Equilibration in Heavy Ion Collisions

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- Motivation
- Transport Equations
- Timescales of Equlibrations
- Summary

Equilibration?

Energy range: few GeV bombarding energies (SIS)

- The degrees of freedom are known
- Transport models give good description of the data
- thermal models?
  (J. Cleymans, H. Oeschler, K. Redlich, J.Phys.G25:281-285,1999)
  particle ratios are good, except for η, for strangeness an extra, somewhat artificial parameter
- No global equlibrium at 0.4 AGeV central $Ru_{44}^{96} + Zr_{40}^{96}$ collisions
  FOPI, Phys.Rev.Lett. 84 (2000) 1120

Spectral equilibration

- medium effects on the spectrum of hadrons (vector mesons)
- how they get on-shell (energy-momentum conservation)
- Field theoretical method
  B. Schenke, C. Greiner, Phys.Rev.C73:034909,2006
- Off-shell transport
  W. Cassing, S. Juchem, Nucl.Phys. A672 (2000) 417
  S. Leupold, Nucl.Phys. A672 (2000) 475
- Spectral equilibration: Markov or memory effect

BUU

- Boltzmann-Ühling-Uhlenbeck equation

$$\frac{\partial F}{\partial t} + \frac{\partial H}{\partial \mathbf{p}}\frac{\partial F}{\partial \mathbf{x}} - \frac{\partial H}{\partial \mathbf{x}}\frac{\partial F}{\partial \mathbf{p}} = \mathcal{C}, \quad H = \sqrt{(m_0 + U(\mathbf{p},\mathbf{x}))^2 + \mathbf{p}^2}$$

- potential: momentum dependent, soft: K=215 MeV

  $U^{nr} = A\frac{n}{n_0} + B\left(\frac{n}{n_0}\right)^{\tau} + C\frac{2}{n_0}\int\frac{d^3p'}{(2\pi)^3}\frac{f_N(x,p')}{1+\left(\frac{\mathbf{p}-\mathbf{p}'}{\Lambda}\right)^2},$

  Teis et al., Z. Phys. 1997

- testparticle method

  $F = \sum_{i=1}^{N_{test}} \delta^{(3)}(\mathbf{x}-\mathbf{x}_i(t))\delta^{(4)}(p-p_i(t)).$

Collision term

- NN $\leftrightarrow$ NR, NN $\leftrightarrow$ $\Delta\Delta$
- baryon resonance can decay via 9 channels
  R $\leftrightarrow$ Nπ, Nη, Nσ, Nρ, Nω, $\Delta\pi$, N(1440)π, KΛ, KΣ
- 24 baryon resonances + Λ and Σ baryons
  $\pi, \eta, \sigma, \rho, \omega$ and kaons
- $\pi\pi \leftrightarrow \rho$, $\pi\pi \leftrightarrow \sigma$, $\pi\rho \leftrightarrow \omega$
- for resonances: energy dependent with
- $\frac{d\sigma^{X\to NR}}{dM_R} \sim A(M_R)\lambda^{0.5}(s, M_R^2, M_N^2)$

Off-shell transport

- Kadanoff-Baym equation for retarded Green-function
  Wigner-transformation, gradient expansion
- transport equation for $F_\alpha = f_\alpha(x,p,t)A_\alpha$
  $A(p) = -2ImG^{ret} = \frac{\hat{\Gamma}}{(E^2-\mathbf{p}^2-m_0^2-\mathrm{Re}\Sigma^{ret})^2+\frac{1}{4}\hat{\Gamma}^2}$,

  W. Cassing, S. Juchem, Nucl.Phys. A672 (2000) 417
  S. Leupold, Nucl.Phys. A672 (2000) 475
- testparticle approximation

Transport equations

- $$\frac{d\vec{X}_i}{dt} = \frac{1}{1-C_{(i)}} \frac{1}{2\epsilon_i} \left[2\,\vec{P}_i + \vec{\nabla}_{P_i} Re\Sigma^{ret}_{(i)} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \vec{\nabla}_{P_i} \Gamma_{(i)} \right]$$

  $$\frac{d\vec{P}_i}{dt} = -\frac{1}{1-C_{(i)}} \frac{1}{2\epsilon_i} \left[\vec{\nabla}_{X_i} Re\Sigma^{ret}_i + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \vec{\nabla}_{X_i} \Gamma_{(i)} \right]$$

  $$\frac{d\epsilon_i}{dt} = \frac{1}{1-C_{(i)}} \frac{1}{2\epsilon_i} \left[\frac{\partial Re\Sigma^{ret}_{(i)}}{\partial t} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \frac{\partial \Gamma_{(i)}}{\partial t} \right]$$

- where $C_{(i)}$ renormalization factor

  $$C_{(i)} = \frac{1}{2\epsilon_i} \left[\frac{\partial}{\partial \epsilon_i} Re\Sigma^{ret}_{(i)} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \frac{\partial}{\partial \epsilon_i} \Gamma_{(i)} \right]$$

  dangerous, $C_{(i)}$ can be 1

  if $C_{(i)} > 0.5$ we use $\frac{1}{1-C_{(i)}} = 1.33(1 + C_{(i)})$

  However $C_{(i)} = 0$ do not change the results substantially

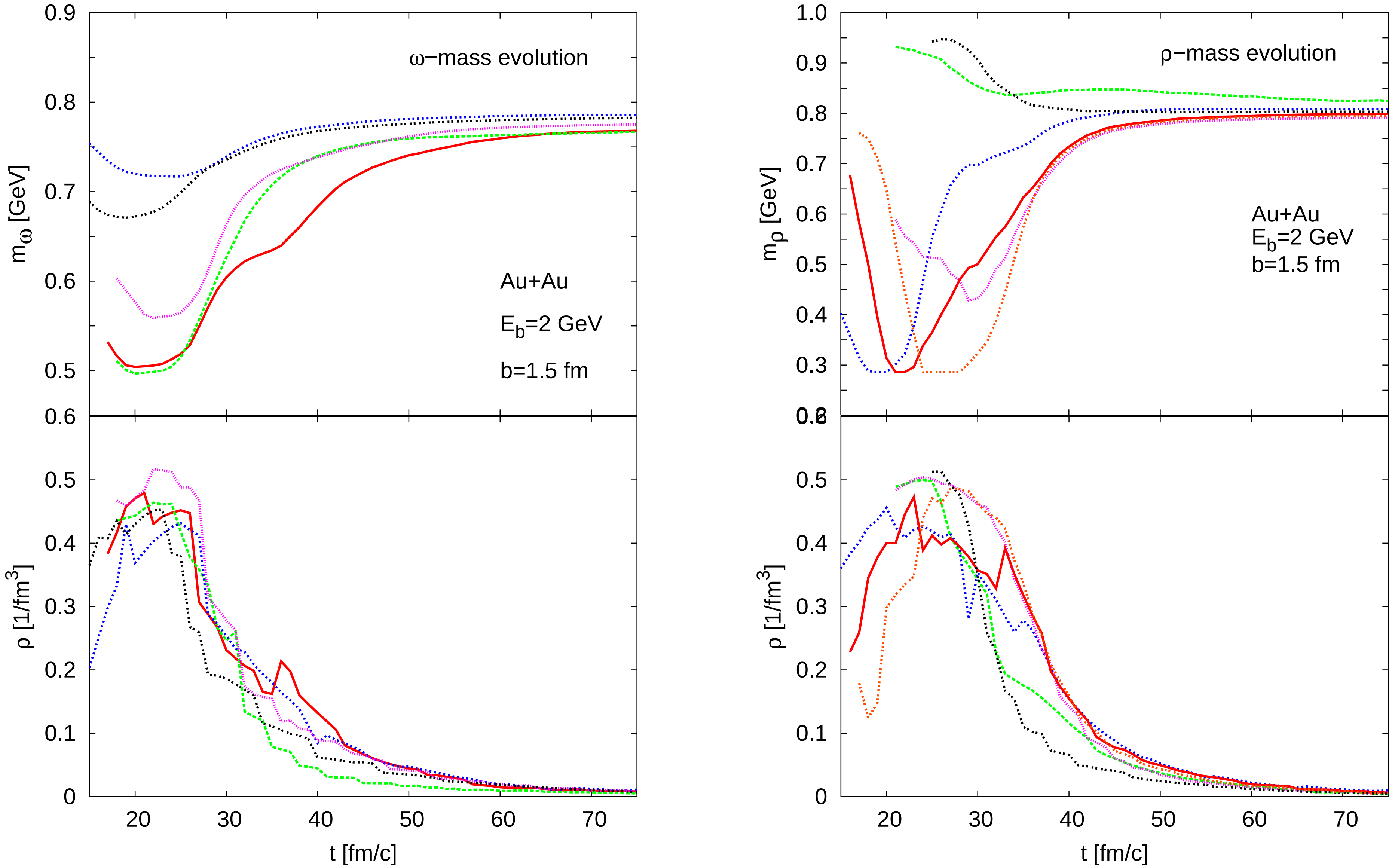
- the last equation can be rewritten as

  $$\frac{dM_i^2}{dt} = \frac{d(\epsilon_i^2 - P_i^2)}{dt} = \frac{dRe\Sigma^{ret}_{(i)}}{dt} + \frac{M_i^2 - M_0^2}{\Gamma_{(i)}} \frac{d\Gamma_{(i)}}{dt}$$

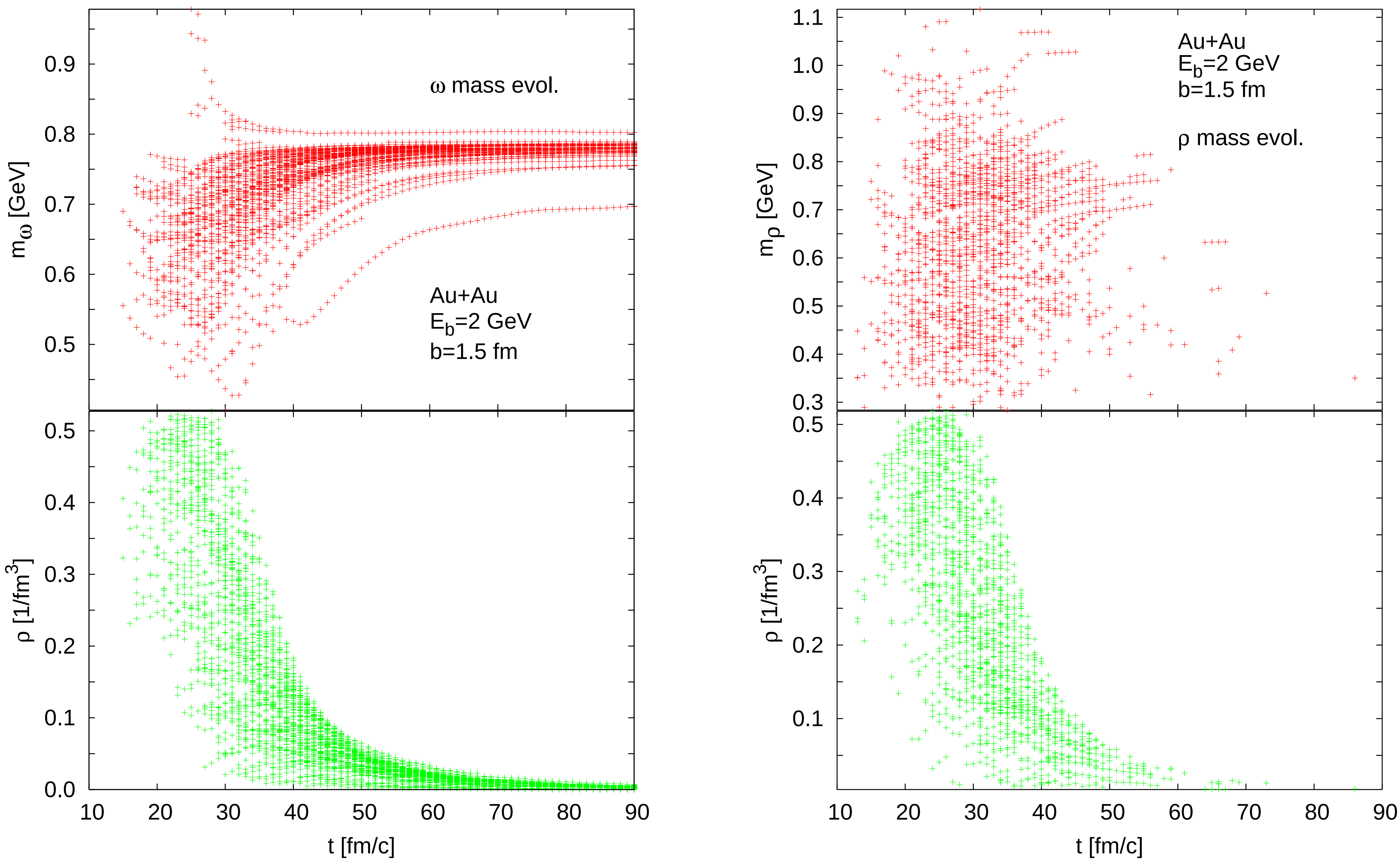
Medium effects

- imaginary part (collisional broadening):
  $\Gamma = \Gamma_{vac} + n v \sigma \gamma$
- real part (mass shift)
  $M = M_{vac} + n/n_o \Delta M$
  $\Delta M_\omega = -50$ MeV, $\Delta M_\rho = -120$ MeV
- danger of double counting
  collision term already contains partly the mixing of mesons with
  resonance-hole excitations
  but sum up only to finite order

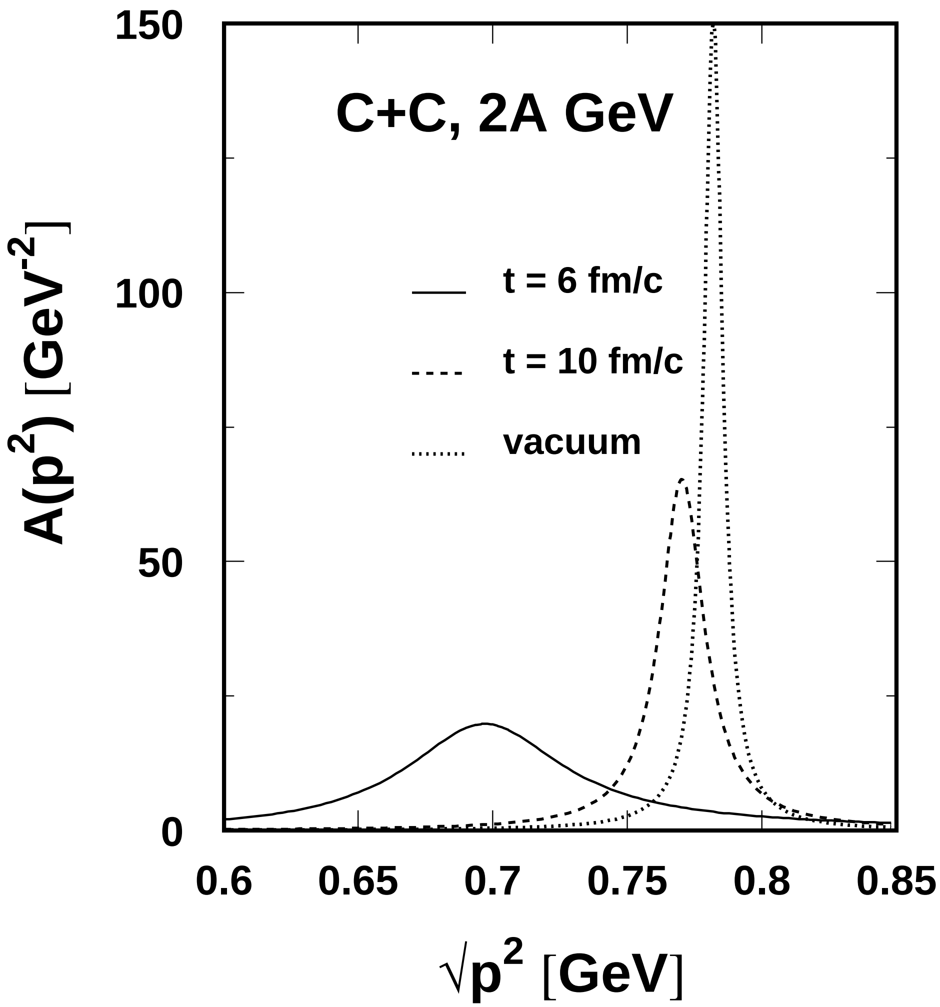
Evolution of masses



Evolution of masses



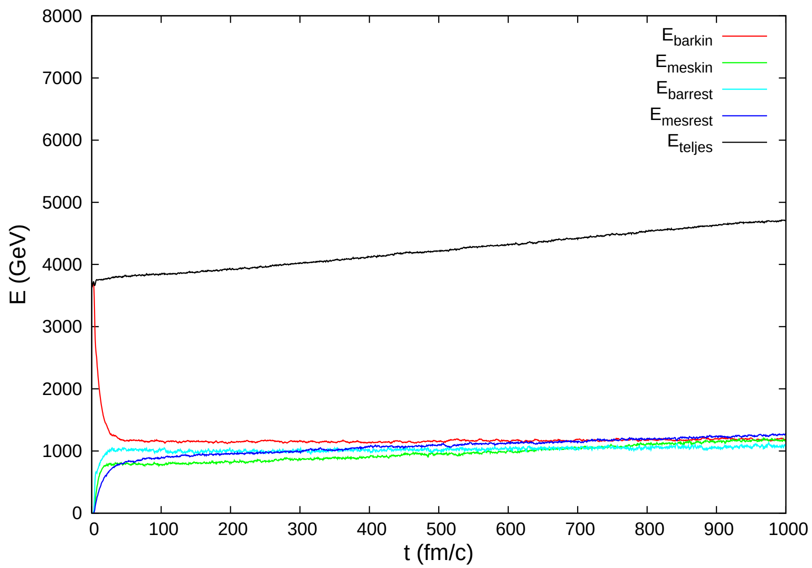
Evolution of the ω spectrum



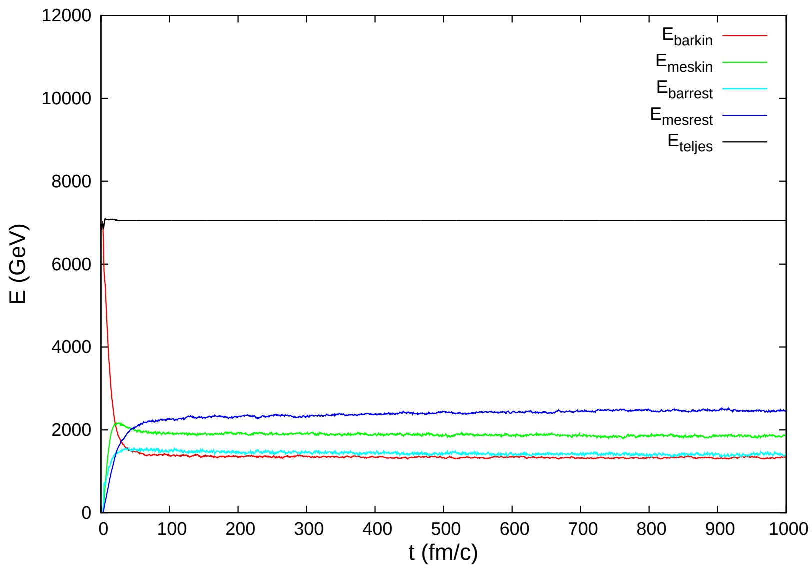
Box

- Box size= 5 fm
- periodicity: Since particles represented by Gaussians, particles out of the box should also be considered by the calculation of the density and potential.
- Intialization by a symmetric collision (temperature and density can be fixed by the colliding energy and masses)
- temperature, chemical potential: compare expectation values of a free resonance gas:
  at a given density, temperature from pion number, average kinetic energy of nucleons ...

Stability



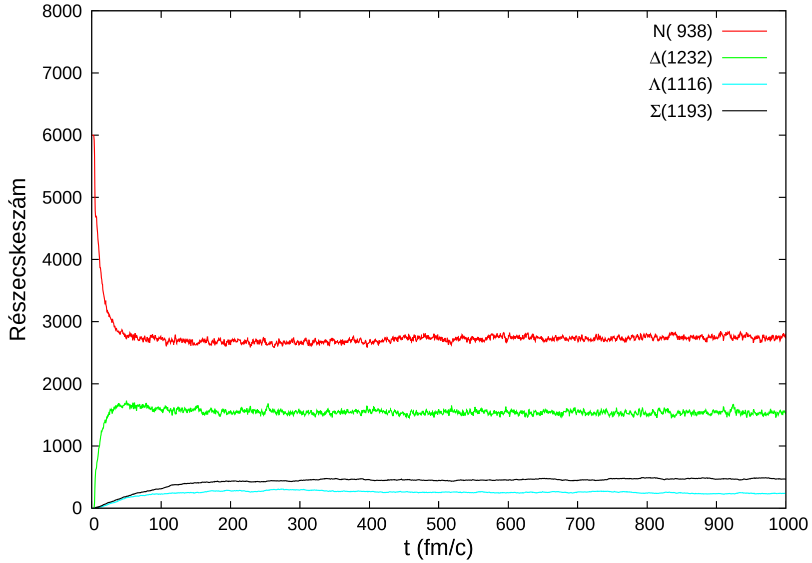
because of the numerical inaccuracy, the energy is not conserved exactly



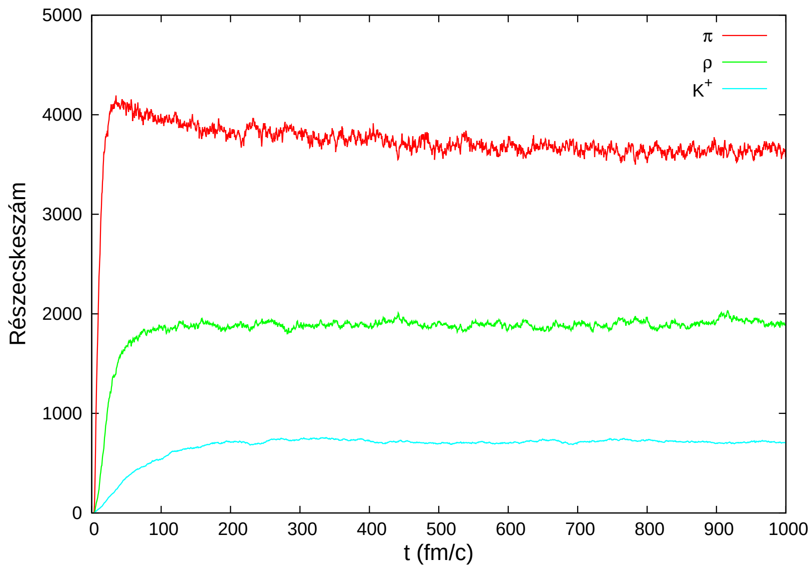
Shift the energy of all baryons by the same amount to restore energy conservation

Chemical equilibration

7 AGeV C+C collision



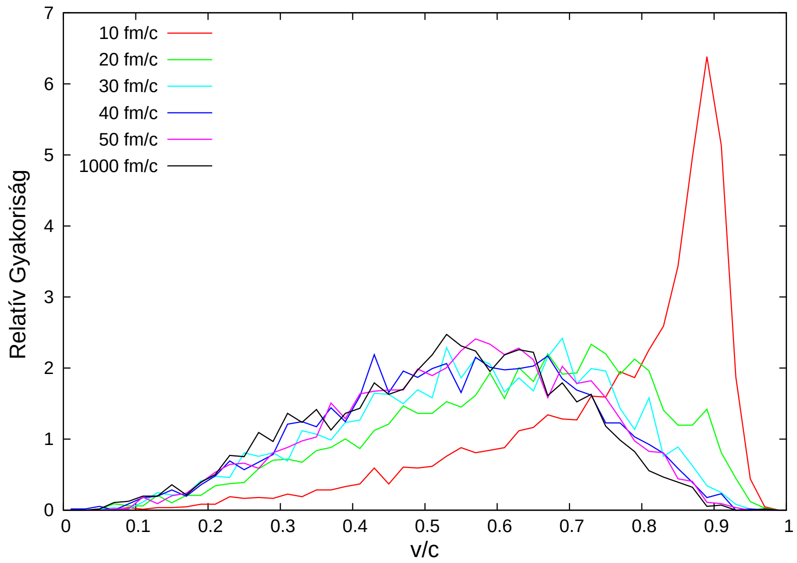
baryons



mesons

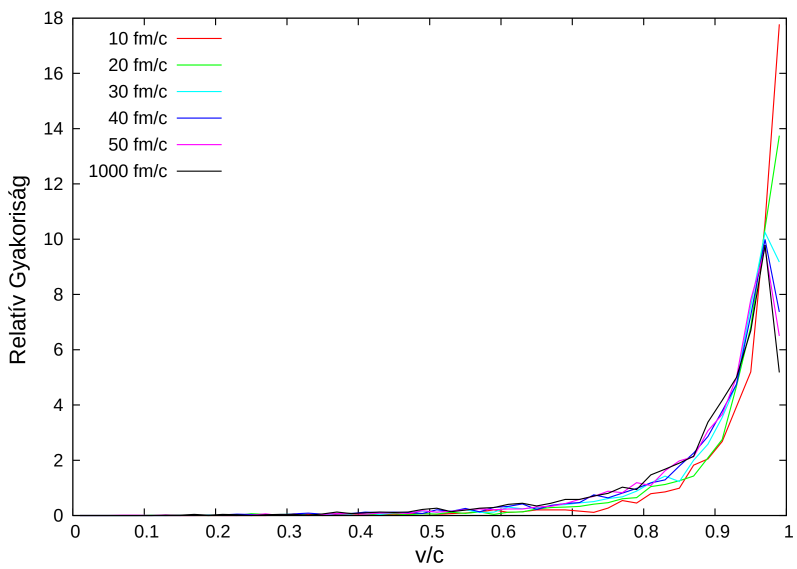
non strange equilibration in 10-20 fm/c
strangness equilibrate in 200-300 fm/c

Thermal equilibration



7 AGeV C+C collision

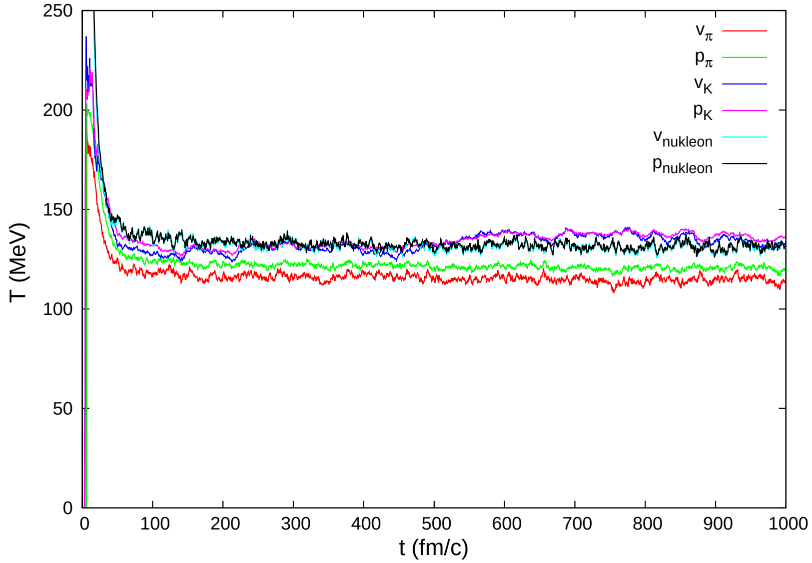
nucleons



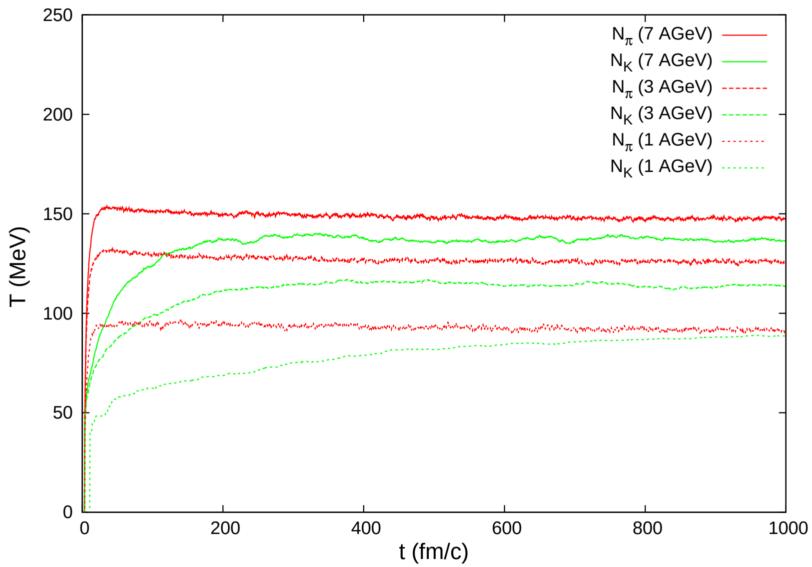
pions

equilibration in 10-20 fm/c

Temperatures



7 AGeV C+C collision



pion, kaon temperatures at 1-7 AGeV

equilibration in 10-20 fm/c

Evolution of mass distribution

Let us study the Cassing-Juchem-Leupold equations in homogenous system, where the selfenergies depend only on density

$$\frac{d}{dt}M_i(t)^2 = \left(\frac{d\mathrm{Re}\Sigma_\mathrm{i}}{dt} + \frac{M_i^2(t) - M_0^2 - \mathrm{Re}\Sigma_\mathrm{i}}{\mathrm{Im}\Sigma_\mathrm{i}}\frac{d\mathrm{Im}\Sigma_\mathrm{i}}{dt}\right).$$

$$\frac{d(M_i^2(t) - M_0^2 - \mathrm{Re}\Sigma_\mathrm{i})}{dt} = \frac{M_i^2 - M_0^2 - \mathrm{Re}\Sigma_\mathrm{i}}{\mathrm{Im}\Sigma_\mathrm{i}}\frac{d\mathrm{Im}\Sigma_\mathrm{i}}{dt}.$$

We have a testparticle with mass of the resonance peak:

$$M_i^2(0) = M_0^2 + \mathrm{Re}\Sigma_\mathrm{i}(0)$$

then

$$M_i^2(t) = M_0^2 + \mathrm{Re}\Sigma_\mathrm{i}(\mathrm{t})$$

no memory effect, the peak position of the mass distribution is always at the peak of the spectral function

If $\mathrm{Im}\Sigma_\mathrm{i}(\mathrm{t})$ monotonous, it can be inverted and

$$\frac{d(M_i^2(t) - M_0^2 - \mathrm{Re}\Sigma_\mathrm{i})}{d(\mathrm{Im}\Sigma_\mathrm{i})} = \frac{M_i^2 - M_0^2 - \mathrm{Re}\Sigma_\mathrm{i}}{\mathrm{Im}\Sigma_\mathrm{i}}.$$

$$\frac{M_i^2(t) - M_0^2 - \mathrm{Re}\Sigma_\mathrm{i}(\mathrm{t})}{M_i^2(0) - M_0^2 - \mathrm{Re}\Sigma_\mathrm{i}(0)} = \frac{\mathrm{Im}\Sigma_\mathrm{i}(\mathrm{t})}{\mathrm{Im}\Sigma_\mathrm{i}(0)}.$$

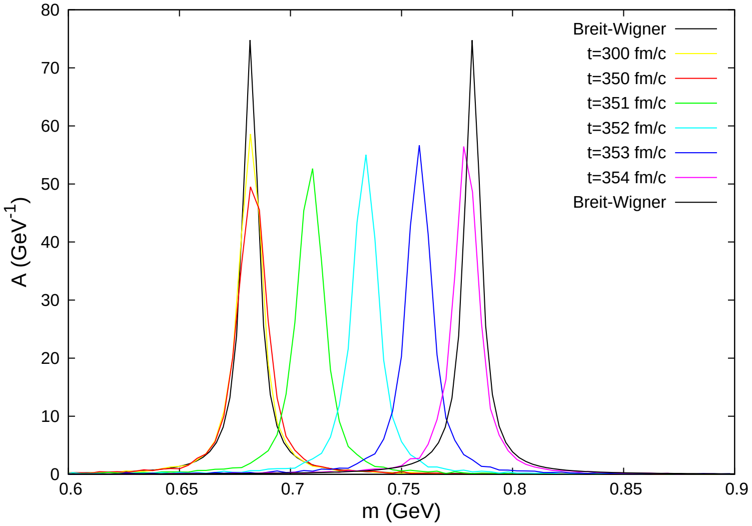
The difference of the testparticle mass from the peak scales with the imaginary part of the selfenergy.

$$\begin{aligned} M_i^2(t) &= M_i^2(0) + (\mathrm{Re}\Sigma_\mathrm{i}(\mathrm{t}) - \mathrm{Re}\Sigma_\mathrm{i}(0)) + \\ &\quad + \frac{\mathrm{Im}\Sigma_\mathrm{i}(\mathrm{t}) - \mathrm{Im}\Sigma_\mathrm{i}(0)}{\mathrm{Im}\Sigma_\mathrm{i}(0)} \left(M_i^2(0) - M_0^2 - \mathrm{Re}\Sigma_\mathrm{i}(0)\right). \end{aligned}$$

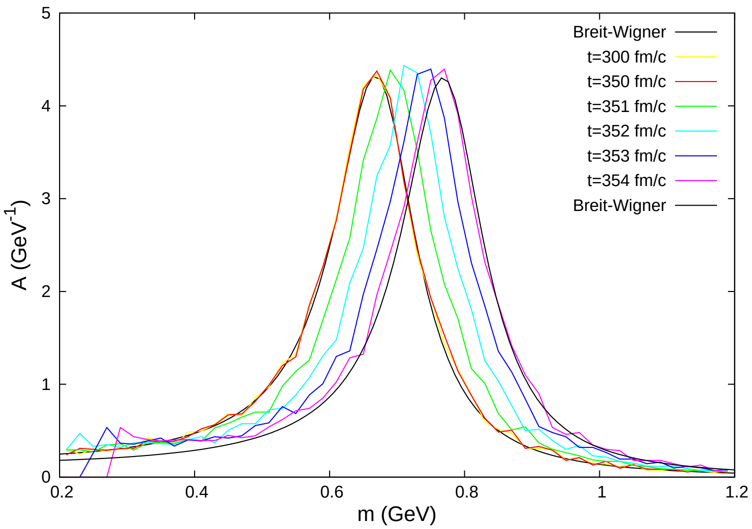
In case of decreasing imaginary part of the selfenergy with time, the testparticle mass gets closer to the peak position.

Evolution of mass distribution in a simulation

the vector meson masses are shifted linearly with density, and change the density linearly from ρ_0 to 0 in 4 fm/c:



ω



ρ

Summary

- BUU with off-shell propagation
- The thermal equilibration can happen during the heavy ion collision
- The chemical equilibration may also happen with the exception of strange particles
- There is no memory effect on the spectrum, the mass distribution follows the spectral one. The off-shell transport equations may not be good for the reason they were derived.

Cross sections

Elastic baryon-baryon cross section is fitted to the elastic pp data

Meson absorption cross sections are given by

$$\sigma_{\pi N \to R} = \frac{4\pi}{p^2}(spin factors)\frac{\Gamma_{in}\Gamma_{tot}}{(s - m_R^2) + s\Gamma_{tot}^2}$$

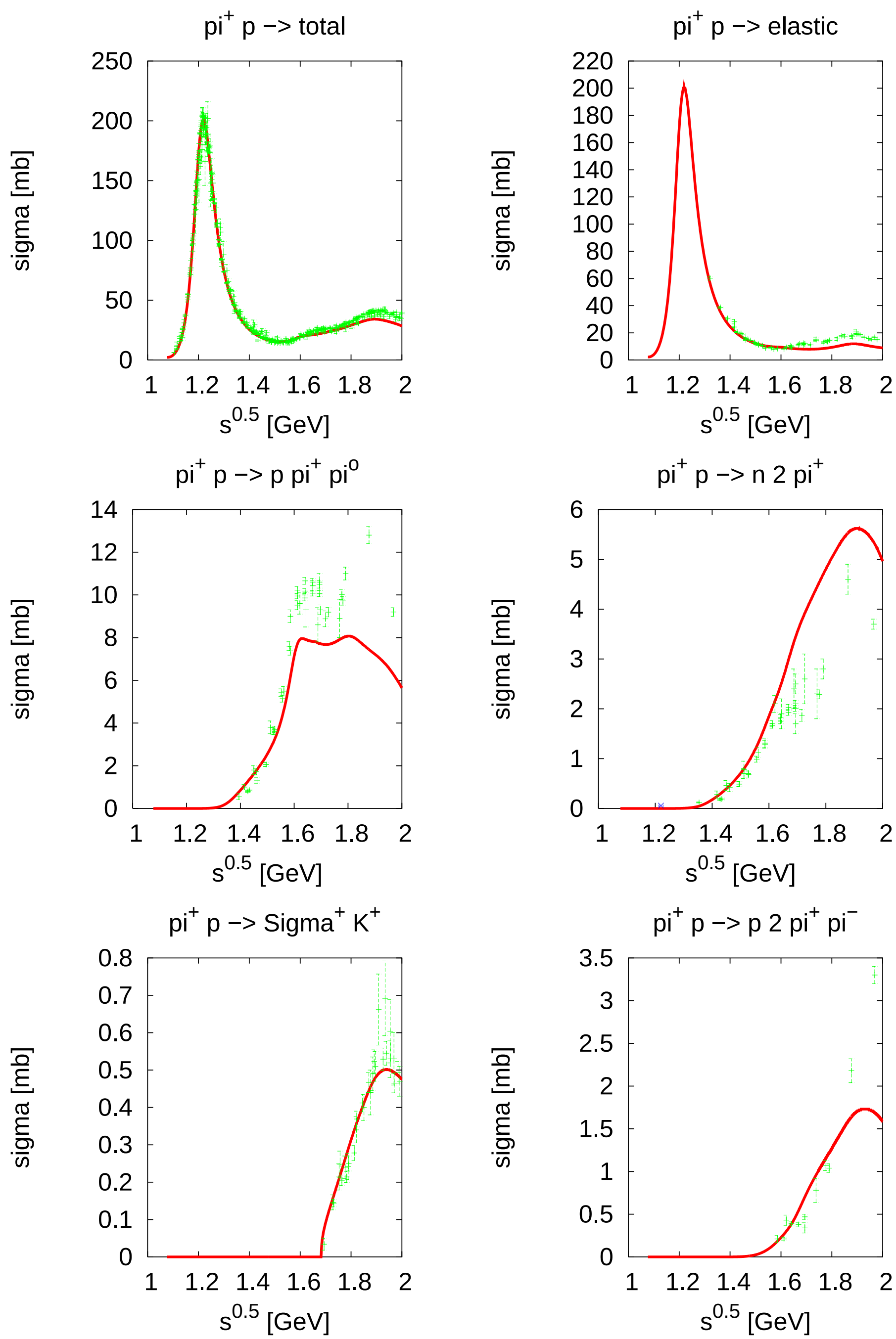
Baryon resonance parameters: mass, width, branching ratios are fitted by describing the meson production channels in πN collisions:

$$\sigma_{\pi N \to NM} = \sum_R \sigma_{\pi N \to R}\frac{\Gamma_{R \to NM}}{\Gamma_{tot}}$$

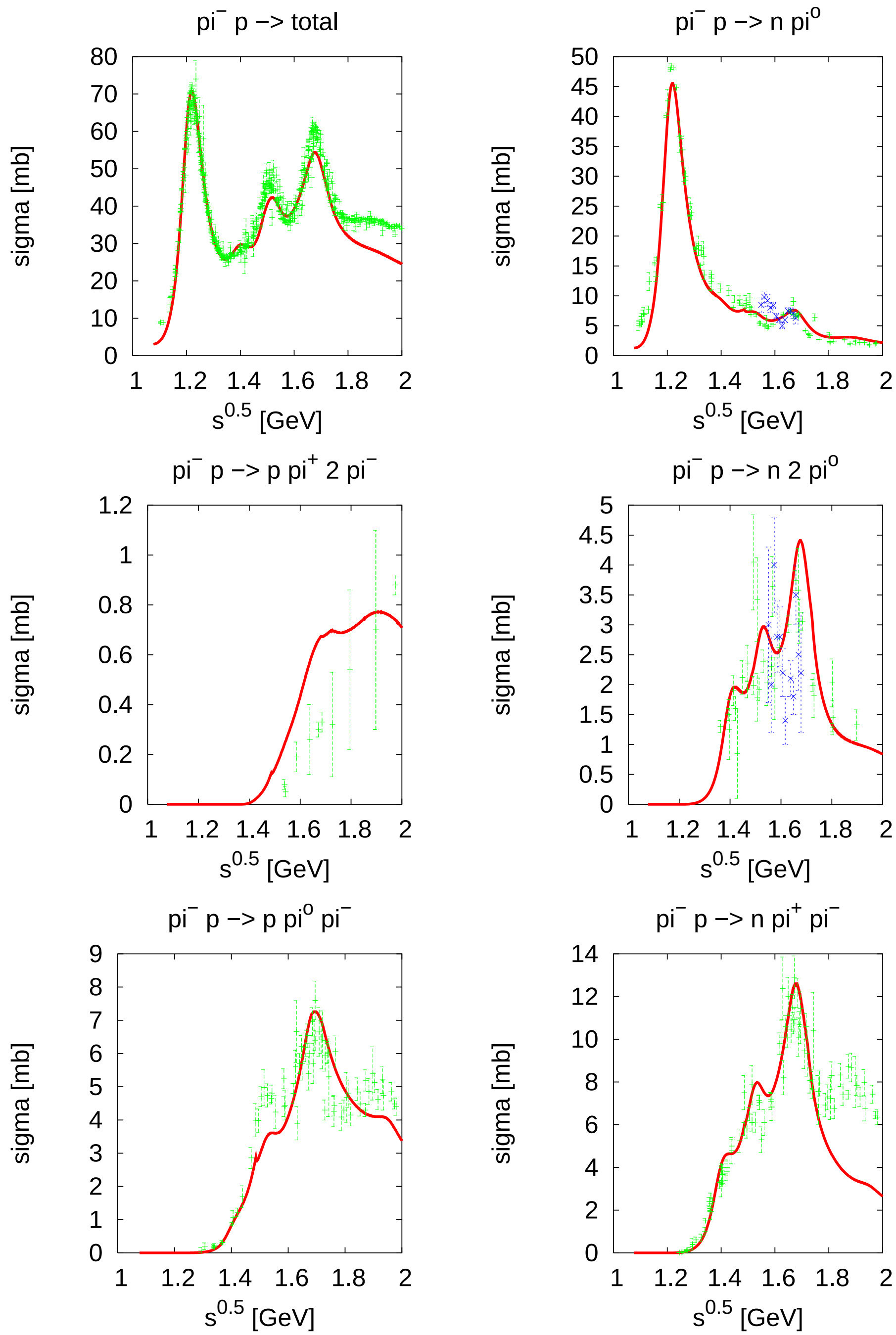
Resonance production cross section $NN \to NR$ is given by the fit of

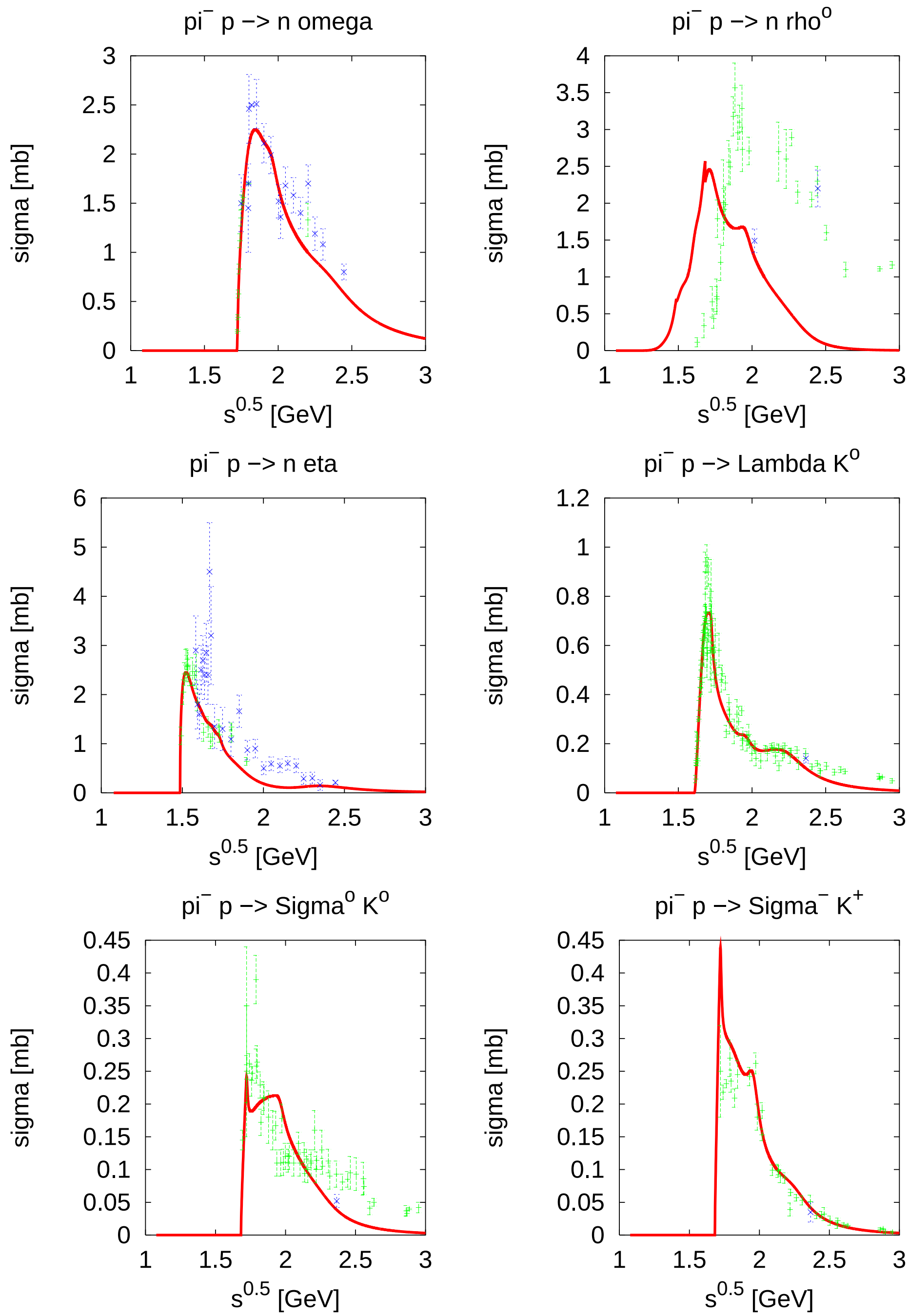
$$\sigma_{NN \to NM} = \sum_R \sigma_{NN \to NR}\frac{\Gamma_{R \to NM}}{\Gamma_{tot}}$$

27 baryons, 6 mesons. Fit is done by the Minuit package (CERN)

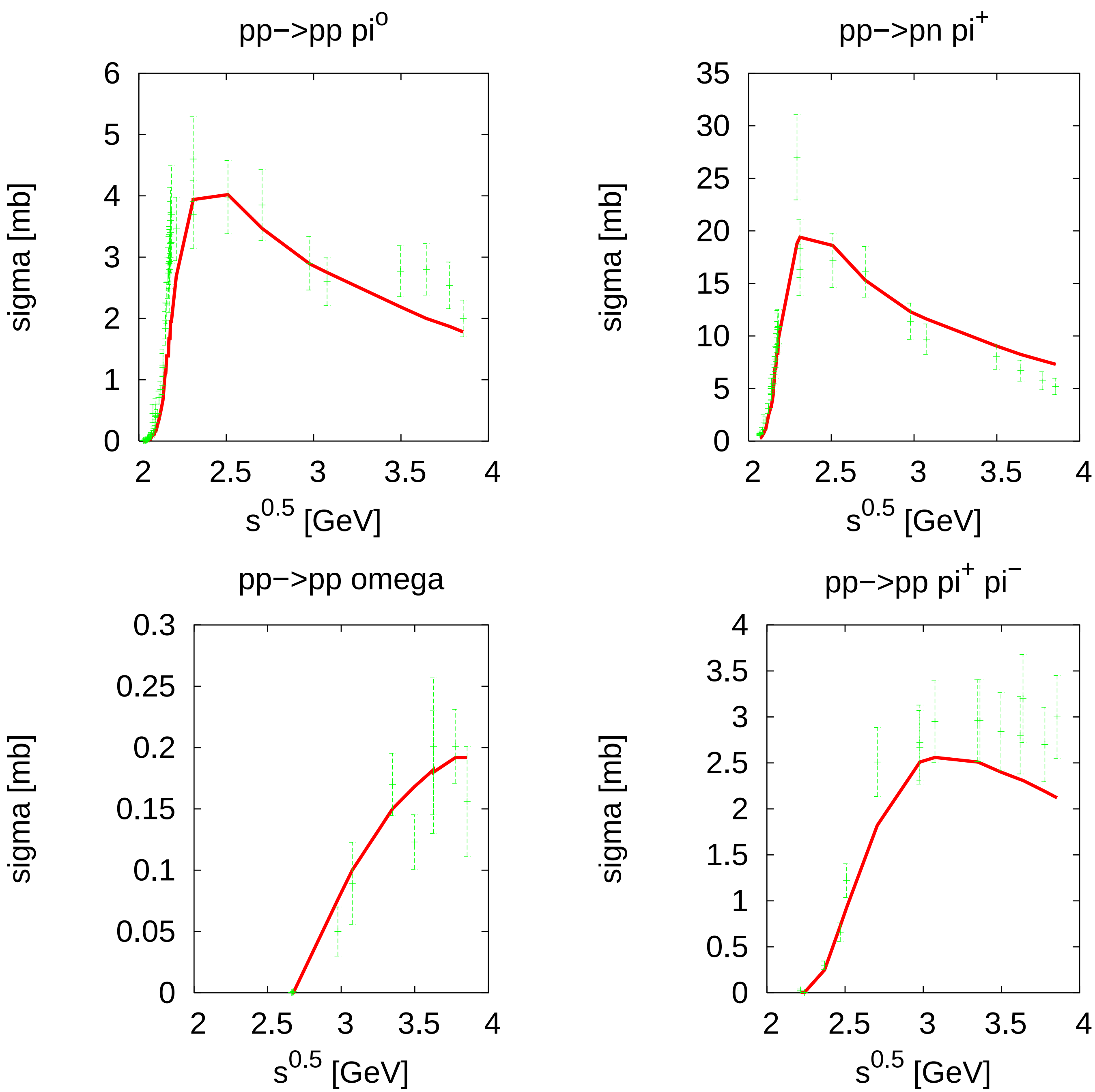


pi+ p -> total
250
200
150
100
50
0
1
1.2
1.4
1.6
1.8
2
sigma [mb]
s0.5 [GeV]
pi+ p -> elastic
220
200
180
160
140
120
100
80
60
40
20
0
sigma [mb]
s0.5 [GeV]
pi+ p -> p pi+ pio
14
12
10
8
6
4
2
0
sigma [mb]
s0.5 [GeV]
pi+ p -> n 2 pi+
6
5
4
3
2
1
0
sigma [mb]
s0.5 [GeV]
pi+ p -> Sigma+ K+
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
sigma [mb]
s0.5 [GeV]
pi+ p -> p 2 pi+ pi−
3.5
3
2.5
2
1.5
1
0.5
0
sigma [mb]
s0.5 [GeV]


pi$^-$ p –> total
sigma [mb]
s$^{0.5}$ [GeV]
pi$^-$ p –> n pi^0
sigma [mb]
s$^{0.5}$ [GeV]
pi$^-$ p –> p pi$^+$ 2 pi$^-$
sigma [mb]
s$^{0.5}$ [GeV]
pi$^-$ p –> n 2 pi^0
sigma [mb]
s$^{0.5}$ [GeV]
pi$^-$ p –> p pi^0 pi$^-$
sigma [mb]
s$^{0.5}$ [GeV]
pi$^-$ p –> n pi$^+$ pi$^-$
sigma [mb]
s$^{0.5}$ [GeV]



pi⁻ p -> n omega
pi⁻ p -> n rho⁰
pi⁻ p -> n eta
pi⁻ p -> Lambda K⁰
pi⁻ p -> Sigma⁰ K⁰
pi⁻ p -> Sigma⁻ K⁺
sigma [mb]
s^0.5 [GeV]



pp−>pp pi^o
pp−>pn pi^+
pp−>pp omega
pp−>pp pi^+ pi^−
sigma [mb]
s^0.5 [GeV]

Kadanoff-Baym Equation

Schwinger-Dyson equation:

$$G = G_0 + G_0 \Sigma G$$

$$\begin{aligned}
G^{11}(1,2) &= G^T(1,2) = \langle T(\phi(1)\phi(2))\rangle \\
G^{22}(1,2) &= G^{AT}(1,2) = \langle \tilde{T}(\phi(1)\phi(2))\rangle \\
G^{21}(1,2) &= G^{<}(1,2) = \langle \phi(2)\phi(1)\rangle \\
G^{12}(1,2) &= G^{>}(1,2) = \langle \phi(1)\phi(2)\rangle \\
G^{r}(1,2) &= \theta(t1-t2)(G^{>}(x1,t1;x2,t2) - G^{<}(x1,t1;x2,t2)) \\
G^{a}(1,2) &= \theta(t2-t1)(G^{<}(x1,t1;x2,t2) - G^{>}(x1,t1;x2,t2))
\end{aligned}$$

After some manipulation: *Kadanoff-Baym equation*:

$$\begin{aligned}
(i\hbar\partial_{t1} - H_0(1))G^{<}(1,2) &= \int d3\, \Sigma^{r}(1,3)G^{<}(3,2) + \int d3\, \Sigma^{<}(1,3)G^{a}(3,2 \\
(i\hbar\partial_{t1} - H_0(1))G^{r}(1,2) &= \delta^4(1,2) + \int d3\, \Sigma^{r}(1,3)G^{r}(3,2)
\end{aligned}$$

Wigner-transformation

- Retarded propagator is not a distribution function
- Wigner transform:

$$r = x1 - x2 \qquad , \qquad R = x1 + x2$$

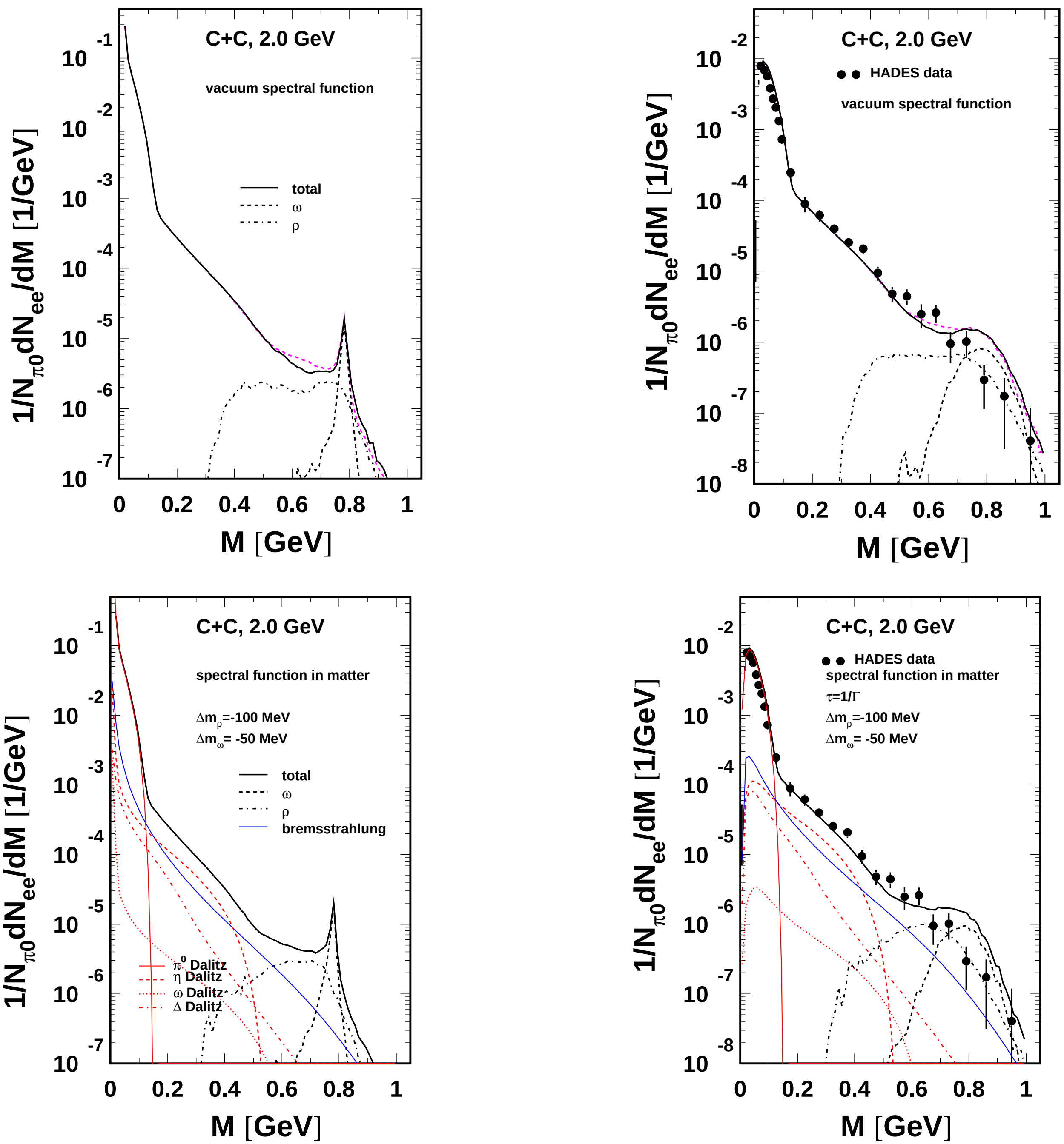
R (center of mass) dependence of propagators and selfenergies are weaker than the r dependence

$$G^r(R,P) = \int d^4r\, G^r(X+r, X-r)$$

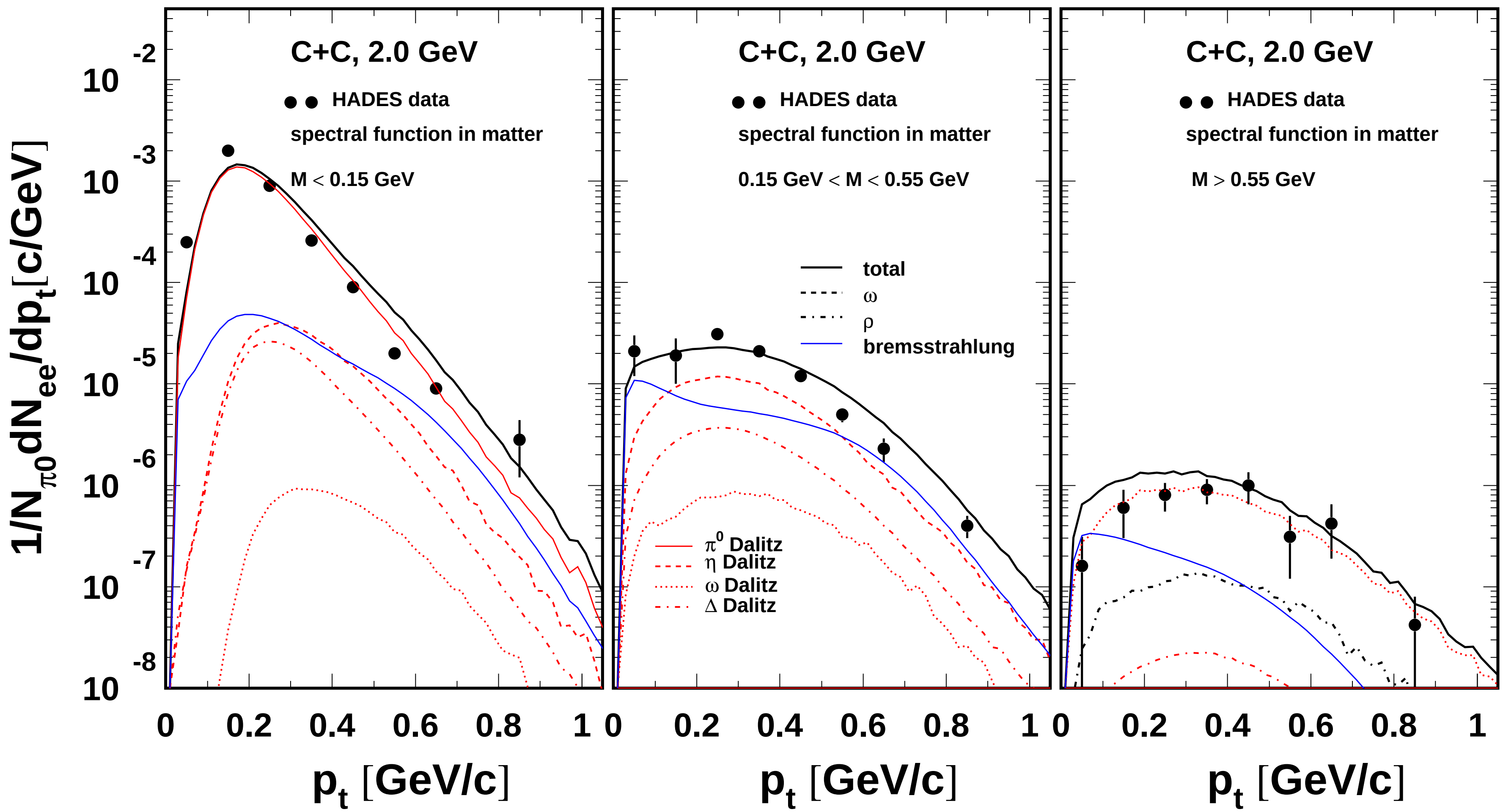
- Gradient expansion in r. Neglect all terms with more than one derivative in R
- transport equation for $F_\alpha = iG^<(R,P) = f_\alpha(x,p,t)A_\alpha$
  $A(p) = -2ImG^r = \frac{\hat{\Gamma}}{(E^2 - \mathbf{p}^2 - m_0^2 - \mathrm{Re}\Sigma^r)^2 + \frac{1}{4}\hat{\Gamma}^2},$

  Cassing, Juchem (2000) and Leupold (2000)
- testparticle approximation

C + C 2 GeV

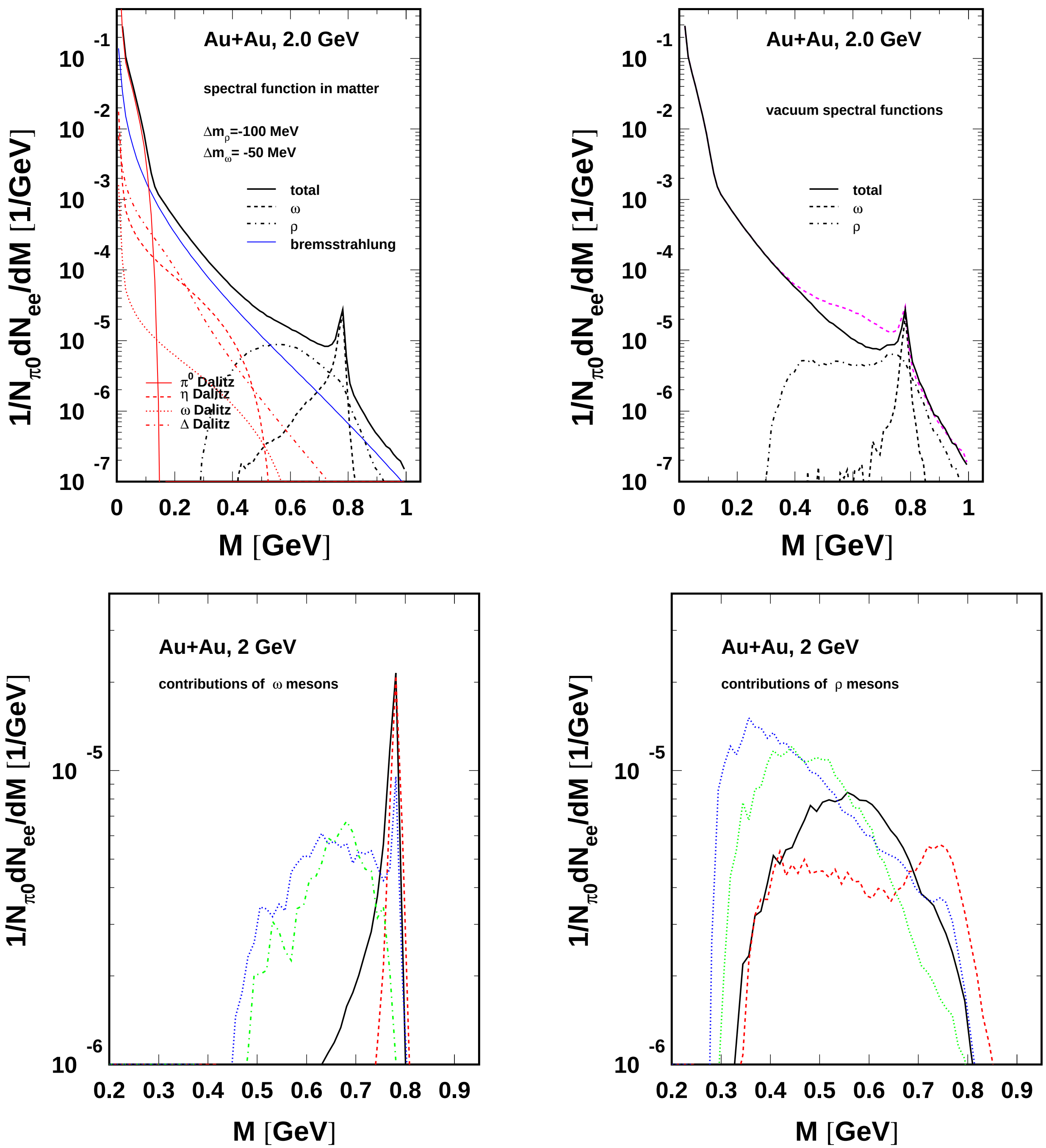


C+C, 2.0 GeV
vacuum spectral function
total
ω
ρ
1/N$_{\pi0}$dN$_{ee}$/dM [1/GeV]
M [GeV]
C+C, 2.0 GeV
HADES data
vacuum spectral function
1/N$_{\pi0}$dN$_{ee}$/dM [1/GeV]
M [GeV]
C+C, 2.0 GeV
spectral function in matter
Δm$_\rho$=-100 MeV
Δm$_\omega$= -50 MeV
total
ω
ρ
bremsstrahlung
π^0 Dalitz
η Dalitz
ω Dalitz
Δ Dalitz
1/N$_{\pi0}$dN$_{ee}$/dM [1/GeV]
M [GeV]
C+C, 2.0 GeV
HADES data
spectral function in matter
τ=1/Γ
Δm$_\rho$=-100 MeV
Δm$_\omega$= -50 MeV
1/N$_{\pi0}$dN$_{ee}$/dM [1/GeV]
M [GeV]



C+C, 2.0 GeV
HADES data
spectral function in matter
M < 0.15 GeV
0.15 GeV < M < 0.55 GeV
M > 0.55 GeV
total
ω
ρ
bremsstrahlung
π0 Dalitz
η Dalitz
ω Dalitz
Δ Dalitz
1/Nπ0dNee/dpt[c/GeV]
pt [GeV/c]

Au + Au 2 GeV

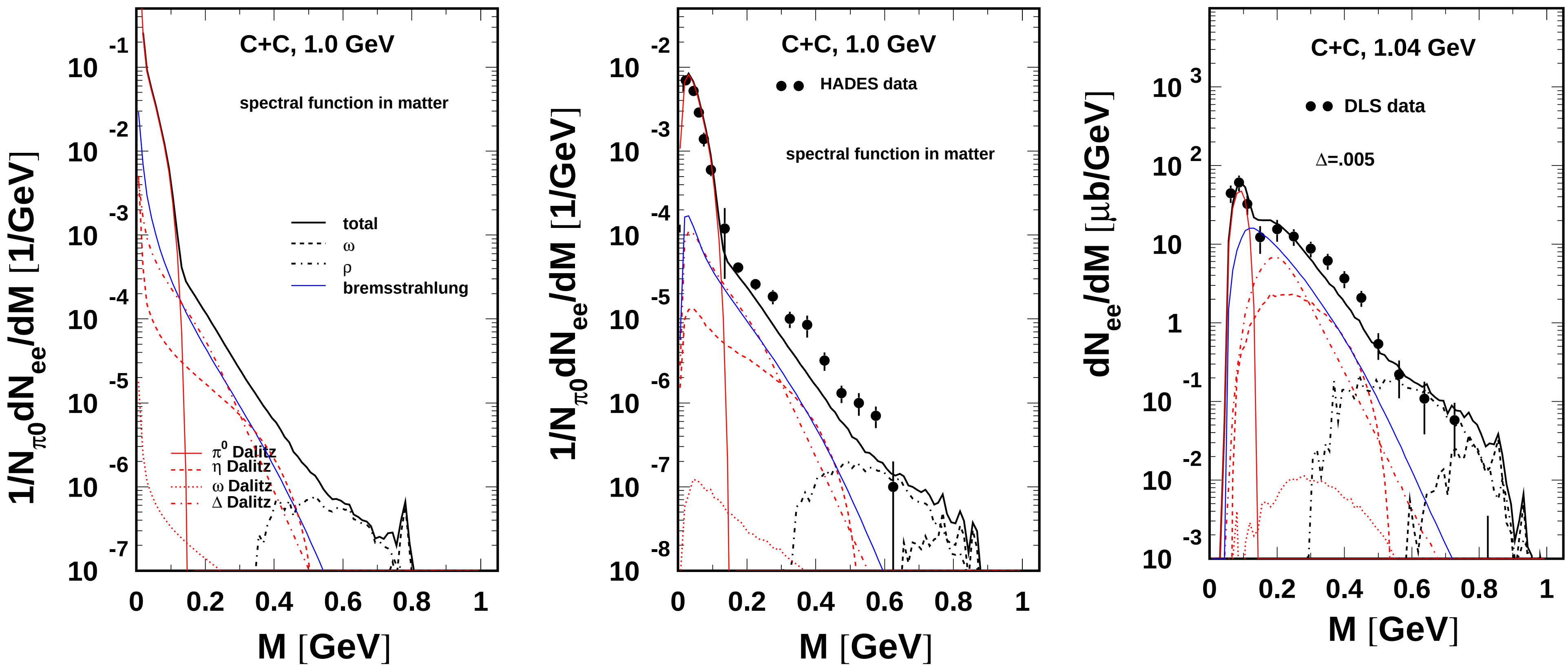


Vacuum

Matter

Static

C + C 1 AGeV



C+C, 1.0 GeV
spectral function in matter
total
ω
ρ
bremsstrahlung
π0 Dalitz
η Dalitz
ω Dalitz
Δ Dalitz
1/Nπ0dNee/dM [1/GeV]
M [GeV]
C+C, 1.0 GeV
HADES data
spectral function in matter
1/Nπ0dNee/dM [1/GeV]
M [GeV]
C+C, 1.04 GeV
DLS data
Δ=.005
dNee/dM [μb/GeV]
M [GeV]