Gibbs paradox and the QCD phase transition

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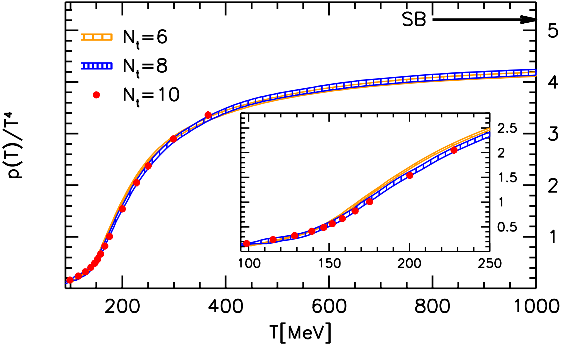

Sz. Borsanyi, G. Endrodi, Z. Fodor, A.J., S. D. Katz, S. Krieg, C. Ratti, K.K. Szabo, JHEP 1011 (2010) 077

Thermodynamics of strongly interacting plasma

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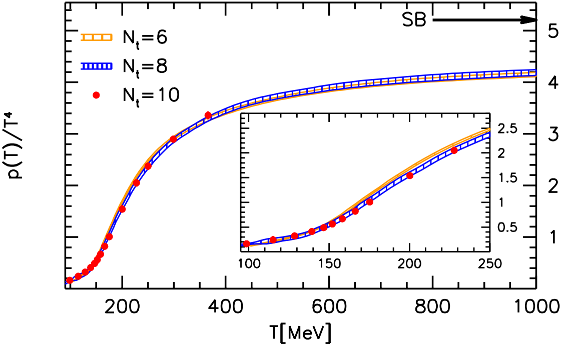

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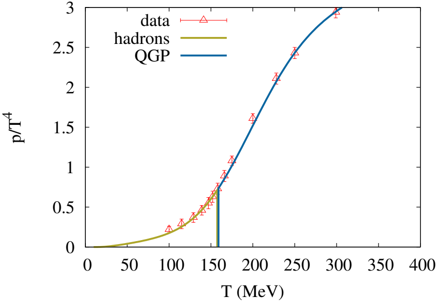
How do we interpret the results?

Thermodynamics of strongly interacting plasma

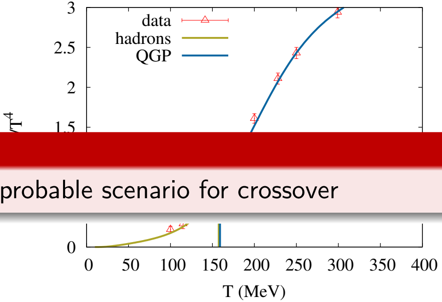
hadron-QGP phase transition



- low temperature: : hadrons
- high temperature: : QGP
- (would-be) critical temperature $T_c = 156\,\mathrm{MeV}$.
- in reality: crossover – everything changes continuously
  still sharp change in the excitations??

Thermodynamics of strongly interacting plasma

hadron-QGP phase transition



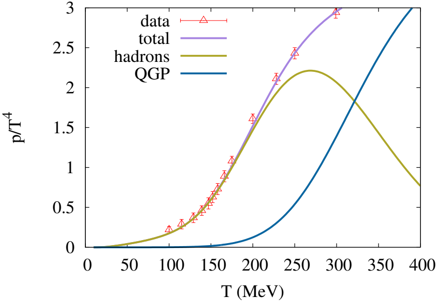
Lesson

Very unprobable scenario for crossover

- low temperature: : hadrons
- high temperature: : QGP
- (would-be) critical temperature $T_c = 156\,\mathrm{MeV}$.
- in reality: crossover – everything changes continuously
  still sharp change in the excitations??

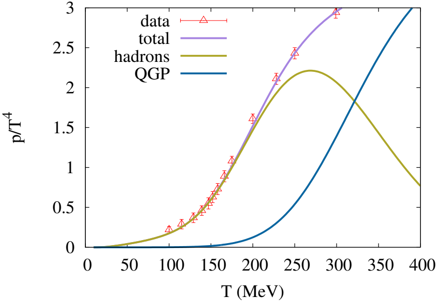
Thermodynamics of strongly interacting plasma

in this talk: Proposal: continuous changes



Thermodynamics of strongly interacting plasma

in this talk: Proposal: continuous changes



- hadrons determine thermodynamics up to $T \lesssim T_c$
- quarks determine thermodynamics for $T \gtrsim 3\,T_c \approx 450\,\mathrm{MeV}$
- hadrons survive T_c, quarks appear continuously
- new phase of matter appears at T_c, but not QGP

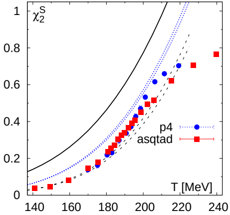
Outlines

Description of QCD thermodynamics: low temperature

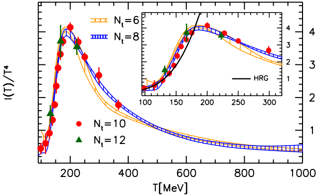
HRG (hadron resonance gas):

$$P = \frac{T}{2\pi^2} \sum_{n=1}^{N} \mp \int_0^\infty dp\, p^2 \ln\left(1 \mp e^{-E(p,m_n)/T}\right)$$

- free hadrons, $\pm$ for bosons/fermions
- masses from experiments (PDG)
- valid to $T < 150 - 180$ MeV



(P. Huovinen and P. Petreczky, Nucl. Phys. A **837**) (26 (2010) [arXiv:0912.2541 [hep-ph]].)

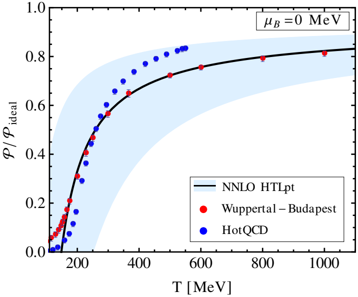


(Sz. Borsanyi, G. Endrodi, Z. Fodor, A.J., S. D. Katz)
(S. Krieg, C. Ratti, K.K. Szabo, JHEP 1011 (2010) 077)

Description of QCD thermodynamics: high temperature

GQP (quark-gluon plasma)

- QCD degrees of freedom
- resummation needed
  DR, HTL – 3-loop
- IR safe quantities
  like P and S
- valid for
  $T \lesssim 250 - 300\,\mathrm{MeV}$



(N. Haque *et.al.* [e-Print: arXiv:1309.3968])

Phase transition regime: observations

$T \in [150, 250-300]\,\mathrm{MeV}$

- crossover (contiuous) phase transition
- hadrons do not disappear at T_c

  (J. Liao, E.V. Shuryak PRD73 (2006) 014509 [hep-ph/0510110])

  MC: hadronic states are observable even at $T \sim 1.5\,T_c$

  (AJ., P. Petreczky, K. Petrov, A. Velytsky, PRD75 (2007) 014506)

- MC: non-quasiparticle-like correlations $T \in [150, 250]$ MeV:
  free HRG, QGP description not possible

  (P. Petreczky, J. Phys. Conf. Ser. **402**, 012036 (2012) [arXiv:1204.4414 [hep-lat]])
  (R. Bellwied, S. Borsanyi, Z. Fodor, S. D. Katz and C. Ratti, [arXiv:1305.6297 [hep-lat]])

Phase transition regime

Puzzles:

- HRG is not stable at large T (Hagedorn instability)
  What happens with the hadrons?
- What happens with the quarks at low T?

Possible explanation:

- hadrons/quarks exist, but have large self-energies

$$m_h \stackrel{T\to\infty}{\longrightarrow} 0, \quad m_{q,g} \stackrel{T\to 0}{\longrightarrow} \infty$$

- leads to small thermal weights $\sim e^{-\beta m} \ll 1$
- BUT: MC data do not support this idea!
  direct mass, and correlation measurments

Alternative picture

melting/dissociation of hadrons

- particle state disappears
- it would explain why we do not see quarks at low energy or hadrons at very high temperatures

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Question

Is it possible to change the number of species without changing the ground state?

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melting/dissociation of hadrons

- particle state disappears
- it would explain why we do not see quarks at low energy or hadrons at very high temperatures

Question

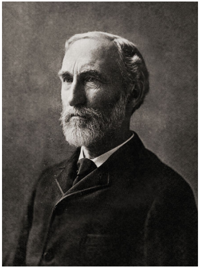
Is it possible to change the number of species without changing the ground state?

Physical example: Gibbs paradox!

Outlines

1. The QCD equation of state
2. Phases of QCD
3. Gibbs paradox
4. Mathematical treatment of a generic spectrum
5. QCD thermodynamics
6. Conclusions

Gibbs paradox: changing number of particle species



J.W.Gibbs (1839-1903)

(J.W.Gibbs, 1875-1878; E.T.Jaynes, 1996)

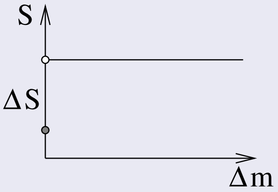
take two (ideal) gases with m, $m + \Delta m$ masses:
initially n_1, V_1 n_2, V_2, p, T
mix them: $V = V_1 + V_2$, $n = n_1 + n_2$
entropy difference (mixing entropy) ($f = n_1/n_2$)

$$\Delta S = -nR(f \log f + (1-f)\log(1-f)) \Rightarrow -nR \log 2, \text{ for } n_1 = n_2,\ V_1 = V_2.$$

- describes change of number of particle species $2 \to 1$
- relies on (in)distinguishability of particles

(not on the change of ground state)

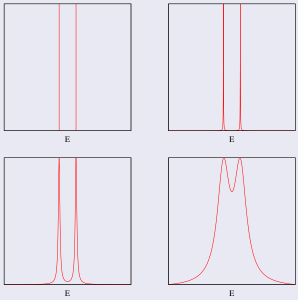
discuntinuous entropy



Gibbs paradox in interacting systems

Without interaction the energy levels (spectral lines) are infinitely thin lines. In interacting gases the spectral lines broaden.

spectrum in interacting gases



- 1st plot: 2 lines
  4th plot: one broad peak (?)
- Gibbs: particles are distinguishable, if a mixed gas can be separated by some means. Going from case 1 to 4 this is harder and harder!
- if $\Gamma \gtrsim \Delta m$ we cannot separate peaks

Outlines

Description of melting

Lesson of the Gibbs paradox

melting $\equiv$ merging spectral lines
more generally: disappearance of a peak from the spectrum
$\Rightarrow$ We need to treat the complete spectrum!

- in Hamiltonian formalism exponential damping
  $\Rightarrow$ $\hat{H} \to \hat{H} - i\gamma$ $\Rightarrow$ loss of unitarity!
  $\Rightarrow$ use Lagrangian formalism
- largest part of interactions is is used to change the spectrum (c.f. HRG: strongly interacting quarks $\to$ weakly interacting bound states)
  $\Rightarrow$ neglect interactions
- spectrum from experiments or from self-consistent methods (SD-equations, 2PI)
  $\Rightarrow$ we start from a given ϱ spectral function

Lagrangian formalism for general spectral functions

for one bosonic (fermionic) component:

$$\mathcal{L} = \frac{1}{2}\Phi^*(p)\mathcal{K}(p)\Phi(p)$$

- unique $\varrho \to \mathcal{K}$ relation: $G_{ret}(p) = \int \frac{d\omega}{2\pi} \frac{\varrho(\omega,\mathbf{p})}{p_0-\omega+i\varepsilon}, \quad \mathcal{K} = \mathrm{Re}\, G_{ret}^{-1}$
- defines a consistent field theory:
  unitary, causal, Lorentz-invariant, $E, \mathbf{p}$ conserving
  (AJ. Phys.Rev. D86 (2012) 085007 [arXiv:1206.0865])
- thermodynamics: $\varepsilon(T) = \langle T^{00} \rangle$, and use thermodynamical relations to obtain pressure.

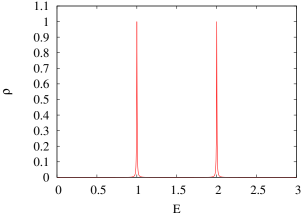
Result

$\varrho \to G_{ret} \to \mathcal{K}$, then

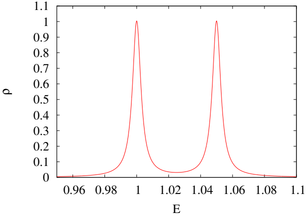
$$\varepsilon = \int \frac{d^4p}{(2\pi)^4} E(p) n(p_0) \varrho(p), \qquad E(p) = p_0 \frac{\partial \mathcal{K}}{\partial p_0} - \mathcal{K}$$

Gibbs paradox for interacting systems

Gibbs paradox in the language of spectral functions?



$\Rightarrow$



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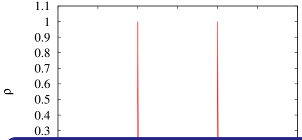

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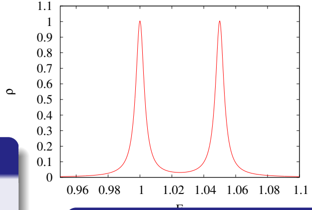
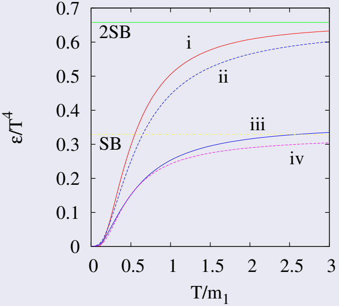


energy density



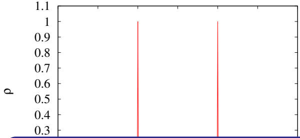
Curves

$m_1 = 1$, $m_2 = 2$, $\Gamma = 0$ or 0.2

- i. $\Gamma = 0$
- ii. independent, finite Γ
- iii. $\Gamma/\Delta m = 0.2$
- iv. one free particle

Gibbs paradox for interacting systems

Gibbs paradox in the language of spectral functions?



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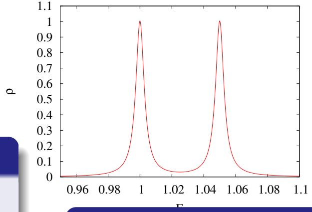

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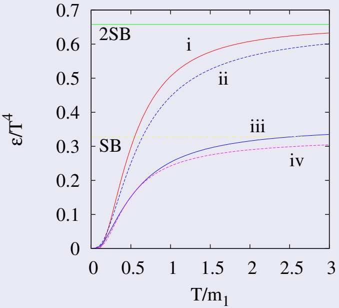

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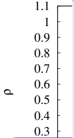
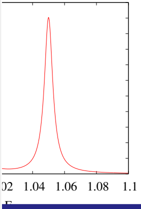
degrees of freedom disappear continuously!

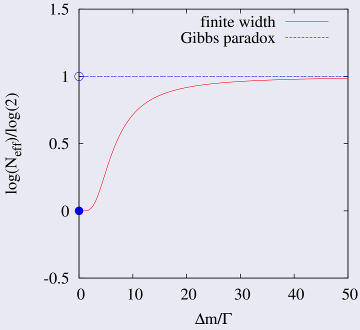
Gibbs paradox for interacting systems

Gibb … nctions?

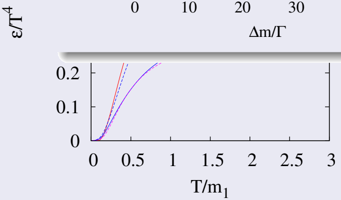




number of species



energ



= 2, $\Gamma = 0$ or 0.2

$= 0$

independent, finite Γ

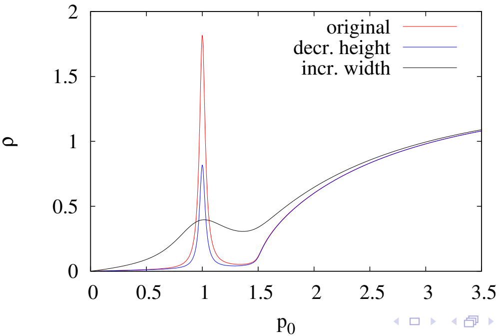
- iii. $\Gamma/\Delta m = 0.2$
- iv. one free particle

degrees of freedom disappear continuously!

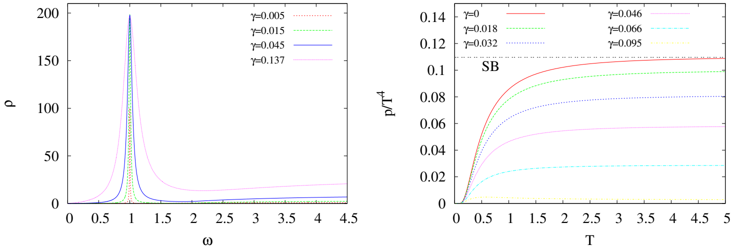
Outlines

Melting hadrons

- spectral function: most important effect is merging quasiparticle and scattering states
  $\Rightarrow \quad \varrho = \text{QP peak} + \text{continuum}$
- thermal variation height and/or width of the QP peak changes mass variation is not too important (especially for very massive hadrons)



Pressure of melting hadrons

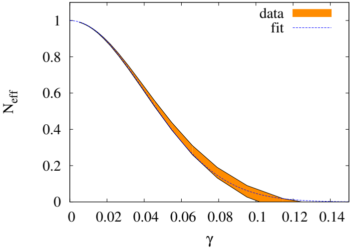


rescaled plots for fixed QP height $\Rightarrow$ correct height from sum rule.

- pressure decreases!
- p/T^4 can be very small even for large QP peak heights
- pressure curves are self-similar

Effective thermodynamical DoF

Thermodynamical dof: $N_{eff}(T) = \frac{p(T,\gamma)}{p(T,\gamma=0)}$



- just slightly temperature dependent (orange band)
- fit: Gaussian $e^{-\frac{\gamma^2}{2\gamma_0^2}}$

pressure of a melting quasiparticle

$$p(T) = N_{eff}(T)p_{ideal}(T) = e^{-c\gamma^2(T)}p_{ideal}(T)$$

QCD thermodynamics: statistical description

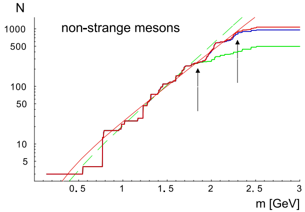
Describe HRG with melting hadrons

- HRG: huge # of hadronic contributions, each small!
  $\Rightarrow$ statistical description is needed
- we need spectra. . . hard to obtain
  $\Rightarrow$ idealized, simplified picture for hadron masses and widths.

(AJ. Phys.Rev. D88 (2013) 065012 [arXiv:1306.2660])

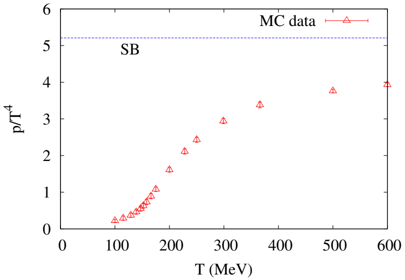
- masses: Hagedorn spectrum
  $$\varrho_{hadr}(m) \sim (m^2 + m_0^2)^a e^{-m/T_H}$$
  several fits (eg. $a = 0$) possible
- width
  $$\gamma^2(T) = \gamma_0^2 + cT^2$$
  consistent with model-calculations

(F. Riek and J. Knoll, NPA **740**, 287 (2004))



(W. Broniowski *et.al.*PRD **70**, 117503 (2004))

Interpreting QCD pressure



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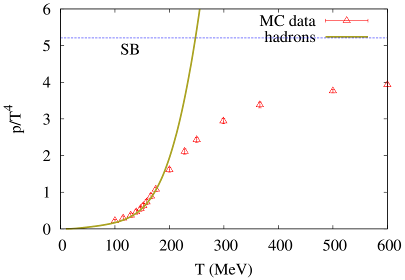

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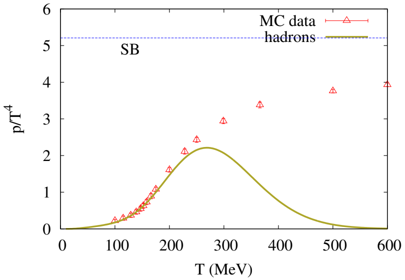

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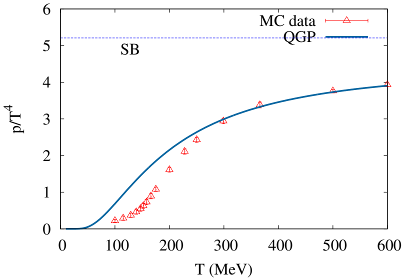

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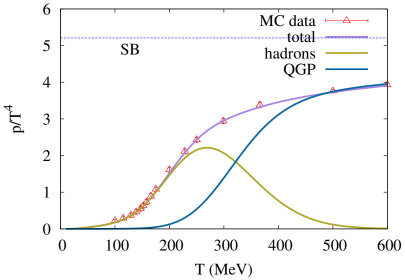
- $T > 300\,\mathrm{MeV}$: QGP parameters (fixed $m_q = 330\,\mathrm{MeV}$, $m_h = 600\,\mathrm{MeV}$)

Interpreting QCD pressure



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- $\gamma^2_{hadr} = (T/T_0)^2$: fit T_0 to avoid large hadron pressure
- $T > 300\,\mathrm{MeV}$: QGP parameters (fixed $m_q = 330\,\mathrm{MeV}$, $m_h = 600\,\mathrm{MeV}$)
- quark and gluon width depends on the number of hadrons $\gamma^2_{QGP} = \gamma^2_0 + cN^{\alpha}_{hadr}$

Outlines

Conclusions

- changing number of particle species:
  - change ground state, or
  - change peaks of the spectrum
- physical example: Gibbs paradox:
  - in ideal gas case: diconsintuity in N_{eff}
  - in interacting case: continuous change in N_{eff}
- application to QCD ($T_c = 156\,\mathrm{MeV}$)
  - for $T < T_c$: HRG
  - for $T > 3T_c$: QGP
  - for $3T_c > T > T_c$: mixed phase with non-quasiparticle spectra
    new phase of QCD matter
- Outlook
  - cross-correlations (through QP-multiparticle cont. overlap)
  - transport coefficients