The Tsallis Distribution at the LHC.

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Heavy Ion Forum
CERN
January 13 2014

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Outline

Transverse Momentum Distribution

STAR, PHENIX, ALICE, CMS, ATLAS use:

$$\frac{d^2N}{dp_T dy} = p_T \times \frac{dN}{dy} \frac{(n-1)(n-2)}{nT(nT + m_0(n-2))} \left(1 + \frac{m_T - m_0}{nT}\right)^{-n}$$

What is the connection with the Tsallis distribution?
Also, the physical significance of the parameters n and T has never been discussed by STAR, PHENIX, ALICE, ATLAS, CMS.

In the grand canonical ensemble the particle number, energy density and pressure are given by

$$
\begin{aligned}
N &= gV \int \frac{d^3p}{(2\pi)^3} \exp\left(-\frac{E-\mu}{T}\right), \\
\epsilon &= g \int \frac{d^3p}{(2\pi)^3} E \exp\left(-\frac{E-\mu}{T}\right), \\
P &= g \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{3E} \exp\left(-\frac{E-\mu}{T}\right),
\end{aligned}
$$

where T and μ are the temperature and the chemical potential, V is the volume and g is the degeneracy factor.

In particular, the particle number is:

$$E\frac{d^3N}{d^3p} = \frac{gVE}{(2\pi)^3}\; e^{-\frac{E-\mu}{T}},$$
$$\frac{d^2N}{m_T dm_T dy} = \frac{gVm_T\cosh y}{(2\pi)^2}\; e^{-\frac{m_T\cosh y-\mu}{T}},$$

at mid-rapidity, $y = 0$ and zero chemical potential this becomes

$$\left.\frac{d^2N}{m_T dm_T dy}\right|_{y=0} = \frac{gVm_T}{(2\pi)^2}\; e^{-\frac{m_T}{T}}$$

m_T scaling, works :
F. Becattini and G. Passaleva, Eur. Phys. J. C23 (2002) 551

Entropy: Tsallis vs Boltzmann

The Boltzmann entropy is given by

$$S^B = -g \sum_i \left[f_i \ln f_i - f_i \right], \tag{1}$$

The Tsallis entropy is given by

$$S_T^B = -g \sum_i \left[f_i^q \ln_q f_i - f_i \right], \tag{2}$$

which uses

$$\ln_q(x) \equiv \frac{x^{1-q} - 1}{1 - q}, \tag{3}$$

often referred to as q-logarithm.
By maximizing the entropy one obtains expressions for particle density, energy density and pressure.

For high energy physics a consistent form of Tsallis statistics for the particle number, energy density and pressure are given by integrals over the Tsallis distribution:

$$f = \left[1 + (q-1)\frac{E-\mu}{T}\right]^{-\frac{1}{q-1}} .$$

$$N = gV\int \frac{d^3p}{(2\pi)^3} f^q, \qquad \epsilon = g\int \frac{d^3p}{(2\pi)^3}\, E\, f^q,$$

$$P = g\int \frac{d^3p}{(2\pi)^3}\,\frac{p^2}{3E}\, f^q, \quad S = -gV\int \frac{d^3p}{(2\pi)^3}\,\left[f^q \ln_q f - f\right].$$

It can be shown that

$$\epsilon + P = Ts + \mu N \tag{4}$$

Thermodynamic consistency

$$dE = -pdV + TdS + \mu dN$$

Inserting $E = \epsilon V$, $S = sV$ and $N = nV$ leads to

$$d\epsilon = Tds + \mu dn$$

$$dP = nd\mu + sdT$$

In particular

$$n = \left.\frac{\partial P}{\partial \mu}\right|_T, \quad s = \left.\frac{\partial P}{\partial T}\right|_\mu, \quad T = \left.\frac{\partial \epsilon}{\partial s}\right|_n, \quad \mu = \left.\frac{\partial \epsilon}{\partial n}\right|_s.$$

are satisfied for the Tsallis distribution.

In the Tsallis distribution the total number of particles is given by:

$$N = gV \int \frac{d^3p}{(2\pi)^3} \left[1 + (q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}} .$$

The corresponding momentum distribution is given by

$$E\frac{dN}{d^3p} = gVE\frac{1}{(2\pi)^3} \left[1 + (q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}} ,$$

which, in terms of the rapidity and transverse mass variables, $E = m_T \cosh y$, becomes (at mid-rapidity for $\mu = 0$)

$$\left.\frac{d^2N}{dp_T\, dy}\right|_{y=0} = gV\frac{p_T m_T}{(2\pi)^2} \left[1 + (q-1)\frac{m_T}{T}\right]^{-\frac{q}{q-1}} ,$$

J.C. and D. Worku, J. Phys. **G39** (2012) 025006; arXiv:1203.4343[hep-ph].

Comparison of the Tsallis form with the STAR, PHENIX, ALICE, ATLAS, CMS distribution

$$\frac{d^2N}{dp_T dy} = gV \frac{p_T m_T}{(2\pi)^2} \left[1 + (q-1)\frac{m_T}{T}\right]^{-q/(q-1)},$$

$$\frac{d^2N}{dp_T dy} = p_T \times \frac{dN}{dy} \frac{(n-1)(n-2)}{nT(nT + m_0(n-2))} \left[1 + \frac{m_T - m_0}{nT}\right]^{-n}$$

For the comparison use the following substitution:

$$n \rightarrow \frac{q}{q-1}$$

$$nT \rightarrow \frac{T + m_0(q-1)}{q-1}$$

After this substitution one obtains

$$
\frac{d^2N}{dp_T\, dy} = p_T \frac{\mathrm{d}N}{\mathrm{d}y} \frac{(n-1)(n-2)}{nT(nT+m_0(n-2))} \left[\frac{T}{T+m_0(q-1)}\right]^{-q/(q-1)} \left[1+(q-1)\frac{m_T}{T}\right]^{-q/(q-1)} .
$$

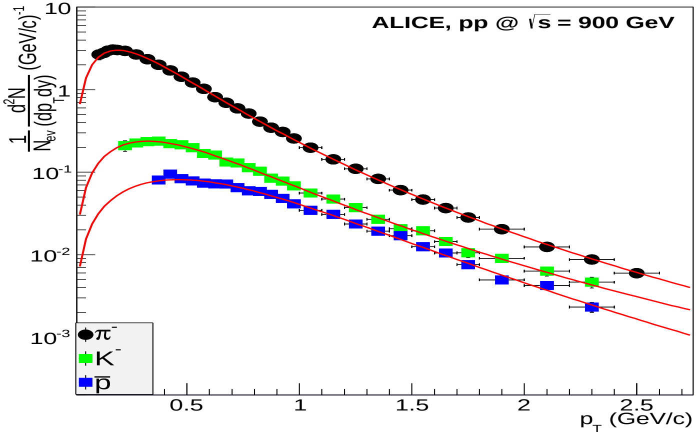
To be compared with

$$
\frac{\mathrm{d}^2N}{\mathrm{d}p_\mathrm{T}\mathrm{d}y} = gV\frac{p_T m_T}{(2\pi)^2}\left[1+(q-1)\frac{m_T}{T}\right]^{-q/(q-1)} .
$$

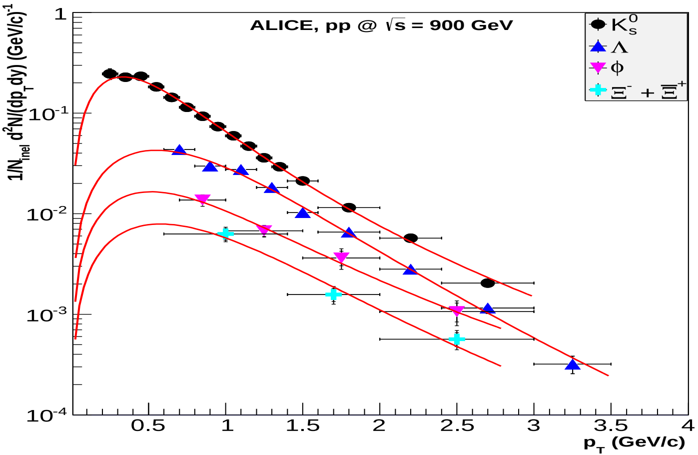
Apart from several constant factors, which can be absorbed in the volume V, only a factor of m_T differs! However, m_0 shouldn't appear as it destroys m_T scaling. The inclusion of the factor m_T leads to a more consistent interpretation of the variables q and T.

Outline

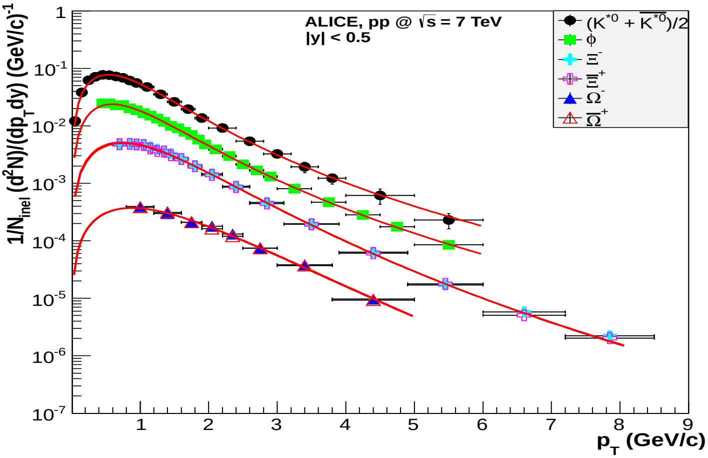
ALICE: Charged particles



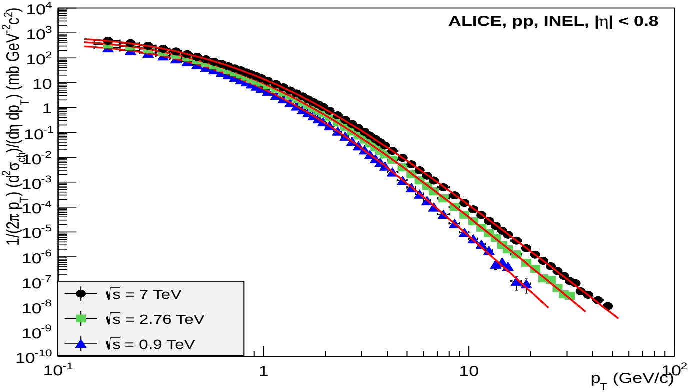
ALICE: Strange particles



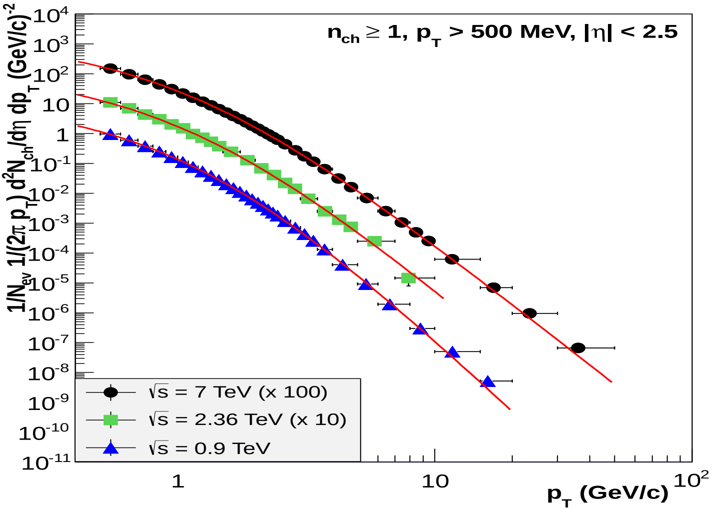
ALICE: 7 TeV


ALICE, pp @ √s = 7 TeV
|y| < 0.5
(K*0 + K̄*0)/2
ϕ
Ξ-
Ξ̄+
Ω-
Ω̄+
1/N_inel (d²N)/(dp_T dy) (GeV/c)^-1
p_T (GeV/c)
1
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
10^-7
1
2
3
4
5
6
7
8
9

ALICE: Differential Cross Sections

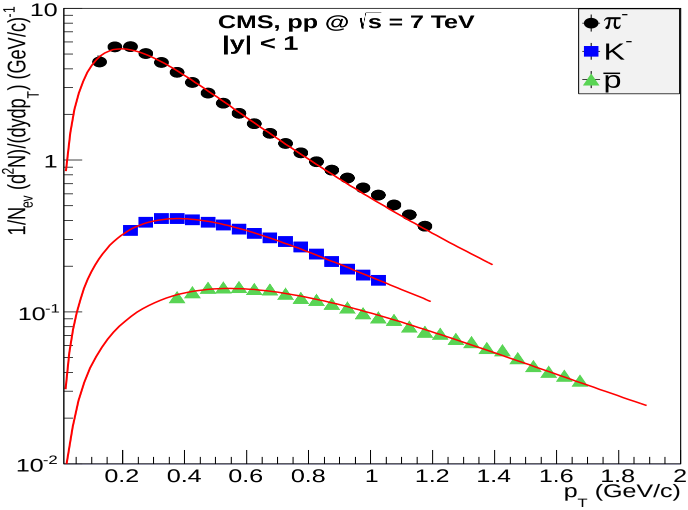


Outline

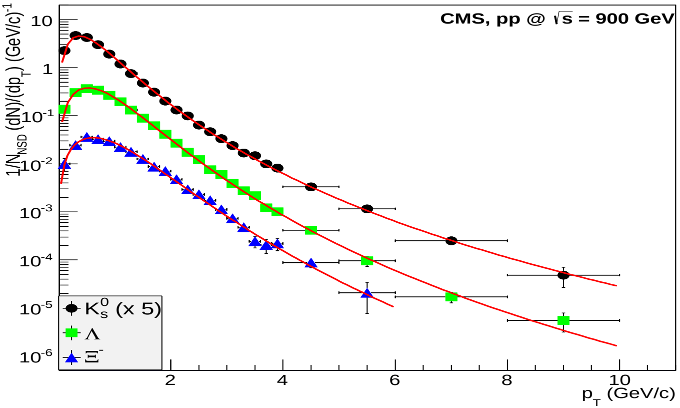

$n_{ch} \geq 1$, p_T > 500 MeV, $|\eta|$ < 2.5
$1/N_{ev}\ 1/(2\pi\ p_T)\ d^2N_{ch}/d\eta\ dp_T\ (GeV/c)^{-2}$
p_T (GeV/c)
$\sqrt{s}$ = 7 TeV (x 100)
$\sqrt{s}$ = 2.36 TeV (x 10)
$\sqrt{s}$ = 0.9 TeV

Outline

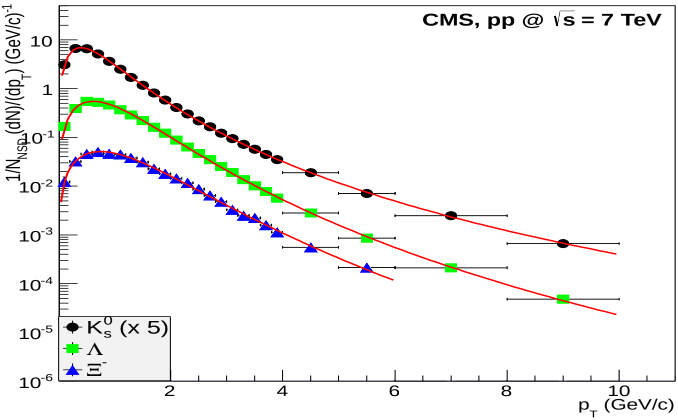
V


CMS, pp @ √s = 7 TeV
|y| < 1
π⁻
K⁻
p̄
1/N_ev (d²N)/(dydp_T) (GeV/c)⁻¹
10
1
10⁻¹
10⁻²
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
p_T (GeV/c)

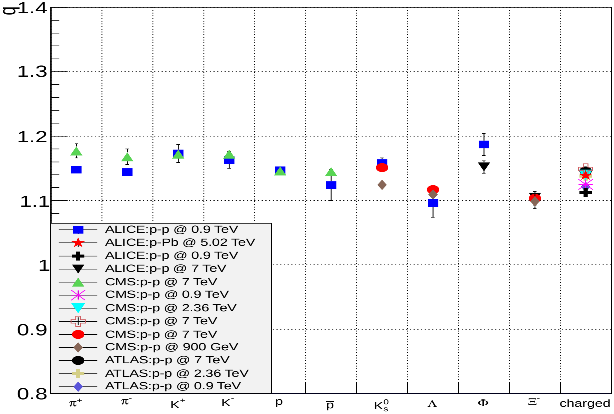
CMS: strange particles in p-p collisions 900 GeV



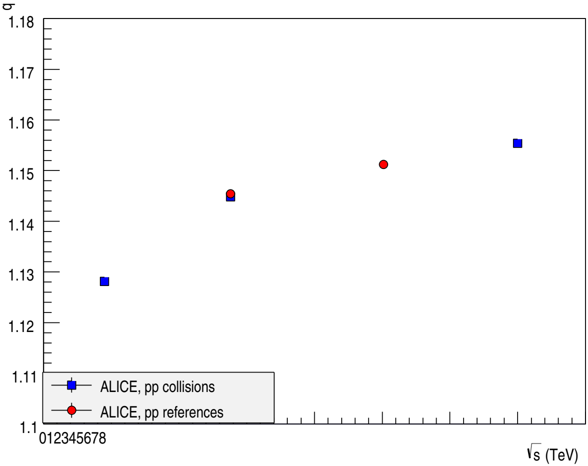
CMS: strange particles in p-p collisions 7 TeV



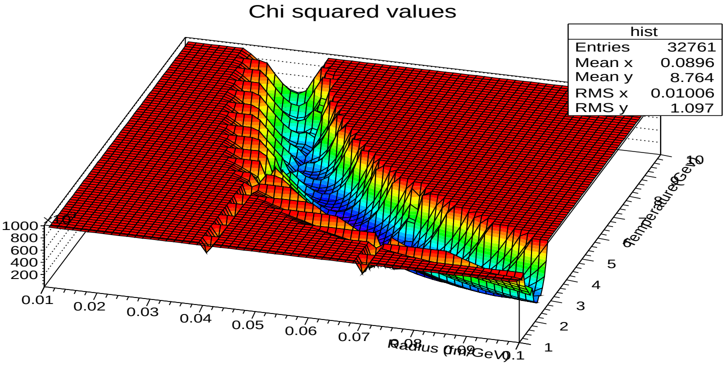
p-p collisions: Summary of results for parameter q



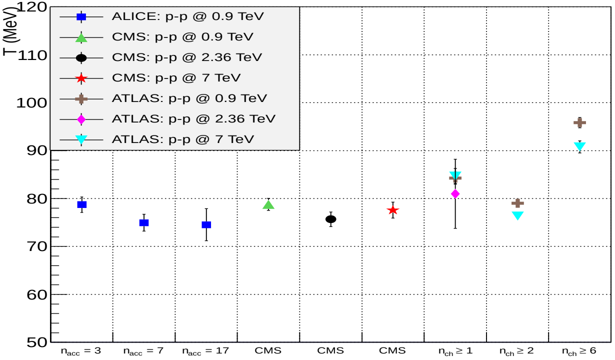
$q$value: increases with beam energy



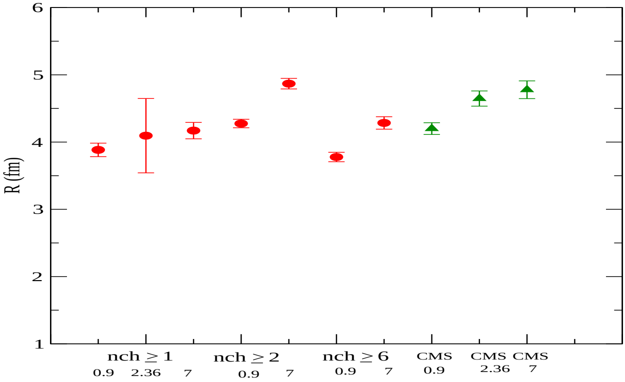
Tsallis: problem in determining parameters T and V



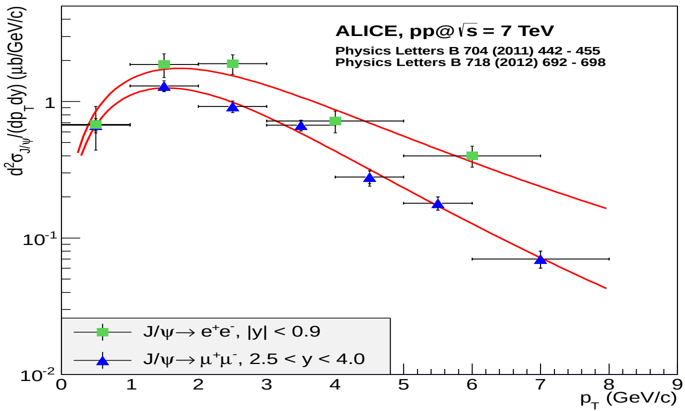
p-p collisions: Summary of results for parameter T



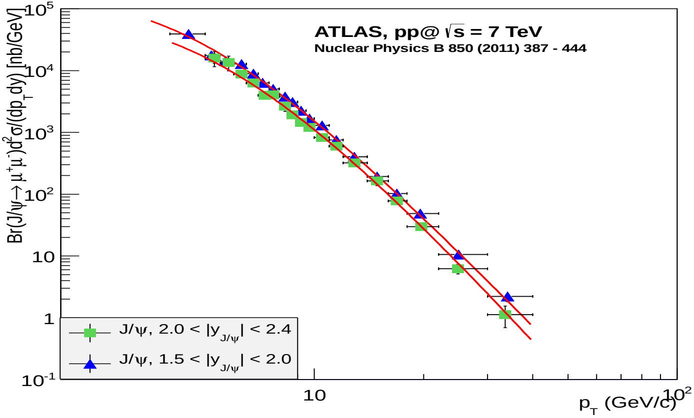
p-p collisions: Summary of results for parameter R



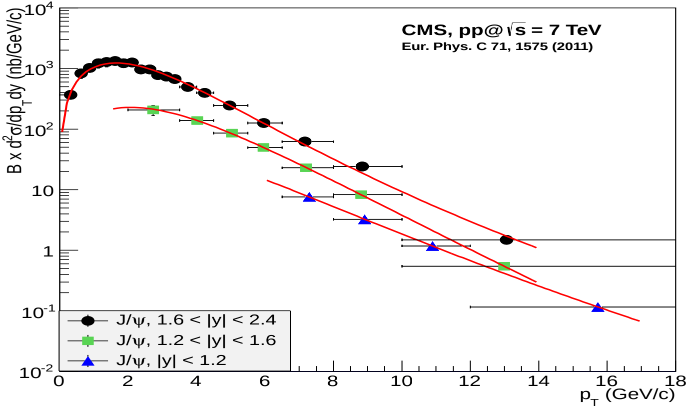
J/ψ Production ALICE



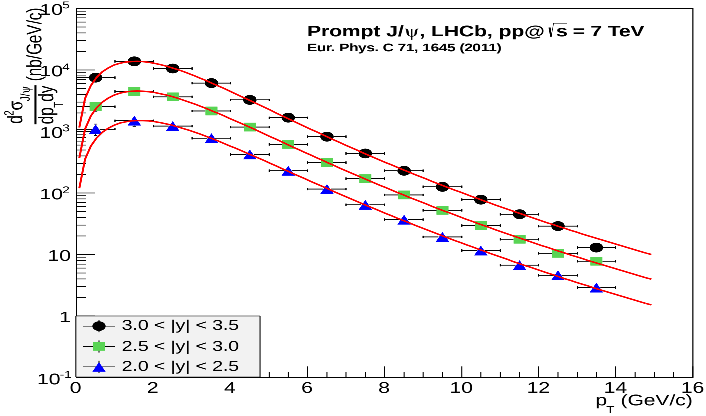
J/ψ Production ATLAS



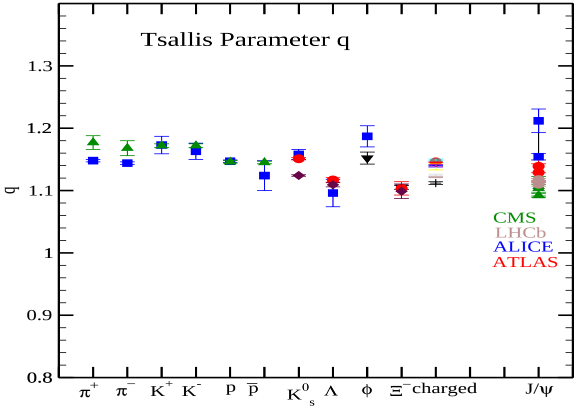
J/ψ Production CMS

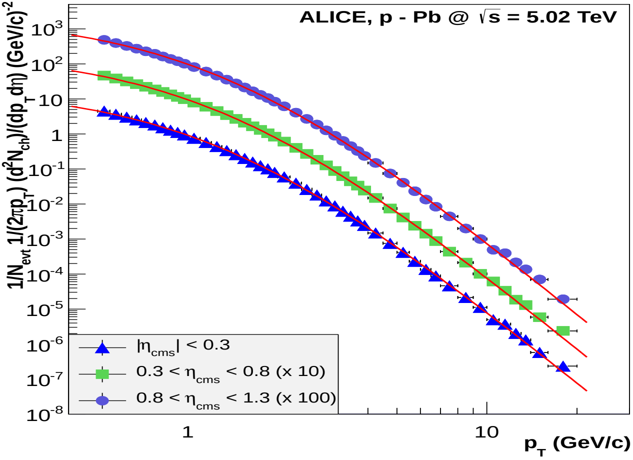


J/ψ Production LHCb



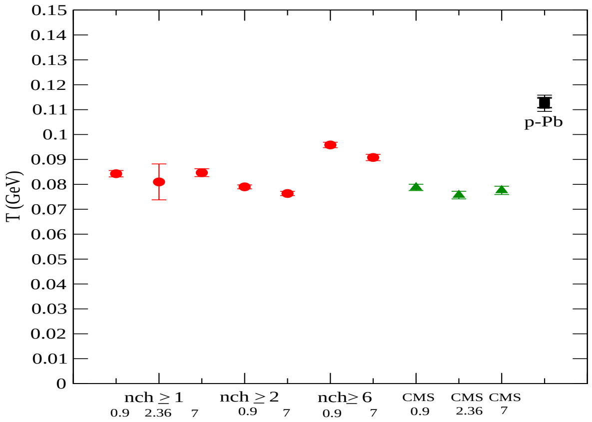
p-p collisions: Summary of results for parameter q
J/ψ Production in p-p collisions: Summary of results



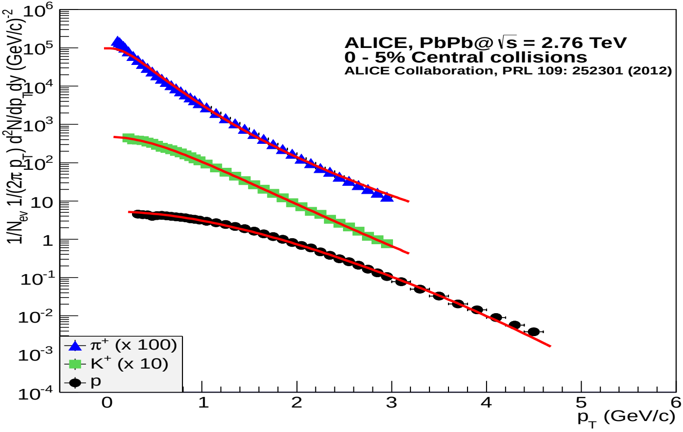


$q = 1.140$ consistent with p-p collisions

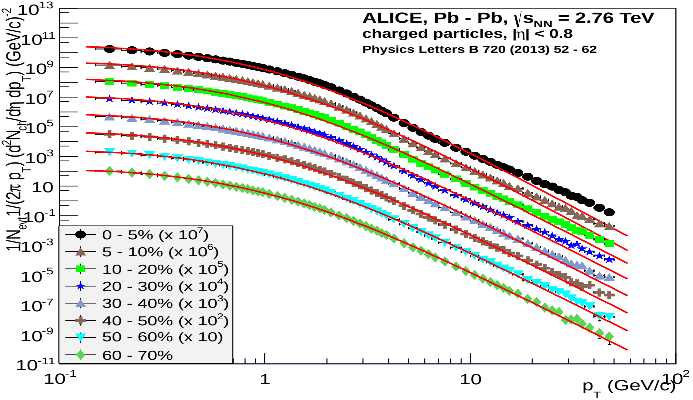
p-p collisions: Summary of results for parameter T

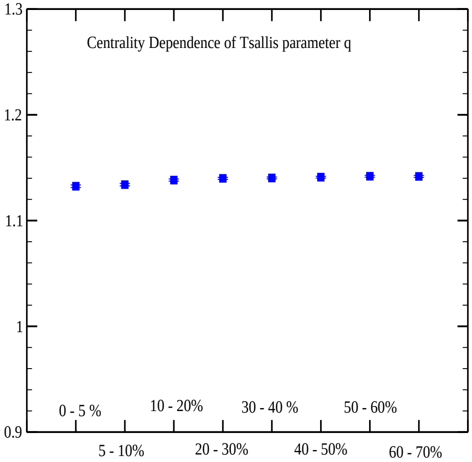


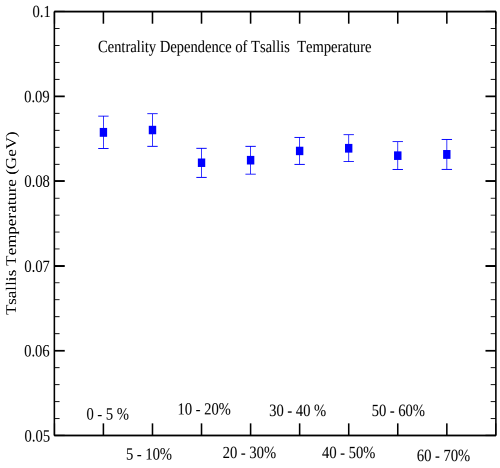
Tsallis Distribution in Pb-Pb

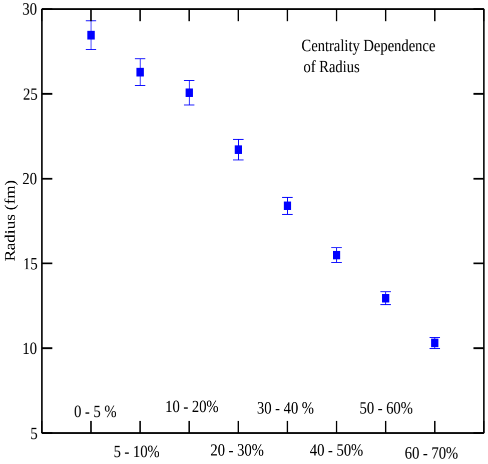


Tsallis Distribution in Pb-Pb




Centrality Dependence of Tsallis parameter q
1.3
1.2
1.1
1
0.9
0 - 5 %
5 - 10%
10 - 20%
20 - 30%
30 - 40 %
40 - 50%
50 - 60%
60 - 70%


Centrality Dependence of Tsallis Temperature
Tsallis Temperature (GeV)
0.1
0.09
0.08
0.07
0.06
0.05
0 - 5 %
10 - 20%
30 - 40 %
50 - 60%
5 - 10%
20 - 30%
40 - 50%
60 - 70%


Centrality Dependence
of Radius
Radius (fm)
30
25
20
15
10
5
0 - 5 %
10 - 20%
30 - 40 %
50 - 60%
5 - 10%
20 - 30%
40 - 50%
60 - 70%

Conclusion:
Use

$$\frac{d^2N}{dp_T dy} = gV\frac{p_T m_T}{(2\pi)^2}\left[1+(q-1)\frac{m_T}{T}\right]^{-\frac{q}{q-1}}, \qquad (5)$$

instead of

$$\frac{d^2N}{dp_T dy} = p_T \times \frac{\mathrm{d}N}{\mathrm{d}y}\frac{(n-1)(n-2)}{nT(nT+m_0(n-2))}\left[1+\frac{m_\mathrm{T}-m_0}{nT}\right]^{-n} \qquad (6)$$

What Next?

- Need to recalculate dN/dy,
- Need to pin down a unique set of values for T, q, V,
- Need good data at low p_T.

Tsallis vs Boltzmann