Multipole solutions in relativistic hydrodynamics
& higher order flow coefficients

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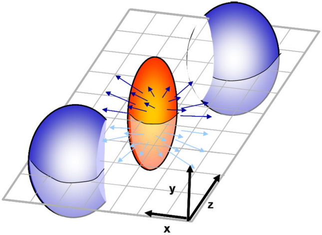
Introduction

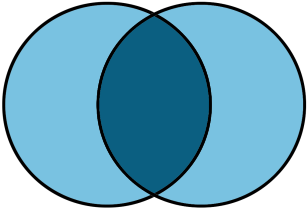
- High energy collisions → Quark-Gluon Plasma
- Present knowledge: nearly perfect fluid, we can use relativistic hydro
- Existing solutions → use naive geometrical initial conditions

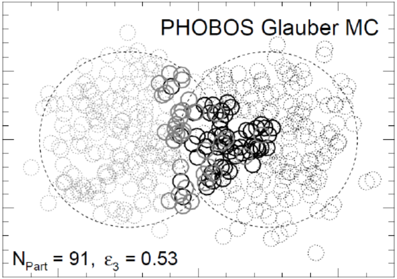


- v_2 measures ellipticity in the momentum distribution

Higher order asymmetry coefficients

- Basic picture: elliptical region → v_2 is the most important
- Finite number of nucleons: initial conditions fluctuate event-by-event
- Observation of individual collisions → higher order flow coefficients arise
- How to handle this?
  - We can implement initial conditions numerically
  - Analytically?





Ellipsoidal solutions

- Based on Csörgő, Csernai et al., Heavy Ion Phys. A21 (2004)
- Self-similarity handled by scaling variable

$$s = \frac{r_x^2}{X(t)^2} + \frac{r_y^2}{Y(t)^2} + \frac{r_z^2}{Z(t)^2}$$
$$u^\mu = \gamma\left(1, \frac{\dot{X}}{X}r_x, \frac{\dot{Y}}{Y}r_y, \frac{\dot{Z}}{Z}r_z\right)$$

- "Good" scaling variables satisfy:

$$u^\mu \partial_\mu s = 0$$

- Most general s:

$$\overline{s} = F(s_x, s_y, s_z)$$

- $s_i = \frac{r_i^2}{t^2}$ or $s_i = \frac{r_i^2}{\tau^2} \rightarrow X(t) = \dot{X}_0 t \rightarrow \dot{X} = \mathrm{const.}$
- Thus we define a group of solutions

Non-elliptical solutions

We can have a special form in the transverse plane:

- Ellipsis in polar coordinates:

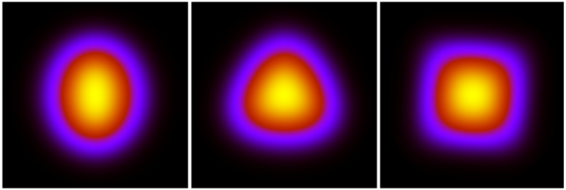
$$\frac{r_x^2}{X(t)^2} + \frac{r_y^2}{Y(t)^2} = \frac{r^2}{R(t)^2}(1 + \epsilon_2 \cos(2\phi))$$

- Modify this to have N-fold symmetry:

$$s = \frac{r^N}{R(t)^N}(1 + \epsilon_N \cos(N\phi))$$

- Even supercomposing of these satisfies hydro equations:

$$s = \sum_N \frac{r^N}{R(t)^N}(1 + \epsilon_N \cos(N\phi))$$



- Still: $(X, Y, R) \sim t$

- One can expand the N-fold solution to the z axis:

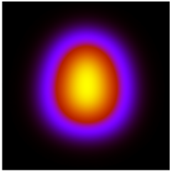
$$s = \sum_N \frac{r^N}{R(t)^N}(1+\epsilon_N\cos(N\phi)) + \frac{z^N}{R^N}$$

- We can be more general:

$$s = \frac{r^K}{R(t)^K}(1+\sum_N \epsilon_N\cos(N\phi)) + \frac{z^M}{R^M}$$

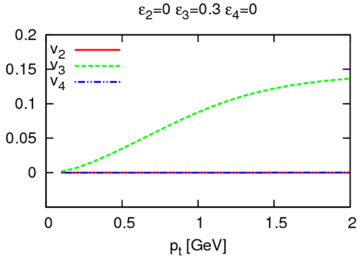
- These are special cases of $\bar{s} = F(s_x, s_y, s_z)$.

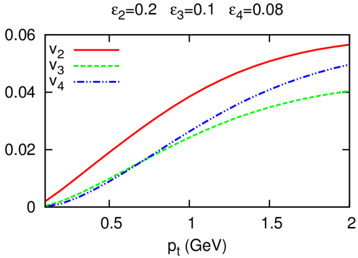
- Phase shifts define Nth order reaction planes
- Works even with $R(t) = R_0 + u_t t \rightarrow$ New group of solutions!



$\epsilon_2 = 0.6 \quad \epsilon_3 = 0.2 \quad \epsilon_4 = 0.1$

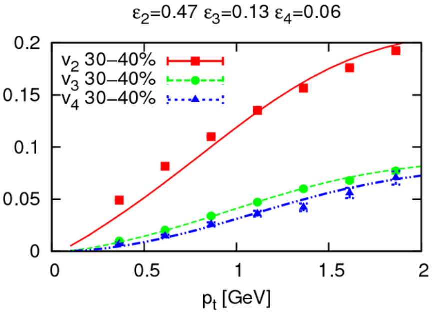
- Self-check: no odd $\epsilon \rightarrow$ no v_{2n+1}
- Azimuthally integrated observables (eg. spectra) are the same
    - We can use earlier results here
- Relative orientation of reaction planes makes no difference
- Higher order coefficients appear naturally





Comparison to data

- PHENIX data of higher order flow coefficients in several centrality bins: Phys. Rev. Lett. 107,252301 (2011)
- Tune ϵ_N asymmetry parameters to data:



Other parameters: Csanád, Vargyas, Eur.Phys.J. A44 (2010)

- Detailed investigation on v_n coefficients and centrality dependence is to follow

Summary

- Higher order coefficients measure the departure from elliptical symmetry
- $v_3, v_4, \ldots$ arise in event-by-event observations
- Manageable numerically
- First analytic hydro solutions to handle initial conditions with superimposed symmetries
- Higher order flow coefficients become calculable with realistic initial geometry