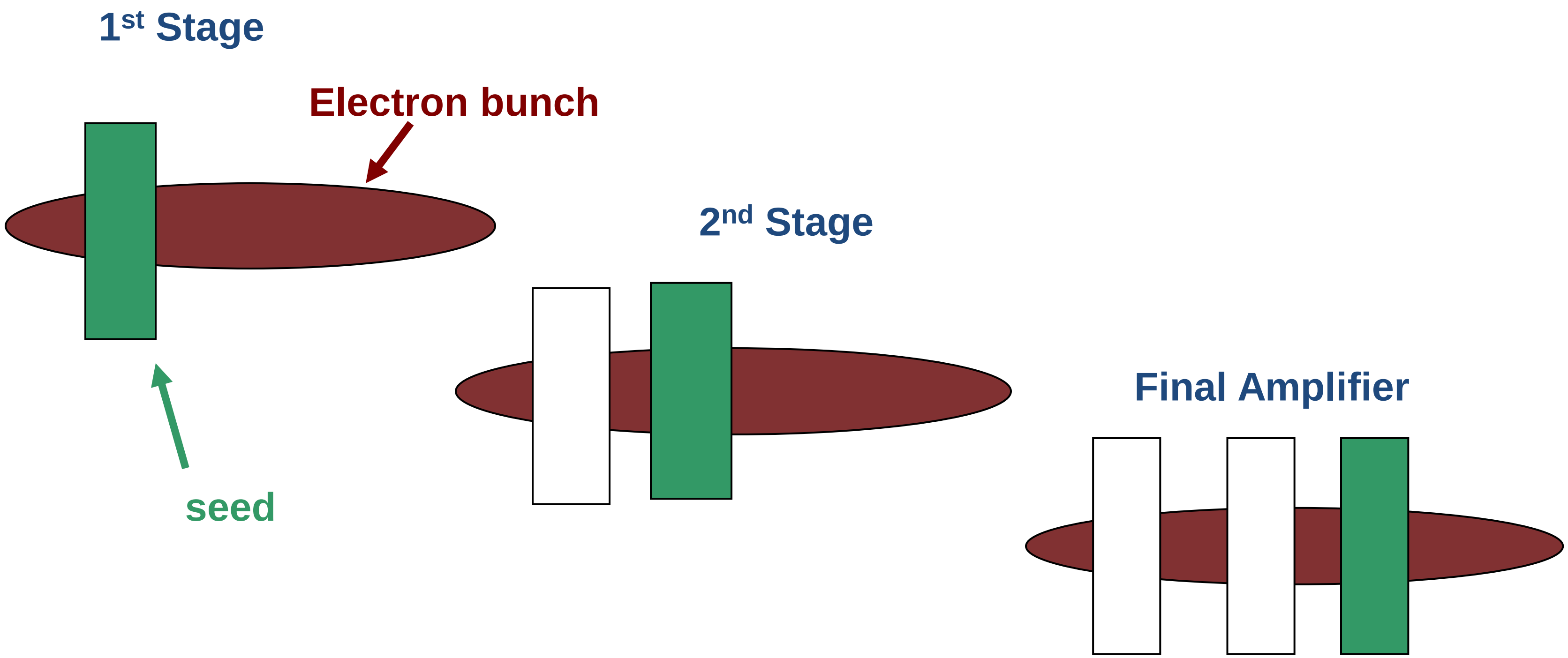
$$\sigma_{\gamma,tot} = \sqrt{\sigma_\gamma^2 + \frac{\Delta\gamma^2}{2}}$$

Linking Bunch, Seed and Undulator

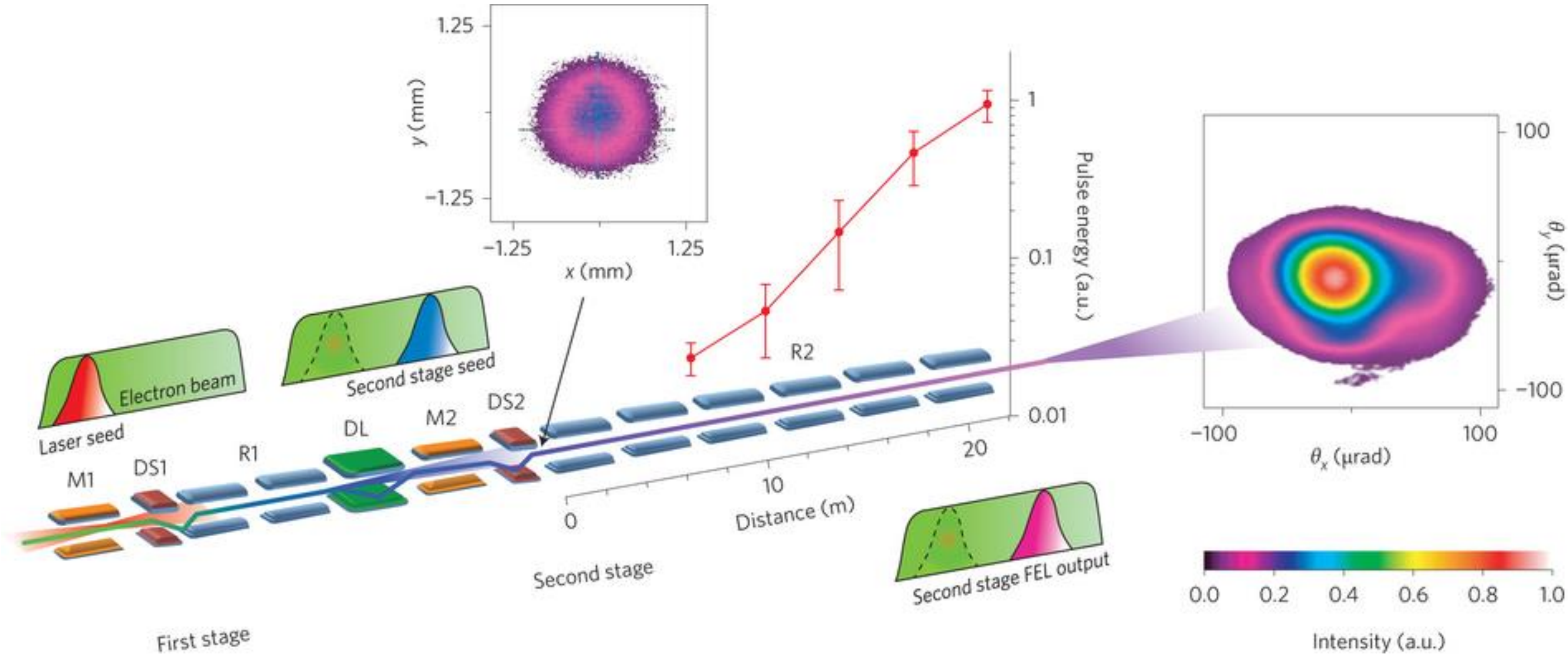


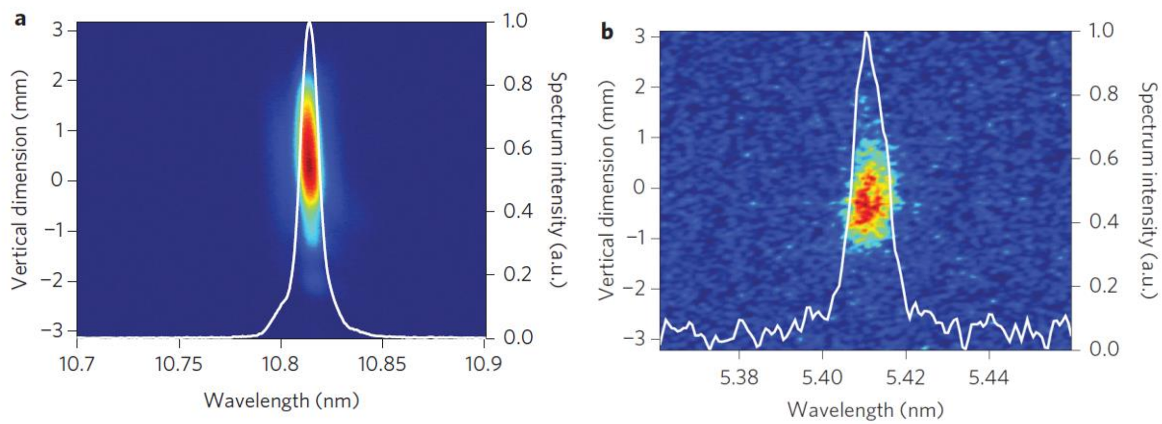
Fresh Bunch Technique

The seeded bunch part is no longer suitable for a further seeding process . → Use a long bunch and shift the interaction region for each stage.



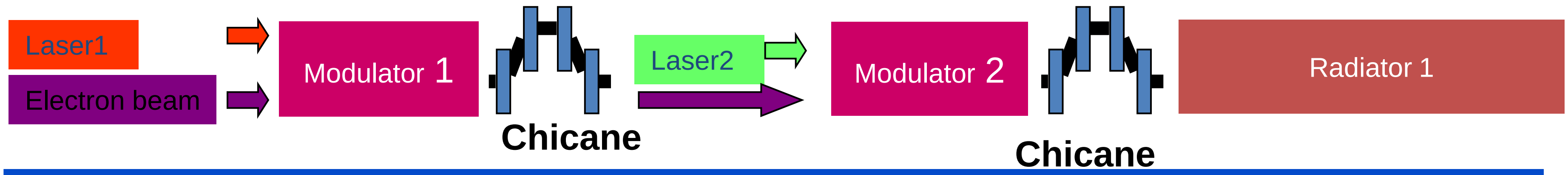
HGHG-FEL Example : FERMI





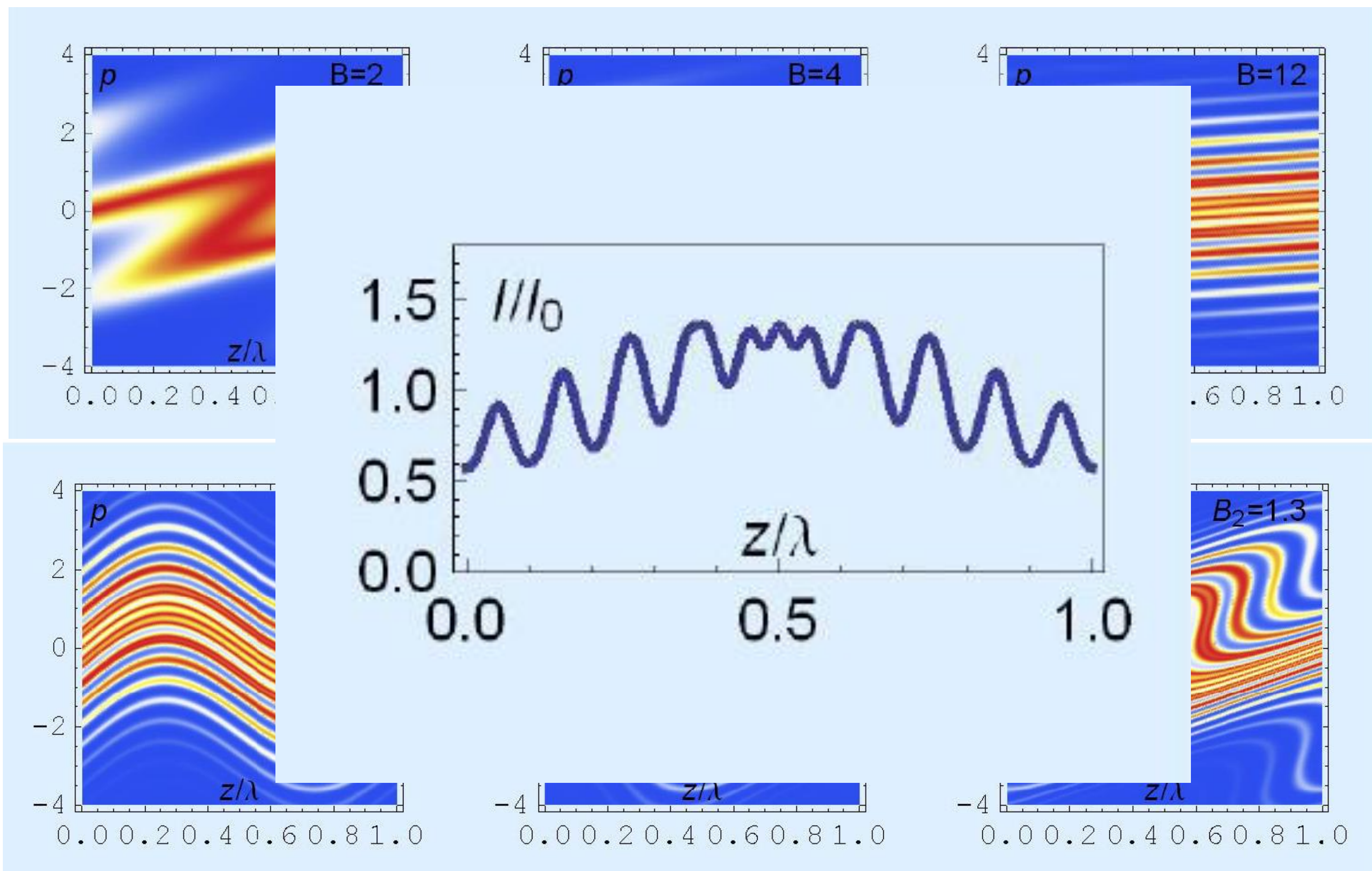
NATURE PHOTONICS | VOL 7 | NOVEMBER 2013 | p913-918

Modulator Cascade – ECHO Scheme



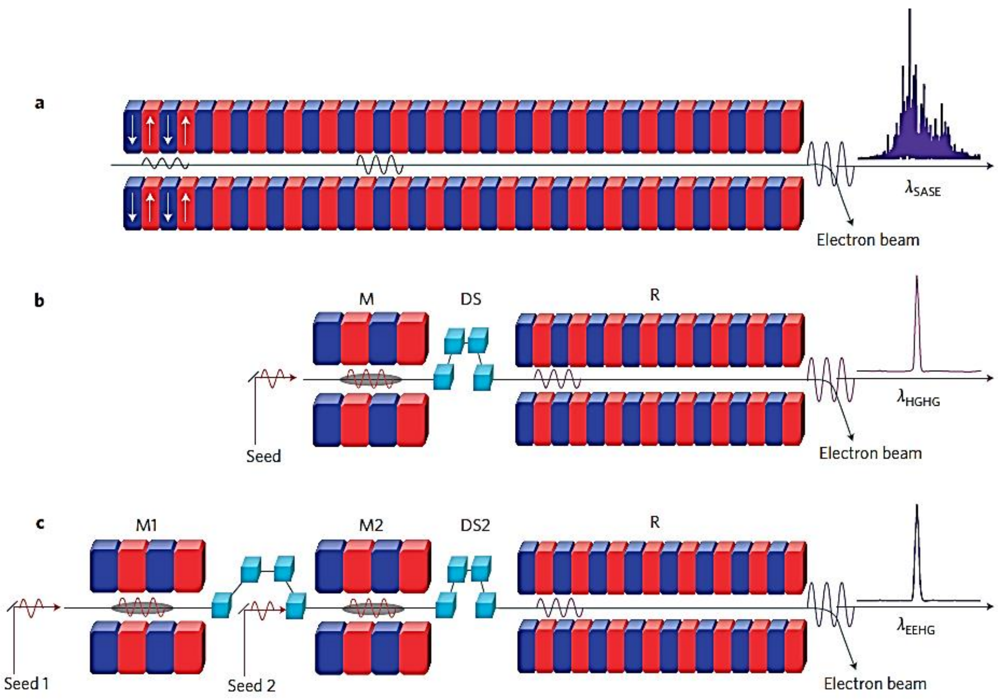
Modulator 1
+ Chicane 1

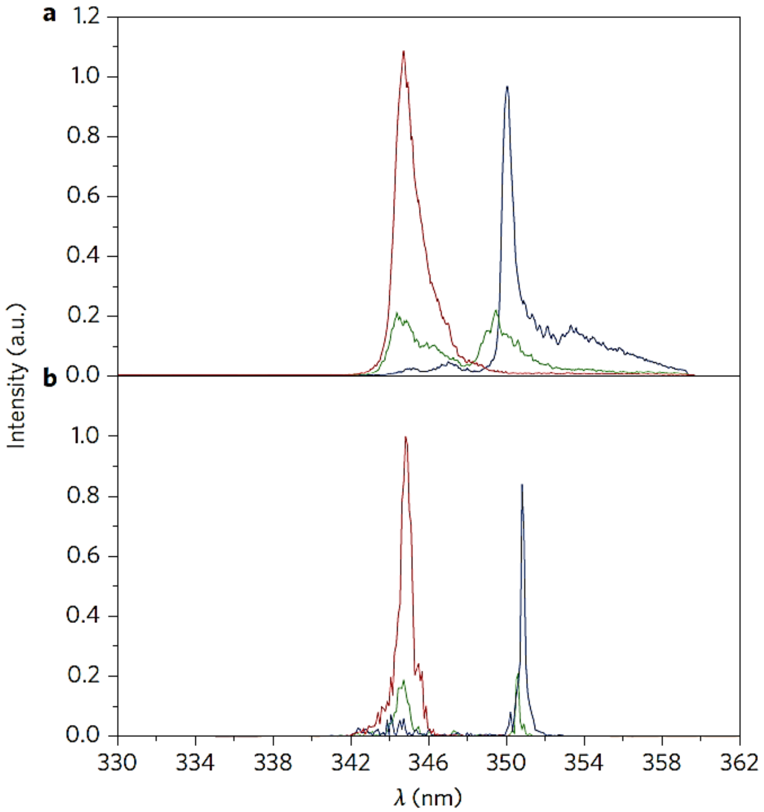
Modulator 2
+ Chicane 2



G. Stupakov
FEL 2009

EEHG Example : SDUV-FEL





NATURE PHOTONICS | VOL 6 | JUNE 2012 | p 360-363

Figure 3 | Spectra for FEL radiation. **a, Experimental results** (red line, HGHG; blue line, EEHG; green line, intermediate state between HGHG and EEHG). **b, Simulation** results (red line, HGHG; blue line, EEHG; green line, intermediate state between HGHG and EEHG).

Signal to Noise Ratio and Seed Power

For successful seeding we have to ensure that

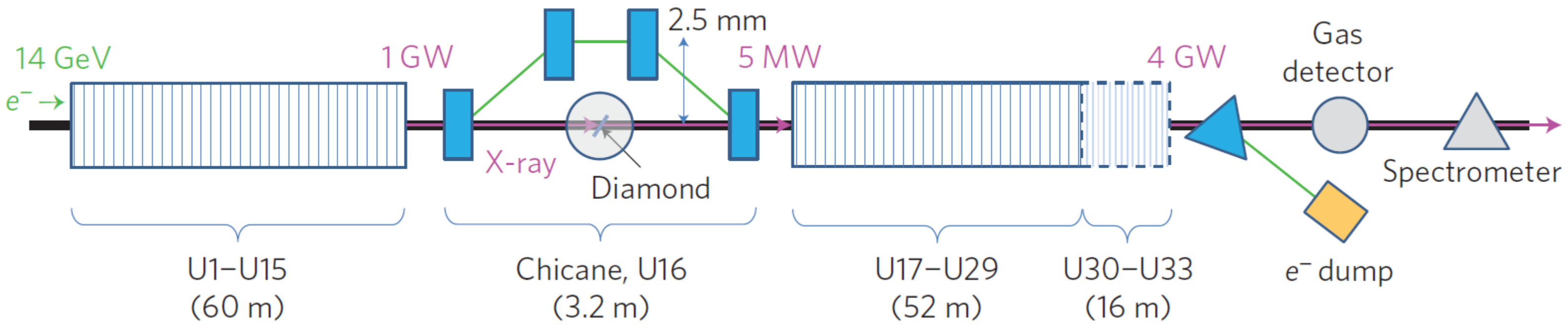
- the phase correlation and pulse length are conserved!
- the shot-noise effects are suppressed**!**

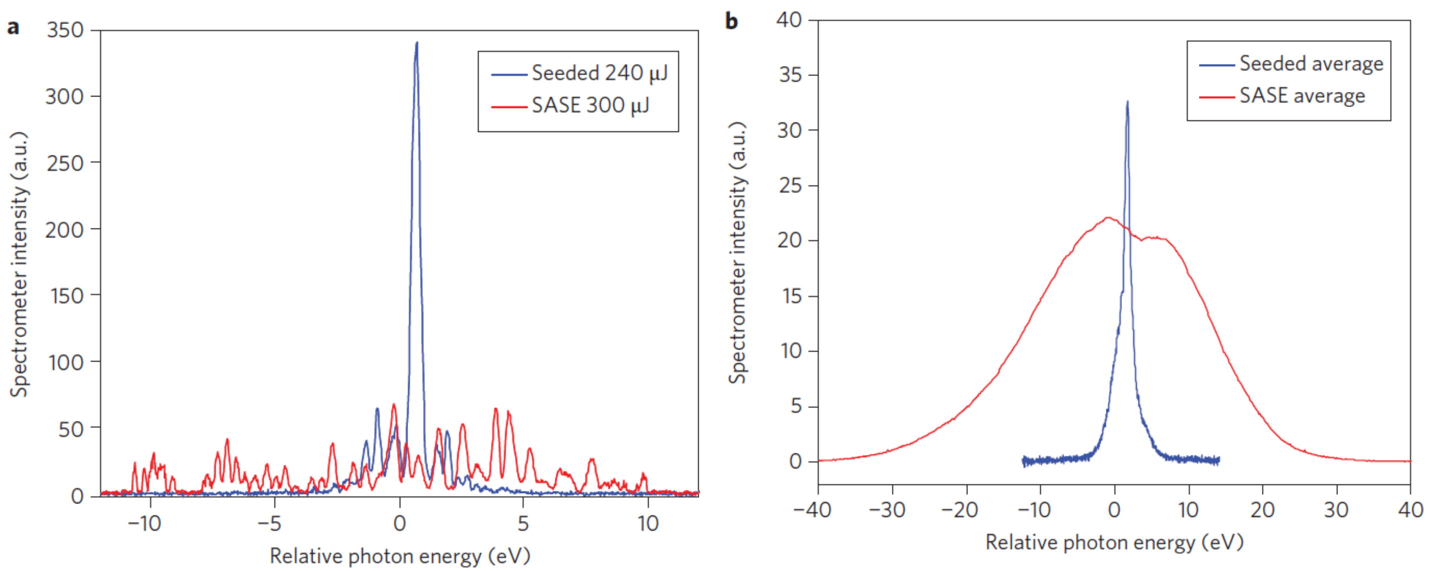
$$\left(\frac{P_s}{P_n}\right)_{out} = \frac{1}{n^2}\left(\frac{P_s}{P_n}\right)_{in}^{*}$$

- Limits the total harmonic number
- High seed power required

* E. Saldin et a., Opt. Comm. 202 (2002) 169

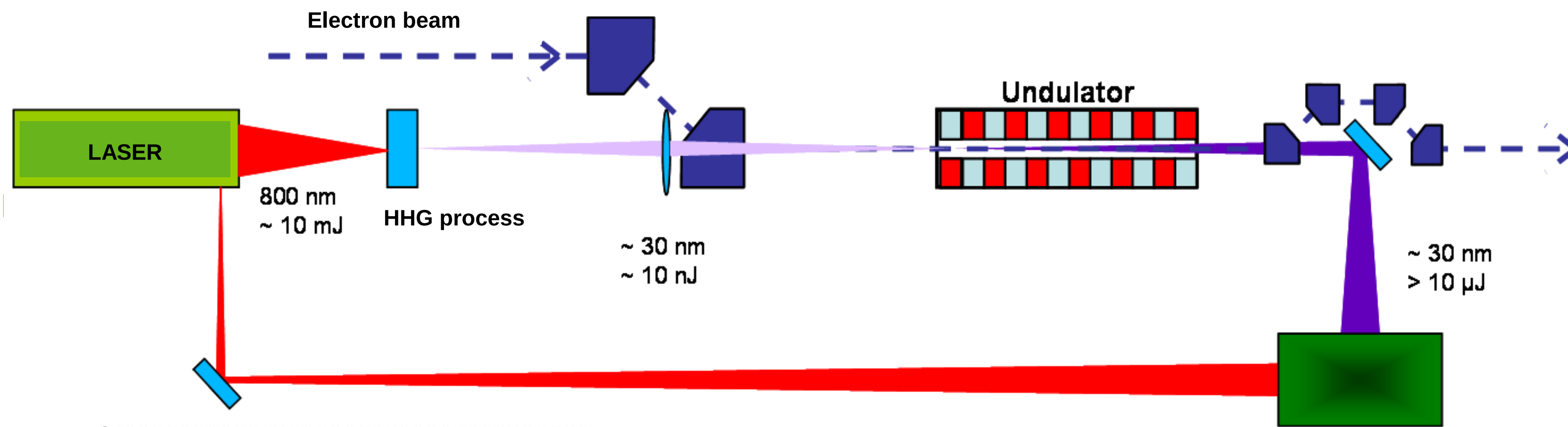
Self-Seeding Example: LCLS

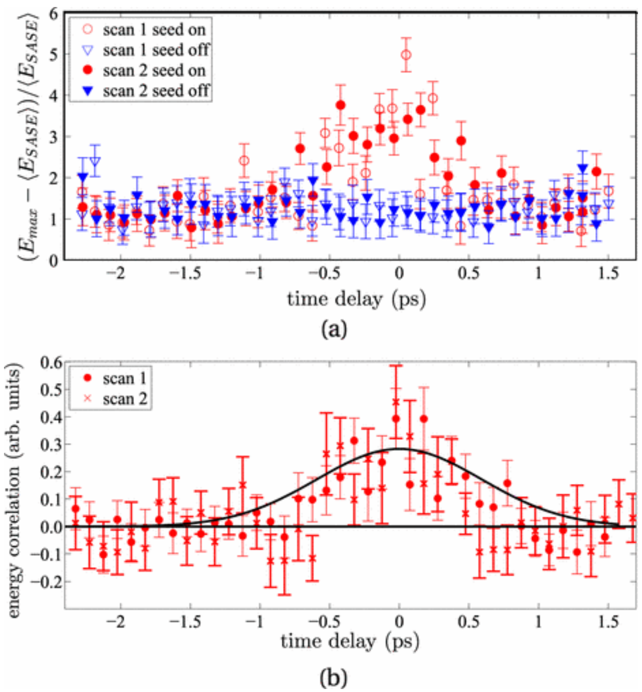


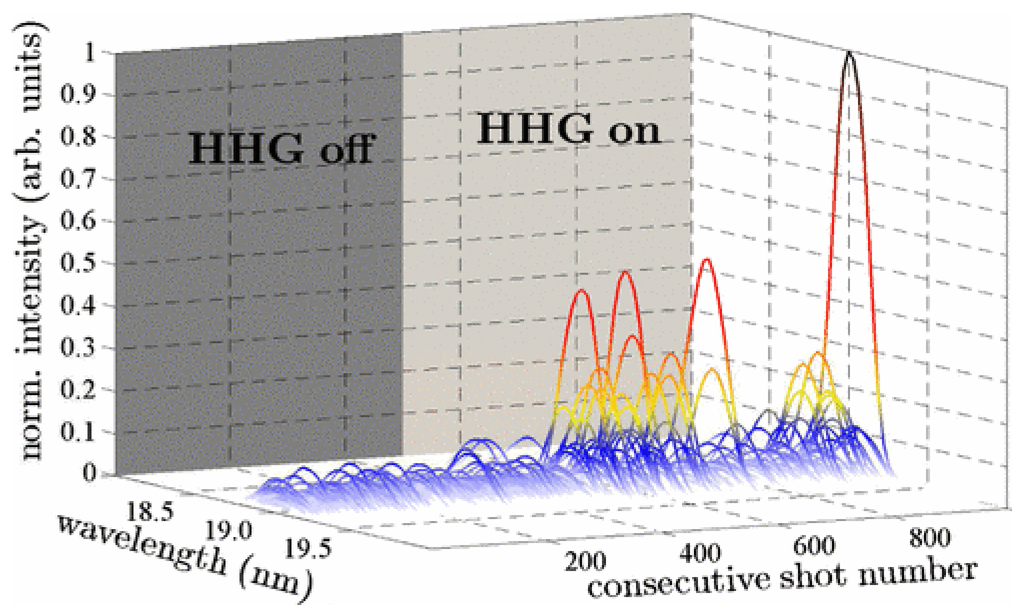


NATURE PHOTONICS | VOL 6 | OCTOBER 2012 | p 693- 698

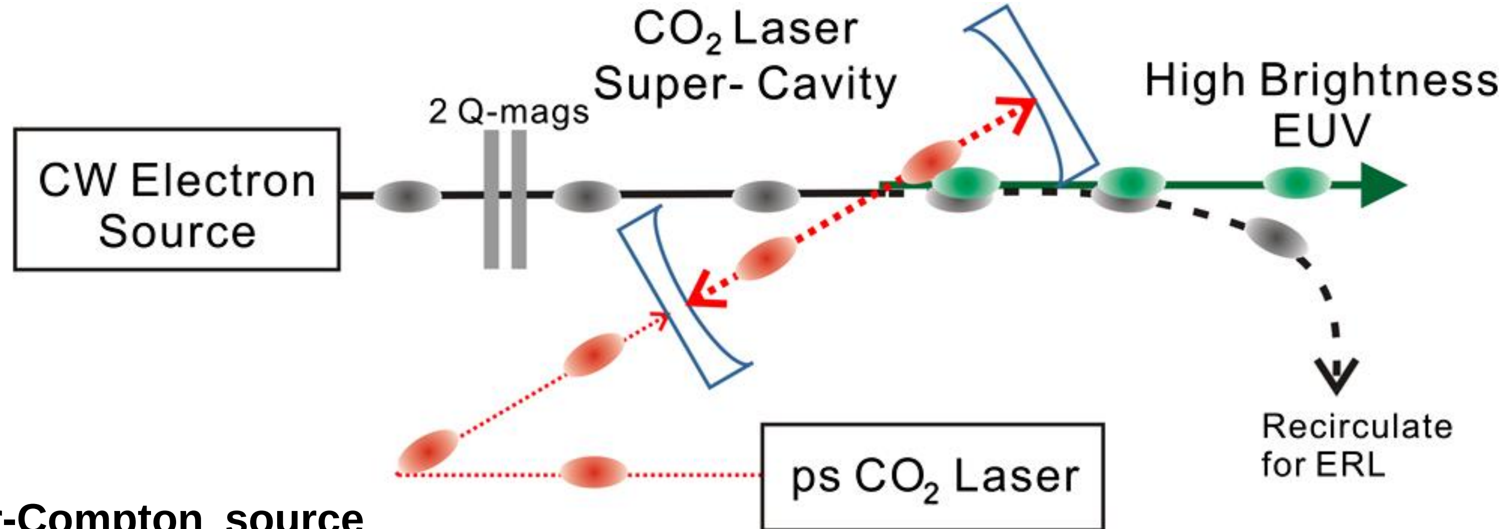
Seeding with HHG-Sources , Example: sFLASH



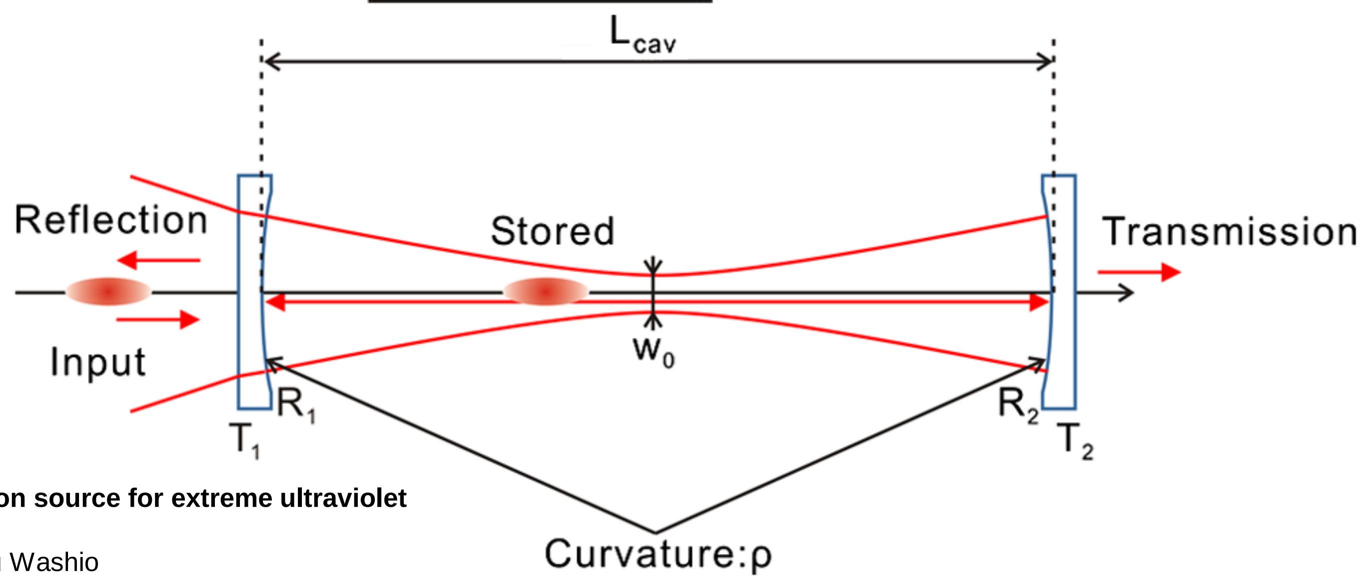




Future Application of LASERs



Laser-Compton source
Design of a clean, high-brightness light source is presented for extreme ultraviolet/soft x-ray (EUV/SXR) lithography research and mask inspection.

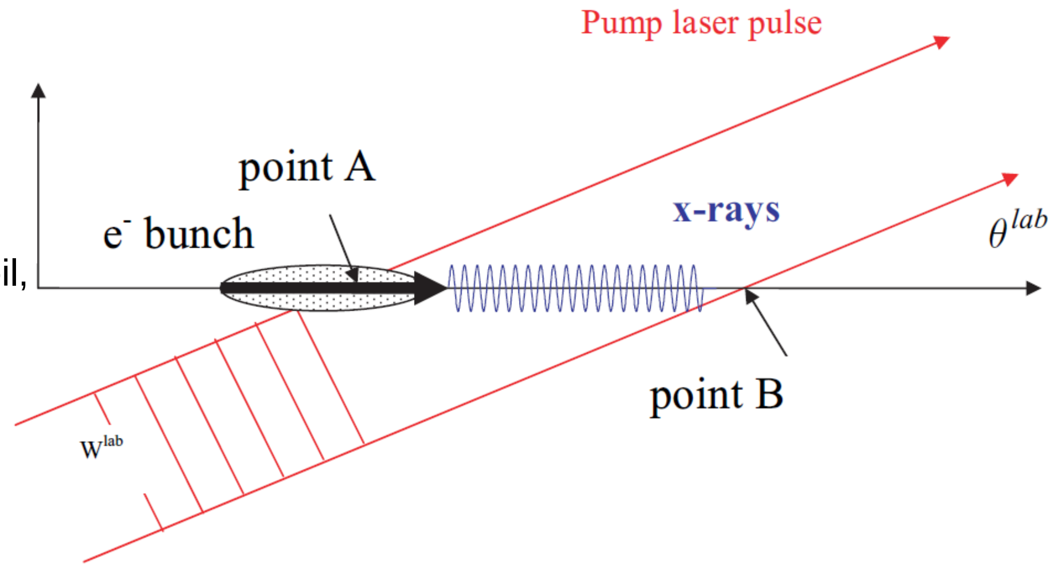


Design of high brightness laser-Compton source for extreme ultraviolet and soft x-ray wavelengths
Kazuyuki Sakaue ; Akira Endo ; Masakazu Washio
J. Micro/Nanolith. MEMS MOEMS. 11(2), 021124 (May 03, 2012).

Future Application of LASERs in FELs?

The wavelength in the electron rest frame

$$\lambda^{\text{elec}} = \frac{\lambda_{\text{pump}}}{\gamma[1-\beta\cos(\theta^{\text{lab}})]}$$



In the lab frame the wavelength, neglecting recoil, of the scattered radiation parallel to the *z*-axis:

$$\lambda_{\text{x-ray}} = \frac{\lambda^{\text{elec}}}{(1+\beta)\gamma} = \frac{\lambda_{\text{pump}}(1-\beta)}{1-\beta\cos(\theta^{\text{lab}})}$$

$$M = \frac{\lambda_{\text{pump}}}{\lambda_{\text{x-ray}}} = \frac{1-\beta\cos(\theta^{\text{lab}})}{1-\beta} \cong (\theta^{\text{lab}}\gamma)^2$$

$$\rho = \left(\frac{K^2\gamma(\lambda_{\text{x-ray}})^2 r_e n^{\text{lab}}}{4\pi}\right)^{1/3}$$

$$\frac{\lambda^{\text{elec}}}{L_G^{\text{elec}}} = 4\pi\sqrt{3}\rho$$

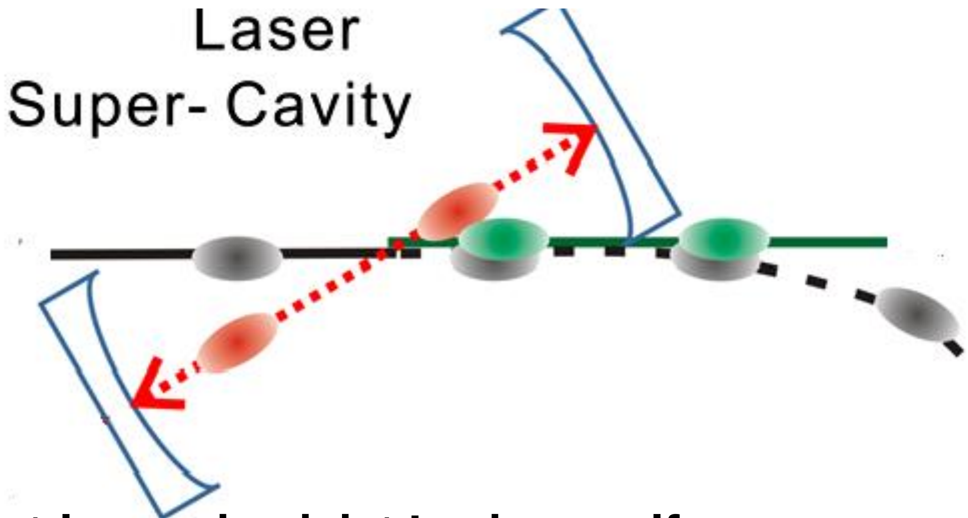
The Lorentz invariant gain per wavelength or per cycle of interaction.

The primary effect of the ponderomotive impulse which occurs when the pump laser pulse starts to overlap with the electron bunch is to decrease the axial velocity of the bunch. This effect has been included using:

$$\gamma^* = \frac{1}{\sqrt{1-(\beta^*)^2}} = \frac{\gamma}{\sqrt{1+K^2}}$$

Nearly copropagating sheared laser pulse FEL undulator for soft x-rays , J. E. Lawler et al., J. Phys. D: Appl. Phys. **46** (2013) 325501 (11pp)

Future Application of LASERs in FELs?



$$c^2 = \frac{1}{\varepsilon_0 \varepsilon \; \mu_0 \mu} = \frac{c_0^2}{\varepsilon\mu} = \frac{c_0^2}{n^2}$$

$$n = n' + i\, n'' = \pm\sqrt{\varepsilon\mu}$$

$$\varepsilon = \varepsilon' + i\, \varepsilon''$$

$$\mu = \mu' + i\, \mu''$$

$$\varepsilon'|\mu| + \mu'|\varepsilon| < 0 \quad \Rightarrow \quad n < 0$$

As a not-laser-physicist I ask myself:

- **How does the developing microbunching change the speed of the light inside the laser-cavity?**
- **Do we expect some kind of equillibrium after some time?**
- **Do we need to adjust the the cavity length fast, slow, at all?**

Permittivity **ε(ω):**

$$\epsilon(\omega) = 1 - \frac{\omega_0^2}{\omega(\omega + \mathrm{i}\gamma)}$$

Damping term

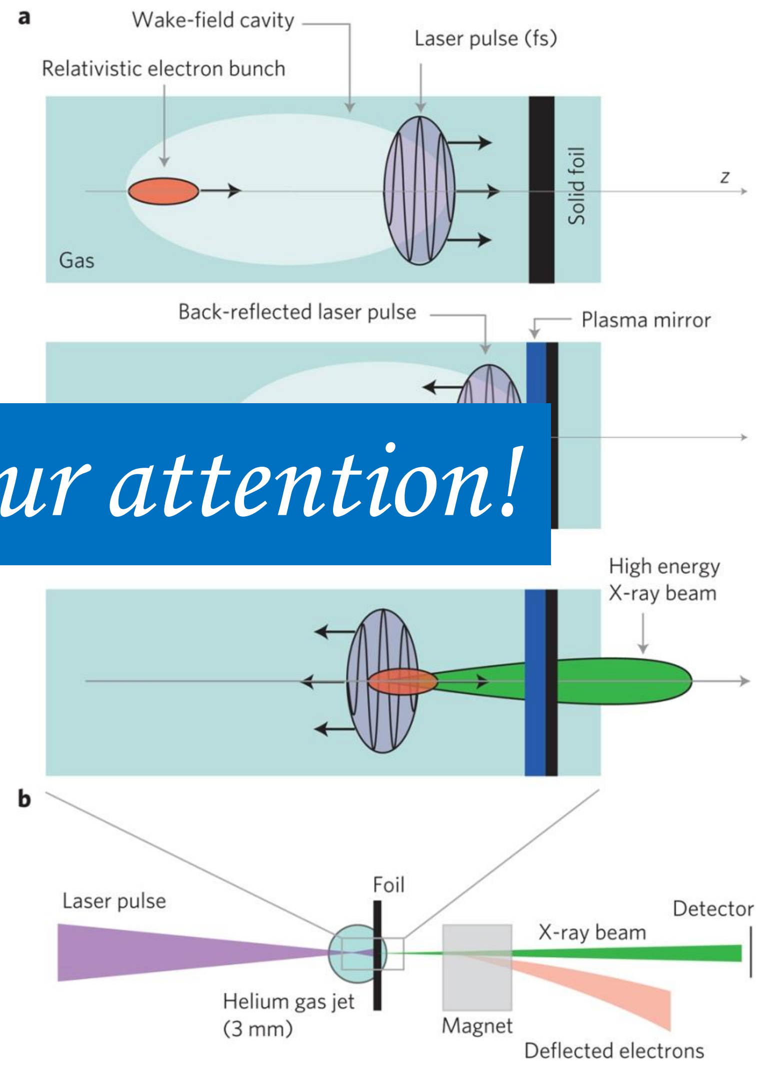
Exciting Future Application of LASERs

All-optical Compton gamma-ray source

K. Ta Phuoc et al.

"Here, we present a simple and compact scheme for a **Compton source based on the combination of a laser-plasma acceleratoı** ... approach is ... spectrum of X-rays extending up to hundreds of keV and with a 10,000-fold increase in brightness over Compton X-ray sources based on conventional accelerators"

Thank you for your attention!



NATURE PHOTONICS | VOL 6 | MAY 2012 | p 308-311