Screening masses from lattice QCD

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In collaboration with

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Zimányi Winter School on Heavy Ion Physics

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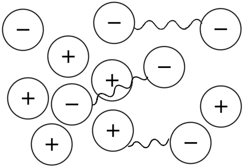
Outline

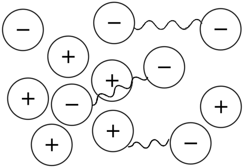
Perturbative results



QED plasma

m_D, Debye screening mass: pole of the photon propagator at finite temperature

Perturbative results



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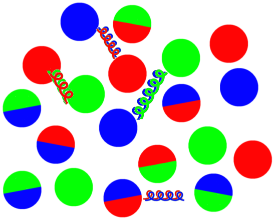

QCD plasma

m_D in QCD: pole of the gluon propagator at finite temperature

Perturbative results

The leading order (LO) result in QCD:

$$m_D^{LO} = \left(\frac{N}{3} + \frac{N_f}{6}\right)^{1/2} gT + \mathcal{O}(g^2 T).$$

The NLO result:

$$m_D^{NLO} = \left(\frac{N}{3} + \frac{N_f}{6}\right)^{1/2} gT + \frac{3}{4\pi} g^2 T \left(\ln \frac{2m_D^{LO}}{m_M} - \frac{1}{2}\right) + \mathcal{O}(g^3 T),$$

where m_M is the mass of magnetic gluons, which cannot be determined in perturbation theory.

We need a nonperturbative definition of the Debye screening mass.

Calculation with **Lattice QCD**

Perturbative results

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Calculation with **Lattice QCD**

Introduction to lattice QCD

Path integral in eucledian space-time

- The QCD Lagrangian is

$$\mathcal{L}_{QCD} = -\frac{1}{4} F_a^{\mu\nu} F^a_{\mu\nu} + \bar{\psi}(\not{D} + m)\psi.$$

- Partition function:

$$Z = \int \mathcal{D}A_\mu \, \mathcal{D}\bar{\psi} \, \mathcal{D}\psi \, e^{-S[A_\mu, \bar{\psi}, \psi]}.$$

- The expectation value of an observable:

$$\langle \mathcal{O}(A_\mu, \bar{\psi}, \psi) \rangle = \frac{1}{Z} \int \mathcal{D}A_\mu \, \mathcal{D}\bar{\psi} \, \mathcal{D}\psi \, \mathcal{O}(A_\mu, \bar{\psi}, \psi) \, e^{-S[A_\mu, \bar{\psi}, \psi]}.$$

- We have to regularize these integrals.

Introduction to lattice QCD

Discretization of space-time

- 4D isotropic hypercubic grid (a is the lattice spacing):

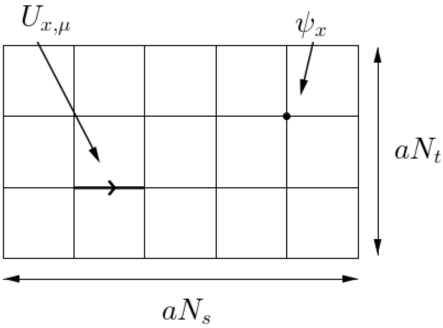
$$\Lambda = a\,\mathbb{Z}^4 = \{x \,|\, x_\mu/a \in \mathbb{Z}\}$$

- lattice size: $N_s^3 \times N_t$
- $\partial_\mu \to \frac{1}{2}(\Delta_\mu^f + \Delta_\mu^b)$, where $\Delta_\mu^f \phi(x) = \frac{1}{a}(\phi(x+a\hat{\mu}) - \phi(x))$
and $\Delta_\mu^b \phi(x) = \frac{1}{a}(\phi(x) - \phi(x-a\hat{\mu}))$
- $\int d^4x \to \sum_n a^4$
- $\int \mathcal{D}\phi = \int \prod_n \mathrm{d}\phi(na)$, and use Monte-Carlo techniques and importance sampling to compute the integrals
- temperature: $T = \frac{1}{N_t a}$

Introduction to lattice QCD

Gauge and fermion fields on the lattice

- link variables for gauge fields: $U_{x,\mu} = e^{iagA_\mu(x)} \in \mathrm{SU}(3)$
- Grassman variables for fermion fields: $\bar{\psi}, \psi$



Gauge action:
closed loops (e.g. plaquette)

Fermion action:
different discretizations
(**staggered**, wilson, overlap, ...)

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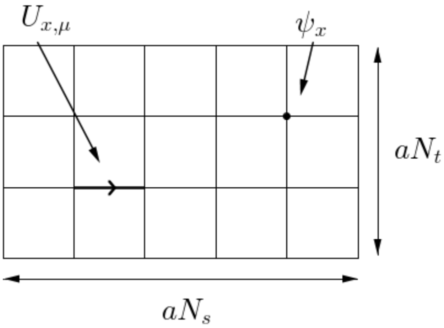

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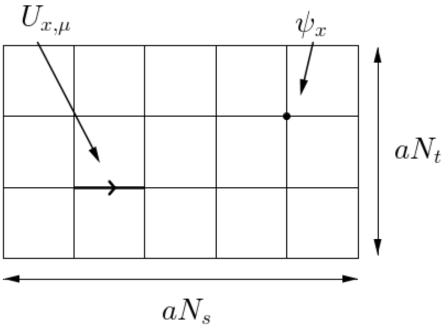

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Screening masses from lattice QCD

How can we study the interaction between two color charges in lattice QCD?

Screening masses from lattice QCD

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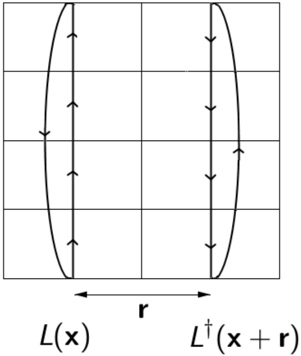
Two static charges:



Polyakov loop:

$$L(\mathbf{x}) = \prod_{x_4=0}^{N_t-1} U_4(\mathbf{x}, x_4).$$

Polyakov loop correlator:

$$C(r) = \left\langle \sum_{\mathbf{x}} \mathrm{Tr} L(\mathbf{x}) \mathrm{Tr} L^\dagger(\mathbf{x}+\mathbf{r}) \right\rangle.$$

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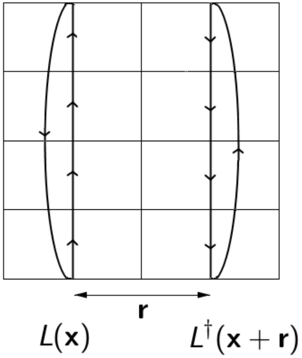

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Euclidean time reflection $\sim$ in real time this is $\mathcal{TC}$

- A_0 is odd,
- $\mathbf{A}$ is even under this symmetry

Screening masses from lattice QCD

Euclidean time reflection $\sim$ in real time this is $\mathcal{TC}$

$$L \xrightarrow{\mathcal{T}} L^\dagger, \quad L \xrightarrow{\mathcal{C}} L^*$$

Then, we define magnetic and electric operators:

$$L_M \equiv \frac{1}{2}(L + L^\dagger), \quad L_E \equiv \frac{1}{2}(L - L^\dagger).$$

They can decomposed into $\mathcal{C}$-even and $\mathcal{C}$-odd operators

$$L_{M\pm} \equiv \frac{1}{2}(L_M \pm L_M^*), \quad L_{E\pm} \equiv \frac{1}{2}(L_E \pm L_E^*).$$

Two generalized gauge-invariant correlation function:

$$C_{M+}(r,T) \equiv \langle \sum_{\mathbf{x}} \mathrm{Tr} L_{M+}(\mathbf{x}) \mathrm{Tr} L_{M+}(\mathbf{x}+\mathbf{r}) \rangle - |\langle \mathrm{Tr} L \rangle|^2$$

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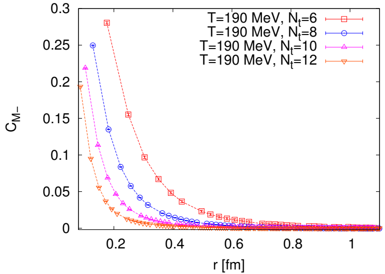
Screening masses from lattice QCD

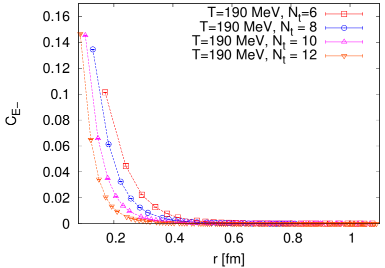
The magnetic correlator:

$$C_{M+}(r,T) \xrightarrow{r\to\infty} \gamma_{M+}(T)\frac{e^{-m_M(T)r}}{r}$$

The electric correlator:

$$C_{E-}(r,T) \xrightarrow{r\to\infty} \gamma_{E-}(T)\frac{e^{-m_E(T)r}}{r}$$





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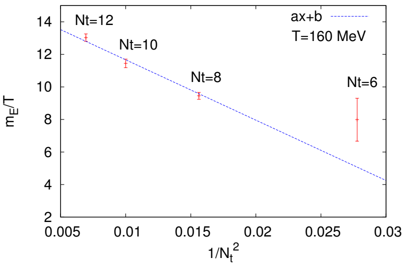

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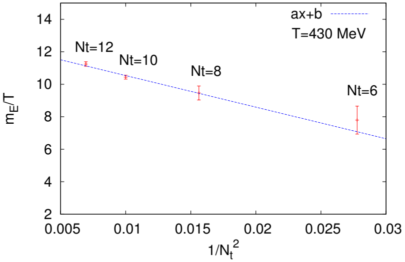
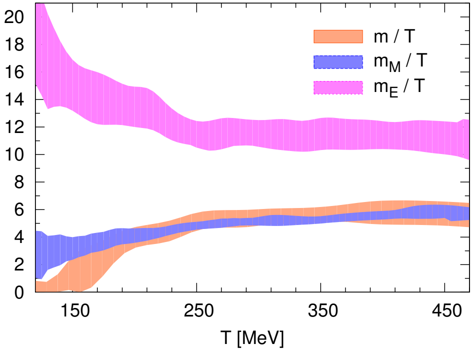


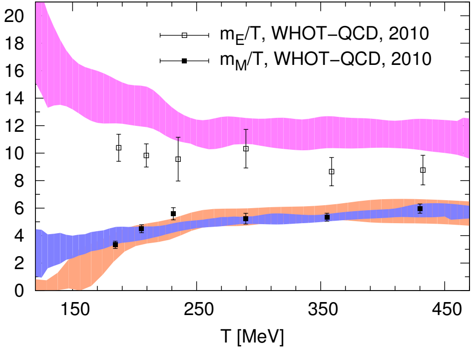
Screening masses from lattice QCD

Continuum results



Screening masses from lattice QCD

Continuum results compared to WHOT-QCD results
(Maezawa et al., Phys.Rev. D81 (2010) 091501)
(single lattice spacing ($N_t = 4, a \geq 0.1$ fm),
non-physical pion mass ($m_{PS}/m_V = 0.8$))



Thank for your attention!

Backup slides

Continuum results compared to perturbation theory

The NLO result:

$$m_D^{NLO} = \left(\frac{N}{3} + \frac{N_f}{6}\right)^{1/2} gT + \frac{3}{4\pi} g^2 T \left(\ln \frac{2m_D^{LO}}{m_M} - \frac{1}{2}\right) + \mathcal{O}(g^3 T).$$

Using our results at a given (high) temperature, then solving this equation for g, one can get $g \sim 4.8$, thus $\alpha \sim 1.8$.

Coupling is not small $\rightarrow$ contradiction.