Heavy quark potential at T>0 from lattice QCD

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- Motivation : the study and interpretation of static meson correlators is much more easier than the analysis of heavy meson correlators
  static correlators → potential model → spectral functions
- Free energy and singlet free energy of static quark anti-quark pair in 2+1 flavor QCD with HISQ action => controlled discretization effects
- Wilson loops at $T>0$ on the lattice
- Wilson loops in HTL perturbation theory
- Extracting the real and the imaginary parts of the potential

in collaboration with Alexei Bazavov, Eur. Phys. J. A49 (2013) 85, work in progress

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EFT, potential models and static energy

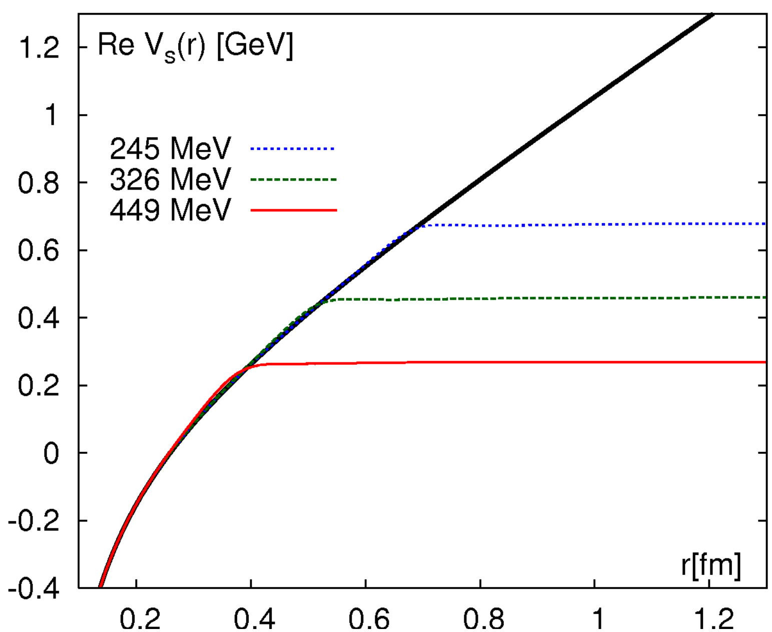
Above deconfinement the binding energy is reduced and eventually E_{bind}~mv^2 is the smallest scale in the problem (zero binding) $mv^2 << \Lambda_{QCD}, 2\pi T, m_D$ => most of medium effects can be described by a T-dependent potential

Potential = Static Energy

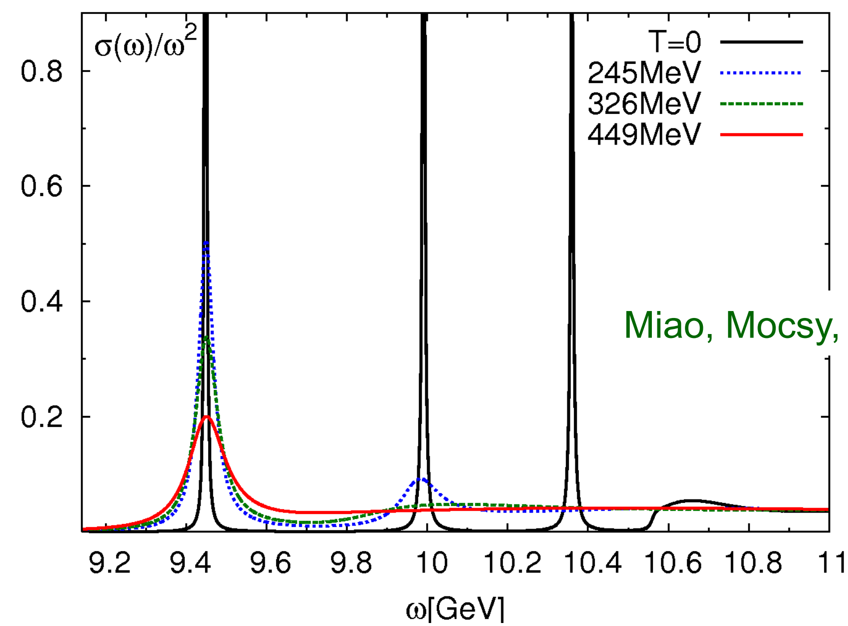
Determine the potential by non-perturbative matching to static quark anti-quark potential calculated on the lattice

Caveat : it is difficult to extract static quark anti-quark energies from lattice correlators => constrain Re $V_s(r)$ by lattice QCD data on the singlet free energy, take Im $V_s(r)$ from pQCD calculations

"Maximal" value for the real part
Mócsy, P.P., 2007



Minimal (perturbative) value for imaginary part
Laine et al, 2007, Brambilla et al, 2008



Static quark anti-quark pair in T>0 QCD

Consider correlation functions of statice meson operators

$$G_1(t,x,y,T)=\langle O(x,y;t)O(x,y;0)\rangle,\quad G_8(t,x,y,T)=\langle O^\alpha(x,y;t)O^\alpha(x,y;0)\rangle$$

for color singlet and adjoint represenation at time $t=1/T$ with

$$O(x,y;t)=\bar\psi(x,t)U(x,y;t)\psi(y,t)$$
$$O^\alpha(x,y;t)=\bar\psi(x,t)U(x,x_0;t)T^\alpha U(x_0,y;t)\psi(y,t)$$

After integration out the static quarks we get

$$G_1(r,T)=\frac{1}{N}\langle \mathrm{Tr}L^\dagger(x)U(x,y;0)L(y)U^\dagger(x,y,1/T)\rangle,$$
$$G_8(r,T)=\frac{1}{N^2-1}\langle \mathrm{Tr}L^\dagger(x)\mathrm{Tr}L(y)\rangle$$
$$-\tfrac{1}{N(N^2-1)}\langle \mathrm{Tr}L^\dagger(x)U(x,y;0)L(y)U^\dagger(x,y,1/T)\rangle, r=|x-y|.$$

$L(x)$ is the temporal Wilson line; on the lattice $L(x)=\prod_{\tau=0}^{N_\tau-1}U_0(x,\tau)$

Alternative choice of $O(x,y;t)$: fix the Coulomb gauge and omit $U(x,y;t)$

For $t\to\infty$ there the choice of meson operators is not important but $t\le 1/T$

Static quark anti-quark pair in T>0 QCD (cont'd)

The color averaged correlator gives the ratio of the partition of partition function of QCD at $T > 0$ with static $Q\bar{Q}$ to partition function without static sources:

$$G(r,T) = \frac{1}{N^2}\langle \mathrm{Tr} L(r) \mathrm{Tr} L^\dagger(0)\rangle = \frac{Z_{QQ}(r,T)}{Z(T)} = e^{-F(r,T)/T}$$

McLerran, Svetitsky 1981

$\rightarrow -T \ln G(r,T) \equiv F(r,T)$ is the excess free energy due to static sources.

$$G(r,T) = \frac{1}{N^2} G_1(r,T) + \frac{N^2-1}{N^2} G_a(r,T) \equiv \frac{1}{N^2} e^{-F_1(r,T)/T} + \frac{N^2-1}{N^2} e^{-F_8(r,T)/T}$$

The spectral represenation of singlet and averaged correlators $(T < T_c)$:

$$G_1(r,T) = \sum_{n=1}^{\infty} c_n(r) e^{-E_n(r)/T}, \qquad G(r,T) = \frac{1}{N^2} \sum_{n=1}^{\infty} e^{-E_n(r)/T}$$

Jahn, Philipsen, 2004

Perturbation theory $(T \gg T_c)$:

$$F_1(r,T) = -\frac{N^2-1}{2N}\frac{\alpha_s}{r} e^{-m_D r} - \frac{(N^2-1)\alpha_s m_D}{2N},$$

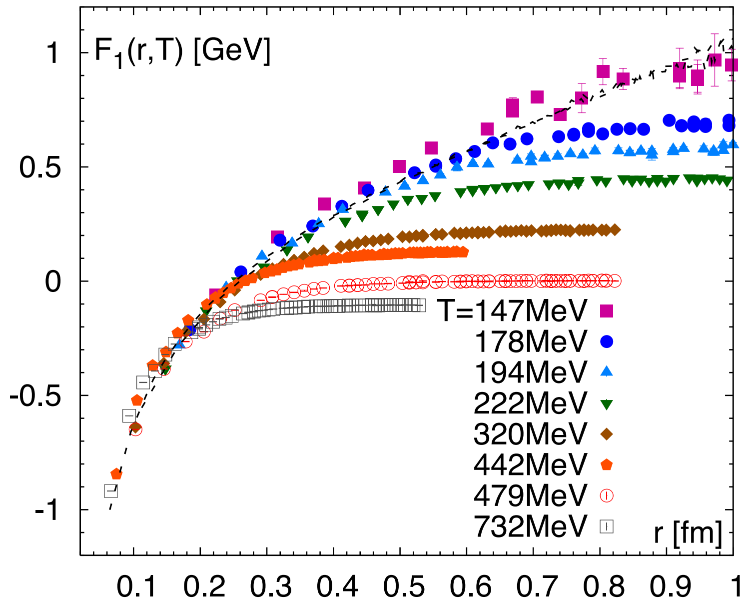
$$F_8(r,T) = +\frac{1}{2N}\frac{\alpha_s}{r} e^{-m_D r} - \frac{(N^2-1)\alpha_s m_D}{2N},$$

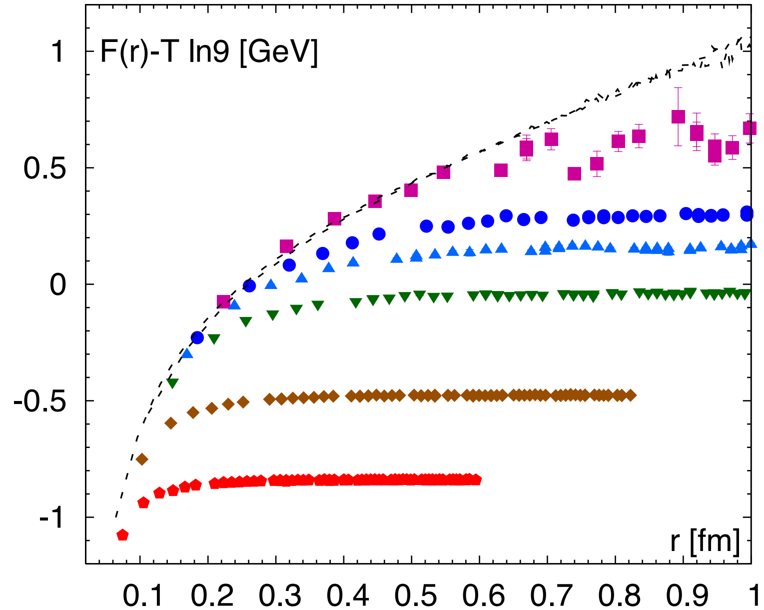
decomposition in terms of F_1 and F_8 can be extended to any order for $rT \ll 1$

Brambilla, Ghiglieri, PP, Vairo 2010

Static quark anti-quark free energy in 2+1f QCD

HISQ action, $24^3 \times 6$, $16^3 \times 4$ (high T) lattices, $m_\pi \simeq 160$ MeV

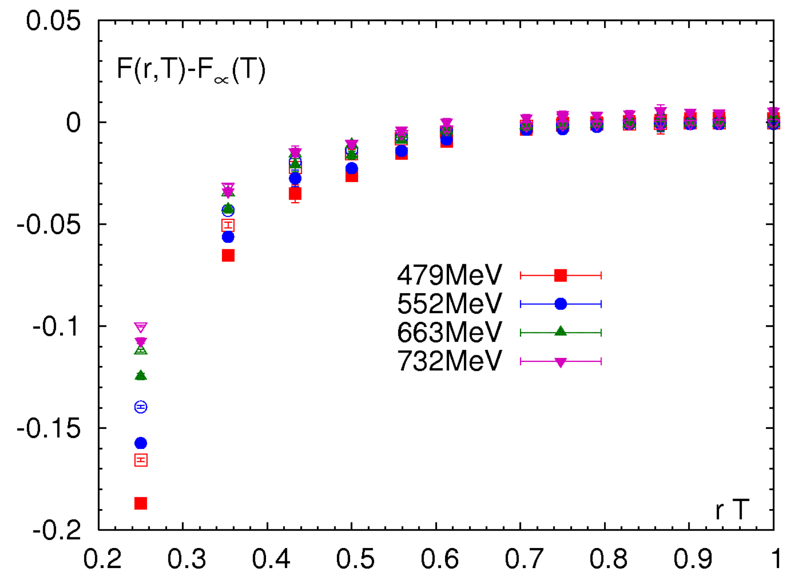




- The strong *T*-dependence for *T<200* MeV is not necessarily related to color screening

- The free energy has much stronger *T*-dependence than the singlet free energy due to the octet contribution

- At high T the temperature dependence of the free energy can be entirely understood in terms of F_1 and Casimir scaling $F_1=-8\ F_8$



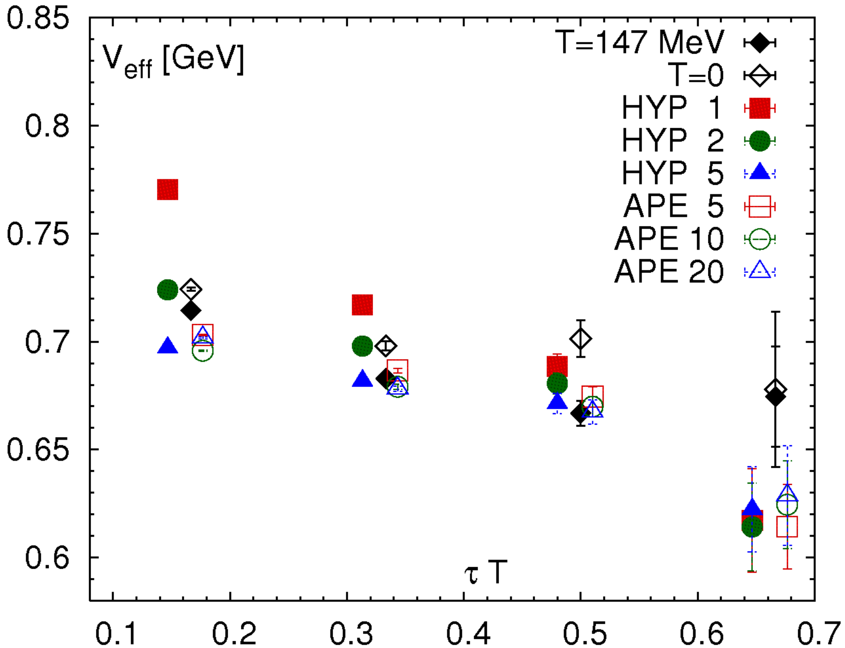
The temperature dependence of the effective potentials

Assume single state dominance → $V_{eff}(\tau,r)$=-ln $W(r,\tau)/\tau$ ➔ $V(r)$ for large τ

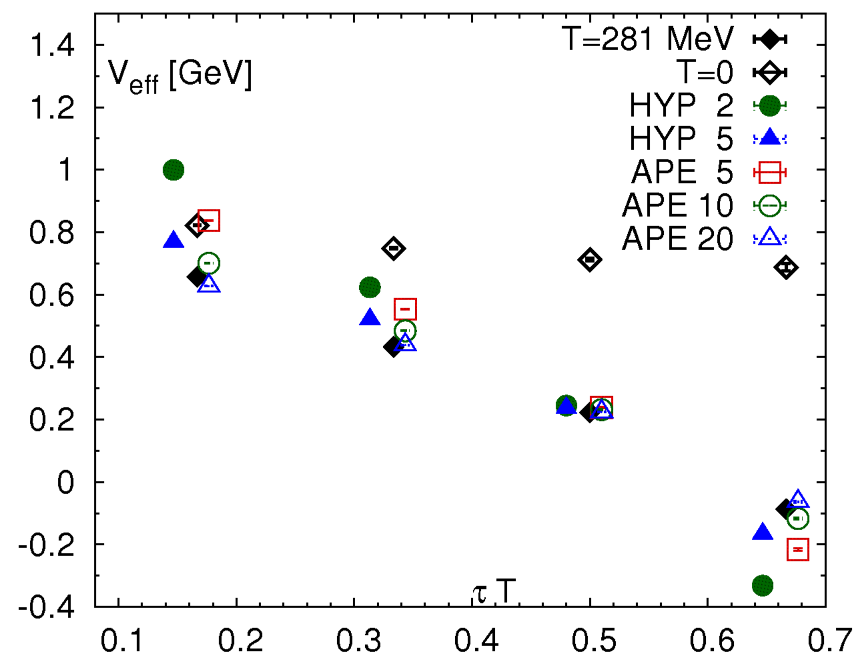
$$\sigma(\omega,T) \sim \delta(\omega - V(r,T))$$

Can be seen at *T=0* in the considered range

r T =1/2



r T =1



- At $T>0$ the effective potential V_{eff} does not reach a plateau but shows an approximately linear decrease in τ, and V_{eff} is always smaller than for *T=0*
- For large enough τ all smeared Wilson loops and Coulomb gauge correlators give the same V_{eff}

⇒ The observed decrease in τ is a physical effect (width ?) independent of the choice of the correlators

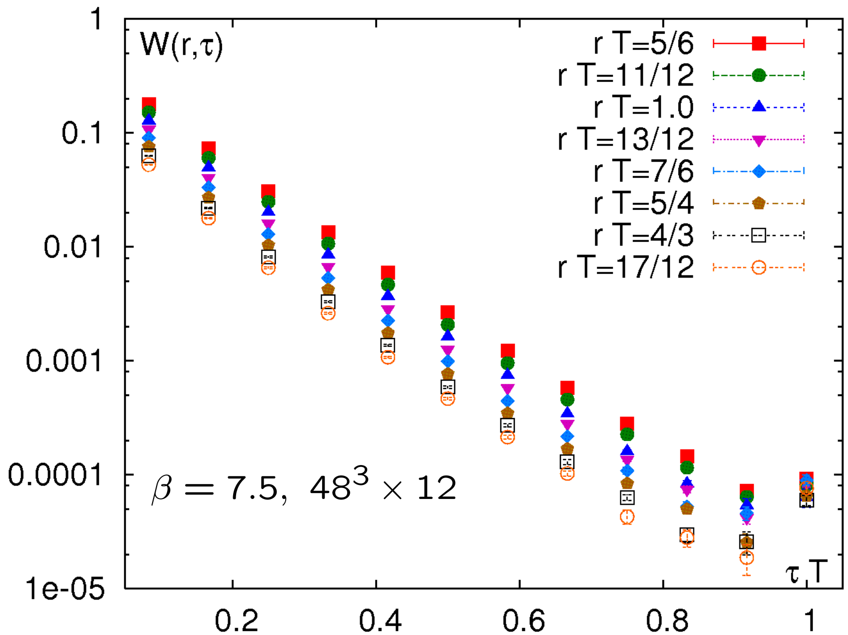
Wilson loops and potential at T>0

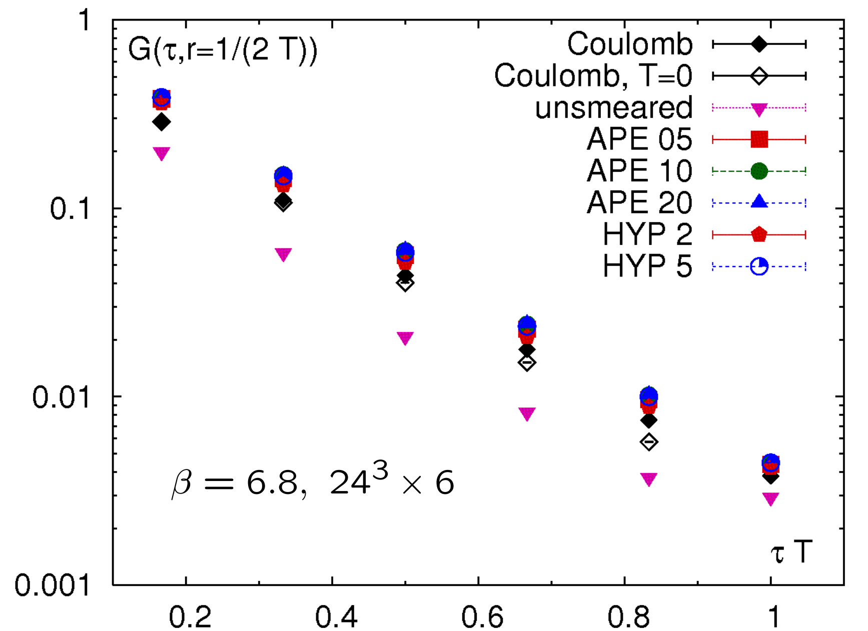
HISQ action, $48^3 \times 12$, $24^3 \times 6$ lattices, $m_\pi \simeq 160$ MeV

$$W(r,\tau) = \int_{-\infty}^{\infty} d\omega \sigma(\omega, T) e^{-\omega\tau}, \tau < 1/T$$

Rothkopf 2009, Hatsuda, Rothkopf, Hatsuda 2011

not related to the free energy !





Choices of the spatial links:

Naïve= un-smeared

smeared

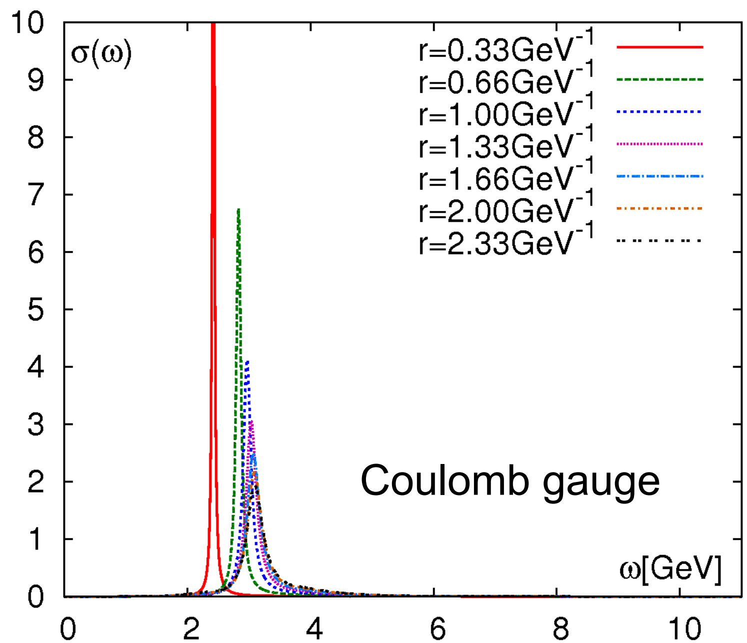
or use Coulomb gauge and *U(x,y)=1*

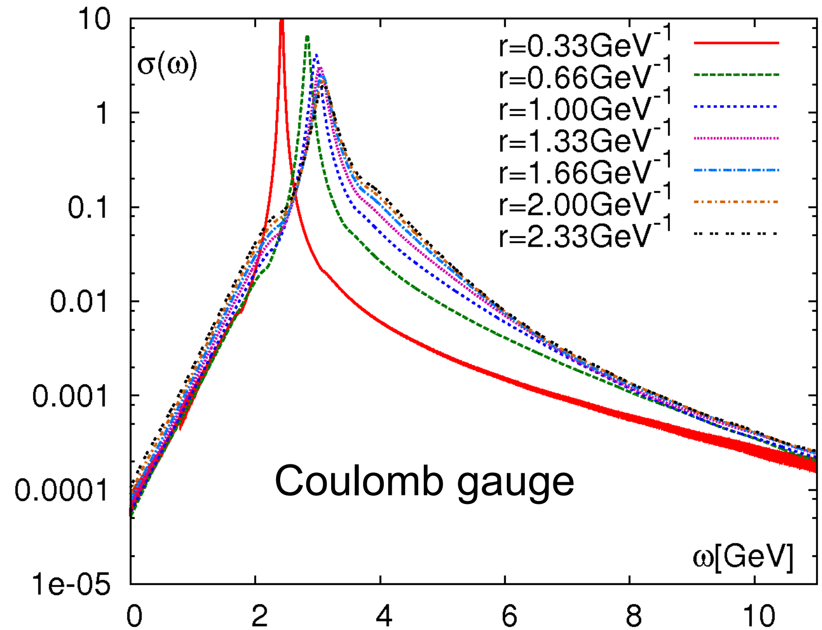
Un-smeared Wilson loops show non-exponential behavior and are suppressed compared to the smeared Wilson loops and Coulomb gauge corr. which decay exponentially, except for $\tau T \sim 1$

Spectral functions in HTL perturbation theory

Perturbative hard thermal loop (HTL) calculations for $T=2.33\ T_c$, $T_c=270$ MeV $(N_f = 0)$:

Burnier, Rothkopf, 2013

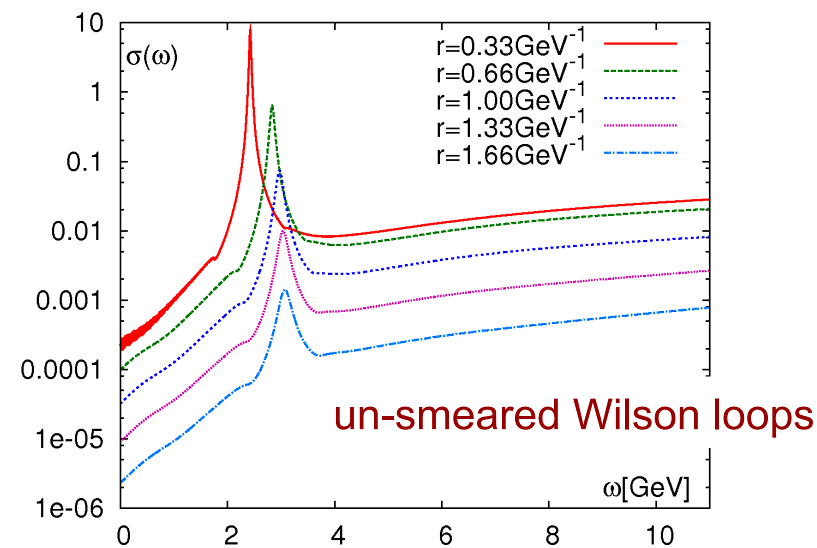




Spectral functions has long tails and non-Lorentzian away from the peak,

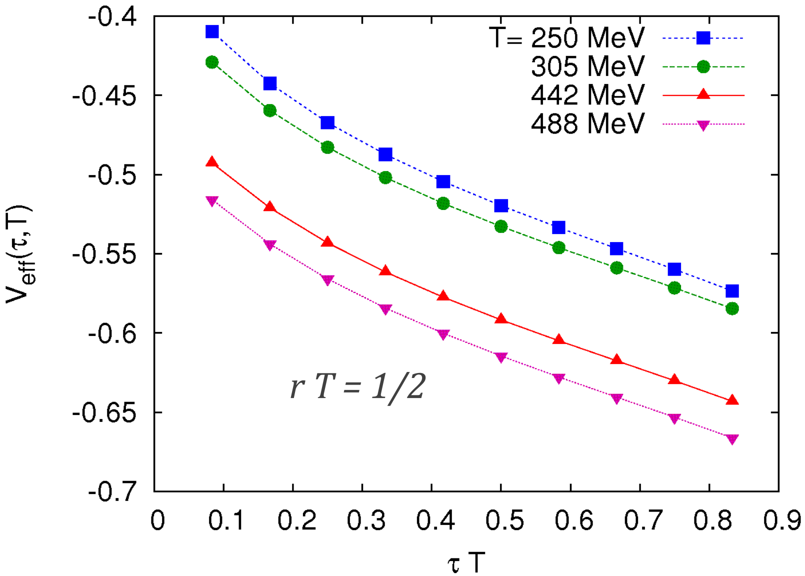
$\Rightarrow$ explanation for the behavior of the Wilson loops and V_{eff} at large times

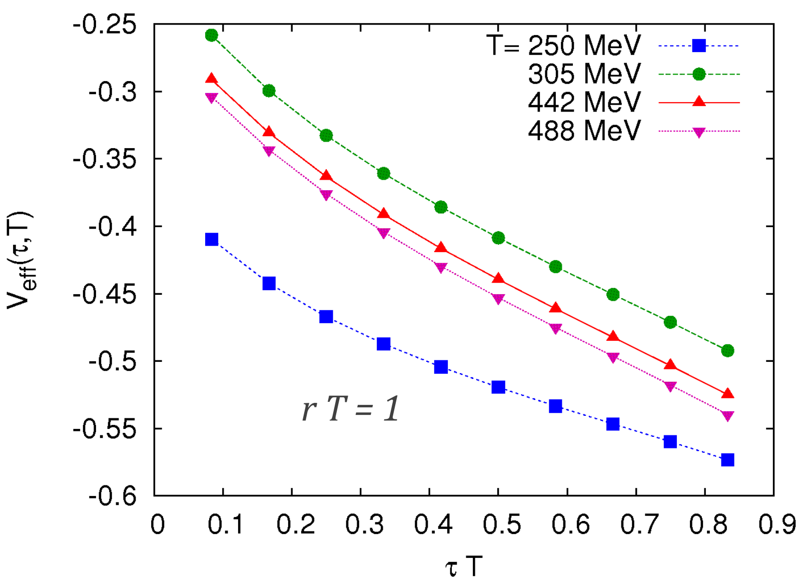
For un-smeared Wilson loops the peak heigh is much suppressed compared to the Coulomb gauge case



Effective potential at high temperatures

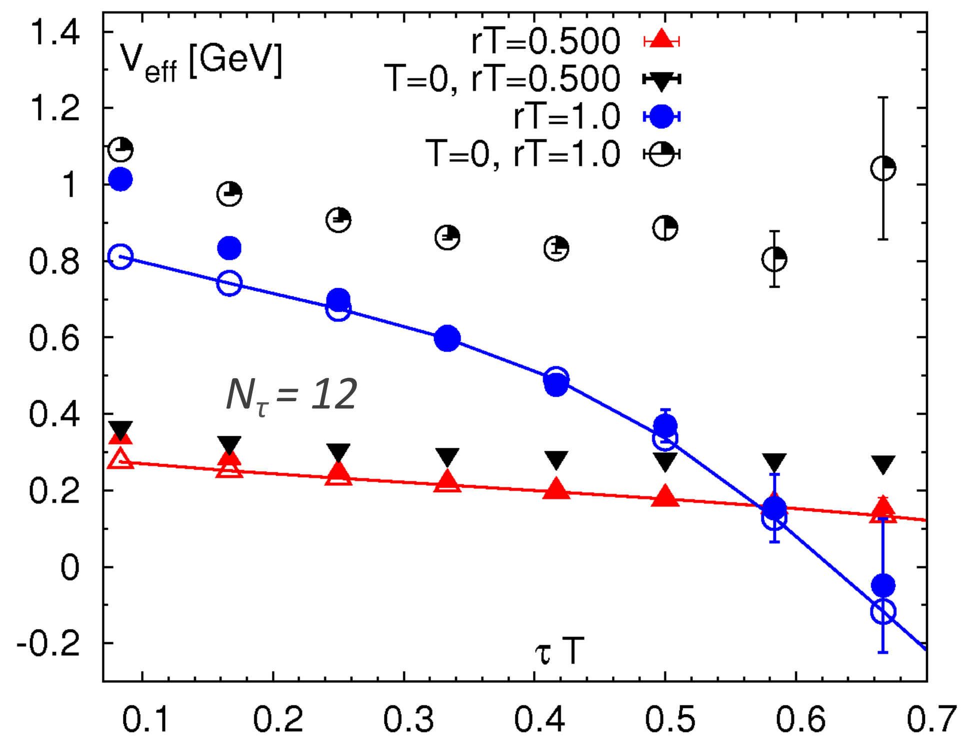
Effective potential V_{eff} in HTL perturbation theory:





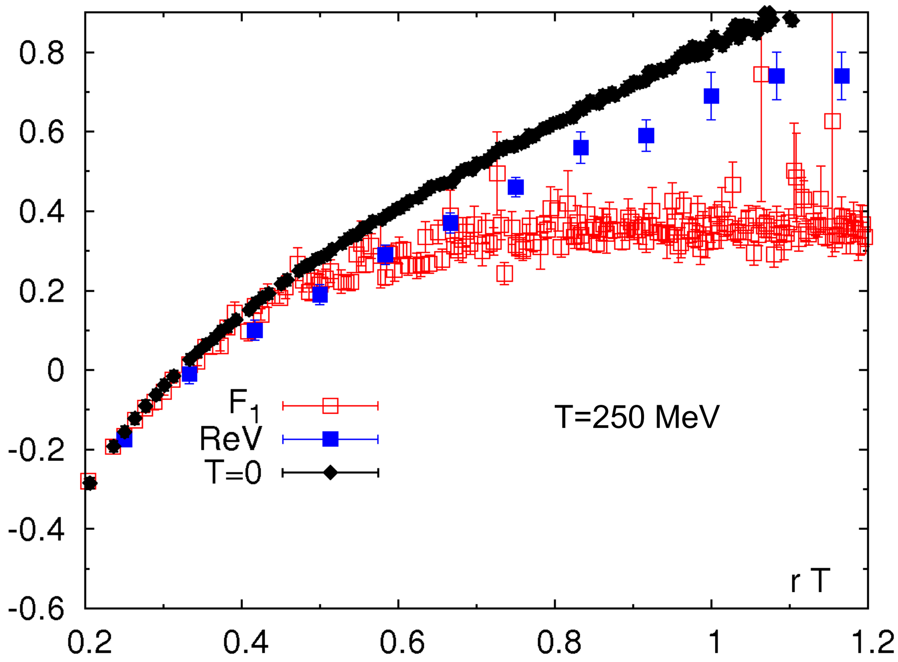
V_{eff} decreases with τ due to the width of the spectral functions, its slope increases with T and the distance r as observed in the lattice calculations

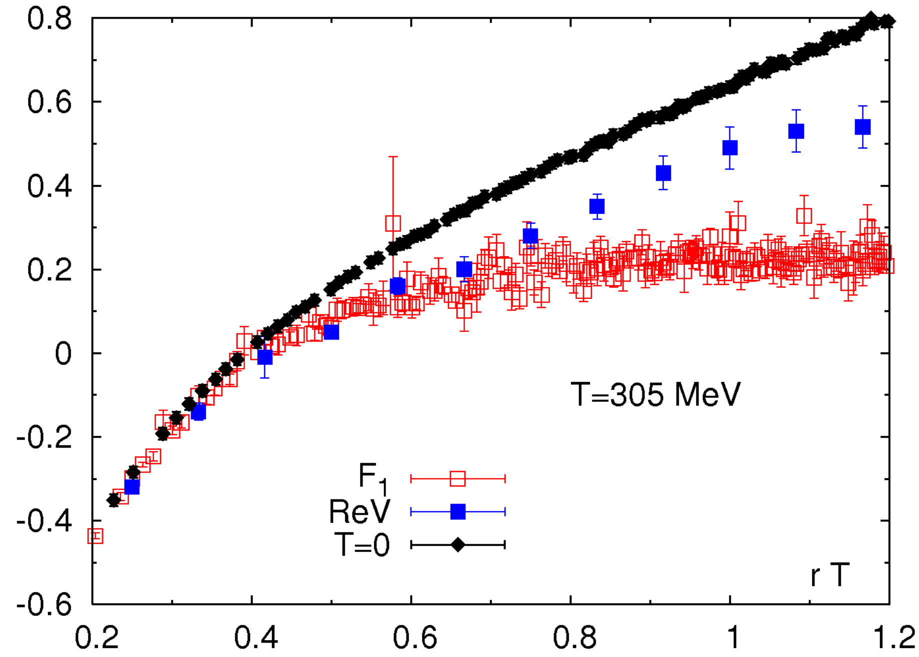
Use $\sigma^{HTL}((\omega\text{-}E)\,\lambda)$ as a 2-parameter fit Ansatz for the lattice results:



Real part of the potential above deconfinement

Results for $N_\tau = 12$ lattices using $\sigma^{HTL}((\omega\text{-}E)\,\lambda)$ as a 2-parameter fit Ansatz:

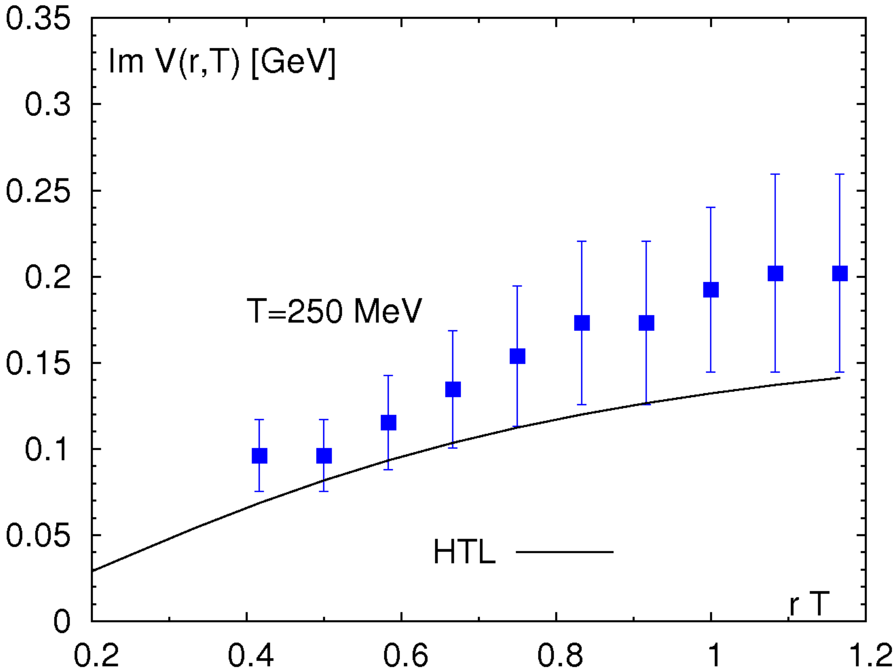


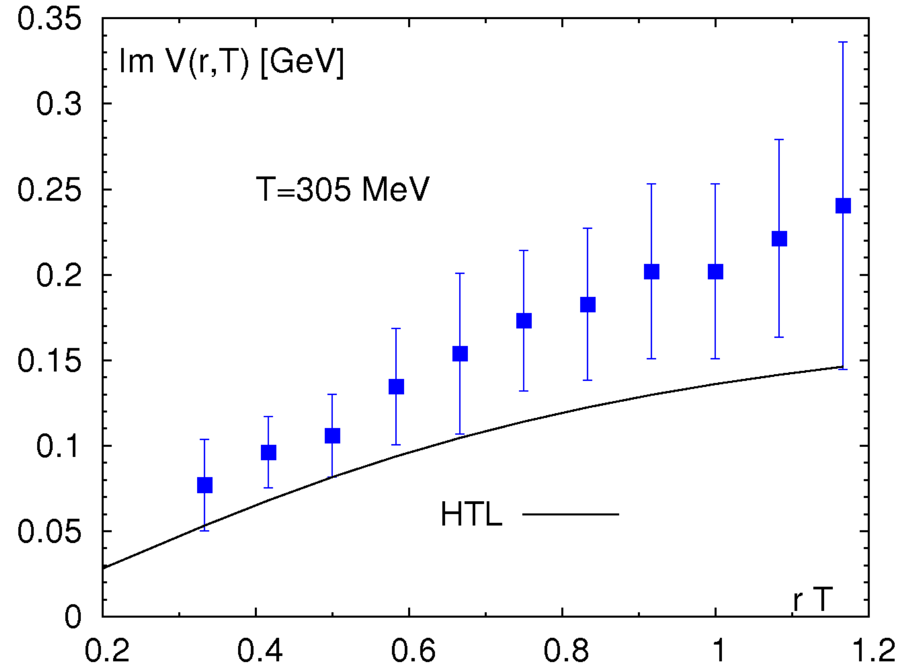


- For $r\,T < 0.7$ the real part of the potential is roughly equal to the singlet free energy
- At larger distances it is between the singlet free energy and the $T=0$ potential
- The real part of the potential saturates at $r\,T \sim 1$ (screening) at a fairly large value (non-perturbative effects)

Imaginary part of the potential above deconfinement

Results for $N_\tau = 12$ lattices using $\sigma^{HTL}((\omega\text{-}E)\,\lambda)$ *as a 2-parameter fit Ansatz:*





- The imaginary part of the potential has large errors as the width of the spectral functions is difficult to extract from the lattice correlators

- The imaginary part increases with r and saturates at $r\,T \sim 1$

- The central value imaginary part of the potential is *(1.5 - 2.0)* larger than in HTL perturbation theory

Summary

- The free energy and singlet free energy of static quark anti-quark pair calculated with HISQ action show strong screening effects for $T>200$ MeV and the results are in agreement with earlier calculations perfromed with p4 action

- The behavior of the Wilson loops calculated on the lattice is qualitatively the same as in HTL perturbation theory

- It is possible to extract the potential at $T>0$ from the Wilson loops using fits based on HTL spectral functions

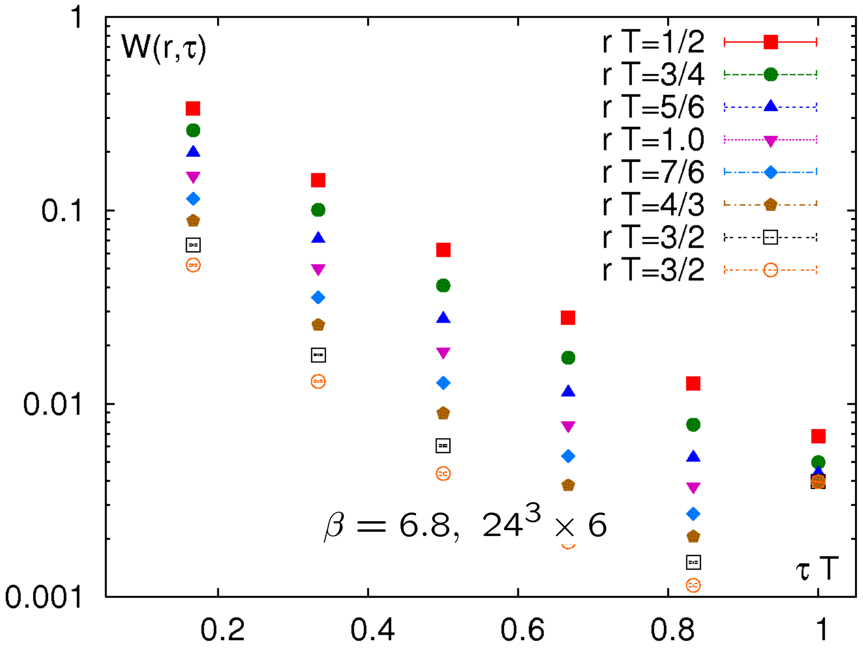
- The potential at $T>0$ shows strong screening effects at for $T>200$ MeV and its real part is larger than the singlet free energy

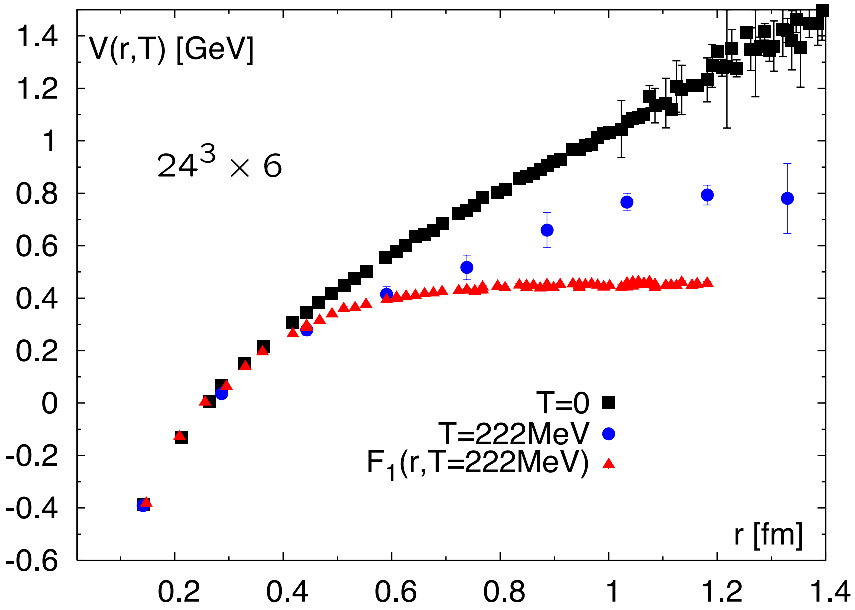
- The imaginary part of the potential is about a factor of two larger than the HTL perturbation theory result

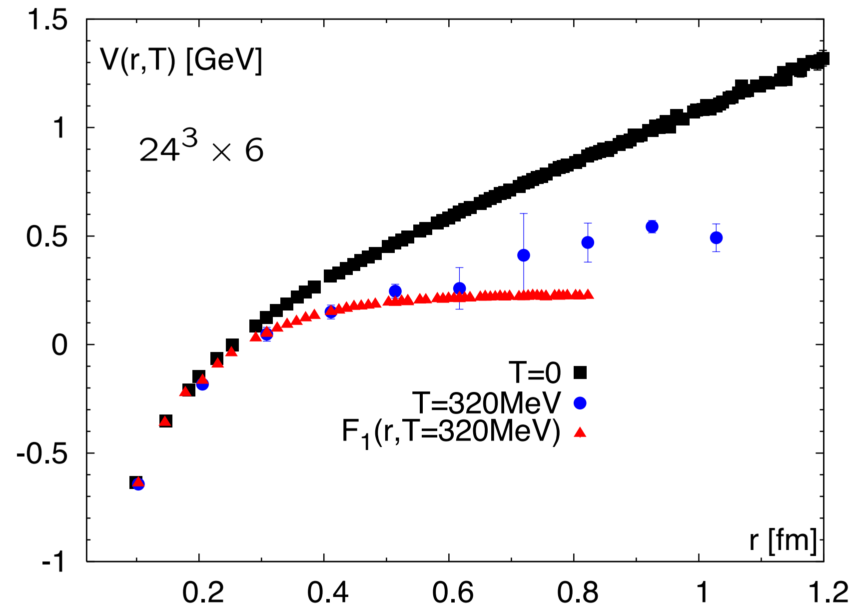
- The potential extracted from the lattice is similar to the phenomenological "maximally binding" potential used in potential model calculations but differs in details

$\Rightarrow$ consequences for the quarkonium spectral functions and melting temperatures ?

Back-up slides







• for *T>200* MeV the potential is significantly modified compared to the *T=0* result

• the potential is larger than the singlet free energy but approaches it from above as the temperature increases

similar behavior in quenched QCD
Burnier, Rothkopf 2012