Rest frames in relativistic thermodynamics and hydrodynamics

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1. Dissipative relativistic fluids
   – Causality, stability, flow-frames, kinetic equilibrium
2. Dissipative fluids and general frames

with T.S. Biró

Basic concepts:

$$T^{ab} = eu^a u^b + q^a u^b + q^b u^a + P^{ab},$$ energy-momentum density

$$N^a = nu^a + j^a.$$ particle number density

$$q^a u_a = j^a u_a = 0, \quad P^{ba} u_a = P^{ab} u_a = 0^b$$

$$T^{ab} = \begin{pmatrix} e & q^i \\ q^j & P^{ij} \end{pmatrix}, \qquad N^a = \begin{pmatrix} n \\ j^i \end{pmatrix}$$

$$a, b \in \{0,1,2,3\}; \quad i, j \in \{1,2,3\}; \quad diag(1,-1,-1,-1)$$

$$\dot{e} = u^a \partial_a e$$

- u^a – velocity field
- e – energy density
- q^a – momentum density or energy current??
- P^{ab} – pressure
- n – particle number density
- j^a – particle current

General, expressed by comoving splitting

$$u_a \partial_b T^{ab} = \dot{e} + e\, \partial_a u^a + \partial_a q^a + u_a \dot{q}^a + u_a \partial_b P^{ab} = 0^a$$ energy balance

$$\partial_b N^b = \dot{n} + n\, \partial_a u^a + \partial_a j^a = 0$$ particle number balance

Dissipative or ideal?

$$P^{ab} = -p\Delta^{ab} + \Pi^{ab} = (-p + \Pi)\Delta^{ab} + \pi^{ab}$$ pressure splitting

Constitutive theory:

Fields:

N^a	4
T^{ba}	10
u^a	3
	Σ17
j^a	3
q^a	3
Π^{ab}	6
	Σ 12

$$q^a u_a = j^a u_a = 0, \quad \Pi^{ba} u_a = \Pi^{ab} u_a = 0^b$$

Equations:

$\partial_a N^a = 0,$	1
$\partial_b T^{ab} = 0^a,$	4

N^a – *particle number density vector*
T^{ab} – energy-momentum tensor
u^a – velocity field

j^a – *particle current*
q^a – energy current??
Π^{ab} – viscous pressure

n, e, u^a – basic fields

Non-equilibrium thermodynamics, second law

Entropy inequality:

$$\partial_a S^a = \dot{s} + s\,\partial_a u^a + \partial_a J^a \geqslant 0$$

Eckart (1940):

$$S^a(T^{ab}, N^a) = s(e,\ n)u^a + \frac{q^a}{T}$$

(Müller)-Israel-Stewart (1969-72):

$$S^a(T^{ab}, N^a) = \left(s(e,\ n) - \frac{\beta_0}{2\mathrm{T}}\Pi^2 - \frac{\beta_1}{2\mathrm{T}} q_b q^b - \frac{\beta_2}{2\mathrm{T}} \pi^{bc}\pi_{bc} \right) u^a +$$

$$+ \frac{1}{T}\left(q^a + \alpha_0 \Pi q^a + \alpha_1 \pi^{ab} q_b \right)$$

Concept of dissipation

constitutive theory – closure by linear relations
thermodynamic fluxes and forces
kinetic theory calculates

$$\Sigma = -j^a \partial_a \alpha - \beta \Pi^{ab} \partial_b u_a + q^a (\partial_a \beta + \beta \dot{u}_a) \geq 0$$

Closure:

$$j^a = \chi \Delta^{ab} \partial_b \alpha ,$$

$$\Pi^{ab} = \nu \partial_c u^c \Delta^{ab} + \eta \Delta^{ac} \Delta^{bd} (\partial_c u_d + \partial_d u_c)/2,$$

$$q^a = \lambda \Delta^{ab} (\partial_b \beta + \beta \dot{u}_b)$$

+ balances

Ideal fluid:

$$q^a = 0^a ,\ j^a = 0^a ,\ \Pi^{ab} = 0^{ab}$$

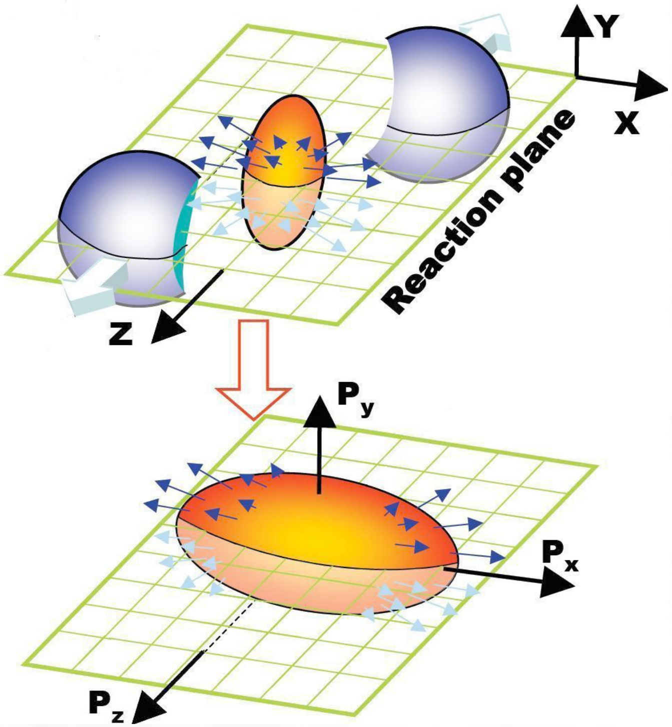
Dissipative relativistic fluids

heavy ion collisions

cosmology

– (quark-)gluon plasma

there is a minimal viscosity



What is viscous?

a need of theories

– *Causality*

hyperbolic or parabolic?

– *Stability* – second law

instable homogeneous equilibrium

– *Velocity* – *flow-frames*

Is there a freedom? (Eckart, Landau-Lifshitz, …) What is ideal?

– *Kinetic theory*

Do we need anything else?

Causality

– infinite speed of signal propagation
– second order time derivatives
– hyperbolic system of equations

Divergence type theories – finite speed is material
(Liu-Ruggeri-Müller, Geroch, Lindblom, Calzetta)

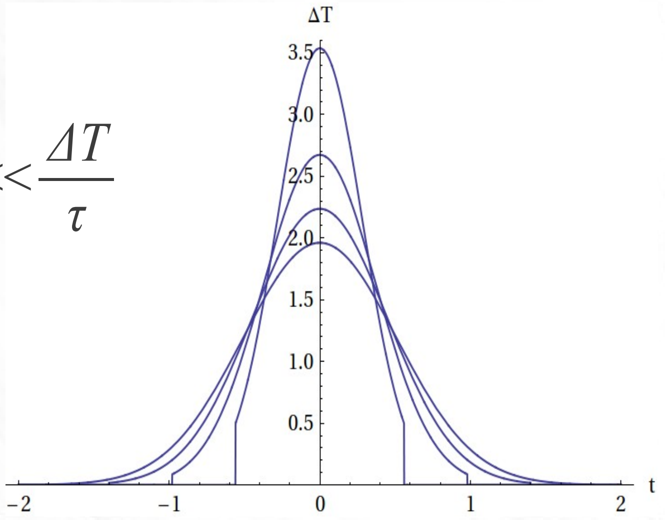
Physical:
Propagation speed of *continuum limit.*
Propagation speed of observable signals.

Example:

$$\partial_t T = -\kappa \, \partial_x^2 T \qquad \partial_x T << \frac{\Delta T}{\xi}, \quad \partial_t T << \frac{\Delta T}{\tau}$$

water at room temperature:

$$v_{max} \approx \frac{\kappa}{\xi} = 14\, m/s$$



Stability

Generic stability: linear stability of homogeneous equilibrium

Instability of first order theories (Hiscock-Lindblom, 1985)
Stability of the (Müller)-Israel-Stewart theory (Hiscock-Lindblom, 1983)

Divergence type theories – built in stability, need of dissipation

Conceptual question:
Thermodynamics is related to stability.

There are conditions.

Stability conditions of the Israel-Stewart theory

(Hiscock-Lindblom 1983,1987)

$$\Omega_1 = \frac{1}{e+p}\frac{\partial e}{\partial p}\Big|_{\frac{s}{n}} = \frac{T}{(e+p)\frac{\partial p}{\partial e}\Big|_n - n\frac{\partial p}{\partial n}\Big|_e} \geq 0,$$

$$\Omega_2 = \frac{1}{e+p}\frac{\partial e}{\partial(s/n)}\Big| \frac{\partial p}{\partial(s/n)}\Big|_{\frac{\mu}{nT}} = \ldots \geq 0,$$

Conditons for the

- EOS
- IS coefficients
- both

+ usual

Eckart frame

$$\Omega_5 = \beta_0 \geq 0, \quad \Omega_8 = \beta_2 \geq 0, \quad \Omega_7 = \beta_1 - \frac{\alpha_1^2}{2\beta_2} \geq 0,$$

$$\Omega_4 = e + p - \frac{2\beta_2 + \beta_1 + 2\alpha_1}{2\beta_1\beta_2 - \alpha_1^2} \geq 0, \quad \Omega_6 = \beta_1 - \frac{\alpha_0^2}{\beta_0} - \frac{2\alpha_1^2}{3\beta_2} - \frac{1}{n^2 T}\frac{\partial T}{\partial(s/n)}\Big|_n \geq 0,$$

$$\Omega_3 = (e+p)\left(1 - \frac{\partial p}{\partial e}\Big|_{\frac{s}{n}}\right) - \frac{1}{\beta_0} - \frac{2}{3\beta_2} - \frac{K^2}{\Omega_6} \geq 0, \quad K = 1 + \frac{\alpha_0}{\beta_0} + \frac{2\alpha_1}{3\beta_2} - \frac{n}{T}\frac{\partial T}{\partial n}\Big|_{s/n} \geq 0.$$

Velocity – flow-frames

What is a fluid? What is moving?

Eckart (material) frame: $$u^a = \frac{N^a}{\sqrt{-N^b N_b}} \rightarrow N^a = n u^a$$

Landau-Lifshitz (energy) frame: $$\hat{u}^a = \frac{E^a}{\sqrt{-E^b E_b}} \rightarrow T^{ab} = \hat{e}\,\hat{u}^a \hat{u}^b + \hat{P}^{ab}$$

Jüttner (thermometer) flow: $$\check{u}^a = \frac{\beta^a}{\sqrt{-\beta^b \beta_b}} \rightarrow \beta^a = \check{\beta}\,\check{u}^a$$

Do we have a choice?

Landau-Lifshitz:

$$N^a = \hat{n}\,\hat{u}^a + \hat{j}^a$$
$$T^{ab} = \hat{e}\,\hat{u}^b\,\hat{u}^a + \hat{P}^{ab} = \hat{e}\,\hat{u}^b\,\hat{u}^a - \hat{p}\,\hat{\Delta}^{ab} + \hat{\Pi}^{ab}$$

Eckart:

$$N^a = n u^a$$
$$T^{ab} = e u^b u^a + q^b u^a + q^a u^b - p \Delta^{ab} + \Pi^{ab}$$

Transformation:

$$u^a = \frac{\hat{u}^a + \hat{z}^a}{\zeta}$$

What is ideal?

$$N_0^a = n u^a$$
$$T_0^{ab} = e u^b u^a - p \Delta^{ab}$$

$$N_0^a = \hat{n}\,\hat{u}^a + j^a$$
$$T_0^{ab} = \hat{e}\,\hat{u}^b\,\hat{u}^a + q^b\,\hat{u}^a + q^a\,\hat{u}^b - p\,\hat{\Delta}^{ab} + \Pi^{ab}$$

$$\hat{n} = \frac{n}{\zeta}, \quad j^a = \frac{n\,\hat{z}^a}{\zeta}, \quad \hat{e} = \frac{e+p}{\zeta^2} - p, \quad q^a = (e+p)\,\hat{z}^a, \quad \Pi^{ab} = \frac{\hat{z}^a\,\hat{z}^b}{e+p}$$

Ideal fluid is a class of N^a, T^{ab}

Entropy production, Gibbs relation are flow-frame dependent.

Kinetic theory → thermodynamics

$$p^a \partial_a f = C(f)$$ Boltzmann equation

$$p^a p_a = m^2$$

Boltzmann gas

Thermodynamic equilibrium = no dissipation:

$$\sigma = \partial_a S^a = \partial_a \left(-\int \frac{d^3 p}{p^0} p^a f (\ln f - 1) \right) =$$

$$\frac{1}{4} \sum_{i,j,k,l} \int \frac{d^3 p_i}{p_i^0} \frac{d^3 p_j}{p_j^0} \frac{d^3 p_k}{p_k^0} \frac{d^3 p_l}{p_l^0} \left(\frac{f_k f_l}{f_i f_j} - \ln \frac{f_k f_l}{f_i f_j} - 1 \right) f_i f_j W_{ij|kl} = 0$$

$$\Leftrightarrow f f_1 = f' f_1' \Leftrightarrow$$

$$f_0(x,k) = e^{\alpha(x) - \beta_b(x) p^b}$$

(local) equilibrium distribution

Thermodynamic relations – normalization

$$f_0(x,p)=e^{\alpha(x)-\beta_b(x)p^b}$$

$$N_0^a=\int p^a f_0$$
$$T_0^{ab}=\int p^b p^a f_0$$

Jüttner distribution AND Jüttner flow-frame

$$\alpha=\frac{\check{\mu}}{\check{T}},\ \beta_a=\frac{\check{u}_a}{\check{T}} \qquad f_0(x,p)=e^{\frac{\check{\mu}-\check{u}_b p^b}{\check{T}}}$$

When calculated frame independently, one obtains:

$$\beta^a=\beta(u^a+w^a) \qquad \check{u}^a=\frac{u^a+w^a}{\sqrt{1-w^2}} \qquad \check{T}=\frac{T}{\sqrt{1-w^2}},\ \check{\mu}=\frac{\mu}{\sqrt{1-w^2}},$$

$$f_0(x,p)=e^{\alpha-\beta_b p^b}=e^{\frac{\mu-(u_b+w_b)p^b}{T}}=e^{\frac{\check{\mu}-\check{u}_b p^b}{\check{T}}}$$

Energy-momentum density:

$$N_0^a = \int p^a f_0 = \check{n}\check{u}^a = nu^a + nw^a$$

$$\check{n} = n\sqrt{1-w^2} = 4\pi m^2 \check{T} K_2\left(\frac{m}{\check{T}}\right) e^{\frac{\check{\mu}}{\check{T}}}$$

$$T_0^{ab} = \int p^a p^b f_0 = \check{e}\check{u}^b\check{u}^a + \check{p}\check{\Delta}^{ab}$$

$$\check{e} = 3\check{n}\check{T} + m\check{n}\frac{K_1\left(\frac{m}{\check{T}}\right)}{K_2\left(\frac{m}{\check{T}}\right)}$$

$$T_0^{ab} = eu^b u^a + q^b u^a + u^b q^a - p\Delta^{ab} + \frac{q^b q^a}{e+p}$$

$$\check{p} = p, \quad e = \frac{\check{e} + p w^2}{1-w^2}, \quad q^a = (e+p) w^a$$

Heat flux:

$$I^a = q^a - \frac{e+p}{n} j^a = 0$$

Requirements:

– Kinetic equilibrium: $q^a = (e + p) w^a$

– Generic stability: physical conditions!

Freedom:

– Flow-frames: arbitrary or fixed?
– *Thermodynamics*

Observation: thermo is flow-frame dependent

Gibbs: $ds + \alpha\, dn = \beta\, de$

entropy production

Thermodynamics in arbitrary frames:

$$S^a + \alpha N^a - \beta_b T^{ab} = \Phi^a$$ objective/covariant starting point

$$\beta^a = \beta\left(u^a + w^a\right)$$ temperature vector

Thermodynamics:

a) $\Phi^a = p\beta^a$ matching $S_0^a + \alpha N_0^a - \beta_b T_0^{ab} = \beta^a p_0$

b) $$ds + \alpha\, dn = \beta_a dE^a + \beta\, p\, w_a du^a$$
$$= \beta\left(u_a + w_a\right) d\left(e u^a + q^a\right) + \beta\, p\, w_a du^a$$
$$= \beta\left(de + w_a dq^a + \underline{\left[\left(e+p\right) w_a - q_a\right] du^a}\right)$$

Kinetic compatible: $q^a = \left(e+p\right) w^a$

$w^\mu = 0 \;\Rightarrow\; ds + \alpha\, dn \neq \beta\, de$ $\qquad$ $w^\mu = 0 \wedge q^a = 0 \;\Rightarrow\; ds + \alpha\, dn = \beta\, de$

$$\beta^a = \beta(u^a + w^a)$$

Entropy production:

$$0 \leq \Sigma = \Pi^{ab}\partial_a\beta_b + q^a\partial\beta_a - j^a\partial_a\alpha + \beta w^a\left(h\dot{u}_a + \dot{q}_a + q_a\partial_b u^b + \partial_b\Pi^b_a\right)$$

$$= \left(\Pi^{ab} - q^a w^b\right)\partial_b\beta_a + \left(q^a - hw^a\right)\left(\partial\beta_a - \beta w_b\partial_a u^b\right) + \left(nw^a - j^a\right)\partial_a\alpha$$

Kinetic : $w^a = \frac{q^a}{h}$

$$\Sigma_K = \left(\Pi^{ab} - \frac{q^a q^b}{h}\right)\partial_b\beta_a + \left(\frac{n}{h}q^a - j^a\right)\partial_a\alpha \geqslant 0$$

Jüttner: $w^a = 0$

$$\Sigma_J = \Pi^{ab}\partial_b\beta_a + q^a\partial\beta_a - j^a\partial_a\alpha \geqslant 0$$

Condition of generic stability: $\left.\frac{\partial\beta}{\partial n}\right|_e > 0$

Entropy production:

$$\Sigma = \left(\Pi^{ab} - \frac{q^a q^b}{h} \right) \partial_b \beta_a + \frac{n}{h} \underbrace{\left(q^a - \frac{h}{n} j^a \right)}_{I^a} \partial_a \alpha \geqslant 0$$

$$\beta^a = \beta (u^a + w^a)$$

$$q^a = (e + p) w^a$$

Chooce of the flow: freedom.
Mixed Eckart–Landau-Lifsic frame:

$$n^a = \frac{N^a}{\|N^a\|} = \frac{n u^a + j^a}{\|N^a\|} \qquad e^a = \frac{E^a}{\|E^a\|} = \frac{e u^a + q^a}{\|E^a\|}$$

$$u^a = \frac{A e^a + B n^a}{O}$$

$\rightarrow j^a \| q^a$

Generic stability with natural conditions.

Conclusions

Temperature is not necessarily parallel to the flow.

Dissipation is spacelike.

$$\beta^a = \beta\left(u^a + w^a\right)$$

Kinetic theory prefers

u-orthogonal parts of the temperature:

$$w^a = \frac{q^a}{h}$$

Generic stability in general frames.

Israel-Stewart theory is not necessary.

Temperature!

VP, Biró, TS., EPJ–ST, 155:201–212, 2008, (arXiv:0704.2039v2).
VP, MMS, 3(6):1161–1169, 2008, (arXiv:07121437).
Biró, TS., VP. EPL, 89:30001, 2010.
VP, EPJ WoC, 13:07004, 2011, (arXiv:1102.0323).
VP, Biró, TS., PLB, 709(1-2):106–110, 2012, (arXiv:1109.0985).
VP, Biró TS., in Proc. of JETC13, arXiv: 1305.3190
VP, Biró TS., under publ. in AIP Proc. Ser. arXiv:1340:5976

Thermodynamics

is

~~amazing.~~

stability?

The idea of Eckart:

$$u^a\partial_b T^{ab} = \dot{e} + e\partial_a u^a + \partial_a q^a + u_a \dot{q}^a + u_a\partial_b P^{ab} = 0^a$$

$$\partial_b N^b = \dot{n} + n\partial_a u^a + \partial_a j^a = 0$$

$$ds + \alpha\, dn = \beta\, de$$

$$\dot{e} = u^a \partial_a e$$

$$J^a = \beta q^a - \alpha j^a$$

$$\partial_a S^a = \dot{s}(e,n) + s\partial_a u^a + \partial_a J^a \geq 0$$

$$-\frac{1}{T}\left(P^{ij} - p\delta^{ij}\right)\partial_i v_j + q^i \partial_i \frac{1}{T} \geq 0$$

$$\sigma_s = -j^a\partial_a\alpha - \beta\underbrace{\left(P^{ab} - p\delta^{ab}\right)}_{\Pi^{ab}}\partial_b u_a + q^a\left(\partial_a\beta + \beta\dot{u}_a\right) \geq 0$$

Eckart term

Thermodynamic relations - normalization

$$f_0(x,k) = e^{\alpha(x) - \beta_\nu(x) k^\nu}$$

$$N_0^\mu = \int k^\mu f_0$$

$$T_0^{\mu\nu} = \int k^\nu k^\mu f_0$$

Jüttner distribution? $\alpha = \frac{\mu}{T}, \beta_\nu = \frac{u_\nu}{T}$ $f_0(x,k) = e^{\frac{\mu - u_\mu k^\nu}{T}}$

$$\partial_\mu N_0^\mu = \partial_\mu \int k^\mu f_0 = \int \left(f_0 k^\mu \partial_\mu \alpha - f_0 k^\nu k^\mu \partial_\mu \beta_\nu \right) =$$

$$\partial_\mu N_0^\mu = N_0^\mu \partial_\mu \alpha - T_0^{\mu\nu} \partial_\mu \beta_\nu$$

$$S_0^\mu := (1-\alpha) N_0^\mu + \beta_\nu T_0^{\mu\nu}$$ Legendre transformation

$$\partial_\mu S_0^\mu = -\alpha \partial_\mu N_0^\mu + \beta_\nu \partial_\mu T_0^{\mu\nu}$$

$$\partial_\mu S_0^\mu + \alpha \partial_\mu N_0^\mu - \beta_\nu \partial_\mu T_0^{\mu\nu} = 0$$

covariant Gibbs relation
(Israel,1963)

Remark: $$\partial_\mu S^\mu + \alpha \partial_\mu N^\mu - \beta_\nu \partial_\mu T^{\mu\nu} = \sigma \geq 0$$

Lagrange multipliers – non-equilibrium

Rest frame quantities:

$$S^\mu = s u^\mu + J^\mu$$

$$N^\mu = n u^\mu + j^\mu$$

$$T^{\mu\nu} = u^\mu E^\nu + q^\mu u^\nu + P^{\mu\nu}$$

$$\boxed{E^\nu = e u^\nu + q^\nu}$$

$$u_\mu u^\mu = 1,\ \Delta^{\mu\nu} = \delta^{\mu\nu} - u^\mu u^\nu;$$

$$u_\mu J^\mu = 0, u_\mu j^\mu = 0;$$

$$u_\mu q^\mu = 0, u_\mu P^{\mu\nu} = P^{\mu\nu} u_\nu = 0.$$

$$\partial_\mu S^\mu + \alpha \partial_\mu N_0^\mu - \beta_\nu \partial_\mu T_0^{\mu\nu} =$$

$$\boxed{\dot{s} + \alpha \dot{n} - \beta_\mu \dot{E}^\mu} + \left(s + \alpha n - \beta_\mu E^\mu\right) \partial_\nu u^\nu -$$

$$\alpha \partial_\mu j^\mu + \beta_\nu \partial_\mu (q^\mu u^\nu + P^{\mu\nu}) = 0$$

$$\dot{s} + \alpha\dot{n} - \beta_\mu \dot{E}^\mu + \left(s + \alpha n - \beta_\mu E^\mu\right)\partial_\nu u^\nu - j^\mu \partial_\mu \alpha + (q^\mu u^\nu + P^{\mu\nu})\partial_\mu \beta_\nu = 0$$

A) $\dot{s} + \alpha\dot{n} - \beta_\mu \dot{E}^\mu = 0$

$$s(n, E^\mu)$$

$$\frac{\partial s}{\partial n} = \alpha = \frac{\mu}{T}$$

deviation from Jüttner

$$\frac{\partial s}{\partial E^\mu} = \beta_\mu = \frac{g_\mu}{T} = \frac{u_\mu + w_\mu}{T} \qquad u^\mu w_\mu = 0$$

Velocity dependence?

$$Tds + \mu dn = \beta_\mu dE^\mu = (u_\mu + w_\mu) d(eu^\mu + q^\mu) = de + w_\mu dq^\mu + (ew_\mu - q_\mu) du^\mu$$

B) $\left(s + \alpha n - \beta_\mu E^\mu\right)\partial_\nu u^\nu - j^\mu \partial_\mu \alpha + (q^\mu u^\nu + P^{\mu\nu})\partial_\mu \beta_\nu = 0$

$$p = Ts + \mu n - \beta_\mu E^\mu = nT$$

ideal gas

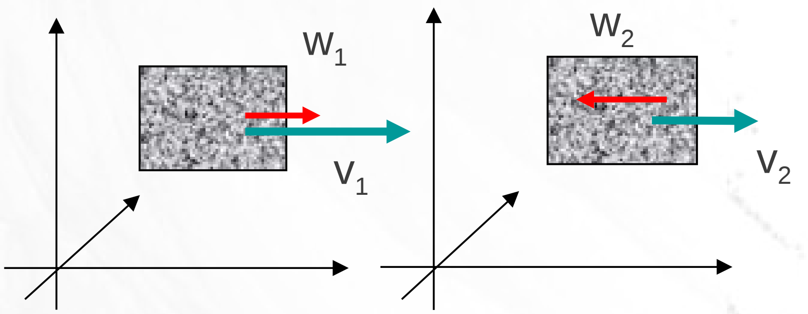
$$\nabla_\nu \mu = 0,\ \nabla_\nu T = 0,\ \nabla_\nu w_\mu = 0$$

rest frame/uniform intensives

$$TdS = (u_a + w_a)\, dE^a \;\Rightarrow\; \frac{(u_1^a + w_1^a)}{T_1} = \frac{(u_2^a + w_2^a)}{T_2}$$

1+1 dimensions:

$$u^a = (\gamma, \gamma v), \quad w^a = (\gamma v w, \gamma w)$$



$$\frac{\gamma_1 (1 + v_1 w_1)}{T_1} = \frac{\gamma_2 (1 + v_2 w_2)}{T_2}$$

$$\frac{\gamma_1 (v_1 + w_1)}{T_1} = \frac{\gamma_2 (v_2 + w_2)}{T_2}$$

$\Rightarrow$

$$\frac{v_1 + w_1}{1 + v_1 w_1} = \frac{v_2 + w_2}{1 + v_2 w_2}$$

$$\frac{\sqrt{1 - w_1^2}}{T_1} = \frac{\sqrt{1 - w_2^2}}{T_2}$$

Transformation of temperatures

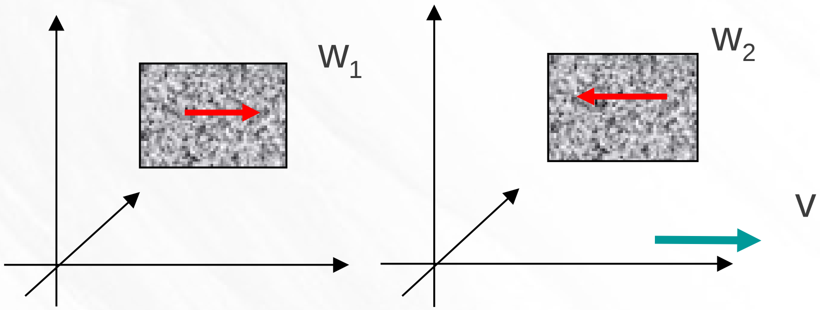
$$\frac{v_1+w_1}{1+v_1w_1}=\frac{v_2+w_2}{1+v_2w_2},\quad \frac{\sqrt{1-w_1^2}}{T_1}=\frac{\sqrt{1-w_2^2}}{T_2}$$

Four velocities: v_1, v_2, w_1, w_2

Relative velocity

(Lorentz transformation)

$$v=\frac{v_2-v_1}{1-v_1v_2}$$

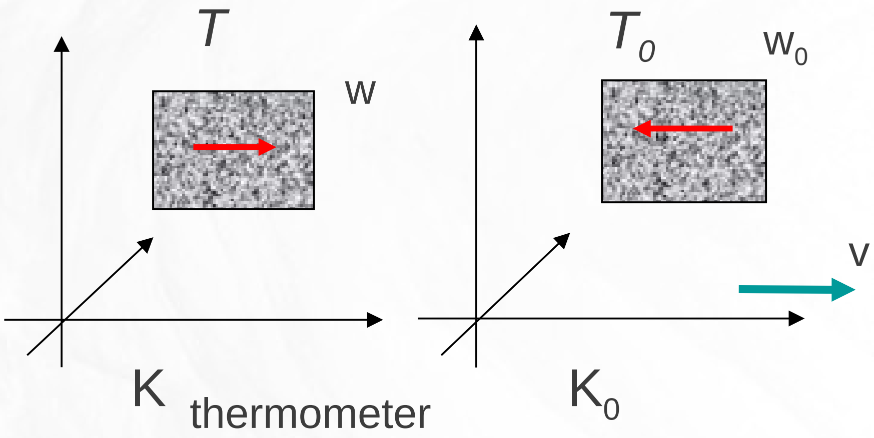


$$w_1=\frac{v+w_2}{1+vw_2}$$

$$\frac{T_1}{T_2}=\frac{\sqrt{1-v^2}}{1+vw_2}$$

general Doppler-like form!

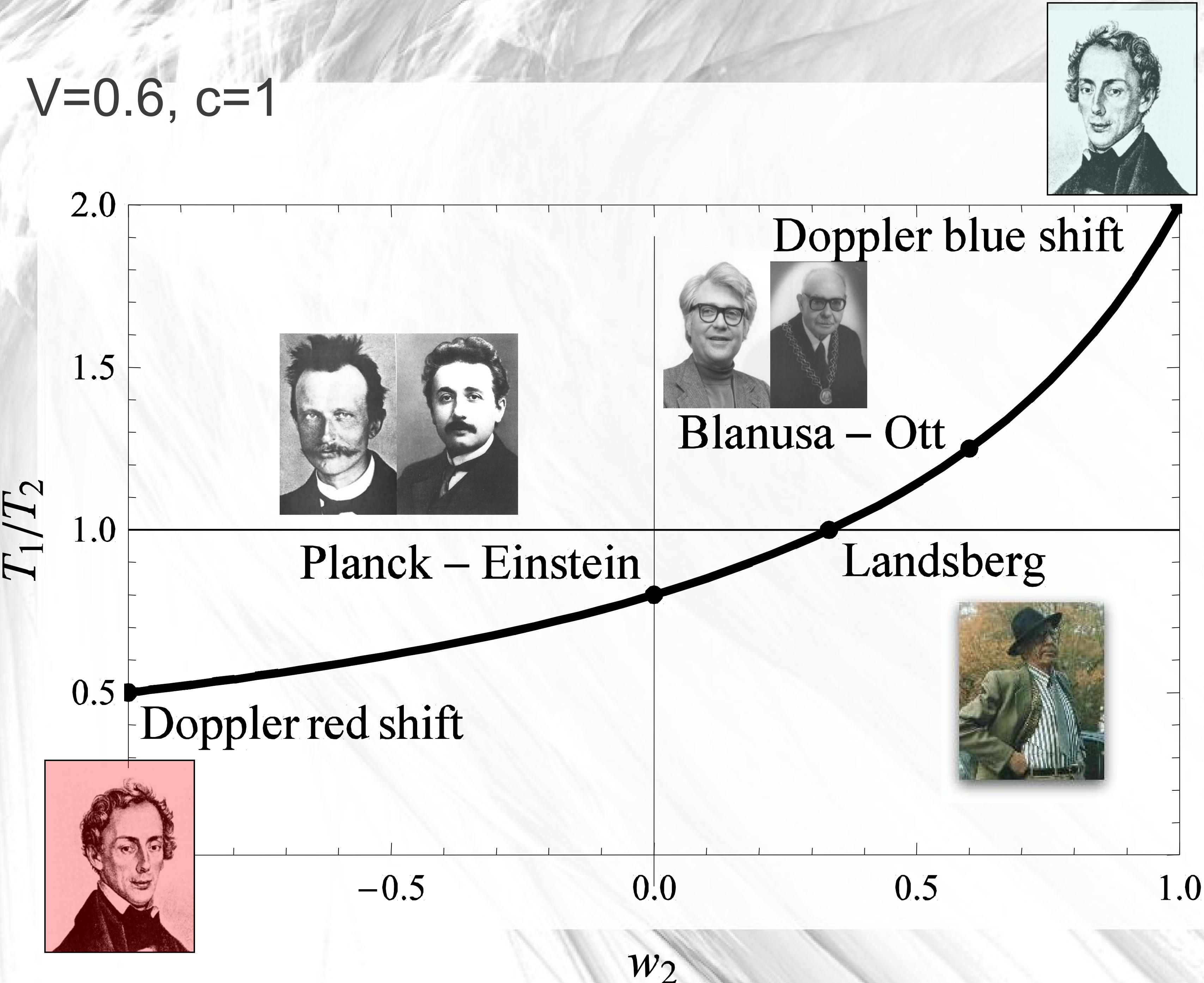
$$\frac{T}{T_0} = \frac{\sqrt{1-v^2}}{1+vw_0}$$



Special:

$w_0 = 0$	$T = T_0/\gamma$	*Planck-Einstein*
$w = 0$	$T = \gamma\, T_0$	*Ott*
$w_0 = 1,\ v > 0$	$T = T_0 \bullet$ *red*	*Doppler*
$w_0 = 1,\ v < 0$	$T = T_0 \bullet$ *blue*	*Doppler*
$w_0 + w = 0$	$T = T_0$	*Landsberg*

V=0.6, c=1



2.0
1.5
1.0
0.5
T_1/T_2
Doppler blue shift
Blanusa – Ott
Planck – Einstein
Landsberg
Doppler red shift
−0.5
0.0
0.5
1.0
w_2

Universal?

Simple, sound and exact formulation.

THE Second Law

Thermodynamic equilibrium is stable.

This is a discipline related exact statement.

- There is entropy: statics.
- (It is concave: statics.)
- Entropy is not decreasing: dynamics.

? Stability

Necessary condition: generic stability – linear stability of homogeneous thermodynamic equilibrium..

Objectivity is easier!

V.P. Thermodynamics of continua: the challenge of universality, in Proc. Of JETC13, arXiv: 1305.3582

Dissipative hydrodynamics

$$\partial_a N^a = \dot{n} + n\partial_a u^a + \partial_a j^a = 0,$$

$$u^a \partial_b T^{ab} = \dot{e} + (e+p)\partial_a u^a + \partial_a q^a + q^a \dot{u}_a - \Pi^{ab}\partial_b u_a = 0,$$

$$\Delta^a_c \partial_b T^{cb} = (e+p)\dot{u}^a + q^a \partial_b u^b + q^b \partial_b u^a + \Delta^a_c(\dot{q}^c + \partial_b \Pi^{cb}) = 0^a,$$

$$q^a = -\lambda \Delta^{ac}\left(\partial_c T + T\dot{u}_c + T\frac{\dot{q}_c}{e}\right),$$

$$\nu^a = -\varsigma \Delta^{ac}\partial_c \frac{\mu}{T},$$

$$\Pi^a_a = P^a_a - p = -\xi \partial_c u^c,$$

$$\Pi^a_b = -2\eta < \partial_b u^a > .$$

< > symmetric, traceless, spacelike

⇒ Generic stability.

CONDITION: thermodynamic stability

(Ván: J. Stat. Mech. (2009), 02054)

Fluid families:

	Nonrelativistic	Relativistic
Local equilibrium (1st order)	Fourier+Navier-Stokes	~~Eckart~~ (1940), Tsumura-Kunihiro (2008)
Beyond local equilibrium (2^{nd} order)	Cattaneo-Vernotte, generalized Navier-Stokes, rheology, etc…	Israel-Stewart (1969-72), Pavón-Jou-Casas-V. (1982), Liu-Müller-Ruggieri (1982), Geroch, Öttinger, Carter,… conformal (2007-08), our (2008), Betz et. al. (2009)

Eckart – Israel–Stewart – Pavón–Jou–Casas-Vázquez:

$$S^a(T^{ab}, N^a) = \left(s(e, n) - \frac{\beta_0}{2T}\Pi^2 - \frac{\beta_1}{2T} q_b q^b - \frac{\beta_2}{2T}\pi^{bc}\pi_{bc} \right) u^a + \frac{1}{T}\left(q^a + \alpha_0 \Pi q^a + \alpha_1 \pi^{ab} q_b \right)$$

(+ order estimates)