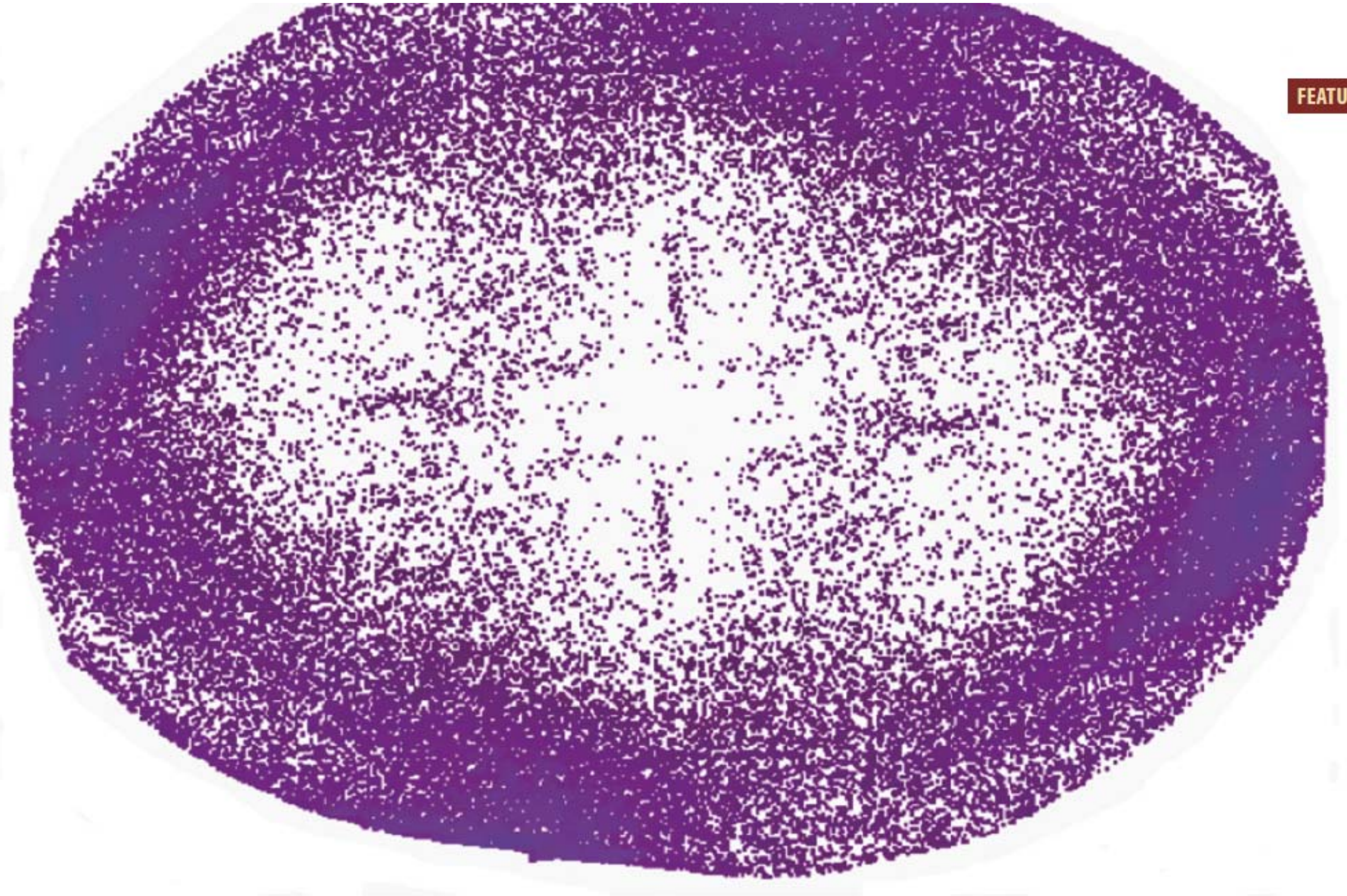
Rotation and Turbulence in Low Viscosity QGP and its Measurement

13. Zimányi WINTER SCHOOL ON HEAVY ION PHYSICS, Dec. 2. - Dec. 6., Budapest, Hungary

Laszlo P. Csernai,
University of
Bergen, Norway

FEATURES



The Quark-Gluon Plasma, a nearly perfect fluid

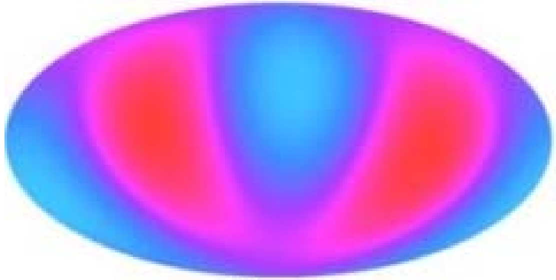
L. Cifarelli[1], L.P. Csernai[2] and H. Stöcker[3] - DOI: 10.1051/epn/2012206

europhysicsnews
THE MAGAZINE OF THE EUROPEAN PHYSICAL SOCIETY

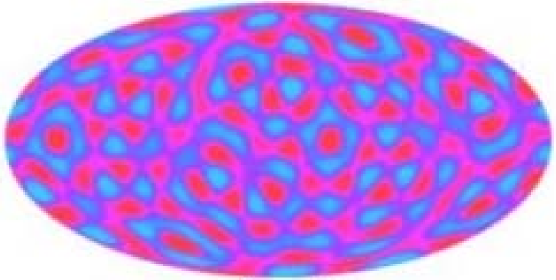
Low viscosity → Fluctuations

oil

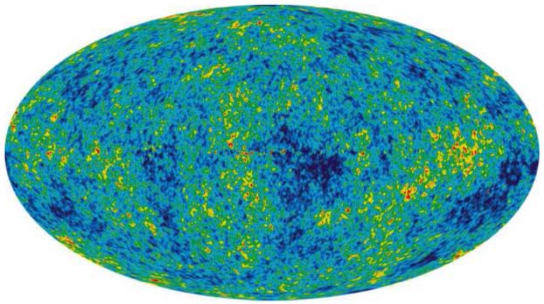
water

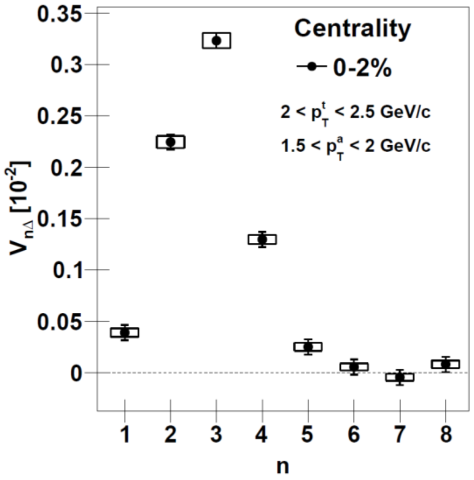


$\ell = 2$

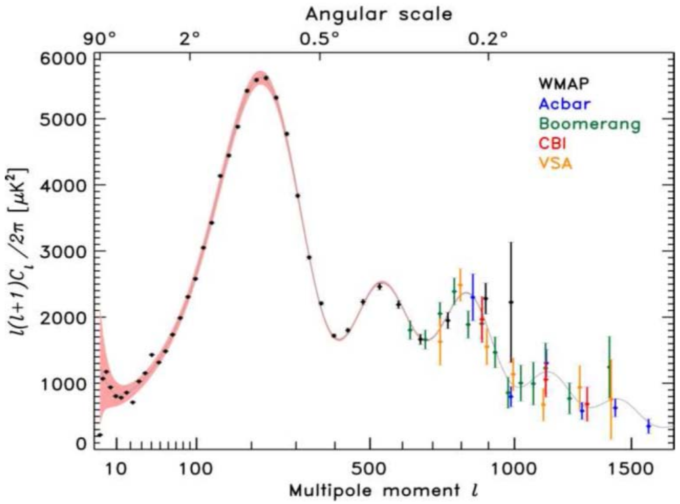


$\ell = 16$

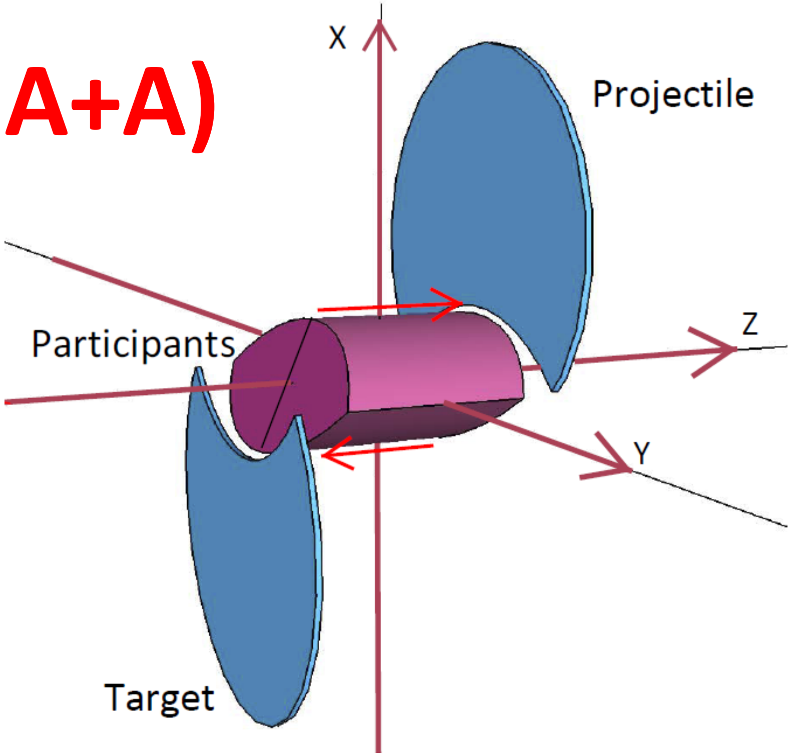




Measurable azimuthal fluctuations up to n=8 are evidence for low viscosity



Peripheral Collisions (A+A)



- Global Symmetries
- Symmetry axes in the global CM-frame:
  - (y ←→ -y)
  - (x,z ←→ -x,-z)
  - Azimuthal symmetry: ϕ-even (cos nϕ)
  - Longitudinal z-odd, (rap.-odd) for $v_{\_odd}$
  - Spherical or ellipsoidal flow, expansion

$$\frac{d^3N}{dy\,dp_t\,d\phi} = \frac{1}{2\pi}\frac{d^2N}{dy\,dp_t}\left[1 + 2v_1(y,p_t)\cos(\phi) + 2v_2(y,p_t)\cos(2\phi) + \cdots\right]$$

$$\frac{d^3N}{dy\,dp_t\,d\phi} = \frac{1}{2\pi}\frac{d^2N}{dy\,dp_t}\left[1 + 2v_1(y-y_{CM},p_t)\cos(\phi-\Psi_{RP}) + 2v_2(y-y_{CM},p_t)\cos(2(\phi-\Psi_{RP})) + \cdots\right]$$

- Fluctuations
- Global flow and Fluctuations are simultaneously present → ∃ interference
  - Azimuth - Global: even harmonics - Fluctuations : odd & even harmonics
  - Longitudinal – Global: v1, v3 y-odd - Fluctuations : odd & even harmonics
  - The separation of Global & Fluctuating flow is a must !! (not done yet)

Collective flow

- There are alternative origins:
- (a) Global collective flow (RP from spectators)
- (b) Asymmetries from random I.S. fluctuations
- (c) Asymmetries from Critical Point fluctuations

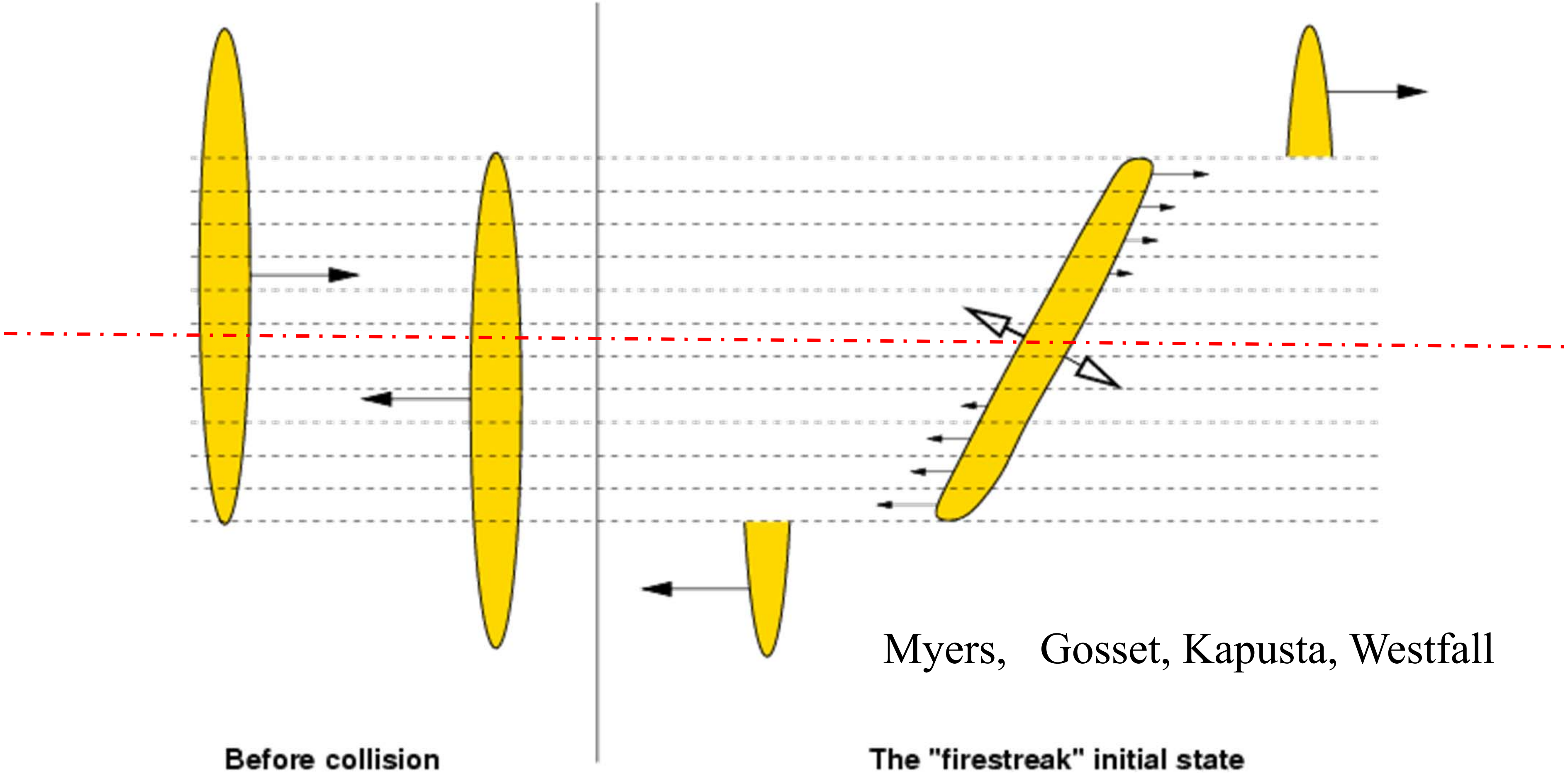
- **Goal is to separate the these**

  ➔ This provides more insight

- How can we see the flow of QGP?

  ➔ Rapid hadronization and freeze-out

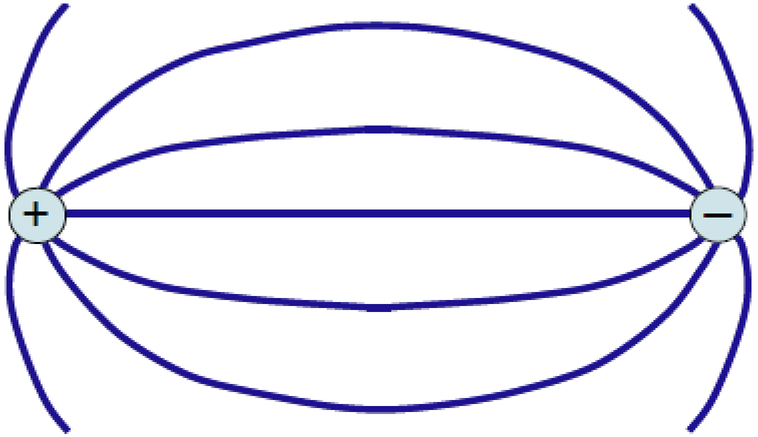
„Fire streak” picture – 3 dim.



Symmetry axis = z-axis. Transverse plane divided into streaks.

Flux – tubes

ED or QED:

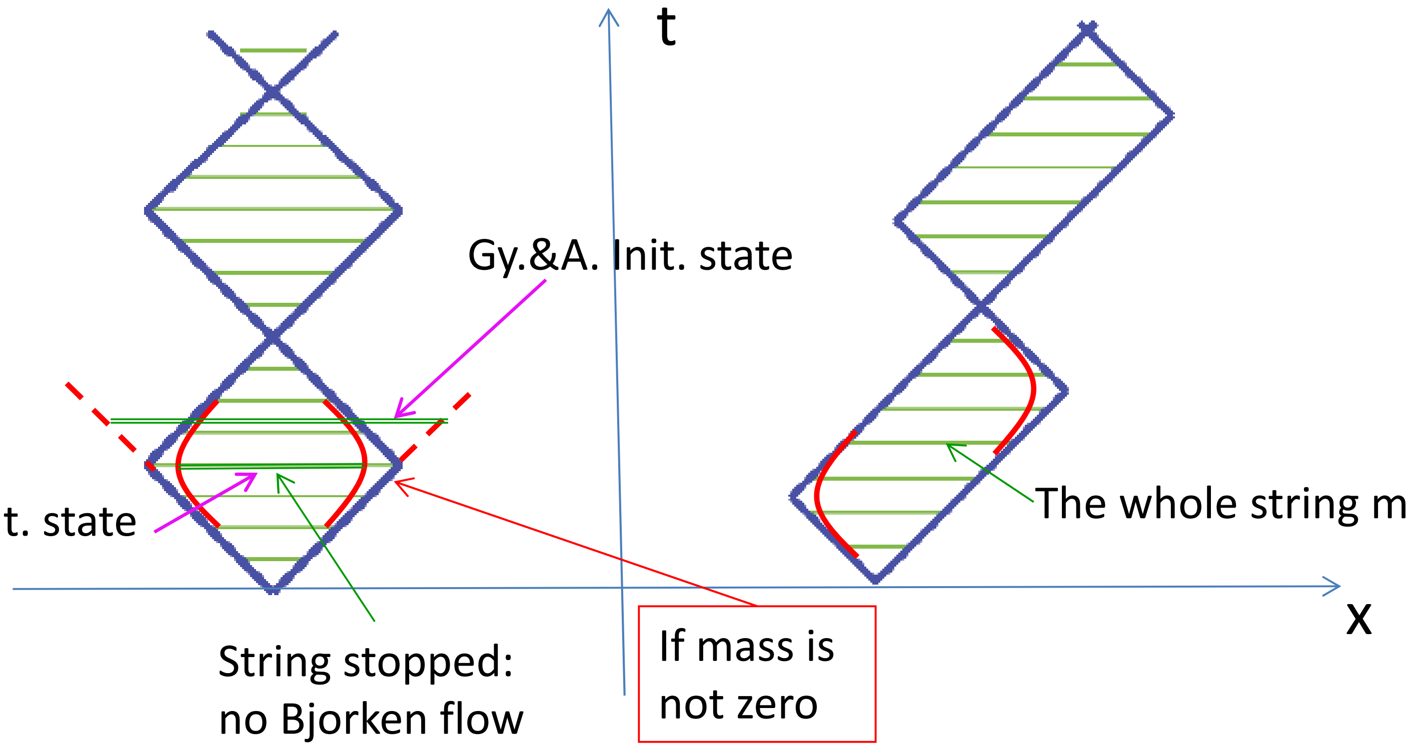


Gluon self-interaction makes field lines attract each other. →
QCD:

→ linear potential → confinement

String model of mesons / PYTHIA

Light quarks connected by string → mesons have ‘**yo-yo**’ modes:



Gy.&A. Init. state

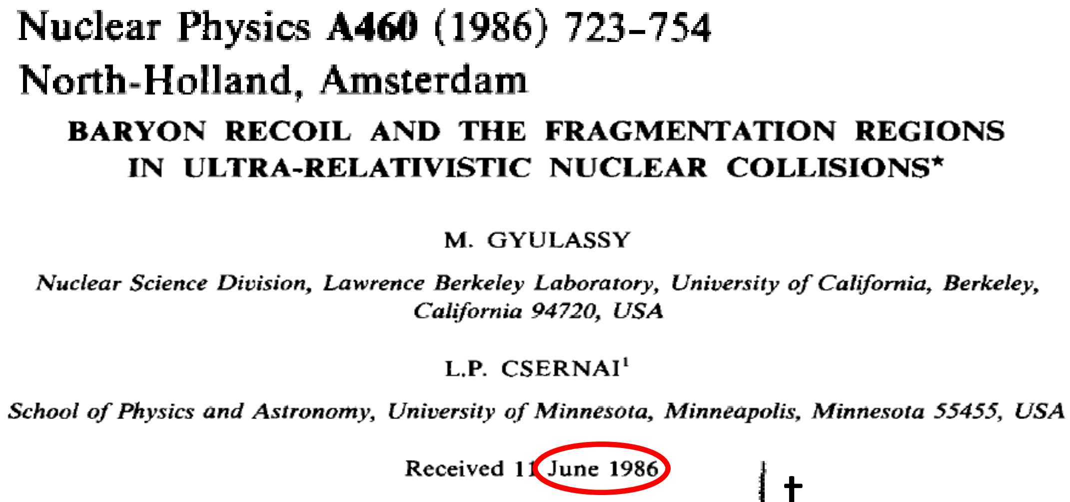
Cs.M.&S. Init. state

The whole string moves

String stopped:
no Bjorken flow

If mass is
not zero

[T. Sjostrand & H.U. Bengtsson, 1984-1987]
PYTHIA



Nuclear Physics A460 (1986) 723–754
North-Holland, Amsterdam

BARYON RECOIL AND THE FRAGMENTATION REGIONS
IN ULTRA-RELATIVISTIC NUCLEAR COLLISIONS*

M. GYULASSY

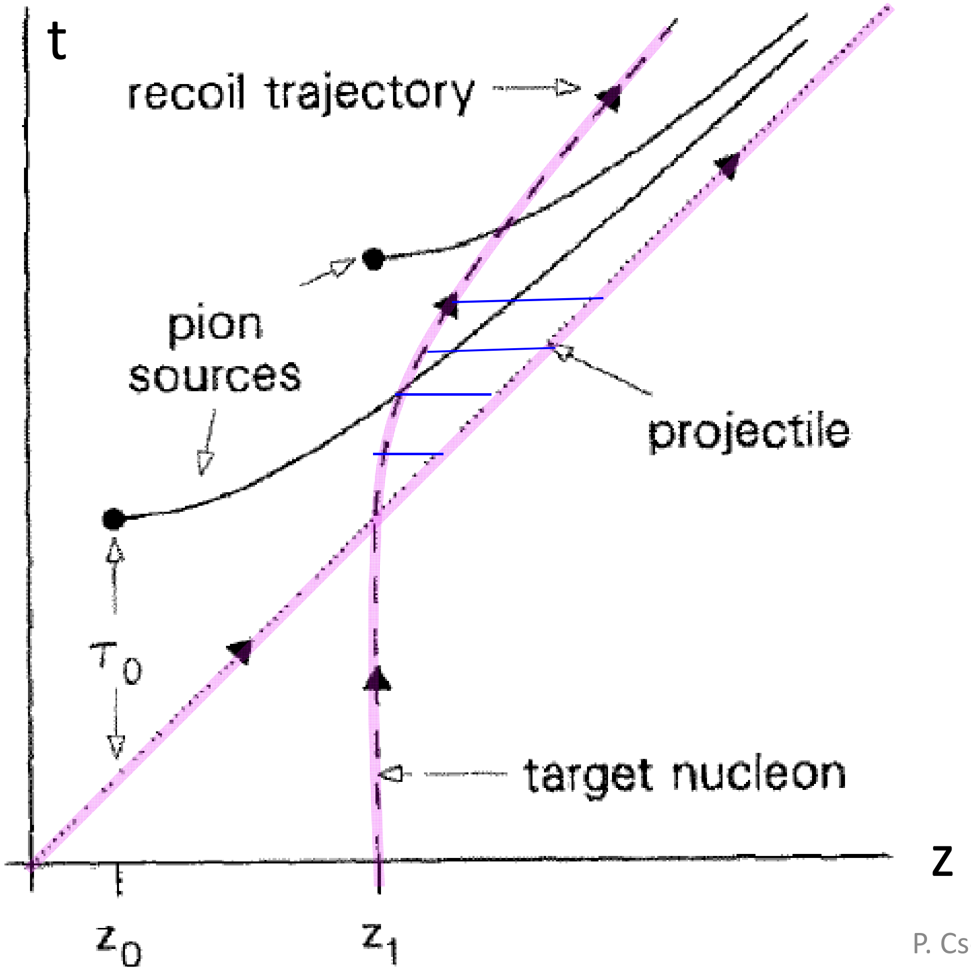
Nuclear Science Division, Lawrence Berkeley Laboratory, University of California, Berkeley,
California 94720, USA

L.P. CSERNAI[1]

School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455, USA

Received 11 June 1986

Yo-yo in the fixed target frame → target recoil → density and energy density increase in the "fragmentation region"



Initial stage: Coherent Yang-Mills model

[Magas, Csernai, Strottman, Pys. Rev. C ‘2001]

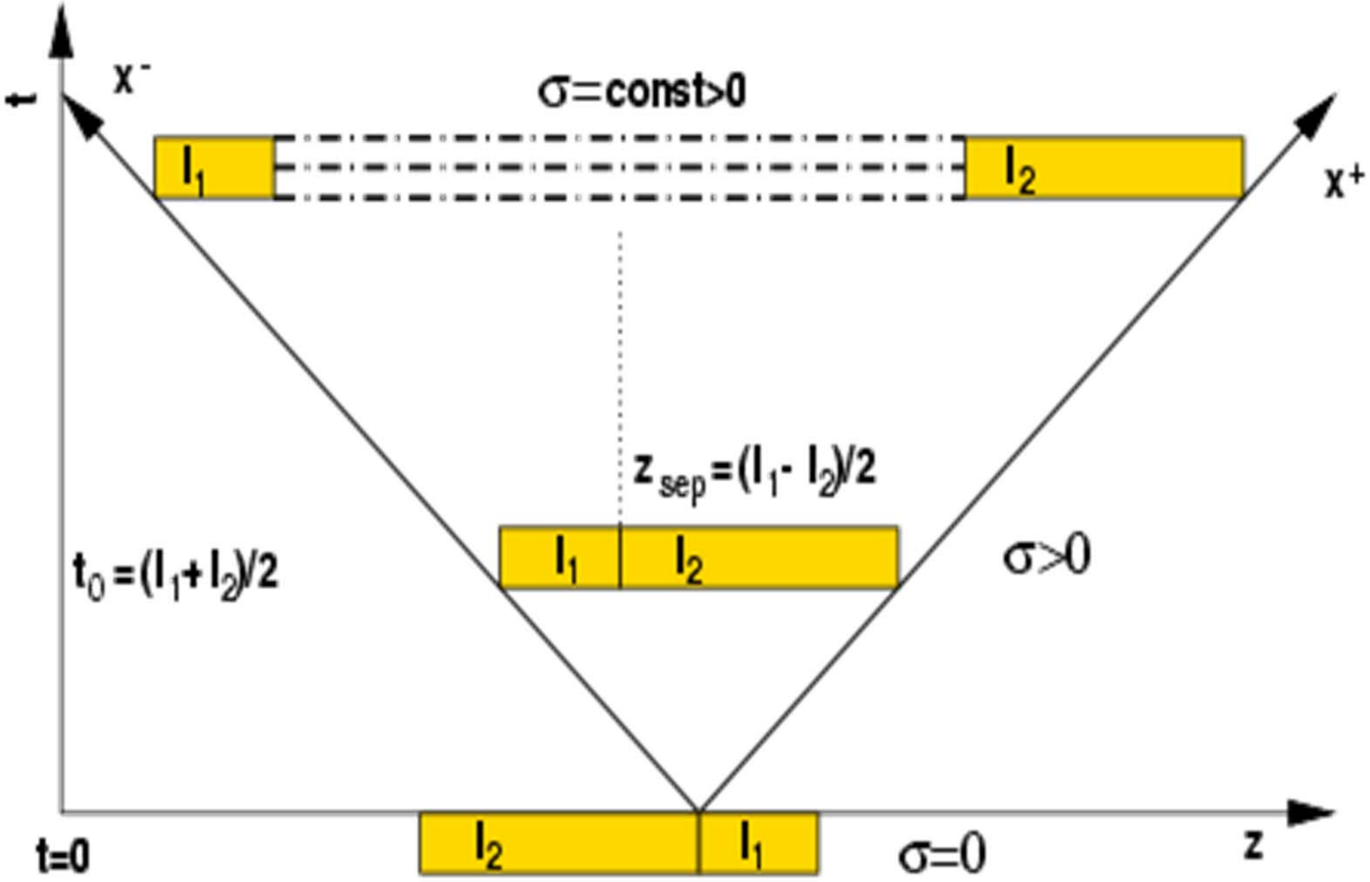
M. Gyulassy, L. Csernai Nucl. Phys. A660 (1986) 723-754.

$$\boxed{\begin{gathered}\partial_\mu T^{\mu\nu} = F^{\nu\mu} n_\mu + \Sigma_\pi^\nu \\ \partial_\mu n^\mu = 0\end{gathered}}$$

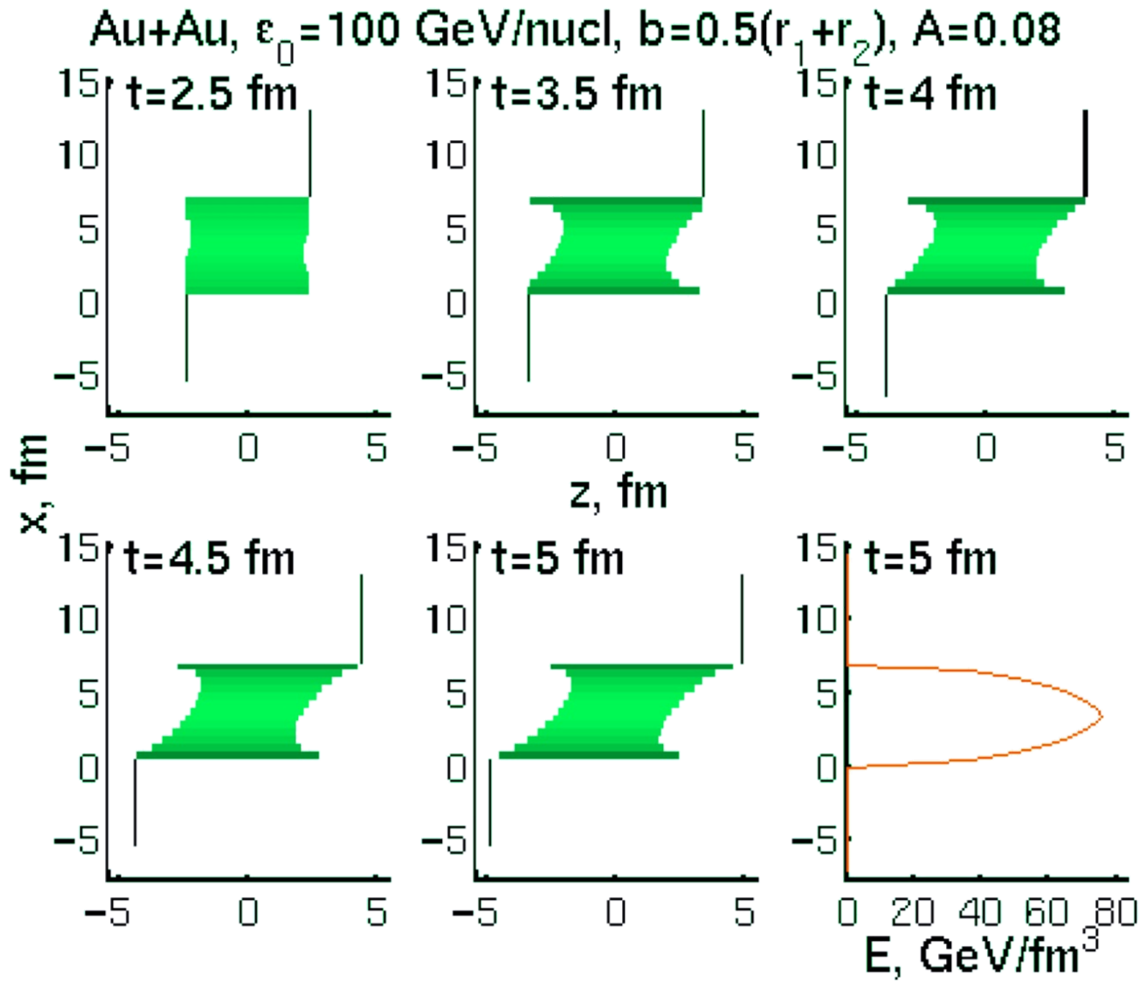
- $T^{\mu\nu} = e_t \left(\left(1 + c_0^2\right) u_t^\mu u_t^\nu - c_0^2 g^{\mu\nu} \right)$
- Σ_π^ν – pion source term.
- $F^{\mu\nu}$ – effective field, describes interaction between target and projectile.

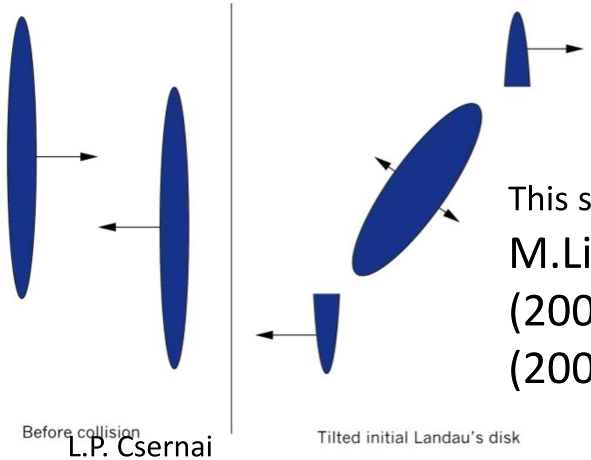
$$F^{\mu\nu} = \begin{pmatrix} 0 & -\sigma \\ \sigma & 0 \end{pmatrix} \quad ,$$

String rope --- Flux tube --- Coherent YM field



Initial State

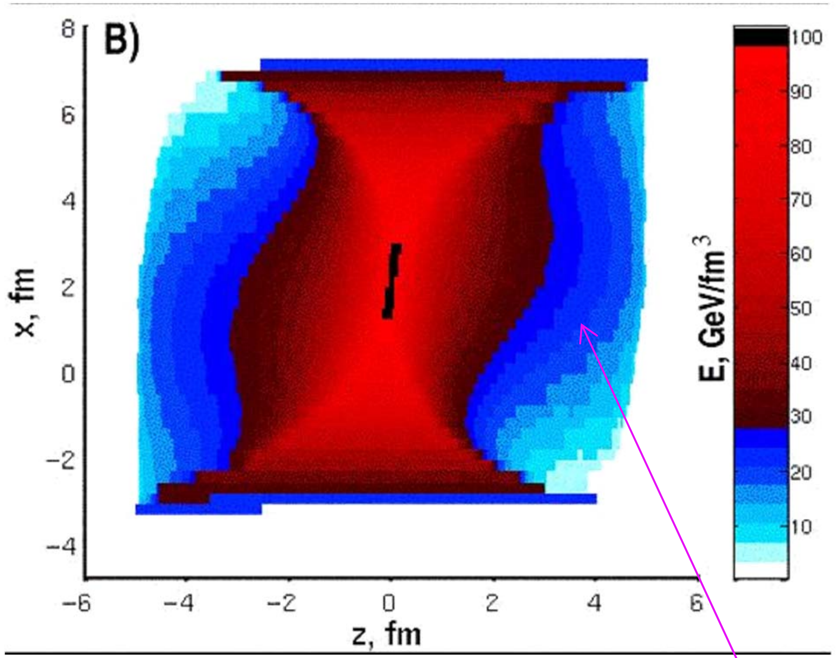




This shape is confirmed by M.Lisa &al. HBT: PLB496 (2000) 1; & PLB 489 (2000) 287.

3rd flow component

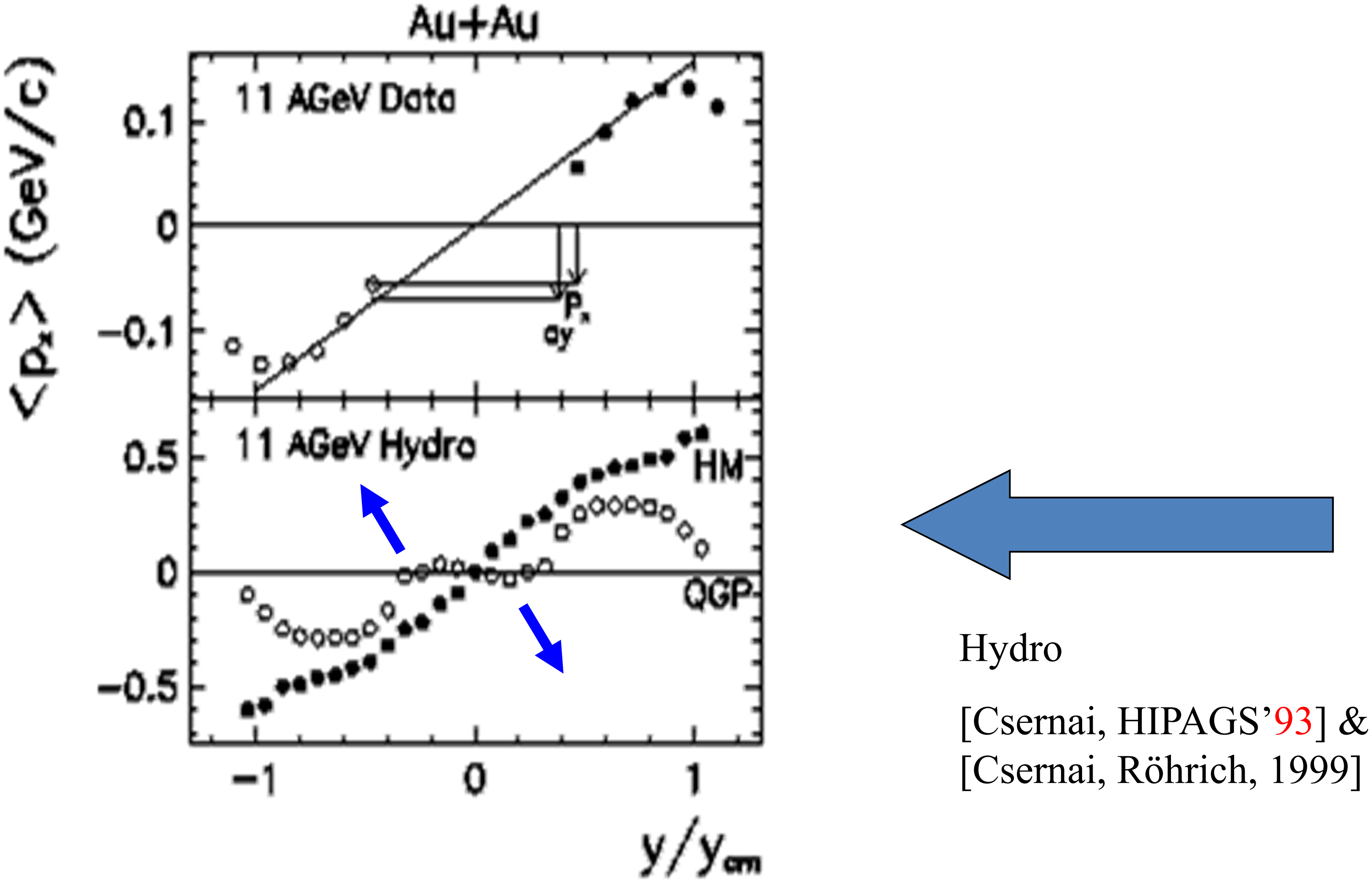
Initial state – reaching equilibrium



Initial state by V. Magas, L.P. Csernai and D. Strottman
Phys. Rev. C64 (01) 014901

Relativistic, 1D Riemann expansion is added to each stopped streak

3rd flow component

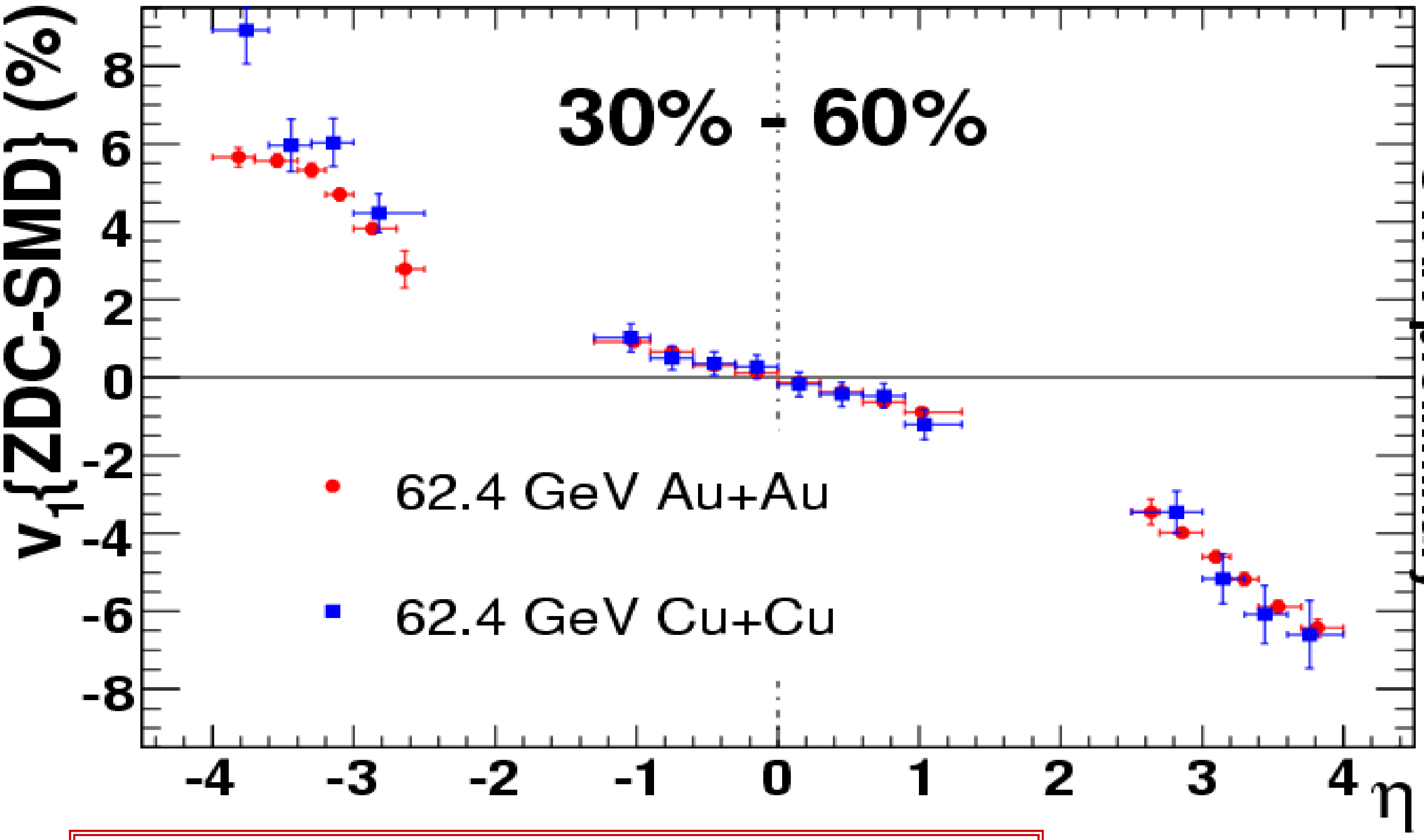


Hydro

[Csernai, HIPAGS'93] &
[Csernai, Röhrich, 1999]

G. Wang / STAR
QM 2006 :

$v_1(\eta)$: system-size dependence

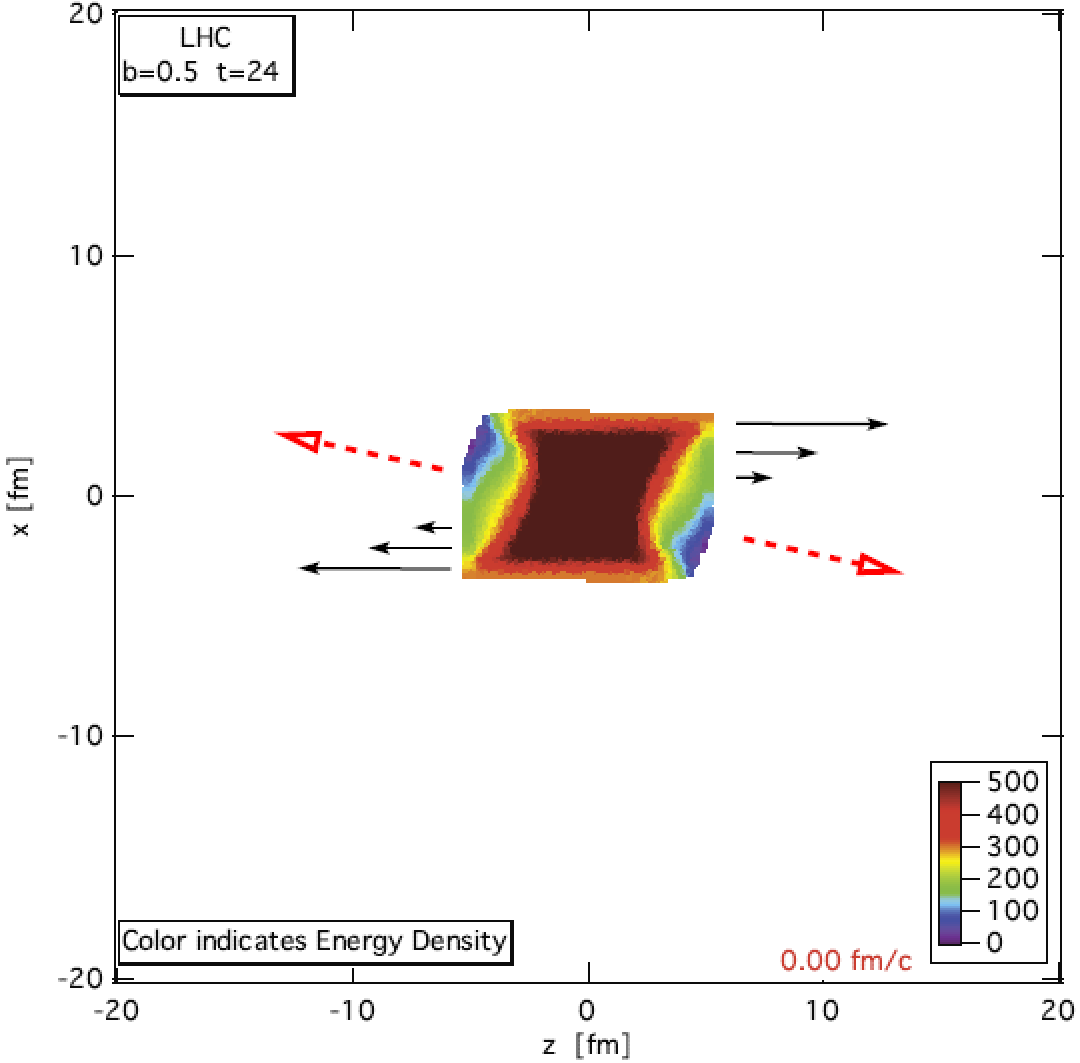


System size doesn't seem to influence $v_1(\eta)$.

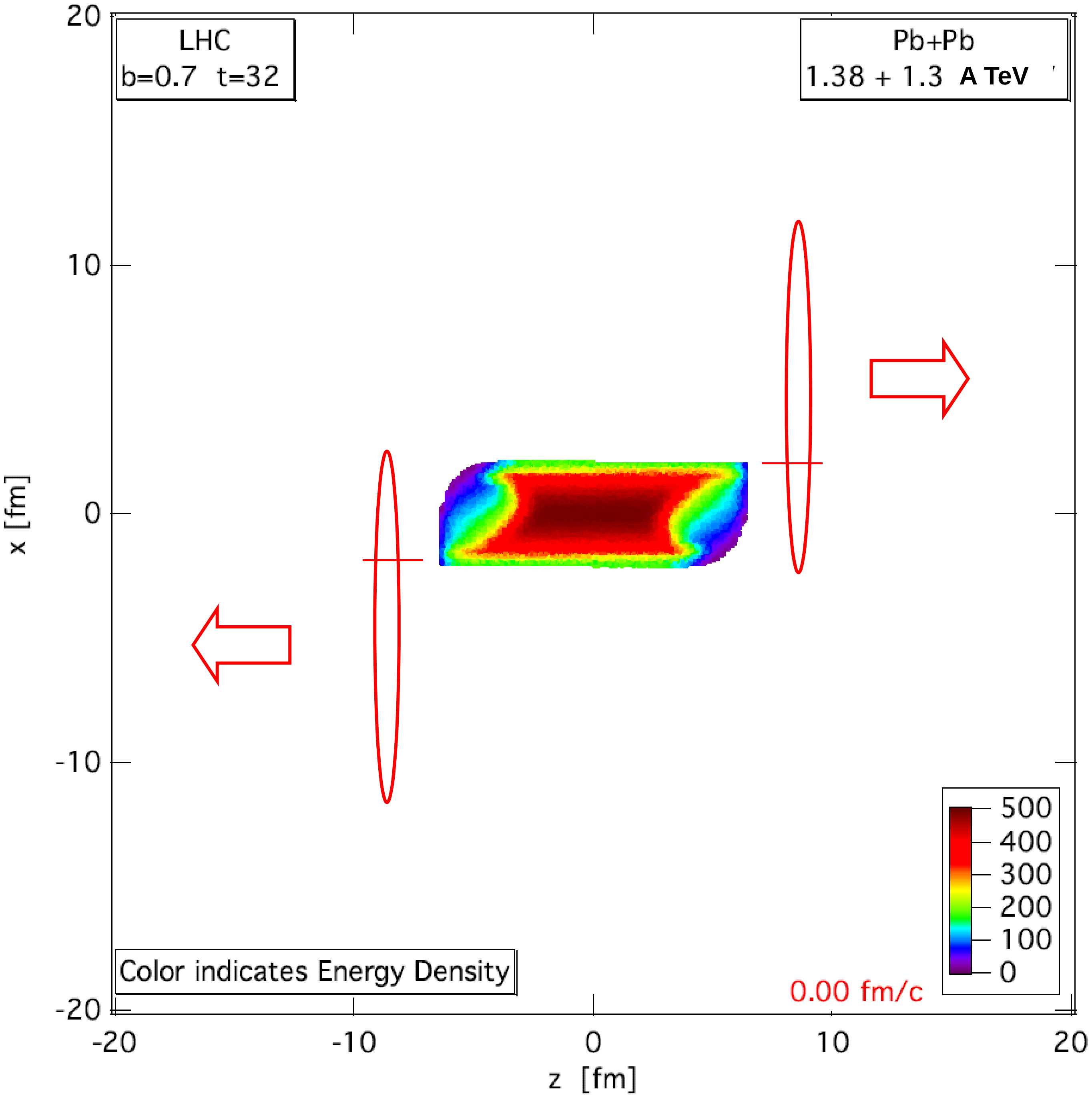
L. P. Csernai,[1,2,3] V. K. Magas,[4] H. Stöcker,[3] and D. D. Strottman[1,3]

PHYSICAL REVIEW C **84**, 024914 (2011)

Anti-flow (v1) at LHC



Initial energy density [GeV/fm3] distribution in the reaction plane, [x,y] for a Pb+Pb reaction at 1.38 + 1.38 ATeV collision energy and impact parameter b = 0.5_bmax at time 4 fm/c after the first touch of the colliding nuclei, this is when the hydro stage begins. The calculations are performed according to the effective string rope model. This tilted initial state has a flow velocity distribution, qualitatively shown by the arrows. The dashed arrows indicate the direction of the largest pressure gradient at this given moment.



PIC-hydro

Pb+Pb 1.38+1.38 A TeV, b= 70 % of b_max

Lagrangian fluid cells, moving, ~ 5 mill.

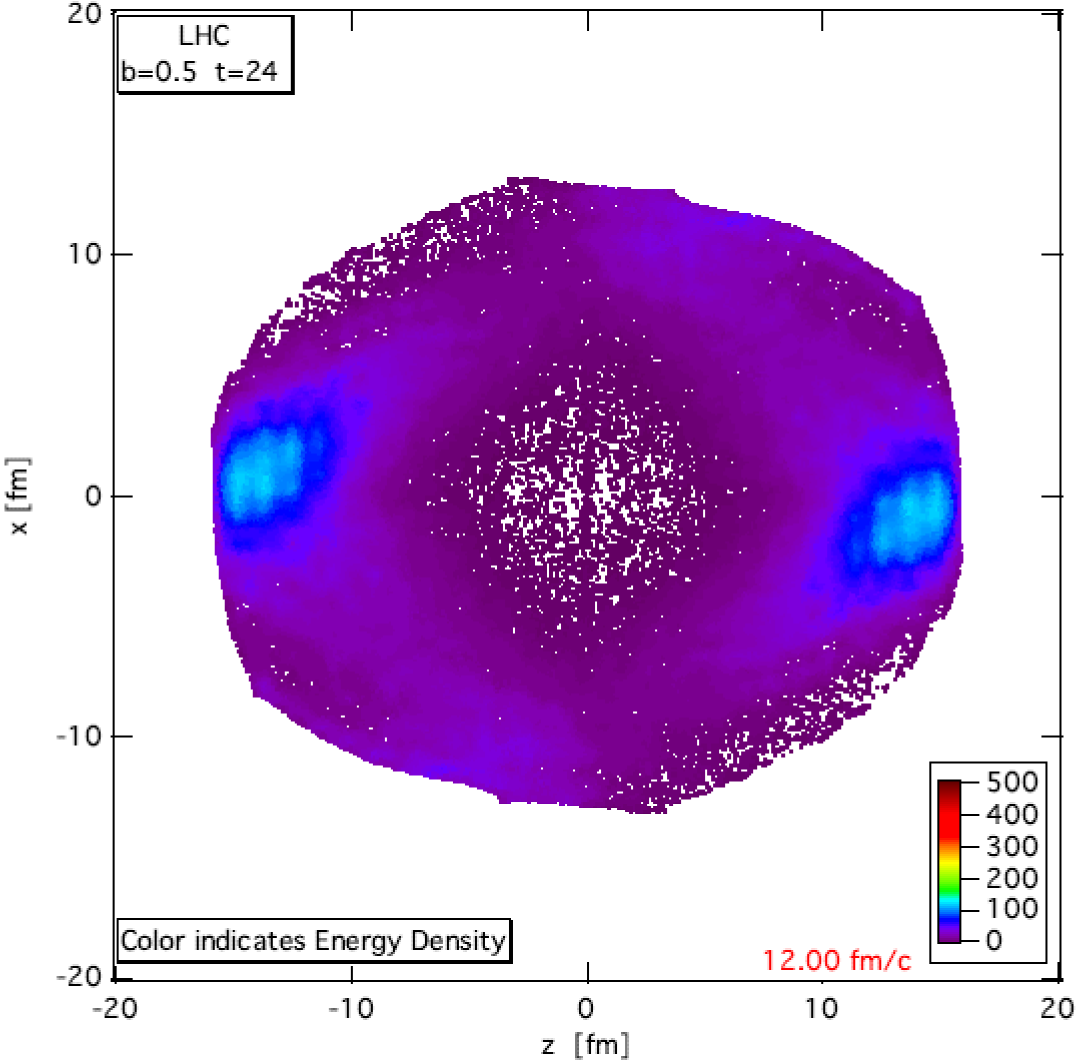
MIT Bag m. EoS

FO at T ~ 200 MeV, but calculated much longer, until pressure is zero for 90% of the cells.

Structure and asymmetries of init. state are maintained in nearly perfect expansion.

..\zz-Movies\LHC-Ec-1h-b7-A.mov

Anti-flow (v1)



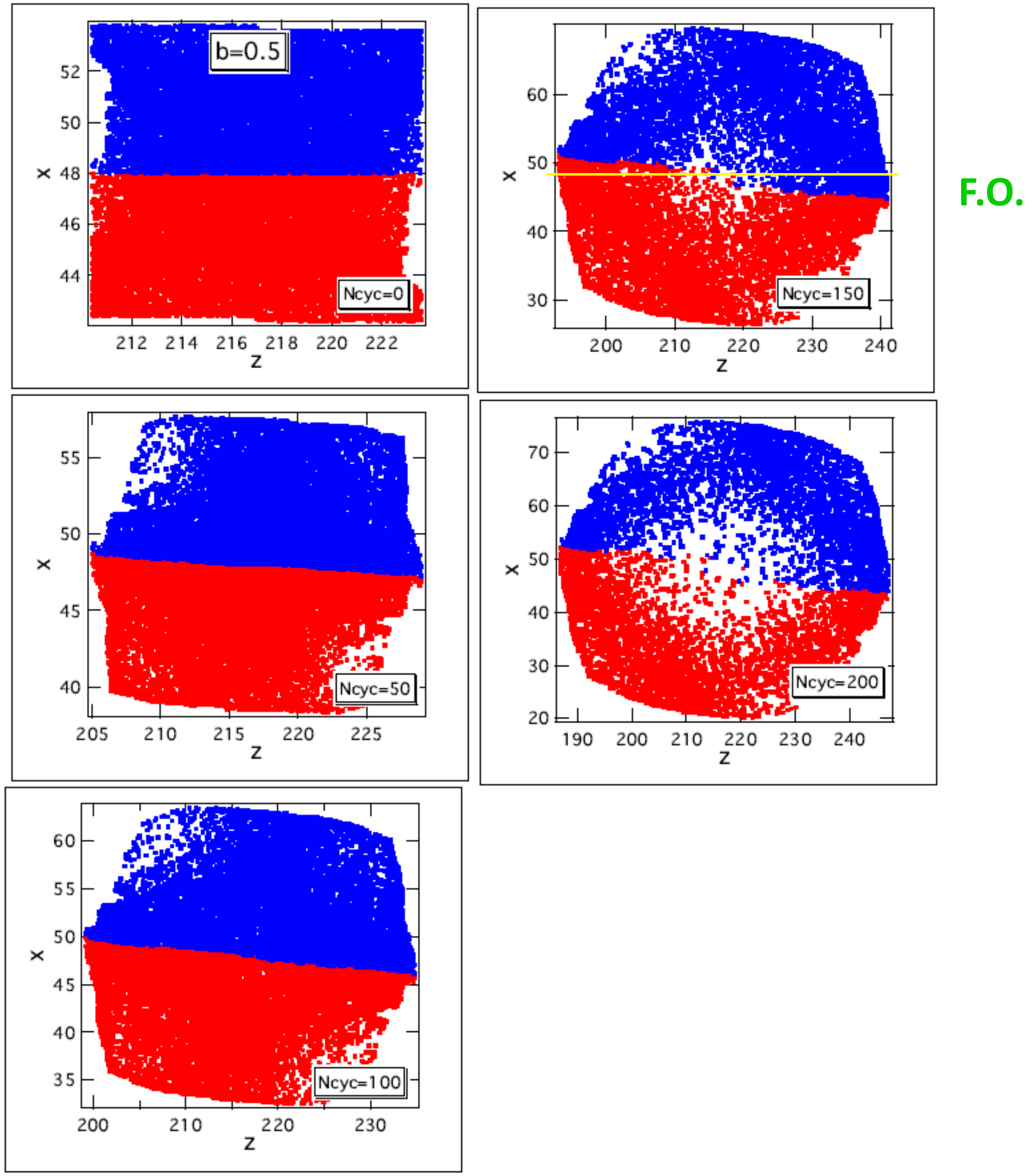
The energy density [GeV/fm3] distribution in the reaction plane, [x,z] for a Pb+Pb reaction at 1.38 + 1.38 A.TeV collision energy and impact parameter b = 0.5b_max at time 12 fm/c after the formation of the hydro initial state. The expected physical FO point is earlier but this post FO configuration illustrates the flow pattern.

[LP. Csernai, V.K. Magas,
H. Stöcker, D. Strottman,
Phys. Rev. **C84** (2011) 02914]

Rotation

The rotation is illustrated by dividing the upper / lower part (blue/red) of the initial state, and following the trajectories of the marker particles.

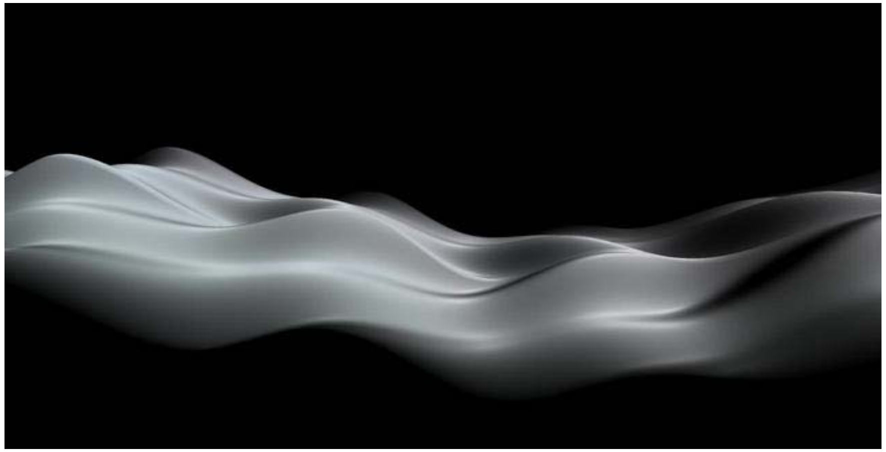
Turbulence ?



Kelvin-Helmholtz Instability (KHI)

- Turbulent fluctuations are common in **air*** and **water***
- Usually ∃ source*
- Usually damped, but weakly
- ∃ quasi-stationary and developing instabilities
- For KHI the source is shear-flow

Low viscosity → Turbulence



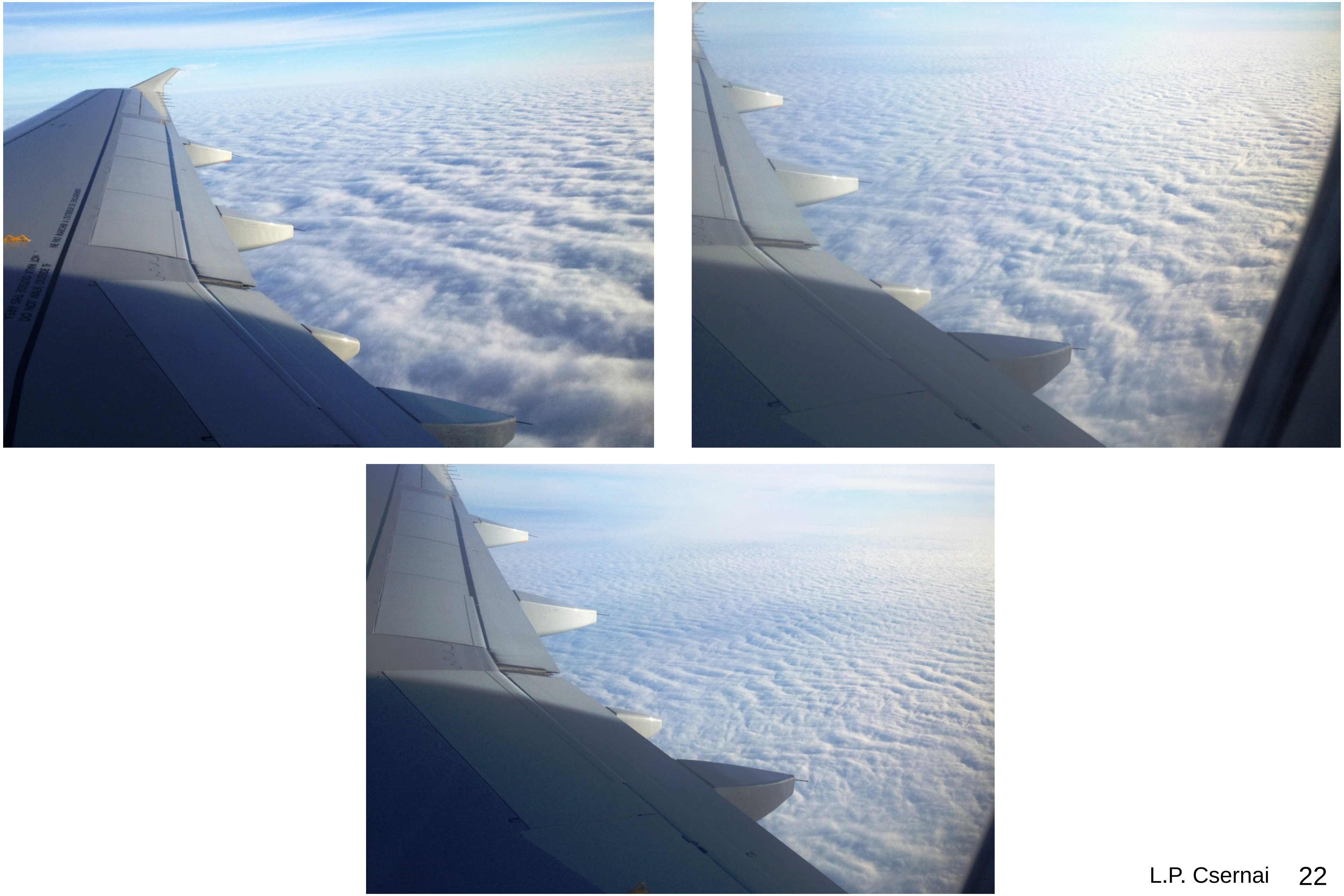
oil

water

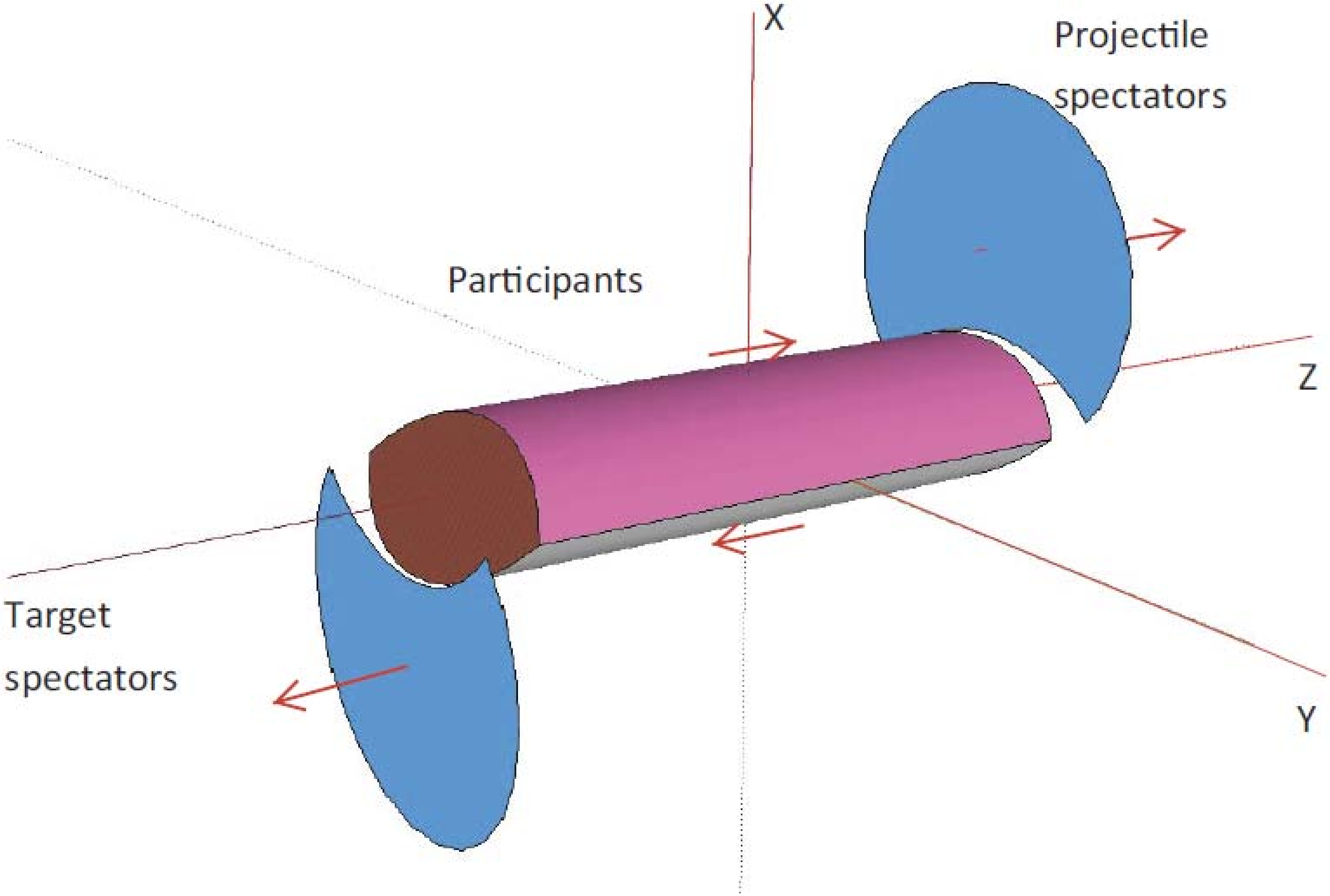
Viscous liquid shows smooth sinusoidal waves, while a non-viscous fluid has sharp, non-sinusoidal waves, leading to turbulence.

A typical turbulent phenomenon is the Kelvin-Helmholtz instability

KHI in air from above

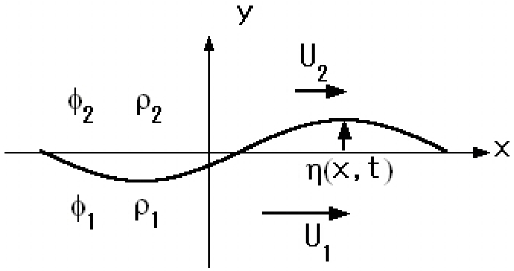


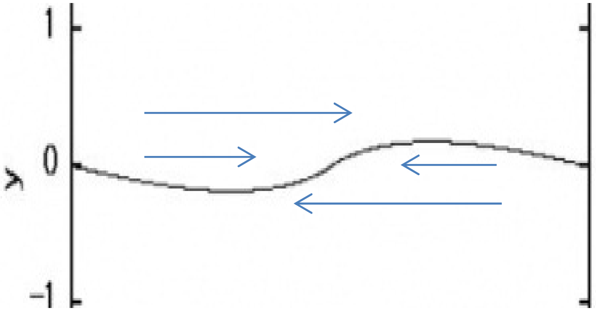
Initial geometry at ultra-relativistic energies



The Kelvin – Helmholtz instability

- Initial, almost sinusoidal waves



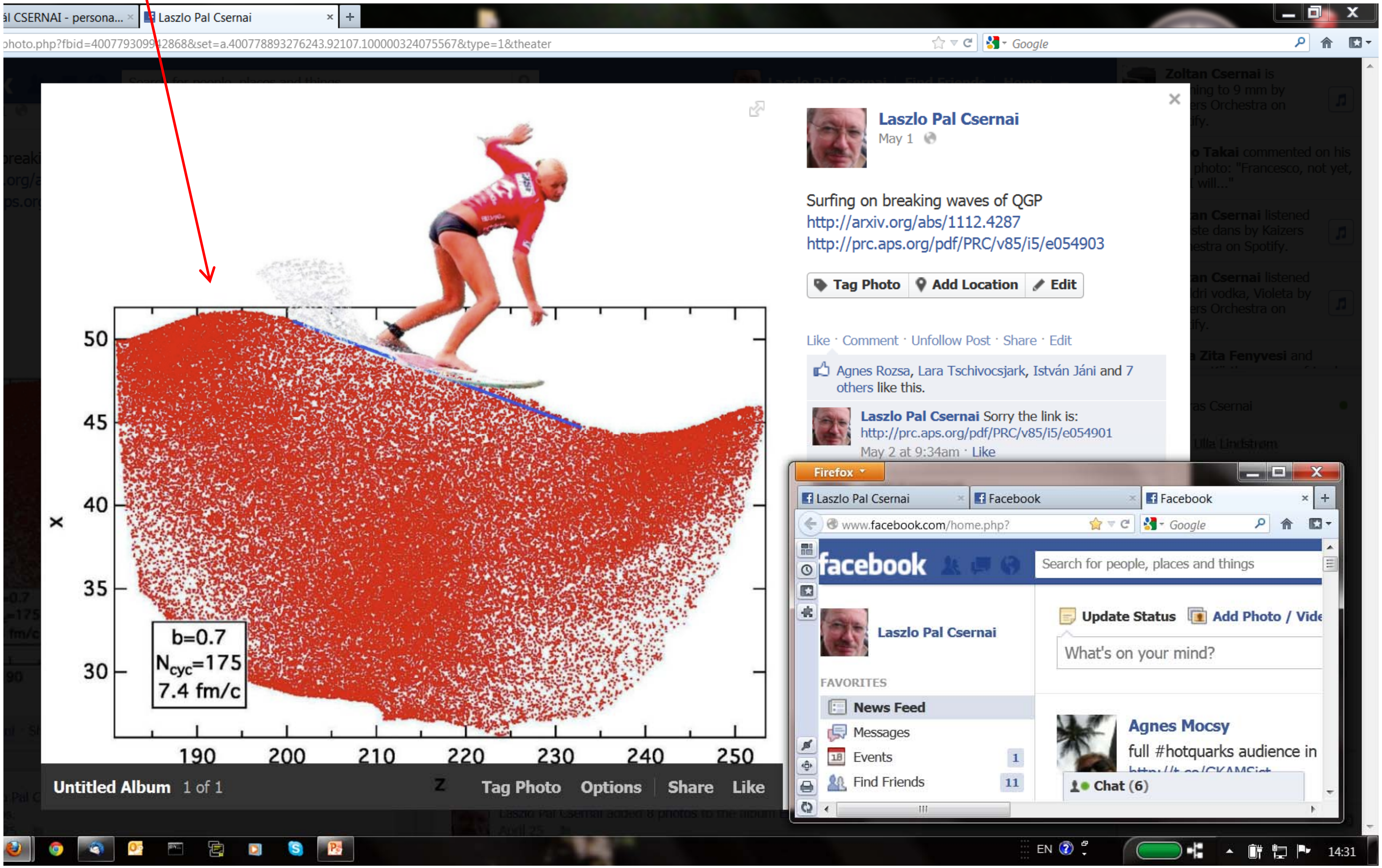


- Well developed, non-linear wave

The interface is a layer with a finite thickness, where viscosity and surface tension affects the interface. Due to these effects singularity formation is prevented in reality. The roll-up of a sheet is observed

[Chihiro Matsuoka, Yong Guo Shi, Scholarpedia]

Kelvin-Helmholtz Instability (KHI)



Laszlo Pal Csernai
May 1

Surfing on breaking waves of QGP
http://arxiv.org/abs/1112.4287
http://prc.aps.org/pdf/PRC/v85/i5/e054903

Like · Comment · Unfollow Post · Share · Edit

Agnes Rozsa, Lara Tschivocsjark, István Jáni and 7 others like this.

Laszlo Pal Csernai Sorry the link is:
http://prc.aps.org/pdf/PRC/v85/i5/e054901
May 2 at 9:34am · Like

Laszlo Pal Csernai

Agnes Mocsy
full #hotquarks audience in

Kelvin-Helmholtz instability in high-energy heavy-ion collisions

L.P. Csernai[1,2,3], D.D. Strottman[2,3], and Cs. Anderlik[4]

PHYSICAL REVIEW C **85**, 054901 (2012)

arXiv:1112.4287v3 [nucl-th]

PIC method !!!

KHI →

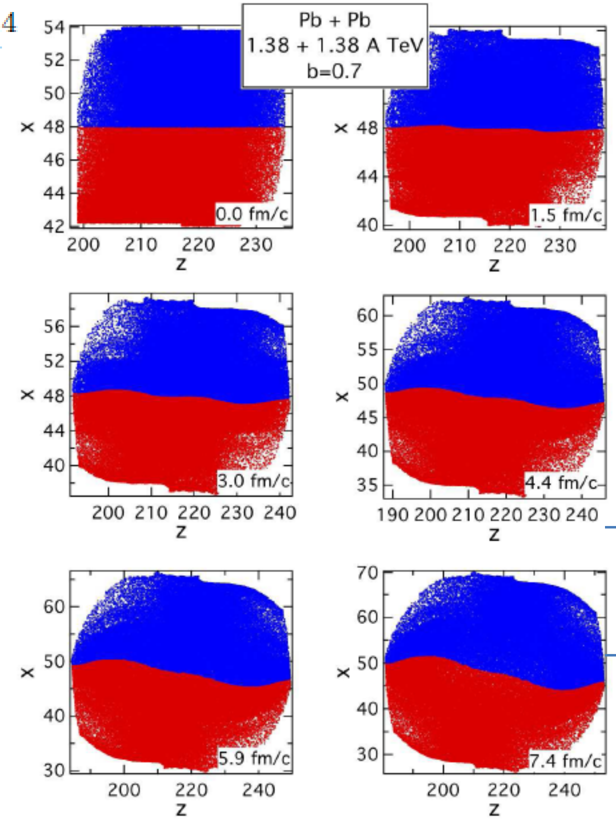
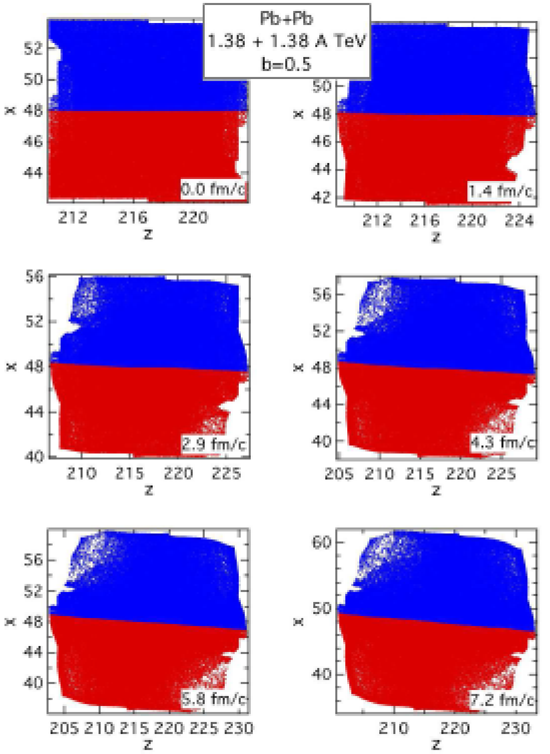


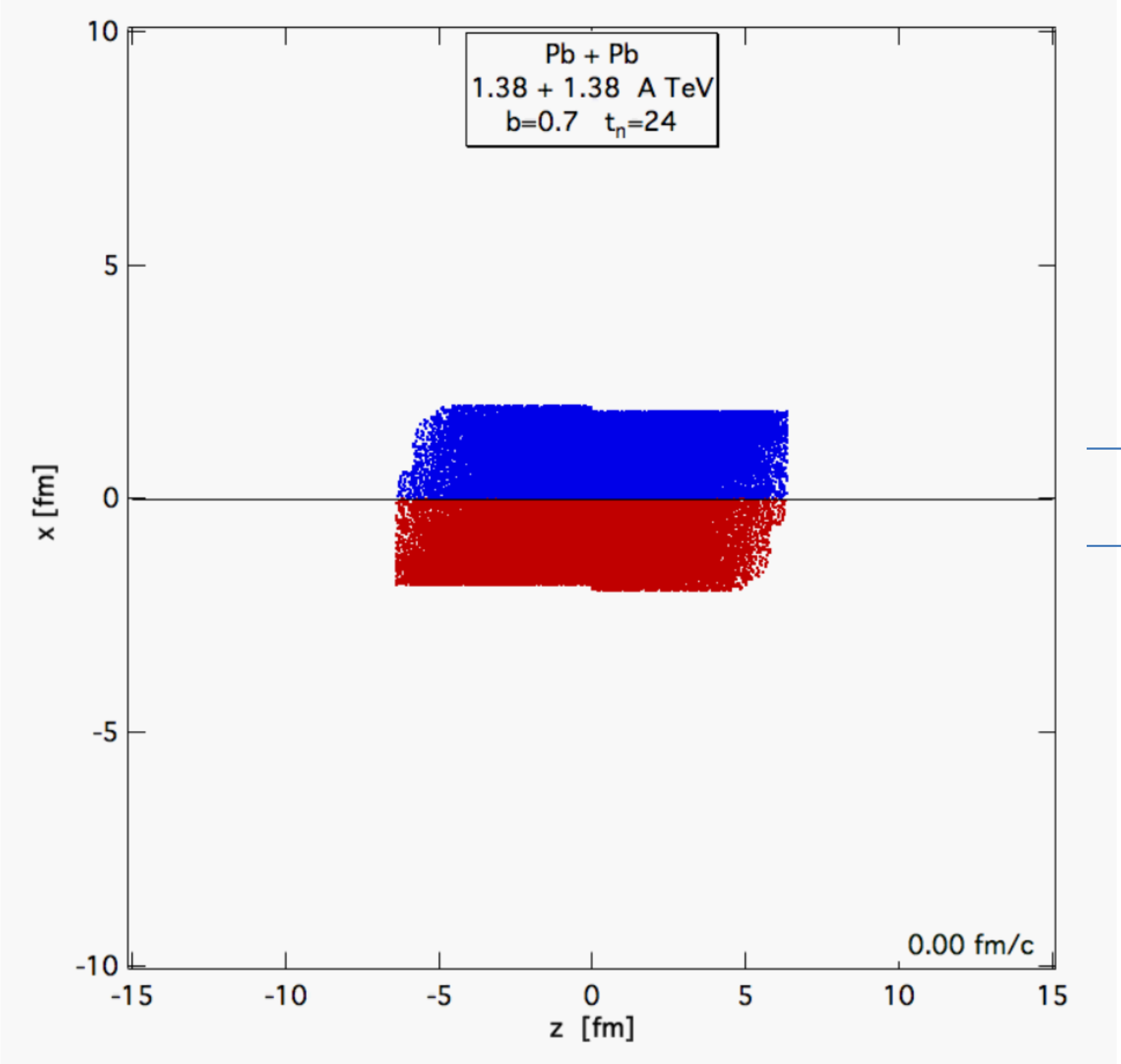
FIG. 1: (color online) Growth of the initial stage of Kelvin-Helmholtz instability in a 1.38A + 1.38A TeV peripheral, $b = 0.7 b_{\max}$, Pb+Pb collision in a relativistic CFD simulation using the PIC-method. We see the positions of the marker particles (Lagrangian markers with fixed baryon number content) in the reaction plane. The calculation cells are $dx = dy = dz = 0.4375$fm and the time-step is 0.04233 fm/c The number of randomly placed marker particles in each fluid cell is 8^3. The axis-labels indicate the cell numbers in the x and z (beam) direction. The initial development of a KH type instability is visible from $t = 1.5$ up to $t = 7.41$ fm/c corresponding from 35 to 175 calculation time steps).

2.4 fm

ROTATION

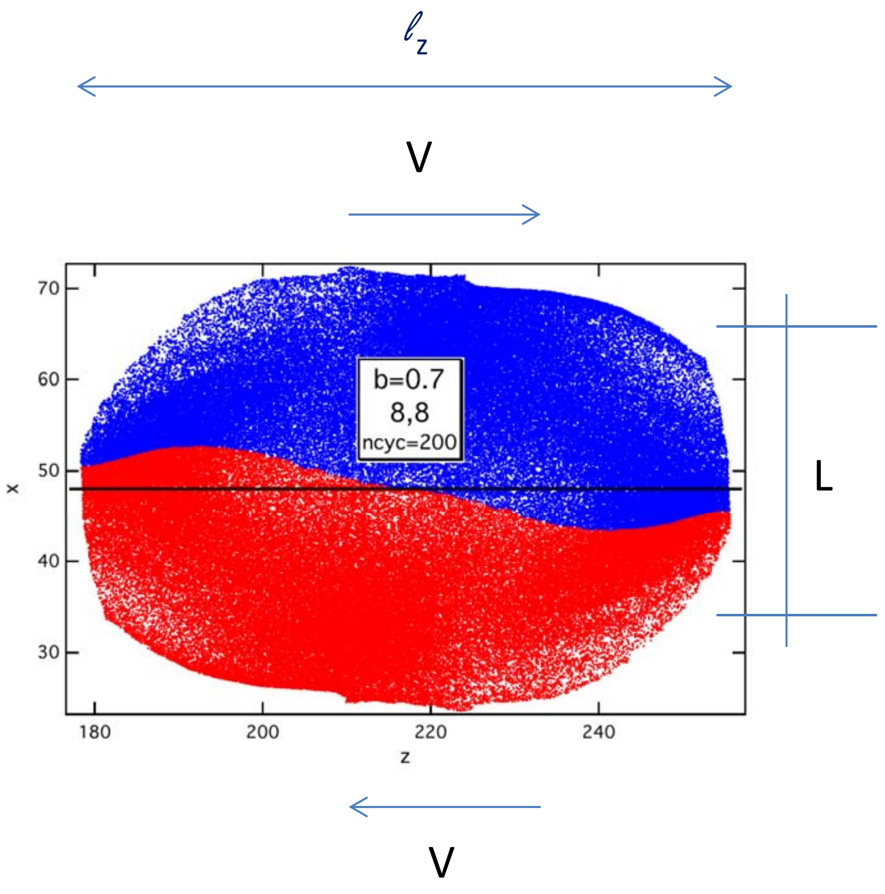


KHI


Pb + Pb
1.38 + 1.38 A TeV
b=0.7 t_n=24
0.00 fm/c
x [fm]
10
5
0
-5
-10
z [fm]
-15
-10
-5
0
5
10
15

2.1 fm

The Kelvin – Helmholtz instability (KHI)



Our resolution is $(0.35\text{fm})^3$ and 8^3 markers/fluid-cell →
~ 10k cells & 10Mill m.p.-s

- Shear Flow:
- L=(2R-b) ~ 4 – 7 fm, init. profile height
- ℓ_z=10–13 fm, init. length (b=.5-.7b_{max})
- V ~ ±0.4 c upper/lower speed →
- Minimal wave number is k = .6 - .48 fm^{-1}
- KHI grows as $\propto \exp(st)$, where $s = kV$ →
- Largest k or shortest wave-length will grow the fastest.
- The amplitude will double in 2.9 or 3.6 fm/c for (b=.5-.7b_{max}) without expansion, and with favorable viscosity/Reynolds no. Re=LV/ν .
- → this favors large L and large V

The Kelvin – Helmholtz instability (KHI)

- **Formation of critical length KHI (Kolmogorov length scale)**
- ∃ critical minimal wavelength beyond which the KHI is able to grow. Smaller wavelength perturbations tend to decay. (similar to critical bubble size in homogeneous nucleation).
- **Kolmogorov**: $\lambda_{Kol} = [\nu^3/\epsilon]^{1/4}$.
- Here $\epsilon = \dot{e}/\rho \;\propto\; T\dot{\sigma}/\rho \;\propto\; \nu,$ is the specific dissipated flow energy.
- We estimated: $\lambda_{Kol} = \begin{cases} 2.1 \;\div\; 5.4 \text{ fm for } b = 0.5 b_{max} \\ 1.4 \;\div\; 3.6 \text{ fm for } b = 0.7 b_{max} \end{cases}$
- It is required that $l_z > \lambda_{Kol}$. → we need $b > 0.5\, b_{max}$
- Furthermore
  Re = 0.3 – 1 for "$\eta/s = 1$" and
  Re = 3 – 10 for "$\eta/s = 0.1$"

PHYSICAL REVIEW C **87**, 034906 (2013)

Flow vorticity in peripheral high-energy heavy-ion collisions

L. P. Csernai,[1] V. K. Magas,[2] and D. J. Wang[1]

$$\Omega(z,x) \equiv w(z,x)\omega(z,x)$$

$$\Theta \equiv \nabla_\mu u^\mu = \partial_\mu u^\mu,$$

$$w_{ik} \equiv (N_{cell}/E_{tot}) \quad E_{ik}.$$

$$\omega^\mu_\nu \equiv \frac{1}{2}(\nabla_\nu u^\mu - \nabla^\mu u_\nu),$$

$$\omega_y \equiv \omega_{xz} \equiv \frac{1}{2}(\partial_z v_x - \partial_x v_z)$$

If $\partial_\tau u^\mu \equiv \dot{u}^\mu = u^\alpha \partial_\alpha u^\mu$ is negligible

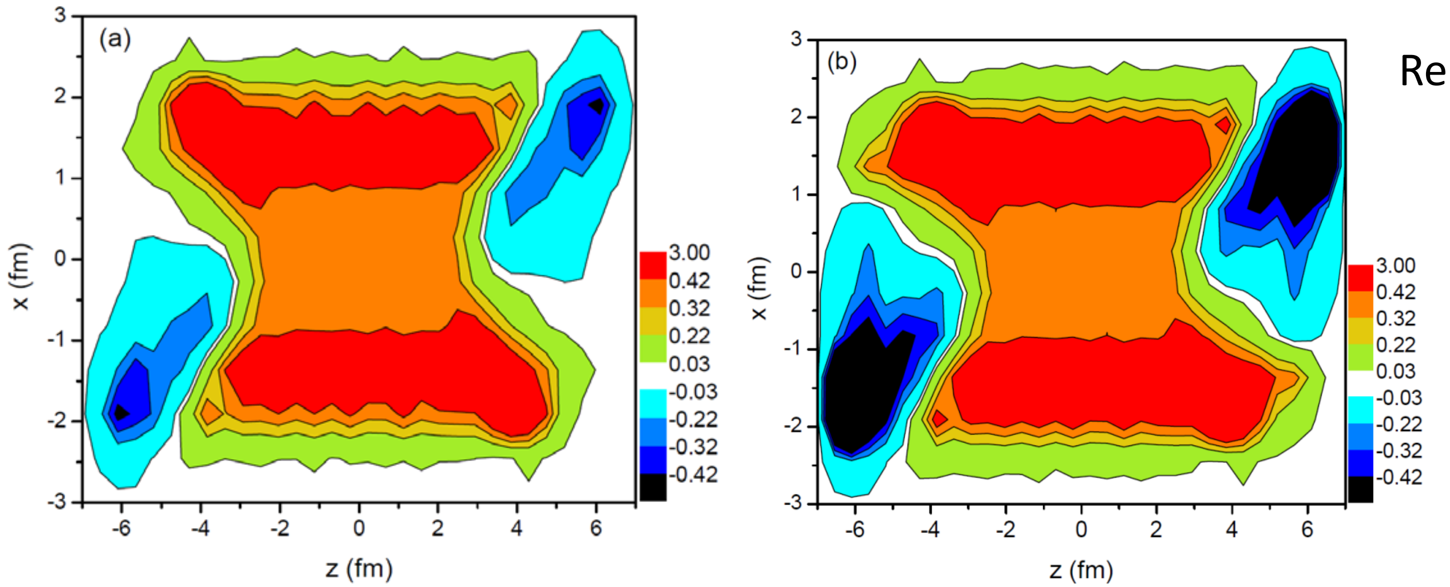
$$\omega^x_z = -\omega^z_x = \frac{1}{2}(\partial_z \gamma v_x - \partial_x \gamma v_z) = \frac{1}{2}\gamma(\partial_z v_x - \partial_x v_z) + \frac{1}{2}(v_x \partial_z \gamma - v_z \partial_x \gamma)$$

$\Omega(z,x)$

Classical

Relativistic

Reaction plane only



Max = 3. c/fm

FIG. 1: The classical (left) and relativistic (right) weighted vorticity, Ω_{zx}, calculated in the reaction, [x-z] plane at t=0.17 fm/c. The collision energy is $\sqrt{s_{NN}} = 2.76$ TeV and $b = 0.7 b_{max}$, the cell size is $dx = dy = dz = 0.4375 fm$. The average vorticity in the reaction plane is 0.1434 / 0.1185 for the classical / relativistic weighted vorticity respectively.

All y-layers

Classical

Relativistic

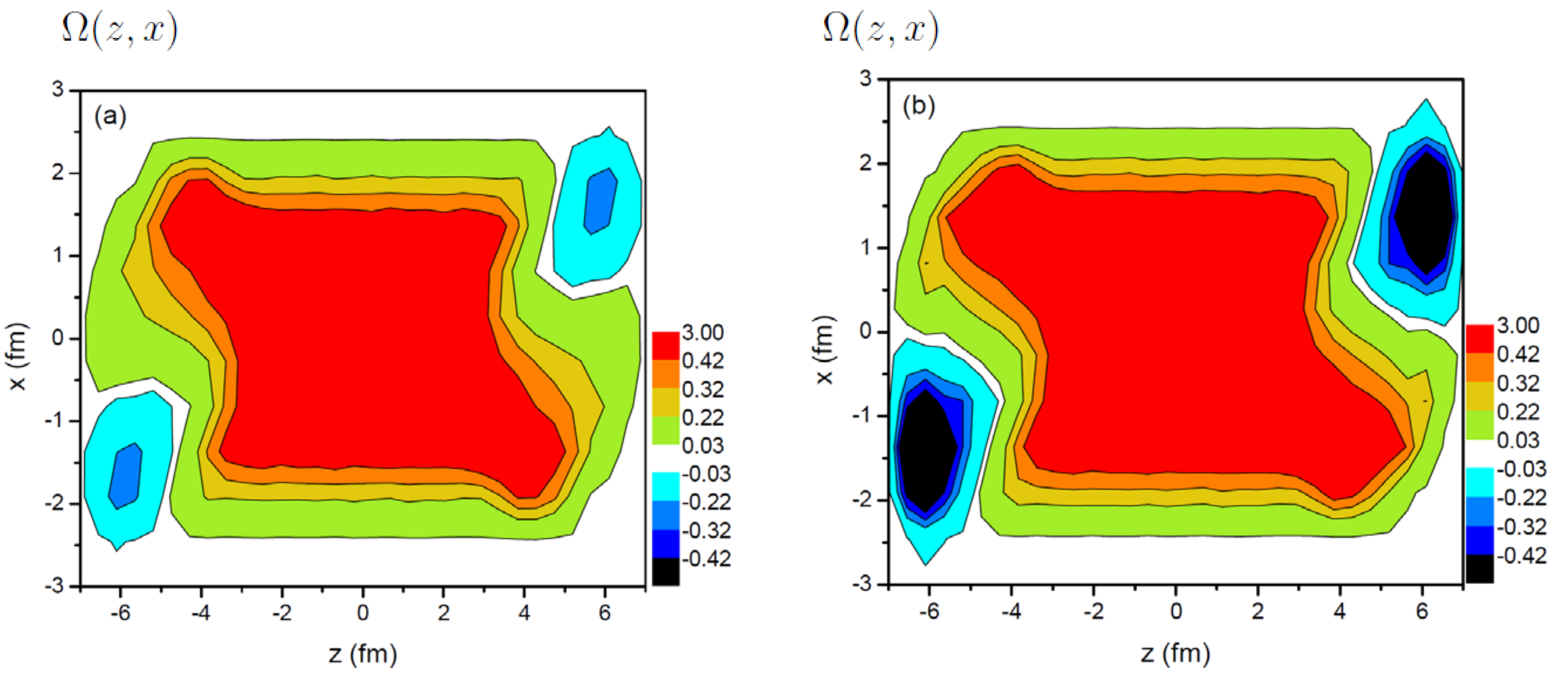


FIG. 4: The classical (left) and relativistic (right) weighted vorticity calculated for all [x-z] layers at t=0.17 fm/c. The collision energy is $\sqrt{s_{NN}} = 2.76$ TeV and $b = 0.7 b_{max}$, the cell size is $dx = dy = dz = 0.4375 fm$.

Classical

Late (FO) time

Relativistic

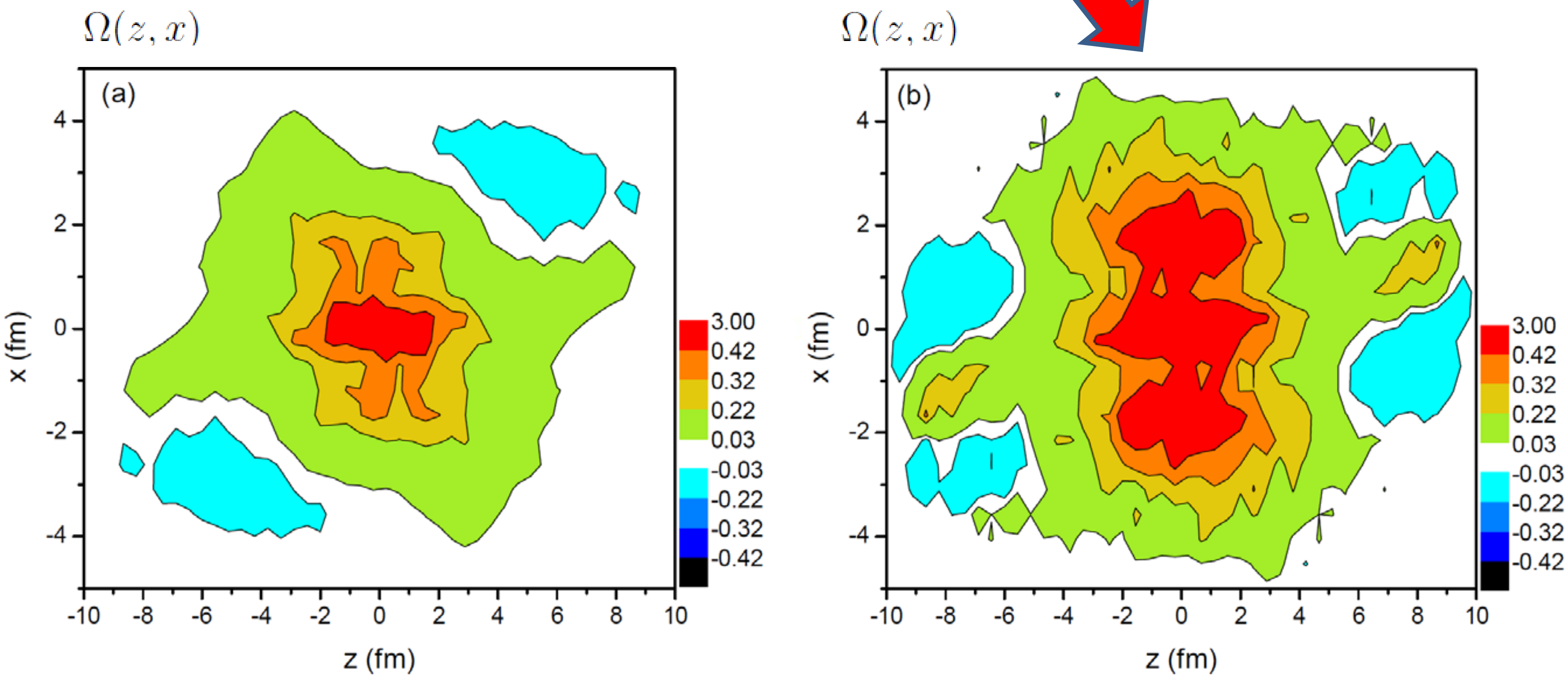


FIG. 5: The classical (left) and relativistic (right) weighted vorticity calculated for all [x-z] layers at t=3.56 fm/c. The collision energy is $\sqrt{s_{NN}} = 2.76$ TeV and $b = 0.7 b_{max}$, the cell size is $dx = dy = dz = 0.4375 fm$. The average vorticity in the reaction plane is 0.0538 / 0.10685 for the classical / relativistic weighted vorticity respectively.

the surface element $S(t)$. Then we can describe the *circulation* along

$$\Gamma(C(t)) = \oint_{C(t)} \mathbf{v} \cdot d\mathbf{l} = \int\int_{S(t)} \vec{\omega} \cdot \mathbf{n}\, dS$$

where ω is the vorticity

$$\vec{\omega} = \ \mathbf{rot\ v}$$

The circulation is conserved for perfect incompressible classical fluids.

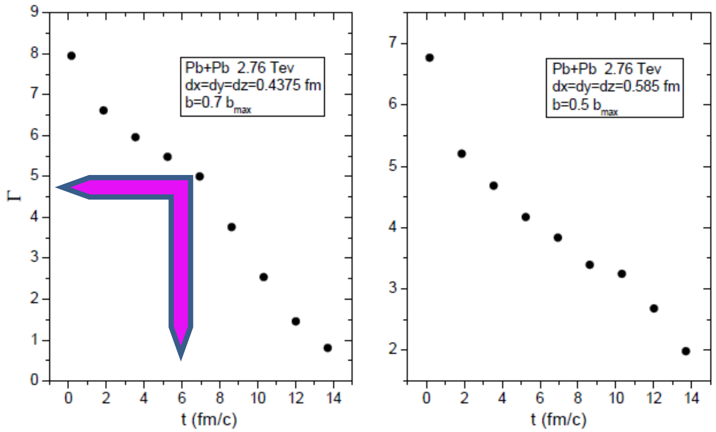
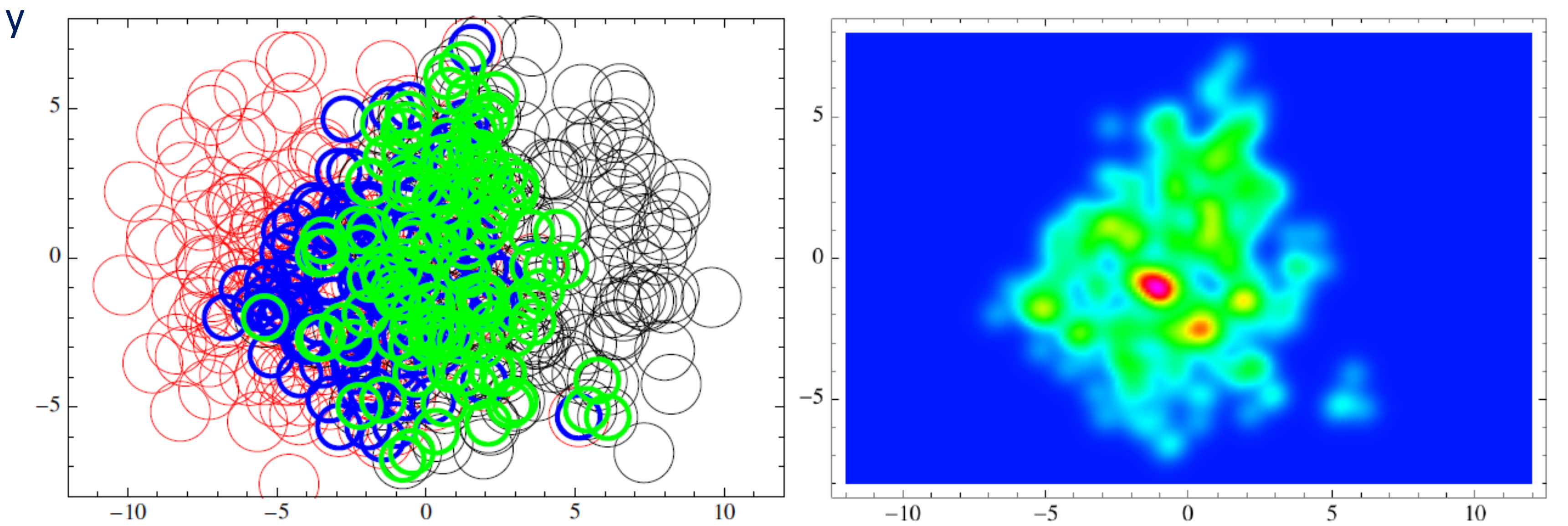


FIG. 7: The time dependence of classical circulation, $\Gamma(t)$, in units of [fm c], calculated for all [x-z] layers and then taking the average of the circulations for all layers. The collision energy is $\sqrt{s_{NN}} = 2.76$ TeV and $b = 0.7 b_{max}$, the cell size is $dx = dy = dz = 0.4375 fm$ (left). For comparison another initial state configuration was also tested for the same collision energy but $b = 0.5 b_{max}$, the cell size is $dx = dy = dz = 0.585 fm$ (right). This configuration shows also the rotation, but due to its less favorable parameters it does not show the KHI. Although at this impact parameter, which is less peripheral the reaction plane has a larger area filled with matter, nevertheless the initial classical circulation is less by about 15%. For the more peripheral case with smaller numerical viscosity the circulation decreases with time faster and the circulation for the two cases becomes equal around $t = 10\,\mathrm{fm/c}$.

Onset of turbulence around the Bjorken flow

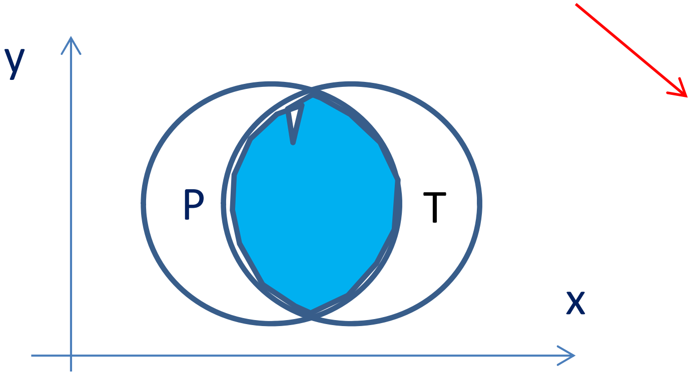
S. Floerchinger & U. A. Wiedemann, JHEP 1111:100, 2011; arXiv: 1108.5535v1



nucleons x [fm]

energy density x [fm]

- Transverse plane [x,y] of a Pb+Pb HI collision at $\sqrt{s_{NN}}$=2.76TeV at b=6fm impact parameter
- Longitudinally [z]: **uniform** Bjorken flow, (expansion to infinity), depending on τ only.

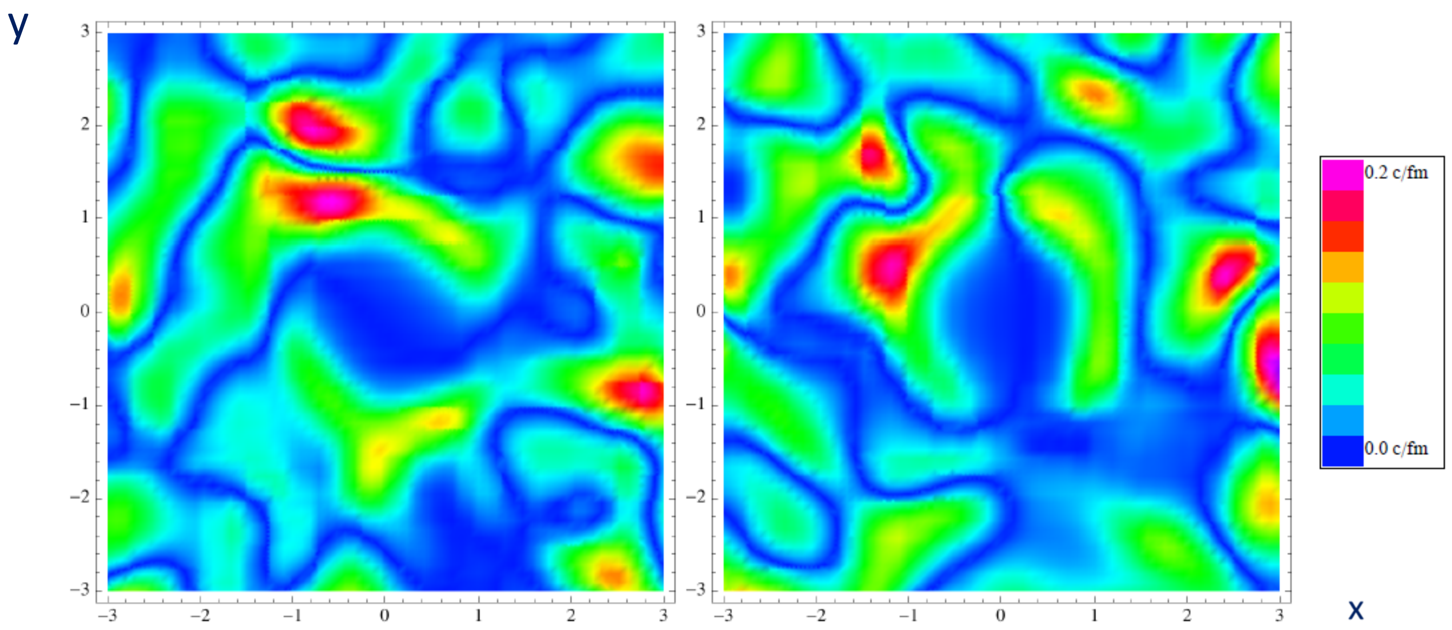


Green and **blue** have the same longitudinal speed (!) in this model. Longitudinal shear flow is omitted.

Onset of turbulence around the Bjorken flow

S. Floerchinger & U. A. Wiedemann,   JHEP 1111:100, 2011; arXiv: 1108.5535v1

Max = 0.2 c/fm



absolute value of vorticity $|\partial_1 u^2 - \partial_2 u^1|$ divergence $|\partial_1 u^1 + \partial_2 u^2|$

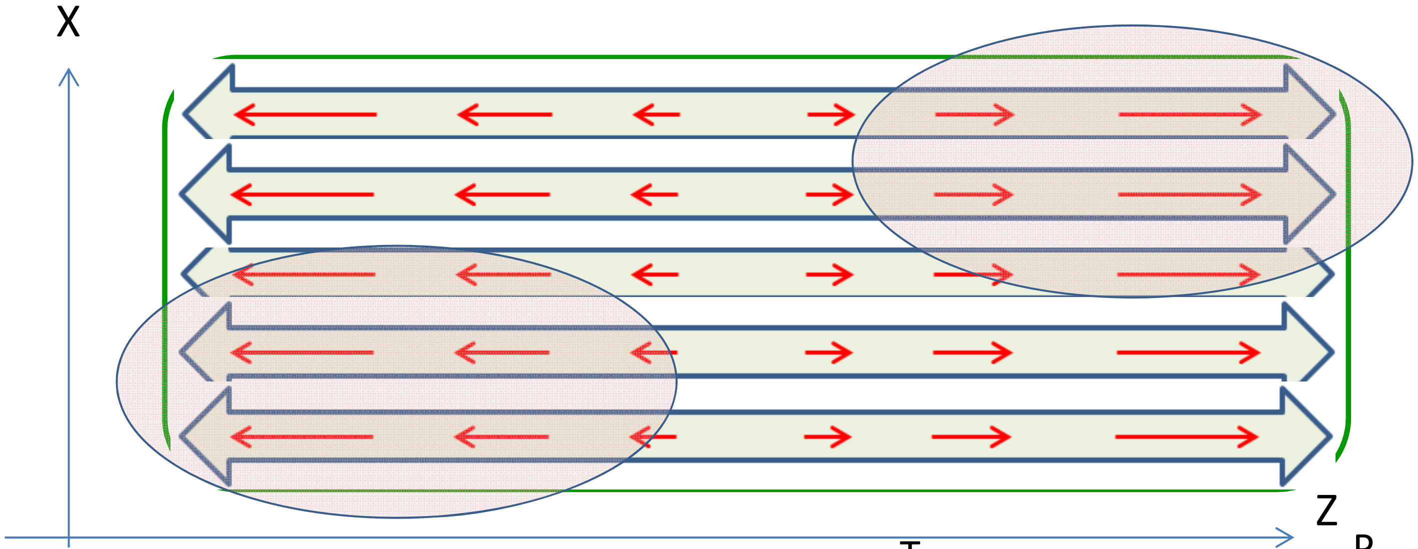
- Initial state Event by Event vorticity and divergence fluctuations.
- Amplitude of random vorticity and divergence fluctuations are the same
- In dynamical development viscous corrections are negligible (→ no damping)
- Initial transverse expansion in the middle (±3fm) is neglected (→ no damping)
- High frequency, high wave number fluctuations **may feed** lower wave numbers

Typical I.S. model – scaling flow

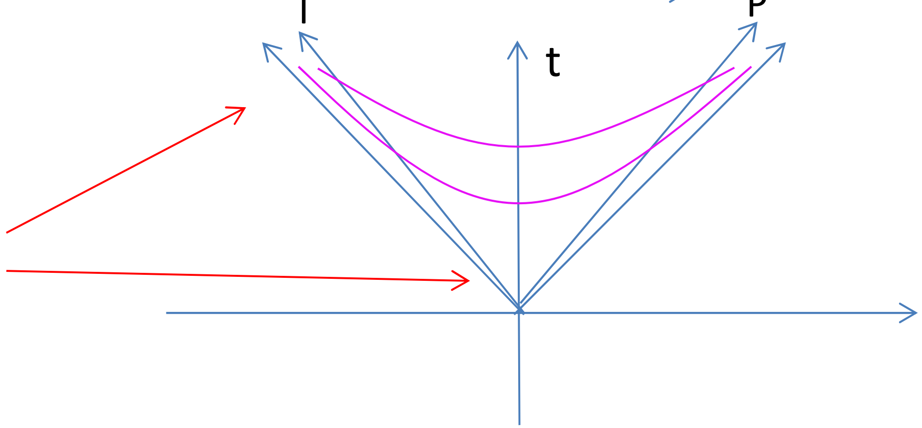
The same longitudinal expansion velocity profile in the whole [x,y]-plane !
No shear flow. **No string tension**! Usually **angular momentum is vanishing**!

$$\omega_y \equiv \omega_{xz} \equiv -\omega_{zx} \equiv \frac{1}{2}(\partial_z v_x - \partial_x v_z)$$

$$\boldsymbol{\omega} \equiv \frac{1}{2}\mathrm{rot}\ \boldsymbol{v} = \frac{1}{2}\nabla \times \boldsymbol{v}$$



Zero vorticity
&
Zero shear!



Such a re-arrangement of the matter density is dynamically not possible in a short time!

Deceleration of high-energy protons by heavy nuclei

L. P. Csernai* and J. I. Kapusta

PHYSICAL REVIEW D
VOLUME 31, NUMBER 11
1 JUNE 1985

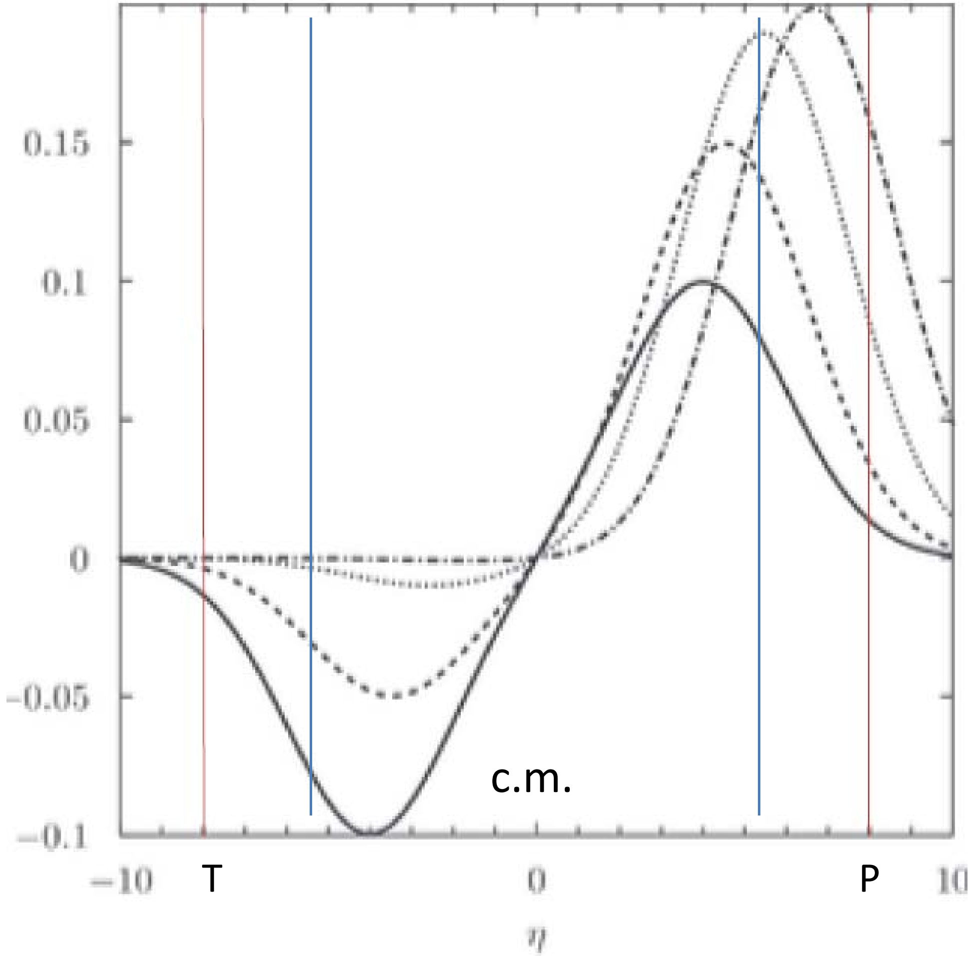
Δy = 2.5

Bjorken scaling flow assumption:

$$v^z = \frac{z}{t}.$$

$$v^z = \tanh \eta$$

$$u^\eta = 0.$$



The momentum distribution, in arbitrary units normalized to the total c.m. energy and momentum. The momentum is zero. Rapidity constraints at projectile and target rapidities are not taken into account! [Philipe Mota, priv. comm.]

Adil & Gyulassy (2005) initial state

x, y, η, τ coordinates → Bjorken scaling flow

PHYSICAL REVIEW C **72**, 034907 (2005)

Considering a longitudinal *“local relative rapidity slope”,* based on observations in D+Au collisions:

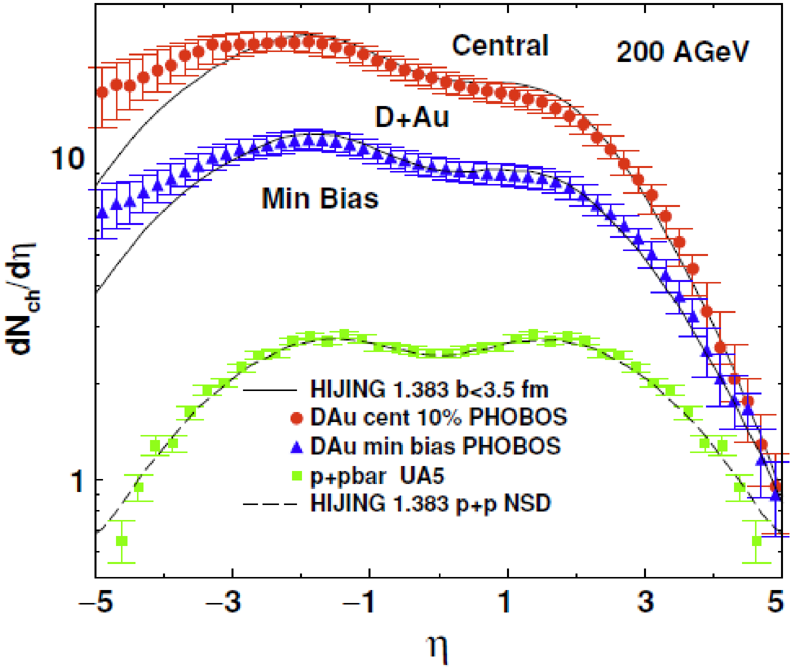
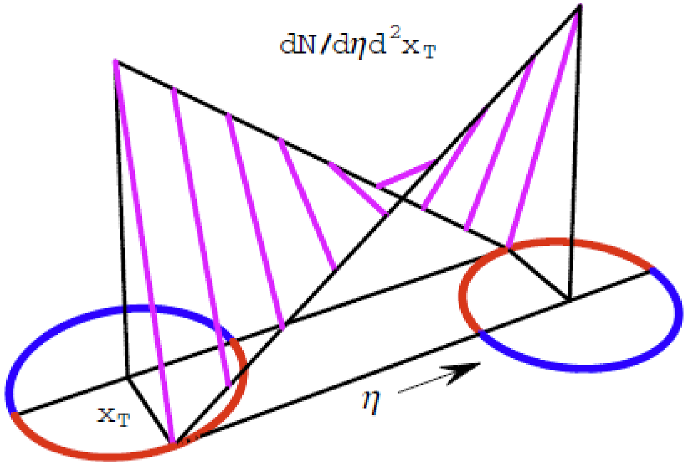
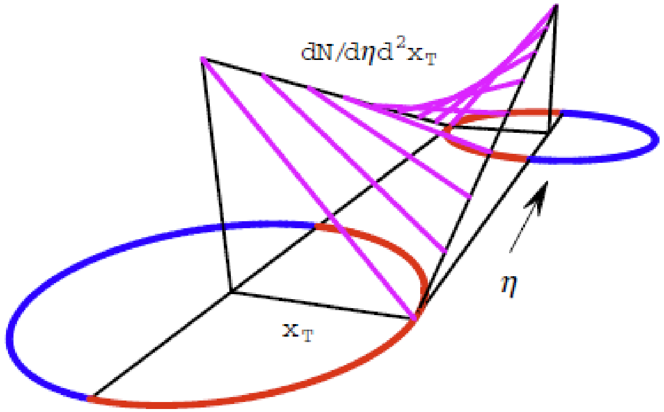


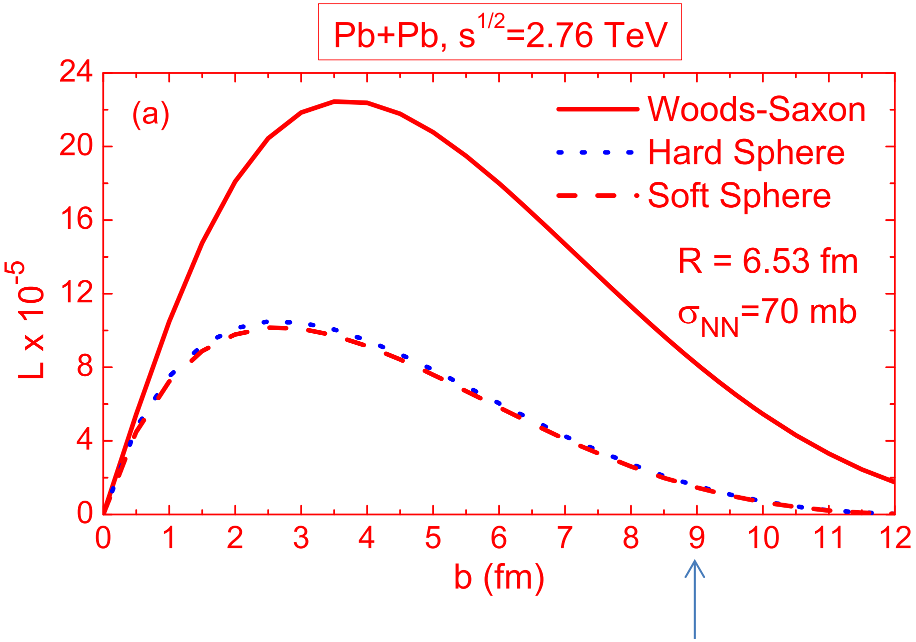
FIG. 2. (Color online) Asymmetric pseudorapidity distributions of charged hadrons produced in D+Au minimum bias and central 0–10% reactions at 200*A* GeV from PHOBOS [12] are compared to $p+\bar{p}$ data from UA5 [13]. The curves show predictions using the HIJING v1.383 code [14,15].

→

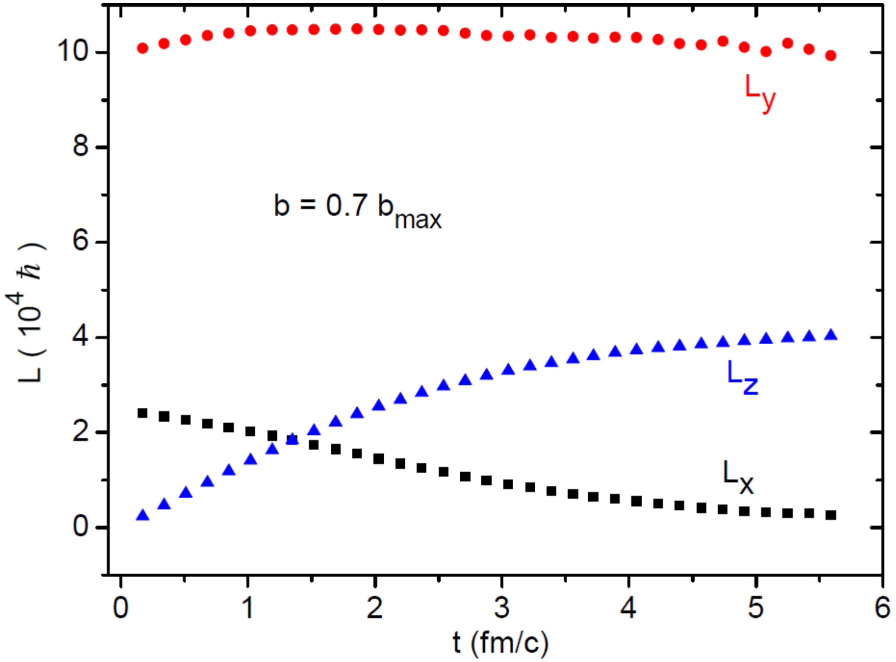




Detecting initial rotation



b = 0.7 b_{max}

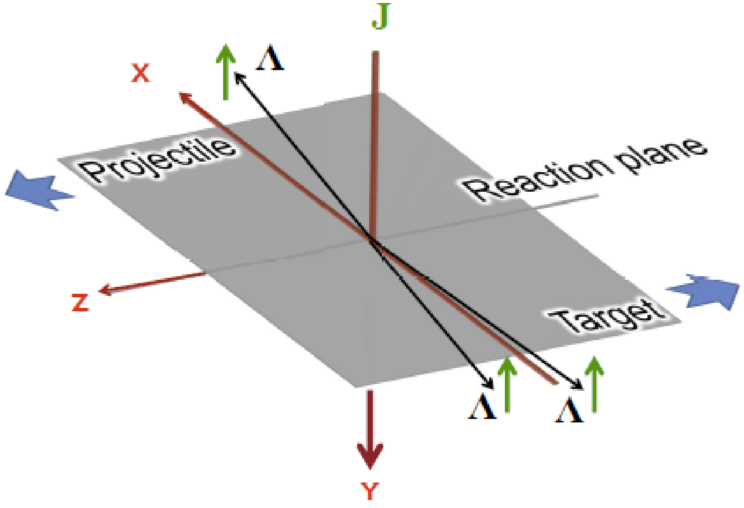


V. Vovchenko, D. Anchishkin, and L.P. Csernai,
Phys. Rev. C **88**, 014901 (2013)

J. H. Gao, S. W. Chen, W. T. Deng, Z. T. Liang, Q. Wang and X. N. Wang, Phys. Rev. C 77, 044902 (2008).

F. Becattini, F. Piccinini, J. Rizzo, Phys. Rev. C 77, 024906 (2008).

Detecting rotation: Lambda polarization



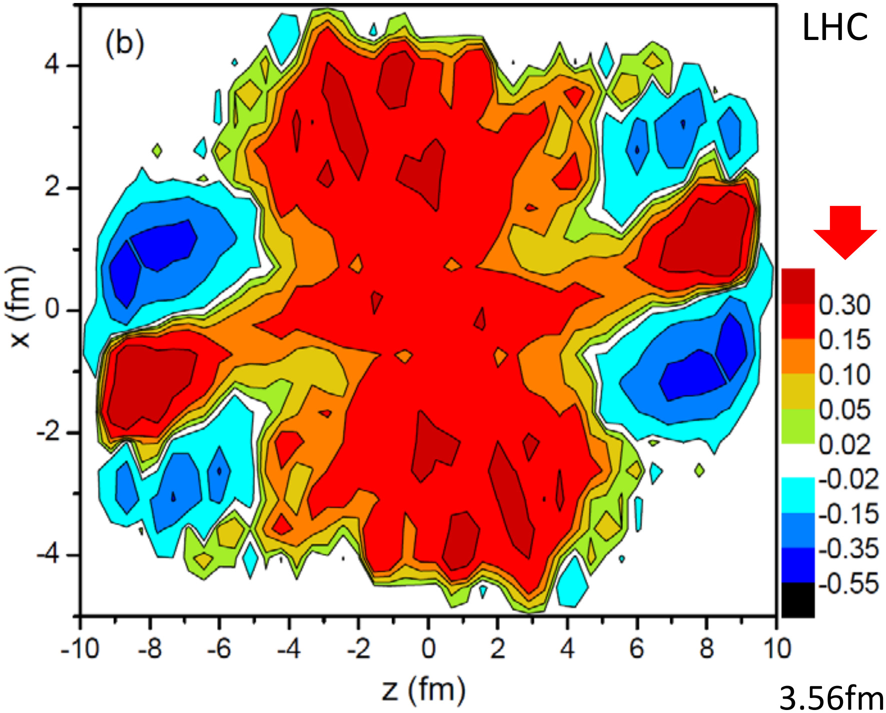
$$\mathbf{\Pi}(p) = \frac{\hbar\varepsilon}{8m}\frac{\int \mathrm{d}V\, n_F\, (\nabla\times\beta)}{\int \mathrm{d}V\, n_F}$$

$$\beta^\mu(x) = (1/T(x))u^\mu(x)$$

← From hydro

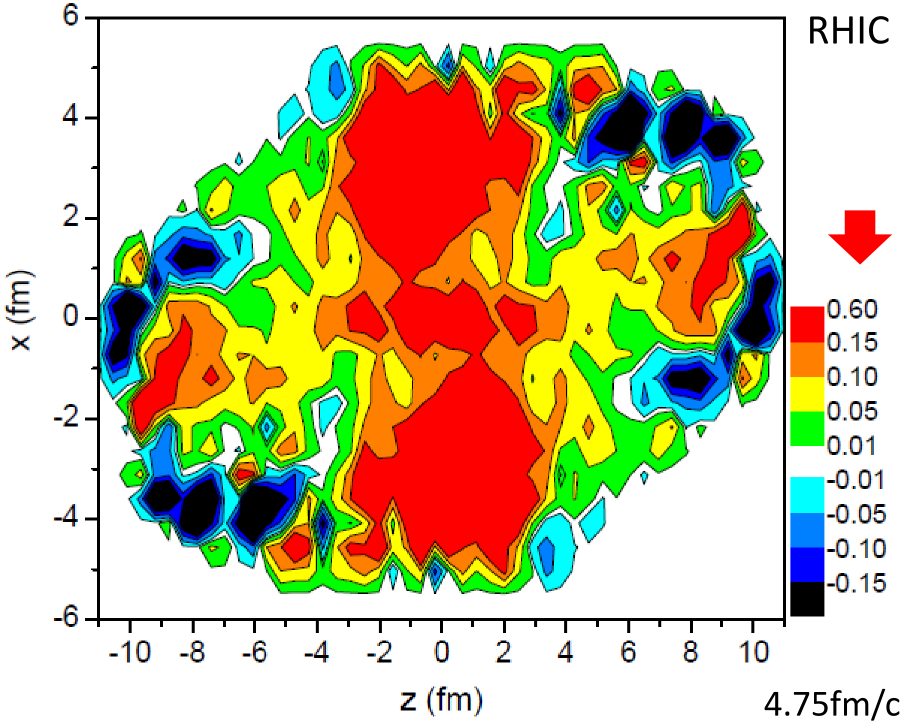
$$\mathbf{\Pi}_0(p) = \mathbf{\Pi}(p) - \frac{\mathbf{p}}{\varepsilon(\varepsilon+m)}\mathbf{\Pi}(p)\cdot\mathbf{p}$$

[F. Becattini, L.P. Csernai, D.J. Wang,
Phys. Rev. C **88**, 034905 (2013)]



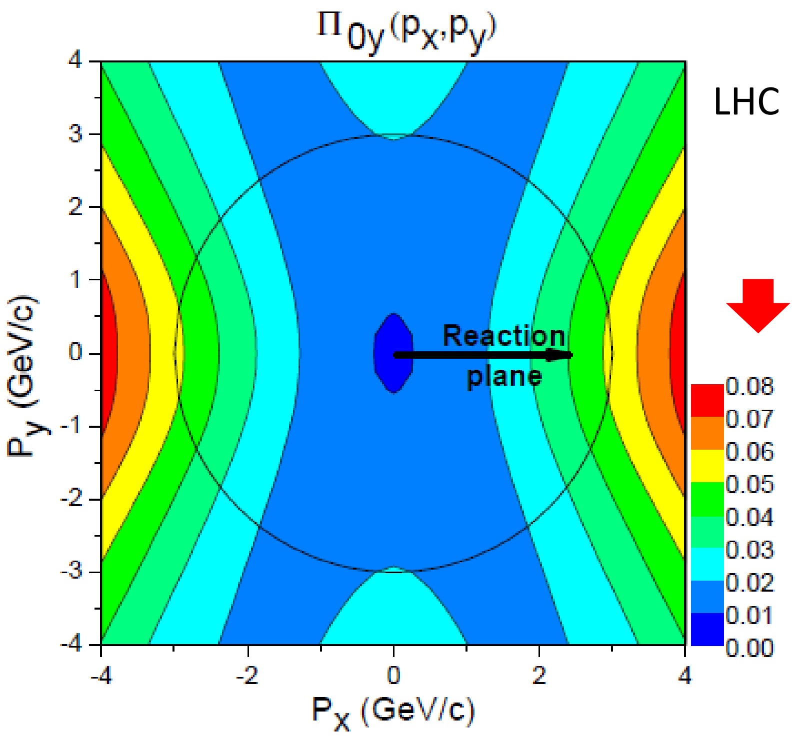
LHC

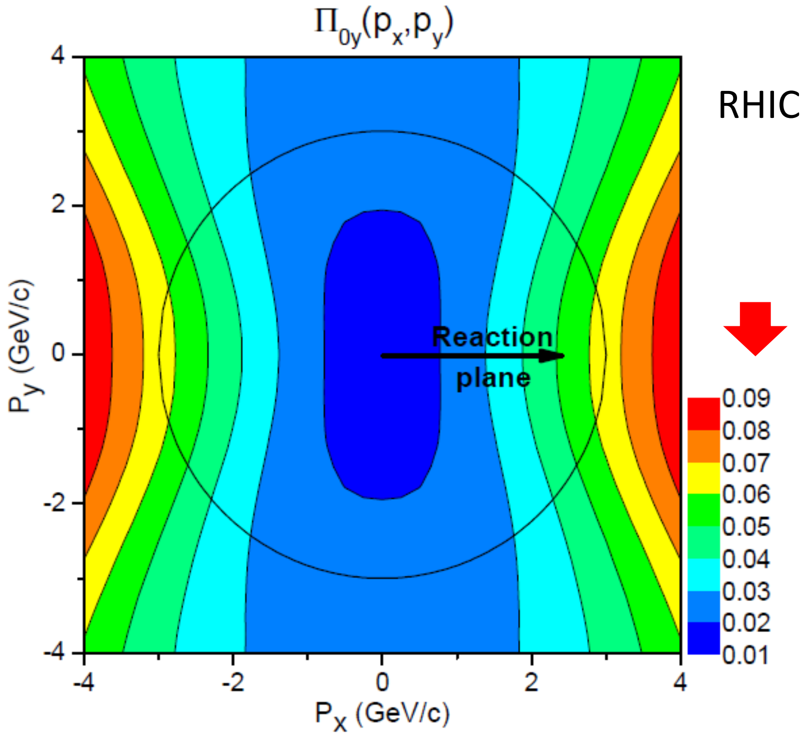
3.56fm/c



RHIC

4.75fm/c





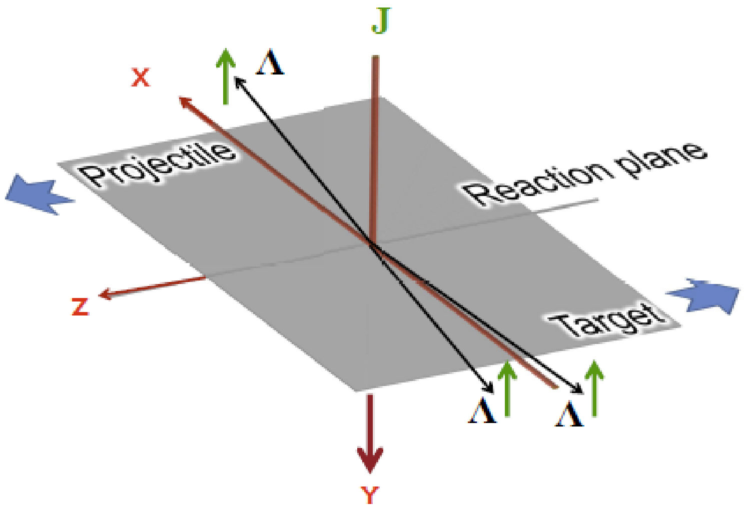
- The **POLARIZATION** of Λ and $\bar{\Lambda}$ due to thermal equipartition with local vorticity is slightly stronger at RHIC than at LHC due to the much higher temperatures at LHC.
- Although early measurements at RHIC were negative, these were averaged over azimuth! We propose selective measurement in the reaction plane (in the +/- x direction) in the EbE c.m. frame. Statistical error is much reduced now, so significant effect is expected at $p_x \geq 3$ GeV/c.

Summary

- FD model: Initial State + **EoS** + Freeze out & Hadronization
- In p+p I.S. is problematic, but ∃ collective flow
- In A+A the I.S. is causing global collective flow
- Consistent I.S. is needed based on a dynamical picture, satisfying causality, etc.
- Several I.S. models exist, some of these are oversimplified beyond physical principles.
- Experimental outcome strongly depends on the I.S.

Thank you

Detecting rotation: Lambda polarization



$$\mathbf{\Pi}(p) = \frac{\hbar\varepsilon}{8m}\frac{\int \mathrm{d}V\, n_F\, (\nabla\times\beta)}{\int \mathrm{d}V\, n_F}$$

$$\beta^\mu(x) = (1/T(x))u^\mu(x)$$ ← From hydro

$$\mathbf{\Pi}_0(p) = \mathbf{\Pi}(p) - \frac{\mathbf{p}}{\varepsilon(\varepsilon+m)}\mathbf{\Pi}(p)\cdot\mathbf{p}$$

[F. Becattini, L.P. Csernai, D.J. Wang, Submitted to Phys. Rev. Lett. arXiv:1304.4427v1 [nucl-th]]