Phase-space view of pair production in inhomogeneous fields

Dániel Berényi [1,2], Sándor Varró [2], Vladimir Skokov [3], Péter Lévai [2]

1, Loránd Eötvös University, Budapest, Hungary
2, Wigner RCP, Budapest, Hungary
3, Brookhaven National Laboratory, Upton, USA

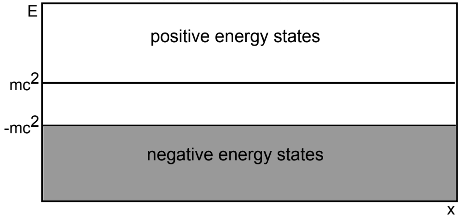
Zimányi School, Budapest, 6. 12. 2013

Table of Contents

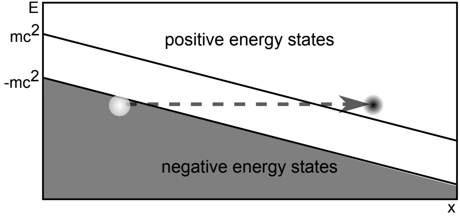
Table of Contents

Phenomenology of pair production from vacuum:

Without external field:



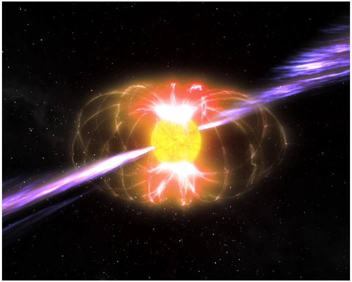
With external field:



QED Motivation

- QED pair production from vacuum was predicted half a century ago, but was not yet observed $\rightarrow$ "Holy Grail of QED".
- Rapid development of laser technology promises its observation in the near future (ELI, IZEST, HiPER, LMJ).
- QED pair production may take place in near miss heavy ion collisions.

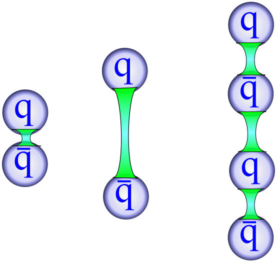
QED pair production near massive astrophysical objects (black holes, magnetars, possible source of gamma ray bursts?)



John Rowe Animations

QCD Motivation

From QCD:
The success of color rope/string models in describing Heavy Ion collisions (e.g. at LHC):



Quark potential is linear with separation: if a $q - \bar{q}$ pair is separating, the interaction creates more and more quark pairs until energy is depleted.

Challanges

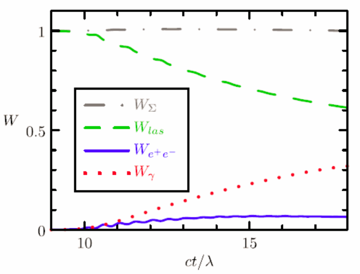
Relevant scales for QED:

- Field strength: $\mathcal{E}_c = \frac{m^2 c^3}{e\hbar} \approx 1.3 \cdot 10^{18} \frac{\mathrm{V}}{\mathrm{m}}$
- Time: $t_c = \frac{\hbar}{mc^2} \approx 1 \cdot 10^{-21} \mathrm{s}$
- Frequency: $\omega_c = \frac{e\mathcal{E}}{mc} \approx 8 \cdot 10^{20} \mathrm{Hz}$ $(\mathcal{E} = \mathcal{E}_c)$
- Spatial gradient: $\partial_r = \frac{mc}{\hbar} \approx 6.6 \cdot 10^{10} \mathrm{m}^{-1}$

For QCD replace *m* with quark mass.

Challanges

QED: The laser beam is absorbed in EM cascades starting from a single pair OR seed particle!



E.N. Nerush et al. PRL 106:035001 (2011).

QCD: The low energy part of the spectrum thermalizes, high pT particles are the main source of information.

Both: Tera- and Peta-scale physics requires Tera- and Peta-scale simulations!

Table of Contents

History

Some analytic results from the history of pair production:

- Homogeneous static electric field. (J. Schwinger)

$$P_{e^+e^-} = \frac{e^2\mathcal{E}^2}{4\pi^3} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left(-\pi \frac{nm^2}{e\mathcal{E}}\right)$$

- Some special analytic time dependent, homogeneous electric fields.

  (V. S. Popov, M. S. Marinov, N. B. Narozhny, A. I. Nikishov, ...)

- Quark and soft gluon production in constant chromoelectric field:

  (G. C. Nayak, P. Nieuwenhuizen)

$$P_{q\bar{q}} = \frac{1}{4\pi^3} \sum_{n=1}^{\infty} \sum_{j=1}^{3} \frac{|g\lambda_j|}{n} \exp\left(-\pi \frac{n(p_T^2+m^2)}{|g\lambda_j|}\right)$$

$$P_g = \frac{1}{8\pi^3} \sum_{n=1}^{\infty} (-1)^{(n+1)} \sum_{j=1}^{6} \frac{|g\lambda_j|}{n} \exp\left(-\pi \frac{np_T^2}{|g\lambda_j|}\right)$$

History

Recently a different approach is gaining attention: *kinetic formulation*

- Transport equation for QCD Wigner operator, later for Abelian plasmas. (D. Vasak, M. Gyulassy, H.-T. Elze)
- Equal time formulation of QED transport equations for the Wigner function (named the Dirac-Heisenberg-Wigner, DHW equations).

  (I. Bialynicki-Birula, P. Górnicki, J. Rafelski)
- Same for scalar QED:

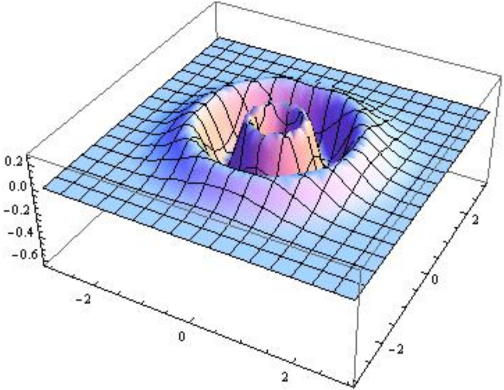
  (C. Best, P. Górnicki, W., Greiner and S. Varró, J. Javanainen)
- Same for SU(N) quark pairs:

  (A. V. Prozorkevich, S. A. Smolyansky, S. V. Ilyin)
- Study of inhomogeneous and time dependent QED particle production (F. Hebenstreit, R. Alkofer, H. Gies)

Wigner function

Tool of description: the Wigner function:

- Quantum analogue of the classical phase space distribution.



Wigner function of an n=3 Fock state.

Wigner function

How it is defined?

- Take the equal time density matrix in terms of 'center of mass' coordinates:

$$\hat{C}(\vec{x},\vec{s},t) = e^{-ie\int_{-1/2}^{1/2}\vec{\mathcal{A}}(\vec{x}+\lambda\vec{s},t)\vec{s}\mathrm{d}\lambda}\left[\Psi(\vec{x}+\frac{\vec{s}}{2},t),\bar{\Psi}(\vec{x}-\frac{\vec{s}}{2},t)\right] \quad (1)$$

- Take the expectation value.
- Fourier transform it w.r.t the coordinate difference:

$$W(\vec{x},\vec{p},t) = -\frac{1}{2}\int e^{-i\vec{p}\vec{s}}\langle 0|\hat{C}(\vec{x},\vec{s},t)|0\rangle \mathrm{d}^3 s \quad (2)$$

Wigner function

- The time derivative of the Wigner function gives us the evolution of the system:
- spin$= 1/2$:

$$D_t W = -\frac{1}{2}\vec{D}_{\vec{x}}[\gamma^0\vec{\gamma}, W] - im[\gamma^0, W] - i\vec{P}\{\gamma^0\vec{\gamma}, W\} \qquad (3)$$

Wigner function

The equation has the following non-local differential operators:
Theoretical approximation: external field is classical (quantum fluctuations are neglected)

$$D_t = \partial_t + e\vec{\mathcal{E}}(\vec{x},t)\vec{\nabla}_{\vec{p}} - \frac{e\hbar^2}{12}(\vec{\nabla}_{\vec{x}}\vec{\nabla}_{\vec{p}})^2\vec{\mathcal{E}}(\vec{x},t)\vec{\nabla}_{\vec{p}} + ... \tag{4}$$

$$\vec{D}_{\vec{x}} = \vec{\nabla}_{\vec{x}} + e\vec{\mathcal{B}}(\vec{x},t)\times\vec{\nabla}_{\vec{x}} - \frac{e\hbar^2}{12}(\vec{\nabla}_{\vec{x}}\vec{\nabla}_{\vec{p}})^2\vec{\mathcal{B}}(\vec{x},t)\times\vec{\nabla}_{\vec{p}} + ... \tag{5}$$

$$\vec{P} = \vec{p} + \frac{e\hbar}{12}(\vec{\nabla}_{\vec{x}}\vec{\nabla}_{\vec{p}})\vec{\mathcal{B}}(\vec{x},t)\times\vec{\nabla}_{\vec{p}} + ... \tag{6}$$

Observables from the Wigner function

For spin-1/2, the 4x4 gamma matrix basis is used:

$$W(x,p,t) = \frac{1}{4}\left[\mathbb{1}\mathbb{s} + i\gamma_5\mathbb{p} + \gamma^\mu\mathbb{v}_\mu + \gamma^\mu\gamma_5\mathbb{a}_\mu + \sigma^{\mu\nu}\mathbb{t}_{\mu\nu}\right] \tag{7}$$

Some components has clear physical interpretation:

- $\mathbb{s}$: Mass density
- $\mathbb{v}_0$: Charge density
- $\vec{\mathbb{v}}$: Current density
- $\vec{\mathbb{p}}\vec{\mathbb{v}} + m\mathbb{s}$: Energy density
- $\vec{\mathbb{a}}$: Spin density

Equations of motion for the spin-1/2 Wigner function

We arrive at a system for 16 unknown real functions:

$$
\begin{aligned}
&D_t\mathbb{s} && && - && 2\vec{P}\cdot\vec{\mathbb{t}}_1 &&= 0 && (8)\\
&D_t\mathbb{p} && && + && 2\vec{P}\cdot\vec{\mathbb{t}}_2 &&= 2m\mathbb{a}_0 && (9)\\
&D_t\mathbb{v}_0 && + && \vec{D}_{\vec{x}}\cdot\vec{\mathbb{v}} && && &&= 0 && (10)\\
&D_t\mathbb{a}_0 && + && \vec{D}_{\vec{x}}\cdot\vec{\mathbb{a}} && && &&= 2m\mathbb{p} && (11)\\
&D_t\vec{\mathbb{v}} && + && \vec{D}_{\vec{x}}\mathbb{v}_0 && + && 2\vec{P}\times\vec{\mathbb{a}} &&= -2m\vec{\mathbb{t}}_1 && (12)\\
&D_t\vec{\mathbb{a}} && + && \vec{D}_{\vec{x}}\mathbb{a}_0 && + && 2\vec{P}\times\vec{\mathbb{v}} &&= 0 && (13)\\
&D_t\vec{\mathbb{t}}_1 && + && \vec{D}_{\vec{x}}\times\vec{\mathbb{t}}_2 && + && 2\vec{P}\mathbb{s} &&= 2m\vec{\mathbb{v}} && (14)\\
&D_t\vec{\mathbb{t}}_2 && - && \vec{D}_{\vec{x}}\times\vec{\mathbb{t}}_1 && - && 2\vec{P}\mathbb{p} &&= 0 && (15)
\end{aligned}
$$

Equations of motion for the Wigner function

Vacuum initial conditions:

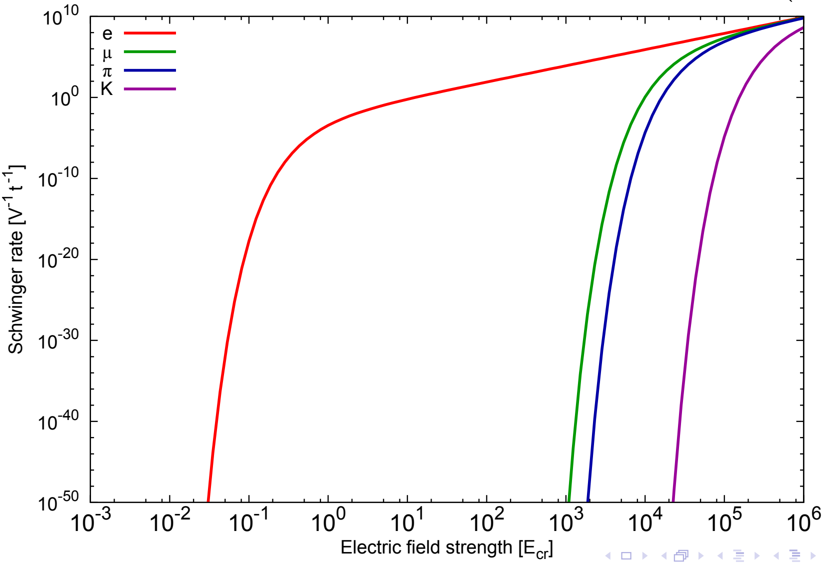
- spin$= 1/2$:

$$\mathbb{s} = -\frac{2m}{\omega}, \qquad \vec{\mathbb{v}} = -\frac{2\vec{p}}{\omega} \tag{16}$$

with $\omega = \sqrt{m^2 + \vec{p}^2}$, and all other components are zero.

Static uniform electric field limit

The Schwinger result is recovered for spin-1/2: $n \simeq \frac{e^2\mathcal{E}^2}{4\pi^3} \exp\left(-\frac{m^2\pi}{e\mathcal{E}}\right)$



The Quantum Kinetic limit

- A special case is when $\mathcal{B} = 0$, and $\mathcal{E}(x, y, z, t) = \mathcal{E}(t)$
- This leads to the Quantum Kinetic equation on f, u, v:

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{e\mathcal{E}\varepsilon_\perp}{\omega^2} v \tag{17}$$

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{1}{2}\frac{e\mathcal{E}\varepsilon_\perp}{\omega^2}(1 - 2f) - 2\omega u \tag{18}$$

$$\frac{\mathrm{d}u}{\mathrm{d}t} = 2\omega v \tag{19}$$

where:

$$\omega^2(\vec{p}, t) = \varepsilon_\perp^2 + \vec{p}_\parallel^2 \tag{20}$$

$$\varepsilon_\perp^2 = m^2 + \vec{p}_\perp^2 \tag{21}$$

$$\vec{p} = (\vec{q}_\perp, q_\parallel - e\mathcal{A}(t)) \tag{22}$$

Quark pair production

The Wigner function based description can be extended to higher symmetries (non-Abelian case) too.

(Quark Wigner function evolution: A.V. Prozorkevich, S.A. Smolyansky, S.V. Ilyin)

- There will be more and more components
- The intermixing of components will be more complex (cannot be reduced to an ODE!)
- If the SU(N) color matrices replaced by unity, the QED equations are recovered.

The hierarchy of pair production models

SU(N) quark equations

$\Downarrow$

color generators $\leftarrow$ identity

$\Downarrow$

U(1) QED DHW equations

$\Downarrow$

Homogeneous electric field

$\Downarrow$

Quantum Kinetic (Vlasov) equations

$\Downarrow$

Static electric field

$\Downarrow$

Schwinger formula

SU(N) scalar diquark equations

$\Downarrow$

color generators $\leftarrow$ identity

$\Downarrow$

U(1) sQED equations

$\Downarrow$

Homogeneous electric field

$\Downarrow$

scalar Quantum Kinetic (Vlasov) equations

$\Downarrow$

Static electric field

$\Downarrow$

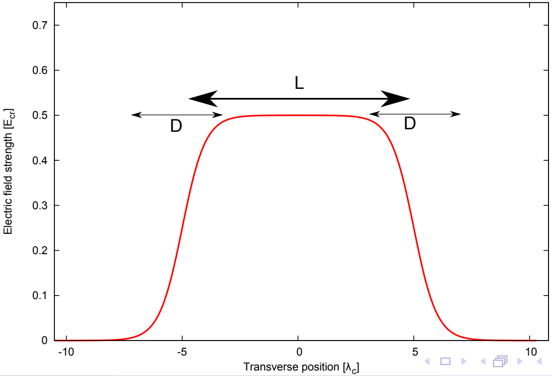
scalar Schwinger formula

Table of Contents

Applications of the inhomogeneous Wigner equations of motion

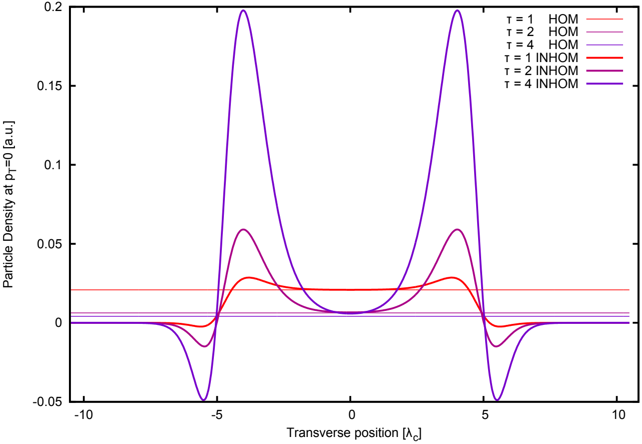
Motivating question: What happens at the surface of the color string? We apply the abelian approximation, and investigate it by the Wigner equations.

Consider an inhomogeneous plateu field in the transverse spatial direction and Sauter-like ($\cosh(t/\tau)^{-2}$) time dependence. ($E_0 = 0.5$).



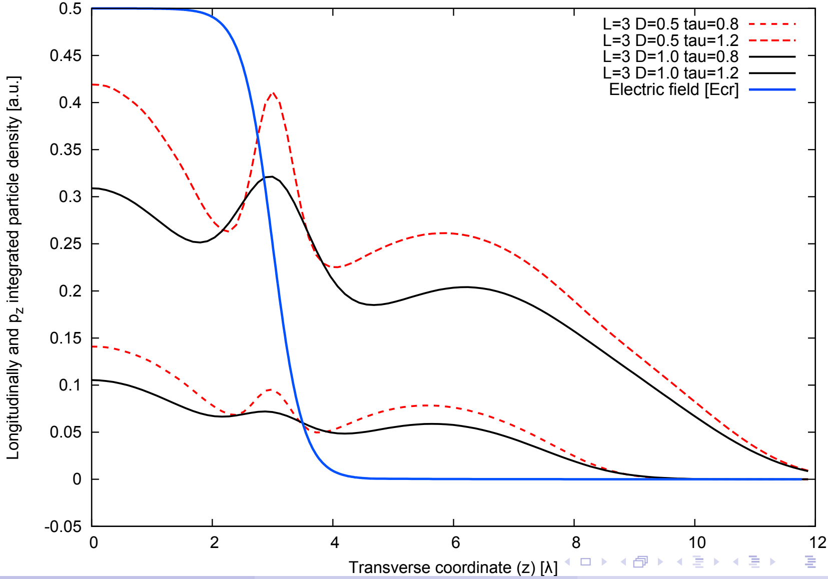
Abelian (spin-1/2) toy model of a color string

The inhomogeneity on the edges significantly increase the particle density! And it is enhanced by longer pulses.

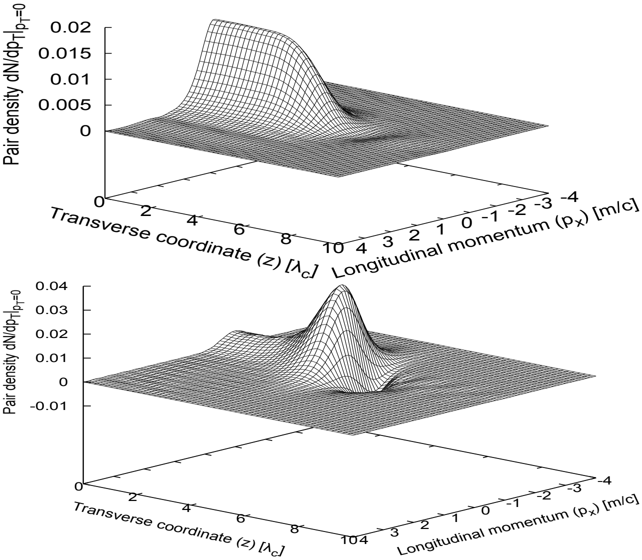


Abelian (spin-1/2) toy model of a color string

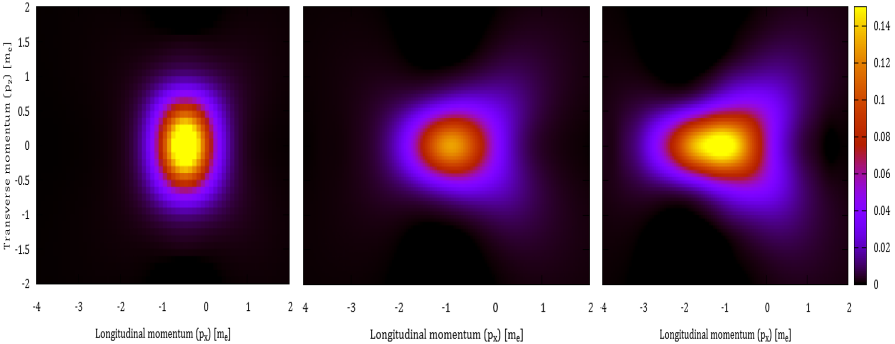
Zoom in:



Abelian (spin-1/2) toy model of a color string



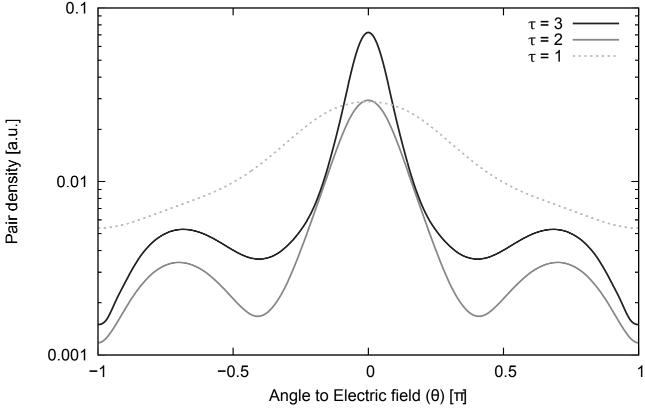
Abelian (spin-1/2) toy model of a color string



(a) $\tau = 1\frac{\lambda_c}{c}$ (b) $\tau = 2\frac{\lambda_c}{c}$ (c) $\tau = 3\frac{\lambda_c}{c}$

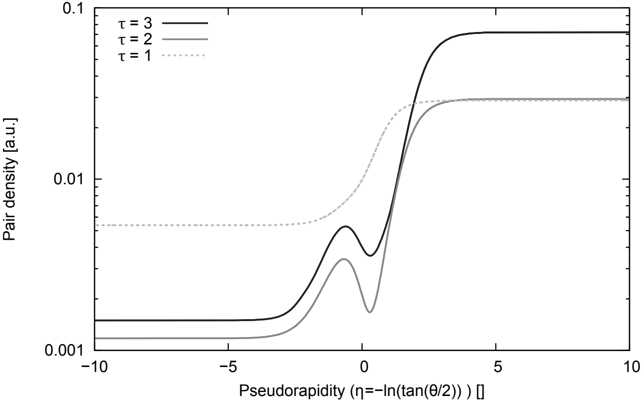
Abelian toy model of a color string

What would a calorimetric experiment see?
Longitudinal angle to the Electric field (θ, with $\theta = 0$ is longitudinal direction, $\theta = \pm\frac{\pi}{2}$ is the transverse direction):



Abelian toy model of a color string

What would a calorimetric experiment see?
Pseudorapidity ($\eta = -\ln(\tan(\frac{\theta}{2}))$):



Abelian toy model of a color string

Observations:

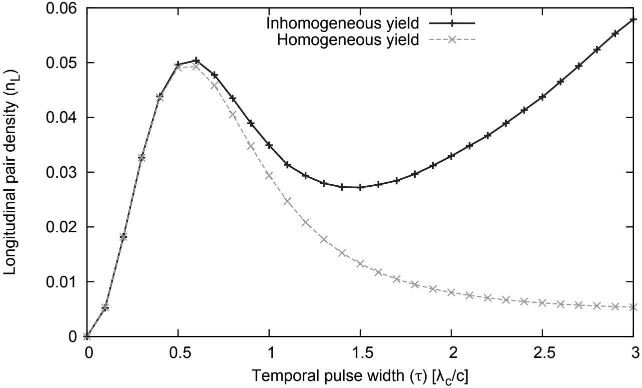
- The inhomogeneity increases the particle yield.
- The increase is further enhanced if the pulse lasts longer (as opposed to the homogeneous case).

As a consequence, homogeneous models of string fragmentation may underestimate the particle production rates.

- The transverse spectra develops distinct high-momenta "shoulders" at the boundaries.

Abelian toy model of a color string

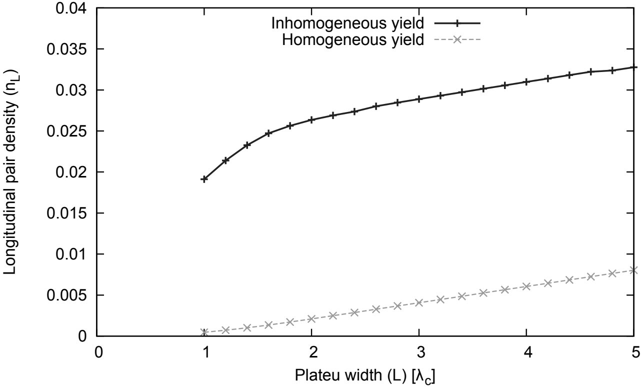
Integrated particle yields:



Again: increase with pulse length!

Abelian toy model of a color string

Integrated particle yields:



Same slope: inhomogeneous pair production is a surface effect!
Predicted by Heisenberg in 1934!

Summary

- We have shown how pair production phenomena connects different areas of high energy physics, from laser physics to heavy ion collisions.
- The evolution of the scalar and spin-1/2 Wigner function in 3+1 dimensions is now possible including space/time dependent field configurations.
- As an application, we discussed an abelian model of a gluon string, focusing on the pair production at the surface of the string and found that homogeneous models may be missing characteristic effects in the spectra and in the particle yield.

Supporters: OTKA Grants No. 77816, No. 104260, No. 106119 and US DoE contract DE-AC02-98CH10886.